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Weberian induction

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Abstract

Various kinds of electric polarization of a rotating metallic disc are discussed. A new kind of induction is predicted by Weber's electrodynamics, but not by Maxwellian electrodynamics. An experimental test to check this new effect is proposed.

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1. Introduction

We discuss in this work different kinds of induction which happen when a metallic disc rotates at a constant angular speed about its axis of symmetry. The disc has a radius r and rotates at a constant angular speed ω relative to the laboratory by external mechanical means. The laboratory can be considered a good inertial frame as regards the effects being analyzed here. We consider the disc in the xy plane of an inertial coordinate system S with its center at the origin O of S. The z axis coincides with the axis of symmetry of the disc, in such a way that the vectorial angular rotation can be written as $\boldsymbol{\omega} = \omega \hat{z}$, where $\hat{z}$ is the unit vector pointing along the positive direction of the z axis. We utilize cylindrical coordinates (ρ, φ, z), with corresponding unit vectors $\hat{\rho}, \hat{\varphi}, \hat{z}$.

The force acting on a charge q in the presence of electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$, is given by (according to Lorentz's force): $\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B} = -q(\nabla\phi + \partial \boldsymbol{A}/\partial t) + q\boldsymbol{v} \times \nabla \times \boldsymbol{A}$, where ϕ is the electric scalar potential and $\boldsymbol{A}$ is the magnetic vector potential where the charge is located. When there are no magnetic fields nor changing magnetic vector potentials this yields $\boldsymbol{F} = q\boldsymbol{E} = -q\nabla\phi$. Performing a line integral of $\boldsymbol{E}$ between points A and B yields the voltage or difference of potential between these points as given by:

$$\Delta\phi = \phi_B - \phi_A = -\int_A^B \boldsymbol{E} \cdot d\boldsymbol{\ell}. \quad (1)$$

The main goal of this Letter is to propose an experimental test to distinguish Lorentz's force from Weber's force. In order to present a feasible experiment,

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we consider several effects which must be taken into account simultaneously.

2. Inertial induction

The first case to be considered here is that of a disc rotating alone in a region free of magnetic fields. Initially the disc is supposed stationary and electrically neutral at all its points. We then rotate it mechanically at a constant rate. As the metallic disc is a good conductor of electricity and has free electrons on its body, this rotation polarizes it electrically between its center $\rho = 0$ and periphery $\rho = r$. That is, its periphery will become negatively charged and its center positively charged, originating an electric field pointing radially outwards. This polarization will grow to the point in which the electric force will have the exact value to keep other free electrons moving in circular orbits around the axis of the disc. Consider a free electron at a distance ρ from the axis of the disc, moving in circular orbit with a centripetal acceleration given by $\boldsymbol{a}_c = -\omega^2\rho\hat{\rho}$. Applying Newton's second law of motion, $\boldsymbol{F} = m\boldsymbol{a}$, yields: $q\boldsymbol{E}_i = -m\omega^2\rho\hat{\rho}$. Here $\boldsymbol{E}_i$ is the electric field due to the polarized charges generated by this inertial effect. This means that the induced radial electric field will grow linearly with distance, pointing radially outwards (remembering that for an electron $q = -e = -1.6 \times 10^{-19}$ C and $m = 9.1 \times 10^{-31}$ kg).

This will generate a voltage or potential difference between the periphery and center of the disc given by:

$$\Delta\phi_i \equiv \phi(r) - \phi(0) = -\int_{\rho=0}^{r} \boldsymbol{E}_i \cdot d\boldsymbol{\rho} = \frac{m\omega^2 r^2}{2e}. \qquad (2)$$

The subscript 'i' refers to the inertial induction related with the rotation of the disc and not connected with any magnetic field.

This inertial induction or inertial *emf* can be measured with a voltmeter at rest in the laboratory touching the center and periphery of the disc by means of sliding contacts. If $r = 10$ cm $= 0.1$ m and if the disc rotates at 3000 r.p.m. ($\omega = 314.16$ rad/s), then $\Delta\phi_i = 2.8 \times 10^{-9}$ V ≈ 3 nV.

To our knowledge the first to point out the possibility of such an effect has been Maxwell in his *Treatise*, [1]. The first measurement of an effect similar to this one has been made by Tolman and Stewart in 1916 [2], and [3]. Recently a much greater precision has been obtained, confirming the findings of Tolman and Stewart [4].

3. Unipolar induction

The induction of current in a test closed circuit when there is variation of current in another closed source circuit was discovered by Faraday in 1831. He also discovered that he could induce a current in the test circuit if the source circuit carried a constant current and there were a relative motion between the two circuits (translatory velocities). He could also induce a current in the test circuit if there were a relative motion between this circuit and a permanent magnet.

In 1832 he discovered what has been named afterwards as unipolar induction. In this case the magnet is at rest in the laboratory and a disc rotates above it at a constant speed. If we close the circuit with a galvanometer or voltmeter connected by sliding contacts at the center and periphery of the disc, a constant current will flow. The magnetic *emf* can be easily calculated in this case utilizing the magnetic force on a moving charge given by $q\boldsymbol{v} \times \boldsymbol{B}$. Here $\boldsymbol{v}$ is the velocity of the charge relative to an inertial frame where there is the magnetic field $\boldsymbol{B}$. To simplify the analysis we will suppose a constant and uniform magnetic field over the area of the disc, pointing in the positive z direction, $\boldsymbol{B} = B\hat{z}$. For a charge at a distance ρ from the center of the disc moving in a circular orbit with an angular velocity $\boldsymbol{\omega} = \omega\hat{z}$ we can write $\boldsymbol{v} = \rho\omega\hat{\varphi}$. With $q = -e$ this yields $q\boldsymbol{v} \times \boldsymbol{B} = -e\rho\omega B\hat{\rho}$. This means that electrons will concentrate at the center of the disc (if we rotate the disc at the opposite direction they will concentrate at the periphery). In the steady state situation (constant $\boldsymbol{\omega}$ and a constant charge at the center or periphery of the disc) there will be created an electric field which will balance this magnetic force, namely: $q\boldsymbol{E}_m = -q\boldsymbol{v} \times \boldsymbol{B}$ (remembering that $q\boldsymbol{E}_i$ has already been supposed to balance $m\boldsymbol{a}$). Here

E_m is the electric field due to the polarized charges generated by this magnetic effect.

According to Eq. (1) the potential difference due to this electric field will be given by

$$\Delta\phi_m \equiv \phi(r) - \phi(0) = \int_{\rho=0}^{r} (\boldsymbol{v}\times\boldsymbol{B})\cdot d\boldsymbol{\rho}$$

$$= \frac{\omega B r^2}{2}. \tag{3}$$

The subscript 'm' refers to magnetic induction.

We can estimate the order of magnitude of this effect supposing a magnetic field of $100\,G = 10^{-2}\,T$. With $r = 0.1$ m and $\omega = 314$ rad/s this yields $\Delta\phi_m = 15.7$ mV, such that $\Delta\phi_m/\Delta\phi_i = 5\times10^6$. This shows that in many practical situations involving typical magnets the inertial induction can be neglected. Even supposing this induction to be due to the earth's magnetic field of $0.5\,G = 5\times10^{-5}\,T$ yields $\Delta\phi_m = 78$ μV. This shows that this effect is still much larger than $\Delta\phi_i$: $\Delta\phi_m/\Delta\phi_i = 2.8\times10^4$.

In any event it is possible to distinguish the two effects observing that the inertial induction is independent of B and is quadratic in ω, while the magnetic induction is linear in ω, changing signs when the direction of rotation changes (while keeping the same magnetic field).

4. Weberian Induction

We now consider that we surround the disc and galvanometer (or voltmeter) by a spherical shell of radius R charged uniformly with a charge Q. The shell and its charges are supposed to be always at rest in the laboratory, even when the disc is put into rotation by mechanical means.

According to Lorentz's force nothing should change, as this shell does not generate any electric nor magnetic field inside itself. That is, no matter the value of the stationary external charge Q, the spherical shell will not exert any force on the stationary or spinning charges of the metallic disc. If there is no magnet, the disc should polarize according to Eq. (2). If there is a magnet inside the shell there should happen as well a magnetic induction given by Eq. (3), no matter the value of the external charge Q.

But according to Weber's electrodynamics there should be a force exerted by the stationary charged spherical shell on any internal electric charge which is accelerated by other forces. Weber's force law (see Chap. 3 of [5]) between two electrical charges in relative motion is given by

$$\boldsymbol{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{r^3}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\ddot{r}}{c^2}\right), \tag{4}$$

where q_1 and q_2 are separated by a displacement vector $\boldsymbol{r}$, $r \equiv |\boldsymbol{r}|$ is the magnitude of $\boldsymbol{r}$, $\dot{r} \equiv dr/dt$ is the relative velocity between the charges, $\ddot{r} \equiv d^2r/dt^2$ is the relative acceleration between them, $\varepsilon_0 = 8.85\times10^{-12}$ $C^2\ N^{-1}\ m^{-2}$ is the permittivity of free space and c is the speed of light.

The associated potential energy is given by

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}\left(1 - \frac{\dot{r}^2}{2c^2}\right). \tag{5}$$

If the two charges are stationary, the force law becomes identical to Coulomb's law.

Weber's electrodynamics is contemporary to Maxwell's work. Weber and Maxwell showed that with Weber's force it was possible to derive Coulomb and Gauss's law, the magnetic circuital law and Faraday's law of induction. For recent discussion of the compatibility of Weber's force and Maxwell's equations see Refs. [6,7].

The terms on Weber's force which depend linearly on the velocities yield the equivalent of $q\boldsymbol{v}\times\boldsymbol{B}$ of Lorentz's force [8,9], and [10].

When we calculate the force between neutral current elements with these two expressions we derive Grassmann–Biot-Savart's law from Lorentz's force, while from Weber's law we derive Ampère's force between current elements. Although Ampère and Grassmann's forces are different when considering only current elements, they yield the same result when calculating the net force on any current element due to a closed circuit of arbitrary form (the current element may belong or not to this closed circuit), see Refs. [11,12] and [13]. This means that to distinguish these two expressions we need to work with open mechanical circuits.

Weber's force has components which depend on the square of the velocity of the test charge q_1, v_1^2, and on its acceleration, $\boldsymbol{a}_1$. Here we concentrate on this last component, which has no analog in Lorentz's force. The Coulombian term and the terms depending on the square of the test charge velocity go to zero after integrating the force of the shell elements on q_1, as they are inverse square components. On the other hand the acceleration term does not go to zero after integration as it falls only as $1/r$.

Suppose, inside the uniformly charged stationary spherical shell with Q, a test charge q is located at time t at $\boldsymbol{r}$, moving with velocity $\boldsymbol{v} = d\boldsymbol{r}/dt$ and acceleration $\boldsymbol{a} = d\boldsymbol{v}/dt$ relative to the origin of the sphere. We can integrate the above equations over the whole charge on the spherical shell. We thus obtain the force on q (see Refs. [5], Sections 7.3 and 7.4, and [10,14,15]):

$$\boldsymbol{F} = \frac{\mu_0 q Q \boldsymbol{a}}{12\pi R}, \tag{6}$$

where $\mu_0 = 4\pi \times 10^{-7}$ kg m C^{-2} is the vacuum permeability.

Integrating Eq. (5) over the whole charge on the spherical shell yields the potential energy for the charge q:

$$U = \frac{qQ}{4\pi\varepsilon_0}\frac{1}{R}\left(1 - \frac{v^2}{6c^2}\right). \tag{7}$$

Note that according to Lorentz's force and Maxwell's equations, the corresponding potential energy is

$$U = \frac{qQ}{4\pi\varepsilon_0}\frac{1}{R}, \tag{8}$$

and the force on q is zero.

As the free electrons have a negative charge and the centripetal acceleration points radially towards the origin, this Weberian force will point radially outwards if $Q > 0$ (or radially inwards if $Q < 0$). This means that according to this force the disc will tend to become negatively charged in its periphery and positively charged at its center if $Q > 0$. In the steady state situation (constant $\boldsymbol{\omega}$ and a constant charge at the center or periphery of the disc) there will be created a Weberian electric field $\boldsymbol{E}_W$ which will balance this Weberian force, namely: $q\boldsymbol{E}_W = -\mu_0 q Q \boldsymbol{a}/12\pi R$. This Weberian electric field is due to the polarized charges generated by Weber's force (6).

According to Eq. (1) this electric field will give rise to a potential difference or voltage given by

$$\Delta\phi_W = \phi(r) - \phi(0)$$
$$= \int_{\rho=0}^{r} \frac{\mu_0 Q \boldsymbol{a}}{12\pi R} \cdot d\boldsymbol{\rho} = -\frac{\mu_0 Q \omega^2 r^2}{24\pi R} \tag{9}$$

To arrive at this result we utilized that $\boldsymbol{a} = -\omega^2\rho\hat{\rho}$. The subscript 'W' refers to Weberian induction.

Choosing the zero potential at infinity, the potential of the charged sphere can be written as $\phi_S = Q/(4\pi\varepsilon_0 R)$. This means that the Weberian induction can be written as $\Delta\phi_W = -\phi_S(\omega^2 r^2/6c^2)$. Comparing it with $\Delta\phi_i$ yields: $\Delta\phi_i/\Delta\phi_W = 3mc^2/e\phi_S$. With the previous values $\omega = 314$ rad/s and $r = 0.1$ m this yields $\Delta\phi_W = -18 \times 10^{-16}\phi_S$. This has an extremely small value for small voltages of the sphere. It will be of the same order of $\Delta\phi_i$ if the sphere is charged to 1.5 MV. As this is attainable in laboratories working with high voltages, it is possible to test the existence of this effect.

5. Experimental test

We can distinguish between $\Delta\phi_i$ and $\Delta\phi_W$ by first spinning the disc inside the uncharged sphere in the absence of any magnetic field. We then charge the spherical shell to plus or minus 1.5 MV and measure the new *emf* between its center and periphery. Nothing should change according to Lorentz's force. On the other hand according to Weber's electrodynamics the net potential should go to zero if $\phi_S = -1.5$ MV or double its value if $\phi_S = +1.5$ MV.

The main difficulty in this experiment is to avoid the magnetic induction described above due to the earth's magnetic field. In any event $\Delta\phi_m$ changes sign according to the direction of rotation while $\Delta\phi_i$ and $\Delta\phi_W$ do not change their signs. Utilizing this aspect it might be interesting to perform the experiment with a sinusoidal rotation, namely: $\omega =$

$\omega_0 \sin(\alpha t)$. The integrated or averaged value of the magnetic induction goes to zero in this case, while the integrated value of the inertial and Weberian induction remain different from zero. Another possibility is to utilize a pair of counter-rotating disks. By connecting at their central parts and their rims respectively the magnetic induction effects will be canceled, since the magnetic induction depends on its rotational direction. On the other hand, the inertial and Weberian induction will remain present as they depend on ω^2.

We propose this as a possible experimental test of Weber's electrodynamics. Experiments like those of Tolman and Stewart, or those of Moorhead and Opat, might be repeated inside a Faraday cage (or Van de Graaf generator) charged to a high potential.

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