Proceedings Conference on Foundations of Mathematics and Physics, Perugia, 1989

Modern Experiments Related to Weber's Electrodynamics(*)

A. K. T. Assis
Departamento de Raios Cósmicos e Cronologia
Instituto de Física "Gleb Wataghin"
Universidade Estadual de Campinas
C. P. 6165
13081 Campinas, S.P.
Brazil

Abstract – We present Weber's force law and the classical results that follow from it. We discuss the historical controversy surrounding Ampère's law of force between current elements versus Grassmann–Biot–Savart's law. Then we make a review of modern experiments related to this topic and to the electric field generated by a steady and stationary neutral current. Finally we analyse some theoretical aspects of Weber's law as its extension through retarded potentials to include electromagnetic radiation, and its relation to alternative interpretations of experiments devised to show the mass variation with velocity.

(*) Work presented at the conference "I Fondamenti della Matematica e della Fisica nel XX Secolo: La Rinuncia all'Intuizione" (Perugia, Italy, 27 - 29 September 1989)

I - Main Results Following from Weber's Law

In order to obtain Ampère's law of force between current elements from an interaction between point charges Weber proposed, in 1846, the following law [1]:

$$\begin{aligned}\vec{F} &= \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\vec{r}_{12}}{r^3}\left[1+\frac{1}{c^2}\left(r\ddot{r}-\frac{\dot{r}^2}{2}\right)\right] = \\ &= \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\vec{r}_{12}}{r^3}\left[1+\frac{1}{c^2}\left(\vec{v}_{12}\cdot\vec{v}_{12}+\vec{r}_{12}\cdot\vec{a}_{12}-\frac{3}{2}(\hat{r}_{12}\cdot\vec{v}_{12})^2\right)\right], \qquad (1)\end{aligned}$$

where $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$, $r = |\vec{r}_1 - \vec{r}_2|$, $\dot{r} = dr/dt$, $\ddot{r} = d^2r/dt^2$, $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$, $\vec{a}_{12} = \vec{a}_1 - \vec{a}_2$, $\hat{r}_{12} = \vec{r}_{12}/r$, and $\vec{F}$ is the force that q_2 exerts on q_1.

Two years later Weber showed that this force can be derived from a velocity dependent potential energy given by

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{\dot{r}^2}{2c^2 r}\right) \qquad (2)$$

Eq. (1) can be obtained from U differentiating it with respect to r and changing the sign, as shown by Weber in 1848. The conservation of energy follows immediately from the mutual potential energy (2) if we add the kinetic energy to it and observe that the result is a constant for any time, that is,

$$\frac{d}{dt}(T+U) = 0, \qquad (3)$$

where $T = m_1 v_1^2/2 + m_2 v_2^2/2$. To obtain Eq. (3) we only need to relate Newton's Second and Third Laws to Eq. (1), namely: $\vec{F} = m_1\vec{a}_1 = -m_2\vec{a}_2$. This was a strong result obtained by Weber because it increased the confidence on his law. He knew his law satisfied Newton's action and reaction law in the strongest form and so the main classical results were maintained: conservation of linear momentum, angular momentum and energy.

We now show how to obtain Ampère's law of force from Weber's law. To this end we suppose two neutral current elements, i.e., $q_{i-} = -q_{i+} (i = 1 \text{ or } i = 2)$ and define $I_i d\vec{\ell}_i = q_{i+}(\vec{v}_{i+} - \vec{v}_{i-})$. Adding the force of the negative and positive charges of $I_2 d\vec{\ell}_2$ on the negative and positive charges of $I_1 d\vec{\ell}_1$ yields

$$d\vec{F} = -\frac{\mu_0}{4\pi} I_1 I_2 \frac{\hat{r}_{12}}{r^2}[2d\vec{\ell}_1\cdot d\vec{\ell}_2 - 3(\hat{r}_{12}\cdot d\vec{\ell}_1)(\hat{r}_{12}\cdot d\vec{\ell}_2)], \qquad (4)$$

where we utilized $c^2 = (\mu_0\varepsilon_0)^{-1/2}$ and $d\vec{F}$ is the force of the element $I_2 d\vec{\ell}_2$ on the element $I_1 d\vec{\ell}_1$. This is Ampère's law of force between current elements. It is also a Newtonian force in the sense that it follows Newton's Third Law in the strongest form, i.e., the forces lies on the line joining the two current elements. It was based on this law that Ampère derived his famous circuital law. Eq. (4) was called by Maxwell as the cardinal formula of electrodynamics and he also called Ampère as the 'Newton of electricity' [2].

Then we can derive from Eq. (1) the cardinal formulas of electrodynamics and of electrostatics (that is, Coulomb's law of force, which is obtained from (1) when $\dot{r} = 0$ and $\ddot{r} = 0$), as was desired by Weber.

Another fundamental law of electromagnetism, Faraday's law of induction, can also be derived from Eq. (1) when the source which induces the current is a closed loop. For the proof of this remarkable fact there are three sources which use slightly different points of departure to arrive at the same conclusion: Maxwell [3], Whittaker [4] and Wesley [5]. The reader should consult these authors in order to have a better feeling of the mathematics and physical concepts involved in the proof.

This completes the classical results obtained through Weber's law. We proceed now to study some modern experiments related to this subject.

II - Ampère $\times$ Grassmann's Law

In 1845 Grassmann proposed the following force that the current element $I_2 d\vec{\ell}_2$ exerts on $I_1 d\vec{\ell}_1$, [6]:

$$
\begin{aligned}
\vec{F} &= \frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2} d\vec{\ell}_1 \times (d\vec{\ell}_2 \times \hat{r}_{12}) = \\
&= -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}[(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r}_{12} - (d\vec{\ell}_1 \cdot \hat{r}_{12})d\vec{\ell}_2] \quad . \qquad (5)
\end{aligned}
$$

If we consider the magnetic field due to $I_2 d\vec{\ell}_2$ as given by Biot–Savart's law, we can write Eq. (5) as $\vec{F} = I_1 d\vec{\ell}_1 \times \vec{B}_2$, where $\vec{B}_2 = \mu_0 I_2 (d\vec{\ell}_2 \times \hat{r}_{12}) \;/\; (4\pi r^2)$.

Contrarily to Eq. (4), Grassmann's force law does not always follow Newton's Third Law, as can be seem when $d\vec{\ell}_1$ is parallel to $\hat{r}_{12}$ and perpendicular to $d\vec{\ell}_2$. Another difference between the two laws is when $d\vec{\ell}_1$ is parallel to $d\vec{\ell}_2$, and both are parallel to $\hat{r}_{12}$. In this case there should be a repulsive force between the two current elements according to (4), and a zero net force according to (5). This divergence of predictions is in the cern of the experiments devised to distinguish and to choose between these two laws. These experiments must rely in a single circuit due to an amazing aspect: although Eqs. (4) and (5) are structurally different, when we integrate for the force exerted by a closed loop in a current element of another circuit, both equations give the same result. For a proof of this fact see Ref. [7]. If $I_1 d\vec{\ell}_1$ is part of the circuit 2 the proof no longer holds and here begins the controversy.

For a critical analysis of the differences and similarities between Ampère and Grassmann's laws, and also of Weber's law in general, we suggest the important book of O'Rahilly [8]. For an alternative theory to Weber's one the reader can see the work of Ritz [9].

The first experiment adduced to prove that Ampère's law of force between current elements is better (more correct) than that of Grassmann is the famous Ampère's bridge experiment [10]. To Maxwell this was not a decisive experiment as it involved a closed loop. Maxwell's point of view was criticized by Pappas [11], who pointed out that the proof of the equivalence between the two laws is only valid when we have two different closed circuits. On the physical side Pappas argued that while the action and reaction occurs between current elements according to Ampère's formula, it must occur between current element and the field if we try to implement Grassmann's formula together with Newton's Third Law. So we can never have a complete equivalence between Eqs. (4) and (5) according to Pappas.

Ampère's bridge (or hairpin) experiment was improved by Tait [12]. His contribution for the clarification of the experiment and also the arguments of Cleveland [13] in favour of Ampère's law were discussed by Graneau [14].

Our review of recent experiments related to Weber's electrodynamics, as implied by one of its consequences, Eq. (4), begins in 1982. In this year Graneau published two papers [15, 16] describing experiments related to this topic. In the first of these two papers Graneau repeats Ampère's hairpin experiment and observes a new effect which corroborates Ampère's law: jet–propulsion in the liquid mercury in the direction of current flow between liquid and solid conductors, and so confirming longitudinal propulsion as should be expected according to (4). In the second paper he discusses experiments with railgun accelerators. In it he uses finite current–element analysis to calculate the force distribution on the system and also the expected acceleration of the gun. In particular he emphasizes that the recoil force should be generated in different points of the stationary railgun circuit according to (4) or to (5). According to Ampère's law the seat of the recoil force should be in the rails and mostly near the projectile, while according to Grassmann's law the recoil force should act in the magnetic field. A neat experimental verification of Ampère's law can be seen in Fig. 3 of Ref. [17], where is shown the buckling of thin rails due to recoil forces. In this paper he also estimates the efficiency of railguns and raises the possibility that the poor efficiency can be due to a mechanical defficiency caused by the distortion of the rails.

In 1980–82 Pappas performed a different experiment which was also in favour of Eq. (4) and against Eq. (5). It was published in 1983 [11]. It is essentially Ampère's experimented with an extra degree of freedom, and can be called the electromagnetic impulse pendulum experiment. In it Ampère's bridge was substituted by a π–shaped aluminium wire suspended by thin threads. As the current was supplied to the circuit the π shape moved forward, but when he changed the direction in which the current was supplied no such movement occurred (see his Figs. 2 and 3). According to Eq. (5) there should be no difference in the two situations, while according to (4) the repulsion between the two parts of the circuit should happen only in the first case. This was an important step in establishing a clear distinction between the two laws through experiments.

In 1983 Graneau published a paper [18] describing a new kind of experiment: the exploding wire phenomena. In it pulsed currents up to 7000 A caused fragmentation of an aluminium wire into 10 to 50 pieces of irregular length. The wire had 1.2 mm diameter and was one meter long. Against the usual explanations that this happened through melting and evaporation of the wire, he obtained strong evidence of tensile breaks as the cause of the explosions. This should be expected according to (4) but not according to (5). He also pointed out in this paper that pinch forces could not cause tensile fracture on the wire. He obtained scanning electron microscope photograph of a fracture face showing tensile fracture in the solid state and thus ruling out the explanation of the melting break. In a later paper [19] Graneau improved his experiment and now bent the wire into a semicircle, which was connected to a capacitor discharge circuit through an arc gap. In this way he suceeded in eliminating the possible explanation of the explosion as being due to Lorentz hoop tension. He also suceed in eliminating many other possible explanations as: thermal shock, longitudinal stress waves, Lorentz pinch–off, bending stresses and material defects. The only remaining explanation for this remarkable phenomena is Ampère tension as given by Eq. (4).

In 1985 Graneau and Graneau presented a new kind of experiment [20] which involved the explosion of a conducting liquid made of salt water when an electric arc current passed through the liquid. He could explain the main aspects of the explosion using longitudinal Ampère forces. In another experiment a 1000 A dc current caused a portion of liquid mercury used to complete the circuit (i.e., close the current) to separate in two symmetric parts and so interrupt the current with an arc. To me there is no other experiment which shows so neatly the longitudinal Ampère tension as this one.

Ternan [21] and Jolly [22] argued that Ampère's force law is equivalent to Grassmann's (sometimes called Biot–Savart's force law, or Lorentz's force law) even in the situation of a single circuit. Pappas and Moyssides presented a different point of view [23] and we let the readers to decide on their arguments. The same can be said about the controversy of Graneau and Ternan [24 - 26]. Graneau presented some theoretical considerations showing that it is not true in general that the Ampère and Lorentz force laws are mathematical identities [27]. Perhaps this will help to clarify the discussion.

Moyssides and Pappas devised an experiment to measure directly the force which arises on a part of a single circuit due to the remaining circuit [28]. They obtained systematically smaller forces than those predicted by Biot–Savart–Lorentz forces. They suggest Ampère's law as a possible explanation for these results.

An improved version of Pappas' electromagnetic impulse pendulum experiment was done by Graneau and Graneau [29]. The advantage of this experiment was that they already had the value of the stored energy of the capacitor bank (6.4 KJ to 25.6 KJ). In this way they could rule out the explanation of the experiment based on transfer of momentum between the pendulum and the field, for if the mechanism were this one the magnetic energy stored in the field would be, according to Graneau, $\simeq 1500$ times the energy stored in the capacitor. Where did this energy come from? The only explanatory mechanism which hadn't this problem was based on Ampère's reaction force (Eq. (4)).

He could even explain the poor momentum imparted to the pendulum based on the elastic distortion of the pendulum structure (this could be accounted for by Ampère's force but not by Grassmann–Lorentz's force).

In the same year Graneau published a review of Ampère–Neumann electrodynamics of metallic conductors [14]. One year earlier he had published a book on the same subject [30]. In these works he details Neumann's deduction of Faraday's law of induction based on Ampère's law (Eq.(4)) and Lenz's law. He describes Neumann's introduction of the electrodynamic potential (magnetic energy) and how we can calculate the correct mechanical forces which are exerted in metallic current circuits based on these ideas. It is also shown how the magnetic vector potential appears naturally in his theory (this can also be seen in Ref. [4]). Graneau shows how to calculate the self inductance of circular and retangular closed loops based on finite element analysis and also how to compute Ampère tension in solid metallic circuits of finite size. He also discusses Hering's longitudinal force experiments [31]. References [14] and [30] are extremely valuable works and should be consulted by anyone interested in this whole subject.

In 1986 Aspden [32] analysed Graneau experiments and related these experiments to anomalous cathode reaction forces found in the vacuum arc. He presents a possible theoretical explanation based in a law which resembles Weber's law. These topics were treated by the author in previous papers [33, 34]. Regarding longitudinal forces in gaseous conductors we can't forget to mention Nasilowski suggestion [35, 36] that as these Ampère forces exist in solid metallic conductors and in liquid conductors, they should also exist in weak gaseous conductors such as welding arc, the switching arc, arcs used in metal furnaces and principally in magnetically confined fusion plasmas.

The theme of wire explosions was treated with a better detail in 1987 [37], where the author showed that rapid thermal expansion cannot explain some experimental facts which occur in the explosions, and so once more replying based on facts some theoretical arguments raised by Ternan [38]. Aspden suggested that the phenomenon could be due to an inductive effect [39] and gave some hints on what to look for in the experiments in order to settle the controversy: to see if the rupture is proportional to I or to I^2, and also to analyse in detail the important factor of the frequency of the pulsed current if rupture occurs after several cycles.

The relation of railgun recoil to relativity was discussed by Graneau [40], who showed that the recoil forces should have its seat, as found experimentaly, on the railheads, as would be expected according to the Ampère–Neumann electrodynamics. It seems that relativistic electromagnetism could not explain these findings.

Whitney argued that relativistic field theory could explain all these experimental facts [41]. Her point of view is that there can be an electric field responsible for these effects, this electric field being due to nonuniform current and charge distributions. As she only gave a qualitative argument and did not show how to explain quantitatively the effects based on this field we will not consider her work any longer. Anyway we give this and another reference of her to the interested reader [42].

In a recent work Graneau revised the concept of alpha–torque forces (a ponderomotive torque which should exist according to Ampère's law but not according to Grassmann's law, and which acts on the atoms of a conductor metal) and many experiments showing their effects [43]. We'd like only to point out here the beautiful experiment of the liquid mercury fountain. For details of this and others experiments we invite the reader to read this nice paper.

We'd like to mention also a paper by Graneau [44] in which he refutes arguments of Christodoulides [45] in defense of relativity.

We conclude this section with an important theoretical work of Graneau [46]. On it he shows that only Ampère's force law agrees in all cases, without exception, with the virtual–work concept. Moreover he shows that the Lorentz's force law is not in general compatible with the virtual–work idea in situations involving a single circuit. Only if the closed circuit has a high degree of symmetry can the compatibility be proved. To show that Ampère's law is always compatible with the virtual–work formula, he derives Ampère's law from the virtual–work formula, and shows that this cannot be done in general for Lorentz's law.

III - Electric Field Due to a Steady Current

The next topic of controversy which arises when we study Weber's law is related to the electric field due to steady currents. To see this we calculate the force of a current element $I_2 d\vec{\ell}_2$ with $q_{2-} = -q_{2+}$ on a charge q_1 using equation (1). This yields

$$\begin{aligned} \vec{F} &= \frac{q_1 q_{2+}}{4\pi\varepsilon_0 c^2}\frac{\vec{r}}{r^3}\Big\{3[\hat{r}\cdot(\vec{v_{2+}}-\vec{v_{2-}})](\hat{r}\cdot\vec{v_1}) - \\ &\quad -2\vec{v_1}\cdot(\vec{v_{2+}}-\vec{v_{2-}}) - \vec{r}\cdot(\vec{a_{2+}}-\vec{a_{2-}}) - \\ &\quad -\frac{3}{2}[(\hat{r}\cdot\vec{v_{2+}})^2 - (\hat{r}\cdot\vec{v_{2-}})^2] + (v_{2+}^2 - v_{2-}^2)\Big\}, \end{aligned} \tag{6}$$

and this result holds for any acceleration of q_1.

So, to have a zero net force in general, we need to have, even when $\vec{v_1} = 0$, $|\vec{v_{2+}}| = |\vec{v_{2-}}|$ and $\vec{a_{2+}} = \vec{a_{2-}}$. As this is not always true we should expect, according to Weber's law, a net force acting on q_1 due to a current element with zero net charge. Supposing q_1 to be at rest ($\vec{v_1} = 0$), a steady current in the circuit 2 ($\vec{a_{2+}} = \vec{a_{2-}} = 0$) and $\vec{v_{2+}} = 0$ (as it happens with solid metallic conductors and with magnets) we get

$$\vec{F} = q_1 \vec{E_M}, \tag{7}$$

where

$$\vec{E_M} = \frac{q_{2+}}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}\left[\frac{3}{2}\frac{(\hat{r}\cdot\vec{v_{2-}})^2}{c^2} - \frac{v_{2-}^2}{c^2}\right], \tag{8}$$

and $\vec{E_M}$ can be called the motional electric field. It must exist if this is a correct theory. Its order of magnitude is of the order v_D^2/c^2, where v_D is the drift velocity of the electrons in the wire.

The first to point out that steady electric currents should exert forces on electrotastic charges at rest, if Eq. (1) is valid and if $\vec{v}_{2+} \neq \vec{v}_{2-}$, was Clausius [47]. The first measurement of the drift speed of conduction charges in metals is due to von Ettingshausen [48], who used the Hall effect to make this measurement. In this way it was shown that only the electrons move when the current flows in a metal.

Due to the smallness of the effect it couldn't have been detected in the past. To my knowlwdge, the first modern experiment devised to detect such a force is due to Edwards [49 - 52]. In one of these papers it is stressed: "Today, techniques for making direct measurements of the second-order electric fields are available. For this reason, the matter can finally be placed on an experimental basis" [51]. On it they show how Maxwell's theory predicts zero electric field due to a steady current of zero net charge. Then they perform an experiment in which a steady current in a superconducting Nb-Ti circular coil is seem to generate a potential (i.e., an electric field). This potential is observed to scale as I^2 and is independent of the direction of the current, as should be expected according to (8). They make several variations of the experiment and in this way they eliminate several possible sources of this potential such as: the self-Hall effect, configurational emf's, nonsteady currents, thermoelectric effects and flux-motion potentials. They conclude the experiment saying that the observed effect is a real one and that it cannot be explained by Maxwell's classical theory.

Curé proposed a liaison between this new electrodynamic field and the formalism of linearized general relativity [53]. He also suggests a modified version of the Millikan oil drop experiment in order to test more clearly the existence of this electric field. To my knowledge this proposal was never realized in practice. I think this could be an important test of Weber's theory.

Bartlett and Ward interpreted the findings of Edwards et al. as a possible change of the value of the electric charge with velocity [54]. Following this idea they developed some very ingenuous experiment to test this hypothesis. In this way they suceeded in putting severe limits on a possible variation of electric charge with velocity.

Bonnet, on the other side, argued that the positive results of Edward's experiment could be due to a particular property of superconductors, namely, that these materials wouldn't radiate in steady state [55]. In this way they didn't consider the acceleration terms in the Lienard-Wiechert fields and obtained effects of the same order as those found by Edwards.

I think some remarks should be made on Bonet's points of view. On the theoretical side there is a growing feeling of serious problems with the Lienard-Wiechert potentials: flaws in the conservation of total charge, doubts on its covariance, difficulties in its meaning etc. This has been stressed recently mainly by Whitney [56, 57]. On the experimental side there is the work of Sansbury [58], who found an electric field due to a steady current in a copper conductor. As the experiment was made at room temperature, it has nothing to do with a superconductor effect. So the explanation of Bonet could not be used in this case.

Unhappily the experiment of Sansbury could only detect without doubts the existence of the field, but it wasn't sensible enough the measure the value of the field but only its lower limit. As the experiment involved a torque bar we can expect many improvements along this line in the future due to the great precisions that can be achieved with torsion balances. I think this is a very important line of research to be followed by others.

It should be remembered here that althought Edwards et al. and Sansbury found an electric field due to a steady current, there is a disparity in their results. Edwards et al. found a motional electric field pointing to the current (as if the current had became negatively charged), as would be expected according to Weber's law and the fact that only the electrons move in a metallic current. On the other side Sansbury found a motional electric field pointing away to the current (as if the current had became positively charged), contrary to Weber's law. I don't know of any explanation for this discrepancy. This is an important issue which deserves better study in order to clarify the situation.

IV – Theoretical Development

We now turn our attention to other aspects of Weber's electrodynamics. From its beginning Weber's force law had a serious limitation: to be an action at a distance theory. Despite this fact it should be remembered here that Weber and Kohlrausch were the first to measure the velocity with which an electric disturbance is propagated along a conducting wire and this was soon recognized to be of the same order as the velocity of light [59, 60]. With the advent of Maxwell's theory, with Hertz experimental confirmation of a finite velocity of propagation for the electromagnetic fields according to Maxwell's theory, and finally with Einstein's theory of relativity, Weber's law was let aside together with its main successes. Also the question of action at a distance in Weber's theory was almost forgotten until recent times.

In 1954 Moon and Spencer analysed Ampère's law of force between current elements and also Weber's law in a series of three papers [61]. In the first of these they discussed Ampère's law and a possible generalization of it. In the second one they studied a law of force between moving charges with the restriction that they were not accelerated. Finally in the third paper of the series they analysed in detail Weber's law and made the great step forward to introduce the retarded time in Weber's law. In this way they arrived at the retarded Weber force and could derive Maxwell's radiation terms (the radiating field of a dipole antenna and so on).

Recently the same kind of reasoning was employed by Wesley [5]. He also used retarded time ($t - r/c$ instead of t) to obtain radiation effects with Weber's law but with an advantage: he made this introduction in the general case, beginning with the potentials, and so his results have a wider application than those of Moon and Spencer.

To arrive at his results be needed to introduce a magnetic scalar potential. In a later work he amplied the scope of the theory following the same line of reasoning [62].

In my opinion the ideas of Moon, Spencer and Wesley are great advances proposed in order to modify Weber's law so that it can include radiation effects and the propagation of light. Much theoretical and experimental work is necessary in this direction but the directions of the main lines of research are already open for must of us. Before closing this section I'd like only to point out a different way of obtaining time delays in action at a distance theory: Graneau's work which appeared in the same year of Wesley's [63]. Graneau's idea is to obtain time delays through an induction law. This is a very interesting and fruitful insight and deserves a better investigation.

We should also express here an important improvement in Weber's law proposed by Phipps [64]. He proposes a potential energy given by

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}\left(1 - \frac{\dot{r}^2}{c^2}\right)^{1/2}. \tag{9}$$

Following the usual procedure he obtains for the force of q_2 on q_1:

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\vec{r}_{12}}{r^3}\left[\left(1 - \frac{\dot{r}^2}{c^2}\right)^{1/2} + \frac{r\ddot{r}}{c^2}\left(1 - \frac{\dot{r}^2}{c^2}\right)^{-1/2}\right]. \tag{10}$$

This force is apropriate for terms varying as $\dot{r}^4/c^4$ and higher and overcomes Helmholtz objection to Weber's law [65 - 66] as it is free of the "negative mass behavior" [67] for all velocities smaller than c. The limit velocity in Eqs. (9) or (10) is c instead of $2^{1/2}c$ as given by Eqs. (1) or (2). Eqs. (9) and (10) are free of the instabilities predicted by Weber's theory and are compatible with all experiments up to this date.

Another subject of this work resulted in a paper recently published [68]. On it we show that using Eq. (1) we arrive that the force inside a stationary capacitor with a uniform electric field will be a function of the velocity of the test charge which is inside it. In this way we suceeded in showing that up to second order in v/c, inclusive, there are two alternative explanations of Bucherer's experiment [69]. This experiment is an extension of Kaufmann's ones [70]. In Bucherer's arrangement a source of β-rays (electrons) was placed in the middle of a large capacitor. Perpendicular to the electric field generated by this capacitor and to the movement of the electrons there was an uniform magnetic field so that the entire set worked as a velocity filter for the electrons. Knowing the intensities of the electric and magnetic fields and collecting the emergent electrons he could verify the mass change with velocity. Now there are two different interpretations of the same results. In the usual one we use Coulomb's law for the force on a charge inside a capacitor, with the mass of the electron changing with velocity according to relativity, and in the second explanation we use Weber's law to calculate all the forces and suppose the electron mass to be a constant for any velocity. After publishing this paper we found an important paper of Zahn and Spees [71] in which the authors make a critical analysis of Bucherer's experiment. In it they found that the velocity filter didn't work for $v/c > 0.7$ due to problems with the resolving power of the apparatus. Thus we can say with certainty that this classical experiment couldn't say anything for terms greater than second order in v/c. Quoting from their paper: "In view of the fundamental importance of such experiments it seems that much is left to be desired." In a later review Faragó and Jánossy showed that in direct experiments on the behaviour of free electrons the experimental errors of the measurements is so high that we can't distinguish between different theoretical formulae [72]. Better results seem the follow from the fine structure doublet separation of hydrogen - like spectra. Recently the fine–structure energy levels of the hydrogen atom was obtained through Weber's potential, Eq. (2), and the usual Schroedinger equation [73]. This was accomplished without mass change with velocity and is an important work as it extends the range of application of Weber's law to quantum phenomena.

A last topic to be touched upon in this work are the many fruitful results we get when applying a Weber force law for gravitation [74]. This yields the observed precession of the perihelion of the planets through an orbit equation different from that of general relativity. We derive the proportionality between inertial and gravitational masses (we don't need to postulate it). We also derive equations of motion similar to Newton's First and Second Laws. We implement Mach's principle according to which all inertial forces are due to gravitational interactions of any body with the rest of the universe. This is accomplished in a strictly relational theory (kinematics equivalent to dynamics) so that we don't need to introduce the concepts of absolute space or of inertial frames.

Acknowledgements – The author wishes to thank Fundação de Amparo à Pesquisa do Estado de São Paulo, FAPESP, and Conselho Nacional de Desenvolvimento Científico e Tecnológico, CNPq (Brasil), for financial support during the last years. The author is grateful to Prof. W. A. Rodrigues Jr. for helpful conversations, stimulus and valuable ideas. The author is also grateful to IMECC, UNICAMP, for secretarial help during the preparation of this work.

References

[1] W. Weber, Abh. Leibnizens Ges., Leipzig (1846), p. 316; Ann. d. Phys. 73, 193 (1848); English translation in Taylor's Scientific Memoirs 5, 489 (1852); W. Weber, Phil. Mag. S. (4), 43 (n. 283), 1 (1872); Wilhelm Weber's Werke, Vols. 1-6 (Springer, Berlin, 1893).

[2] J. C. Maxwell, A Treatise on Electricity and Magnetism (Dover, New York, 1954), Article 528.

[3] Ref. [2], Articles 856-860.

[4] E. T. Whittaker, A History of the Theories of Aether and Electricity – The Classical Theories (Humanities Press, New York, 1973), pp. 198-205.

[5] J. P. Wesley, Spec. Sci. Technol. 10, 47 (1987).

[6] H. Grassmann, Ann. d. Phys. 64, 1 (1845).

[7] R. A. R. Tricker, Early Electrodynamics – The First Law of Circulation (Pergamon, Oxford, 1965), pp. 55-58.

[8] A. O'Rahilly, Electromagnetic Theory – A Critical Examination of Fundamentals (Dover, New York, 1965, 2 volumes), Chapters IV and X (this book was formerly titled "Electromagnetics", Longmans, Green & Company, 1938).

[9] W. Ritz, Ann. Chim. Phys. 13, 145 (1908); W. Ritz, Oeuvres (Gauthier – Villars, Paris, 1911).

[10] Ref. [2], Article 687.

[11] P. T. Pappas, Nuovo Cimento 76 B, 189 (1983).

[12] P. G. Tait, Phil. Mag. 21, 319 (1861).

[13] F. F. Cleveland, Phil. Mag. 25, 416 (1936).

[14] P. Graneau, Fortschr. Phys. 34, 457 (1986).

[15] P. Graneau, Nature 295, 311 (1982).

[16] P. Graneau, J. Appl. Phys. 53, 6648 (1982).

[17] P. Graneau, J. Appl. Phys. 62, 3006 (1987).

[18] P. Graneau, Phys. Lett. 97A, 253 (1983).

[19] P. Graneau, J. Appl. Phys. 55, 2598 (1984).

[20] P. Graneau and P. N. Graneau, Appl. Phys. Lett. 46, 468 (1985).

[21] J. C. Ternan, Appl. Phys. Comm. 57, 1473 (1985).

[22] D. C. Jolly, Phys. Lett. 107A, 231 (1985).

[23] P. T. Pappas and P. G. Moyssides, Phys. Lett. 111A, 193 (1985).

[24] J. G. Ternan, J. Appl. Phys. 57, 1743 (1985).

[25] P. Graneau, J. Appl. Phys. 58, 3638 (1985).

[26] J. G. Ternan, J. Appl. Phys. 58, 3639 (1985).

[27] P. Graneau, Phys. Lett. 107A, 235 (1985).

[28] P. G. Moyssides and P. T. Pappas, J. Appl. Phys. 59, 19 (1986).

[29] P. Graneau and P. N. Graneau, Nuovo Cimento 7D, 31 (1986).

[30] P. Graneau, Ampère-Newmann Electrodynamics of Metals (Hadronic Press, Nonantum, 1985).

[31] C. Hering, J. Franklin Inst. 194, 611 (1921).

[32] H. Aspden, IEEE Trans. Plasma Sci. PS-14, 282 (1986).

[33] H. Aspden, Phys. Lett. 111A, 22 (1985).

[34] H. Aspden, Lett. Nuovo Cimento 44, 689 (1985).

[35] J. Nasilowski, IEEE Trans. Magnet. 20, 2158 (1984).

[36] J. Nasilowski, Phys. Lett. 111A, 315 (1985).

[37] P. Graneau, Phys. Lett. A 120, 77 (1987).

[38] J. G. Ternan, Phys. Lett. A 115, 230 (1986).

[39] H. Aspden, Phys. Lett. A 120, 80 (1987).

[40] P. Graneau, J. Phys. D 20, 391 (1987).

[41] C. K. Whitney, Phys. Lett. A 128, 232 (1988).

[42] C. K. Whitney, Am. J. Phys. 56, 871 (1988).

[43] P. Graneau, Electronics & Wireless World, June 1989, p. 556.

[44] P. Graneau, Phys. Lett. A 137, 87 (1989).

[45] C. Christodoulides, J. Phys. A 20, 2037 (1987).

[46] P. Graneau, Nuovo Cimento 78B, 213 (1983).

[47] R. Clausius, J. für Math. 82, 86 (1877).

[48] A. von Ettingshausen, Ann. d. Phys. 11, 432 (1880); ibid. p. 1044.

[49] W. F. Edwards, Measurement of an Electric Field due to Conduction Currents (Utah State Univ. Press, Logan, Utah, 1974).

[50] W. F. Edwards, Bull. Am. Phys. Soc. 20, 630 (1975).

[51] W. F. Edwards, C. S. Kenyon and D. K. Lemon, Phys. Rev. D 14, 922 (1976).

[52] D. K. Lemon and W. F. Edwards, Bull. Am. Phys. Soc. 23, 40 (1978).

[53] J. C. Curé, Phys. Lett. B 116, 158 (1982).

[54] D. F. Bartlett and B. F. L. Ward, Phys. Rev. D 16, 3453 (1977).

[55] G. Bonnet, Phys. Lett. A 82, 465 (1981).

[56] C. K. Whitney, Phys. Essays 1, 15 (1988); ibid., p. 18 (1988).

[57] C. K. Whitney, Hadronic J. 10, 289 (1987); ibid. 11, 101 (1988); ibid. 11, 147 (1988); ibid. 11, 257 (1988).

[58] R. Sansbury, Rev. Sci. Instrum. 56, 415 (1985).

[59] W. Weber and F. W. Kohlrausch, Ann. d. Phys. 99, 10 (1856).

[60] Ref. [4], p. 232.

[61] P. Moon and D. E. Spencer, J. Franklin Inst. 257, 203 (1954); ibid. p. 305; ibid. p. 369.

[62] J. P. Wesley, Phys. Essays, a review to be published May (1990).

[63] P. Graneau, Hadronic J. 10, 145 (1987).

[64] T. E. Phipps Jr., private communication to be published.

[65] H. v. Helmholtz, J. für Math. 72, 57 (1870); ibid. 75, 35 (1873); ibid. 78, 273 (1874); Phil. Mag. 44, 530 (1872); Monatsberichte Akademie der Wissenschaften zu Berlin, 400 - 418 (1875).

[66] A. I. Miller, Albert Einstein's Special Theory of Relativity (Addison–Wesley, Reading, MA, 1981), pp. 92 - 93.

[67] Ref. [4], pp. 201-4 and 233-5.

[68] A. K. T. Assis, Phys. Lett. A 136, 277 (1989).

[69] A. H. Bucherer, Ann. d. Phys. 28, 513 (1909).

[70] W. Kaufmann, Gott. Nachr., p. 143 (1901); ibid. p. 291 (1902); ibid. p. 90 (1903); Ann. der Phys. 19, 487 (1906).

[71] C. T. Zahn and A. H. Spees, Phys. Rev. 53, 511 (1938).

[72] P. S. Faragó and L. Jánossy, Nuovo Cimento 5, 1411 (1957).

[73] J. P. Wesley, private communication to be published.

[74] A. K. T. Assis, Found. Phys. Lett. 2, 301 (1989).