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The Meaning of the Constant c in Weber's Electrodynamics

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Abstract

In this work it is analysed three basic electromagnetic systems of units utilized during last century by Ampère, Gauss, Weber, Maxwell and all the others: The electrostatic, electrodynamic and electromagnetic ones. It is presented how the basic equations of electromagnetism are written in these systems (and also in the present day international system of units MKSA). Then it is shown how the constant c was introduced in physics by Weber's force. It is shown that it has the unit of velocity and is the ratio of the electromagnetic and electrostatic units of charge. Weber and Kohlrausch's experiment to determine c is presented, emphasizing that they were the first to measure this quantity and obtained the same value as that of light velocity in vacuum. It is shown how Kirchhoff and Weber obtained independently of one another, both working in the framework of Weber's electrodynamics, the fact that an electromagnetic signal (of current or potential) propagate at light velocity along a thin wire of negligible resistivity.

Key Words: Electromagnetic units, light velocity, wave equation.

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1 Introduction

One of the most fundamental constants in physics is the ratio of electromagnetic and electrostatic units of charge, c. Its origin in Weber's electrodynamics is traced in this work, along with its first measurement and the first time the wave equation describing an electromagnetic disturbance was obtained. Initially the basic systems of units utilized during last century for describing electromagnetic quantities is presented, along with a short review of Weber's electrodynamics and the measurement of c. How the equation of telegraphy was first derived is also considered. These are the main goals of this work.

2 Systems of Units

According to Newton's first and second definitions in the *Principia*, [New34], the quantity of motion is the mass of a body times its velocity. His second law of motion states: "The change of motion is proporcional to the motive force impressed; and is made in the direction of the right line in which that force is impressed." Representing this force vectorially by $\vec{F}$, the mass by m and the velocity by $\vec{v}$, the second law can be written as

$$\vec{F} = K_1 \frac{d(m\vec{v})}{dt} , \tag{1}$$

where K_1 is a constant of proportionality. Considering this constant to be the unitless number 1 and supposing a constant mass during the motion yields Newton's second law in the usual form

$$\vec{F} = m\vec{a} , \tag{2}$$

where $\vec{a} = d\vec{v}/dt$ is the acceleration of the body. If the force $\vec{F}$ is constant during the time t, this equations yields $\vec{a} = \vec{F}/m = constant$ and $\vec{v} = \vec{v}_o + \vec{a}t$, where $\vec{v}_o$ is the initial velocity of the body.

The unit force is then that constant force which when it acts upon the unit of mass imparts to this mass a unit of velocity in unit of time, [Web72, see p. 2].

Usually the basic units of mechanics are chosen to be the mass, length and time. Weber used to consider milligrams, millimeters and seconds as his basic units.

Newton's law of universal gravitation of 1687 states that this force is proportional to the product of the masses m and m', and inversely proportional to the square of their distance r. If $\hat{r}$ is the unit vector pointing from m' to m, the force on m due to m' is given by

$$\vec{F} = -K_2 \frac{mm'}{r^2} \hat{r} . \tag{3}$$

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This force is along the straight line connecting the masses and is always attractive. The constant K_2 allows for different systems of units. If we take this constant $K_2 = 1$ unitless, then combining Eq. (3) with Eq. (2) yields the unit of mass as deduced (or derived) from the units of time and length, see [Max54, Article 5, pp. 3-5].

The force on a particle of mass m can be written as

$$\vec{F} = m\vec{g} = m\left(-\sum \frac{K_2 m'}{r^2}\hat{r}\right) . \tag{4}$$

Here $\vec{g}$ is called the gravitational field acting on m due to all the masses m'. It is the force per unit mass.

According to Coulomb's law of 1785, the force between two point charges e and e' is proportional to their product and inversely proportional to the square of their distance r:

$$\vec{F} = K_3 \frac{ee'}{r^2}\hat{r} , \tag{5}$$

where the force is along the straight line connecting the charges and is repulsive (attractive) if $ee' > 0$ ($ee' < 0$). Two equal charges $e = e'$ are said to have unit magnitude when they exert upon one another a unit force when separated by a unit distance.

The first system of units applicable to electric quantities to be considered here is the electrostatic system of units. In this system $K_3 = 1$ unitless and the unit (or dimension) of the charges e and e' is called electrostatic unit, *esu*.

The force on a charge e can be written as

$$\vec{F} = e\vec{E} = e\left(\sum \frac{K_3 e'}{r^2}\hat{r}\right) . \tag{6}$$

Here $\vec{E}$ is called the electric field acting on e due to all the charges e'. It is the force per unit charge.

Coulomb also obtained in 1785 the force between two magnetic poles p and p' separated by a distance r as given by a similar expression. In the case of long thin bar magnets, the poles are located at the extremeties. Usually a north pole of a bar magnet (which points towards the geographic north of the earth) is considered positive and a south pole negative. Coulomb's expression for the force between magnetic poles is given by:

$$\vec{F} = K_4 \frac{pp'}{r^2}\hat{r} . \tag{7}$$

Once more two equal magnetic poles $p = p'$ are said to have unit magnitude when exert a unit of force when separated by a unit distance. There will be a force of repulsion (attraction) when $pp' > 0$ ($pp' < 0$). It is also along the straight line connecting the poles.

The second system of units utilized during last century is the electromagnetic system of units. In it $K_4 = 1$ unitless and the unit (or dimension) of p and p' is called electromagnetic unit, *emu*.

The force on a magnetic pole p can be written as

$$\vec{F} = p\vec{B} = p\left(\sum \frac{K_4 p'}{r^2}\hat{r}\right) . \tag{8}$$

Here $\vec{B}$ is called the magnetic field acting on p due to all the poles p'. It is the force per unit magnetic pole.

Between 1820 and 1826 Ampère obtained the force between two current elements. He was led to his researches after Oersted great discovery of 1820 that a current carrying wire afects a magnet in its vicinity. Following Oersted discovery Ampère decided to consider the direct action between currents. From his experiments and theoretical considerations he was led to his force expression.

If the circuits carry currents i and i' and the current elements separated by a distance r have lengths ds and ds', respectively, Ampère's force is given by

$$d^2\vec{F} = K_5 \frac{ii'dsds'}{r^2}\hat{r}\left(3\cos\theta\cos\theta' - 2\cos\varepsilon\right)$$

$$= K_5 \frac{ii'}{r^2}\left[3(\hat{r}\cdot d\vec{s})(\hat{r}\cdot d\vec{s}\,') - 2(d\vec{s}\cdot d\vec{s}\,')\right] . \tag{9}$$

The constant K_5 allows for different systems of units. In this expression θ and θ' are the angles between the positive directions of the currents in the elements and the connecting right line between them, ε is the angle between the positive directions of the currents in the elements, $\hat{r}$ is the unit vector connecting them, $d\vec{s}$ and $d\vec{s}\,'$ are the vectors pointing along the direction of the currents and having magnitude equal to the length of the elements.

After integrating this expression Ampère obtained the force exerted by a closed circuit C' where flows a current i' on a current element $id\vec{s}$ of another circuit as given by:

$$d\vec{F} = id\vec{s} \times \left(K_5 \oint_{C'} \frac{i'd\vec{s}\,' \times \hat{r}}{r^2}\right) . \tag{10}$$

A simple example is given here. Integrating this expression to obtain the force dF exerted by a long straight wire where flows a current i' acting on a current element ids at a distance ℓ to the wire and parallel to it is given by

$$dF = 2K_5 \frac{ii'ds'}{\ell} , \tag{11}$$

pointing from the current element to the wire (supposing that both currents flow in the same direction).

The physical connection between magnetic pole and current was given by Oersted experiment. That is, he observed that a galvanic current orients a small magnet in the same way as others magnets (or the earth) do. From Ampère's force law we can obtain a mathematical connection between these two concepts. This is done writing Eq. (10) as

$$d\vec{F} = id\vec{s} \times \vec{B} \ , \tag{12}$$

where $\vec{B}$ is called the magnetic field generated by the closed circuit C'. It is only possible to call it a magnetic field by Oersted's experiment. That is, the force exerted on a unit magnetic pole located at the same place as $id\vec{s}$ by the current carrying circuit C' is given by this magnetic field. This means that p and ids have the same units. Comparing the magnetic field of this equation with that of Eq. (8) yields

$$K_4 = K_5 \ . \tag{13}$$

Alternatively we can compare a magnetic pole and a galvanic current (or connect the constants K_4 and K_5) considering the known fact described by Maxwell in the following words: "It has been shewn by numerous experiments, of which the earliest are those of Ampère, and the most accurate those of Weber, that the magnetic action of a small plane circuit at distances which are great compared with the dimensions of the circuit is the same as that of a magnet whose axis is normal to the plane of the circuit, and whose magnetic moment is equal to the area of the circuit multiplied by the strength of the current", [Max54, Article 482, p. 141]. By magnetic action we can understand the force of the small circuit or of the small magnet on another magnetic pole. We may also say that the magnetic field exerted by this small circuit is the same as that generated by the small magnet, provided that

$$p\ell\hat{\ell} = iA\hat{u} \ . \tag{14}$$

Here i is the current of the small plane circuit of area A and normal unit vector $\hat{u}$, p is the magnetic pole of the small magnet of length ell and $\hat{\ell}$ points from the south to the north pole, $p\vec{\ell}$ being the magnetic moment of the magnet. As ℓ has the unit of length and A has the unit of length squared, the ratio of p/i has the unit of length.

Ampère who obtained for the first time a mathematical expression for the force between current carrying circuits utilized what is called the electrodynamic system of units. In this system $K_4 = K_5 = 1/2$ unitless and the currents are measured in (or its units and dimensions are) electrodynamic units. On the other hand, in the electromagnetic system $K_4 = K_5 = 1$ unitless and the currents are measured in electromagnetic units, see [Tri65, pp. 25, 51, 56 and 73].

The electrodynamic system of units was adopted by Ampère but has been since then abandoned. In any event it is relevant to compare the currents in electrodynamic and in electromagnetic measures. Let us represent the strengths of the currents in electrodynamic measure by j and j', and the same currents in electromagnetic measure by i and i'. By the fact that $K_5 = 1$ in the electromagnetic system and that $K_5 = 1/2$ in the electrodynamic system we have from Eq. (9): $jj'/2 = ii'$ or $j = \sqrt{2}i$, if there is the same current in both wires ($i = i'$ and $j = j'$). If we wish to compare the unit current in electromagnetic measure with the unit current in electrodynamic measure, we can consider the previous example of two parallel wires carrying the same current, Eq. (11). The force per unit length (dF/ds') between them if they are separated by a unit distance is given by 2 force units per length unit if $i = i' = 1$ unit electromagnetic current, remembering that $K_5 = 1$ in electromagnetic measure. On the other hand, if $j = j' = 1$ unit electrodynamic current, $dF/ds' = 1$ force unit per length unit, if they are separated by a unit distance, remembering that $K_5 = 1/2$ in electrodynamic measure. This means that in order to generate the same effect as one electromagnetic unit of current (that is, to have the same force between the wires), it is necessary to have $\sqrt{2}$ electrodynamic units of current. Hence the unit current adopted in electromagnetic measure is *greater* than that adopted in electrodynamic measure in the ratio of $\sqrt{2}$ to 1, [Max54, Article 526, p. 173] and [Tri65, p. 51]. That is, although $j = \sqrt{2}i$, a unit electromagnetic unit of current is equal to (has the same effect of, or generates the same force of) $\sqrt{2}$ units of electrodynamic current.

The connection between the electric currents (or between the units of charge) in electrostatic and in electromagnetic units is considered below.

3 Weber's Electrodynamics

In order to unify electrostatics (Coulomb's force of 1785) with electrodynamics (Ampère's force between current elements of 1826) and with Faraday's law of induction (1831), Wilhelm Weber proposed in 1846 the following force between two point charges e and e' separated by a distance r:

$$\vec{F} = K_3 \frac{ee'}{r^2}\hat{r}\left(1 - \frac{a^2}{16}\dot{r}^2 + \frac{a^2 r\,\ddot{r}}{8}\right) , \tag{15}$$

In this equation $\dot{r} = dr/dt$, $\ddot{r} = d^2r/dt^2$ and a is a constant which Weber only determined 10 years later. The charges e and e' may be considered as localized at $\vec{r}_1$ and $\vec{r}_2$ relative to the origin O of an inertial frame of reference S, with velocities and accelerations given by, respectively, $\vec{v}_1 = d\vec{r}_1/dt$, $\vec{v}_2 = d\vec{r}_2/dt$, $\vec{a}_1 = d\vec{v}_1/dt$ and $\vec{a}_2 = d\vec{v}_2/dt$. The unit vector pointing from 2 to 1 is given by $\hat{r} = (\vec{r}_1 - \vec{r}_2)/|\vec{r}_1 - \vec{r}_2|$. In this way $r = |\vec{r}_1 - \vec{r}_2| = \sqrt{(\vec{r}_1 - \vec{r}_2)\cdot(\vec{r}_1 - \vec{r}_2)}$, $\dot{r} = \hat{r}\cdot(\vec{v}_1 - \vec{v}_2)$ and $\ddot{r} = [(\vec{v}_1 - \vec{v}_2)\cdot(\vec{v}_1 - \vec{v}_2) - (\hat{r}\cdot(\vec{v}_1 - \vec{v}_2))^2 + (\vec{r}_1 - \vec{r}_2)\cdot(\vec{a}_1 - \vec{a}_2)]/r$. Weber wrote this equation with $K_3 = 1$ unitless and without vectorial notation.

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By 1856 Weber was writing this equation with c instead of $4/a$. But Weber's $c = 4/a$ is not our $c = 3\times 10^8\ m/s$, but $\sqrt{2}$ this last quantity. To avoid confusion with our c, and following the procedure adopted by Rosenfeld, [Ros57], we will call Weber's $4/a$ by c_w. This means that by 1856 Weber was writing his force law as the middle term below (the term on the right hand side is the modern rendering of Weber's force with the present day value of c):

$$F = K_3 \frac{ee'}{r^2}\left(1 - \frac{\dot{r}^2}{c_w^2} + \frac{2r\,\ddot{r}}{c_w^2}\right) = K_3 \frac{ee'}{r^2}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2}\right) . \qquad (16)$$

If there is no motion between the point charges, $\dot{r} = 0$ and $\ddot{r} = 0$, Weber's law reduces to Coulomb's force.

Weber knew in 1846 Coulomb's force between point charges and Ampère's force between current elements. He arrived at his force from these two expressions and a connection between current and charges. A description of his procedure can be found in his work and also in Maxwell and Whittaker's books: [Web66], [Max54, Chapter XXIII] and [Whi73, pp. 201-203]. Here we follow the opposite approach, namely: we begin with Weber's force in order to arrive Ampère's force.

Consider then the force between two current elements, 1 and 2. The positive and negative charges of the first one are represented by de_{1+} and de_{1-}, while those of element 2 are de_{2+} and de_{2-}. Supposing that they are electrically neutral yields $de_{1-} = -de_{1+}$ and $de_{2-} = -de_{2+}$. Adding Weber's force exerted by the positive and negative charges of the neutral element 1 on the positive and negative charges of the neutral element 2 yields (see [Ass94, Section 4.2]):

$$F = K_3 \frac{de_{1+}de_{2+}}{r^2}\frac{1}{c^2}\left\{3[\hat{r}\cdot(\vec{v}_{1+} - \vec{v}_{1-})][\hat{r}\cdot(\vec{v}_{2+} - \vec{v}_{2-})]\right.$$

$$\left. - 2(\vec{v}_{1+} - \vec{v}_{1-})\cdot(\vec{v}_{2+} - \vec{v}_{2-})\right\} . \qquad (17)$$

In order to arrive at Ampère's force from this expression we need a relation between current and charge. The commonly accepted definition of current is the time rate of change of charge, that is, a current is the amount of charge transferred through the cross section of a conductor per unit time:

$$i = \frac{de}{dt} . \qquad (18)$$

If the charge is measured or expressed in electrostatic, electromagnetic or electrodynamic units, the current will also be measured or expressed in electrostatic, electromanetic or electrodynamic units, respectively (see [Max54, Articles 231, 626 and 771]).

Applying this definition in Eq. (9) and comparing it with Eq. (5) yields a relation betwen the dimensions of K_3 and K_5. That is, the ratio K_3/K_5 has the unit of a velocity squared. It is independent of the units of electric and magnetic quantities and is a fundamental constant of nature.

The idea that galvanic currents are due to the motion of charges is due to Fechner and Weber, in 1845-1846. They supposed the currents to consist of an equal amount of positive and negative charges moving in opposite directions with the same velocity relative to the wire, [Whi73, p. 201]. Nowadays we know that the usual currents in metallic conductors are due to the motion of only the negative electrons. But it has been possible to show that it is possible to derive Ampère's force from Weber's one even without assuming Fechner's hypothesis, [Wes90], [Ass90] and [Ass94, Section 4.2].

Utilizing Eq. (18) and $\vec{v} = d\vec{s}/dt$ in Eq. (17) yields

$$d^2\vec{F} = \frac{K_3}{c^2}\frac{ii'}{r^2}\left[3(\hat{r}\cdot d\vec{s})(\hat{r}\cdot d\vec{s}\,') - 2(d\vec{s}\cdot d\vec{s}\,')\right] . \tag{19}$$

This will be Ampère's force (9) provided $K_3/c^2 = K_5$, that is:

$$c = \sqrt{\frac{K_3}{K_5}} . \tag{20}$$

Integrating Eq. (19) for the force exerted by an infinitely long straight wire carrying a constant i' acting on a current element ids parallel and at a distance ℓ to it is given by

$$dF = 2\frac{K_3}{c^2}\frac{ii'ds'}{\ell} . \tag{21}$$

If we are utilizing electrostatic units ($K_3 = 1$ unitless), the force per unit length (dF/ds') between them if they are separated by a unit distance is given by $2/c^2$ force units per length unit if $i = i' = 1$ electrostatic unit. On the other hand by Eq. (11) we saw that in electromagnetic units if we have $i = i' = 1$ electromagnetic unit than dF/ds' will be given by 2. For the current in electrostatic units generate the same force per unit length its magnitude needs to be given by c units. This means that c is the ratio of electromagnetic and electrostatic units of current, or the ratio of electromagnetic and electrostatic units of charge.

For this reason we can write

$$de_{\text{electromagnetic measure}} = \frac{de_{\text{electrostatic measure}}}{c} . \tag{22}$$

Alternatively we might also say that c is the number of units of statical electricity which are transmitted by the unit electric current in the unit of time. That is, if we have two equal unit electrostatic charges separated by a unit distance, they exert a unit force on each other by Eq. (5). By Eqs. (22) and (5) we can also write $F = c^2ee'/r^2$, where e and e' are the charges in electromagnetic units ($K_3 = c^2$ in electromagnetic measure). If we have two equal unit electromagnetic charges separated by a unit distance they exert on each other a force of magnitude c^2 units of force. In order to generate a unit force (as two unit electrostatic forces do), we need to have $e = e' = c$ electromagnetic units.

Analogously the constant $c_w = \sqrt{2}c$ is the ratio of the electrodynamic and electrostatic units of charge.

Charges are usually obtained in electrostatic units, measuring directly the force between charged bodies. Currents, on the other hand, are usually obtained in electromagnetic units. That is, it is measured the force between current carrying circuits or the deflection of a galvanometer (torque due to the forces between current carrying conductors). Alternatively it can be measured the torque or deflection of a small magnet due to a current carrying wire. But in order to know the numerical value of K_3/K_5 it is necessary to measure electrostatically the force between two charged bodies, discharge them and measure this current electromagnetically. Then it will be possible to express currents (and charges) measured in electromagnetic units in terms of currents (and charges) expressed in electrostatic units.

The first measurement of c_w was performed by Weber and Kohlrausch in 1856, who found $\sqrt{2}c = 4.39 \times 10^8 m/s$, such that $c = 3.1 \times 10^8 m/s$. This was one of the first quantitative connections between electromagnetism and optics. Discussions of this measurement can be found in: [Kc57], [Ros57], [Woo68], [Ros73], [Woo76], [Wis81], [Har82], [JM86, pp. 144-6 and 296-7] and [D'A96].

In the International System of Units MKSA we have the basic units for length, mass, time and electric current as given by meter (m), kilogram (kg), second (s) and Ampère (A). Forces are expressed in the unit Newton ($1N = 1kgms^{-2}$) and electric charges in Coulomb ($1C = 1As$). In this system the constants discussed in this work are given by: $K_1 = 1$ unitless and $K_2 = G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$. Moreover, $K_3 = 1/(4\pi\varepsilon_o)$, where $\varepsilon_o = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$ is called the permittivity of free space. The constant $K_4 = K_5 = \mu_o/(4\pi)$, where μ_o is called the vacuum permeability. By definition its value is given by $\mu_o = 4\pi \times 10^{-7}\ kgmC^{-2}$. In this case the units of p and p' are $Am = Cms^{-1}$. And finally $K_6 = 1$ unitless. The constant c is related with ε_o and μ_o by $c = 1/\sqrt{\mu_o\varepsilon_o}$. Of these three constants (ε, μ_o and c), only one is measured experimentally. The value of μ_o is given by definition, with ε_o and c related by the equation above.

4 Propagation of Electromagnetic Signals

We now discuss Kirchhoff's work who arrived at the telegraphy equation in 1857. He worked with Weber's action at a distance theory. He has three main papers related directly with this, one of 1850 and two of 1857, all of them have been translated to English: [Kir50], [Kir57] and [GA94]. Weber's simultaneous and more thorough work was delayed in publication and appeared only in 1864. Both worked independently of one another and predicted the existence of periodic modes of oscillation of the electric current propagating at light velocity in a conducting circuit of negligible resistance.

From now on we will utilize the international system of units MKSA.

In his first paper of 1857, Kirchhoff considered a conducting circuit of circular cross section which might be open or closed in a generic form. He wrote Ohm's law taking into account the free electricity along the surface of the wire and the induction due to the alteration of the strength of the current in all parts of the wire:

$$\vec{J} = -g\left(\nabla\phi + \frac{\partial\vec{A}}{\partial t}\right) \; , \tag{23}$$

where $\vec{J}$ is the current density, g is the conductivity of the wire, ϕ is the electric potential and $\vec{A}$ the magnetic vector potential. He calculates ϕ integrating the effect of all surface free charges:

$$\phi(x,y,z,t) = \frac{1}{4\pi\varepsilon_o}\int\int\frac{\sigma(x',y',z',t)da'}{|\vec{r}-\vec{r}\,'|} \; . \tag{24}$$

where $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ is the point where the potential is being calculated, t is the time, σ is the free surface charge. After integrating over the whole surface of the wire of length ℓ and radius α he arrived at:

$$\phi(s,t) = \frac{\alpha\sigma(s,t)}{\varepsilon_o}\ln\frac{\ell}{\alpha} \; , \tag{25}$$

where s is a variable distance along the wire from a fixed origin.

The vector potential $\vec{A}$ he obtains from Weber's formula given by

$$\vec{A}(x,y,z,t) = \frac{\mu_o}{4\pi}\int\int\int\left[\vec{J}(x',y',z',t)\cdot(\vec{r}-\vec{r}\,')\right](\vec{r}-\vec{r}\,')\frac{dx'dy'dz'}{|\vec{r}-\vec{r}\,'|^5} \; . \tag{26}$$

Here the integration is through the volume of the wire.

After integrating this expression he arrived at

$$\vec{A}(s,t) = \frac{\mu_o}{2\pi}I(s,t)\ln\frac{\ell}{\alpha}\hat{\alpha} \; , \tag{27}$$

where $I(s,t)$ is the variable current.

Considering that $I = J\pi\alpha^2$ and that $R = \ell/(\pi g\alpha^2)$ is the resistance of the wire, the longitudinal component of Ohm's law could then be written as

$$\frac{\partial\sigma}{\partial s} + \frac{1}{2\pi\alpha}\frac{1}{c^2}\frac{\partial I}{\partial t} = -\frac{\varepsilon_o R}{\alpha\ell\ln(\ell/\alpha)}I \; . \tag{28}$$

In order to relate the two unknowns σ and I Kirchhoff utilized the equation for the conservation of charges which he wrote as

$$\frac{\partial I}{\partial s} = -2\pi\alpha\frac{\partial \sigma}{\partial t} \ . \tag{29}$$

By equating these two relations it is obtained the equation of telegraphy, namely:

$$\frac{\partial^2 \xi}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} = \frac{2\pi\varepsilon_o R}{\ell \ln(\ell/\alpha)}\frac{\partial \xi}{\partial t} \ , \tag{30}$$

where ξ can represent I, σ, ϕ or the longitudinal component of $\vec{A}$.

If the resistance is negligible, this equation predicts the propagation of signals along the wire with light velocity.

Although in this derivation the interaction between any two charges is given by Weber's action at a distance law, the collective behaviour of the disturbance propagates at light velocity along the wire. This is somewhat similar to the propagation of sound waves derived by Newton or the propagation of signals along a stretched string obtained by d'Alembert. In all these cases classical newtonian mechanics was employed, without time retardation, without displacement current and without any field propagating at a finite speed. Although the interaction of any two particles in all these cases was of the type action at a distance, the collective behaviour of the signal or disturbance did travel at a finite speed.

In these cases there is a many body system (molecules in the air, molecules in the string or charges in the wire) in which the particles had inertia. Is it possible to derive the propagation of electromagnetic signals in vacuum, like in radio communication, by an action at a distance theory? I believe the answer to this question is positive. In practice there is never only a two body system. In any antenna there are many charged particles. Even if the material medium between two antennae is removed, there is always a gas of photons in the space between them. The action at a distance between the charges in both antennae with one another and with the gas of photons in the intervening space may give rise to a collective behaviour which is called electromagnetic radiation propagating at light velocity. Moreover, by Mach's principle the distant universe must always be taken into account. After all, the inertial properties of any charge is due to its gravitational interaction with the distant matter in the cosmos. At the moment I am working on this topic of antennae with Weber's electrodynamics. I am extending Kirchhoff's analysis to consider the case of waveguides, coaxial cables, dipole antennae and other situations dealing with open mechanical electromagnetic circuits.

5 Conclusion and Discussion

The constant c (or $c_w = \sqrt{2}c$) was introduced in electromagnetic theory by Weber in 1846. His goal was to unify electrostatics (Coulomb's force) with electrodynamics (Ampère's force) in a single force law. It is the ratio of electromagnetic (or electrodynamic) and electrostatic units of charge. Weber was also the first to measure this quantity working together with Kohlrausch. Their work is from 1856 and they obtained $c = 3.1 \times 10^8\ m/s$ (or $c_w = 4.4 \times 10^8\ m/s$). Weber and Kirchhoff were also the first to obtain the equation of telegraphy describing the propagation of electromagnetic signals along wires. In the case of negligible resistance they obtained the wave equation with a characteristic velocity given by c. These were some of the first connections between electromagnetism and optics as the value of light velocity was known to be $3 \times 10^8\ m/s$, the same value obtained for c by Weber and Kohlrausch's experiment.

It should be stressed that the works of Weber and Kirchhoff in 1856-57 were published before Maxwell wrote down his equations in 1861-64. When Maxwell introduced the displacement current $(1/c^2)\partial\vec{E}/\partial t$ he was utilizing Weber's constant c. He was also aware of Weber and Kohlrausch's measurement that c had the same value as light velocity. He also knew Weber and Kirchhoff's derivation of the telegraphy equation yielding the propagation of electromagnetic signals at light velocity. For a detailed work describing the link between Weber's electrodynamics and Maxwell's electromagnetic theory of light we recommend D'Agostino's paper, [D'A96].

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