On the Electromagnetic and Electrostatic Units of Current and the Meaning of the Absolute System of Units – For the 200th Anniversary of Wilhelm Weber's Birth

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1. Introduction

In this work it is discussed the absolute system of units introduced by *C. F. Gauss* (1777–1855) and *W. Weber* (1804–1891). Biographies of *Gauss* and *Weber* and discussions of their researches with many references can be found in the works of *Reich*,[1] *Wiederkehr* (1967),[2] *Woodruff*,[3] *Jungnickel* and *McCormmach*,[4] *Archibald*,[5] *d'Agostino*[6] and *Wiederkehr* (1997).[7]

To many physicists and authors of textbooks on electromagnetism the meaning of this absolute system of units is unknown or at least is not mentioned. This work follows the development of the absolute system of units concentrating on the important magnitudes, especially on the electric current intensity and the definition of its units. The important experimental work of *W. Weber* and *R. Kohlrausch* is also discussed as it gave a decisive impulse to the new approach of electromagnetism due to *J. C. Maxwell* (1831–1879) and to his electromagnetic theory of light. *Gauss* and *Weber* didn't write any dimensions to their various numerical results arising from their experiments, but from their texts these dimensions can be extracted. These dimensions are utilized here due to their clarity. The theory of dimensions, as first stated by *Fourier*, was applied to electromagnetism by *Maxwell*. In his

1 *K. Reich*: Carl Friedrich Gauß – 1777/1977. Heinz Moos Verlag, München, 1977.

2 *K. H. Wiederkehr*: Wilhelm Eduard Weber – Erforscher der Wellenbewegung und der Elektrizität (1804–1891), volume 32 of Grosse Naturforscher, H. Degen (ed.). Wissenschaftliche Verlagsgesellschaft, Stuttgart, 1967.

3 *A. E. Woodruff*: Weber, Wilhelm Eduard. In C. C. Gillispie, editor, Dictionary of Scientific Biography, vol. 14, p. 203–209, New York, 1976. Scribner.

4 *C. Jungnickel* and *R. McCormmach*: Intellectual Mastery of Nature – Theoretical Physics from Ohm to Einstein, volume 1–2. University of Chicago Press, Chicago, 1986. See especially Vol. 1, Chaps. 3, 6 and 7; and Vol. 2, Chap. 17.

5 *T. Archibald*: Energy and the mathematization of electrodynamics in Germany, 1845–1875. Archives Internationales d'Histoire des Sciences, 39:276–308, 1989.

6 *S. D'Agostino*: Absolute systems of units and dimensions of physical quantities: a link between Weber's electrodynamics and Maxwell's electromagnetic theory of light. Physis, 33:5–51, 1996.

7 *K. H. Wiederkehr*: Carl Friedrich Gauss (1777–1855) und Wilhelm Weber (1804–1891). In K. v. Meÿenn, editor, Die Grossen Physiker, Vol. 1, p. 357–370 and 522–524. Verlag C. H. Beck, München, 1997.

Treatise on Electricity and Magnetism (1873) symbols for dimensions are already employed.[8] In order to shed light upon the historical background on which the works of *Gauss* and *Weber* were built, we will present briefly the development of the most important concepts in the theory of electricity. They originated in part at the time of frictional electricity. The construction of some important measuring instruments will also be mentioned. Two important references which can be mentioned here are the books by *Whittaker*[9] and *Heilbron*.[10]

Before discussing the subject we present briefly the nomenclature and meaning of the terms which will be employed here. The dimensions of physical magnitudes (quantities) like mass, length and time will be represented by capital letters: M, L and T, respectively. For derived magnitudes the dimensions will be expressed in terms of these basic ones. The dimensions of velocity and acceleration, for instance, can be expressed as, respectively, LT^{-1} and LT^{-2}. It is possible to choose different units of measure for each dimension. For length, for instance, there is the meter, m, the centimeter, cm, the millimeter, mm, the foot, ft, the mile, mi, etc. For mass there is the kilogram, kg, the gram, g, the milligram, mg, etc. An algebraic symbol representing a physical magnitude stands normally for its numerical value and for its unit of measure. A certain mass m, for instance, can be expressed as 3 kg or as 3000 g. The numerical value and the unit of measure which are implicit in the measurement of a physical magnitude can be represented by braces and square brackets.[11] In this case, for instance, if $m = 3$ kg, then $\{m\} = 3$ and $[m] = \text{kg}$.

2. The Science of Electromagnetism Before Gauss and Weber

At the time of static electricity it was already possible to distinguish between the quantity of electricity and tension. Here we quote only three names: *Henry Cavendish* (1731–1810), an ingenious scientist, mathematically versed, but a solitary man who did not publish much and for this reason he had a limited influence upon the others. *Giambatista Beccaria* (1716–1781), follower of *B. Franklin*, had influence upon *Volta*. *Alessandro Volta* (1745–1827), was already prior to his work about galvanic electricity a renowned experimental physicist, also in frictional electricity. He discovered the electrophorus and the straw electrometer.

With the Leyden phial (1745) it was necessary to distinguish between the spark gap or discharge and its intensity. The spark gap indicated the magnitude of the tension, measured with an electrometer (straw electrometer or electrometer of gold leaves). The angle of separation of the leaves or straws was a measure for the tension. The amount of charge expressed itself in the intensity of the spark and also in the amount of physiological effects (shock) and in the heating of thin discharge wires. One of the main merits of *Volta* was to transfer these concepts also to galvanic electricity, a type of low voltage electricity. He

8 *J. C. Maxwell*: A Treatise on Electricity and Magnetism. Dover, New York, 1954. Original publication in 1873. See especially Vol. 2, Chap. X, Dimensions of electric units, paragraphs 620–629.

9 *E. T. Whittaker*: A History of the Theories of Aether and Electricity, volume 1: The Classical Theories. Humanities Press, New York, 1973. First published 1910, revised and enlarged edition 1951, reprinted 1958.

10 *J. L. Heilbron*: Electricity in the 17th & 18th Centuries. University of California Press, Berkeley, 1979.

11 IUPAP: Symbole, Einheiten und Nomenklatur in der Physik. Physik Verlag, Weinheim, 1980, p. 1.

showed that the galvanic fluid is not a special kind of "animal electricity" nor a kind of "vital principle". To the production of galvanic electricity, which in essence is identical to frictional electricity, it is only necessary two different metals and a conducting fluid. *Volta* could measure the relatively small tensions with the help of his electrophorus, which can also be utilized as a condenser or capacitor, and of his parallel plate capacitor. After charging the capacitor he lifted the upper plate, which was connected with an electroscope. With this separation between the plates the tension increases and the straws or leaves of the electroscope are much more separated from one another as before. In this way *Volta* set up his well-known contact series of metals. According to *Volta* the source for the production of electricity was the contact of two different metals, a supposition that later on was corrected.

In 1800 *Volta* created the pile which carries his name (voltaic pile, couples of disks of copper and zinc in contact, with each couple separated from the next by a disk of moistened salty pasteboard) and his so-called crown of cups. In this way he created sources of electricity which could produce relatively strong and constant currents, despite their small tension. The door for a new era in electricity was open. *Volta*, a real child of his time, could contribute to a rapid dissemination of this knowledge with his communication capabilities and his charisma, see *Teichmann* (1972)[12] and (1973).[13] The concepts of tension, current and resistance were qualitatively developed by *Volta* and their dependence on one another was also partially recognized. He had already the qualitative relationship between current I, charge Q and time t, namely: $I \sim Q/t$. In 1821 *Davy* obtained experimentally that the conducting power of a wire is directly proportional to its sectional area, whatever its form, and inversely proportional to its length, see footnote 9, p. 90. In 1826 *G. S. Ohm* obtained that the current which flows in a wire is proportional to the conductivity of the wire and to the difference of the electroscopic forces at the terminations of the wire. *Cavendish* and *Volta* had already suspected of a dependence between these basic magnitudes: tension, current and resistance.

Between the reasons which prevented the attainment of quantitative relationships was on one hand the lack of a reliable source of current and tension which might be kept constant for a long period of time, and on the other side a precise measuring instrument for the current. This was changed with the discovery of electromagnetism in 1820 by *H. C. Oersted* (1777–1851), see footnotes 12 and 13. In his experiment a magnetized needle is deflected by an electric current. *Oersted* belonged to the circle of romantic natural philosophers, who postulated a transformation of all natural forces on one another, which were then seen as different forms of a single primary force. In this way these natural philosophers conducted important preparatory work for the future discovery of the principle of the conservation of energy.

A. M. Ampère (1775–1836) was the most original and successful scientist working in this new area of electromagnetism. He showed experimentally the equivalence between the effects of a bar magnet and those produced by a solenoid carrying a constant current. He wanted to reduce even the terrestrial magnetism, which attracted the attention of many scientists during this time, to currents of electricity flowing below the surface of the earth. In general all magnetic phenomena should be produced by the flow of electricity, according

12 *J. Teichmann*: Zur Entwicklung von Grundbegriffen der Elektrizitätslehre, insbesondere des elektrischen Stromes bis 1820. RETE – Strukturgeschichte der Naturwissenschaften, 2:63–91, 1972.

13 *J. Teichmann*: Zur Entwicklung von Grundbegriffen der Elektrizitätslehre, insbesondere des elektrischen Stromes bis 1820. Dr. H. A. Gerstenberg, Hildesheim, 1973.

to *Ampère*. In a current carrying conductor, positive and negative electricities flow in opposite directions, according to *Ampère*'s conception. He researched the electrodynamic forces between current carrying conductors and discovered that, for example, two parallel conductors attract (repel) one another when their currents flow in the same (opposite) direction, the opposite of what is observed in electrostatics. Between 1820 and 1826 he obtained his well-known fundamental law of electrodynamics, expressing the force between two current elements depending on their positions and orientations. This is an action at a distance law, that is, the force doesn't need any time to cross the intermediary space between the current elements and doesn't need an intermediary medium to operate. His proof of the fundamental law utilizing equilibrium conditions was not conclusive. To measure currents he created the astatic galvanometer. It is composed of two magnetic bars of the same length and with equally strong magnetism, which hang in a string. The poles of both magnets were opposite to one another, in order to eliminate the influence of the terrestrial magnetism. One of the bars was located inside a coil (multiplicator), in order to increase the force of deflection. The article of *Wilhelm Weber* of 1846,[14] represented a continuation of *Ampère*'s work. With the electrodynamometer constructed by *Weber* it was possible to prove incontestably *Ampère*'s fundamental law for closed currents. Along the way defined *Weber* the absolute electrodynamic measure of current, one of the main subjects of this work.

After this short introduction we are now in a position to follow the main steps followed by *Gauss* and *Weber*.

3. Gauss and the Absolute System of Units

Gauss presented in 1832 a very important paper which is considered as the basis of the absolute system of magnetic units: The intensity of the earth's magnetic force reduced to absolute measurement. The original work was published in Latin in 1841 and is reprinted in his Collected Works.[15] Two different German translations appeared in 1833[16] and 1894.[17] There is also an unpublished English translation, see *Gauss* (1995)[18] and *Hecht* (1996).[19]

14 *W. Weber*: Elektrodynamische Maassbestimmungen – über ein allgemeines Grundgesetz der elektrischen Wirkung. Abhandlungen bei Begründung der Königl. Sächs. Gesellschaft der Wissenschaften am Tage der zweihundertjährigen Geburtstagfeier Leibnizen's herausgegeben von der Fürstl. Jablonowskischen Gesellschaft (Leipzig), p. 211–378, 1846. Also in Wilhelm Weber's Werke, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893) p. 25–214.

15 *C. F. Gauss*: Intensitas vis magneticae terrestris ad mensuram absolutam revocata. Commentationes Societatis Regiae Scientiarum Goettingensis Recentiores, 8:3–44, 1841. Delivered before the Society in 15 December 1832. Also in C. F. Gauss's Werke, Vol. 5, p. 79–118 (Königliche Gesellschaft der Wissenschaften (ed.), Göttingen, 1867).

16 *C. F. Gauss*: Die Intensität der erdmagnetischen Kraft, zurückgeführt auf absolutes Maass. Annalen der Physik und Chemie, J. C. Poggendorff (ed.), 28:241–273 and 591–615, 1833.

17 *C. F. Gauss*: Die Intensität der erdmagnetischen Kraft auf absolutes Maass zurückgeführt. In E. Dorn, editor, Ostwald's Klassiker der exakten Wissenschaften, Vol. 53. Wilhelm Engelmann Verlag, Leipzig, 1894. Translation by Kiel, notes by E. Dorn.

18 *C. F. Gauss*: The intensity of the earth's magnetic force reduced to absolute measurement. Translated from the German by Susan P. Johnson, edited by L. Hecht, unpublished, 1995.

19 *L. Hecht*: Experimental apparatus and instrumentation. 21st Century, 9(3):35–40, 1996.

As it is difficult to maintain magnetic standards which keep their properties unaltered over long periods of time, *Gauss*' idea was to base the definition and measurement of the magnetic properties on the mechanical standards and units of measurements of mass, length and time (to *Gauss* and *Weber* these mechanical units were the milligram, the millimeter and the second). As regards the force law, *Gauss* utilized *Newton*'s second law of motion (1687) in the form

$$f = \frac{d(mv)}{dt} = ma. \tag{1}$$

Here f is the net force acting on a body of mass m which moves with velocity v relative to an inertial frame of reference, producing an acceleration a.

The law for magnetostatic force was due to *John Michell*, *Tobias Mayer* and *Augustin Coulomb* between 1750 and 1785, see footnote 9, pp. 56–60. It describes the interaction between magnets or between magnetic fluids (magnetic charges or poles). *Coulomb* worked with thin and long magnets, so that the magnetic charges might be considered as concentrated on their ends, the poles of the magnets. Performing experiments with a torsion balance he could find an expression describing the interaction between these magnetic poles (by convention a north charge is considered as positive and a south charge as negative): "The magnetic fluid acts by attraction or repulsion in a ratio compounded directly of the density of the fluid and inversely of the square of the distance of its molecules."[20] *Gauss* took the proportionality factor in this force as being the number one dimensionless. The force between two magnetic fluids μ and μ' separated by a distance r acting along the straight line connecting them (attraction when they have opposite signs and repulsion when they have the same sign) was then written as

$$f = \frac{\mu\mu'}{r^2}. \tag{2}$$

By combining Eqs. (1) and (2) *Gauss* defined that there will be a unit of magnetic fluid when two equal magnetic fluids separated by a unit distance repel one another with a unit force, that is, a force which produces a unit acceleration in a unit mass. From the equation (1) and (2) it is evident that the dimension of magnetic charge μ will be given by $M^{1/2}L^{3/2}T^{-1}$. This shows that it will be based only on mechanical standards.

Gauss defined a unit intensity of the magnetic force (or a unit magnetic field as we would say today), as the intensity of the magnetic field which acting on a unit magnetic fluid generates a unit force. Representing this magnetic field by [illegible], this means that the force on a magnetic charge can be written as $\vec{f} = \mu\vec{b}$, acting along the direction of $\vec{b}$ if $\mu > 0$ or opposite to it if $\mu < 0$. According to this definition the dimension of the magnetic field is given by $M^{1/2}L^{-1/2}T^{-1}$.

Gauss considered in the *Intensity* the magnetic moment $\vec{M}$ of a magnet. In the simplest case it is defined as the product of the distance between the magnetic poles of a (ideal) magnet by the amount of magnetic fluid in the north (positive) pole, pointing from the negative to the positive pole. The direction of $\vec{M}$ is called the magnetic axis of the body, which will point along the direction of the magnetic field around it when the magnet is in

20 *A. Coulomb*: Second memoir on electricity and magnetism. In W. F. Magie, editor, A Source Book in Physics, p. 413–420, New York, 1935. McGraw-Hill. Original publication in French in 1785.

equilibrium. In more general terms *Gauss* presented this concept in Section 5 of the *Intensity* as follows: Let dm be the quantity of free magnetism in one particle with coordinates relative to three orthogonal axes as represented by x, y and z. By definition the magnetic moment of the body is given by $\vec{M} = M_x\hat{x} + M_y\hat{y} + M_z\hat{z} = \iiint \vec{r}dm$, where the integral is over the whole body. The torque exerted by a uniform magnetic field $b = |\vec{b}|$ upon a magnet of magnetic moment $M = |\vec{M}|$ is given by $Mb \sin \theta$. Here θ is the angle between the direction of $\vec{M}$ and the direction of $\vec{b}$. The dimension of magnetic moment is then given by $[M] = M^{1/2}L^{5/2}T^{-1}$.

Beginning with Eq. (2) it is possible to derive the torque exerted by a bar magnet upon another bar magnet. Consider a magnet of magnetic moment M_1 centered on the origin of a coordinate system with its magnetic axis pointing along the positive horizontal y direction. This first magnet is supposed fixed in the laboratory as regards its position and orientation. A second magnet with magnetic moment M_2 is supposed along the horizontal x axis pointing along the negative x direction. The center of this second magnet is supposed fixed in the laboratory, but this second magnet can rotate around an axis passing through its center and pointing along an axis parallel to the vertical z axis. The center of both magnets are separated by a fixed distance r, supposed much greater than their lengths. The torque experienced by the second magnet due to the first magnet, relative to an axis passing through the center of the second magnet and orthogonal to the xy plane, is given by M_1M_2/r^3.

As there are no magnetic poles isolated in nature, *Weber* suggested an alternative definition of magnetic magnitudes.[21] He considered a thought experiment in which he disregarded the magnetic field of the earth. The basic entity would be the magnetic moment M of a bar magnet. In the configuration above the torque exerted by the first magnet upon the second is given by M_1M_2/r^3, in such a way that the dimension of M is given by the relation obtained above, namely (taking into account that the dimension of torque is that of force times a length): $[M] = M^{1/2}L^{5/2}T^{-1}$. According to *Weber*, there will be a unit of magnetic moment in two equal bar magnets when they are in the configuration above, if the torque exerted by the first upon the second behaves relative to the unit measure of torque, as $1/r^3$. That is, if their centers are separated by a unit distance, there will be a unit torque. *Weber* then defines the measure for the intensity of the earth's magnetism, or the unit of the magnetic field as we would say today. As we have seen, the torque exerted by a magnetic field b upon a magnet of magnetic moment M is given by $Mb \sin \theta$, where θ is the angle between the direction of $\vec{M}$ and the direction of $\vec{b}$. A unit of magnetic field is then according to *Weber* the magnetic field which acting on a bar magnet with unit magnetic moment exerts a unit torque when the magnetic axis of the magnet is orthogonal to the direction of the magnetic field.

In a work published in 1840 *Gauss* discussed other force laws decreasing as the inverse square of the distance.[22] Following what he had made with magnetism he wrote these forces also with a dimensionless numeric coefficient equal to one, beginning with the gravitational

21 *W. Weber*: Elektrodynamische Maassbestimmungen insbesondere Widerstandsmessungen. Abhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften, mathematisch-physische Klasse, 1:199–381, 1852. Also in Wilhelm Weber's Werke, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), p. 301–471. See especially pp. 320–1 of Vol. 3 of W. Weber's *Werke*.

22 *C. F. Gauss*: Allgemeine Lehrsätze in Beziehung auf die im Verkehrten Verhältnisse des Quadrats der Entfernung wirkenden Anziehungs- und Abstossungs-kräfte. In C. F. Gauss and W. Weber, editors, Resul-

law as obtained by *Newton* in 1687. The mutual force between two ponderable molecules of masses m and m' separated by a distance r would be given by

$$f = \frac{mm'}{r^2}. \tag{3}$$

Gauss said that a similar law would be valid for the interaction between two electric fluids. It can then be written as

$$f = \frac{ee'}{r^2}. \tag{4}$$

Here e and e' are the point charges separated by the distance r. Following this suggestion, *Weber*[23] (with English translation in footnote 24,[24] see *Hecht*[25]) defined the electrostatic unit of charge utilizing this expression: "The unit of electrical fluid is determined in electrostatics by means of the force, with which the free electricities act on each other at a distance. If one imagines two equal amounts of electricity of the same kind concentrated at two points, whose distance is the unit of length, and if the force with which they act on each other repulsively, is equal to the unit of force, then the amount of electricity found in each of the two points is the measure or the unit of free electricity."

With these definitions the dimensions of magnetic pole, mass and electrical charge will be the same: $M^{1/2}L^{3/2}T^{-1}$.

For the force f exerted by a current element of length ds acting on a supposed magnetic fluid μ (intensity of the magnetic pole) when they are separated by a distance r *Gauss* utilized in this last work *Biot-Savart*'s law (1820), namely, (see footnote 22, p. 198):

$$f = \frac{\mu i_{em} ds \sin\theta}{r^2}. \tag{5}$$

Here θ is the angle between the direction of the current element and the straight line connecting the current element with the magnetic pole. Moreover, the direction of the force is orthogonal to ds and to the straight line connecting the two bodies. In this expression i_{em} is the intensity of the current in what was later called the electromagnetic system of measure. Although *Gauss* did not include explicitly this current intensity when presenting this law in this work, it is obvious that he had it in mind. This is also evident from a posthumously published work from the period 1833–1836[26] in which he presented the

tate aus den Beobachtungen des magnetischen Vereins im Jahre 1839, p. 1–51. Weidmannschen Buchhandlung, Leipzig, 1840. Also in Carl Friedrich Gauss Werke, Vol. 5, p. 195–242 (Königlichen Gesellschaft der Wissenschaften (ed.), Göttingen, 1867). See especially pp. 197–8.

23 *W. Weber* and *R. Kohlrausch*: Über die Elektricitätsmenge, welche bei galvanischen Strömen durch den Querschnitt der Kette fliesst. Annalen der Physik und Chemie, J. C. Poggendoff (ed.), 99:10–25, 1856. Also in Wilhelm Weber's Werke, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), p. 597–608.

24 *W. Weber* and *R. Kohlrausch*: On the amount of electricity which flows through the cross-section of the circuit in galvanic currents. Translated from the German by Susan P. Johnson, edited by L. Hecht, unpublished, 1996.

25 *L. Hecht*: The significance of the 1845 Gauss-Weber correspondence. 21st Century, 9(3):22–34, 1996.

26 *C. F. Gauss*: Carl Friedrich Gauss's Werke, volume 5, Königlichen Gesellschaft der Wissenschaften zu Göttingen (ed.), Göttingen, 1867. See Schering's comments on p. 637.

dimension of the specific intensity of the galvanic current as given by $M^{1/2}L^{1/2}T^{-1}$, see footnote 26, p. 630. And this follows directly from the expression above.

By combining *Newton*'s second law of motion, Eq. (1), with the law of universal gravitation in the form of Eq. (3), *Gauss* could also obtain the dimension of mass in terms of length and time, namely: L^3T^{-2}, see footnote 26, p. 630.

4. The Units of Electrical Current in Absolute Measure

In order to measure electric current, it is necessary to utilize the measurable effects produced by it. *Wilhelm Weber* considered the following possibilities: (A) the force and torque exerted by a current upon a magnet; (B) the decomposition of water; (C) the force and torque between two current carrying circuits; and (D) the thermic effect upon metals.

(A) As we have seen, *Oersted* discovered in 1820 that a wire carrying a constant current deflects magnets in its neighbourhood. *Biot* and *Savart* presented in the same year a mathematical law for the magnetic force as given by Eq. (5). As was common at that time, *Weber* called this expression the fundamental law of electromagnetism, as we can see in footnote 14, especially in Vol. 3, p. 82 of *W. Weber*'s *Werke*.

With this mathematical law it is then possible to calculate the net force and torque exerted by a closed carrying circuit upon a magnet. *Ampère*, in particular, proved in the period 1820–26 a very important theorem which was utilized by *Weber* as the basis of his definition of the absolute electromagnetic unit of current: an electric current is equivalent, in its magnetic effects, to a distribution of magnetism on any surface terminated by the circuit, the axes of the magnetic molecules being everywhere normal to this surface, see p. 88 of footnote 9. Utilizing the equivalence of a closed current and bar magnetism it is possible to transfer the Gaussian absolute measure of magnetism to electric currents. *Weber* discussed this in several papers and we give here the corresponding pages of his collected works (between parenthesis the years of publication):[27] Vol. 2, pp. 171–2 (1839) and 202–3 (1840); Vol. 3, pp. 14–16 (1841), 19–23 (1842), 80–87 and 173 (1846), 277 and 297–8 (1851), 321, 358–368 and 451–5 (1852), 599–600 (1856), 611–4, 642–3 and 648–9 (1857); and Vol. 4, pp. 24–6 (1862), 336–7 (1875), 422, 437 and 442 (1880), 586–7 and 597–609 (1894 posthumously). According to *Weber*'s definition:[28] "As an absolute unit of intensity, can be understood the intensity of that current which, when it circulates through a plane of the magnitude of the unit of measure, exercises, according to electro-magnetic laws, the same action at a distance as a bar-magnet which contains the unit of measure of bar magnetism."[29] This unit of measure of bar magnetism means here a unit magnetic moment. Moreover, the bar-magnet should be considered as orthogonal to the plane of the equivalent

27 *W. Weber*: Wilhelm Weber's Werke, W. Voigt, E. Riecke, H. Weber, F. Merkel and O. Fischer (Eds.), volume 1 to 6. Springer, Berlin, 1892–1894.

28 *W. Weber*: Messungen galvanischer Leitungswiderstände nach einem absolutem Maasse. Annalen der Physik und Chemie, J. C. Poggendorff (ed.), 82:337–369, 1851. Also in Wilhelm Weber's Werke, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), p. 276–300. See especially pp. 277 of Vol. 3 of W. Weber's *Werke*.

29 *W. Weber*: On the measurement of electric resistance according to an absolute standard. Philosophical Magazine, 22:226–240 and 261–269, 1861. See especially p. 227.

current carrying loop. With this definition (or directly from *Biot-Savart*'s law in the form of Eq. (5)) it is possible to see that the absolute dimension of current in electromagnetic measure is given by that of magnetic moment divided by an area: $M^{1/2}L^{1/2}T^{-1}$.

In 1851 and 1852 *Weber* introduced the absolute electromagnetic units of electromotive force (tension) and of resistance, see especially pp. 276–7 of Vol. 3 of *W. Weber*'s *Werke* in footnote 28 (with English translations in footnote 29) and footnote 21. For the electromagnetic absolute unit of measure of electromotive force he utilized *Faraday*'s law of electromagnetic induction (1831) and defined "that electromotive force which the unit of measure of the earth's magnetism exerts upon a closed conductor, if the latter is so turned that the area of its projection on a plane normal to the direction of the earth's magnetism increases or decreases during the unit of time by the unit of surface," see footnote 29, p. 227. For the electromagnetic absolute unit of resistance he utilized *Ohm*'s law (1826) and his electromagnetic unit of current. The definition runs as follows (our words in square brackets): "that resistance can be taken as unit of measure, which a closed conductor possesses in which the unit of measure of electromotive force produces the unit of measure of [electric current] intensity," see footnote 29, p. 226.

The instrument of measure which *Weber* utilized was the tangent galvanometer created by *Cl. S. M. Pouillet* (1790–1868) in 1837.[30] It consisted of a ring through which flowed the current. The plane of the ring was along the plane of the magnetic meridian. In the center of the ring there was a compass needle in an horizontal plane with an angular graduated scale (null angle in the plane of the ring). The current intensity is proportional to the tangent of the angle of deflection of the compass needle. *Weber* showed that it was possible to perform absolute measures of current intensity utilizing this instrument. It is necessary here the precise knowledge of the intensity of the horizontal component of the earth's magnetism. *Gauss* had already devised and performed precise measures to this end. Beyond the tangent galvanometer *Weber* constructed also a measuring instrument which didn't possess any magnetic needle. It consisted of a coil in a bifilar suspension in which circulated the same current as that flowing in the surrounding fixed ring. In this coil acted the current of the surrounding ring and also the horizontal component of the earth's magnetism.[31] This is the origin of the moving coil galvanometer which is utilized even today.

In the middle of the XIXth century it arised an urgent necessity for precise electric units of measure due to the nascent transmission of information through terrestrial and submarine cables. In 1861 the British Association and the Royal Society of London created a Commission led by *William Thomson* which should create a standard measure of resistance. They adopted the system of units created by *W. Weber*. However for the unit of resistance they chose a value with an appropriate order of magnitude different from that of *Weber*, which were closer to the values encountered in the practice. The unit of resistance was called

30 *W. Weber*: Messung starker galvanischer Ströme bei geringem Widerstande nach absolutem Maasse. In C. F. Gauss and W. Weber, editors, Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1840, p. 83–90. Weidmannschen Buchhandlung, Leipzig, 1841. Also in Wilhelm Weber's Werke, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), p. 6–12.

31 *W. Weber*: Ueber das elektrochemische Aequivalent des Wassers. In C. F. Gauss and W. Weber, editors, Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1840, p. 91–98. Weidmannschen Buchhandlung, Leipzig, 1841. Also in Wilhelm Weber's Werke, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), p. 13–18.

Ohmad, later on changed to Ohm. For the preparation of the standard Ohm, *Thomson* reverted to a method devised by *Weber*, improving it. The results were presented at the Report of the Meeting of the British Association for 1863. However, the precision was not what one could wish, and after the electrical congress of Paris in 1881 the physicists of several nations occupied themselves with the practical and precise preparation of the required Ohm in absolute unit. For references on these topics, see *Jenkin*,[32] *Wiedemann*,[33] *Stille*,[34] *Smith* and *Norton Wise*.[35]

(B) Even before the discovery of the magnetic action of a galvanic current, a chemical action had been discovered. Due to his fixation in his theory of metallic contact, *Volta* could not offer relevant contributions to this area of knowledge. It was different with the advocates of a chemical theory of the galvanic elements. One of those was *Johann Wilhelm Ritter* (1776–1810), who discovered the decomposition of water through the electric current in 1803. Another was *Michael Faraday* (1791–1867), who together with *Ritter* and *Davy* is considered as one of the founders of electrochemistry. In 1833 and 1834 *Faraday* presented his two laws of electrolytic decomposition. According to him the chemical forces (affinity) are strongly connected with the electrical forces, both being equivalent to one another. He created the gasvoltmeter and the voltameter for the precipitation of silver or copper. In the voltameter it is electrolytically precipitated a precise amount of metal through a definite amount of electrical charge. In the establishment, or more precisely in the realization of the units for charge and intensity of current, played the voltameter an important role. However, it is not appropriate for the measurement of the instantaneous intensity of current. *Faraday* did not admit the existence of a substantial electric fluid (charge) in conductors: "If we adopt the atomic theory or phraseology, then the atoms of bodies which are equivalents to each other in their ordinary chemical action, have equal quantities of electricity naturally associated with them. But I must confess I am jealous of the term *atom*; for though it is very easy to talk of atoms, it is very difficult to form a clear idea of their nature, especially when compound bodies are under consideration."[36] The same point of view was later on expressed by *Maxwell* in his *Treatise*: "The electrification of a molecule, however, though easily spoken of, is not so easily conceived", see footnote 8, paragraph 260, p. 380. To *Faraday* the electric current is a type of axis of force inside the conductors.

Faraday's laws of electrolysis led *Weber* to present an electrolytic measure of current intensity. Once more we give here the main pages of his collected works where he discussed this topic, see footnote 27: Vol. 3, p. 6 (1841), pp. 13–17 (1842), pp. 598–600 (1856), 612–4 and 649–651 (1857), Vol. 4, p. 88 (1862), p. 437 (1880) and 597–8 (1894 posthumously). According to *Weber*'s definition, one unit of current in electrolytic measure is the current intensity which decomposes a unit mass of water in the unit of time.

32 *H. C. F. Jenkin*: Über die neue von der British Association adoptierte elektrische Widerstandseinheit. Annalen der Physik und Chemie, 126:369–387, 1865.

33 *G. Wiedemann*: Die Lehre von der Elektricität, volume 4, 2nd edition. Friedrich Vieweg und Sohn, Braunschweig, 1893–1898. See especially pp. 633–728.

34 *U. Stille*: Messen und Rechnen in der Physik. Friedrich Vieweg & Sohn, Braunschweig, 1955. See especially pp. 206–237.

35 *C. Smith* and *M. Norton Wise*: Energy & Empire – A biographical study of Lord Kelvin. Cambridge University Press, Cambridge, 1989. See especially pp. 684–698.

36 *M. Faraday*: Experimental Researches in Electricity, volume 45, p. 257–866 of Great Books of the Western World. Encyclopaedia Britannica, Chicago, 1952. See especially paragraph 869.

In a work of 1841, published in 1842, *Weber* could compare these two definitions by observing the deflection of a current carrying coil due to the magnetic field of the earth, while simultaneously water was being decomposed by this current.[37] His results were that the electromagnetic measure of current was to the electrolytic measure of current as 0.009376 to one, or as 1 to $106\frac{2}{3}$. That is, one electromagnetic unit of current is $106\frac{2}{3}$ times smaller than the electrolytic unit of current, $I_{em} = I_{el} / 106\frac{2}{3}$.

(C) Working between 1820 and 1826 *Ampère* obtained an expression for the force between two current elements. As was common at that time, *Weber*[38] called it the fundamental law of electrodynamics[39] (we write i_{ed} and i'_{ed} instead of Weber's i and i' to let it clear the distinction of this current with the previous ones): "When two elements of a current, the lengths of which are α and α', and the intensities i_{ed} and i'_{ed}, and which are at the distance r from each other, so that the direction in which the positive electricity in both elements moves, form with each other the angle ε, and with the connecting right line the angles θ and θ', the magnitude of the force with which the elements of the current reciprocally act upon each other is determined by the expression

$$-\frac{\alpha\alpha' i_{ed} i'_{ed}}{r^2}\left(\cos \varepsilon - \frac{3}{2}\cos \theta \cos \theta'\right), \tag{6}$$

and repulsion or attraction occurs according as this expression has a positive or negative value."

Ampère could also integrate this force to obtain the force of a closed circuit acting upon a current element of another circuit, or the force between two closed current carrying circuits. *Ampère*'s work was the basis for Weber's electrodynamic definition of current, a subject which he discussed in several papers. Once more we present here the main pages of his collected works where he discussed this topic in more details, see footnote 27: Vol. 3, p. 9 (1841); 69–87 (1846); 237–8 (1848); 358–368 (1852); 600 (1856); 612–4 and 652 (1857); Vol. 4, 597–8 (1894 posthumously). *Weber* considered *Ampère*'s force between two equal current elements (both of equal length α and carrying the same current i_{ed}) in the special case in which they are parallel to one another and both of them orthogonal to the straight line connecting them. In the above expression the force reduces to $a^2 i_{ed}^2 / r^2$. The force is then proportional to the square of the length of the current element. Suppose $\alpha = 1$ unit of length. Then there will be a unit intensity of electrodynamic current when these two parallel current elements separated by a unit distance attract or repel one another with a unit force. According to *Weber*'s definition, there will be an electrodynamic unit of current when these two elements, separated by a unit distance, exert a force on one another which is to the unit

37 *W. Weber*: Messung starker galvanischer Ströme nach absolutem Maasse. Annalen der Physik und Chemie, J. C. Poggendorff (ed.), 55:27–32, 1842. Also in Wilhelm Weber's Werke, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), p. 19–23.

38 *W. Weber*: Elektrodynamische Maassbestimmungen. Annalen der Physik und Chemie, J. C. Poggendorff (ed.), 73:193–240, 1848. Also in Wilhelm Weber's Werke, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), p. 215–254. See especially p. 237 of Vol. 3 of W. Weber's *Werke*.

39 *W. Weber*: On the measurement of electro-dynamic forces. In R. Taylor, editor, Scientific Memoirs, vol. 5, p. 489–529, New York, 1966. Johnson Reprint Corporation. Original published in 1852. See especially pp. 510–511.

force as α^2 is to the unit of area, see footnotes 14 (especially pp. 80–81 of Vol. 3 of *W. Weber*'s *Werke)* and 21 (especially p. 360 of Vol. 3 of *W. Weber*'s *Werke*).

In his work of 1846 Weber could compare the electromagnetic measure of current intensity (number expressing the current strength), i_{em}, with the electrodynamic one, i_{ed}, obtaining that the first is $\sqrt{2}$ smaller than the second, see footnote 14, especially pp. 80–7 of Vol. 3 of *W. Weber*'s *Werke*. That is, all measured electromagnetic current intensities must be multiplied by $\sqrt{2}$ in order to obtain the same current intensities expressed in electrodynamic measure (or to reduce them to electrodynamic measure of current intensities). To this end he made a theoretical comparison in a specific geometry of the torque exerted between two small magnets utilizing Eq. (2), the torque exerted by a closed current carrying plain circuit on a small magnet utilizing Eq. (5), and the torque exerted between two small closed carrying circuits utilizing Eq. (6). For the first case he obtained $(mm'/r^3)\sin\,\delta\sqrt{1+3\cos^2\psi}$, where m and m' are the magnetic moments of the first and second magnets separated by a distance r much greater than their sizes, ψ is the angle of the magnetic axis of the first magnet relative to the straight line connecting the two magnets, and δ is the angle which the magnetic axis of the second magnet forms with the direction for which there is no torque between them. For the second expression he obtained the same expression as above but now with $i_{em}\lambda$ replacing m, where i_{em} is the current in electromagnetic measure flowing around an area λ orthogonal to the magnetic axis of the first magnet. For the third case he obtained the same expression as in the first case, but now with mm' being replaced by $i_{ed}\lambda i'_{ed}\lambda'/2$. Here i_{ed} and i'_{ed} are the currents in electrodynamic measure flowing around areas λ and λ', which planes are orthogonal to the magnetic axis of the first and second magnets, respectively. These three expressions can only agree with one another (that is, to produce the same measurable torque) if $m = i_{em}\lambda = i_{ed}\lambda/\sqrt{2}$.

Wilhelm Weber wanted also to test experimentally this relation between the electrodynamic and electromagnetic measures of current intensity. He measured the Amperian electrodynamic force and current intensity with the electrodynamoter which he had built (1846), and measured with a magnetometer the electromagnetic current intensity. The electrodynamometer was composed of an immovable external coil and a suspended internal movable bifilar coil with mirror. The magnetometer utilized by *Weber* was the already tested transportable magnetometer which he had built in 1838, see footnote 27, Vol. 2, p. 89. Here the bar magnet, which was suspended by a torsion wire with a mirror, was not as usual surrounded by a coil (multiplier). The multiplier was outside from the instrument. For the deflection of the magnet relative to the magnetic meridian it is valid in an extended sense the same laws as for the tangent galvanometer. The observations were made through the mirror utilizing scale and telescope. Both instruments were not damped, and the deflection was to be obtained from the change in the oscillations. It should be mentioned that the electrodynamoter was also useful for measures of alternating current because the current flows along both coils and the change in the direction of the current happens simultaneously in both coils. For the then arising electrotechnology the electrodynamoter was an almost indispensable measuring device, see footnote 27, Vol. 3, pp. 36, 37, 55, 57, 87 and 91. However, *Weber* had to rest satisfied with an error of 6% with his verification, as this was not originally foreseen, see footnote 27, Vol. 3, p. 91.

In 1852 *Weber* discussed this question further and made an important remark when comparing these two definitions, see footnote 21, especially pp. 358–365 of Vol. 3 of *W.*

Weber's Werke. He distinguished between the number expressing the current strength in electromagnetic and in electrodynamic units, k and i, respectively, with the electromagnetic and electrodynamic units in which they were measured, K and J, respectively. By expressing the same magnitude in both systems of measurements he obtained $kK = iJ$. As $k/i = 1/\sqrt{2}$ he got $K/J = \sqrt{2}/1$. This means that one electromagnetic unit of current is $\sqrt{2}$ times larger than the electrodynamic unit of current. That is, the ratio of the measured current strengths behaves oppositely to the ratio of the current units of measurement.

We can understand this by comparing a measurement of a distance d in two different systems of units, for instance in meters and in centimeters. We can express it as a number, $\{d\} = n$ or n', times a unit of measurement, $[d] = u$ or u': $\{d\}[d]$ can be written as nu or as $n'u'$. With $n = 3$, $n' = 300$, u = meter = m and u' = centimeter = cm, we get: $d = 3$ m = 300 cm. This shows that in this case $n/n' = u'/u = 1/100$.

Representing the number expressing the current strength in electromagnetic and electrodynamic measures by $\{i_{em}\}$ and $\{i_{ed}\}$ and the corresponding units of measurement by $[I_{em}]$ and $[I_{ed}]$, respectively, yields: $\{i_{em}\}[I_{em}] = \{i_{ed}\}[I_{ed}]$, such that $\{i_{em}\} = \{i_{ed}\}/\sqrt{2}$ and $[I_{em}] = \sqrt{2}/[I_{ed}]$. Although these two current definitions have the same dimension, $M^{1/2}L^{1/2}T^{-1}$, they do not have the unit of measure.

In 1857 *Kohlrausch* and *Weber* suggested to rewrite *Ampère*'s force (6) in terms of the electromagnetic current (substituting $i_{ed}i'_{ed}/2$ of this expression by $i_{em}i'_{em}$):[40]

$$-\frac{\alpha\alpha' \, i_{em} i'_{em}}{r^2}(2\cos\varepsilon - 3\cos\theta\cos\theta'). \tag{7}$$

It is then possible to work only with Eqs. (5) and (7) in order to deal with the forces and torques between current carrying circuits as well as with the forces and torques between currents and magnets without needing to worry about multiplying or dividing the current strengths nor its units by $\sqrt{2}$. This suggestion was eventually adopted and the electrodynamic system of units has since then been abandoned.

In order to compare the electromagnetic and electrodynamic units of current it is of interest to consider the force per unit length exerted between two parallel straight wires of infinite length carrying the currents i and i' when they are separated by a distance d. Integrating Eqs. (6) and (7) it is obtained this force per unit length, namely:

$$\frac{i_{ed} i'_{ed}}{d} = 2\frac{i_{em} i'_{em}}{d}. \tag{8}$$

This force is attractive (repulsive) if the currents flow in the same (opposite) directions.

From this expression it is possible to present another definition for a unit of current: There will be a unit of electrodynamic (electromagnetic) current flowing in two parallel straight wires separated by a unit distance when they exert on one another a unit force (2 units of force) per unit length. The adoption of the electromagnetic unit of current is the

40 *R. Kohlrausch* and *W. Weber*: Elektrodynamische Maassbestimmungen insbesondere Zurückführung der Stromintensitäts-Messungen auf mechanisches Maass. Abhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften, mathematisch-physische Klasse, 3:221–290, 1857. Also in Wilhelm Weber's Werke, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), p. 609–676. See especially pp. 613–4 of Vol. 3 of W. Weber's *Werke*.

origin of this factor 2 which exists until today in the international MKSA System of units, apart from powers of 10. For instance, in the MKSA System the Ampère is defined as the current that, separated in two parallel conductors by a distance of one meter, results in a force on each conductor of 2×10^{-7} N per meter of length of each conductor.

(D) Already in 1805 it was observed that a galvanic current heats the conductor through which it is flowing.[41] Also by the discharge of the Leyden phial it had been observed the effect of heat in the wires and with the help of the air thermometer quantitative experiments had been carried out. In 1841 discovered *James Prescott Joule* (1818–1889) the law of heat production: the amount of heat evolved in unit of time in a metallic wire is proportional to the resistance of the wire multiplied by the square of the current strength, see footnote 9, pp. 211–212. *Alexandre Edmond Becquerel* (1820–1891) and *Heinrich Friedrich Emil Lenz* (1804–1865) confirmed in 1843 and 1844 *Joule*'s law.

Weber discussed in some details the experiments of *Joule*, *Becquerel* and *Lenz* related with the production of heat and the increase of temperature in current carrying resistive wires in the last section of his work of 1862 about galvanometry,[42] and in his work "About the motions of electricity in bodies of molecular constitution."[43] In an appendix introduced in 1864 for a paper written much earlier, but published only posthumously in 1894, *Weber* presented a clear definition for an electrothermic unit of current:[44] there will be an electrothermic unit of current flowing in a platin wire with a unit cross-section when the temperature of the wire increases one unit in the unit of time. *Weber* himself did not consider the electrolytic and thermic definitions of current as absolute ones in a more restricted sense, as they depend on specific properties of the substance through which the current is flowing (decomposibility or specific resistance), as he discussed in pages 597–8 of this work. The electromagnetic and electrodynamic definitions, on the other hand, can be applied independent of the properties of the conductors through which flows the current.

In his work of 1862 *Weber* discussed the transformation of the current work in heat, analyzing the production of heat in a resistive conductor, see footnote 42, especially pp. 91–96 of Vol. 4 of *Weber*'s *Werke*. By the flow of the electric particles (electric atoms of two types, positive and negative) the particles are accelerated (by the applied electromotive force) and decelerated (by collision) during the passage of one molecule to the other. The energy is transferred to the last molecule. By the mechanical theory of heat, the temperature of the conductor is by this means increased.

41 *G. Wiedemann*: Die Lehre von der Elektricität, volume 2. Friedrich Vieweg und Sohn, Braunschweig, 1894. See pp. 174–178 and 198–202.

42 *W. Weber*: Zur Galvanometrie. Abhandlungen der Königl. Gesellschaft der Wissenschaften zu Göttingen, mathematische Klasse, 10:3–96, 1862. Also in Wilhelm Weber's Werke, Vol. 4, H. Weber (ed.), (Springer, Berlin, 1894), p. 17–96. See especially pp. 77–96 of Vol. 4 of W. Weber's *Werke*.

43 *W. Weber*: Ueber die Bewegung der Elektricität in Körpern von molekularer Konstitution. Annalen der Physik und Chemie, J. C. Poggendorff (ed.), 156:1–61, 1875. Also in Wilhelm Weber's Werke, Vol. 4, H. Weber (ed.), (Springer, Berlin, 1894), p. 312–357. See especially pp. 336–9 of Vol. 4 of W. Weber's *Werke*.

44 *W. Weber*: Ueber die Einrichtung des Bifilargalvanometers. In H. Weber, editor, Wilhelm Weber's Werke, Vol. 4, p. 584–615, Berlin, 1894. Springer. See especially pp. 584 (footnote), 597–8 and 608 of Vol. 4 of W. Weber's *Werke*.

5. The electrostatic and mechanical units of current

Beyond presenting definitions of a current based on the effects it produces, *Weber* gave also a definition based on its cause, namely, on the amount of charges flowing through the cross-section of a conductor in a time interval. Here are the main parts of his collected works where he discussed this, see footnote 27: Vol. 3, p. 40 (1846), p. 152 (1856), pp. 366–7 (1852), p. 592 (1855), p. 598 (1856), pp. 615 and 648–652 (1857); Vol. 4, pp. 337 and 350 (1875). In the absolute electrostatic measure of current intensity, one unit of electric charge flows in a second over the cross-section of a conductor. It will be represented here by i_{es}. It should be observed that *Weber* assumed the Fechnerian hypothesis of a double current in a conductor (positive and negative charges moving with equal and opposite velocities relative to the wire). His mechanical measure of current intensity is not identical with the absolute electrostatic measure of current intensity. In one unit of *Weber*'s mechanical current flows in one second a positive and also a negative electrostatic unit of charge through the cross-section: "This measure, which will be called the mechanical measure of current intensity, thus sets as the unit, the intensity of those currents which arise when, in the unit of time, the unit of free positive electricity flows in the one direction, an equal amount of negative electricity in the opposite direction, through that cross-section of the circuit," see footnote 23, especially p. 598 of Vol. 3 of *W. Weber*'s *Werke*, with English translation in footnote 24. It will be represented here by i_{mech}. The dimensions of i_{es} and of i_{mech} are the same, namely, the dimension of electric charge ($M^{1/2}L^{3/2}T^{-1}$) divided by time (T): $M^{1/2}L^{3/2}T^{-2}$. From this it follows that the electrostatic and mechanical dimensions of current are different from the electromagnetic and electrodynamic dimensions of current. The ratio of the dimensions of the electrostatic and electromagnetic currents is that of a velocity, namely: $M^{1/2}L^{3/2}T^{-2}/M^{1/2}L^{1/2}T^{-1} = LT^{-1}$

When there is one unit of current in mechanical measure flowing in a circuit, there will be then 2 units of current in electrostatic measure: $1I_{mech} = 2I_{es}$. As $i_{es}I_{es} = i_{mech}I_{mech}$ this yields: $i_{es} = 2i_{mech}$ and $I_{es} = I_{mech}/2$. Although the electrostatic and mechanical definitions of current have the same dimension, they do not have the same unit of measure.

In order to compare the mechanical with the other units of current (electromagnetic, electrodynamic and electrolytic), *Weber* and *Kohlrausch* performed a decisive experiment in 1854–6.[45] See also footnotes 23 (with English translation in footnote 24) and 40. Detailed descriptions of this experiment with relevant discussions and references can be found in *Kirchner*,[46] footnote 4 (especially Vol. 1, pp. 145–6), footnote 6 and in *Widerkehr*.[47]

Here we follow the train of thought of *Weber* by the consideration of the relation between the "mechanical" unit of current measure I_{mech} and the electrostatic unit of measure I_{es}, the electromagnetic unit of measure I_{em}, the electrodynamic unit of measure I_{ed} and their

45 *W. Weber*: Vorwort bei der Übergabe der Abhandlung: Elektrodynamische Maassbestimmungen, insbesondere Zurückführung der Stromintensitäts-Messungen auf mechanisches Maass. Berichte über die Verhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physische Klasse, 17:55–61, 1855. Also in Wilhelm Weber's Werke, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), p. 591–596.

46 *F. Kirchner*: Determination of the velocity of light from electromagnetic measurements according to W. Weber and R. Kohlrausch. American Journal of Physics, 25:623–629, 1957.

47 *K. H. Wiederkehr*: Wilhelm Weber und Maxwells elektromagnetische Lichttheorie. Gesnerus, Part. 3/4, 51:256–267, 1994.

numerical values. In the experiment from 1855 he and Kohlrausch realized this measure. The result led to the light velocity and to a connection between electric and optical phenomena. However it should be noted that at that moment *Weber* and *Kohlrausch* did not recognize this important cue.

Already in the first publication in the series of "Electrodynamic Measurements" with the subtitle "On a general fundamental law of electrical action" (1846), mentioned *Weber* that a measure of the electric current intensity could be defined utilizing the amount of electricity flowing through the cross section of the conductor, see footnote 27, Vol. 3, p. 41. At this time it seemed to him that an experimental realization was not yet possible, especially as regards the electrostatic measures. And he considered impossible at that point an experimental determination of the drifting velocity of the particles of electricity. In the same publication of 1846 he deduced, beginning with *Ampère*'s fundamental law for the force between current elements, Eq. (6), the relation $i_{ed} = aeu$, see footnote 27, Vol. 3, p. 152. *Weber* wrote i instead of i_{ed}, but we are putting the electrodynamic subscript to clarify its meaning. In this relation e is the amount of positive charge per unit length inside the conductor where flows the current i_{ed}, u is the drifting velocity of these charges according to *Weber*, and a is a proportionality factor. He then derived his fundamental law describing the force between two charges e and e' separated at time t by a distance r as (see footnote 27, Vol. 3, p. 157):

$$\frac{ee'}{r^2}\left(1-\frac{a^2}{16}\frac{dr^2}{dt^2}+\frac{a^2}{8}r\frac{d^2r}{dt^2}\right). \tag{9}$$

In his second publication in the series of "Electrodynamic Measurements" of 1852 he specified in more details his fundamental law and introduced the constant c, see footnote 27, Vol. 3, p. 366. To avoid confusion with the present meaning of c (that is, $c = 3 \times 10^8\ m/s$ = light velocity in vacuum) and following the suggestion of *Rosenfeld*, we will represent Weber's constant c by c_W.[48] According to *Weber*'s definition, $1/c_W = a^2/16$, in such a way that his fundamental force law took the following form:

$$\frac{ee'}{r^2}\left(1-\frac{1}{c_W^2}\frac{dr^2}{dt^2}+\frac{2r}{c_W^2}\frac{d^2r}{dt^2}\right). \tag{10}$$

To *Weber* (see footnote 27, Vol. 3, pp. 366–367) this fundamental constant c_W had the meaning of a specific relative velocity between the interacting charges, such that if the two charges were moving along a straight line with this constant relative velocity, there would not be any action of one charge upon the other.[49]

Already at the time of the joint work with *Gauss* had *Weber*, as already stated, defined in 1841 the absolute electromagnetic current intensity, in connection with the method of pure magnetic measures developed by *Gauss*. In the electrodynamic measure of current intensity created by *Weber* forces between electric currents are now utilized. It is also drawn upon the

48 *L. Rosenfeld*: The velocity of light and the evolution of electrodynamics. Il Nuovo Cimento, Supplement to vol. 4:1630–1669, 1957.

49 *K. H. Wiederkehr*: Wilhelm Webers Stellung in der Entwicklung der Elektrizitätslehre. Dissertation, Hamburg, 1960. See especially pp. 130–131.

magnetic effect in both measures of current. In the "mechanical" or electrostatic measures of current *Weber* searched for the cause, that is, for the motion of the particles of electricity. He chose the denotation "mechanical", because in electrostatics the amount of electricity is determined by means of a mechanical acceleration, see footnote 27, Vol. 3, p. 365 and 366.

For the derivation of the relation between mechanical and electrodynamic measures of current *Weber* utilized the same algebraic equation as for the previous consideration of the connection from electromagnetic and electrodynamic measures of current, namely $kK = kJ$, see footnote 27, Vol. 3, pp. 361 and 367. But now K represents the unit of current intensity in mechanical measure and J the unit of current intensity in electrodynamic measure. We could write this relation as $i_{mech}I_{mech} = i_{ed}I_{ed}$. By the transformation of *Ampère*'s fundamental law in his own fundamental law Weber had obtained the relation $i_{ed} = aeu$, see footnote 27, Vol. 3, p. 152. This means that the electric current i_{ed} is proportional to eu, with a as the proportionality constant, u the velocity of the particles of electricity in the current and e the amount of electricity per unit length (not the amount of electricity, see footnote 27, Vol. 3, p. 152). From the comparison of the earlier formulation of his fundamental law of 1846 (see footnote 27, Vol. 3, p. 157) with the formulation of 1852 he obtained $a = 4/c_W$, c_W being Weber's constant or *Weber*'s limit velocity. From $i_{ed} = aeu$ it is obtained $i_{ed} = 4eu/c_W$. From $kK = iJ$ or $i_{mech}I_{mech} = i_{ed}I_{ed}$ and utilizing that $k = eu$ it can be obtained $i_{ed} = 4k/c_W$ or $i_{ed} = 4i_{mech}/c_W$. From this expression it follows that $k = c_W i_{ed}/4$ or $i_{mech} = c_W i_{ed}/4$, see footnote 27, Vol. 3, p. 367.

It should be observed that k (or i_{mech}) is not anymore a pure numerical value, as was the case in the previous considerations. The reason is that c_W is a velocity and for this reason has a dimension. According to the modern conception a physical magnitude is represented through a numerical value and a unit of measure with dimension. This is not the case anymore with kK. For this reason we wish to distinguish between the pure numerical value of *Weber*'s limit velocity, written here as $|c_W|$, and this velocity itself, written as c_W, which possesses a dimension.

Weber's instruction for the transformation of a current intensity measured electrodynamically into a mechanically measured one leads then to a correct result. His "rule" runs as follows, see footnote 27, Vol. 3, p. 367: We must multiply the numerical value of the electrodynamic measure of current with $c_W/4$, in order to obtain the corresponding magnitude of the current intensity in mechanical measure. That is, $i_{ed}I_{ed}$ corresponds to $i_{ed}|c_W|I_{mech}/4$.

We want to transfer these prescriptions for the transformation of a measured electromagnetic current intensity into a measured electrostatic one. As we will see, *Weber* and *Kohlrausch* obtained experimentally in 1856 that $c_W = 439450 \times 10^6$ mm/s, see footnote 27, Vol. 3, pp. 605 and 652. This corresponds to $\sqrt{2}$ times the light velocity in vacuum, c_L, so that we will write $c_W = \sqrt{2}\,c_L$. Previously it had been obtained $i_{ed}I_{ed} = i_{em}I_{em}$, $i_{ed} = \sqrt{2}\,i_{em}$ and $I_{ed} = I_{em}/\sqrt{2}$. This means that $i_{em}I_{em}$ corresponds to $i_{ed}\sqrt{2}\,|c_L|\,\sqrt{2}\,I_{mech}/4$. As $I_{mech} = 2I_{es}$, we obtain that $i_{em}I_{em}$ corresponds to $i_{em}|c_L|I_{es}$. The numerical value i_{em} must be multiplied with the value of light velocity (in the absolute system of units which is being utilized, here for example mm or m), in order to arrive at the numerical value in the electrostatic system. Forming the quotient of electrostatic and electromagnetic measured intensities of current, yields $i_{es}I_{es}/i_{em}I_{em} = c_L$ or $c_L i_{em}I_{em} = i_{es}I_{es}$. That is, $i_{es} = |c_L|i_{em}$. When $i_{em} = 1$ and when we utilize the units of measure m, kg and s, then the numerical value of c_L is given by $|c_L| = 3 \times 10^8$. That is, one electromagnetic unit of current intensity corresponds to 3×10^8

electrostatic units of current intensity. If in this quotient instead of the current intensities we place the amounts of charge, then we obtain once more the light velocity, as the numerator and denominator need only to be multiplied by the unit of time. And this is exactly the assertion of the *Kohlrausch-Weber*'s experiment. Therefore in a conductor there must flow an enormous amount of electricity (as regards the relation to the electrostatic effects). As it is known today, they flow with a velocity of the order of millimeter per second.

Let us consider once more the dimension for the unit of the amount of electricity. The dimension for the measured electrostatic charge is $M^{1/2}L^{3/2}T^{-1}$, as obtained from *Coulomb*'s force combined with *Newton*'s second law of motion. The dimension for the measured electromagnetic charge is $M^{1/2}L^{1/2}$. It can be obtained from the formula for the current intensity obtained with a tangent galvanometer. Forming the quotient of the electrostatic and electromagnetic measured charges, yields the dimension of a velocity, LT^{-1}. The experiment gave for *Weber*'s constant $c_W = 439450 \times 10^6$ mm/s, see footnote 27, Vol. 3, pp. 605 and 652. This means that $c_L = c_W/\sqrt{2} = 3.1074 \times 10^8$ m/s.

In order to carry out the experimental determination of the mechanical unit of current intensity, *Weber* found in *Rudolf Kohlrausch* an appropriate collaborator. *Kohlrausch* possessed already a great experience in electrostatic measurements, a masterly skill. In 1853 he constructed the sine electrometer.[50] The experiment is here shortly described.

In a Leyden jar was gathered a great amount of electricity utilizing an electrical machine. The tension in the jar amounted to some 30000 Volts, as *Friedrich Kohlrausch* (son of *Rudolf Kohlrausch*) later on calculated, see the introduction and notes of which he wrote to this experiment in footnote 50, pp. 132–141, esp. p. 139. The Leyden jar was discharged through a tangent galvanometer, which had here the function of a ballistic galvanometer (in this way it was measured the amount of electricity in electromagnetic measure). For the electrostatic determination of the amount of charge which later on would flow through the windings of the galvanometer, *Kohlrausch* proceeded by steps. He utilized a great sphere of known diameter covered with tinfoil, his sine electrometer and a Coulombian torsion balance. In the first place the tinfoil sphere was charged by putting it in touch for a short time with the knob of a Leyden jar. With the sine electrometer it was determined the relation between the charges in the Leyden jar and in the tinfoil sphere. The obtained relation was approximately 30:1. The stationary sphere of the Coulombian torsion balance was put in touch with the tinfoil sphere, so that the electric charge was divided between these spheres according to the relation of the radii of the spheres. After that the mobile sphere of the torsion balance was similarly charged. The repulsive force of both spheres in the torsion balance was determined by the torsion. In this way it was possible to find the amount of electricity remaining in the Leyden jar which flowed through the tangent galvanometer. Here we will not discuss the several experimental details and the supplementary theoretical refinements for the improvement of the experiment. The final result which they obtained was that $c_W = 439450 \times 10^6$ mm/s, that is, $c_W/\sqrt{2} = 3.1074 \times 10^8$ m/s, which is essentially the same value as light velocity in vacuum, see footnote 27, Vol. 3, pp. 605 and 652.

50 *W. Weber* and *R. Kohlrausch*: Über die Einführung absoluter elektrischer Maße. In S. Balke, H. Gericke, W. Hartner, G. Kerstein, F. Klemm, A. Portmann, H. Schimank, and K. Vogel, editors, Ostwalds Klassiker der exakten Wissenschaften, new series, Vol. 5. Friedrich-Vieweg & Sohn, Braunschweig, 1968. Commented by F. Kohlrausch and K. H. Wiederkehr. See especially pp. 14–18: Rudolf Kohlrausch, Leben und Wirken.

This experiment was a masterpiece for its time. The experiments which were carried out later on with the same goal (the physicists spoken here from a "v"-determination) were hardly more precise, see footnote 47, especially p. 264 and footnote 33, p. 755.

This experiment essentially completed the works of *Gauss* and *Weber* of setting up the absolute system of units for magnetic and electromagnetic magnitudes.

Kirchhoff was the first to recognize the connection between this measurement and the light velocity. In his theoretical result published in 1857 he obtained that the propagation velocity of an electric perturbation in a wire of negligible resistivity would be given by the experimental value of the ratio of electrostatic and electromagnetic units of current.[51] In this paper he wrote (our words in square brackets):[52] "The velocity of propagation of an electric wave is here found to be $= c/\sqrt{2}$ [$= c_W/\sqrt{2} = 3.1 \times 10^8$ m/s], hence it is independent of the cross section, of the conductivity of the wire, also, finally, of the density of the electricity: its value is 41950 German miles in a second, hence very nearly equal to the velocity of light *in vacuo*."

Also *B. Riemann* and *J. C. Maxwell* interpreted the quotient as the light velocity in vacuum. For *Maxwell* the outcome of the experiment by *Kohlrausch* and *Weber* of 1855 was one of the main basis for the setting-up of his electromagnetic theory of light of 1861, see *Wiederkehr* (footnote 47), *Assis* (1994),[53] *d'Agostino* (footnote 6) and *Assis* (2000).[54]

6. The Modern "Absolute" Units of the MKSA System

With *Faraday* and *Maxwell* happened in the second half of the XIXth century a change in the understanding of the electric and magnetic phenomena. Instead of the forces between charges and current elements, electric and magnetic field lines transforming into one another were introduced (field theory, theory of action by contact). Already at the beginning of the XIXth century, after an exchange of ideas with *Ampère*, *Faraday* had described circular forces or, more precisely, circular lines of magnetic force around a current carrying conductor, see *Williams* (1965),[55] (1966),[56] (1985)[57] and *Wiederkehr* (1991).[58] As an integral part of these conceptions, the electric and magnetic field constants ε_0 and μ_0 (vacuum permittivity and permeability) were introduced. According to many physicists the special character of the electric phenomena could not be well represented by the system of three basic units of *Gauss* and *Weber* with their basic mechanic units. An additional specific electric basic unit was

51 *G. Kirchhoff*: Ueber die Bewegung der Elektricität in Leitern. Annalen der Physik, 102:529–544, 1857. Reprinted in G. Kirchhoff's Gesammelte Abhandlungen (Barth, Leipzig, 1882), p. 154–168.

52 *G. Kirchhoff*: On the motion of electricity in wires. Philosophical Magazine, 13:393–412, 1857.

53 *A. K. T. Assis*: Weber's Electrodynamics. Kluwer Academic Publishers, Dordrecht, 1994. ISBN: 0-7923-3137-0. See especially Section 3.1.

54 *A. K. T. Assis*: The meaning of the constant c in Weber's electrodynamics. In R. Monti, editor, Proc. of the Int. Conf. Galileo Back in Italy II, p. 23–36, Bologna, 2000. Soc. Ed. Andromeda.

55 *L. P. Williams*: Michael Faraday – A Biography. Chapman & Hall, London, 1965.

56 *L. P. Williams*: The Origins of Field Theory. Random House, New York, 1966.

57 *L. P. Williams*: Faraday and Ampère: a critical dialogue. In D. Gooding and F. A. J. L. James, editors, Faraday Rediscovered, p. 83–104. Stockton Press, New York, 1985.

58 *K. H. Wiederkehr*: Faradays Feldkonzept und Hans Christian Oersted. Physikalische Blätter, 47:825–830, 1991.

introduced. In the development of a system with four basic units worked the Italian engineer *Giovanni Giorgi* (1871–1950) years before the first world war. He is considered as the creator of the MKSA-System (meter, kilogram, second, Ampère), see footnote 34, pp. 218–225. After the electrical international congress of 1881, in which appropriate powers of 10 had been accepted in the absolute electromagnetic system, several suggestions were presented in the following international congresses for the realization of the important units of electromotive force, current and resistance (normal element, silver voltameter, mercury column). However it was shown that the magnitudes defined by these means didn't coincide any longer with the mechanical magnitudes. So, for instance, the mechanical unit of work, Joule or Newton times meter, didn't coincide anymore with the electrical unit of work, Volt times Ampère times second, as it should be according to the creators of the absolute triple system of units. The impulse to introduce an "absolute" electric unit came from the Americans during the twenties of last century, in order to obtain once more a compatibility with the mechanical units. This was obtained with the MKSA-System, also called *Giorgi* System of units. In this system the "absolute" (absolute only in the sense of the system with four basic units) Ampère was defined utilizing the ponderomotive force between two current carrying conductors: the Ampère is defined as the current that, separated in two parallel conductors by a distance of one meter, results in a force on each conductor of 2×10^{-7} N per meter of length of each conductor. In this definition it appears notably the factor 2 and not the factor 1 as with *Gauss* and *Weber*. The reason for this is the presence of *Weber*'s electromagnetic unit of current intensity, as we have seen earlier. The four basic units of the MKSA-System are a part of the International System of Units adopted during the 11th General Conference for Measures and Weight of 1960. The dimensions of the magnitudes in the *Giorgi*-System are integral powers of the basic units. In the old triple system there were also fractional units.

As a tribute to *Weber*'s pioneering work the unit of magnetic flux in the International System of Units is called "Weber" (abbreviated Wb). The suggestion for this was first made by *R. Clausius* in 1882 and finally adopted in 1935 after the meeting of the International Electrotechnical Commission, see footnote 2, pp. 135–137.

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Abstract

We present the development of the absolute system of units concentrating on the fundamental works of C. F. Gauss and W. Weber. A greater emphasis is given to the different units of electric current due to their central role in this development.

Zusammenfassung

Zu Beginn wird ein kurzer historischer Rückblick gegeben über die Entwicklung der drei Grundbegriffe Spannung, Strom und Widerstand. Mit der Entdeckung des Elektromagnetismus war eine praktikable Möglichkeit zur Messung der elektrischen Stromstärke gegeben.

Gestreift werden auch die bahnbrechenden Arbeiten von Ampère, sein „Eletrodynamisches Fundamentalgesetz" und seine These, alle magnetischen Erscheinungen auf das Fließen von Elektrizität zurückzuführen. Grundlage und aller so genannten absoluten Messungen und Maßsysteme ist die „Intensitas" (1832) von Gauss. Die magnetische Feldstärke wurde mit den mechanischen Grundgrößen Masse, Länge und Zeit erfaßt, und das Dreier-System für magnetische Größen geschaffen. Ausgangspunkt waren die erdmagnetischen Messungen mit den Gauss'schen Hauptlagen. Auch die nötigen Meßgeräte wurden dazu konstruiert. Experimentelle Unterstützung erhielt Gauss von dem jungen Wilhelm Weber. Er nahm ganz die Ideen zu absoluten Maßsystemen von Gauss auf, und es war ein wesentlicher Teil seines Lebenswerkes, diese Ideen mit der Schaffung des absoluten elektromagnetischen und absoluten elektrodynamischen Maßsystems in die Tat umgesetzt zu haben. In der Zeit des Göttinger „Magnetischen Vereins" definierte Weber bereits die absolute elektromagnetische Stromstärkeeinheit (1842). Als Meßinstrument wurde die Tangentenbussole benutzt. Gauss und Weber erreichten eine Meßgenauigkeit, wie sie atronomische Beobachtungen besaßen. Nach seiner Entlassung als Einer der Göttinger Sieben fand W. Weber in Leipzig als Nachfolger von G. Th. Fechner eine neue Stelle als Ordinarius für Physik. Hier schrieb er seine Arbeit über das „Allgemeine Grundgesetz der elektrischen Wirkung"; (1846, Fernwirkungsgesetz). Es war die erste Abhandlung in der Reihe „Elektrodynamische Maassbestimmungen". Zusammen mit Franz Neumann prägte für die nächsten drei Jahrzehnte Weber die Elektrodynamik auf dem Festland. Mit seinem Elektrodynamometer bewies Weber direkt die Gültigkeit des Ampèreschen Fundamentalgesetzes. Für das absolute elektrodynamische Maßsystem gab Weber den Zusammenhang mit dem absoluten elektromagnetischen Maßsystem. Die Definition des Ampere als SI-Einheit geht auf Webers elektrodynamische Einheit zurück, und der sonst nicht übliche Faktor 2 bei der Definition wird so verständlich.

Auf dem Elektrikerkongreß 1881 in Paris adoptierte man Webers absolute elektromagnetische System mit geeigneten Zehnerpotenzen der Maßgrößen. Als die Nahwirkungstheorie (Feldphysik) die Elektrodynamik Weberscher Prägung ablöste, fügte man den drei mechanischen Grundgrößen noch eine vierte, typisch elektrische Grundgröße, das Ampere hinzu (MKSA-System). Leider führten die unterschiedlichen Verwirklichungen des Ampere in einigen Staaten zu, wenn auch geringen Ungenauigkeiten.. Dies wurde durch die heutige absolute SI-Einheit beseitigt. Man ging dabei wieder auf Gauss zurück, damit das mechanische und das elektrische Energiemaß wieder genau übereinstimmten.

Gauss und Weber führten auch das absolute elektrostatische Maßsystem ein. Zusammen mit R. Kohlrausch bestimmte W. Weber das Verhältnis von absolut elektrostatisch und absolut elektromagnetisch gemessener Elektrizitätsmenge. Benutzt wurde eine Coulombsche Drehwaage und eine Tangentenbussole als Stoßgalvanometer. Das Ergebnis bei der Quotientenbildung war die Lichtgeschwindigkeit, was sowohl die Zahlengröße als auch die Dimension betraf. Gauss und Weber schrieben hinter ihre Zahlenergebnisse noch keine Dimensionen. Aber dem Text kann man diese entnehmen. In dieser Arbeit sind zum besseren Verständnis immer die Dimensionen hinzugefügt. Auf W. Webers Vorstellung eines symmetrischen elektrischen Doppelstromes, den er von Fechner übernommen hatte, beruht sein „mechanisches Maß" der Stromstärke. Es war doppelt so groß wie die absolute elektrostatische Stromstärkeeinheit. Deswegen erhielten Weber und Kohlrausch nur die Hälfte der Lichtgeschwindigkeit und sie sahen auch nicht den Zusammenhang mit der

Lichtgeschwindigkeit, die damals schon recht genau bestimmt worden war. Anders dagegen J. Cl. Maxwell; er interpretierte den Quotienten sofort als Lichtgeschwindigkeit. Das Kohlrausch-Werbersche Ergebnis war für ihn dann eine der Hauptsäulen bei der Entwicklung der elektromagnetischen Lichttheorie.

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