Ampère's Electrodynamics

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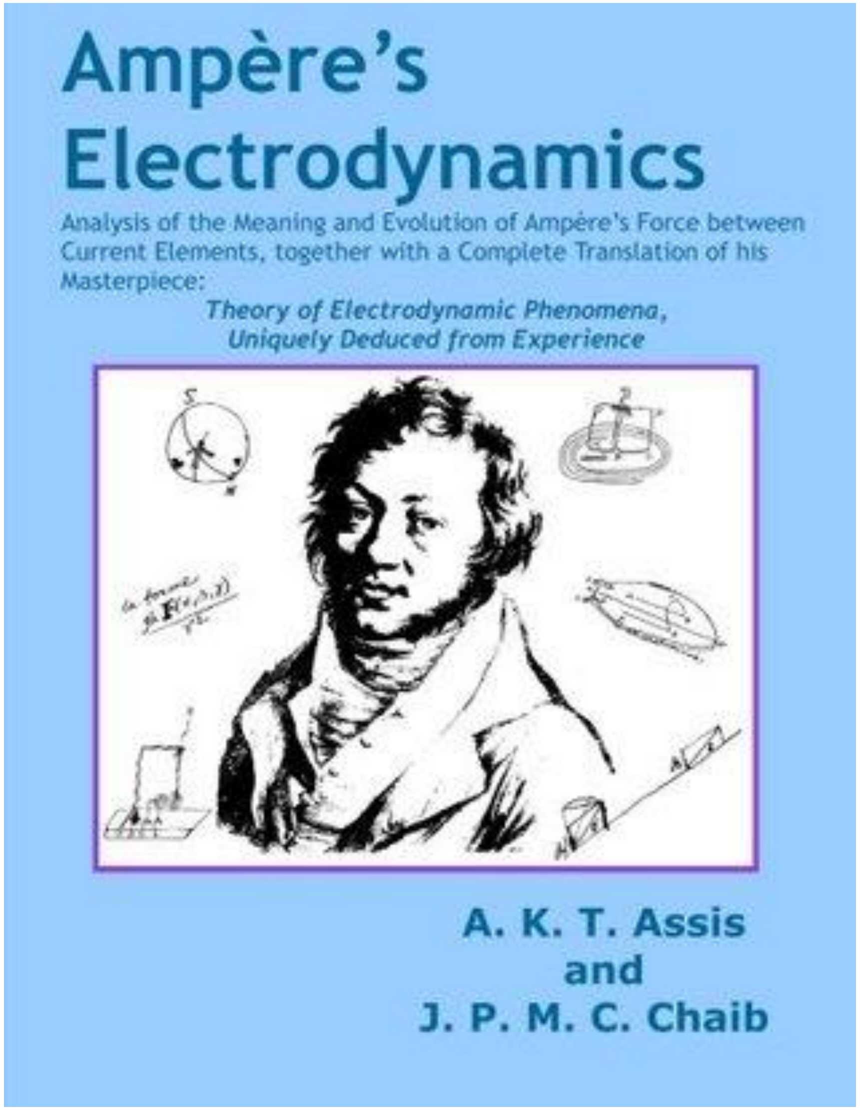

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Force laws:

Newton, gravitational masses (1687):

$$F \; \alpha \; \frac{m_1 m_2}{r^2}$$

Coulomb, electric charges (1785):

$$F \; \alpha \; - \frac{q_1 q_2}{r^2}$$

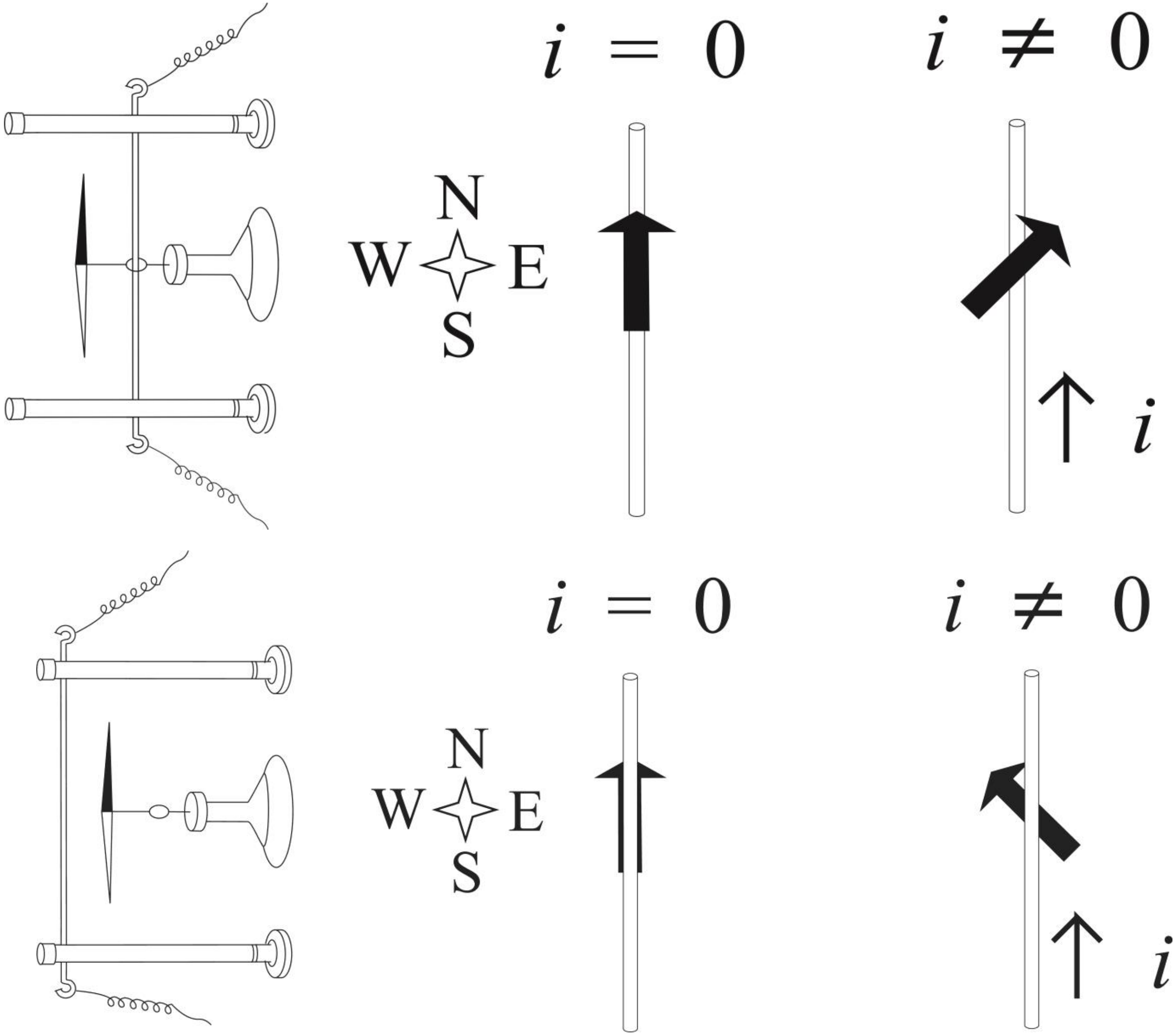
Coulomb, magnetic poles (1785):

$$F \; \alpha \; - \frac{p_1 p_2}{r^2}$$

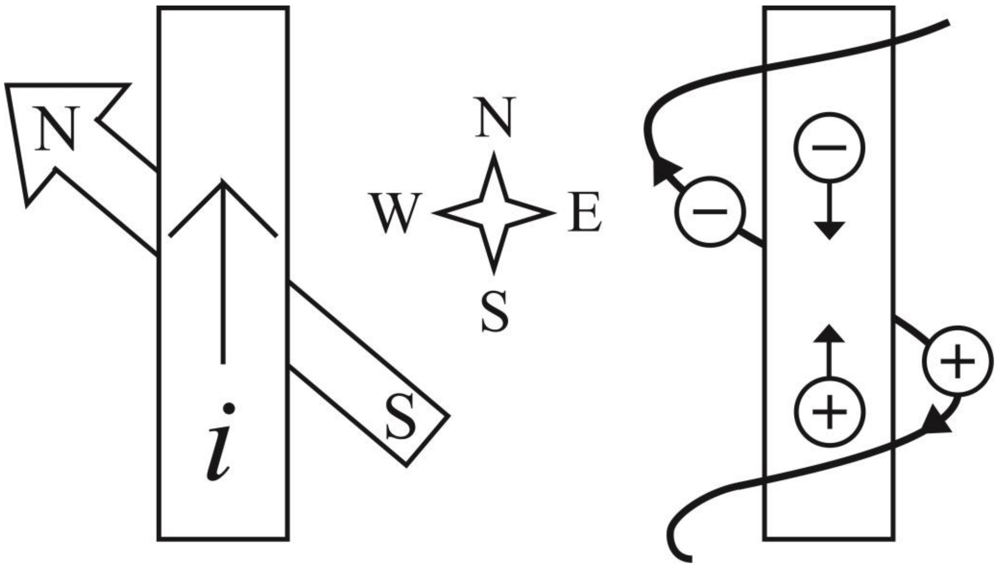
Ampère, electric currents (1822):

$$F \; \alpha \; \frac{i_1 d\ell_1 i_2 d\ell_2}{r^2}\left(\cos\varepsilon - \frac{3}{2}\cos\alpha\cos\beta\right)$$

Oersted's experiment (1820)

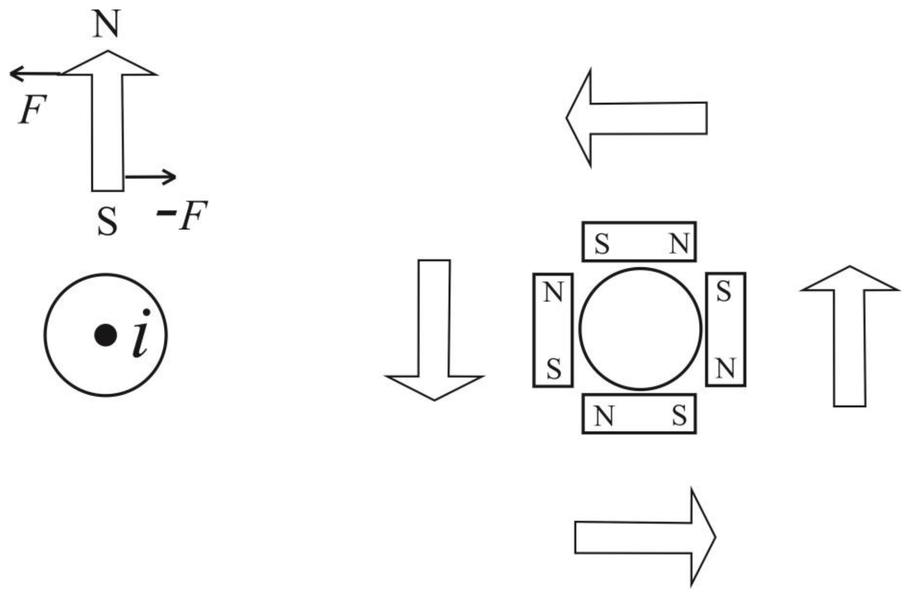


Oersted’s interpretation based on the helical flow of charges <u>outside</u> the current carrying wires.



“Negative electricity propels the North pole, but does not act on the South pole. Positive electricity propels the South pole, but does not act on the North pole.”

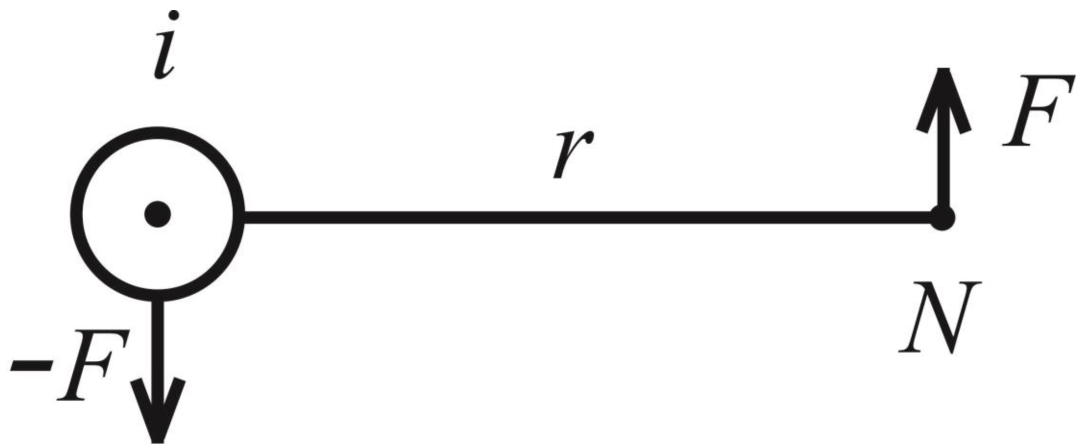
Biot and Savart’s interpretation (1820) of Oersted’s experiment:



“The wire is made magnetic by voltaic current.”

The microscopic transverse magnets at the surface of the wire would act on the external magnetized needle.

Faraday's interpretation (1821) of Oersted's experiment based on transverse forces between the current and the magnetic pole.



These transverse forces would generate an internal or primitive couple (torque) on the system.

In 1820 Ampère was 45 years old. Letter to his son from September 1820:

"I regret for not sending this letter three days ago, but all my time has been taken up by an important circumstance in my life. Ever since I heard for the first time about the discovery by M. Oersted, professor at Copenhagen, of the action of galvanic currents on the magnetized needle, I have been thinking continuously on this subject, and the only thing I have been doing is to write a great theory about this phenomenon and about all those phenomena already known about the magnet, and to perform experiments suggested by this theory, all of which have been successful and made me know several new facts."

In 1820, in order to interpret Oersted’s experiment, Ampère made two crucial and original hypotheses, which turned out to be extremely fruitful:

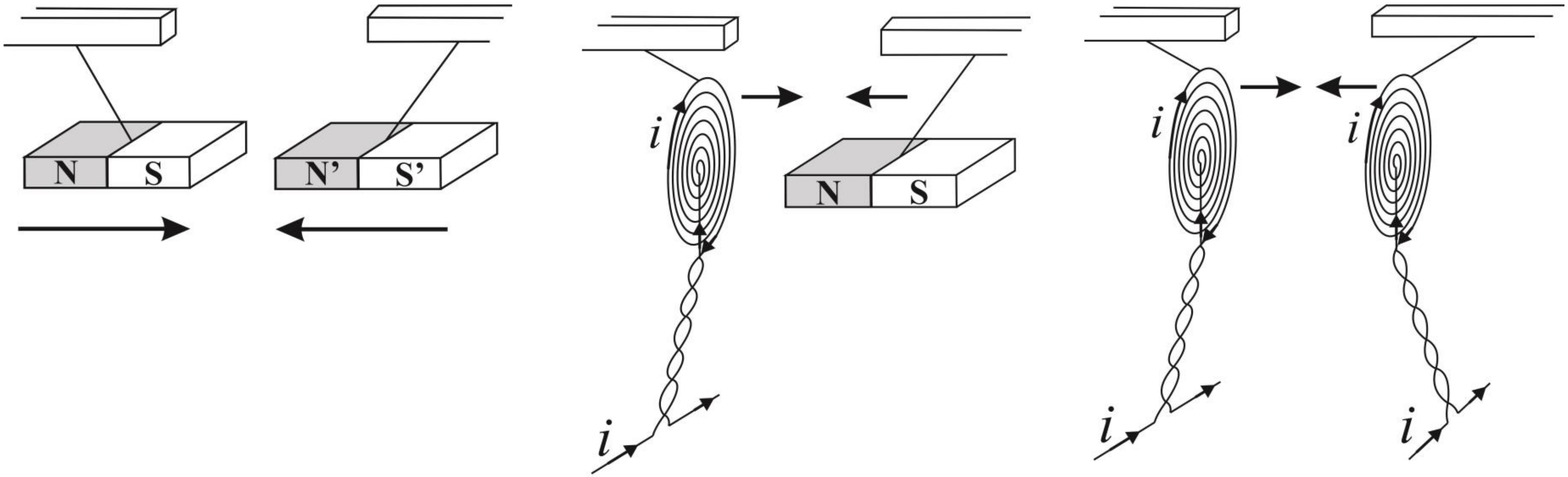
(I) He assumed the existence of forces of attraction and repulsion between current carrying conductors. Nobody had observed these forces before him.

(II) He assumed the existence of electric currents inside magnets and inside the Earth.

He then tried to model electrodynamically a magnet and the Earth by appropriate current carrying wires.

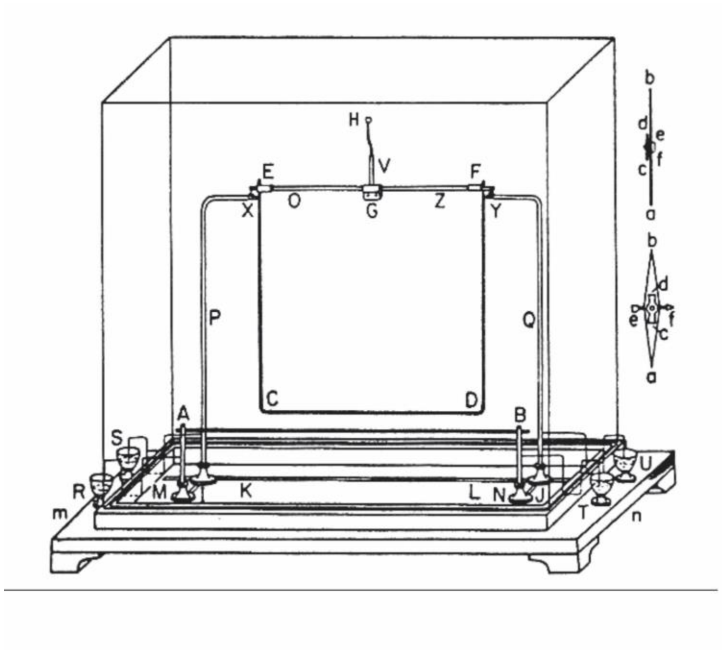
He also tried to find the force law acting between two current elements.

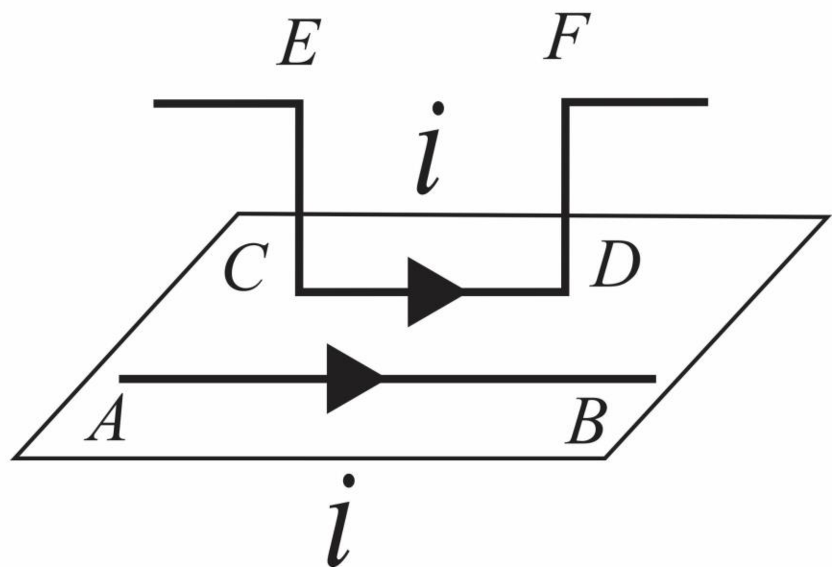
Ampère initially modelled a magnet as a spiral current loop:



"It was in this way that I discovered that two electric currents attract each other when they flow in the same direction and repel each other in the other case."

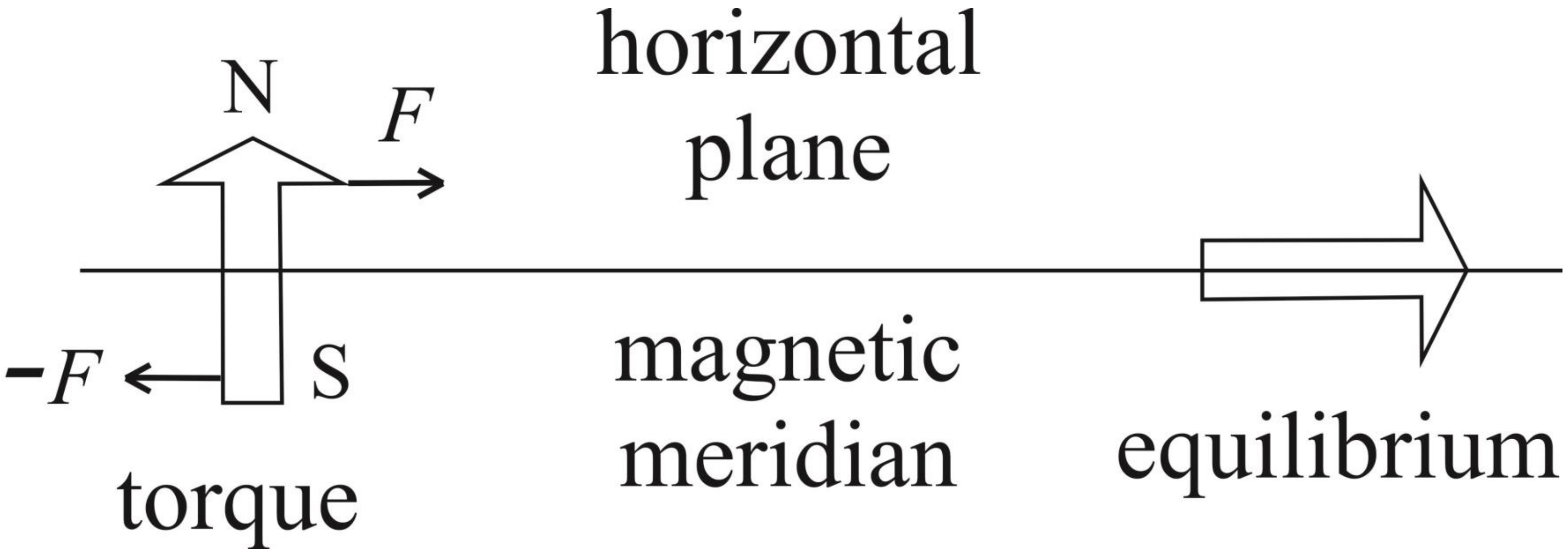
He then performed his most famous experiment showing the attraction and repulsion between two parallel straight wires carrying steady currents.



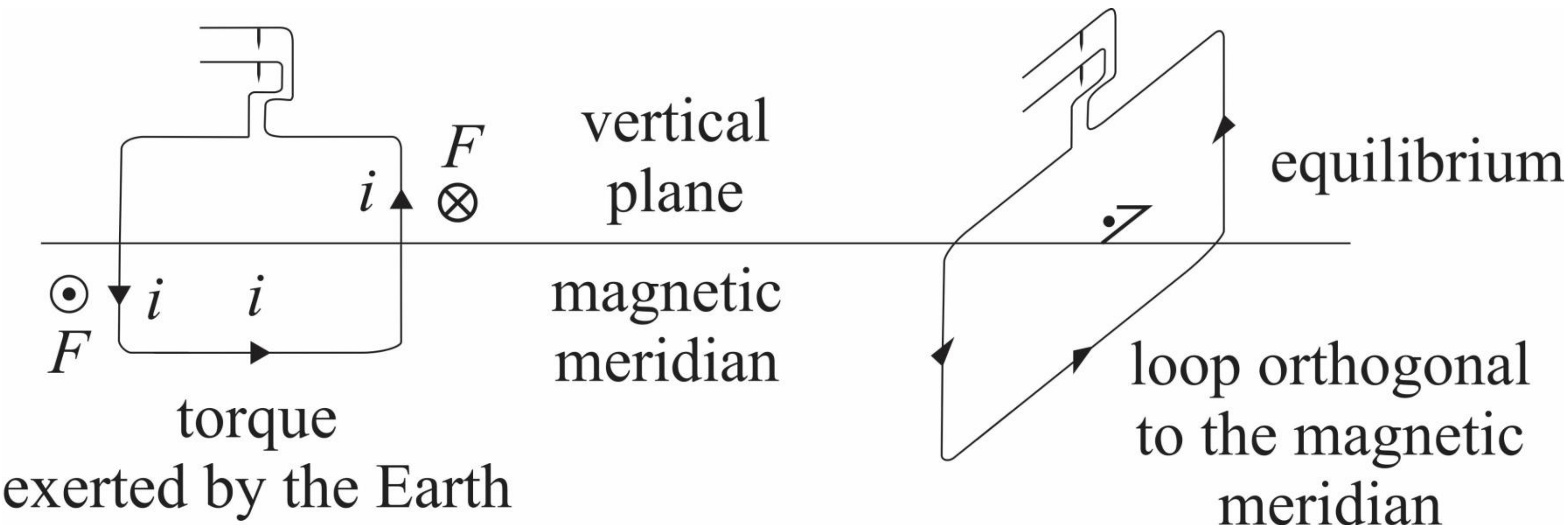


This device is the basis of the current balances available in all didactic physics laboratories.

Directive torque exerted magnetically by the Earth on a compass needle:



Ampère’s 1820 experiment showing an electrodynamic analogue of the directive torque exerted magnetically by the Earth on a compass needle. The current carrying loop can turn around its vertical axis of symmetry. The Earth exerts an electrodynamic torque on the current loop.



Ampère introduced the null method in physics.

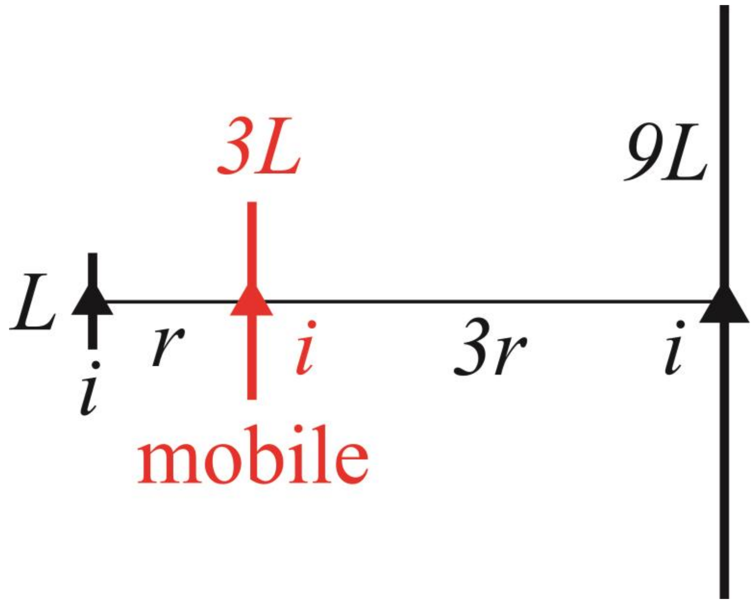
A mobile circuit is placed between other two circuits.

There are certain configurations in which equilibrium takes place (zero net force or zero net torque on the mobile circuit).

By analyzing these situations, he obtained in 1822 the final expression of his force law between two current elements.

$$F \;\alpha\; \frac{i_1 d\ell_1 i_2 d\ell_2}{r^n} f(\alpha, \beta, \varepsilon)$$

$$F_{left} \;\alpha\; \frac{L \cdot 3L}{r^n}$$



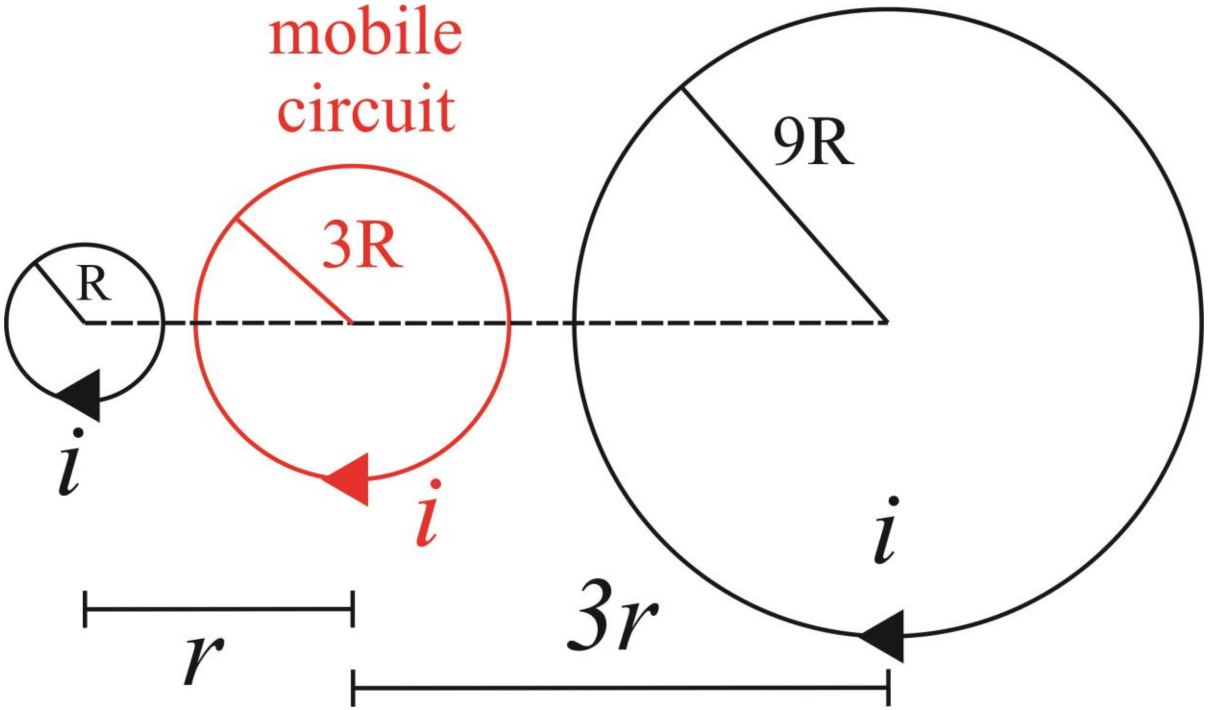
$$F_{right} \;\alpha\; \frac{3L \cdot 9L}{(3r)^n}$$

If $n < 2$ the mobile circuit moves to the right.

If $n = 2$ the mobile circuit remains at rest.

If $n > 2$ the mobile circuit moves to the left.

Case of equilibrium of the law of similarity:



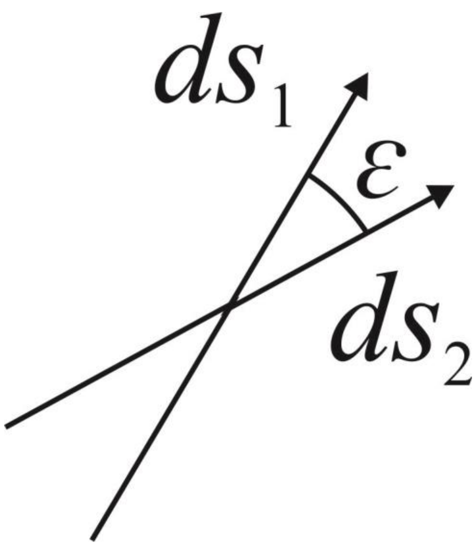
Experimental observation: The mobile circuit remains at rest.

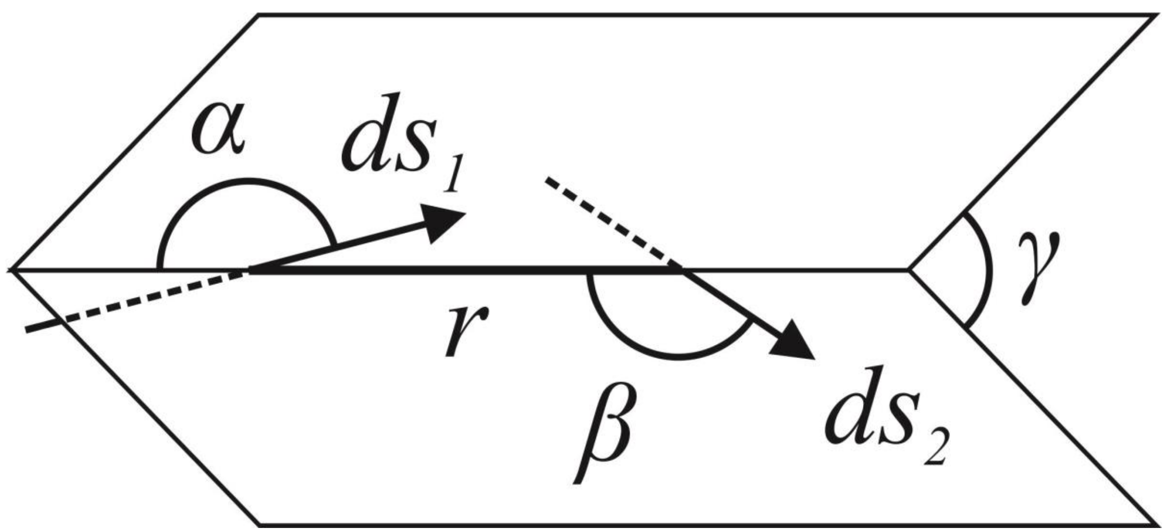
Ampère’s conclusion: The force between two current elements is inversely proportional to the square of their distance.

Ampère's final force law (1822)

$$F \; \alpha \; \frac{i_1 d\ell_1 i_2 d\ell_2}{r^2}\left(\sin\alpha\sin\beta\cos\gamma - \frac{1}{2}\cos\alpha\cos\beta\right)$$

$$= \frac{i_1 d\ell_1 i_2 d\ell_2}{r^2}\left(\cos\varepsilon - \frac{3}{2}\cos\alpha\cos\beta\right)$$

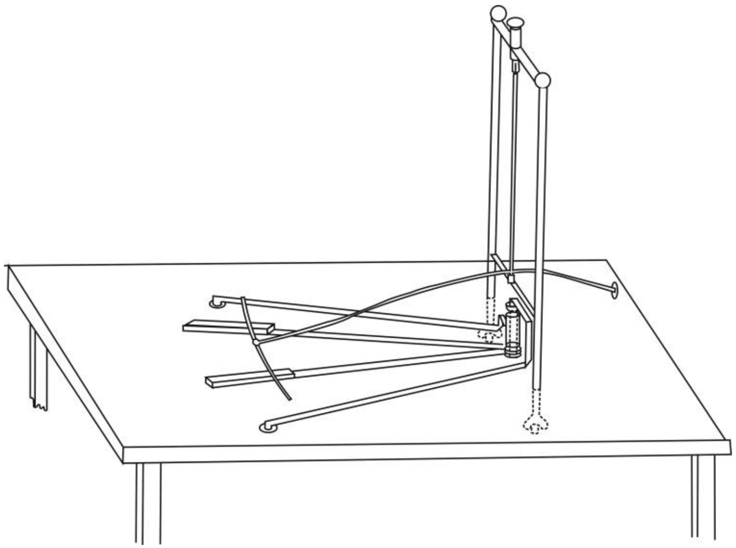


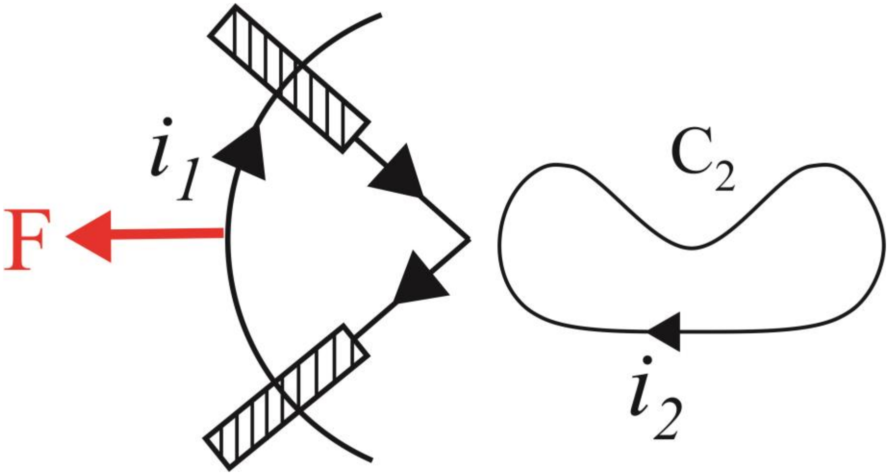


Ampère’s force law in modern vectorial notation and in the International System of Units MKSA:

$$F^{A}_{2\,\mathrm{in}\,1} = -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}\left[2(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - 3(\hat{r}\cdot d\vec{\ell}_1)(\hat{r}\cdot d\vec{\ell}_2)\hat{r}\right]$$

Experimental case of equilibrium of the nonexistence of tangential force:

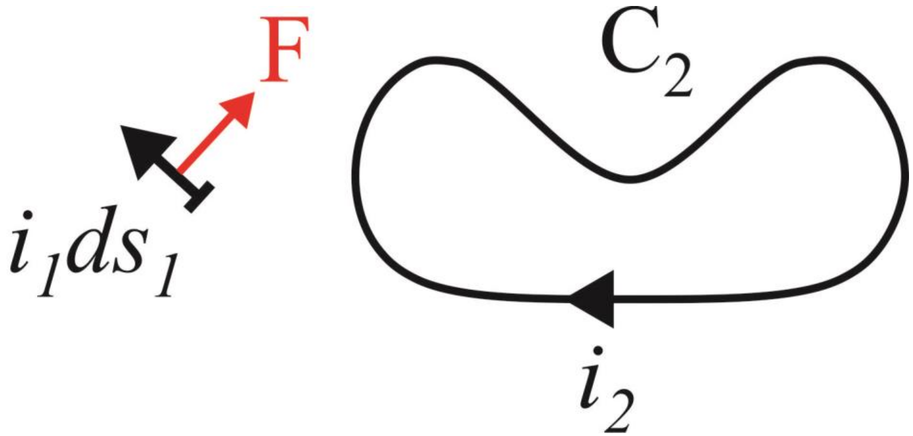




Ampère’s conclusion:

The force exerted by a closed circuit 2 of arbitrary shape acting on an external current element 1 of another circuit is always orthogonal to the direction of this element.

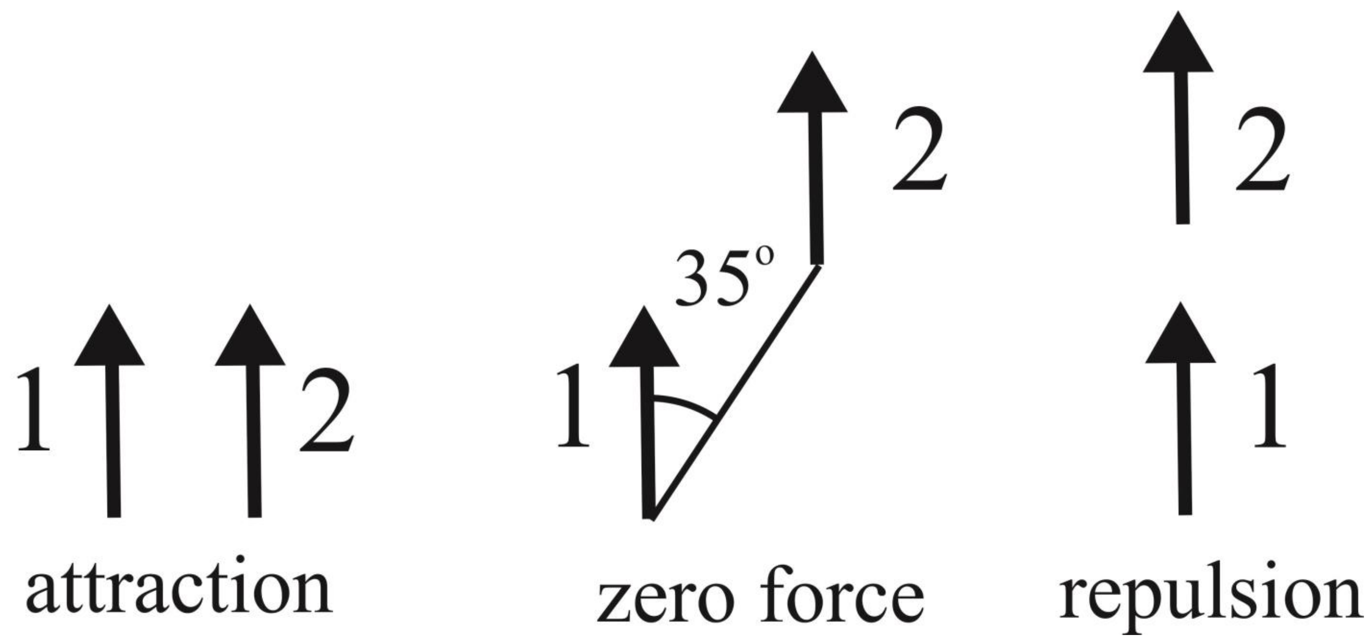
Ampère also proved theoretically that the force exerted by a closed circuit of arbitrary shape acting on an external current element of another circuit is always orthogonal to this element:



$$F^{A}_{2\,\mathrm{in}\,1} = -\oint \frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}\left[2(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - 3(\hat{r}\cdot d\vec{\ell}_1)(\hat{r}\cdot d\vec{\ell}_2)\hat{r}\right]$$

$$= I_1 d\vec{\ell}_1 \times \left(\frac{\mu_0}{4\pi}\oint \frac{I_2 d\vec{\ell}_2 \times \hat{r}}{r^2} \right)$$

Grassmann in 1845 didn't like the following property of Ampère's force:



Ampère had proven that the force exerted by a closed circuit of arbitrary shape acting on an external current element is always orthogonal to this element:

$$F^{A}_{C\,2\,\mathrm{in}\,1} = I_1 d\vec{\ell}_1 \times \left(\frac{\mu_0}{4\pi} \oint \frac{I_2 d\vec{\ell}_2 \times \hat{r}}{r^2} \right)$$

Grassmann then assumed that this expression should be valid for the force between current elements. In this way he created in 1845, theoretically (he never performed any experiment), his force law:

$$F^{G}_{2\,\mathrm{in}\,1} = I_1 d\vec{\ell}_1 \times \left(\frac{\mu_0}{4\pi} \frac{I_2 d\vec{\ell}_2 \times \hat{r}}{r^2} \right)$$

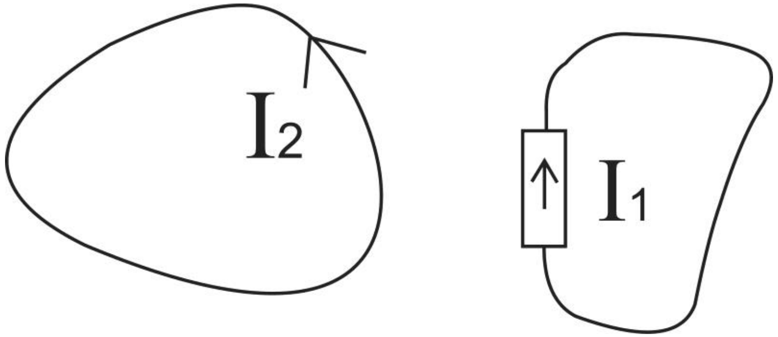
Differences between Ampère (1822-26) and Grassmann's (1845) force laws:

$$\vec{F}^{A}_{2\,\text{in}\,1} = -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}\left[2(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - 3(\hat{r}\cdot d\vec{\ell}_1)(\hat{r}\cdot d\vec{\ell}_2)\hat{r}\right]$$

$$\vec{F}^{G}_{2\,\text{in}\,1} = I d\vec{\ell}_1 \times d\vec{B}_2 = I_1 d\vec{\ell}_1 \times \left(\frac{\mu_0}{4\pi}\frac{I_2 d\vec{\ell}_2 \times \hat{r}}{r^2}\right)$$

$$= -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}\left[(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - (d\vec{\ell}_1 \cdot \hat{r})d\vec{\ell}_2\right]$$

Equivalence between Ampère and Grassmann's forces:



$$F^{A}_{C2\,\text{in ds}1} = F^{G}_{C2\,\text{in ds}1} = I_1 d\vec{\ell}_1 \times \left(\frac{\mu_0}{4\pi} \oint \frac{I_2 d\vec{\ell}_2 \times \hat{r}}{r^2} \right)$$

Therefore, we cannot distinguish these two forces by performing experiments with two or more closed circuits.

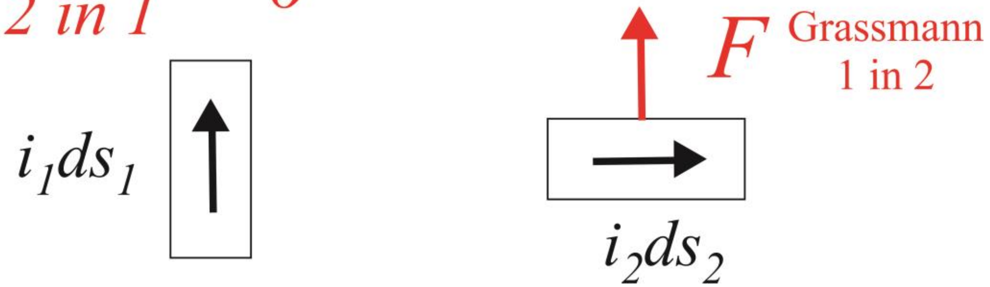
Ampère versus Grassmann with orthogonal current elements:

with Ampère's force:

$$F_{1\ in\ 2} = F_{2\ in\ 1} = 0$$

with Grassmann's force:

$$F_{2\ in\ 1} = 0$$



Therefore, Grassmann's force does not comply with Newton's action and reaction law.

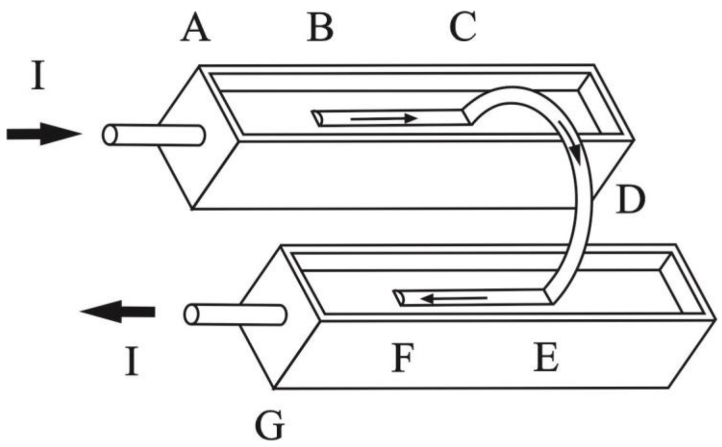
Ampère versus Grassmann with parallel current elements:

with Grassmann's force:

$$F_{1\ in\ 2} = F_{2\ in\ 1} = 0$$

Ampère X Grassmann:

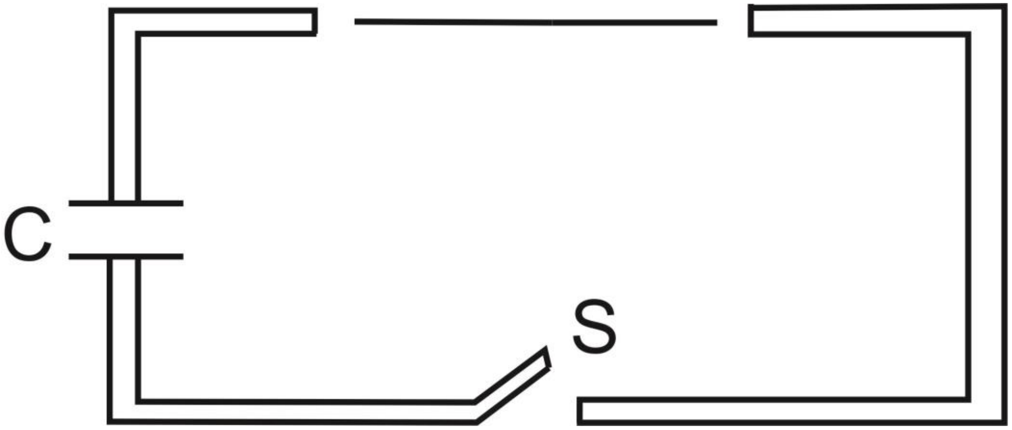
Ampère was so surprised with this result, that in 1822 he devised an experiment to test this prediction:

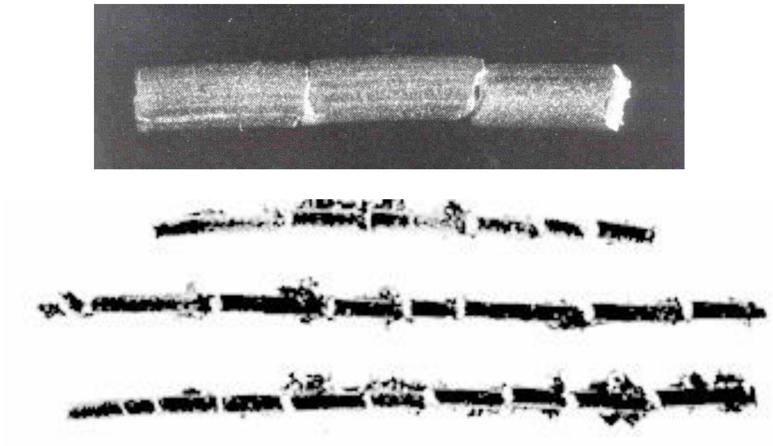


According to Ampère, the observed force on the bridge pointing towards the right is due to the repulsion between portions AB and BC, together with the repulsion between portions EF and FG.

These repulsions don't exist with Grassmann's force.
In order to explain this experiment with Grassmann's force, it is necessary to consider the force exerted by the bridge on itself.

Ampère versus Grassmann with the exploding wire phenomenon:





Peter and Neal Graneau explain the wire fragmentation when we close the circuit as being due to Ampère’s tension.
That is, Ampère’s repulsive forces acting between parallel and collinear current carrying elements.

Tricker, Early Electrodynamics (1965):

"In the theory of gravitation, Newton was already provided withe a knowledge of a range of the phenomena, mainly through the medium of Kepler's laws.

Ampère had to discover the laws as well as provide the theory, and thus do the work of Tycho Brahe, Kepler and Newton rolled into one."

Maxwell in 1873 when comparing the forces between current elements of Ampère (1826), Grassmann (1845) and two other expressions created by Maxwell himself:

Treatise on Electricity and Magnetism, Article 527:

"Of these four different assumptions that of Ampère is undoubtedly the best, since it is the only one which makes the forces on the two elements not only equal and opposite, but in the straight line which joins them."

Maxwell's general assessment of Ampère's work:

Treatise on Electricity and Magnetism, Article 528:

"The experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science. The whole, theory and experiment, seems as if it had leaped, full grown and full armed, from the brain of the `Newton of electricity.' It is perfect in form, and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electro-dynamics."

$$F_{2\,\mathrm{in}\,1}^{A} = -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}\left[2(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - 3(\hat{r}\cdot d\vec{\ell}_1)(\hat{r}\cdot d\vec{\ell}_2)\hat{r}\right]$$

Despite these statements, Ampère's force between current elements disappeared from the textbooks.

We believe the main reason for this fact is that modern electromagnetic theory is based not only on Maxwell's equations, but also on Lorentz's force law which is compatible with Einstein's theory of relativity. Beginning with Lorentz's force we deduce only Grassmann's force, but not Ampère's force.

On the other hand, beginning with Weber's force we deduce only Ampère's force, but not Grassmann's force.

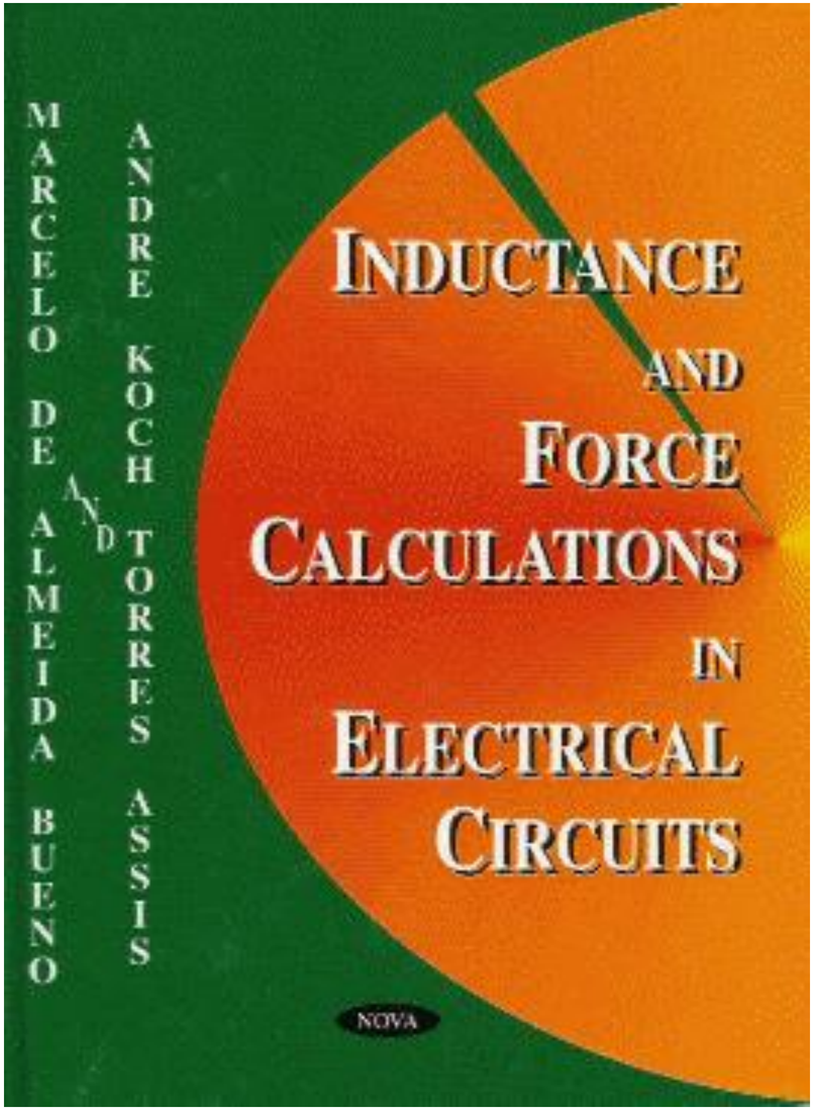
Lorentz (1895) ⟶ Grassmann (1845):

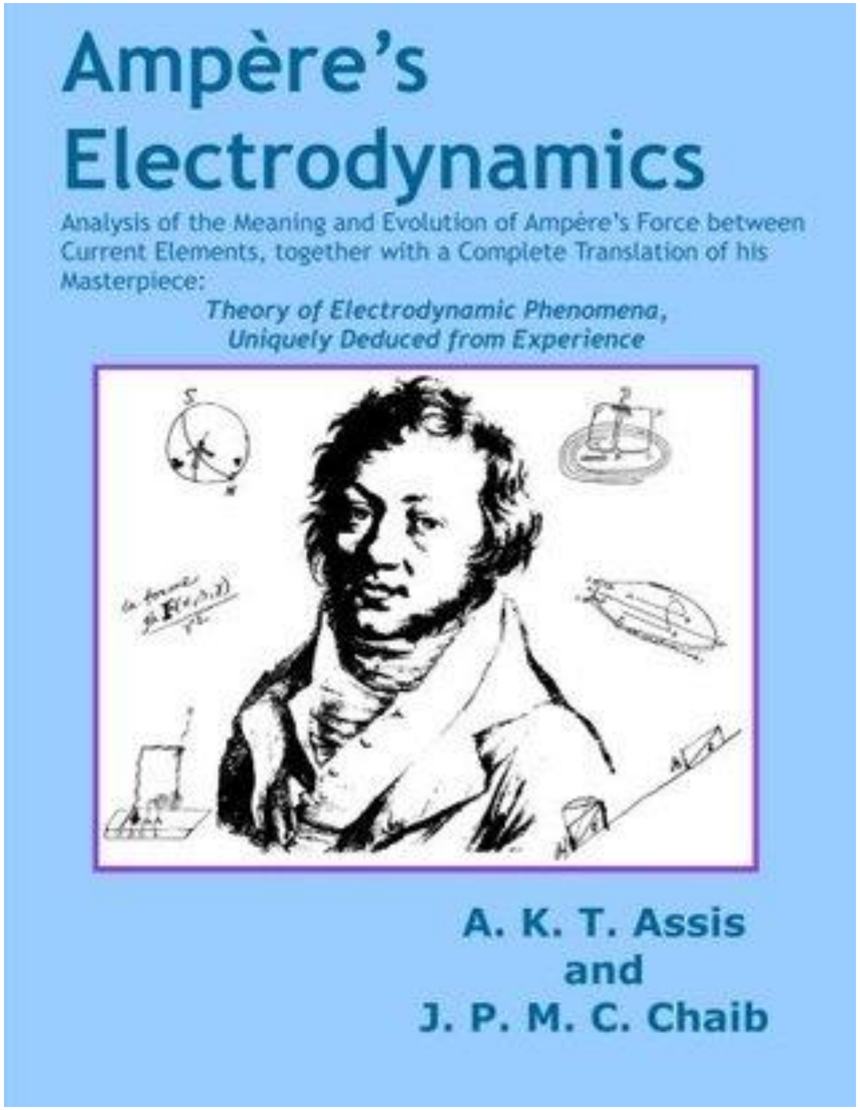
$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \quad \Rightarrow \quad \vec{F}^{G}_{2\,\text{in}\,1} = Id\vec{\ell}_1 \times d\vec{B}_2 = I_1 d\vec{\ell}_1 \times \left(\frac{\mu_0}{4\pi} \frac{I_2 d\vec{\ell}_2 \times \hat{r}}{r^2} \right)$$

Weber (1846) ⟶ Ampère (1822):

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2} \right)$$

$$F^{A}_{2\,\text{in}\,1} = -\frac{\mu_0}{4\pi} \frac{I_1 I_2}{r^2} \left[2(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - 3(\hat{r} \cdot d\vec{\ell}_1)(\hat{r} \cdot d\vec{\ell}_2)\hat{r} \right]$$





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Conclusion

Ampère's electrodynamics is extremely powerful.

In the last few years there has been a renewed interest in Ampère's electrodynamics due to novel experiments and new theoretical developments.

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