Empedocles, Newton, the Centrifugal Force and Their Bucket Experiments

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Abstract: We present some similar experiments presented by Empedocles (c. 490 - c. 430 B. C.) and Isaac Newton (1642-1727) when dealing with deep dynamical problems. In particular, we discuss how the centrifugal force acting on a rotating body can balance the gravitational attraction acting on it. We present their rotating bucket experiments illustrating the effects of the centrifugal force. We also discuss how Ernst Mach presented an interpretation different from the Newtonian one as regards the origin of this centrifugal force in rotational motions.

1 – Dedication

This paper is dedicated to the memory of Tomas E. Phipps Jr. (1925-2016). He was greatly interested in Mach's principle and the origin of inertia.

2 – Introduction

Empedocles (c. 490 - c. 430 B. C.) was a Greek pre-Socratic philosopher. He proposed the cosmogenic theory of the four elements (earth, water, air and fire). He also proposed forces called Love and Strife which would mix as well as separate the elements. We can draw modern analogies for these ideas. For instance, the 94 naturally occurring elements in the periodic table of the chemical elements. Likewise, we have attractive and repulsive forces of interaction acting between material bodies (electrostatic forces, for instance).

In this work we draw an analogy between two famous experiments with rotating buckets. One was made by Empedocles and the other by Isaac Newton (1642-1727).

3 – Empedocles's Bucket Experiment

Pierre Duhem (1861-1916) presented in Chapter VII (The Choice of Hypotheses) of his 1906 book *The Aim and Structure of Physical Theory* some old ideas related to dynamics, gravitation and rotational motion. In particular, he discussed the origins of the idea that the centrifugal force might balance the force of gravity in rotating systems of bodies:[1]

> Aristotle reported to us that Empedocles explained the stationary position of the earth by means of the rapid rotation of the heavens; "thus does it happen with water contained in a bucket which is being swung around; even when the bottom of the bucket is above the water, the water does not fall; the rotation prevents it from doing so."

Duhem was quoting Aristotle's work *On the Heavens*, Book II, Chapter 13, Lines [9-21], Section 295^a. Figure 1 illustrates this experiment.

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[1] [1, pp. 248-249].

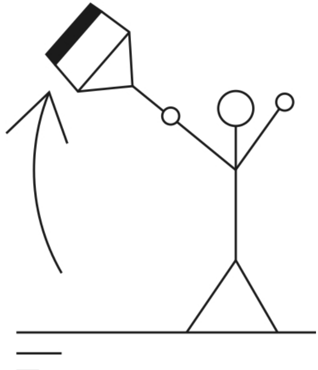

Figure 1: Empedocles's bucket experiment.

The English translation by J. L. Stocks runs as follows:[2]

> If, then, it is by constraint that the earth now keeps its place, the so-called [10] 'whirling' movement by which its parts came together at the centre was also constrained. (The form of causation supposed they all borrow from observations of liquids and of air, in which the larger and heavier bodies always move to the centre of the whirl. This is thought by all those who try to generate the heavens to explain why the earth came together [15] at the centre. They then seek a reason for its staying there; and some say in the manner explained, that the reason is its size and flatness, others, with Empedocles, that the motion of the heavens, moving about it at a higher speed, prevents movement of the earth, as the [20] water in a cup, when the cup is given a circular motion, though it is often underneath the bronze, is for this same reason prevented from moving with the downward movement which is natural to it.)

Instead of "bucket" or "cup", Tigner preferred the expression "ladle":[3]

> ...if it is by constraint that the earth now remains where it is, also [by constraint] it [originally] came together, being carried to the middle by the vortex. (For this they all agree is the cause, arguing from what happens in liquids and in air, where the larger and heavier things always

[2] [2, p. 386].
[3] [3].

> travel to the middle of a vortex. Wherefore all those who generate the heaven [i.e., offer generative accounts of it] say the earth came together in the middle. Then they seek the cause of its remaining there. And some [e.g., Anaxagoras] say, in the manner explained above, that its flatness and extent are the cause; others, like Empedocles, say that the motion of the heaven, revolving in a circle and going so fast, prevents motion of the earth, like the liquid in ladles; for when liquid is swung round in a circle in a ladle [by the long handle], it often comes below the bronze, yet does not fall down, as is its nature to do, for the same reason.)

He also presented a picture of a bronze ladle from the fifth century B.C., similar to our Figure 2. Tigner even performed the experiment of twirling a ladle full of water without spilling, provided it was allowed to swing freely.

Figure 2: Representation of a ladle from the fifth century B.C.

Tigner presented arguments according to which Empedocles's experiment was not intended to explain why the earth remains in its place. Probably the demonstration would seem more apt to explain why the heavenly bodies do not fall towards the earth, assuming they were heavy as any rock. Tigner quoted another passage of *On the Heavens* supporting this point of view, Book II, Chapter 1, Lines [18-26], Section 284[a], namely:[4]

> Wherefore neither must one agree with the tale of the ancients; they say that it [the heaven] stands in the need of maintenance of some Atlas. And

[4] [3].

> in this respect those who contrived the account seem to have the same notion as those later: for, [conceiving] all of the upper bodies as having weight and being earthy, they made animate necessity a support for it, in the mythical way. But neither must we conceive it in this way, nor again [such that,] because of the rotation, so quick a motion befalls them that they are maintained during all this time against their proper weight, as Empedocles says.

Stocks's translation of Aristotle's text runs as follows:[5]

> Hence we must not believe the old tale which says [20] that the world needs some Atlas to keep it safe–a tale composed, it would seem, by men who, like later thinkers, conceived of all the upper bodies as earthy and endowed with weight, and therefore supported it in their fabulous way upon animate necessity. We must no more believe that than follow Empedocles when he says that the world, by being whirled [25] round, received a movement quick enough to overpower its own downward tendency, and thus has been kept from destruction all this time.

We agree with Tigner on this interpretation of the meaning of Empedocles's bucket experiment. When the bucket spins fast enough, the water does not fall down. Likewise, if the heavenly bodies are rotating fast enough around the earth, they would not decrease their distance to the center of the earth, despite their gravitational attraction.

A rock released above the earth falls to the ground. The same happens with water or any other heavy material. In Empedocles's bucket experiment, on the other hand, the water in the spinning bucket does not fall, provided the bucket spins fast enough relative to the ground. That is, the outward centrifugal force balances the downward force of gravity. Likewise, if the heavenly bodies are endowed with weight, they should fall towards the ground if released at rest relative to the ground, just like a free falling apple. However, if they move fast enough tangentially

[5] [2, pp. 375-376].

around the earth, the centrifugal force might keep them at a constant distance from the earth.

4 – Rotational Motion in Newtonian Mechanics

Newton's second law of motion in modern vector notation and in the International System of Units is given by $\vec{F} = m\vec{a}$. Here $\vec{F}$ is the net force acting on the body of inertial mass m which is moving relative to an inertial frame of reference with an acceleration $\vec{a}$. When it describes a circular orbit of radius r and velocity $\vec{v}$, its centripetal acceleration is given by $\vec{a} = -v^2\hat{r}/r$, where $v = |\vec{v}|$ is the magnitude of its velocity and $\hat{r} = \vec{r}/r$ is the unit vector pointing radially away from the center of the orbit along the position $\vec{r}$ of the particle.

4.1 – Lowest Velocity of a Rotating Bucket

Newton's law of universal gravitation acting between particles of mass m_1 and m_2 separated by a distance r is given by $\vec{F} = Gm_1m_2\hat{r}/r^2$. This force is attractive. Here $G = 6.67\times10^{-11}m^3/(kgs^2)$ is the constant of universal gravitation. Suppose body 1 is a free falling apple and body 2 is the Earth with mass $m_2 = 5.98\times10^{24}kg$. Let $R = 6.37\times10^6 m$ be the Earth's radius. Combining Newton's second law of motion, $F = m_1a$, with his law of universal gravitation, $F = Gm_1m_2/R^2$, yields the free fall acceleration of an apple as given by $a = g = Gm_2/R^2 = 9.8m/s^2$.

We now consider Empedocles's bucket experiment according to Newtonian mechanics. When the bucket is describing a circular orbit of radius r and velocity v in a vertical plane close to the Earth's surface we have $F = mv^2 / r$. Here F is the magnitude of the centripetal force acting on the body of mass m in each point of the orbit. Consider the highest point of the orbit. The lowest velocity with which the body can move at this point is such that the only force acting on it is the downward weight of the body. Equating the free fall acceleration g with the centripetal acceleration v^2 / r at this point yields: $mg = mv^2 / r$. That is, $v = \sqrt{gr} = \sqrt{9.8r}$. Supposing the distance r between the center of the circular orbit and the bottom of the bucket to be $r = 1m$ yields the lowest velocity at the top of the circular motion as given by $v = \sqrt{9.8} = 3.1m / s$. If the bucket were moving with a smaller velocity, like $2m / s$, water would fall down of the bucket before it reached the highest point. If the bucket were moving with a higher velocity, like $4m / s$, then the water would press the bottom of the bucket at the highest point of the orbit.

4.2 – A Geostationary Apple

We now present a quantitative calculation of a centrifugal force balancing the attractive gravitational force acting on a spinning body according to modern Newtonian mechanics, in order to quantify Empedocles's insight with modern knowledge.

When we release an apple of mass m at the surface of the Earth, it falls freely towards the ground with an acceleration of $9.8m / s^2$. However, an apple in the equatorial plane can remain at rest relative to the ground, geostationary. To this end it should be rotating together with the earth relative to the frame of distant stars with a period T of one sidereal day. Moreover, it must be located at a certain distance r to the center of the earth in such a way that its centripetal acceleration is given by the gravitational force per unit mass exerted by the earth of mass m_2. In Newtonian mechanics these conditions are given by $Gm_2 / r^2 = v^2 / r = 4\pi^2 r / T^2$, where $v = 2\pi\, r / T$ is the tangential velocity of the apple around the earth, relative to the frame of distant stars. With $T = 1$ sidereal day $= 23h\, 56m\, 4s = 86{,}164s$ we obtain $r = \left[Gm_2 T^2 / \left(4\pi^2\right)\right]^{1/3} = 4.22 \times 10^7\, m$. As the earth's radius is given by $R = 6.37 \times 10^6\, m$, the apple should be orbiting at a distance of $3.58 \times 10^7\, m$ above sea level in order to remain geostationary, that is, with a period of one sidereal day.

In this example the apple will not move relative to the ground and will not decrease its distance to the center of the earth, despite the gravitational attraction exerted by the earth. In the earth's frame of reference we can say that the gravitational attraction exerted by the earth on the apple is balanced by the centrifugal force acting on the apple.

5 – Newton's Bucket Experiment

In his masterpiece *Principia* (1687) Isaac Newton presented his famous bucket experiment. He wanted to distinguish the relative motion of one body relative to another body, from the absolute motion of one body relative to empty space. In the Scholium at the beginning of the book he gave a specific name to empty space, namely, *absolute space*:[6]

> Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies; and which is commonly taken for immovable

[6] [4, pp. 6-7], [5, pp. 6-8] and [6, p. 5].

> space; such is the dimension of a subterraneous, an aerial, or celestial space, determined by its position in respect of the Earth.

Newton's absolute space has no relation to anything external. The absolute position of an apple, for instance, is not related to its position relative to the ground, to the sun, nor relative to the frame of fixed stars. Therefore Newton's absolute space is equivalent to empty free space.

He then presented his bucket experiment in order to distinguish relative rotation from absolute rotation, our emphasis:[7]

> *The effects which distinguish absolute from relative motion are, the forces of receding from the axis of circular motion. For there are no such forces in a circular motion purely relative, but in a true and absolute circular motion, they are greater or less, according to the quantity of motion*. If a vessel, hung by a long cord, is so often turned about that the cord is strongly twisted, then filled with water, and held at rest together with the water; thereupon, by the sudden action of another force, it is whirled about the contrary way, and while the cord is untwisting itself, the vessel continues for some time in this motion; the surface of the water will at first be plain, as before the vessel began to move; but after that, the vessel, by gradually communicating its motion to the water, will make it begin sensibly to revolve, and recede by little and little from the middle, and ascend to the sides of the vessel, forming itself into a concave figure (as I have experienced), and the swifter the motion becomes, the higher will the water rise, till at last, performing its revolutions in the same times with the vessel, it becomes relatively at rest in it. *This ascent of the water shows its endeavor to recede from the axis of its motion; and the true and absolute circular motion of the water, which is here directly contrary to the relative, becomes known, and may be measured by this endeavor*. At first, when the relative motion of the water in the vessel was greatest, it produced no endeavor to recede from the axis; the water showed no tendency to the circumference, nor any ascent towards the sides of the vessel, but remained of a plain surface, and therefore its true circular motion had not yet begun. But afterwards, when the relative motion of the water had decreased, the ascent thereof

[7] [4, pp. 10-11], [5, pp. 11-12] and [6, p. 152].

towards the sides of the vessel proved its endeavor to recede from the axis; and this endeavor showed the real circular motion of the water continually increasing, till it had acquired its greatest quantity, when the water rested relatively to the vessel. *And therefore this endeavor does not depend upon any translation of the water in respect of the ambient bodies, nor can true circular motion be defined by such translation*. There is only one real circular motion of any one revolving body, corresponding to only one power of endeavoring to recede from its axis of motion, as its proper and adequate effect; but relative motions, in one and the same body, are innumerable, according to the various relations it bears to external bodies, and, like other relations, are altogether destitute of any real effect, any otherwise than they may perhaps partake of that one only true motion. [...]

Newton's bucket experiment is illustrated in Figure 3:

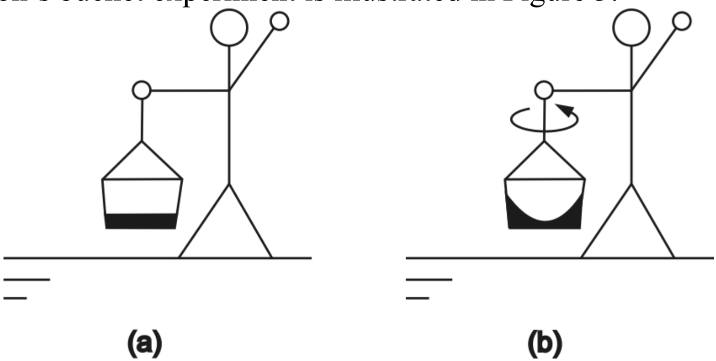

Figure 3: Newton's bucket experiment. (a) Bucket and water and rest relative to the ground, with a plain surface of water. (b) Bucket and water rotating together around the axis of the bucket with a constant angular velocity relative to the ground, producing a concave surface of water.

Newton observed the concavity of the water surface when it was rotating together with the bucket around the axis of the bucket, with a constant angular velocity relative to the ground. According to Newton, this concavity was a measure of the absolute rotation of the water relative to absolute space, that is, relative to empty free space. This concavity was not due to the rotation of the water relative to the bucket, relative to the ground, nor relative to the frame of distant stars.

6 – Mach's Interpretation of Newton's Bucket Experiment

Ernst Mach (1838-1916) presented a completely different interpretation of Newton's bucket experiment in his 1883 book *The Science of Mechanics*. According to him, the concavity of the water was due to its rotation relative to the distant bodies of the cosmos, like the fixed stars. That is, Mach was against Newton's interpretation that this concavity was due to the rotation of the water relative to absolute space or relative to empty free space. One example of his point of view (our emphasis):[8]

> If, in a material spatial system, there are masses with different velocities, which can enter into mutual relations with one another, these masses present to us forces. We can only decide how great these forces are when we know the velocities to which those masses are to be brought. *Resting* masses too are forces if *all* the masses do not rest. Think, for example, of Newton's rotating bucket in which the water is not yet rotating. If the mass m has the velocity v_1 and it is to be brought to the velocity v_2, the force which is to be spent on it is $p = m(v_1 - v_2)/t$, or the work which is to be expended is $ps = m(v_1^2 - v_2^2)$. *All masses and all velocities, and consequently all forces, are relative*. There is no decision about relative and absolute which we can possibly meet, to which we are forced, or from which we can obtain any intellectual or other advantage. When quite modern authors let themselves be led astray by the Newtonian arguments which are derived from the bucket of water, to distinguish between relative and absolute motion, they do not reflect that the system of the world is only given *once* to us, and the Ptolemaic or Copernican view is *our* interpretation, but both are equally actual. *Try to fix Newton's*

[8] [7, p. 279] and [6, p. 245].

> *bucket and rotate the heaven of fixed stars and then prove the absence of centrifugal forces*.

A few pages later he wrote:[9]

> Let us now examine the point on which Newton, apparently with sound reasons, rests his distinction of absolute and relative motion. If the Earth is affected with an *absolute* rotation about its axis, centrifugal forces are set up in the Earth: it assumes an oblate form, the acceleration of gravity is diminished at the equator, the plane of Foucault's pendulum rotates, and so on. All these phenomena disappear if the Earth is at rest and the other heavenly bodies are affected with absolute motion round it, such that the same *relative* rotation is produced. This is, indeed, the case, if we start *ab initio* from the idea of absolute space. But if we take our stand on the basis of facts, we shall find we have knowledge only of *relative* spaces and motions. *Relatively, not considering the unknown and neglected medium of space, the motions of the universe are the same whether we adopt the Ptolemaic or Copernican mode of view. Both views are, indeed, equally correct*; *only the latter is more simple and more practical*. The universe is not *twice* given, with an Earth at rest and an Earth in motion; but only *once*, with its *relative* motions, alone determinable. It is, accordingly, not permitted us to say how things would be if the Earth did not rotate. We may interpret the one case that is given to us, in different ways. If, however, we so interpret it that we come into conflict with experience, our interpretation is simply wrong. *The principles of mechanics can, indeed, be so conceived, that even for relative rotations centrifugal forces arise*.

In our book *Relational Mechanics and Implementation of Mach's Principle with Weber's Gravitational Force* we show how to implement quantitatively these ideas of Ernst Mach utilizing Weber's gravitational force.[10]

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[9] [7, pp. 283-284] and [6, p. 245].
[10] [6].

7 – Conclusion

It is remarkable that Empedocles and Newton utilized similar phenomena, a rotating bucket, when dealing with equivalent dynamical problems. It is also fascinating to discuss Newton and Mach's opposite interpretations related to the origin of the centrifugal force.

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