THE REDSHIFT REVISITED

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Abstract. We analyse the history of modern cosmology based on the redshift phenomenon and on the cosmic background radiation (CBR). We show the models of different authors for the interpretation of the redshift and how the tired light models predicted the correct value of 2.7 K temperature previous to Gamow and collaborators.

Key words: Cosmology, Cosmological Redshift, History of Cosmology, Cosmic Background Radiation

1 Introduction

The origin of the redshift of stellar sources, galaxies and quasars has been discussed for a long time. The majority of the work on this subject interpreted the redshift phenomenon as a Doppler effect associated with the recession of the sources (mainly galaxies). This interpretation leads directly to the idea of the big bang as most galaxies present a redshift and only a few of them located nearby present a blueshift.

In this article we discuss ideas presented in some recent work (Reber, 1986; Arp, 1987; Assis, 1992) which show an alternative interpretation that is also consistent with the data. Moreover, we make a historical analysis of the subject presenting discordant voices of this paradigm of big bang.

2 Different Views of the History of Modern Cosmology

In his interesting article, "How cosmology became a science," Stephen G. Brush makes a historical analysis of two models of modern cosmology: the big bang one and the steady state theory of Hoyle, Narlikar and Gould, (Brush, 1992). According to him, the discovery of the cosmic background radiation, the CBR, in 1965, was the decisive factor in favour of the standard cosmological model of the big bang against the steady state theory. The CBR spectrum has been found to be equivalent to the spectrum of a blackbody

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with a characteristic temperature of 2.7 K. As the steady state theory did not predict such a temperature, while the big bang did, this discovery settled the question in favour of the big bang, according to Brush.

The main characters in Brush's history are Gamow and his collaborators, Alpher and Herman, who had predicted the correct value of the temperature of cosmic space prior to the discovery of Penzias and Wilson. He mentions briefly the work of A. Eddington, written in 1926, where he evaluated the temperature of interstellar space as 3.2 K, (Eddington, 1988a). Although Eddington's work arose much earlier than Gamow's estimates in the period 1949 to 1961, it has a problem, at least according to Brush: "Eddington did not propose a specific procedure for testing his prediction." Later on we will return to this point.

Beyond Eddington and the works of Gamow and collaborators, and of Dicke and Hoyle, Brush only considers the work of Andrew Mackellar, who utilized the levels of excitation of the cyanogen molecule (CN) in intergalactic space to evaluate the temperature of the intergalactic medium. In this remarkable work he obtained the value of 2.3 K, in 1941, without the big bang conjecture.

We emphasize Brush's paper here, not only because of its importance in the field of the history of modern cosmology, but also because of its impact on popular perceptions; *e.g.*, his articles published in *Scientific American.* But his work is similar to statements found in almost all textbooks on this subject. This paper discusses a line of development of the history of the cosmic background radiation that is usually neglected by most authors.

3 A Steady State Theory Without Expansion and Without Continuous Creation of Matter

Brush's paper and the work of most cosmologists usually compare only two models of the universe: the big bang model and the steady state theory of Hoyle, Bondi and Gold. These two models have an important aspect in common: both accept the interpretation of the cosmological redshift as being due to a Doppler effect. So both theories accept the expansion of the universe without further question. But there is a third model of the universe, developed in this century by scientists like Regener, Nernst (the father of the third law of thermodynamics), Finlay-Freundlich, and the Nobel laureates Max Born and Louis de Broglie. Unfortunately, this third model is always nearly neglected in the textbooks and is virtually unknown to todays physicists and astrophysicists. It is the work of the above notables that we wish to rescue.

The model developed by these authors has in common an interpretation of the cosmological redshift as being due to some kind of interaction of the photon in its journey from a distant galaxy to the earth. These explanations are usually called "tired light" theories. What is generally unappreciated is that these authors predicted the correct value of the temperature characteristic of the CBR prior to the works of Gamow and his collaborators. This means that the discovery of Penzias and Wilson can not be considered decisive in favour of the big bang because an alternative model had also predicted the correct value of the temperature.

First, let us go back to Eddington's book of 1926, (Eddington, 1988a). The remarkable aspect of the temperature of interstellar space as 3.2 K is that it was due, according to him, to the total radiation field emitted by the stellar sources being counterbalanced by the incident radiation over them and being absorbed by them. This is typical of an equilibrium situation. Moreover, he utilized Stephan-Boltzmann's law, according to which the total flux F emitted by a black body is given by

$$F = \sigma T^4 \tag{1}$$

where σ is Stephan-Boltzmann's constant ($\sigma = 5.67 \times 10^{-8} W m^{-2} K^{-4}$).

Later on, Eddington changed his cosmological views and accepted the idea of an expanding universe (he even wrote the book *The Expanding Universe*, in 1933, (Eddington, 1988b)). But at least his prediction of 1926, of a temperature of 3.2 K, was not based on an expanding universe.

This utilization of Stephan-Boltzmann's law, characteristic of a black body spectrum, is an extremely important element in the works of Regener, Nernst, and Finlay-Freundlich.

In 1933 Regener, (Regener, 1933), analysing the energy of cosmic rays arriving on earth, wrote:

> Ein Himmelskörper, der die zur Absorption der Ultrastrahlung notwendige Dimension hat (...) wird sich durch die Ultrastrahlung erwärmen. Die Erwärmung wird proportional der zugestrahlten Ultrastrahlungsenergie S_U und der Oberfläche O sein. Er wird sich so lange erwärmen, bis die emittierte Wärmestrahlung, bei schwarzer Strahlung also $= \sigma T^4 O$, ebensogroβ geworden ist. Es ergibt sich die Endtemperatur $T = \sqrt[4]{S_U/\sigma}$. Das gibt nach Einsetzung der Zahlenwerte 2.8 K.[1]

Following this work Nernst presented a remarkable paper in 1937, (Nernst, 1937). Nernst believed in a stationary universe. Making reference to Regener's work he commented:

[1] "A celestial body which has the sufficient dimensions to absorb the cosmic radiation (...) is heated by means of this cosmic radiation. The heating results to be proportional to the energy S_U from the cosmic radiation and to the surface O [of the body]. It is heated until it emits the same amount of radiation, so that the black radiation becomes equal to $\sigma T^4 O$. The final temperature is determined by $T = \sqrt[4]{S_U/\sigma}$ and is found to be 2.8 K" (free translation).

> In der soeben erwähnten wichtigen Arbeit von Regener findet sich die Angabe, daβ im Universum ein die Kosmiche Strahlung absorbierender Körper sich bis auf 2.8^o abs. erwärmen müβte.[2]

Nernst, utilizing Regener's work, advocated a model of a boundless universe, homogeneous in large scale and without expansion. He suggested an equation to explain the light absorption by cosmic dust or something similar, due to a decrease of the luminous quantum of energy, resulting in the reddening of the photon:

$$-d(h\nu) = H(h\nu)dt \tag{2}$$

where h is Planck's constant ($h = 6.6 \times 10^{-34} Js$), ν is the light frequency and H is Hubble's constant. One of his conclusions in this article is that the cosmological redshift is not due to a Doppler effect.

In 1954 Finlay-Freundlich discussed the redshift of the spectral lines of B-stars and O-stars belonging to the Orion Nebula group, (Finlay-Freundlich, 1954). He analysed the influence of the gravitational potential over the results of observed redshifts. He summarized his results of the B-stars stating: "The B-stars in Orion nebula show a systematic redshift relative to the lines in the nebula amounting to at least $+10 km/s$. This value is, by a factor of the order of ten, larger than the redshift predicted by the theory of relativity." Freundlich found, for O-stars, that the redshifts result to be about $+18 km/s$. Analysing binary systems of stars he found redshifts larger by a factor of 10 to 20 than the predicted by general relativity (gravitational redshift). He says about this fact:

> It is quite improbable that they are produced by a systematic motion of the stars in the Orion Nebula group relative to the nebula itself, or by a systematic motion of the O-stars relative to the B-stars in the same cluster. (...) We see thus that the large redshifts reveal a physical effect which cannot be interpreted either as a gravitational displacement or as a true recession effect.

Trying to explain the observed redshifts, Freundlich suggests an interesting hypothesis:

> I propose to introduce as an additional hypothesis that light passing through deep layers of intense radiation field, loses energy — perhaps due to photon-photon interaction — and that the energy loss is proportional both to the density of the radiation field and to the length of path of the light through the radiation field.

Freundlich writes thus an empirical formula to explain these redshifts:

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[2] "In Regener's important work cited above it is found the fact that, in the universe, a celestial body that absorbs cosmic radiation must be heated until 2.8^o K" (free translation).

$$\frac{\triangle\nu}{\nu} = -AT^4 l \tag{3}$$

where $\triangle\nu$ is the change of frequency of the line, ν is the original frequency, A is a constant, T is the temperature of the radiation field and l is the length of path traversed by light through the radiation field. The constant A is obtained when we have $l = 10^7 cm$, $\triangle\nu/\nu = -3.3 \times 10^{-5}$ and $T = 20000K$ for a B-star temperature. Therefore the value of A is $2 \times 10^{-29} K^{-4} cm^{-1}$.

Freundlich applies his formula to the explanation of the redshift of the sun, A-stars, supergiant M-stars, Wolf-Rayet stars, and white dwarfs, with great success.

With these results, Freundlich compared the cosmological redshift and stellar redshifts (for instance, B-stars). He then applied his formula to the cosmological redshift. In his analysis, Freundlich derived a blackbody temperature for intergalactic space. The two extremum values obtained by Freundlich's formula for the mean temperature of intergalactic space were $T = 1.9K$ and $T = 6.0K$.

Finlay-Freundlich concluded his article writing:

> We may have, therefore, to envisage that the cosmological redshift is not due to an expanding universe, but to a loss of energy which light suffers in the immense lengths of space it has to traverse coming from the most distant star systems. That intergalactic space is not completely empty is indicated by Stebbins and Whitford's discovery (1948) that the cosmological redshift is accompanied by a parallel unaccountable excess reddening. Thus the light must be exposed to some kind of interaction with matter and radiation in intergalactic space.

TABLE I

Predictions of the temperature of the CBR according to different models of the universe and different authors.

year	stationary universe	big bang	temperature
1926	Eddington		3.2 K
1933	Regener		2.8 K
1937	Nernst		2.8 K
1949		Alpher and Hermann	$T \geq 5\ K$
1953		Gamow	7 K
1954	Finlay-Freundlich		$1.9\ K \leq T \leq 6.0\ K$
1961		Gamow	50 K

4 Gamow's Different Predictions of the Temperature of the CBR

In his paper, Finlay-Freundlich references Gamow's 1953 temperature of $7K$, a value obtained from thermodynamical considerations, for the mean temperature of intergalactic space. Freundlich did not mention Alpher and Hermann's paper of 1949, (Alpher and Hermann, 1949). These authors, collaborators of Gamow, wrote:

> (the present density of radiation, $\rho_{r''} \cong 10^{-32} g/cm^3$) corresponds to a temperature now of the order of $5^o K$. This mean temperature for the universe is to be interpreted as the background temperature which would result from the universal expansion alone. However, the thermal energy resulting from the nuclear energy production in stars would increase this value.

So, according to these authors, the temperature characteristic of this radiation should be **at least 5 K.**

In 1961 Gamow published a revised edition of his popular book *The Creation of the Universe*, (Gamow, 1961). This is the last work of Gamow known to us, where he discusses the temperature of interstellar space prior to the discovery of the CBR by Penzias and Wilson in 1965. There is only one place in the book where he mentions the temperature of the CBR. Let us quote in full these important paragraphs (our emphasis):

> The relation previously stated between the value of Hubble's constant and the mean density of the universe permits us to derive a simple expression giving us the temperature during the early stages of expansion as the function of the time counted from the moment of maximum compression. Expressing that time in seconds and the temperature in degrees (see Appendix, pages 142-43), we have:
>
> $$\text{temperature} = 1.5 \times 10^{10}/[\text{time}]^{1/2}$$
>
> Thus when the universe was 1 second old, 1 year old, and 1 million years old, its temperatures were 15 billion, 3 million, and 3 thousand degrees absolute, respectively. Inserting the present age of the universe ($t = 10^{17}$ *sec*) into that formula, we find
>
> $$T_{\text{present}} = 50 \text{ degrees absolute}$$
>
> *which is in reasonable agreement with the actual temperature of interstellar space.* Yes, our universe took some time to cool from the blistering heat of its early days to the freezing cold of today!

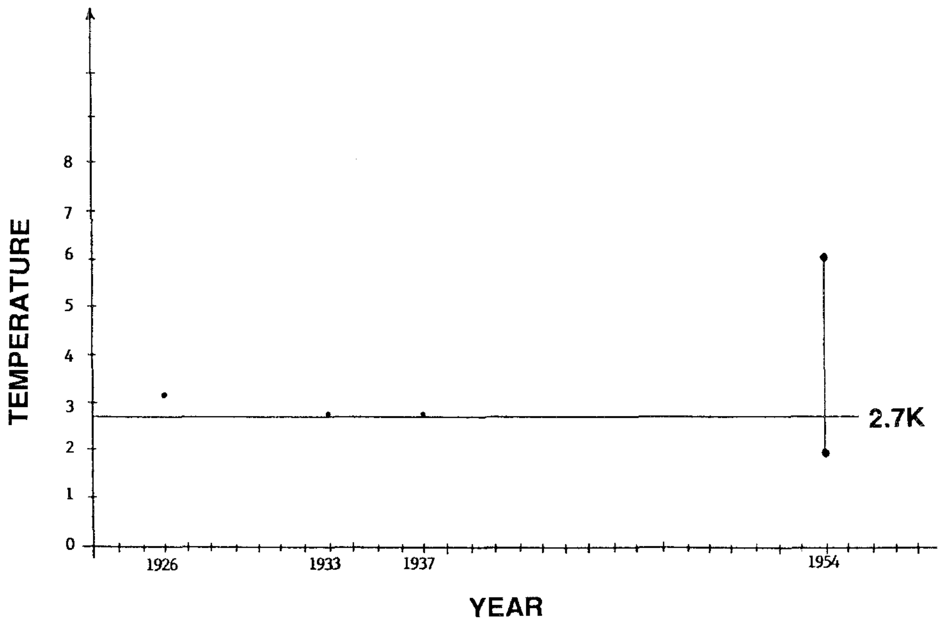

Fig. 1. Predictions of the temperature of the CBR based on a universe in dynamical equilibrium without expansion (for short called stationary universe): Eddington, 1933 [see Eddington 1988] ($T = 3.2\ K$); Regener, 1933 ($T = 2.8\ K$); Nernst, 1937 ($T = 2.8\ K$); and Finlay, 1954 ($1.9\ K \leq T \leq 6.0\ K$). The known observational value of 2.7 K is also shown.

While the theory provides an exact expression for the temperature in the expanding universe, it leads only to an expression with an unknown factor for the density of matter, in fact, one can prove (see Appendix) that

$$[\text{density of matter}] = \text{constant}/[\text{time}]^{3/2}$$

We see in Chapter III that the value of that constant may be obtained from the theory of the origin of atomic species.

This value of 50 K is, obviously, very different from that obtained by Penzias and Wilson in 1965, namely, $T = 3.5 \pm 1.0\ K$, (Penzias and Wilson, 1965). A trajectory of the estimates of the temperature of cosmic space year by year, by Gamow and collaborators, diverges away from the value finally measured in 1965. In Figure 1 we plot, in chronological order, the predictions based on a non-expanding universe according to Eddington, Regener, Nernst

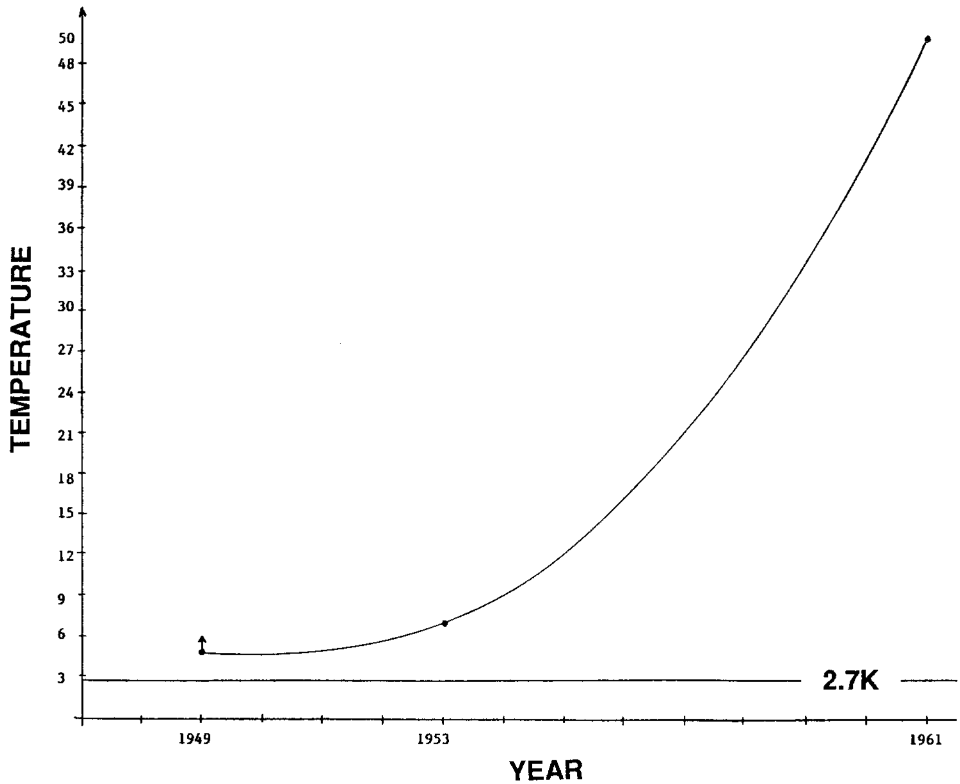

Fig. 2. Predictions of the present value of the temperature of the CBR according to Gamow and collaborators (1949: $T \geq 5\ K$, 1953: $T = 7\ K$, 1961: $T = 50\ K$). It is also presented the known observational value of 2.7 K.

and Finlay-Freundlich. In Figure 2 we present, in a chronological order, the predictions of the temperature of the CBR according to Gamow and collaborators. This data is collected in Table 1.

It is relevant to remember here a letter sent by Gamow to Arno Penzias, in 1965 (which was curiously dated 1963). This letter was reproduced in Penzias's article, entitled "Cosmology and microwave astronomy," (Penzias, 1972). We reproduce it here again:

"Sept 29$^{\text{th}}$ 1963

Dear Dr. Penzias,

Thank you for sending me your paper on 3 K radiation. It is very nicely written except that "early history" is not "quite complete". The theory of, what is now known, as, "primeval fireball", was first developed by me in 1946 (Phys. Rev. 70, 572, 1946; 74, 505, 1948; Nature 162, 680, 1948). The prediction of the numerical value of the present (residual) temperature could be found in Alpher & Hermann's paper (Phys. Rev. 75, 1093, 1949) who estimate it as 5 K, and in my paper (KongDansk. Ved. Sels 27 n^o 10, 1953) with the estimate of 7 K. Even in my popular book *Creation of the Universe* (Viking 1952) you can find (on p. 42) the formula $T = 1.5 \times 10^{10}/t^{1/2}$ K, and the upper limit of 50 K. Thus, you see the world did not start with almighty Dicke.

Sincerely,

G. Gamow"

This letter, as we have seen, does not correspond to the true facts. Gamow, in the revised edition of his book of 1952, published in 1961, calculated a temperature **equal to 50 K**. Thus, Gamow did not estimate in this work an **upper limit** of 50 K.

The chronology of predictions of the temperature of the CBR suggests a different history than that presented in cosmological textbooks and in articles written about cosmology.

In this regard, we quote another part of Penzias paper, (Penzias, 1972):

> It is beyond the scope of this contribution to weigh the various theoretical explanations of the 3^o K. Still the unique claim of the hot evolving universe theory is that it predicted the background radiation before the fact. At the 4^{th} "Texas" Symposium on Relativistic Astrophysics, George Gamow was the chairman of the session on Microwave Background Radiation. He ended his remarks with a comment which, to the best of my recollection, went, "If I lose a nickel, and someone finds a nickel, I can't prove that it's my nickel. Still, I lost a nickel just where they found one." The applause was loud and long.

As a matter of fact Gamow did not lose a single coin, but many of them. Moreover, these coins had different values, in a divergent series relative to the correct value found at a latter date. It is even more remarkable that other people had lost nickels much closer to where they were later found to be, and at an earlier date than Gamow!

5 Discussion and Conclusion

Two other important authors in the subject of a non-expanding universe are Max Born, (Born, 1954), and Louis de Broglie, (de Broglie, 1966). Max Born showed that the theory of Finlay-Freundlich (photon-photon collisions as the cause of redshift) was scientifically sound. When discussing the cosmological redshift in this paper, Max Born made a remarkable prediction: "Thus the redshift is linked to radio-astronomy." This was written eleven years before the discovery of the CBR by Penzias and Wilson, (Penzias and Wilson, 1965), utilizing a horn reflector antenna built to study radio astronomy.

Despite this fact Max Born never stated, to the best of our knowledge, that he did not believe in the expanding universe, he did not feel at all confortable with the big bang theory, as indicated by the following quotations from his book *Einstein's Theory of Relativity*, (Born, 1962):

> The reader may get the impression that modern cosmology has strayed from the sound empirical road into a wilderness where statements can be made without fear of observational check. Indeed, this can be said of the theories just sketched, particularly as the mixed feeling of admiration and slight disgust which they produce is enhanced by the almost fanatical assurance with which they are advertised by their authors. Unfortunately but rather naturally, this state of affairs has been used by different ideologies to claim one of these theories as a confirmation of their dogma and to anathematize the other. (Born, 1962), p. 369

> Views of this kind, preached as dogma, are foreign to the spirit of science, and each of them can be refuted by showing that it does not take all aspects into account. Those who welcome the idea of a "beginning" forget that all one can assuredly say is that this is a state of high density of matter quite distinct from the distribution of isolated stars known to us; one may doubt that in this state the notions of space and time are applicable, because these notions are intimately related to the dispersed system of stars. The "beginning" refers only to our ability to describe the state of things in terms of accustomed concepts. Whether there was a creation from nothing is not a scientific question, but a matter of belief and beyond experience, as the old philosophers and theologians like Thomas Aquinas knew. (Born, 1962), p. 369

Louis de Broglie states a "photon aging" due to a continuous loss of energy by the photon. A more detailed discussion about these two authors can be found in (Assis, 1992) and (Assis, 1993).

In this paper, we have presented another view of the history of cosmology; one that is very different from that presented by Brush. We have emphasized the fact that their exists a large body of research, from a number of notable physicists, that is critical of the Doppler effect as the explanation of cosmic redshifts. It should also be emphasized that there are numerous papers on the topic of anomalous redshift observations (see, for instance, (Reboul, 1981), for a list of 772 untrivial redshifts). Anomolous redshifts cannot easily be explained by the Doppler model. But if the redshift is not due to a Doppler effect, what is its origin?

Finlay-Freundlich believed in a photon-photon interaction in the intense radiation fields of the stars. Marmet believes in a redshift produced by inelastic collisions of photons on atoms and molecules. Reber and Kierein pointed out the Compton effect (interaction photon-electron). Vigier and Monti proposed the resistivity of intergalactic medium. Arp believes in an effect due to the age of celestial bodies. For further discussion of these models and for the references, see (Assis, 1992) and (Assis, 1993).

With respect to the Compton effect (scattering of photons by free electrons), it is known that the variation of wavelength is given by:

$$\lambda - \lambda_o = \frac{h}{mc}(1 - \cos\theta) \ , \tag{4}$$

where λ is the scattered wavelength of the photon, λ_o is the incident wavelength, h is Planck's constant, m is the electron's mass, c the speed of light and θ is the angle between incident and scattered photon.

Therefore, if the cosmological redshift is due to a Compton effect, we would have for each interaction the following contribution to the redshift:

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{\lambda}\frac{h}{mc}(1 - \cos\theta) \ . \tag{5}$$

The constants h, m and c do not depend on λ. Then we are led to conclude that the cosmological redshift should scale as $1/\lambda$, but this is not observed. As a matter of fact Hubble's constant seems to be independent of wavelength. It thus seems improbable that the cosmological redshift is due to a Compton effect.

What is the real mechanism that produces the observed values for the redshift? This question continues to be a great mystery. A possible answer could arise in the future from a stationary model of the universe.

We close this article with three quotations by Hubble, as given by Reber, (Reber, 1986):

> "Light *may* lose energy during its journey through space, but if so, we do not yet know how the energy loss can be explained."
>
> "The disturbing features are all introduced by the recession factor, by the assumption that red-shifts are velocity-shifts. The departure from linear law of red-shifts, the departure from uniform distribution, the curvature necessary to restore homogeneity, the excess material demanded by the curvature; each of these is merely the recession factor in another form. These elements identify a unique model among the array of possible expanding worlds, and, in this model, the restriction in time-scale, the limitation of spatial dimensions, the amount of unobserved material, is each equivalent to the recession factor.
>
> On the other hand, if the recession factor is dropped, if redshifts are not primarily velocity-shifts, the picture is simple and plausible. There is no evidence of expansion and no restriction of the time-scale, no trace of spatial curvature and no limitations of spatial dimensions."

"We seem to face, as once before in the days of Copernicus, a choice between a small, finite universe, and a universe indefinitely large plus a new principle of nature."

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