Coulomb's Memoirs on Torsion, Electricity, and Magnetism Translated into English

Andre Koch Torres Assis
Louis L. Bucciarelli

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The portrait on the cover is an oil on canvas of Charles-Augustin de Coulomb (1736-1806) made in 1894 by the French painter Louis Hierle (1856-1906). It is located in the Palace of Versailles. It is based on an oil painting of Coulomb made between 1803 and 1806 by an unknown artist. Some authors attribute the original portrait to Hippolyte Lecomte (1781-1857). It shows Coulomb in the military blue coat of the *Corps Royal du Génie*. He is holding his famous torsion balance and a scientific work. This is the most famous image of Coulomb.

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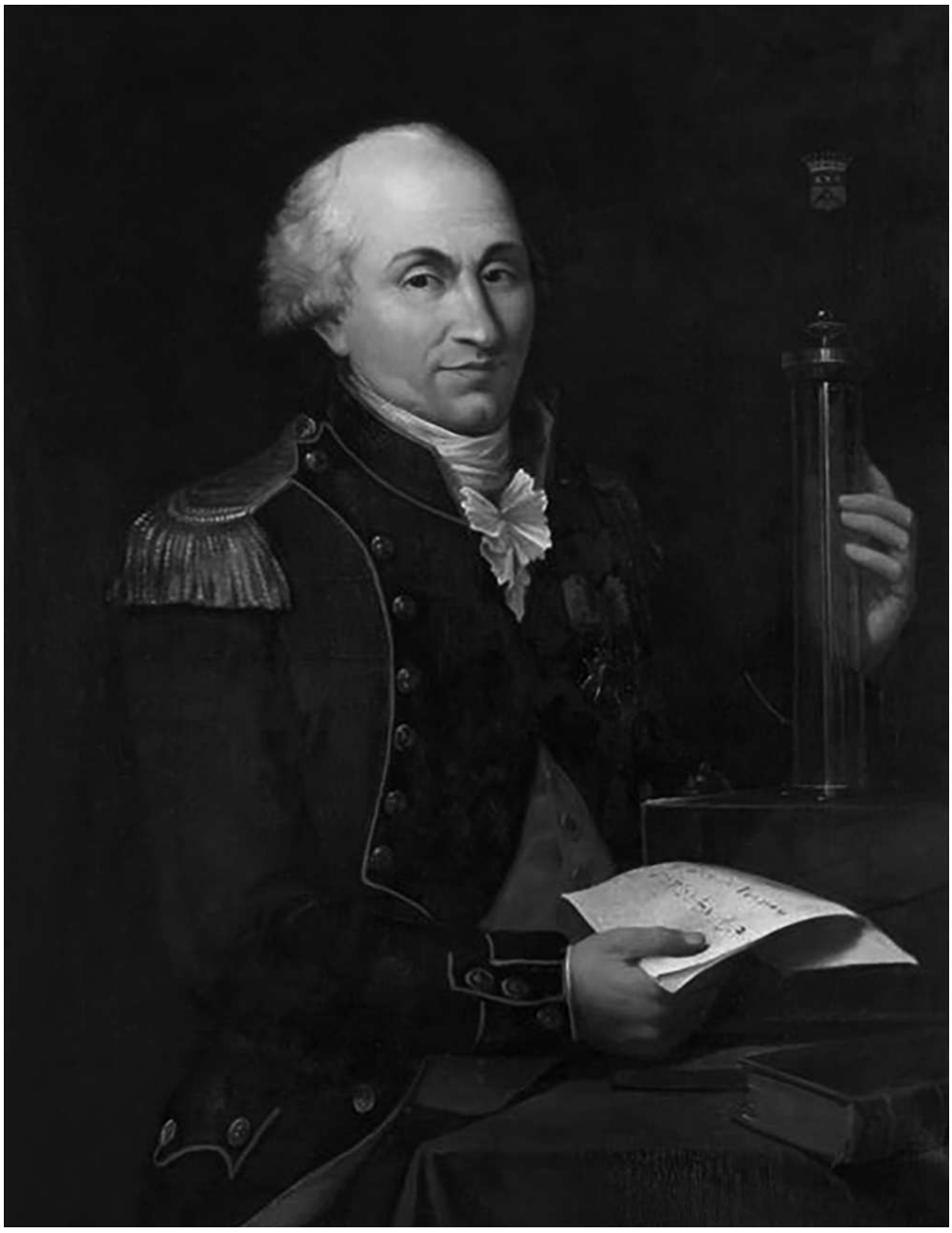

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Foreword

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I am pleased to write the Foreword to this expanded, English edition of Charles Augustin Coulomb's memoirs in Electricity, Magnetism and Torsion. The Editor, A. K. T. Assis, is well known not only for his monographs in Electrodynamics, Mechanics and History of Electricity, but for his body of important critical editions in several languages of the works of classical physicists. Coulomb's memoirs were supposed to be published following his death in 1806, but this came not to be, except for a partial collection of his mechanical and civil papers published in 1821.[1] Following the naming of electrical units in 1881, the French Society of Physics authorized physicist Alfred Potier to publish a collection of Coulomb's papers. These were published beginning in 1884,[2] sometimes in truncated form. Professor Assis has done a masterful job of including the omissions in the Potier edition and has included several other papers of interest in Coulomb's Electricity, Magnetism and Torsion work. This English edition is expanded from Assis' recent edition in Portuguese,[3] especially with his enhanced critical notes and comments as well as those of Louis L. Bucciarelli, a Professor (Emeritus) of Engineering and Technology Studies at MIT. The expanded bibliography is superb. Assis describes his own decades-long research in the action-at-a-distance worldview in physics from Isaac Newton, through Coulomb, André-Marie Ampère and then Wilhelm Weber. This is contrasted to the worldview of Fields, other than Central Forces, which came to prominence in Physics by the mid-nineteenth century. Professor Assis concerns himself with Experiment and Measurement in all its details and difficulties, from antiquity through the early modern period, and how standards differed over the ages. The results of Experiment in Science are frequently contested, sometimes for decades as was the case with some of Coulomb's results.

Many years ago, soon after the publication of my biography of Coulomb,[4] I was invited to speak at the Barnard-Columbia University History of Physics laboratory of Samuel Devons. During my visit and in a lengthy discussion, Devons emphasized the considerable difficulties he experienced in duplicating Coulomb's experiments with the Torsion balance — the extreme sensitivity to air currents, temperature fluctuations, sound volumes, motional perturbations, among other factors. It is quite difficult to reproduce exactly the circumstances

[1] [Coulomb, 1821].
[2] [Potier, 1884].
[3] [Assis, 2022].
[4] [Gillmor, 1971a].

of a classical experiment. One example is that when Potier compared Coulomb's data for wires used in his Torsion balance, Potier used "modern" brass and iron wire where rigidity and hardness varied from the eighteenth-century harpsichord wires (*cuivre jaune*, *laiton*, *fer*) used by Coulomb. For the iron, the differences included the low carbon, high phosphorus content in the old wire. And in general, several properties of wire are sensitive to the degree of their reduction.[5] German, Italian, and some English followers contested Coulomb's electric balance results for several decades, as extensively discussed by modern authors mentioned by Assis (Heering, Blondel, Wolff, Martinez, Buchwald.) The differing reception of Coulomb's results was not simply experimental, but philosophical, just as Coulomb saw the mid-eighteenth century worldviews of Newtonian forces at a distance versus Cartesian vortices and effluvia.

Coulomb was a military engineer and served in half a dozen different locations over more than twenty years. Whatever personal papers he may have been able to transfer and move with him basically are missing. His widow auctioned all his instruments within six months after his death in 1806. His physical research notes and papers were given to J. B. Biot for presentation and publication and Biot utilized some of these in his work published in 1816.[6] The location of these Coulomb papers is unknown as are the locations of all his instruments. In reading the Assis translation, as one who donated several years to reading Coulomb's manuscripts and correspondence in seven or eight military, maritime, engineering and scientific French archives, I find excitement in places, as if I am hearing Coulomb speaking. I heartily applaud Professor Assis' new English edition.

[5] [Goodway and Savage, 1992].

[6] [Biot, 1816a], [Biot, 1816b], [Biot, 1816c] and [Biot, 1816d].

Chapter 1

Introduction to the Translations of Coulomb's Memoirs on Torsion, Electricity, and Magnetism

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The portrait on the cover is an oil on canvas of Charles-Augustin de Coulomb (1736-1806) made in 1894 by the French painter Louis Hierle (1856-1906). It is located in the Palace of Versailles. It is based on an oil painting of Coulomb made between 1803 and 1806 by an unknown artist. Some authors attribute the original portrait to Hippolyte Lecomte (1781-1857). It shows Coulomb in the military blue coat of the *Corps Royal du Génie*. He is holding his famous torsion balance and a scientific work. This is the most famous image of Coulomb.

This book presents the English translation of Coulomb's main works on torsion, electricity, and magnetism. A Portuguese translation of all of these works was published in 2022.[7] Most of these works, although not all of them, had been reprinted in 1884 by the French Society of Physics.[8] In this French edition of 1884 some papers were only partially reprinted. In the present English translation I present a complete translation of all these works, with commentary, together with some other papers which were not included in the 1884 French edition.

The words between square brackets, [ ], were introduced by myself in order to facilitate the comprehension of some sentences. Coulomb's footnotes are represented by [Note by Coulomb]. The Notes by Alfred Potier (1840-1905), the editor of Coulomb's works published in 1884, are represented by [Note by Potier]. The Notes by L. L. Bucciarelli are represented by [Note by Bucciarelli]. All other footnotes were introduced by myself.

[7] [Assis, 2022].
[8] [Potier, 1884].

Coulomb's main biographies were written by Charles Stewart Gillmor.[9] Christine Blondel, Bertrand Wolff and John Heilbron also produced extremely important studies related to Coulomb's life and to several of his works.[10]

1.1 My Motivations to Undertake These Commented Translations of Coulomb's Memoirs

A. K. T. Assis

My first motivation is related to the importance of Coulomb's work. His force between electrified bodies appears in almost all textbooks on electricity. An analogous expression which he obtained for the force between magnetic poles is also mentioned in several publications. Between 1788 and 1793 he published his famous 7 Memoirs on electricity and magnetism containing these results and many other extremely important facts. To arrive at these experimental results he utilized a torsion balance. He had obtained the law of torsion of metal wires in 1784. Despite the relevance of his works, only few of his papers have been fully translated. His paper on the force of torsion of metal wires was translated in 2001.[11] His First Memoir on electricity has been translated in its entirety into German, English and Portuguese.[12] His second, third and fourth major Memoirs have been translated in full only into German and Portuguese.[13] Beyond the complete Portuguese translation of his most important papers on torsion, electricity, and magnetism,[14] I know of only a few partial English translations which will be mentioned in the appropriate Chapters of this work. The English translations presented in this book are intended to fill this gap.

Another motivation arises from my philosophical conceptions and my own lines of research. I work with direct interaction between bodies separated spatially from one another. That is, I do not utilize concepts like the gravitational field, the electric field and the magnetic field. This action at a distance approach was initiated by Isaac Newton (1642-1727) with his law of universal gravitation in which the gravitational force between two particles is a central force proportional to the product of their masses and inversely proportional to the square of their distance, following the principle of action and reaction. This approach was extended by Coulomb when dealing with the interaction between electrified bodies: that is, a central force directly proportional to the product of the electric charge of these particles

[9] [Gillmor, 1971b] and [Gillmor, 1971a].

[10] [Blondel, 1982, Chapter I], [Blondel and Dörries (Editors), 1994], [Blondel, 1995], [Heilbron, 1999, pp. 468-477 and 494-500], [Blondel and Wolff, 2007], [Blondel and Wolff, 2008a], [Blondel and Wolff, 2008b], [Blondel and Wolff, 2009], [Blondel and Wolff, 2011c], [Blondel and Wolff, 2011e], [Blondel and Wolff, 2011d], [Blondel and Wolff, 2011b], [Blondel and Wolff, 2011a], [Blondel and Wolff, 2011f], [Blondel and Wolff, 2013d], [Blondel and Wolff, 2013a], [Blondel and Wolff, 2013b], [Blondel and Wolff, 2013e], [Blondel and Wolff, 2013c], [Blondel and Wolff, 2014] and [Blondel and Wolff, 2015]. See also [Figueras, 1995].

[11] [Coulomb, 1787], with English translation in [Coulomb, 2012a] and [Coulomb, 2012b], and Portuguese translation in [Assis, 2022]. See also [Bucciarelli and Buchwald, 2001].

[12] [Coulomb, 1788b]. Complete translations into German, [Coulomb, 1890b], English, [Coulomb, 2012c], and Portuguese, [Assis, 2022].

[13] [Coulomb, 1788d], [Coulomb, 1788e] and [Coulomb, 1788c]. Complete German translations in [Coulomb, 1890e], [Coulomb, 1890a] and [Coulomb, 1890d]. Complete Portuguese translations in [Assis, 2022].

[14] [Assis, 2022].

and inversely proportional to the square of their distance. He also obtained an analogous expression for the force between magnetic poles (a central force proportional to the product of their magnetic pole intensities and inversely proportional to the square of their distance). This research project was followed by the force between current elements obtained by André-Marie Ampère (1775-1836). Once again there is a central force pointing along the straight line connecting the current elements and satisfying the principle of action and reaction. It is proportional to the product of the current intensities in the two elements, to the product of their infinitesimal sizes, and also inversely proportional to the square of their distance. Moreover, it depends on the angle between the two current elements, and also on the angles between each current element and the straight line connecting them. The most important development of this line of research was made by Wilhelm Eduard Weber (1804-1891). Weber's electrodynamic force between two electrified particles depends not only on their distance r, but also on their relative velocity dr/dt and on their relative acceleration d^2r/dt^2. Once more there is a central force following the principle of action and reaction. Weber unified the laws of Coulomb, Ampère and Michael Faraday (1791-1867) with his force law of 1846. Moreover, the laws of Newton, Coulomb and Weber are compatible with the principles of the conservation of linear momentum, angular momentum and energy.

Ampère and Weber's electrodynamics have been practically abandoned in physics teaching for more than a century. Instead of their approach, physics has followed the conception of Faraday and James Clerk Maxwell (1831-1879) based on electric and magnetic fields. This led to a neglect of the philosophical approach of Newton, Coulomb, Ampère and Weber based on a direct force between the interacting particles. The fundamental works of Ampère and Weber have been forgotten in general by the scientific community. I have been making a huge effort to make this action at a distance approach known to students and scientists. To this end I am translating their main works, with commentary. I made a Portuguese translation of Newton's masterpiece *Principia* (*Mathematical Principles of Natural Philosophy*).[15] I also translated into Portuguese his book *Opticks*.[16] Together with João Paulo M. d. C. Chaib, I published translations into English and Portuguese, with commentary, of Ampère's first paper of electrodynamics from 1820, together with his masterpice of 1826, *Théorie des Phénomènes Électro-dynamiques, Uniquement Déduite de l'Expérience*. These translations were published in the book *Ampère's Electrodynamics — Analysis of the Meaning and Evolution of Ampère's Force between Current Elements, together with a Complete Translation of His Masterpiece: Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience*.[17] With the help of several colleagues, I edited the translation from German into English, with commentary, of Weber's main works. They were published in the book *Wilhelm Weber's Main Works on Electrodynamics Translated into English*. Volume 1: Gauss and Weber's Absolute System of Units.[18] Volume 2: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday.[19] Volume 3: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity.[20] Volume 4: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation.[21] In this translation

[15] [Newton, 1990], [Newton, 2008] and [Newton, 2010].
[16] [Newton, 1996].
[17] [Assis and Chaib, 2011] and [Assis and Chaib, 2015]. See also [Chaib, 2009].
[18] [Assis (editor), 2021a].
[19] [Assis (editor), 2021b].
[20] [Assis (editor), 2021c].
[21] [Assis (editor), 2021d].

we also included 5 papers by Carl Friedrich Gauss (1777-1855), a translation of some letters exchanged between Gauss and Weber, 1 paper by Weber and Friedrich Wohler (1800-1882), 2 papers by Weber and Rudolf Kohlrausch (1809-1858), 1 paper by Gustav Theodor Fechner (1801-1887), 1 paper by Johann Christian Poggendorff (1796-1877), 1 paper by François Felix Tisserand (1845-1896), 2 papers by Carl Neumann (1832-1925), and 3 papers by Gustav Kirchhoff (1824-1887).

In the last 35 years my main topics of research have been Newton and Coulomb's force laws, Ampère's force between current elements and Weber's law applied to electromagnetism and gravitation. I have published several books on these subjects:

- Weber's Electrodynamics.[22]
- Relational Mechanics.[23]
- Inductance and Force Calculations in Electrical Circuits.[24]
- The Electric Force of a Current: Weber and the Surface Charges of Resistive Conductors Carrying Steady Currents.[25]
- Ampère's Electrodynamics — Analysis of the Meaning and Evolution of Ampère's Force between Current Elements, together with a Complete Translation of His Masterpiece: Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience.[26]
- Weber's Planetary Model of the Atom.[27]
- Relational Mechanics and Implementation of Mach's Principle with Weber's Gravitational Force.[28]
- Wilhelm Weber's Main Works on Electrodynamics Translated into English.[29]
- Commented Translation into Portuguese of Coulomb's Main Works on Electricity and Magnetism.[30]

My papers on these subjects can be found in my homepage.

With this English translation and commentary of Coulomb's main works on electricity, magnetism, and torsion I hope to make this fundamental line of research better known to the scientific community and to students in general. In this way his original researches and philosophical conceptions can be explored and developed experimentally and also theoretically.

[22] In English: [Assis, 1994]. In Portuguese: [Assis, 1992], [Assis, 1995] and [Assis, 2015a].

[23] In English: [Assis, 1999a]. In Portuguese: [Assis, 1998a] and [Assis, 1999b].

[24] In English: [Bueno and Assis, 2001]. In Portuguese: [Bueno and Assis, 1998] and [Bueno and Assis, 2015].

[25] In English: [Assis and Hernandes, 2007]. In Portuguese: [Assis and Hernandes, 2009]. In German: [Assis and Hernandes, 2013].

[26] In English: [Assis and Chaib, 2015]. In Portuguese: [Assis and Chaib, 2011].

[27] In English: [Assis et al., 2011]. In Portuguese: [Assis et al., 2014]. In German: [Assis et al., 2018].

[28] In English: [Assis, 2014]. In Portuguese: [Assis, 2013].

[29] Volumes 1 to 4: [Assis (editor), 2021a], [Assis (editor), 2021b], [Assis (editor), 2021c] and [Assis (editor), 2021d].

[30] In Portuguese: [Assis, 2022].

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Natural Sciences of Hamburg University, Germany, the Technical University of Dresden, Germany, and the Alexander von Humboldt Foundation of Germany for granting me three Research Fellowships which allowed me to work in Germany during the years 2001-2002, 2009 and 2014. During this period I collected a huge amount of bibliographic material related to these translations of Coulomb's Memoirs.

Roy Keys, the Editor of Apeiron, has supported me for many years. He made an excellent editorial work for this book.

1.2 Parts, Particles, or Molecules; Fluids One or Two?

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I first encountered Coulomb in a serious way when preparing for teaching a course at MIT together with Jed Buchwald back at the beginning of this century.[31] The course confronted students with the task of replicating two of Coulomb's experiments, both now recognized for their importance in improving physicists' understanding of the laws governing the interaction of charged electric bodies and the behavior of wires — of any cylinder for that matter — when subject to twist, to torsion. It was necessary to translate the two memoirs "Theoretical and Experimental Research on the Force of Torsion, and on the Elasticity of Metal Wires"[32] and "A First Memoir on Electricity and Magnetism"[33] for student reference in the construction and execution of both experiments.

The students were able to successfully carry out the experiments on torsion, reproducing Coulomb's results sufficient to appreciate his accomplishment. They were less successful in replicating his experiment on the law governing the repulsion of two like-charged bodies. (The paucity of data justifying Coulomb's law of electrostatic repulsion in this experiment is to be contrasted with the thoroughness of his experiments on the torsion of wires.) The apparatus he constructed and used in his work on torsion was *sine qua non* for all of his subsequent experimental work in electricity and magnetism. It is why his Memoir on torsion is included in this volume.

In assisting Professor Assis in the translation of the other Memoirs, I found myself unsettled on occasion by Coulomb's description of the materials supporting and enabling the electric and magnetic phenomena he so skillfully put to the test. In particular, he writes of "parties" which I translated simply as "parts" while some historians have seen them as

[31] [Bucciarelli and Buchwald, 2001].

[32] [Coulomb, 1787], with English translation in [Coulomb, 2012a] and [Coulomb, 2012b], and Portuguese translation in [Assis, 2022].

[33] [Coulomb, 1788b] with partial English translation in [Coulomb, 1935a]. There are complete English, German and Portuguese translations in [Coulomb, 2012c], [Coulomb, 1890b] and [Assis, 2022], respectively. This work was presented in 1785 to the French Academy of Sciences and published in 1788.

"particles". Coulomb seems to prefer "molecule" rather than "particule" for the elementary ingredients of a material, the (small) interacting constituents of a body, a preference reflected in his statement of his proposition: "Le Fluide magnétique agit par attracion ou répulsion, suivant la raison composée directe de la densité du fluide, & la raison inverse du carré des distances de ses molécules" found in his Second Memoir.[34]

I suspect Coulomb's introduction of the word "molecule" was influenced by the way his contemporaries, in particular, Laplace, had used the concept of (sensible) forces at (insensible) distances to explain various phenomena.[35] Evidence for this conjecture is found in a long footnote 'On the attractive and repulsive action of bodies, according to the law of distance' in Coulomb's 2nd Memoir.[36] There he suggests that "the cohesion, the elasticity and all the chemical affinities" may be due to attraction and/or repulsion between elements of the body "...in a relation very close to the inverse ratio of the cube of the distances".

Coulomb subsequently came to see "molecules" as important in explaining how, when a magnet is broken into pieces, each piece acts as a little magnet.[37]

And as to the question whether magnetism was due to one fluid [OEpinus] or two fluids, Coulomb allows that:[38] "These two hypotheses explain equally well, and in the same way, all the magnetic phenomena" albeit with "some difficulties." Although he prefers two fluids, Coulomb appears quite open to the possibility that different hypotheses might serve to explain electric phenomena and magnetism. In the Sixth Memoir, in a Section titled "Of the Two Types of Electricity,"[39] he notes that a one-fluid theory of electricity (again OEpinus) "gives, as regards the calculation, the same results as that of the two fluids" but he prefers two-fluids "because it seems to me contradictory to admit at the same time in the parts of the bodies, an attractive force in inverse ratio to the square of the distances demonstrated by universal gravity, and a repulsive force in the same inverse ratio of the square of the distances; a force which would necessarily be infinitely large, relatively to the attractive force due to gravity." He concludes this Section with

> As these two explanations have only a greater or lesser degree of probability, I warn, in order to protect the theory that will follow from any systematic dispute, that in the supposition of the two electric fluids, I have no other intention than to present with the fewest possible elements the results of the calculation and of the experiment, and not to indicate the true causes of electricity.

Like Newton, Coulomb is not making any hypothesis as to the true causes of electricity and magnetism.[40] His forte is experiment.

[34] [Coulomb, 1788d, p. 593] and Section 14.3, page 244 of this volume.
[35] [Bucciarelli and Dworsky, 1980, Chapter 4].
[36] See footnote 556 on page 238 of this volume.
[37] See Assis' Section 4.4, "Magnetic Researches", on page 42 of this volume.
[38] See page 135 of this volume.
[39] See Section 24.40 on page 418 of this volume.
[40] Coulomb's use of two fluids and of molecules as substrata for his explanation of electric and magnetic phenomena brought to mind the two ways engineers can analyze the flow of fluids, i.e. either by following the particles of the fluid (Lagrangian view) or by sitting at a point and figuring the mass flow, the velocity, at that location (an Eulerian view). See https://en.wikipedia.org/wiki/Lagrangian_and_Eulerian_specification_of_the_flow_field.

Chapter 2

Notice of the French Society of Physics when Coulomb's Works were Reprinted in 1884

Notice[41]

The Board of the French Society of Physics, following a proposal by Mr. Joubert,[42] its general secretary, has decided that it would be appropriate and in agreement with the intention of the founders of the Society, when there are available financial means, to publish a series of Memoirs related to physics, focusing our attention on reprinting in particular the Memoirs which were published around a century ago and which are difficult to find.

During the session of January 1883, the Society approved this proposal. Moreover, it decided that the choice of Memoirs to be published and the order of their publication would be decided by this Board. This Board, considering that Coulomb's Memoirs form the foundation of our present knowledge on electricity and magnetism, that the Collection of Memoirs of the old Academy [French Academy of Sciences], where the most important Memoirs had been published, is not widely known, decided that Coulomb's Memoirs should begin this series of reprints. Although many subjects are treated in these Memoirs, spanning electricity, magnetism and fluid resistance, they form a set in which the method and instrumentation are the same. The Board decided, therefore, to put them together, while proposing, in the following Volumes,[43] to bring together Memoirs relating to a single subject and written by different authors.

The publication of the present Volume has been entrusted to Mr. Potier.

[41] [Potier, 1884, pp. v-vi].

[42] Jules François Joubert (1834-1910).

[43] Volumes II and III were published in 1885, edited by Joubert, dealing with Electrodynamic Memoirs, [Joubert, 1885a] and [Joubert, 1885b].

Chapter 3

Introduction by Alfred Potier to the 1884 Reprint of Coulomb's Works

A. Potier[44]

Coulomb (Charles-Augustin de) was born on June 14, 1736, in Angoulême, to a family of magistrates. In his youth he showed a decided interest in mathematical sciences, joined the army military corps and was sent to Martinique, remaining there for nine years. Among the various projects on which he worked, he had to study several mechanical questions applied to constructions and buildings. His works on these topics earned him the title of Correspondent of the *Académie des Sciences*.

He then returned to France and, in 1779, shared the prize in the Academy of Science contest for the best method of making magnetic compasses with Van Swinden;[45] in 1781 he received the prize for the theory of simple machines. In this work he presented his classical experiments on friction.[46]

Called to Paris in 1781, he was nominated a Member of the Academy and worked actively on the laws of magnetism and electricity. During the period 1784-1789 he wrote the fundamental Memoirs on the laws of torsion, the laws of electric and magnetic interactions, together with the distribution of electricity and magnetism.

When the Revolution took place [1789], he was lieutenant-colonel of the Génie, general Intendant of the fountains of France, and was superintendent of the *Intendance des Plans et Reliefs*. He resigned from all of these positions. The Academy had been suppressed; he had been excluded from his membership on the Commission of Weights and Measures. Finally, forced to leave Paris by the law that expelled all nobles, he moved to the Blois region, together with his friend Borda.[47] He returned to Paris when the Institute was founded,[48] being appointed General Inspector of Studies. His health had been declining for some time when he died on August 23, 1806.[49]

Delambre,[50] when presenting his eulogy in 1807, announced that Coulomb's works would

[44] [Potier, 1884, pp. vii-xiii]. I inserted in footnotes some additional information related to Potier's text.
[45] Jan Hendrik van Swinden (1746-1823). See also [Licoppe, 1995].
[46] See, for instance, [Coulomb, 1821].
[47] Jean-Charles de Borda (1733-1799).
[48] The Academy of Sciences was abolished in 1793, being succeeded by the Institute of France in 1795.
[49] Coulomb's best biography with a detailed study of his works was written by Gillmor, [Gillmor, 1971a], see also [Gillmor, 1971b].
[50] Jean-Baptiste Joseph Delambre (1749-1822).

be published, that a note had been found in his papers indicating the order in which his several Memoirs should be placed. This project, which was to be carried out by Biot,[51] was not completed. In any event, its completion remains timely.

No doubt scientific progress, particularly in mathematical physics, will make some of Coulomb's reasonings seem less than rigorous and even singular; however, the laws which he deduced from these experiments are exactly the reason for this progress. As regards magnetism, Coulomb established the law of actions and reactions; as regards electricity, he not only established these laws, but showed that the internal surface of an electrified conductor is not charged with electricity, and that the electric density [on the external surface] is, in each point, proportional to the force;[52] he also noticed the sudden variation of the electric force, when passing from a point belonging to the surface to an external point which is infinitely close [to the surface]. Finally he demonstrated the proportionality to the speed of the resistance of fluids coming from their viscosity or internal friction and discovered the laws of torsional elasticity. His Memoirs therefore deserve the title of fundamental.

As regards especially electricity, his work is frequently reduced to the discovery of the law of attractions and his utilization of the proof plane.[53] Normally it is attributed to Biot, for instance, the experiment in which a metal sphere is completely discharged when it is maintained inside a conductor formed of two mobile hemispheres.[54] However, this experiment has been described by Coulomb in his Sixth Memoir (1788), page 382 of this Volume,[55] so clearly [presented] that we can not doubt that Coulomb modified the experimental conditions; it shows that the external envelop does not need to have the shape of the internal conductor. Coulomb returns several times (pages 319 and 349)[56] to this question and shows that the absence of electricity in the interior of the conductor is a necessary consequence of the law of repulsion. His demonstration is not rigorous, but Poisson,[57] in 1812, still did not know the modern classical demonstration.

Also to be found in Coulomb's Memoirs are all the elements of the demonstration of the proportionality of the force [acting] on a point [on the surface] of the conductor in relation to the [surface charge] density at this point. Initially in the [Sixth] Memoir of 1788 (page 383 of this Volume)[58] he presented the value of the attraction of a uniformly charged sphere [acting] on a point belonging to this surface, and the value of the attraction on an external point, and multiplied the density by the value 2π in the first case and [by the value] 4π in the second [case].[59] Moreover, in his theory of the proof plane (§XLIV and §XLV of the same Memoir),

[51] Jean-Baptiste Biot (1774-1862).

[52] That is, the electric force acting on an element of charge located on the surface of an electrified conductor in equilibrium is proportional to the surface charge density at this point.

[53] A detailed description of Coulomb's proof plane can be found in Section 7.2 (Charge Collectors) of Volume 1 of the book *The Experimental and Historical Foundations of Electricity*, [Assis, 2010b], [Assis, 2010a], [Assis, 2011], [Assis, 2015b] and [Assis, 2017]; together with Section 2.6 of Volume 2 of the same book, [Assis, 2018b], [Assis, 2018a] and [Assis, 2019].

[54] The internal metal sphere is discharged when it comes into contact with one of the external hemispheres, or when it touches this hemisphere through a metal wire.

[55] In the original, [Potier, 1884, p. ix]: *dans son cinquième Mémoire (1788), page 233 de ce Volume.* By a slip Potier wrote Fifth Memoir. However, this specific experiment appears on Coulomb's Sixth Memoir. Page 233 of Potier's book corresponds to page 382 of this English translation. See, in particular, Section 24.4 of the Sixth Memoir.

[56] Pages 178 and 205 of Potier's book, [Potier, 1884].

[57] Siméon Denis Poisson (1781-1840).

[58] Corresponding to page 233 of Potier's book, [Potier, 1884].

[59] See Section 23.4 on page 376 for a discussion of the factor 2π in Coulomb's formulas. See also foot-

he indicates clearly that the action of an electrified body on an external point infinitely close to it is twice the action of the surface element which is infinitely close, an action that is given by $2\pi y$, in which y is the [surface charge] density.[60] Coulomb regularly utilizes these formulas in the calculations of the Fifth and Sixth Memoir. Moreover, although the theorem which is being discussed here is nowhere explicitly enunciated, Sir W. Thomson[61] does not hesitate to call *Coulomb's Theorem* the proposition that the electric force, [acting] on an external point infinitely close to a conducting surface, is the product of 4π and the surface [charge] density in the neighborhood of this point.[62]

Beyond the Memoirs reproduced here, Coulomb left some manuscripts which were in Biot's hands, who summarized them in his *Traité de Physique*.[63] It was deemed unnecessary to reproduce the Memoir in which Coulomb examined if magnets act in other substances beyond iron, steel, nickel and cobalt; his conclusion was that traces of iron, insensitive to chemical analysis, would suffice to give to the metals which he studied (gold, silver, lead, copper and tin) the observed magnetism. Also not included, as foreign to the goal of the Society of Physics, a *Mémoire sur la Statique des voûtes*, the *Recherches sur les moyens d'exécuter sous l'eau toutes sortes de travaux hydrauliques, sans employer aucun épuisement*, his *Théorie des machines simples*, and his *Recherches sur les moulins à vent*.

Coulomb has always been concerned with the absolute value of the forces which he measured. His estimations, in the Memoirs before 1789, are given in the old system of measures. We have inserted, in parenthesis, after each value given by Coulomb, the value of the magnitude measured in units of the CGS system: that is, the number in parenthesis presents the lengths in centimeters, the masses in grammes and the forces in dynes. Here are the elements utilized for these calculations:[64]

1 toise = 6 feet = 72 inches = 864 lines = 194.9 cm.[65]
1 pound = 16 ounces = 9216 grains (mass) = 489.5 g.[66]
1 pound = 16 ounces = 9216 grains (force) = 480200 dyn.[67]

notes 796, 849 and 850 on pages 351, 383 and 383, respectively.

[60] That is, charge by unit area.

[61] William Thomson (1824-1907).

[62] For a discussion of Coulomb's theorem see also [Gillmor, 1971a, p. 209], [Heilbron, 1999, pp. 495-496] and [Blondel and Wolff, 2011c].

[63] [Biot, 1816a], [Biot, 1816b], [Biot, 1816c] and [Biot, 1816d].

[64] In the original:

1 toise = 6 pieds = 72 pouces = 864 lignes = 194,9 cm.
1 livre = 16 onces = 9216 grains (masse) = 489,5 gr.
1 livre = 16 onces = 9216 grains (force) = 480200 dynes.

[65] An English foot corresponds to 30.48 cm, while a French foot utilized in this book corresponds to 32.48 cm = 0.3248 m. The modern English inch corresponds to 2.540 cm. The French inch utilized in this book corresponds to 2.707 cm = 0.02707 m. An English line corresponds to 0.212 cm. The French line utilized in this book corresponds to 0.226 cm = 0.00226 m.

[66] That is, in terms of mass we have 1 pound = 489.5 g = 0.4895 kg; 1 ounce = 30.59 g = 0.03059 kg and 1 grain = 0.05311 $g = 5.311 \times 10^{-5}$ kg.

[67] That is, in terms of force we have 1 pound = 480200×10^{-5} newtons = 4.80200 N; 1 ounce = 0.300125

Coulomb represents the ratio of the circumference to the diameter sometimes by $c/2$, sometimes by $\varphi/2$, sometimes by $180°$.[68] To comply with modern usage, this ratio has always been represented by π, and this letter has not been employed for any other purpose; the utilization of the symbol $\int$ has been reserved for the expressions containing differentials.[69] The symbols a^2, R^2, ..., have been used instead of aa, RR. When, to avoid repetitions, we considered that we should remove a portion of Coulomb's text, we have always indicated this fact with small characters;[70] the same character was utilized for the observations suggested by the text; the numbering of paragraphs and Sections, on the other hand, has been maintained.

As regards the figures, which form separated plates in the Memoirs of the Academy, we have reproduced a certain number of them in wood, and inserted them into the text, in particular the purely geometric figures. Plates I and VII are photographic reproductions made on zinc of the plates of the *Mémoires de l'Académie*. The figures of the Memoir of 1789 had to be grouped in a different way to satisfy some format requirements; they were engraved in copper by Mr. Pérot, which reproduced the character with as much fidelity as the photography; they form Plate VIII.

It seemed useful to compare Coulomb's experiments with the results of Poisson's calculations, relative to the distribution of electricity on two conducting spheres and to the division of electricity between these two spheres, when they are put in contact. Therefore, side by side the numbers obtained by Coulomb, we placed Poisson's values deduced from his calculations and which are, to some extent, equally verifications of the fundamental law.

Poisson's work can be found in the Memoirs of the Institute for the year 1811, although read only on May 19 and August 3, 1812, for the first part, and on September 6, 1813, for the second part.[71]

By adopting the analysis of Laplace[72] for the attraction of spheroids, Poisson shows that the distribution on the surface of an ellipsoid should be such that a force and, consequently, the three partial derivatives of the function $V = \sum \frac{m}{r}$ should be null in the interior of the ellipsoid or, according to the modern language, the potential should be constant in the interior. However, he solves this problem only for an ellipsoid which is just a little different from a sphere and recovers the known solution that the electric layer should be contained between two similar ellipsoids. He then calculates, neglecting the square of the eccentricity, the force exerted on an electric mass infinitely close to the external surface, finding it proportional to the thickness.

> "It is natural to think", he says,[73] "that this is a general result, and that it also takes place on the surface of a conducting body of arbitrary shape; however, although this proposition seems very simple, it would be very difficult to demonstrate it by the

N; and 1 grain $= 5.2105 \times 10^{-4}$ N.

[68] See, for instance, the footnotes 275, 334 and 1065 on pages 120, 150 and 476, respectively.

[69] Potier, in particular, substitute in several places Coulomb's integration symbol $\int$ by the summation symbol $\sum$. In this English translation I have maintained the integration symbol $\int$ utilized by Coulomb.

[70] In this English translation I included the portions of Coulomb's text which had been suppressed by Potier in his reprint of 1884. That is, all papers presented here have been completely translated into English. When Potier presented some sentences which he himself had written in the middle of Coulomb's text, I decided to include Potier's text as a footnote indicated by [Note by Potier].

[71] [Poisson, 1812a], [Poisson, 1812b], [Poisson, 1813] and [Poisson, 1814].

[72] Pierre-Simon de Laplace (1749-1827).

[73] [Poisson, 1812a, p. 232].

formulas for the attraction of spheroids; this is one of those cases in which we should supplement the imperfect analysis by some direct consideration".

As a matter of fact, he presents a demonstration of this theorem, which Laplace communicated to him, which is just a more rigorous reproduction of the considerations utilized by Coulomb.

Like Coulomb, he establishes that the action of a conductor on an external point infinitely close [to the surface] is twice the action of the infinitely close portion of the surface, which is supposed to be limited by a plane parallel to the tangent plane. But a spherical conductor, uniformly charged, of which this infinitely small surface is a part, would exert a repulsion $4\pi y$, where y denotes the [surface charge] density. This force does not depend on the radius of the sphere. We can imagine for the normal as many planes as we wish, separated by dihedral angles ε, and replace each element of the surface of the conductor by an element considered on a sphere of convenient radius; the normal component of the attraction of this element is, therefore, $\frac{\varepsilon}{2\pi} \times 2\pi y = \varepsilon y$, no matter the radius of the sphere. The total attraction of the infinitely small surface, limited by a plane parallel to the tangent plane, will be then $y \sum \varepsilon$ or $2\pi y$, and the total electric force will be $4\pi y$.

The constant value of the potential in the interior of a conductor and the proportionality between the [surface charge] density and the force, or the derivative of the potential along the normal to the surface of the conductor, are the only two theorems utilized by Poisson; at this time, he had not yet demonstrated that the absence of electricity in the interior of conducting bodies, in electric equilibrium, is a consequence of the fundamental law.

The question of the attraction or repulsion between two electrified spheres is intimately connected to the distribution [of charges on the surfaces of the spheres]; it presents a practical interest related to the measure of the amounts of electricity or of the potentials. Poisson did not deal with these topics, but we can easily deduce from his formulas how to calculate the mutual action between two spheres.

It is thought that the goal proposed by the French Society of Physics would be served by inserting, after the reprint of Coulomb's Memoirs, a Note summarizing Poisson's Memoir, which is not well known, together with the recent works of Sir W. Thomson on the same subject. Sir W. Thomson takes for granted some general theorems, which are classic nowadays and which can be found in the general and specialized physics treatises; it was deemed unnecessary to reproduce the demonstration of these theorems. The reader who wants to study this question should consult, beyond the *Reprint of Papers* of Sir W. Thomson,[74] Plana's Memoirs in the VIIth Volume of the second series of the *Mémoires de l'Académie de Turin*;[75] these works contain numerical tables which can be reconstructed utilizing the formulas presented in the *Annexes*.[76]

[74] [Thomson, 1884].

[75] [Plana, 1845] and [Plana, 1854].

[76] These annexes summarizing the theoretical works of Poisson and Thomson were not included in this English translation.

Chapter 4

The Background to Coulomb's Researches

A. K. T. Assis

4.1 Electric Researches

In Coulomb's time there were two main systems or sets of ideas to explain electric phenomena.

One of these systems was based on attractive and repulsive forces acting at a distance between electrified bodies, in analogy with the law of universal gravitation due to Isaac Newton (1642-1727). It followed Newton's ideas presented in his two main publications, namely, the book *Mathematical Principles of Natural Philosophy*,[77] and in the book *Opticks*.[78] In this model the electrified bodies act directly on one another, without the mediation of any material agent between them.

In this Newtonian system there were two sub-systems. One of these sub-systems assumed the existence of two active kinds of electricity (or the existence of two active electric fluids). It was introduced mainly by Charles F. d. C. Du Fay (1698-1739) and Robert Symmer (c. 1707-1763).[79] Originally Du Fay called the two kinds of electricity vitreous and resinous, believing they were connected with the materials on which these phenomena were first detected. As the research was developed by other scientists, it was realized that any specific kind of material might be charged with the two kinds of electricity, depending on the material with which it was rubbed. Nowadays what was called vitreous electricity is called positive electricity. In this case we also say that a body has been positively electrified. What was called resinous electricity, on the other hand, is now called negative electricity. In this case we also say that a body has been negatively electrified. Du Fay also discovered the principle that bodies charged

[77] In English: [Newton, 1934] and [Newton, 1999]. In Portuguese: [Newton, 1990], [Newton, 2008] and [Newton, 2010].

[78] In English: [Newton, 1979]. In Portuguese: [Newton, 1996] and [Assis, 1998b].

[79] [Du Fay, 1733], [Du Fay, 1734] with Portuguese translation in [Boss and Caluzi, 2007]; [Symmer, 1759] and [Mitchell, 1759]. See also Sections 4.3 (Du Fay Recognizes Electrical Repulsion as a Real Phenomenon) and 5.2 (Du Fay Discovers Two Kinds of Electricity) of Volume 1 of the book *The Experimental and Historical Foundations of Electricity*, [Assis, 2010b], [Assis, 2010a], [Assis, 2011], [Assis, 2015b] and [Assis, 2017]; together with Section 1.2 (The Triboelectric Series) of Volume 2 of the same book, [Assis, 2018b], [Assis, 2018a] and [Assis, 2019]. See, moreover, [Fontenelle, 1741], [Heilbron, 1976], [Aepinus, 1979, pp. 198-202], [Heilbron, 1979], [Heilbron, 1981b], [Heilbron, 1982], [Borvon, 1994], [Benguigui, 1995], [Heilbron, 1999], [Borvon, s d], [Borvon, 2009] and [Blondel and Wolff, 2012].

with the same kind of electricity repel one another, while bodies charged with opposite kinds of electricity attract one another.

In the other Newtonian sub-system one assumes the existence of only one kind of electricity (or the existence of a single electric fluid). This second Newtonian sub-system is due to Benjamin Franklin (1706-1790).[80] Nowadays many expressions used in the science of electricity are due to Franklin like "plus and minus" or "positive and negative". Franklin was one of the most important scientists responsible for the establishment and utilization of the law of the conservation of electricity (or conservation of electric charges). Some expressions which he utilized to designate electric particles (or electrified particles) were "electric fire", "electric matter" and "electric fluid". He assumed the existence of a single electric fluid, instead of working with two kinds of electricity. He believed that there was a normal quantity or density of this electric fluid in all substances. A body positively electrified would be due to the fact that it had more electric fluid than the normal amount, that is, an excess relative to the normal density of electric fluid. A body negatively electrified, on the other hand, would be due to the fact that it had less electric fluid than the normal amount, that is, a shortage or deficiency relative to the normal density of electric fluid. Franklin emphasized, with his conception of the conservation of electricity, that electric charge was not generated nor produced in the friction between two substances, or in any other electrification process. The only thing that happened during friction was a redistribution or transference of electrified particles, of electric fluid or of electric charge. That is, one of the bodies received the same amount of electric fluid which had been lost by the other body.

Nowadays we adopt Franklin's nomenclature, while working with a different paradigm, that is, accepting the existence of two different kinds of electricity (or the existence of two types of electric charge, positive and negative). In this modern perspective we also accept the conservation of electricity. But now we say that in any electrification process beginning with two neutral bodies, one of these bodies becomes charged with one kind of electricity, while the other body becomes charged with the same amount of electricity of the other kind.

Franklin also assumed that there was repulsion between two electric fluids (or between two particles of electric matter). There would also be an attraction between the electric fluid and common matter. This common matter might be a body of any nature (water, glass, metal, wood etc.). When there is a normal amount of electric fluid distributed on common matter, this body (composed of common matter together with the electric fluid) will not act electrically on another body in the same state. Two positively electrified bodies will repel one another due to the excess of electric fluid in each one of them. However, Franklin's model leads to a singular assumption. One knows that there is repulsion not only between two positively electrified bodies, but also between two negatively electrified bodies. According to Franklin, a negatively electrified body has less electric fluid than the normal amount. Therefore, to explain with Franklin's model the repulsion between two negatively electrified bodies, it was necessary to assume repulsion between two particles of common matter, that is, between two arbitrary material bodies. For instance, it was necessary to assume repulsion between two pieces of metal, between two pieces of glass, between a piece of metal and a piece

[80] [Franklin, 1769], [Franklin, 1904], [Franklin, 1941], [Cohen, 1966], [Cohen, 1996], [Heilbron, 1999, Chapter XIV], [Morse, 2004b], [Morse, 2004a], [Silva and Pimentel, 2006], [Silva and Pimentel, 2008], [Blondel and Wolff, 2013c], [Moura and Bonfim, 2017], [Moura, 2018], [Moura, 2019], [Moura, 2020] and [Moura, 2023]. See also Section 11.7 (Gray, Franklin, the Power of Points and the Electric Nature of Lightning) of Volume 2 of the book *The Experimental and Historical Foundations of Electricity*, [Assis, 2018b], [Assis, 2018a] and [Assis, 2019].

of glass, etc. This singular assumption of Franklin's model always bothered many scientists, as it seemed to be against Newton's theory of universal gravitation according to which matter attracts matter with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. Heilbron, for instance, in several portions of his book *Electricity in the 17th and 18th Centuries*, discussed how several scientists tried to deal with the minus-minus repulsion in Franklin's system, that is, how they interpreted repulsion between two negatively electrified bodies.[81]

Franz Ulrich Theodor Aepinus (1724-1802) was one of the main scientists who adopted Franklin's point of view according to which there was a single electric fluid. He was a German scientist who worked in Russia for many years and performed original research on electricity and magnetism. His main work was published in 1759, *Essay on the Theory of Electricity and Magnetism*.[82] Aepinus had a great influence on Coulomb and is quoted in his publications.

The other system or set of ideas utilized to explain electric phenomena was based on the contact between material bodies. In this case the electric attractions and repulsions were explained by mechanical impulses produced by the collision of fluids or particles against the electrified bodies. In this model the electrified bodies would not interact directly with one another. The interaction was transmitted by fluids or particles emitted and absorbed by these bodies.

Several different models were proposed to explain by contact the interaction between two electrified bodies which are separated spatially from one another. Normally these models were based on some kind of mechanism or by mechanical contact, that is, assuming the existence of other bodies or of other particles beyond the two electrified bodies. Some of these models assumed the existence of material effluvia which were emitted and absorbed by electrified bodies. These effluvia or emanations, when they came in contact with the other electrified body, somehow would exert attractive and repulsive forces on this second body. These effluvia might be a stream of material particles or corpuscles emitted and absorbed by electrified bodies. In other models there would exist material vortices or whirlpools circulating around electrified bodies. These material vortices would exert attractive and repulsive forces on other electrified bodies when there was a contact between these vortices and these other electrified bodies. In another model there would exist a material atmosphere around each electrified body. When a second electrified body was located in the atmosphere of the first electrified body, it would feel an attractive or repulsive force exerted by this atmosphere. In other models the electrified bodies would emit a filament or some sticky matter that would exert attractive forces in other electrified bodies when these filaments or sticky matter came in touch with them.

I will not present the details of all these mechanical models. Nor will I indicate the names of the authors who defended these ideas of action due to material contact. There are a large number of scientists with many different explanatory models. I recommend the works of J. L. Heilbron and R. W. Home for anyone interested in these ideas.[83]

In his publications Coulomb always opposed these conceptions of electric effluvia or electric atmospheres to explain electric phenomena by mechanical contact.[84] He continued

[81] [Heilbron, 1999, pp. 337, 356-357, 372, 377-378, 387-389, 396, 446].

[82] [Aepinus, 1759] with English translation in [Aepinus, 1979]. See also [Heilbron, 1981a] and [Blondel and Wolff, 2013c].

[83] [Heilbron, 1979], [Heilbron, 1981c], [Heilbron, 1982], [Heilbron, 1999]; [Aepinus, 1979, pp. 3-224], [Home, 1981] and [Home, 1992].

[84] [Gillmor, 1971a, pp. 193-196, 205 and 214-219], [Gillmor, 1971b], [Aepinus, 1979, pp. 215-217],

the researches of John Michell (1724-1793), Aepinus and Johan Carl Wilcke (1732-1796) in favour of a Newtonian system based on attractive and repulsive forces acting directly at a distance to explain electric and magnetic phenomena.[85]

Coulomb accepted the Newtonian system to explain electric phenomena. At the same time he maintained that the sub-system of two fluids was mathematically equivalent to the sub-system of a single fluid to explain electric phenomena. Despite this fact, he preferred the system based on the existence of two electric fluids. For instance, in his Sixth Memoir on electricity and magnetism, he said the following in Section 24.40 when discussing the two kinds of electricity:[86]

> Whatever may be the cause of electricity, all its phenomena will be explained, and the calculation will be found to conform to the results of experiments, assuming two electric fluids, the parts of the same fluid repelling each other in inverse proportion to the square of the distances, and attracting the parts of the other fluid in the same inverse ratio of the square of the distances. This law was found by experiment for electric attraction and repulsion, in the First and Second Memoirs on Electricity, *volume of the Academy of 1785*;[87] according to this supposition, the two fluids in the conducting bodies always tend to unite until there is equilibrium, that is to say, until by their meeting, the attractive and repulsive forces compensate each other. It is the state in which all bodies are found in their natural state; but if by any operation whatsoever, a superabundant quantity of one of the electric fluids is passed into an insulated conducting body, it will be electrified, that is to say, it will repel the electric parts of the same nature, and will attract the electric parts of another nature than the superabundant fluid with which it is charged. If the electrified conducting body is brought into contact with another insulated conducting body, it will share with it the superabundant electric fluid in the proportions indicated in this Memoir and those which precede; but if it is made to communicate with a non-insulated body,[88] it will lose in an instant all its electricity, since it will share it with the globe of the Earth, whose dimensions relatively to it are infinite.
>
> Mr. Aepinus has supposed in the theory of electricity,[89] that there was only one electric fluid, whose parts mutually repelled each other and were attracted by the parts of bodies with the same force as they repelled each other. But to explain the state of bodies in their natural situation, as well as the repulsion in the two kinds of electricity, it is necessary to suppose that the molecules of bodies repel each other with the same force as they attract electric molecules, and that these electric molecules repel each other. It is easy to perceive that the supposition of Mr. Aepinus gives, as regards the calculation, the same results as that of the two fluids. I prefer that of the two fluids which has already been proposed by several physicists, because it seems to me contradictory to admit at the same time in the parts of the bodies, an attractive force in inverse ratio to the square of the distances demonstrated by universal gravity,

[Heilbron, 1999, pp. 95-96] and [Blondel and Wolff, 2011a].

[85] [Michell, 1750], [Michell and Canton, 1752], [Aepinus, 1759] with English translation in [Aepinus, 1979], [Wilcke, 1766]. See also [Grozier, 2013].

[86] Section 24.40 of [Coulomb, 1791]. This work is translated in Chapter 24.

[87] These Memoirs are translated in Chapters 11 and 14, respectively.

[88] That is, if it is grounded when it comes into contact directly with the Earth, or with a conducting body connected to the Earth.

[89] See footnote 182 on page 73. See also [Aepinus, 1759] with English translation in [Aepinus, 1979].

and a repulsive force in the same inverse ratio of the square of the distances; a force which would necessarily be infinitely large, relatively to the attractive action from which gravity results.

The supposition of the two fluids is, moreover, in conformity with all the modern discoveries of chemists and physicists, who have made us acquainted with different gases whose mixture in certain proportions suddenly and completely destroys their elasticity; an effect which cannot take place without something equivalent to a repulsion between the parts of the same gas which constitutes their elastic state, and to an attraction between the parts of the different gases which makes them suddenly lose their elasticity.

As these two explanations have only a greater or lesser degree of probability, I warn, in order to protect the theory that will follow from any systematic dispute, that in the supposition of the two electric fluids, I have no other intention than to present with the fewest possible elements the results of the calculation and of the experiment, and not to indicate the true causes of electricity. I will refer to the end of my work on electricity, the examination of the principal systems to which electric phenomena have given rise.

4.2 Coulomb's Electric Terminology

In this Section I present some expressions utilized by Coulomb to refer to electric fluids.

In the Second, Third and Sixth Memoir Coulomb utilized the expressions "electric mass", "mass of the electric fluid" and "mass of electricity" in order to express the total amount of charge contained in an electrified sphere.[90] Certainly these expressions were influenced by Newton's law of universal gravitation according to which the gravitational force is proportional to the product of the masses of the interacting bodies.[91]

An electrified particle (or an electric particle) is sometimes called an "electrified molecule" (or an "electric molecule") (*molécule électrique*).[92] Sometimes Coulomb refers to an electric particle in French as a "*partie électrique*".[93]

Many times Coulomb will utilize the expressions "electric density" or "density of an electric fluid" when referring to an electric body or electric molecule.[94] In some situations this concept of an electric density (or density of an electric fluid) will refer to what is called nowadays the amount of electric charge of a body or the amount of electric charge of each electrified particle. In the Fifth and Sixth Memoirs he studies the distribution of charges on the surface of conducting bodies. Normally in these situations these expressions (electric density or density of an electric fluid) will refer to the surface charge density, namely, to the amount of charge per unit area at each point on the surface of these conductors.

[90] See, for instance, footnotes 528, 530, 663, 689, 843 and 847 on pages 230, 231, 290, 300, 382 and 383, respectively. See also [Gillmor, 1971a, pp. 175, 187 and 191].

[91] [Newton, 2008, Book III].

[92] See, for instance, footnotes 622 and 912 on pages 257 and 419, respectively.

[93] See footnote 909 on page 419. See also footnotes 647, 730 and 733 on pages 283, 321 and 322, respectively. See also [Gillmor, 1971a, pp. 159, 201 and 217].

[94] See, for instance, pages 257, 284 and 318.

4.3 Some Basic Concepts of Magnetism

The local geographic meridian, usually called simply meridian, is a great circle passing by this place, containing the North and South poles of the diurnal rotation of the Earth relative to the set of fixed stars.

Consider a compass composed of a magnetized needle ns which is free to rotate in a horizontal plane around a vertical axis passing through the center of the needle. When it is released at rest in an arbitrary orientation, usually it will begin to oscillate around a certain direction and will point along this direction when it comes to rest relative to the ground. The straight line indicated by this needle in equilibrium (or the vertical plane passing through this needle) is called the local magnetic meridian. By convention, the extremity of a needle pointing approximately toward the geographic North pole of the Earth, is called the North pole of a needle, n. The South pole of the needle, s, is located at the other extremity of the needle. Let us consider two horizontal straight lines beginning at the center of the needle. One of these straight lines points along the needle toward its North pole n when the needle is in equilibrium at rest relative to the ground. The other straight line points from the center of the needle toward the geographic North pole of the Earth. The magnetic declination is the angle φ on the horizontal plane between these two straight lines: that is, the angle between the direction indicated by the North end of a magnetized compass needle, and the direction along a meridian passing through the geographic North pole, Figure 4.1.

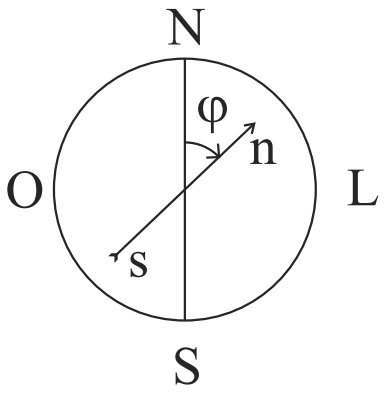

Figure 4.1: Magnetic declination.

By convention, declination is positive when magnetic North n is East of the true North N, and negative when it is to the West. That is, $\varphi > 0$ when the North pole of the needle points clockwise relative to the geographic North pole of the Earth and $\varphi < 0$ when it points anti-clockwise. This angle varies depending on the location on the Earth's surface. It also changes over time. For instance, the magnetic declination in October 2021 in Paris was approximately $+1°$, while in São Paulo it was approximately $-21°$.

The knowledge of the magnetic declination has always been extremely important in navigation and for orientation in general. In Coulomb's time it was known that the value of the magnetic declination in any specific location changes over the hours, days and years. It was important to know this phenomenon in greater detail. A needle supported over a pivot does not always point along the true magnetic meridian if there is a large friction at the point of support which prevents the free motion of the needle. Another method of suspension is the thread-suspended compass in which the needle is suspended horizontally by its center of gravity by a vertical thread. This method was known to Francesco Lana

de Terzi (1631-1687) in 1686, Lous in 1773 etc.[95] Coulomb began his magnetic researches studying the diurnal variation of the declination. He utilized thin silk threads to support his needle. However, the torsion of the thread may prevent the orientation of the compass along the true magnetic meridian. His studies on the force of torsion, or rotational moment, seem to have originated on these researches. He began studying hair and silk thread, and later on studied the torsion of metal wires.

A magnetized needle ns which is free to rotate on a vertical plane around a horizontal axis passing through its center of gravity is called a dip needle, Figure 4.2.

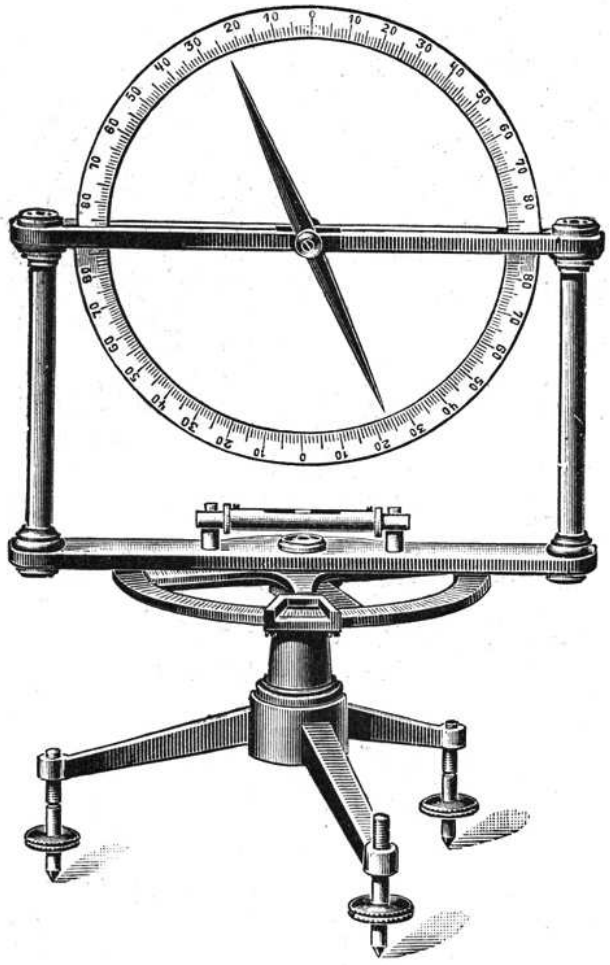

Figure 4.2: Dip needle or dip circle, [Müller-Baden (Editor), 1905, Fig. 265].

It is utilized with the horizontal axis of rotation of the needle orientated perpendicularly to the local magnetic meridian. When the dip needle is released at rest in an arbitrary orientation, normally it will begin to oscillate in a vertical plane around a specific direction, pointing along this direction when it comes to rest in equilibrium relative to the ground. The magnetic inclination θ, also known as magnetic dip θ, is the angle that the North pole n of the needle makes with the downward side of the horizontal plane, Figure 4.3.

By convention, $\theta > 0$ when the North pole of the needle points below the horizon, while a negative value of θ happens when n remains above the horizon. The magnetic dip changes

[95] [Gillmor, 1971a, pp. 141-142 and 162-165], [Hackmann, 1995] and [Heilbron, 1999, pp. 185-192 and 469].

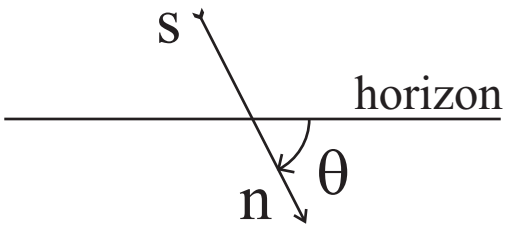

Figure 4.3: Magnetic inclination or magnetic dip.

at different points on the Earth's surface. It also changes over time in any specific location. For instance, the dip angle of Paris in October 2021 was approximately $+64°$, while in São Paulo it was approximately $-39°$.

4.4 Magnetic Researches

In Coulomb's time there were two systems or sets of ideas to explain magnetic phenomena, one based on the ideas of René Descartes (1596-1650) and another one based on Newtonian conceptions.

Descartes presented his model in the book *Principles of Philosophy* published in Latin in 1644. A French translation was published in 1647 under Descartes' supervision.[96]

Descartes tried to explain magnetic interactions (like the interaction between two magnets or the interaction between the Earth and a magnet) through a mechanism or by mechanical contact. That is, assuming that the action was due to contact and collision between material bodies. To this end he had to assume the existence of other material bodies (or particles) beyond the two magnets that were separated from one another. He assumed the existence of streams of particles passing through each magnetized body and circulating around it like vortices. He believed that the Earth, natural magnets, magnetized iron and steel would have two sets of pores or axial channels along their magnetic poles. One of these sets of channels would accept only right-handed particles, while the other set of channels would accept only left-handed particles. These particles would describe vortices or whirls around the magnetized bodies, entering through one extremity of these bodies and leaving from the other extremity. They would then circulate around the magnetized bodies until entering again through the first extremity. An example of these vortices appears on Figure 4.4.

In this case the Earth is represented by the central sphere D. The right-handed particles might enter the Earth, for instance, by its South pole A. They would pass through the Earth by internal channels, leaving the Earth at the North pole B. These particles would then circulate around the Earth, entering it again once more through A. They might, for example, describe the trajectory $ABFGEA$ or the trajectory $ABFHEA$. The left-handed particles, on the other hand, would move in the opposite direction, that is, passing through the Earth from B to A.

In this Figure 4.4 we can also see several spherical magnets I, K, L, M and N. The particles circulating around the Earth would pass through the pores or channels of these magnets. By colliding with these channels, they would orientate these magnets.

Non magnetic bodies, or bodies which cannot be magnetized, would be substances which

[96] [Descartes, 1647] with Portuguese translation in [Descartes, 1997] and [Descartes, 2002]. See also [Andrade, 2013].

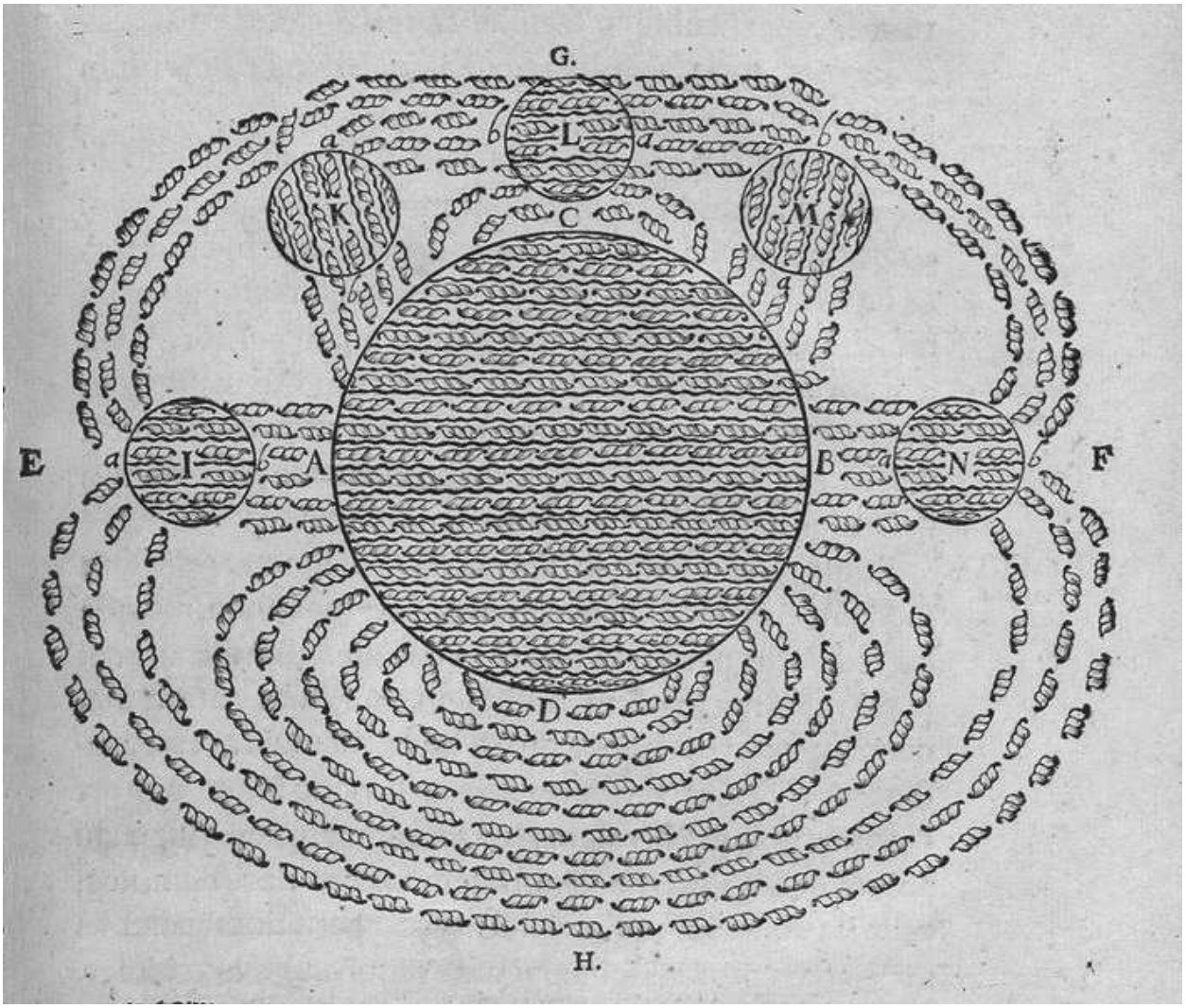

Figure 4.4: Descartes' vortices, [Descartes, 1647, Plate XIX, Figure 1].

did not have these internal channels. Due to this fact they would not react to the presence of the Earth or to the presence of magnets in their neighborhood.

In Coulomb's time this Cartesian theory of magnetism was still very popular. In 1742 and 1744 the Royal Academy of Sciences in Paris organized a competition for the best explanations for magnetism. There were no winners. Then in 1746 it was presented once more. There were now three prize winning essays due to Leonhard Euler (1707-1783); Étienne François Dutour de Salvert (1711-1789), also known as Du Tour; Daniel Bernoulli (1700-1782) and Jean Bernoulli, also known as Johann Bernoulli.[97] These authors followed Descartes ideas according to which magnetism was due to a vortex of a material fluid (or a flux of material particles) entering each magnet by one end and exiting at the other end, circulating around the magnetized body. The interaction between two magnets causing, for instance,

[97] [Euler, 1752], [Du Tour, 1752] and [Bernoulli and Bernoulli, 1752]. According to Home, [Aepinus, 1979, p. 145], the joint work between the Bernoulli's was made by Daniel and his father Jean I (1667-1748), also known as Johann I. According to Gillmor and Heilbron, [Gillmor, 1971a, pp. 177, 193 and 288 n. 10] and [Heilbron, 1999, pp. 31, 88, 93 and 575], this joint work was made by Daniel and his brother Jean II (1710-1790), also known as Johann II. In the published essay the authors appear as Daniel and Jean Bernoulli, so that it is not clear if the second author was Jean I or Jean II, [Bernoulli and Bernoulli, 1752, p. 115].

their mutual orientation, would be due to the action of the vortices of material fluid of one magnet interacting with the pores or channels of the other magnet, and vice versa. A discussion of these Cartesian systems to explain magnetism can be found in Gillmor's work.[98]

The second main system or set of ideas to explain magnetic phenomena in Coulomb's time was the Newtonian system. In this case magnetic phenomena are explained by assuming the existence of attractive and repulsive forces acting between magnetic particles. These forces would act directly at a distance along the straight line connecting the particles, following the principle of action and reaction.

There were two main sub-systems in this Newtonian approach. One of them assumed the existence of two kinds of magnetic fluids. One kind was called austral fluid, South magnetic fluid or simply South fluid. The other kind was called boreal fluid, North magnetic fluid or simply North fluid. Fluids of the same kind would repel one another, while fluids of opposite kind would attract one another. Anton Brugmans (1732-1789) and Johan Carl Wilcke (1732-1796) followed these ideas.[99]

The other sub-system in this Newtonian approach was adopted by Aepinus. In this second sub-system one assumes the existence of a single type of magnetic fluid. It was an adaptation of the electric system advocated by Franklin in which there would be a single electric fluid in nature. As regards magnetism, this approach assumes the existence of a single kind of magnetic fluid which repels other magnetic fluid, but attracts particles of iron, steel and their compounds. When there is a normal or natural amount of magnetic fluid in iron or steel, we say that these bodies are not magnetized. During magnetization of a steel bar by any procedure, part of the magnetic fluid would move to a portion of the bar, increasing its normal amount, while the other portion of the bar would remain with less magnetic fluid than the normal quantity. Aepinus called the portion of a magnetized steel bar in which there is an abundance of magnetic fluid (having more fluid than the normal amount) the positive pole. The negative pole would be the portion of the bar deprived of magnetic fluid (having less fluid than the normal amount).

However, it was known that magnetic poles of the same type repel one another, while opposite poles attract one another. To explain this fact Aepinus was led to a curious or singular assumption, namely, he assumed the existence of a repulsion between the negative portions of two magnetized steel bars (that is, between the portions with less magnetic fluid than the normal amount). Therefore he was obliged to assume a repulsive force between two iron particles.

But according to Newton's law of universal gravitation, matter attracts matter. Therefore this property of Aepinus' system was strange to many authors, as it seemed to go against Newtonian ideas. This violation bothered several physicists, including Coulomb.

Coulomb always opposed the use of the Cartesian system to explain magnetic phenomena.[100] He presented his points of view against vortices especially in his works of 1777 and 1793. The prize winning paper of 1777, published in 1780, is translated in Chapter 5. See, in particular, Section 5.6. His work of 1793 is the Seventh Memoir on electricity and magnetism, translated in Chapter 26. See, in particular, Section 26.25.

For instance, in his 1777 prize winning Memoir he said the following:[101]

[98] [Gillmor, 1971a, pp. 176-177, 193-194 and 214-219].

[99] [Brugmans, 1765], [Wilcke, 1766], [Gillmor, 1971a, pp. 180 and 215-216] and [Aepinus, 1979, pp. 200, 209-210 and 216-217].

[100] [Gillmor, 1971a, pp. 177-179, 193, 214-215 and 218] and [Blondel and Wolff, 2011a].

[101] See page 60 of this English translation.

> Therefore it seems that it follows from experiment, that it is not vortices which produce the different magnetic phenomena; and that, to explain them, we must necessarily resort to attractive and repulsive forces of the nature of those which we are obliged to make use of to explain the weight of bodies and celestial physics.

Coulomb attacked the Cartesian system and defended the Newtonian system based on attractive and repulsive forces acting at a distance. Initially he was in favor of Aepinus' sub-system which assumed the existence of a single magnetic fluid. In any event, he emphasized that both Newtonian approaches (that of Aepinus based on the existence of a single magnetic fluid and that of Brugmans and Wilcke which assumed the existence of two magnetic fluids) were mathematically equivalent in the explanation of magnetic phenomena. His final choice in favor of the Newtonian sub-system containing two magnetic fluids took place only in a paper read in 1799, after the publication of his Seventh Memoir on electricity and magnetism. In this paper of 1799 he said the following:[102]

> It follows from these experiments that, whatever the cause of magnetic phenomena, all these phenomena could be explained and subjected to calculation, assuming in the steel laminae or in their molecules, two magnetic fluids, the parts of each fluid repelling each other in direct proportion to their density, and in inverse proportion to the square of their distance, and attracting the molecules of the other fluid in the same ratio; so that each lamina of iron or steel contains in each molecule, before being magnetized, a sufficient quantity of both fluids to saturate or balance each other, that the two fluids thus combined no longer exert any action on each other.

When one reads the works of Coulomb and other scientists of the same period, one realizes that they assumed that any specific portion of a magnet or magnetized needle might contain a single type of magnetic fluid. That is, this portion might possess more North fluid than South fluid (if one assumes the existence of two magnetic fluids), or this portion might be positively magnetized (if one assumes the existence of a single magnetic fluid, with this portion of the material having more magnetic fluid than the normal amount). When a steel needle was magnetized by another magnet, these authors assumed that the magnetic fluid might move along the needle, concentrating in one portion of the needle and leaving the other portion with a deficiency of fluid.

Consider a magnetized cylindrical piece of steel $A0B$. Figure 4.5 shows ideal distributions of the volume density ρ of North and South magnetic fluids as a function of the longitudinal coordinate x. Configuration (a) presents a simplified model in which the left half $A0$ has a constant magnetic density $-\rho_0$, while the right half $0B$ has a constant magnetic density $+\rho_0$. Configuration (b) presents another model which was assumed by some of Coulomb's contemporaries in which the density increased linearly from the center toward each end of the magnet, being positive in one side and negative in the other side. Configuration (c) presents the model in which the North and South fluids were concentrated only at the extremities of the magnet.

Coulomb tried to estimate the distribution of magnetic fluid along a magnetized needle, with the density of magnetic fluid varying from point to point. If this were the case, it would be possible to separate the two magnetic fluids by breaking the needle at its center. One half of the needle would have an excess of North fluid relative to the South fluid in

[102] See page 499, Chapter 28 of this English translation. See also [Gillmor, 1971a, p. 216].

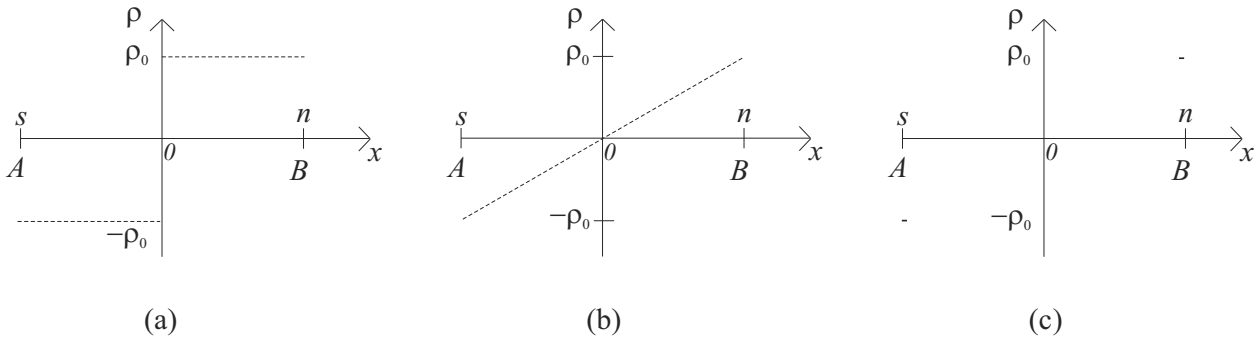

Figure 4.5: Distributions of magnetic fluids along a magnetized cylinder $A0B$. (a) Constant and opposite volume densities $\pm\rho_0$ in each half of the cylinder. (b) Densities increasing linearly from the center of the cylinder toward each extremity. (c) North and South magnetic fluids concentrated only at the extremities of the magnet.

the two-fluid model, or it would be positively magnetized in the one-fluid model (having an excess of fluid in relation to the normal amount). The other half of the needle would have an excess of South fluid relative to the North fluid in the two-fluid model, or it would be negatively magnetized in the one-fluid model (having a deficiency of fluid in relation to the normal amount). Therefore, by separating the two halves of the broken needle, the Earth might exert a net force different from zero when acting on each half of the needle. However, this does not take place in reality. Experiments show that any piece of a broken magnet feels only a pair of equal and opposite forces exerted by the Earth and producing a torque on it. That is, there is no net force exerted by the Earth and acting on any piece of a broken magnetized needle. Petrus Peregrinus, also known as Pierre de Maricourt, knew since 1269 that when one breaks a magnet in two halves, each half becomes a new magnet with two equal and opposite magnetic poles.[103]

Probably due to this fact, in 1777 Coulomb began thinking in molecular terms:[104]

> Each point of a magnet or of a magnetized bar can be regarded as the pole of a tiny magnet...

In his Seventh Memoir, in Section 26.29, Coulomb called attention to the fact that when a magnet is broken into pieces, each piece acts as a little magnet. In Section 26.30 he then developed a new theory of magnetism based on molecular polarization. In this model the North fluid, for instance, cannot move along a magnetized needle, but only inside each steel molecule, the same happening with the South fluid. These fluids could not pass from one molecule to another. In the natural state these fluids would be joined together and the molecule would not be magnetized. When a steel needle is magnetized, there would be a separation of the two magnetic fluids only inside each steel molecule, so that these molecules would become magnetically polarized. His words:[105]

> I believe that we could reconcile the result of the experiments with calculation by making some changes in the hypotheses; here is one which appears able to explain all the magnetic phenomena of which the preceding experiments have given precise

[103] [Peregrinus, 1904], [de Grave and Speiser, 1975], [Sparavigna, 2016] and [Martins, 2017].
[104] See Article 23, page 72, and [Gillmor, 1971a, pp. 180-181 and 216-217].
[105] See page 479 of this English translation and also [Gillmor, 1971a, pp. 180-181 and pp. 216-218].

measurements. It consists in supposing in Mr. OEpinus' system that the magnetic fluid is contained in each molecule or integral part of the magnet or the steel; that the fluid can be transported from one extremity to the other of this molecule, which gives each molecule two poles, but that this fluid cannot pass from one molecule to another. Thus, for example, if a magnetized needle was of a very small diameter, or if, *Figure 7*, each molecule could be regarded as a small needle whose North end would be united to the South end of the preceding needle, there would be only the two ends n and s of this needle which would give signs of magnetism, because it would be only at the two ends where one of the poles of the molecules would not be in contact with the opposite pole of another molecule.

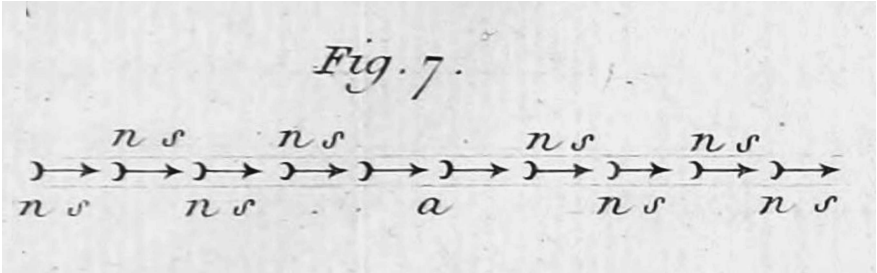

Figure 4.6: Coulomb's model of molecular magnetic polarization.

Coulomb's model was later important to J.-B. Biot, S. D. Poisson and René Just Haüy (1743-1822).[106] Poisson, for instance, said the following:[107]

> This opinion, very singular upon first glance, is, however, that which has generally prevailed.

Coulomb's molecular polarization model was also important to André-Marie Ampère (1775-1836) and Augustin-Jean Fresnel (1788-1827). However, these two authors changed the idea of molecular polarization to the assumption that magnetism was due to microscopic electric currents flowing in a plane normal to the magnetic axis of each molecule or particle of a magnetized substance. A discussion of Ampère's molecular currents can be found in the book *Ampère's Electrodynamics — Analysis of the Meaning and Evolution of Ampère's Force between Current Elements, together with a Complete Translation of His Masterpiece: Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience*.[108]

4.5 Coulomb's Magnetic Terminology

In French the magnetic fluid is called "*fluide magnétique*" or "*fluide aimantaire*".[109] The boreal or North fluid is called "*fluide boréal*", while the austral or South fluid is called "*fluide austral*".[110]

A "magnetic particle" or "magnetic molecule" is called "*molécule magnétique*", "*molécule aimantaire*" or "*partie magnétique*".[111] This magnetic particle or molecule would be a parti-

[106] [Gillmor, 1971a, pp. 175 and 217-218].
[107] [Poisson, 1822, p. 250] and [Gillmor, 1971a, p. 218].
[108] [Assis and Chaib, 2011] and [Assis and Chaib, 2015]. See also [Chaib, 2009].
[109] See, for instance, footnotes 147, 148 and 310.
[110] See footnotes 306 and 307.
[111] See, for instance, footnotes 181, 595, 606, 624, 998 and 1110.

cle having only one kind of magnetic fluid. It would be equivalent to what is called nowadays a magnetic monopole.

The extremities of magnetized needles are called magnetic poles. The extremity pointing approximately toward the geographic North pole of the Earth is called a North pole, "*pôle boréal*", "*pôle boréal N*", "*pôle N*" or "*pôle nord*". The extremity pointing approximately towards the geographic South pole is called a South pole, "*pôle austral*", "*pôle austral S*", "*pôle S*" or "*pôle sud*".[112] The magnetic pole of a needle is also called a center of action, "*centre d'action*", center of magnetic action, "*centre d'action magnétique*", or center of gravity of the curve of magnetic densities, "*centre de gravité de la courbe des densités magnétiques*".[113]

Sometimes Coulomb refers to the "density of magnetic fluid" or to the "magnetic density".[114] In some situations these expressions refer to the total amount of magnetic fluid contained in a particle or in a magnetic pole. In other situations these expressions refer to the linear, surface or volume density of magnetic fluid, that is, to the amount of magnetic fluid per unit length, area or volume, respectively.

The quantity of magnetic fluid in a particle is also indicated by the expressions "mass intensity", "magnetic intensity" and "mass of the magnetic fluid".[115]

Many times he utilizes the expression "*momentum magnétique*". In some situations this expression may refer to the "magnetic moment" of a magnetized needle, that is, to the degree of magnetization of this needle. In other situations it can refer to the "magnetic torque" exerted by the Earth on this needle. This torque is proportional to the magnetic moment or to the magnetization of the needle.

4.6 Magnetization Methods: Magnetization by Contact, Simple Touch, Divided Touch and Double Touch

To contextualize Coulomb's work, I present here some magnetization methods.[116] These methods were used to produce artificial magnets. The main methods in Coulomb's time, with several variations, were called magnetization by contact, single touch, divided touch and double touch. John Michell (1724-1793) created the expressions single and double touch in 1750.[117]

Magnetization by contact is very old and was well known much before Coulomb. Petrus Peregrinus, for instance, in his letter on the magnet of 1269 mentioned that an iron or steel needle can be magnetized by touching a natural magnet.[118] When the North pole of a magnet touches the extremity of a needle for some time, this extremity becomes a South pole, and vice versa. Consider a non magnetized piece of iron. When it touches a strong magnet for some time, or simply remains close to this magnet for some time, this piece of iron becomes

[112] See, for instance, pages 51, 56, 70 and 242.
[113] See footnotes 568, 1043 and 1046.
[114] See, for instance, pages 137, 248, 257, 467 and 479.
[115] See footnotes 129, 998 and 1072.
[116] [Brewster, 1837, Chapter X: Account of the different methods of making artificial magnets, pp. 283-320], [Aepinus, 1979, pp. 179-182], [Dhogal, 2008, pp. 88-89], [Ricker III, 2011] and [Martins, 2017, Notes 23 and 45].
[117] [Michell, 1750, pp. 8 and 36-37] and [Michell and Canton, 1752].
[118] [Peregrinus, 1904], [de Grave and Speiser, 1975], [Verschuur, 1996, p. 9], [Bacha and Vannucci, 2014], [Sparavigna, 2016] and [Martins, 2017].

magnetized. However, when separated from the strong magnet, it loses all of its magnetism. If the same procedure is repeated with a piece of steel, this piece will remain magnetized for a long time after being separated from the magnet. The extremity of the steel object which touched the North pole of the magnet becomes a South pole, while the other extremity of the object becomes a North pole, Figure 4.7.

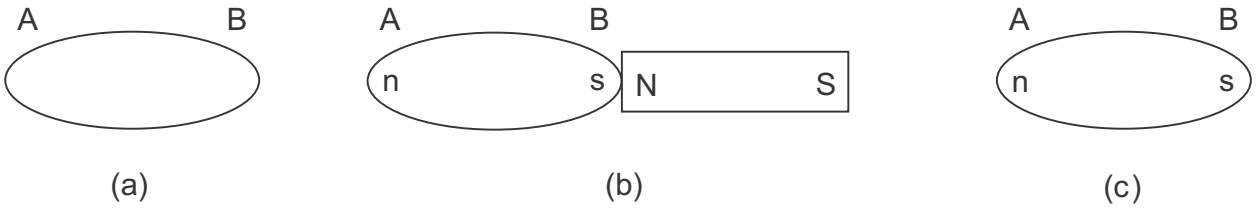

Figure 4.7: Magnetization by contact. (a) An initially non magnetized steel object AB. (b) Magnetization of AB when it remains in contact with a strong magnet NS for some time. (c) The object AB remains magnetized after being separated from the magnet.

Peregrinus seems to have been the first person to utilized the word "pole" in relation to magnetism. He also seems to have been the first scientist to mention that opposite poles attract one another, showing also three methods to localize the poles of a spherical magnet.[119]

Figure 4.8 presents a simplified representation of the three magnetization methods when there is motion between the magnet and the object to be magnetized.

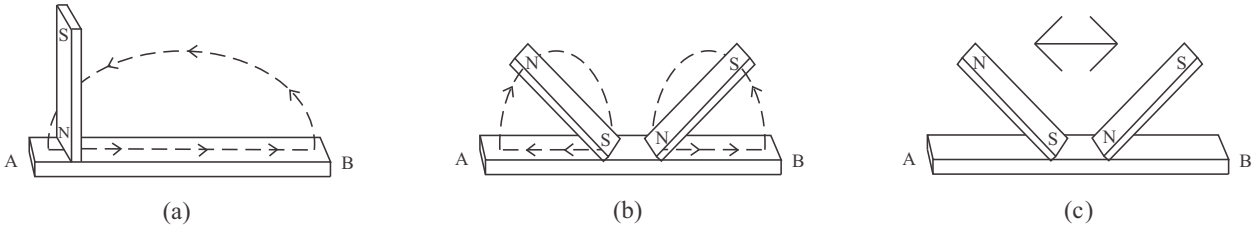

Figure 4.8: (a) Magnetization by single touch, (b) by divided touch, and (c) by double touch.

In the single touch method a single pole of a magnet is utilized to magnetize a steel needle or bar. This method was also known long before Coulomb. Normally a single pole of a magnet is rubbed in one direction along a steel bar AB in order to magnetize it, Figure 4.8 (a). This process is repeated many times on both faces of the bar AB, increasing its magnetization, until it reaches a saturation point. Two magnets are utilized in the divided touch method. They are placed with their opposite poles on the center of the steel bar AB and move in opposite directions towards the extremities of the bar, Figure 4.8 (b). This process is also repeated many times on both faces of the bar. The double touch method also utilizes two magnets. However, in this case both magnets remain together with a fixed distance between their extremities, as they are rubbed in both directions along the bar, Figure 4.8 (c). That is, instead drawing the poles apart at each stroke, they are held a small fixed distance apart and moved together back and forth several times along the bar.

We now present these three methods in greater detail.

Suppose we want to magnetize a steel bar AB. In the single touch method this bar is fixed on a support on the laboratory and a permanent magnet NS is placed vertically above it, Figure 4.9 (a). This magnet is rubbed on the surface of the bar, always in the same direction

[119] [Araújo, 2010] and [Martins, 2017].

(for instance, from A to B) with the same pole of the magnet (N for example) sliding on the bar. When the magnet reaches the end B, it is raised, placed at A and the procedure is repeated a few times. The same procedure is done on both faces of the bar. At each repetition this process increases the magnetization of the bar, until it reaches a saturation point when the acquired magnetism no longer increases. At the end of this process extremity A of the steel bar acquires a North pole n, while extremity B acquires a South pole s, Figure 4.9 (b):

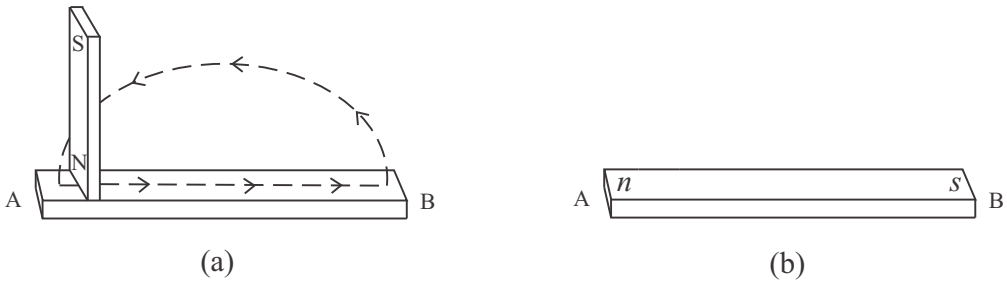

Figure 4.9: Single touch method. (a) Magnet NS is rubbed a few times against the steel bar AB. (b) Extremity A acquires a North pole n, while extremity B acquires a South pole s.

In a variation of this process the magnet NS is rubbed in an inclined orientation relative to the bar, Figure 4.10.

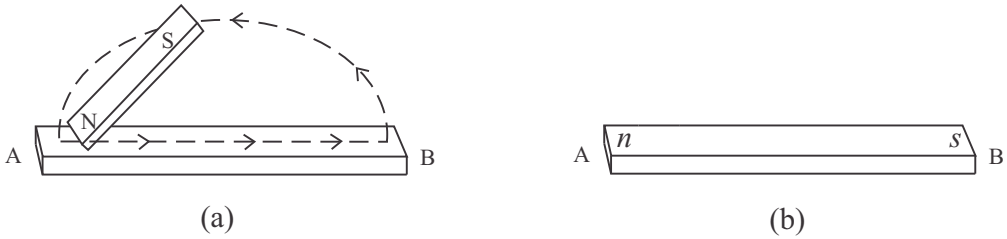

Figure 4.10: Single touch method. (a) The inclined magnet NS is rubbed a few times against the steel bar AB. (b) Extremity A acquires a North pole, while extremity B acquires a South pole.

It is also possible to keep the magnet at rest relative to the laboratory, while the steel bar is rubbed against it. Consider a bar AB with its extremity A above the North pole of a vertical magnet NS. We rub the bar against the North pole of the magnet, from A to B, Figure 4.11 (a). When extremity B reaches the end of the magnet, we raise the bar, place extremity A above the North pole and repeat this process a few times. At the end of this procedure extremity A acquires a North pole n, while extremity B acquires a South pole s, Figure 4.11 (b).

During the XVIIIth century there was a revolution in magnetization methods. New magnetization procedures were devised, creating stronger and more homogeneous magnets. These developments were essential for Coulomb's researches.

The divided touch method due to Gowin Knight (1713-1772) was utilized during the 1740s. The final result was presented to the *Royal Society*, although the method was kept secret during his life for commercial reasons, being published only after his death.[120] It is

[120] [Knight, 1744a], [Knight, 1744b], [Knight, 1746], [Knight, 1754], [Brewster, 1837, Chapter X: Account of the different methods of making artificial magnets], [Aepinus, 1979, pp. 158-160, 179 and 390], [Fara, 1995] and [Fara, 1999].

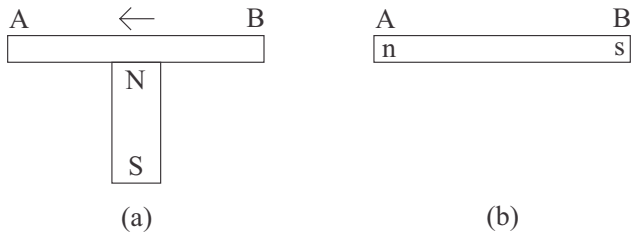

Figure 4.11: Single touch method. (a) The bar AB is rubbed from A to B against the North pole of a magnet. (b) Extremity A acquires a North pole, while extremity B acquires a South pole.

shown in Figure 4.12.

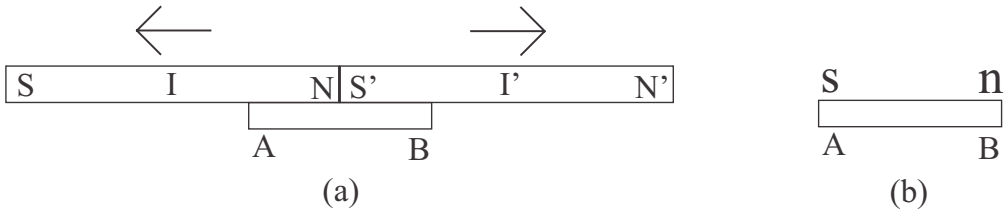

Figure 4.12: Knight's procedure.

The steel bar AB to be magnetized is placed under the opposite poles of magnets I and I', Figure 4.12 (a). These poles are then rubbed in opposite directions against the bar, the pole N moving towards A and the pole S' moving towards B. This procedure is repeated a few times. At the end of the procedure, extremity A acquires a South pole s, while extremity B acquires a North pole n, Figure 4.12 (b).

Figure 4.13 illustrates the divided touch method due to Henri-Louis du Hamel du Monceau (1700-1782), known as Duhamel's method.[121]

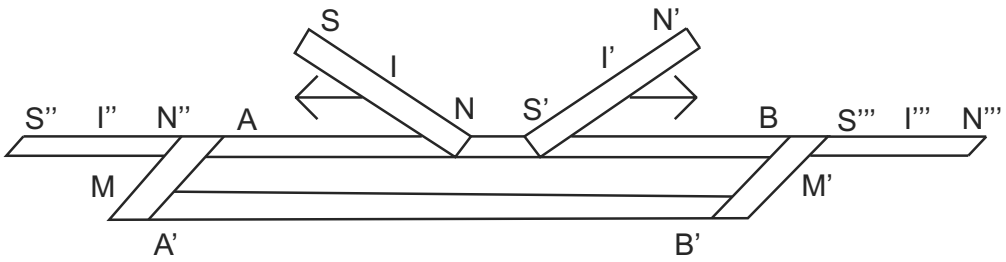

Figure 4.13: Duhamel's method.

AB and $A'B'$ are two steel bars to be magnetized. They remain parallel to one another between two pieces of soft iron M and M', called the armors or armatures, orthogonal to them and forming a rectangle. Two magnets I'' and I''' are aligned with AB with their poles as indicated in Figure 4.13. He utilizes two strong magnets I and I' with opposite lower poles. Pole N of magnet I is placed toward pole N'' of magnet I'', while pole S' of magnet I' is placed toward pole S''' of magnet I'''. Magnets I and I' can be orthogonal or inclined relative to bar AB, as shown in the Figure. They are rubbed against this bar by being slid in opposite directions toward the extremities of the bar. Magnets I and I' are again placed

[121] [Du Hamel, 1745], [Du Hamel, 1750], [Brewster, 1837, pp. 285-287] and [Aepinus, 1979, pp. 179-180].

at the center of AB and the whole procedure is repeated a few times, Figure 4.13. Lamina AB is turned upside down and this procedure is repeated to magnetize its lower side.

The same method is applied with bar $A'B'$, but now magnet I' is placed with its lower pole S' toward A', while magnet I is placed with its lower pole N toward B'. Moreover, magnets I'' and I''' are placed along $A'B'$ with inverted poles. Magnets I and I' are rubbed against $A'B'$ by being slid in opposite directions a few times toward its extremities, Figure 4.14.

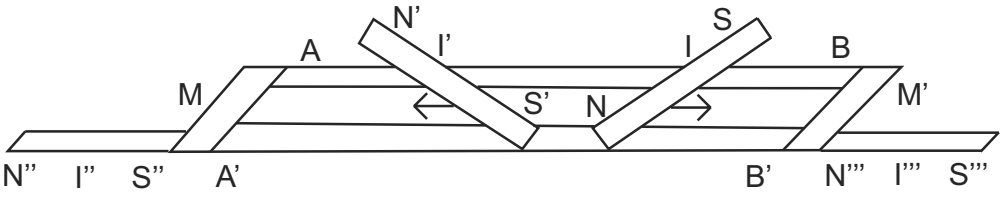

Figure 4.14: Duhamel's method.

It is also possible to replace magnets I and I' by two sets of magnetized laminae. In each set the poles of the same type are fixed together, forming a single magnet.

At the end of Duhamel's method, bar AB becomes magnetized with poles n and s, while bar $A'B'$ becomes magnetized with poles n' and s', as indicated in Figure 4.15.

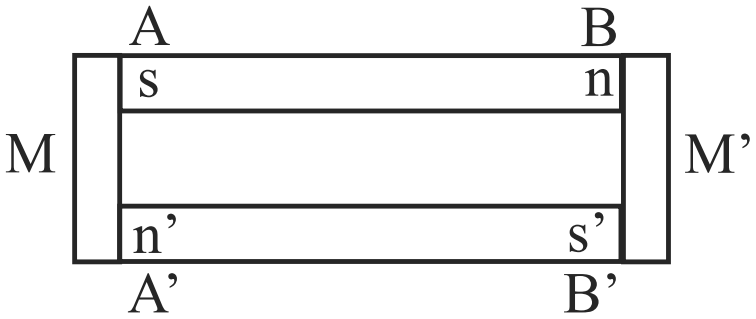

Figure 4.15: Final result of Duhamel's method.

Figure 4.16 illustrates the double touch method due to Michell and John Canton (1712-1772).[122]

A set of three magnetized laminae CD is placed over bar AB to be magnetized. This bar can be divided into three pieces as shown in the Figure, or in six pieces as described by Michell. Let us assume that the South pole of laminae CD is located at D, touching AB. Another set of magnetized laminae EF is placed with its opposite pole on AB. With our assumption we would have the North pole of these laminae EF located at F, touching AB. There is a small separation between the lower poles D and F. This system of six laminae is slid together a few times back and forth between the extremities A and B of the steel bar, with a fixed separation maintained between the two sets of three laminae. Bar AB is turned upside down and the same procedure is repeated a few times. At the end of the process, the bar AB acquires a North pole n at extremity A and a South pole s at extremity B, Figure 4.17 (a) and (b).

[122] [Michell, 1750], [Canton, 1752a] with German translation in [Canton, 1752b], [Michell and Canton, 1752], [Brewster, 1837, pp. 287-288], [Aepinus, 1979, pp. 180-182 and 373], [Fara, 1995] and [Reich and Roussanova, 2022].

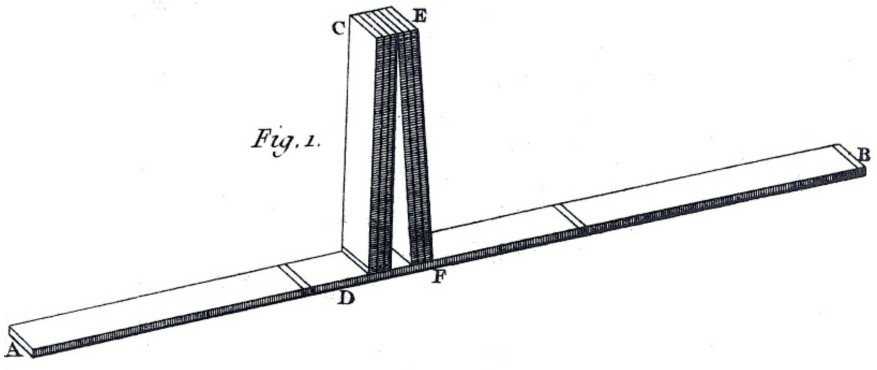

Figure 4.16: Michell's double touch method.

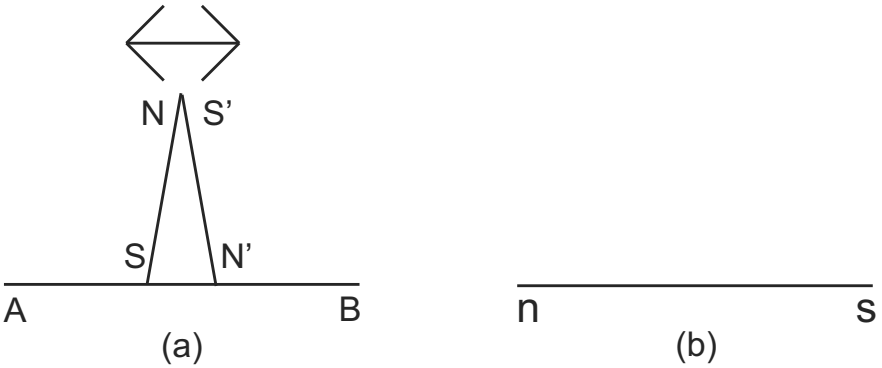

Figure 4.17: Illustration of Michell's method.

Figure 4.18 illustrates the method of double touch due to Aepinus (1724-1802) and Antheaulme.[123]

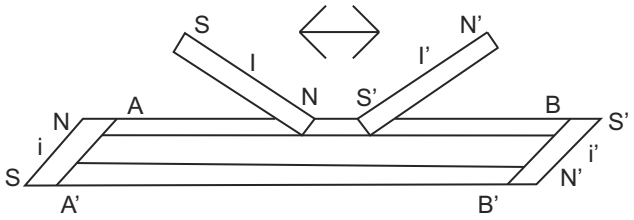

Figure 4.18: Aepinus' double touch method.

Aepinus placed the two steel bars to be magnetized between two magnets i and i' with inverted poles, Figure 4.18, instead of placing them between two iron pieces as in Duhamel's method. Magnets I and I' utilized to magnetize the steel bars were placed above the center of one of these bars. However, instead of being almost orthogonal to this bar as in Michell's method, they were now very inclined relative to it, with inclinations of only 15 or 20 degrees, with lower poles of opposite kinds. This inclination was the great advancement relative to Michell's method. There was a fixed small separation between magnets I and I'. Moreover, instead of I and I' being moved in opposite directions toward the extremities of the bar, magnets I and I' were now moved together back and forth between the extremities of the

[123] [Aepinus, 1759] with English translation in [Aepinus, 1979, pp. 181-182 and 380-383], [Antheaulme, 1760] and [Brewster, 1837, pp. 291-292].

bar. These magnets I and I' remained inclined relative to the bar, while a small piece of wood connecting their lower extremities maintained them at a constant distance while they were rubbed against the bar. The bar AB was turned upside down and the whole procedure was repeated. The same procedure was performed with bar $A'B'$, but now magnet I' was placed toward A', while magnet I was placed towards B'.

At the end of the process, extremity A acquired a South pole s, while extremity B acquired a North pole n. Bar $A'B'$ became magnetized with opposite poles, Figure 4.19.

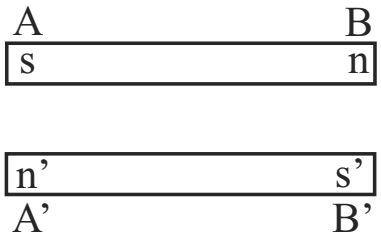

Figure 4.19: Final result of Aepinus' method.

It is also possible to replace the two magnets I and I' with two sets of magnetized laminae. In each set the laminae are fixed to one another with poles of the same kind placed together.

Coulomb improved the double touch method due to Antheaulme and Aepinus.[124] With his procedure Coulomb obtained long artificial magnets with strong, well defined poles. These artificial magnets were essential in order to obtain the force law between magnetic poles. For instance, in his Second Memoir he utilizes cylindrical magnetized bars 54 to 68 cm long (20 to 25 inches) with a diameter of 0.3 cm (1.5 line). They were made of an excellent steel, uniformly magnetized following the double touch method of Michell and Canton, as improved by Antheaulme and Aepinus. Coulomb showed that their centers of action, or magnetic poles, were concentrated in very small regions located approximately at 2.3 cm (10 lines) from each extremity. He then utilized his torsion balance to measure the force between two magnetic poles and showed that it was inversely proportional to the square of their separation.

[124] [Aepinus, 1979, pp. 179-182] and [Heilbron, 1999, pp. 91-96, 373 and 469-470].

Chapter 5

Investigations of the Best Method of Making Magnetized Needles, of Suspending Them, of Ensuring that They are in the True Magnetic Meridian; Finally, of Explaining Their Regular Diurnal Variations

Coulomb[125]

While all the parts of the Earth are united by their respective needs and by the exchange of excess goods; while whole armies and nations span and inhabit the seas: scholars as for their love of the public good as for their wisdom, offer to the researches of physicists and mathematicians, the perfection of the instrument which directs the paths of ships; which, placed in the center of a vast and uniform horizon, traces a line whose direction is known: it is to serve humanity and our homeland to respond to their views and to focus our attention on such a useful object.

5.0.1 Definitions and Principles

1. If we suspend a magnetized needle by its center of gravity, around which we suppose that it can turn freely in all directions, it will take a fixed direction, so that, if we move it away from this direction, it will always be brought back to it while oscillating.

If, through the direction of this needle, we pass a vertical plane, this plane will be the meridian of the compass, or in other words the magnetic meridian. The angle formed by this plane with the true meridian of the world,[126] will be the declination of the compass.

[125] [Coulomb, 1780] with Portuguese translation in [Assis, 2022]. This work published in 1780 shared the 1777 award from the French Academy of Sciences related to the investigations of the best method of making magnetic needles, [Gillmor, 1971a, pp. 176-182].

[126] That is, with the local geographic meridian, the vertical plane that contains the Earth's axis of rotation in relation to the stars.

If through the point of suspension of the needle, we pass a horizontal plane, the angle formed by the direction of the needle with this plane, will be the inclination of the compass.

One distinguishes in the magnetic needles, their extremities with the names of *poles*. The extremity which points almost to the [geographic] North is called the boreal pole. The extremity which points roughly towards the [geographic] South is called the austral pole.[127] The poles of the same name of different magnets or needles appear to exert on each other a repulsive force. The poles of different names appear to exert an attractive force.

Steel blades are only able to take on a certain degree of magnetism which they cannot exceed. Having reached this point, they are said to be magnetized to saturation.

5.0.2 First Fundamental Principle

2. If, after having suspended a needle by its center of gravity, we move it away from the direction that it takes naturally, it is always brought back by forces which act parallel to that direction and which are different for different points along the needle, but which are the same for each of these points separately, in whichever orientation the needle is placed in relation to its natural direction; so that a magnetized needle always experiences the same action, in any position, due to the magnetic forces of the Earth.[128]

Development of This Principle

The globe of the Earth is a natural magnet which, by its action, produces the orientation of the compass. If we suppose that the magnetic forces are attractive or repulsive forces placed inside the globe of the Earth, the centers of these forces will be at a distance which may be regarded as infinite relative to the length of the compass. But, as the action of the attractive or repulsive forces, depends on the nature and the intensity of the masses,[129] and on a function of the distance, the distance being able to be supposed the same, in any position which we place the compass, and each point of this compass taken separately, experiences no variation with change of position with respect to the constitution of its parts, it follows that each of the points of the needle will be solicited by a force whose direction will always be the same, and the intensity will be independent of the orientation of the needle.

The experiments of Mr. Musschenbroek,[130] and those of Wiston,[131] cited by the same

[127] In the original: *pole boréal* and *pole austral*. The word *boreal* refers to things that belong to the North of the Earth or that are relative to the Northern hemisphere. The word *austral* refers to things that belong to the South of the Earth or that are relative to the Southern hemisphere. Nowadays we call the North Pole of a bar magnet (or of a compass) to its extremity that points approximately towards the geographic North pole of the Earth. Therefore, Coulomb's boreal pole would nowadays be called the North pole of the needle, while its austral pole would be called the South pole of the needle.

Some other authors have adopted a definition opposite to Coulomb's for the denomination of the poles of a magnet, calling the tip of the needle that is directed approximately towards the geographic North pole of the Earth as the austral pole of the needle.

There has never been a consensus on the name of the magnetic poles of a magnet.

[128] A discussion and illustration of this first fundamental principle can be found in Section 6.1.

[129] In the original: *intensité des masses*. Newton's gravitational force of attraction is proportional to the product of the masses of the interacting particles. In this case Coulomb is referring to the magnetic force exerted by the Earth on the different points of a magnetized needle. What he calls here the "intensity of the masses" seems to be referring to the product of the quantities of magnetic fluid contained in the particles that would be interacting by magnetic forces.

[130] Pieter van Musschenbroek (1692-1761) was a Dutch scientist.

[131] Maybe Coulomb was referring to William Whiston (1667-1752), an English natural philosopher.

author, support the theory. Mr. Musschenbroek (*Dissertatio de magnete*, Exp. CIII),[132] found that when a dip needle[133] oscillated in a vertical plane other than that of the magnetic meridian, the forces which produced the oscillations in the different planes were between them like the cosines of inclination, formed by the natural direction of the needle with these planes. Now if we suppose that the particular force which solicits each point of the compass always has the same direction, and that its action is independent of the orientation of the needle, it will result from this supposition and from the principle of the decomposition of the forces, that the forces which will make the magnetic needle oscillate in planes inclined to the natural direction, will be like the cosines of the angles which these planes form with this direction. This being confirmed by experiment, it follows that the established principle is legitimate.

Another fact that we have every day before our eyes; proves again, it seems to me, this principle in an indisputable way. When we suspend on the tip of a pivot an ordinary needle of declination, if it was in balance, before being magnetized,[134] it will cease to be so when magnetized, the boreal part will be heavier than the austral part,[135] and we will be obliged, in order to restore the balance, either to add a small counterweight to the austral part, or to reduce the weight of the boreal part. Therefore these forces are dependent on the magnetic power which increases the gravity of the boreal part, or which decreases that of the austral part. But when the balance has been restored by a small counterweight, if the compass is in a horizontal position naturally directed in its magnetic meridian, and if this compass is rotated horizontally, it will continue, left to itself, to remain horizontal in all the positions to which it finds itself brought by its oscillatory movement. Therefore the magnetic force[136] increases the weight of the boreal part, or decreases the weight of the austral part by the same amount, in whatever orientation this compass is found with respect to its magnetic meridian, therefore the orientation of the compass does not influence the action of the different magnetic forces.

5.0.3 Second Fundamental Principle

3. The magnetic forces of the terrestrial globe that attract the different points of a compass needle act in two opposite ways. The boreal part of the needle is attracted towards the boreal pole of the magnetic meridian.[137] The austral part[138] of the needle is attracted in the opposite direction. Whatever may be the law according to which these forces act, the sum of the forces which attract the needle towards the boreal pole is exactly equal to the sum of the forces which attract the austral pole of the needle in the opposite direction.[139]

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[132] [Musschenbroek, 1754].

[133] In the original: *aiguille d'inclinaison*. This expression can also be translated as dip circle or inclination needle.

[134] That is, if this non-magnetized needle were to remain in equilibrium pointing in a horizontal direction.

[135] If we have a dip circle in Paris, it is observed that, in equilibrium, the boreal pole of the magnetized needle (with Coulomb's definition, equivalent to the North pole of the compass) remains below the horizon, while the austral pole of the needle remains above the horizon.

[136] In the original: *force aimantaire*.

[137] In the original: *La partie bóreale de la boussole est attirée vers le pole boréal du méridien magnétique*. The boreal part of a compass needle is nowadays called its Northern magnetic part. When the needle is in equilibrium, this part will point to the Earth's magnetic South pole, which is close to the Earth's geographic North Pole.

[138] In the original: *La partie australe*. That is, the austral or Southern part of the needle.

[139] A discussion and illustration of this second principle can be found in Section 6.2 on page 140.

Development of This Principle

Mr. Musschenbroek (*Dissertatio de magnete*, Exp. XXVI) found that a steel blade weighed before being magnetized, and after being magnetized, did not change its weight at all. However precise he was in conducting his experiments, they always gave him the same result. Thus if follows from this experiment and from the principles of statics that for all the forces which act on a magnetized needle, when these forces are decomposed into horizontal and vertical components, the sum of the vertical forces must be null.

From another perspective, we know that when we float a magnetic needle on a small piece of cork, it is directed along the magnetic meridian, but the center of gravity of the whole system soon reaches a state of rest; however if the sum of the horizontal forces is not zero; if, for example, the sum of the forces which pull towards the [Earth's] boreal pole were greater than the sum of the forces which act in the opposite direction, the center of gravity of the system should move towards the [geographic] North with a continuous movement.

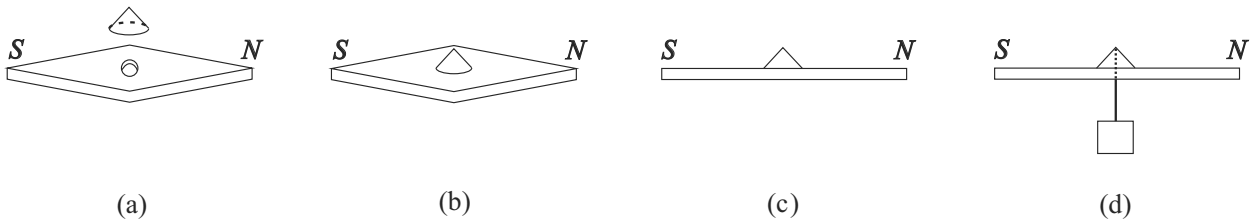
We conclude from these two experiments, that since the sum of the force components in the horizontal plan directed along the magnetic meridian is null, just as the sum of the vertical forces, it follows that the sum of the forces which act along the natural direction of the compass, is also zero.

As to the possibility that the cohesion of water might destroy the effect of horizontal forces in this last experiment,[140] here is a fact which seems to me undeniable.

A very light wooden ruler AB (*Figure 1*), pierced in the middle C and furnished at this point with a compass cap,[141] was suspended, by means of this cap, on a pivot in the same way as we suspend a declination needle.[142,143]

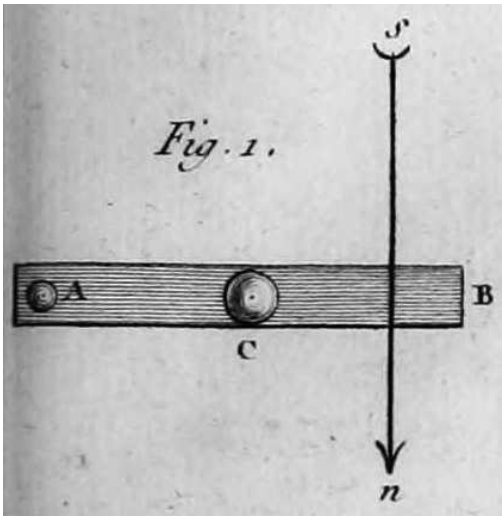
[140]That is, this person could claim that the resultant magnetic force on the compass is not zero, but the needle would not move due to the frictional force exerted by the water, which would cancel the magnetic force exerted by the Earth.

[141]In the 1777 original: *chappe de boussole*, [Coulomb, 1780, p. 172]. In the 1884 text edited by Potier, this expression appears as *chape de boussole*, [Potier, 1884, p. 7]. The compass cap is a small hollow cavity, in the form of a cup, cover, cone or concave spherical cap (like a thimble), to receive the pivot or axis around which the compass rotates. In an ordinary compass, this cap is fixed or welded to the top face of the magnetic blade of the compass, around the hole in its center, with the concave part facing downwards. It can be made, for example, of metal or agate. Figure (a) of this footnote illustrates a magnetic needle SN with a hole in the center and a conical cap above it. In (b) the cover fastened around the hole is seen in perspective. In (c) the needle with the cap is seen from the side. In (d) the needle can be seen from the side with the cap resting on the tip of a pin.



[142]In the original: *aiguille de déclinaison*.

[143]In this experiment the cap is fixed above a hole at the middle of a wooden ruler as shown in Coulomb's Figure 1.



A [magnetized] needle sn, was attached to the end of this ruler, and formed a right angle with it, a small counterweight was placed at A, so that the whole system was balanced horizontally, and could move freely on the tip of a pivot, around the center of suspension C.[144] After the oscillations had died down, the needle sn found itself directed along the magnetic meridian, that is to say, along the same line, as if it had been supported on the tip of a pivot at its center E.[145] Here is the explanation and the result of this experiment. The forces which act on this needle, when it is in its magnetic meridian, are directed along its length: now, since experiment shows us that the compass reaches its state of rest, when it is directed along the magnetic meridian, it follows that the boreal and austral forces having the same lever arm; cannot be in balance unless they are equal.

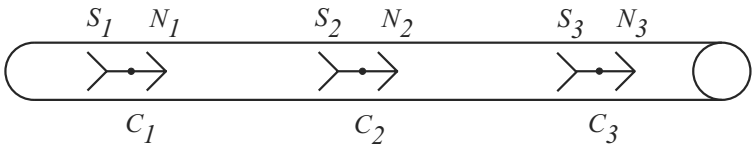
I repeated this experiment on a very large number of needles magnetized to saturation or not, having only one magnetic center, or having a greater number.[146] I consistently found the same result. This experiment will be even more exact, by suspending the small wooden ruler with silk threads, as I will explain later in this Memoir.

[144]The letters s and n on the needle refer to its magnetic poles. That is, this is a magnetized needle with s being its South pole and n being its North pole. The magnetic needle sn is fixed on the ruler. Both the ruler and the needle sn are horizontal. This system can rotate around the small concave cap fixed in the center C of the wooden ruler, supported on a pivot or vertical axis.

[145]This center E of the needle sn was not indicated in Coulomb's original Figure 1.

[146]In the original: *centre aimantaire*. The expression "centre aimantaire" is being translated here as magnetic center. In the case of an ordinary homogeneously magnetized compass needle, the magnetic center is at the midpoint between its magnetic poles, that is, at the midpoint between the North and South poles of the compass. It is possible to magnetize a steel blade such that it has more than two magnetic poles. For example, it might have three pairs of magnetic poles, namely, N_1 and S_1, N_2 and S_2, as well as N_3 and S_3. In this case this blade will have three magnetic centers, C_1, C_2 and C_3. The center C_1 will be at the midpoint between N_1 and S_1, the center C_2 will be at the midpoint between N_2 and S_2, while C_3 will be at the midpoint between N_3 and S_3.

There is a simple way to produce a wooden skewer or plastic straw with three magnetic centers. It is enough to attach along its length three magnetized needles, as in the Figure of this footnote:



5.0.4 General Corollary

4. We can, it seems to me, conclude from these two principles that the direction of a magnetized needle cannot depend on a stream of fluid which, moving rapidly along the magnetic meridian, forces the needle, by its impulsion, to align itself along this meridian. For, by the first established principle, the needle always experiences the same action from the magnetic fluid,[147] whatever angle it forms with its natural direction, which should be the direction of the stream of magnetic fluid.[148] However, according to all that we can know of the laws of the impulses of the fluids, they act differently, according to how the bodies they strike are posed and whether they present a less or greater surface to the direction of the stream. Thus, since experiment teaches us that the magnetic forces of the terrestrial globe act equally on the needle in all orientations, this action cannot come from a stream of fluid.

In the second place, from the second principle, the sum of the forces which act on the needle, is equal in the two opposite directions; it follows then that, if we want to make the direction of the needle depend on the impulse of a fluid, it is necessary to imagine opposite streams which will also act in opposite directions without mutually destroying each other. Such hypotheses seem to be rejected by physics, as being too contrary to the principles of mechanics.

Therefore it seems that it follows from experiment, that it is not vortices which produce the different magnetic phenomena; and that, to explain them, we must necessarily resort to attractive and repulsive forces of the nature of those which we are obliged to make use of to explain the weight of bodies and celestial physics.

5.1 Chapter I. Formulas which Derive from All the Forces, Either Active or Coercive, Which Can Influence the Orientation of a Needle in Equilibrium in a Horizontal Plane

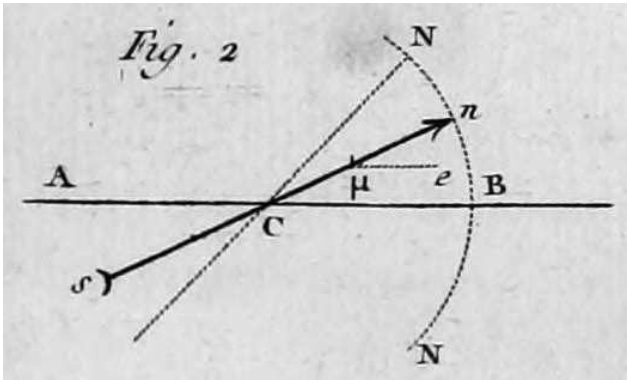
5. When a declination needle balanced in a horizontal plane, can turn freely around its point of suspension; if it is removed from its magnetic meridian, it will be brought back there by the [Earth's] magnetic force, which acts on each point of this needle; and its movement will be retarded by all the coercive forces, coming either from the friction of the [compass'] cap on its pivot, or from the torsion of the silk thread, to which we can suppose the compasses suspended, or finally from the resistance of the air, in which the compass makes its oscillations. We do not consider here the errors which may arise from the position of the point of suspension, and from the imperfection of the pivots and the caps.[149] We will come back to this later.

Of these different coercive forces, all of which tend to destroy the motion of the oscillating needles: some are constant and depend either on friction or on the cohesion of the air: others also depend on the friction and cohesion of the air; but increase with speed; so that the *moment*[150] of all the coercive forces will be represented by a quantity $(A + Fu)$ where A

[147] In the original: *fluide magnétique*.
[148] In the original: *fluide aimantaire*.
[149] That is, for now Coulomb will focus on errors or deviations due to the resistance of air.
[150] In the original: *momentum*. This expression can also be translated by torque, moment of force or rotational force.

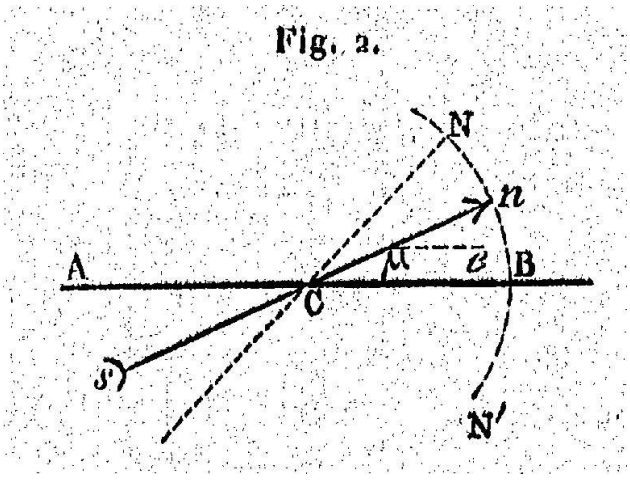
being a constant quantity, Fu will be a function of the angular velocity.

6. Let AB (*Figure 2*) be the true meridian of a declination needle, from which it is assumed to be displaced, at the beginning of its movement, by the angle $BCN = B$, the point C being the point of suspension, which deviates very little from the center of gravity and from the magnetic center, in homogeneous blades magnetized to saturation.[151]



When the needle has arrived at n, let the angle $NCn = S$, the angle $nCB = B - S$, the angular velocity $= u = dS/dt$, the magnetic force μe, which acts on any point μ (decomposed along the horizontal plane), parallel to the magnetic meridian $= \varphi$, $C\mu = r$, $CN = l$, the

[151] In Figure 2 the letter N at the bottom should be replaced by N', as corrected in the 1884 reprint of Coulomb's work:



moment of the magnetic force of the point μ, will be represented by[152]

$$\varphi\mu r\sin(B-S)\ .$$

If $R=(A+Fu)$ represents the *moment* of all the coercive forces, we will have, for the total *moment*, around the point C, the quantity $\int\varphi\mu r\sin(B-S)-R$; but, when the needle has arrived at its state of rest, the active and coercive forces must be in equilibrium; thus we will have, for the needle's error,[153]

$$\sin(B-S)=\frac{R}{\int\varphi\mu r}\ ;$$

and, when the deviation angle is small,[154] we will have $(B-S)=R:\int\varphi\mu r$; thus, to have the most advantageous dimensions of a needle, it is necessary, when we know the quantity R and the quantity $\int\varphi\mu r$, integrated for all the length of the needle, to ensure that the angle $(B-S)$ be a *minimum*.

7. Now let us move on to the oscillatory movement: we will need it in the following, either to compare the magnetic force of different needles, or to compare the magnetic force with the coercive force.

The *moment* of all the forces which produce the acceleration of the needle when it has arrived at the point n, is, as we have just seen in the previous Article, expressed by $\int\varphi\mu r\sin(B-S)-R$; but the acceleration of the point μ, or the small arc traversed by this point, is expressed by rdu; thus we will have, in naming dt the element of time,[155]

[152] In the original 1777 paper this equation appears as, [Coulomb, 1780, p. 175]:

$$\varphi\mu C\mu\frac{\sin(B-S)}{rayon}\ .$$

In 1884 Potier wrote this equation as, [Potier, 1884, p. 11]:

$$\varphi\mu r\sin(B-S)\ .$$

In this English translation I will follow Potier's modern notation. That is, if θ represents an angle, then the sine and cosine of that angle will be represented by $\sin\theta$ and $\cos\theta$, instead of Coulomb's original notation, $\frac{\sin\theta}{radius}$ and $\frac{\cos\theta}{radius}$.

What Coulomb calls $\varphi\mu$ in this equation, is the horizontal component of the Earth's magnetic force acting on the point μ of the needle which is at a distance r from the axis of rotation. The component of this force normal to the direction of the needle is given by $\varphi\mu\sin(B-S)$. Therefore, the torque it produces on the needle is given by $\varphi\mu r\sin(B-S)$.

[153] In the original: *l'erreur de l'aiguille*. That is, the error or deviation of the needle in relation to the magnetic meridian. This deviation is caused by frictional or coercive forces that prevent it from returning exactly to the magnetic meridian.

[154] The angle of departure of the compass from the magnetic meridian is given by $B-S$. When $B-S\ll 1$ radian, we have $\sin(B-S)\approx(B-S)$.

[155] In the next equation, μ on the right side represents an infinitesimal element of inertial mass, where r is the distance from this element of mass to the vertical axis of rotation. Therefore, the moment of inertia of the entire needle about this axis of rotation is given by $I=\int\mu r^2$. The integration here should be thought of as a volumetric integration over the entire volume of the needle. The angular velocity around this axis is given by $u=dS/dt=\dot{S}$, where S is the angle NCn of Figure 2. Therefore, $du/dt=d^2S/dt^2=\ddot{S}$ represents the angular acceleration. The torque around the axis of rotation is given by $\tau=\int\varphi\mu r\sin(B-S)-R$. The equation given by Coulomb is then the well-known second law of motion of mechanics applied to rotation, given by $\tau=I\ddot{S}$. That is, the torque acting on the needle is equal to its moment of inertia times its angular acceleration.

$$\left[\int \varphi\mu r \sin(B-S) - R\right] dt = du \int \mu r^2 \ ;$$

whence, by integrating this quantity, after having substituted, in place of dt, its value $dS : u$, and noticing that u vanishes, when $S = 0$, we have[156]

$$\int \varphi\mu r \left[\cos(B-S) - \cos B\right] - \int RdS = \frac{u^2}{2} \int \mu r^2 \ .$$

8. If the angle B is very small, this is the only case we will need in the sequel, we will have[157]

$$\cos(B-S) - \cos B = \frac{1}{2}(2BS - S^2) \ ;$$

so the equation reduces to

$$\int \varphi\mu r(2BS - S^2) - 2\int RdS = u^2 \int \mu r^2 \ .$$

9. If we make $u = 0$, we find

$$2B - S = \frac{2\int RdS}{S\int \varphi\mu r} \ ;$$

and if R were a constant quantity, we would have

$$2B - S = \frac{2A}{\int \varphi\mu r} \ ;$$

thus when the needle, after having traversed the arc NB, goes back to N',[158] letting the arc BN' be B', we will have

$$B - B' = \frac{2A}{\int \varphi\mu r} \ ;$$

which always gives, on the assumption of constant coercive forces, the same quantity for the difference of the descending and ascending arcs.

[156] In the original article this equation appears as, [Coulomb, 1780, p. 175]:

$$\int (\varphi\mu r)\frac{(\cos(B-S) - \cos B)}{rayon} - \int RdS = \frac{uu}{2} \int r^2\mu \ .$$

As stated in footnote 152, I am following Potier and replacing Coulomb's expression $\frac{\cos\theta}{radius}$ with $\cos\theta$, where θ represents any angle.

[157] When the angle B is very small, also $B - S$ will be very small. Assuming B expressed in radians, the following approximations can then be made: $\cos B \approx 1 - B^2/2$ and $\cos(B-S) \approx 1 - (B-S)^2/2$. Using these approximations, we arrive at the next result presented by Coulomb.

[158] See footnote 151 on page 61.

10. If we suppose $R = A + Fu$, we will then have, however small the velocity u, $B - B'$ greater than

$$\frac{2A}{\int \varphi\mu r} \ .$$

This consideration will suffice in what follows to prove that the resistance of the air cannot produce a sensible error in the orientation of the needle.

11.[159] When, in the previous equation, we suppose $R = 0$, we obtain the approximate equation

$$u^2 = \frac{\int \varphi\mu r}{\int \mu r^2} \left(2BS - S^2\right) \ ;$$

from which

$$\left(\frac{\int \varphi\mu r}{\int \mu r^2}\right)^{1/2} dt = \frac{dS}{\sqrt{2BS - S^2}} \ .$$

Or

$$\int \frac{dS}{\sqrt{2BS - S^2}}$$

is the angle whose radius is B, and S the sine-verse; quantity equal to $\pi/2$, when $S = B$; thus, by naming T the time of a total oscillation,[160] we will have

$$T \left(\frac{\int \varphi\mu r}{\int \mu r^2}\right)^{1/2} = \pi \ .^{[161]}$$

12. If we want to compare the magnetic force with gravity: letting g represent this force, we have[162]

[159] I present in Section 6.3 on page 141 a detailed discussion of this Article 11.

[160] In the original: *en nommant T le temps d'une oscillation totale.*

[161] See Section 6.3 on page 141 for a discussion of the meaning Coulomb gave to the expression "time of a total oscillation", along with a deduction of this formula presented by him.

[162] Although Coulomb calls g the "force" of gravity, he is referring to the free-fall acceleration at the Earth's surface, that is, the gravitational force per unit mass. Suppose we have a simple pendulum of length λ that makes small oscillations in a vertical plane due to the gravitational action of the Earth, starting from rest at an angle θ_o. In the next equation T' represents the time for the pendulum to go from θ_o to $-\theta_o$, see Section 6.3 on page 141. So T' is then given by:

$$T' = \frac{1}{2}\left(2\pi\sqrt{\frac{\lambda}{g}}\right) = \pi\sqrt{\frac{\lambda}{g}} \ .$$

In the original article Coulomb wrote this relationship as follows, [Coulomb, 1780, p. 176]:

$$T' \left(\frac{g}{\lambda}\right)^{1/2} = (180)^\circ \ .$$

$$T' \left(\frac{g}{\lambda}\right)^{1/2} = \pi$$

for the oscillations of a pendulum, whose length is λ; thus, if we want the time T' to be isochronous with the oscillations of the magnetic needle, we will make

$$\frac{g}{\lambda} = \frac{\int(\varphi\mu r)}{\int \mu r^2} \ ,$$

hence

$$\lambda = g\frac{\int \mu r^2}{\int \varphi\mu r} \ .$$

Assuming that the compass is a blade of a uniform width and thickness, naming δ the transverse section of this blade, and l half of its length, we will find[163]

$$\int \mu r^2 = 2\delta l \cdot \frac{l^2}{3} \ ;$$

but $2\delta l$ represents the mass of the needle, which, multiplied by the force of its gravity g, equals its weight P; thus

$$\int \mu r^2 = \frac{Pl^2}{3g} \ ,$$

and therefore

$$\lambda = \frac{Pl^2}{3\int \varphi\mu r} \ .$$

13. If we look for a weight Q, which, placed at the end of the lever l, has the same *moment* as the magnetic force of the needle, we will have

Potier, [Potier, 1884, p. 13], replaced it with

$$T' \left(\frac{g}{\lambda}\right)^{1/2} = \pi \ .$$

[163] In the next equation Coulomb is presenting the moment of inertia of a blade in the shape of a parallelepiped of length 2ℓ, width a and thickness e, such that $\ell \gg a$ and $\ell \gg e$. Let M be the mass of this needle and $\rho = M/(2\ell \cdot a \cdot e)$ its volumetric mass density. Consider an axis perpendicular to the plane $2\ell \cdot a$ passing through the center of the needle. The moment of inertia of this blade relative to this axis is given by

$$I = \int_i \mu_i r_i^2 = \frac{M(2\ell)^2}{12} = \frac{M\ell^2}{3} \ .$$

What Coulomb calls δ or cross section of this blade is its linear mass density, or else its volumetric mass density multiplied by the area ae of the cross section of the blade, namely:

$$\delta = \frac{M}{2\ell} = \rho ae \ .$$

Assuming a mass density equal to unity, $\delta = ae$. In this case, the mass of the needle is represented by $M = 2\delta\ell$, as mentioned by Coulomb shortly thereafter.

$$Ql = \int \varphi \mu r \ ;$$

but, as a result of the previous Article,

$$\int \varphi \mu r = \frac{Pl^2}{3\lambda} \ ;$$

thus $Q = \frac{Pl}{3\lambda}$; quantity which is the same as that found by Mr. Euler,[164] in the work which competed for the Prize [of the French Academy of Sciences], in 1743,[165] where this geometer, dividing the magnetic force into two parts, which act in opposite directions, at the two extremities of the needle, found for each [of these parts the value] $Q = Pl : 6\lambda$.

5.2 Chapter II. Theoretical and Experimental Determination of the Magnetic Forces

13*.[166] Mr. Musschenbroek (*Dissert. de Magnete*, Exp. CVII) says that, in the oscillations of magnetic blades, the square of the time in which a certain number of vibrations take place, is in a compound ratio of the length of the blades and their weight; which, expressed algebraically, gives

$$T^2 = mlP \ ,$$

T expressing the time of a certain number of vibrations, m being a constant coefficient, $2l$ the length, and P the weight of the needle; but we have found, in the preceding Articles,

$$T^2 = \pi^2 \frac{\int \mu r^2}{\int \varphi \mu r} = \pi \frac{Pl^2}{3g \int \varphi \mu r} \ ;$$

thus, by comparing this value of T^2 with the experiment, it results from this comparison the equation

$$mlP = \pi^2 \frac{Pl^2}{3g \int \varphi \mu r} \ ,$$

from which

$$\int \varphi \mu r = \pi^2 \frac{l}{3gm} = Ql \ ;$$

thus, by comparing the experiments of Musschenbroek with the theory of oscillations, we would find that the total *moment* of the magnetic forces of a blade, whatever the dimensions of this blade, would always be equal to a constant weight, multiplied by the length of the blade.

[164] Leonhard Paul Euler (1707-1783) was a Swiss scientist.

[165] [Euler, 1752].

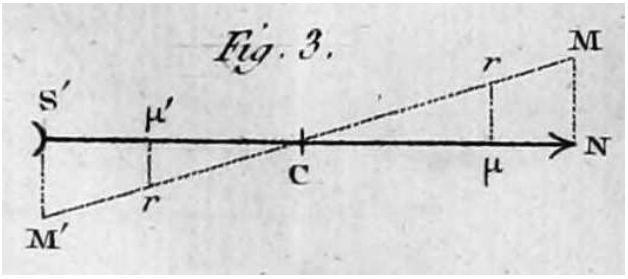
[166] In the reprint of Coulomb's work of 1884, Potier used the number 13 twice in the Articles of this work, [Potier, 1884, pp. 14-15]. We are following the original work and naming this second Article 13 by the number 13*, [Coulomb, 1780, p. 178].

14. From there we would conclude that at equal lengths, the friction of the needles of declination, which turn on a pivot, increasing according to a law of weight, and the *moment* of the magnetic action being always a constant quantity, the lightest compasses would be the best.

15. It would also result that if the *moment* of friction increased in direct proportion to the weights, it would increase, all else being equal, like the lengths of the compasses: now the *moment* of the magnetic force, increases in the same proportion. Thus, the ratio of the *moment* of the magnetic forces, to the *moment* of the frictions, being constant, the same error in the orientation of the needles would always result.

16. If we wanted to seek, according to the same formulas, founded on the experiments of Musschenbroek, the law of the magnetic forces of different points of the needles, here is how we could go about it.

Let $S'N$ *(Figure 3)* represent a needle, whose magnetic center is at C, that is to say, whose point C is such that all points μ of the part CN, experience a boreal force,[167] while all the points μ'[168] of the part CS' experience an austral force.[169]

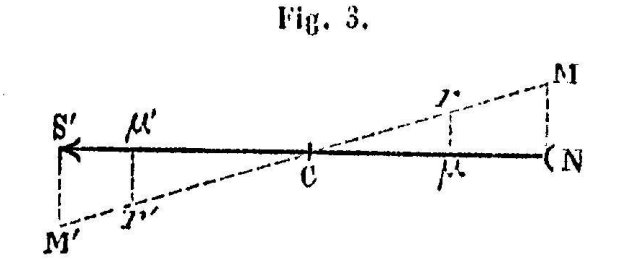


If the ordinate μr represents the force on the point μ, and if, through the extremities

[167]In the original: *force boréale*. That is, these points have a boreal fluid or North fluid. It can also be said that the points μ of the CN part suffer a magnetic force that points approximately towards the geographic North pole of the Earth.

[168]In the original text, μ appears here instead of μ'.

[169]In the original: *force australe*. That is, these points have an austral fluid or a Southern fluid. It can also be said that the points μ' of the CS' part suffer a magnetic force that points approximately towards the geographic South pole of the Earth. In Figure 3 the letter r at the bottom should be replaced by r'. This was corrected in the 1884 reprint of Coulomb's work. The problem is that when this Figure was remade in 1884, the tip and tail of the magnetized needle were inverted in relation to the original Figure:



of all the ordinates, we pass a line MCM', this line will be the locus[170] of all the magnetic forces, and will cut the needle at a point C, which will be the magnetic center. But, by the second principle, the sum of the boreal forces is equal to the sum of the austral forces, from which we conclude that the area CMN = the area $CM'S'$.

But experiment shows us that in homogeneous needles, magnetized to saturation, the magnetic center is in the middle of the needles: thus, if we suppose that the quantity φ, [that is,] the magnetic force of the point μ, is expressed by $nl^q r^k$, n being a constant coefficient, l half the length of the needle, and r the distance from point μ to point C, q and k the powers of l and r, we will derive from this [assumption] that $\int \varphi\mu r = Ql$, Q being a constant weight,

$$\int \varphi\mu r = \int n\delta l^q r^{k+1} dr = \frac{n\delta l^q r^{k+2}}{k+2} \; ;$$

and, when $r = l$, we will have

$$\frac{2n\delta l^{q+k+2}}{k+2} = Ql \; ;$$

however, as this equation must be satisfied, and as Q is a constant quantity, it is necessary that $(q + k + 1) = 0$, or $(q + k) = -1$; thus, the force φ, for the end N, being nl^{q+k}, we always have at the end of the needles, $\varphi = (n/l)$, whatever the value of q or of k.

17. If we suppose with most of the authors who have dealt with magnetism, that the boreal and austral forces on the different points μ of the needle, vary as the distances $c\mu$ of these points from the magnetic center,[171] in this case $\varphi = nl^q r$ and $q = -2$; thus $\varphi = nr/l^2$, and the locus MCM', will be a straight line.

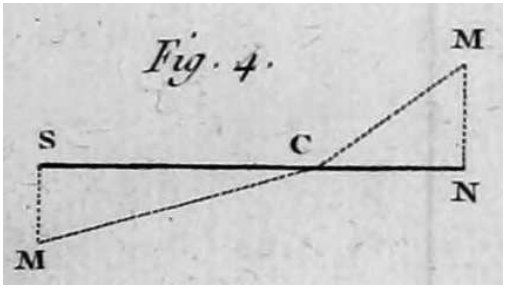
18. Although there are several experiments which seem to agree in proving that the forces of the different points of a blade are proportional to the distances of these points from the magnetic center, there arises a difficulty which must, it seems to me, make us wary of this hypothesis: one sees, in truth quite easily, that when a needle is magnetized to saturation, the magnetic center being in the middle of the blade, the geometric locus of the magnetic forces, can be represented by two equal triangles, opposed at the [magnetic] center and linked by the same equation: but we have mastered [the way] to move this magnetic center towards the ends of the blade, using the practice prescribed by Mr. le Monnier (*Loi du Magnétisme*, page 107).[172] If we suppose that this center is in another point than the middle of the blade, then the boreal forces will be represented *(Figure 4)*, by a triangle CMN, and the austral forces by a triangle $M'CS$ *(Figure 4)*.[173]

[170] In the original: *le lieu géométrique*. In geometry, a locus (Latin word for place or location) is a set of all points, whose location satisfies or is determined by one or more specified conditions. This set of points is commonly a line, a line segment, a curve or a surface.

[171] That is, if the density of magnetic fluid at each point is assumed to be proportional to the distance from that point to the center of the needle. With that assumption the magnetic force exerted by the Earth on each point on the needle will also be proportional to the distance of that point from the center of the needle.

[172] Pierre Charles Le Monnier (1715-1799) was a French astronomer. His work on the laws of magnetism was published in two parts between 1776 and 1778, [Le Monnier, 1778]. See also [Licoppe, 1995].

[173] In Figure 4 the letter M at the bottom should be replaced by M', as corrected in the 1884 reprint of



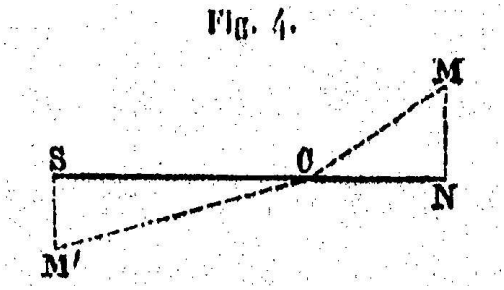
The law of continuity requires that the two triangles be similar, or that MCM' be a straight line: but it results from the first principle, that the sum of the austral forces must be equal to the sum of the boreal forces: thus it is necessary, to satisfy this principle, that the two triangles are equal, which is incompatible with the similarity of the two triangles, when the line NC will be larger or smaller than CS: thus, the hypothesis of the magnetic forces of the different points of the needle, proportional to the distance of these points, cannot be admitted.

5.2.1 New Experiments to Determine the Directing Force of Magnetized Blades

19. If Musschenbroek's experiments were more numerous; if the theory of magnetism had been carried in his time to the degree it has reached; the authority of this author in physics having such great weight I would have blindly adopted the simple formulas which result from his work, but it will be easy to see that they are incompatible with the theory of magnetism, when we expose what repeated tests have shown over the past few years, regarding the way in which the magnetic virtue is communicated: I will draw consequences from these that I believe are interesting for the subject I am dealing with.

20. When the pole of a magnet is placed on the end of a steel blade (*Figure 5)* at n,[174] if it is, for example, the austral pole of the magnet, which touches the point n, a part nC of this blade takes on a boreal force,[175] while the other Cs, takes on an austral force; and

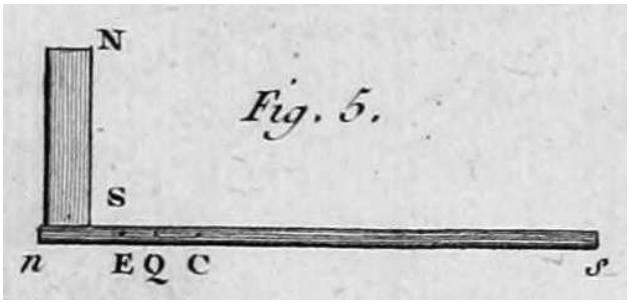
Coulomb's work:



[174]In the original article, n' appears here instead of n.

[175]In this experiment it must be assumed that the steel blade was not initially magnetized. After the magnet NS has been on one end of the blade for some time, with its South pole touching the blade, it is observed that the piece nC of the steel blade is magnetized with a boreal fluid (that is, with a North magnetic fluid, or with a North pole). The piece nC acquires a fluid of a type opposite to the pole of the magnet that was touching this part of the steel blade.

the center C, which separates the boreal part from the austral part, which has no magnetic force, is called the magnetic center, or center of indifference.[176]



If the pole S of the magnet is made to slide along the blade, the center of indifference C approaches the point s; the austral force of the extremity s increases first, until the pole of the magnet has reached a point E, then it decreases until the pole has reached a point Q where it is zero. It then becomes boreal, and continues to increase until the austral pole S of the magnet has reached the point s: what is said in relation to the point s will also take place for the point n, its force initially boreal, will increase, will decrease, will become null, then austral, while the pole of the magnet traverses the length of the blade.

What we have just found for the austral pole of the magnet, will also take place, *vice versa*, using the boreal pole N.

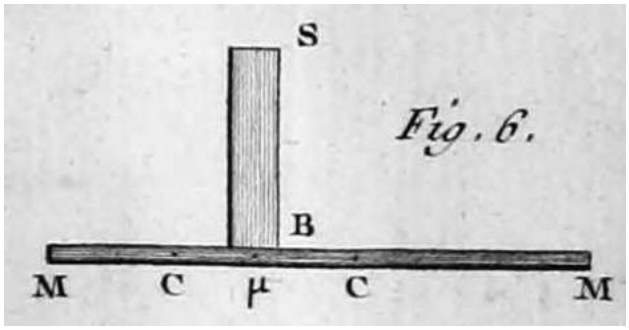
These experiments have been made by several authors; we find the most detailed information of it in a work by Vanswiden, (*Tentamina theoriae mathematiae de phaenomenis magneticis*);[177] it is conceivable that, in the operation that we have just detailed, when the magnetic force of the ends n or s becomes zero; in this case the center of indifference of this blade, falls at the extremities of the blade.

21. In general, the pole of a magnet being applied to a point μ *(Figure 6)* of a blade, communicates to this point a force of a name contrary to that of the pole of the magnet which touches the point of the blade.[178]

[176]The magnetic center is the non-magnetized point of the steel blade ns. Assuming that the austral pole (the modern South pole) of the magnet is on the point n of the steel blade, it is observed that the part nC is magnetized with a boreal fluid or North fluid, while the part Cs is magnetized with an austral fluid or South fluid.

[177]Jean Henri van Swinden (1746-1823) was a Dutch physicist and mathematician. The work cited by Coulomb was published in 1772.

[178] In Figure 6 the letters C and M on the right should be replaced by C' and M', respectively, as corrected in the 1884 reprint of Coulomb's work, namely:

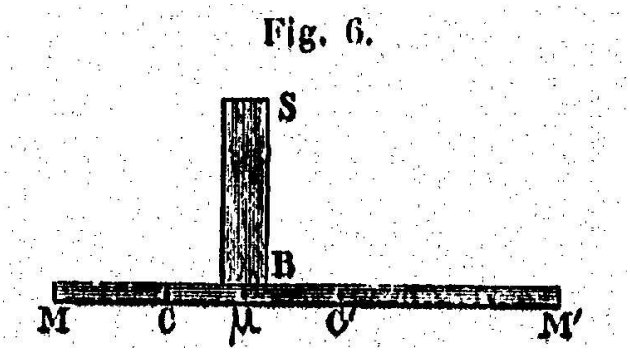


For example, if it is the boreal pole of the magnet which touches the point μ, this point μ will take on an austral force: the same thing will take place with all the surrounding points, which will all take on an austral force, this force will always decrease to the points C and C', which will be the magnetic centers: the extremities CM and $C'M'$ will have boreal forces. It will most often happen that the shortest extremity $\mu M'$,[179] will have an austral force, and that the blade will only divide into two parts by a center C: it could also happen that it divides into three and four parts by several magnetic centers, which depends on the nature of this blade, its dimensions, and the strength of the magnet.

If the pole N of the magnet is made to slide along the blade, the magnetic centers will traverse this blade; but the point on which the pole N will be found, will always receive a force of a name contrary to this pole.

22. From these experiments it results that since the pole of a magnet always produces on the part of the blade where it is applied, a force of a name different from the pole which touches; if we join together two blades magnetized to saturation, by uniting the poles of the same name; whatever be the cause of their action, they tend to produce on each other a force of a name contrary to that with which they are endowed: thus, the effect of this action must diminish the polar force of each of these blades.

Consequently, the magnetic force of each longitudinal element of an artificial magnet necessarily decreases as its thickness increases: thus, the ratio of the total force of two artificial magnets of the same length, but of unequal thickness, both magnetized to saturation, will



Moreover, the letter B at the lower end of the vertical magnet represents a boreal pole, that is, a North pole. In Figure 5, this boreal pole was represented by the letter N for North pole.

[179] In order for Coulomb's original text to conform to his Figure 6 as indicated in footnote 178, we must replace the expression "shortest extremity" with "longest extremity" here. Or else we keep this expression "shortest extremity $\mu M'$," only then the letters C and M on the left side of Coulomb's original Figure 6 have to be replaced by C' and M'.

be smaller than that of their mass.

23. If, instead of making the pole of a magnet touch a steel blade, we present it only one or two lines away [0.226 or 0.452 cm], we will observe the same phenomena as in Article 21; but the degree of magnetism which the blade will acquire will be less than in the first case.

Thus, each point of a magnet or of a magnetized bar can be regarded as the pole of a tiny magnet, which tends to produce in the other points of this bar a force of a name contrary to that which it has itself;[180] and the effect of this action is all the greater, as the intensity of the force of the point which acts, is greater, and as its distance from the points on which it acts, is less; thus the magnetic force of a magnet, depends on the reciprocal action which all the points of this magnet exert one on the other.

24. If we develop the preceding reasoning, it will be seen that since the action experienced by a magnetic point necessarily increases as the intensity of the force of the other points that form the blade increases, as the number of points that act is greater, and as they exert their action at a shorter distance: the closer the points of an artificial magnet are brought together by the shape of this magnet, the greater the action which the different parts exert on each other, tending to destroy their reciprocal forces, will be considerable, and consequently the smaller the force of each point.

Thus, in two blades of the same weight and of the same length, the magnetism will be greater, in the one whose width will be greater, because the longitudinal fibers will be more isolated in the wider blade.

Thus, if a blade is separated into two parts, each of them magnetized to saturation will in particular receive a greater degree of magnetism than when they were united.

Thus of all the shapes, the cylindrical shape being for steel rods, that where the parts of equal length, for the same weight, are brought closer together, will also be that where the mutual action of the magnetic parts[181] will be the greatest, and consequently that whose magnetism will be the least.

By continuing to follow the same analogies, we will find that the points of the surface of a blade will necessarily be endowed with a more considerable magnetic force than the points of the interior of this blade, since the interior parts are touched on all sides by elements which tend to destroy their magnetic force; whereas, in the surfaces, there is only one side which is in contact.

It will also be found that the corners of magnetized rods are the parts that will take on the greatest degree of magnetism, because they are the parts that are the most isolated.

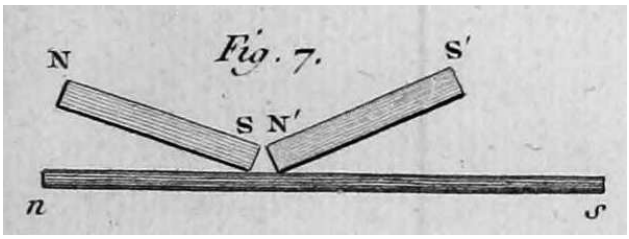
Finally, we will conclude that, for the same thickness, the ends of a long blade magnetized to saturation, and whose magnetic center is in the middle, will have less force than the ends of a small blade; since, in the first, there are more parts that act, than in the second, etc.

[180] Original sentence: *qui tend à produire dans les autres points de cette lame une force d'un nom contraire à celui qu'il a lui-même.* The meaning of this sentence is that each point of a magnetized bar tends to produce, in the other points of this bar, a magnetic pole of a name contrary to that which the point has itself.

[181] In the original: *parties aimantaires.* This expression can also be translated as elementary magnetic parts, [Gillmor, 1971a, p. 217]. Another possible translation is "magnetic particles" (die magnetischen Theilchen), [Coulomb, 1890c, p. 37].

25. From these reflections, we can draw a multitude of consequences, on the choice of magnetized blades in the construction of compasses: but, before engaging in this discussion, we will report on several experiments, which will help us to develop this theory in a more sure and precise manner.

26. To magnetize to saturation the blades, we used in the experiments that follow two steel bars, the length of which was 12 inches (32.48 cm), the width one inch (2.71 cm), and the thickness 5 lines (1.128 cm). We magnetized them, by the method of the double touch, such as it is prescribed by Messrs. Antheaume and OEpinus:[182] it consists *(Figure 7)*, in tilting the two artificial magnets on the blade that we want to magnetize; so that the austral pole S of the bar NS is only one or two lines [0.226 or 0.452 cm] from the boreal pole N' of the bar $N'S'$.



The two magnets are slid in this orientation from one end of the blade to the other: when the blade is thin and only seven to eight inches long [18.95 to 21.66 cm], it is rare that it is not magnetized to saturation, after seven to eight rather slow rubbings on each of the faces: we can be sure that the blade is magnetized to saturation if, suspended horizontally, it continues to make the same number of oscillations at the same time, however many times they are rubbed again, or whether you use other magnets than the first.[183]

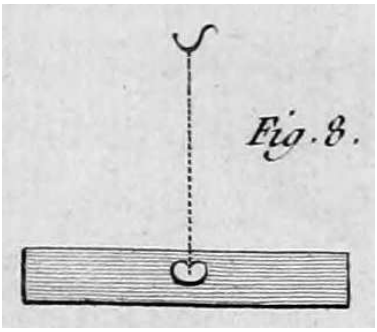
In all the experiments, very pure steel of the same grain was used: all the blades were taken from a German saw, of a more or less uniform thickness: but care was taken to flatten it for a long time while cold under the hammer; experience has shown that this is the only way to obtain consistent results, and to avoid inequalities which are due to the dissimilarity of the position of the parts, and for which no hypothesis can account.

When a blade was magnetized to saturation, it was suspended from the side horizontally,[184] by a very flexible silk [thread], at the end of which was attached a little wax which was stuck to this blade *(Figure 8)*.

[182]Franz Ulrich Theodor Aepinus (1724-1802) was a German physicist who carried out original research on electricity and magnetism. His main work was published in 1759, *Essay on the Theory of Electricity and Magnetism*, [Aepinus, 1759] with English translation in [Aepinus, 1979]. See also Section 4.1, [Heilbron, 1981a] and [Blondel and Wolff, 2013c]. Antheaulme published in 1760 the work *Sur les Aimans Artificiels*, [Antheaulme, 1760]. The double touch method is discussed in Section 4.6.

[183]See Section 4.6 on page 48.

[184]In the original: *on la suspendait de champ horizontalement.* Let a blade in the form of a parallelepiped with length L, width W and thickness T be such that $L > W > T$. It is said to be suspended sideways when the length L and thickness T are horizontal, while the width W is vertical.



We were assured by experiments, which will be explained later, that the torsion of the silk could not influence the time of the oscillations:[185] we carefully counted the time that the blade took to make 20 oscillations; each operation was repeated twice, then one inch [2.7 cm] was cut from each side of the blade; the remainder being magnetized to saturation, the same operations were carried out as on the first blade.

The blade oscillated in a well-closed box, so that the currents of air which prevailed in the room did not disturb the experiments: this precaution is above all indispensable, when we [are heating the room] with a fire.

5.2.2 Experiments to Determine the Magnetic Force of the Blades with Regard to Their length

First Experiment

27. The blade was three lines wide (0.677 cm): one foot long (32.48 cm), weighed 288 grains (15.30 g);[186] it made these oscillations:

Lengths	Duration of 20 oscillations
16 inches (43.31 cm)	231”
12 inches (32.48 cm)	180
10 inches (27.07 cm)	154
8 inches (21.65 cm)	126
6 inches (16.24 cm)	98
4 inches (10.83 cm)	80

Second Experiment

The blade was eight lines wide (1.805 cm): a length of one foot (32.48 cm) weighed 976 grains (51.83 g), it made these oscillations:

[185] After the blade is in equilibrium, it is rotated through a certain angle around the vertical axis that coincides with the silk thread and released from rest. Coulomb's experiments showed that the number of oscillations it performs per unit of time does not depend on this initial angle. In the next experiments Coulomb will not even mention what those initial angles were.

[186] Assuming a steel blade with a mass density of $8 \times 10^3\ kg/m^3$ its thickness would be 0.087 cm.

Lengths	Duration of 20 oscillations
16 inches (43.31 cm)	254”
12 inches (32.48 cm)	202
8 inches (27.07 cm)	154
4 inches (21.65 cm)	104

Third Experiment

This blade was twelve lines wide (2.71 cm); a length of one foot (32.58 cm) weighed 1105 grains (58.70 g); it made these oscillations:

Lengths	Duration of 20 oscillations
16 inches (43.31 cm)	250”
12 inches (32.48 cm)	205
8 inches (27.07 cm)	153
4 inches (21.65 cm)	110

Result of These Three Experiments

In the first experiment, for a blade 12 inches long and 3 lines wide; we have 20 oscillations in 180”. In the same experiment, we have, for a blade 4 inches in length, and having moreover the same dimensions as the preceding one, 20 oscillations in 80”: thus, the difference in time for 20 oscillations in these two blades, is 100”.

In the second experiment, we have, for a blade 8 lines wide and 12 inches long, 20 oscillations in 202”: for this same blade reduced to 4 inches, we find 20 oscillations in 104”: thus, the time difference, for a decrease in length equal to 8 inches, is found for 20 oscillations 98”.

In the third experiment, we find, for a blade 12 lines wide and one foot long, 20 oscillations in 205”: we have, by the same experiment, for a 4-inch blade, 20 oscillations in 110”, which gives, for a decrease of 8 inches, for the time of 20 oscillations, a decrease of 95”.

By actually comparing these three results, we see that an equal reduction in the lengths gives, more or less, the same reduction in the time of oscillations:[187] thus, the width of the blades influences this decrease only a very little.

If we compare, in each particular experiment, the diminution of the time of the oscillations, with the shortening of the blades, we will see that this time decreases, more or less, by quantities which are proportional to the shortening of the blades.

We see again, by these experiments, that the total time of the oscillations is greater, at equal thicknesses and lengths, for the wide blades than for the narrow blades. This is evident in comparing the results of the first experiment with the third; this is what the theory predicts: the second experiment compared with the third, seems to give a contrary result; but, if we note that the second blade, although narrower than the third, is proportionally heavier, and consequently thicker, we will see that they give a result conforming exactly to the theory.

[187] In the original: *dans le temps des oscillations.*

Fourth Experiment

28. In this experiment, we tried to determine if, by increasing the thickness of the blades, the increase in the time of oscillation would continue to be proportional to the increase of the blade length, as in the previous Articles.

The blade used in this experiment was of the same nature as the preceding ones: it was three lines wide [0.677 cm], like that of the first experiment; but its thickness was a little more than threefold, and the 12 inches in length [32.48 cm] weighed 936 grains (49.71 g): it made these oscillations:

Lengths	Duration of 20 oscillations
12 inches [32.48 cm]	229"
10 inches [27.07 cm]	208
8 inches [21.65 cm]	176
6 inches [16.24 cm]	151
4 inches [10.83 cm]	128

Result of This Experiment

If we subtract in this experiment from the time that a 12-inch blade takes to make 20 oscillations, the time that a blade 4 inches in length employs to make the same 20 oscillations; we obtain 101", a quantity almost exactly the same as that which we found by the first experiment. Thus, it appears that the thickness does not contribute anything to the increase in the time of the oscillations, which is always proportional to the increase in the lengths.

29. Considering now the results of all the preceding experiments, it will easily be seen that the time T, of a certain number of oscillations, can always be represented for blades of uniform thickness and width, by a quantity $(A + ml)$ where A expresses a function of the thickness and the width, and (ml) is the product of a constant coefficient by the length l, the quantity A will increase as the width and thickness increase: it will be greater for a cylindrical rod than for any other shape.

The constant coefficient m, will depend on the nature of the steel and the degree of magnetism to which it is susceptible. This coefficient will be greater, as the steel or the iron will be less susceptible to magnetism. In commercial wires, we find on average that a reduction of eight inches in length [21.65 cm] produces a reduction of 120" for 20 oscillations: let us now determine the quantity A.

5.2.3 Experiments Relating to the Width of the Blades

Fifth Experiment

We tried, in this experiment, to find a relationship between the time of the oscillations and the width of the blades: we therefore took a blade 4 inches long [10.83 cm] and one inch wide [2.71 cm], which we divided exactly into 36/3 lines [2.707 cm]: this blade, of a uniform thickness, weighed 378 grains (20.08 g); after having been magnetized to saturation, it was suspended like the blades of the preceding Articles, and the time required to make 20 oscillations was determined. Then part of its width was cut off; the remaining part was

magnetized again to saturation, and the time it took to make 20 oscillations was measured, continuing this operation while gradually decreasing the width of the blade, we obtained:

	Width of the blades	Duration of 20 oscillations	A
First trial	36/3 lines (2.707 cm)	114"	65"
Second trial	22/3 lines (1.654 cm)	99	50
Third trial	13/3 lines (0.997 cm)	83	34
Fourth trial	7/3 lines (0.526 cm)	74	25
Fifth trial	3/3 line (0.226 cm)	68	19

Result of This Experiment

The general expression for the time of the oscillations, is represented by the quantity $(A+ml)$: now a decrease of 8 inches in the length [21.65 cm] of the blades, produces (*Experiments* 1 , 2, 3), for 20 oscillations, a decrease of 98", a quantity about average between 100 and 95, given by the first and the third experiment. Thus, the blade having here 4 inches in length, ml will be equal to 49", and the general expression will become $T = (A + 49'')$: thus, by subtracting everywhere, in this experiment, 49" from the time of the 20 oscillations, we will have the designated quantity A at the end of each trial.

But we have just seen, in the previous Article, that this quantity is equal to a function of the width and the thickness: thus, if this function can be represented by a single term, we will have $A = nL^{\mu}E^{\nu}$, n being a constant coefficient, L^{μ} a power μ of the width, and E^{ν} a power ν of the thickness;[188] and since in our tests the thickness is constant, we must find the values of A proportional to L^{μ}: thus, comparing two blades of a different width L and L' with the quantities that correspond to them, A and A', we will have

$$A : A' :: L^{\mu} : L'^{\mu} ,$$

from which

$$\frac{A}{A'} = \left(\frac{L}{L'}\right)^{\mu} \qquad \text{and} \qquad \mu = \frac{\log\left(\frac{A}{A'}\right)}{\log\left(\frac{L}{L'}\right)} .$$

It is easy now, by substituting in place of A, and in place of L, their numerical values obtained in each trial, to discover the quantity μ.

First and Fifth Trial

A blade 36/3 lines wide, gives $A = 65''$.
A blade 3/3 line wide, gives $A = 19''$.

It results from these two trials:

$$\mu = \frac{\log\left(\frac{65}{19}\right)}{\log\left(\frac{36}{3}\right)} = 0.4951 .$$

[188] Due to a lapse in the original text, we have here power E of the thickness instead of power ν of the thickness.

First and Fourth Trial

A blade 36/3 lines wide, gives $A = 65''$.
A blade 7/3 lines wide, gives $A = 25''$.

It results from these two trials:

$$\mu = \frac{\log\left(\frac{65}{25}\right)}{\log\left(\frac{36}{7}\right)} = 0.5835 \ .$$

First and Third Trial

It results from these two trials $\mu = 0.6363$.

First and Second Trial

It results from these two trials $\mu = 0.5330$.

Although the value of μ is not perfectly equal in all these comparisons; however the differences are too slight for them to be attributed to anything other than the imperfection in doing the experiments; and we can, without appreciable error in practice, suppose $\mu = 1/2$.

Similar experiments, made with blades of six and eight inches in length, gave me the same results, and the quantity μ never differed by 1/6 from its value 1/2. We must not, moreover, hope in these experiments for greater exactness, a few disparate parts suffice to produce these differences.

5.2.4 Experiments Related to the Thickness of the Blades

Sixth Experiment

31. To have a complete theory of magnetized blades, the only question remaining was to determine how their thickness increased the time of oscillations. Here are different tests that we have made to complete the story.

First trial. A blade 4 inches long, 3 lines wide, weighing 310 grains (16.46 g), was magnetized to saturation; it made 20 oscillations in 136", which gives $A = 87''$.

Second trial. The surface of the blade from the previous test was filed down, without reducing its width: this blade, reduced to 200 grains (10.60 g) and magnetized to saturation, made 20 oscillations in 112", which gives $A = 63''$.

Third trial. Reduced by the same operation, to 104 grains [5.52 g] or to a third of its first thickness, it made 20 oscillations in 79", whence $A = 30''$.

Fourth trial. Reduced on the grinding wheel to 64 grains (3.40 g), it made 20 oscillations in 70" from which $A = 21''$.

Fifth trial. Reduced to 33 grains (1.75 g), it made 20 oscillations in 60"; from which $A = 11''$.

Result of This Experiment

The thicknesses of the blades are between them, in these five trials, approximately like the numbers 3, 2, 1, 2/3, 1/3, the corresponding quantity A is expressed by the numbers 87, 63, 30, 21, 11, which differ very little from being in the same ratio as the first:[189] thus, we can deduce that the quantity A increases proportionally to the thickness; and the general formula $T = A + ml$, which expresses the time of a certain number of oscillations, will become

$$T = \left(nL^{1/2}E + ml\right) \ .$$

5.2.5 Compound Blades

Of the Degree of Magnetism of Several Blades Joined Together

32. In order to render a more exact account of the agreement of the theory of magnetism with experiment, to be able to penetrate into the interior of the magnetized bars, we joined several blades which touched exactly at all the points of their surface. They were fixed together at their extremities and at their center, by three small very light silk ties: the bundles thus composed were magnetized to saturation; they were suspended and made to oscillate, to obtain their degree of magnetism; then decomposing these bundles, we made each blade oscillate separately, in order to be able to compare them with each other. Here is the result of some of these experiments.

Seventh Experiment

33. *First trial.* A single blade four inches long, three lines wide, weighing 108 grains [5.74 g], made 20 oscillations in 80" hence $A = 31''$.

Second trial. Two blades of the same dimensions as the first, were joined together as precisely as possible, they formed a single blade that was twice as thick as the first one, and which weighed 218 grains; it made 20 oscillations in 114", hence $A = 65''$.

Third trial. Three blades united in the same way as the two previous ones, made 20 vibrations in 139", hence $A = 90''$.

Fourth trial. Five blades together made 20 oscillations in 190", hence $A = 141''$.

Fifth trial. Eight blades together made 20 oscillations in 242", hence $A = 193''$.

Result of This Experiment

It follows from this experiment, compared with the sixth, that a bundle of blades takes on nearly the same degree of magnetism as a single blade of the same shape, and of the same weight, consequently the quantity A, is proportional to the thicknesses. This is what results again from the first three trials. But if we compare the first and the fifth trial, we will find in the quantity $A = nL^{\mu}E^{\nu}$, that

$$\nu = 0.7783 \ ,$$

a quantity smaller than the unity: from which it appears necessary to conclude, that when the thickness is significant, everything else being equal, the quantity A increases in a lesser ratio than the thicknesses; but this remark, which seems to introduce a second term into the

[189] That is, the quantities A are to each other as the ratio of the thicknesses.

function of the widths and thicknesses which represent the quantity A, can only apply to very thick bars, and not compass blades, which the theory of magnetism taught us must be large and light.

Eighth Experiment

34. To obtain the magnetic force of the different blades united in the preceding experiment, I have broken down the bundles, and I have caused each blade to oscillate separately.

First trial. A bundle of three blades, which made 20 oscillations in 139", having been decomposed, the two blades of the surfaces made their 20 oscillations, one in 100"; the other in 114"; the center blade showed almost no sign of magnetism.

Second trial. The bundle of eight blades, which made 20 oscillations in 242", being decomposed, made for each particular blade, the same number of oscillations in the following order:

		Duration of 20 oscillations
I	Surface blade	91"
II	Blade	231
III	Blade	278
IV	Blade	211
V	Blade	222
VI	Blade	237
VII	Blade, reversed poles	237
VIII	Surface blade	90

Third trial. As I suspected that the magnetic matter in the two previous tests was in a forced state because the bundles were decomposed only a few hours after having been magnetized, here is what we did to determine the magnetism of each blade, when it would have reached a stable state.

We took a bundle formed of five blades, from which we had carefully removed all the magnetism, before they were brought together. This bundle was then magnetized to saturation; it made 20 oscillations in 190"; we left this bundle for two months without disuniting it, so that, if the magnetic matter found itself in a forced state, it would have time to distribute itself, as in its natural state: at the end of two months, we sought to determine the magnetic force of each blade, and here is what we found.

All the blades being united, [made] 20 oscillations in 196".

The bundle of five blades decomposed:

		Duration of 20 oscillations
I	Surface blade	105"
II	Blade	438
III	Blade	340
IV	Blade	320
V	Surface blade	98

Result of This Experiment

The magnetic force of each of the blades of the preceding experiments, being in inverse ratio of the square of the times of their oscillations, it follows that the magnetic force of the interior blades, is much less than that of the surfaces; it even sometimes happens, when decomposing the bundles, that the poles of one or more blades are reversed; this is what I noticed in the second trial for the seventh blade.

The last trial, which was done with great care, proves to us that the magnetic force of the interior parts of the magnetized bar is almost zero compared to the magnetic force of the surfaces.

We must however observe, that the magnetic *moment* of the interior blades,[190] is probably not the same one, when the blades are united;[191] and, when they are divided, I have almost always found, by calculating the magnetic *moment* of each particular blade, that the sum of these *moments*, was greater than the *moment* of the bundle before separation, which probably arises from the fact that the magnetic state of each blade, depending on the mutual action of all the blades which compose the bundle, this state changes when the blades are separated.

Ninth Experiment

On the Magnetic Force in the Inner Parts of the Blades

35. Between two blades eight lines wide (1.805 cm) and four inches long (10.83 cm), each weighing 244 grains (12.97 g), a third blade of the same dimensions was inserted, but divided according to its length into three other blades; the blade of the center was 4 lines wide, that of the two edges each had two lines: the blade of 4 lines was consequently placed in the center of the bundle which was magnetized to saturation. Here is the result, which seemed to me relevant to the theory of magnetism.

The bundle made 20 oscillations in 172".

By decomposing the bundle:

The blades of eight lines of each surface, made 20 oscillations in 123".

The blades of 2 lines which formed the edges of the central blade, [made] 20 oscillations in 124".

The central blade, 4 lines wide, made 20 vibrations in 128"; but its poles were in a situation contrary to those of the bundle: so that its boreal extremity was located at the austral extremity of the bundle.

Result of This Experiment

It evidently follows from this experiment, that it can often happen that the central parts of magnetized bars have a force of a name contrary to that of the parts which adjoin them.[192]

[190] In the original: *momentum magnétique des lames intérieures*. Coulomb is referring here to the magnetization of the inner blades. The magnetic torque exerted by the Earth on these magnetized blades is proportional to the magnetization of the blades.

[191] That is, by bringing together several magnetized blades, Coulomb found that the magnetism of the blades that were in the center decreased in relation to their magnetism when they were isolated from the other blades.

[192] That is, they may have a type of magnetic fluid with a name contrary to the fluid of neighboring parts.

36. We have made a large number of experiments of the same kind, either by joining several laminae to increase the thicknesses, or by joining the same laminae, according to their width. We have also composed bundles, with very fine steel wires: but all these experiments, which seemed to us suitable for enriching the theory of magnetism, do not have a sufficiently direct relationship with the main subject of this Memoir, to merit including here. It is the same with all the experiments that we have been able to make with blades, which were first magnetized separately to saturation, and with which we then formed bundles, when we came to separate them, the magnetic force of the central blades had almost disappeared, or at least was hardly greater than in the experiments of Article 34.

5.2.6 Thoughts on the General Formula $T = \left(mL^{1/2}E + nl\right)$

37. It can be assured that the formula $T = mL^{1/2}E + nl$ has been confirmed by a large number of experiments, and that it has always predicted the results as exactly as can be expected in practice.

We are now going to compare it with the formulas of oscillatory motion, determined in Article 7, and we will draw the consequences which relate to our object.

We found (Article 11):

$$\int \varphi\mu r = \frac{\pi^2 \int \mu r^2}{T^2} \ .$$

On the other hand, we have, for blades of uniform width and thickness, assuming the density equal to unity,[193]

$$\int \mu r^2 = \frac{2LEl^3}{3} \ .$$

Substituting these two values, and setting $K = 2\pi^2/3$, we will have

$$\int \varphi\mu r = \frac{K \cdot LE \cdot l^3}{\left(mL^{1/2}E + nl\right)^2} \ .$$

Here is what this equation implies:

38. The *moment* of the magnetic force $\int \varphi\mu r$, will increase with the length of the blade, and will become infinite when this length is infinite.

39. This same *moment* will grow as the width L increases; and when this width is infinite, it will be equal to $Kl^3/(m^2E)$.

40. If we differentiate this equation by making only E variable, we will find, for the *maximum* of $\int \varphi\mu r$:

$$\frac{dE}{E} = \frac{2mL^{1/2}dE}{mL^{1/2}E + nl} \qquad \text{and} \qquad E = \frac{nl}{mL^{1/2}} \ .$$

[193] See footnote 163 on page 65.

41. If we divide $\int \varphi \mu r$ by the cross section LE, we will have the mean *moment* of the magnetic force of each longitudinal fiber, of which we can suppose the lamina formed; which will give

$$\frac{\int \varphi \mu r}{LE} = \frac{Kl^3}{\left(mL^{1/2}E + nl\right)^2} \ ,$$

a quantity which will increase as the quantity l increases, which will become infinite with this quantity, which will also increase, as L or E decrease, and which will be equal to Kl/n^2, when L or E will be zero; which gives the *moment*, in the latter case, proportional to the length of the blade. We find here the formula of Mr. Musschenbroek, which is only true when L or E can be assumed to be infinitely small, or when the quantity $mL^{1/2}E$ can be neglected; but yet, in this case, the weight does not matter; it does not enter into this expression; which is again contrary to the theory of this author.

5.3 Chapter III. Experiments and Theory on the Twisting Force of Hair and Silk Threads. Comparison of These Forces with the Magnetic Force. Air Resistance in Very Slow Movements. Construction of a New Declination Compass, Suitable for Observing Diurnal Variations

42. All the means that can be used to suspend a declination needle necessarily entail inconveniences. If it is suspended by a silk thread, or by a hair, the compass will always have to employ a certain force to twist them; and if the silk thread is supposed to be twisted, when the needle is on its true [magnetic] meridian, the silk will make an effort to drag it in another direction.

If the needle is supported, by means of a cap,[194] on the tip of a pivot, however perfect this cap, however hard the tip of the pivot, the cap will press with all the weight of the needle onto the tip of the pivot: any pressure generates friction; thus, as soon as the *moment* of the magnetic force, is equal to the *moment* of the friction, the needle will not experience any [unbalanced] force tending to re-establish it along its magnetic meridian.[195]

Besides the difficulties which the means of suspension present, there is another which arises from the cohesion of the air. Any fluid has some tenacity among its parts; thus, in order that a body which is immersed in it may change its position, the force which draws it from its state of rest must necessarily be greater than the resistance which this tenacity opposes to it. But we will see presently that the resistance due to cohesion is small compared to the magnetic force, and that it can be neglected.

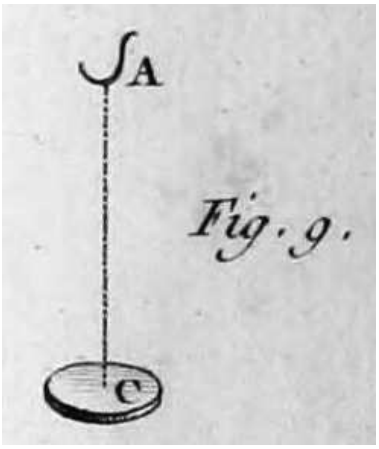
[194] See footnote 141 on page 58.
[195] That is, friction will prevent it from orienting itself along its magnetic meridian.

5.3.1 On the Twisting Force of Hair and Silk Threads

43. We cannot quote here the experiments of any author:[196] but those which we are going to report are so simple, so easy to repeat, that I hope that they will deserve some confidence.

First Experiment

I suspended *(Figure 9)*, with a hair six inches long (16.24 cm), a round piece of copper eight lines in diameter (1.80 cm), and weighing 50 grains (2.66 g), so that it was supported by its center C, and that its plane was horizontal. I have turned this plate around its center C, without disturbing it from its horizontal position; the wire AC always remaining vertical; left to itself, it assumed, while oscillating, a rotational movement, around its center C.



We measured the time of each oscillation, and we found that whether this plate made one, two and up to six or seven revolutions per oscillation, the time of each oscillation was constant and equal to $\frac{16}{2}''$.[197]

Result of This Experiment

When a body suspended by a thread or a hair is left to itself, it soon comes to a state of rest, in which the thread, which supports it, makes no effort to make it rotate in any direction. This state is what can be called the natural situation of the hair. But if, the center of gravity remaining motionless, we make the body rotate around this center; as it moves from its state of rest, the hair will twist, and in twisting it will make an effort to reestablish itself in its natural situation: now we find, in this experiment, that this effort produces oscillations whose time is constant, whatever the initial angle of rotation: thus, the forces of torsion, which bring a body back to its natural situation, are necessarily proportional to the angle

[196] That is, Coulomb is being original here, as he has not encountered any other scientist who has studied the twisting force of threads.

[197] As will be seen in Section 6.3 on page 141, this time interval of 8 s to perform each oscillation is the time for the torsion thread to start at rest at an initial angle θ_o around the vertical axis, with that angle measured from a certain horizontal reference line fixed in the laboratory, until it comes to rest at angle $-\theta_o$ at the other side of this reference line. The total period required to begin at θ_o and to arrive again at this initial angle θ_o is 16 s. What Coulomb found is that the 8 s time for each oscillation was constant, no matter if $\theta_o = 360°$, $720°$, ..., or even $\theta_o = 7 \times 360° = 2520°$.

of torsion.[198]

Second Experiment

44. We sought in this experiment if the weight of the body supported by the hair, influenced the force of torsion. Here is what we find.

First trial. A single plate of the same dimensions as in the preceding experiment, suspended from a six-inch hair, makes one oscillation in $\frac{16}{2}''$.

Second trial. Under this first plate, a second plate absolutely similar to the first was glued with a little wax: the two pieces joined together made one oscillation in $\frac{22}{2}''$.

Third trial. A third plate, joined to the other two, made each oscillation in $\frac{27}{2}''$.

Fourth trial. A fourth plate joined together like the preceding ones, makes each oscillation in $\frac{30}{2}''$.

Fifth trial. A fifth [plate] joined together makes each oscillation in $\frac{35}{2}''$.

Sixth trial. A sixth piece [joined together makes each oscillation] in $\frac{39}{2}''$.

Seventh trial. A seventh [plate joined together makes each oscillation] in $\frac{42}{2}''$.

Result of This Experiment

The torsion force, being in consequence of the preceding Article, proportional to the angle of twist, if the difference of the weights does not change this force, it will be the same in each test, and T^2 will be proportional to $\int \mu r^2$, T being the time of one oscillation,[199] and $\int \mu r^2$ the sum of the products [of the masses] of all the points of the plate, and the square of their distance from the center of rotation C, but the plates being all equal, $\int \mu r^2$ is as the number of plates used in each test; thus, it is only a question of seeing if T^2 is proportional to the number of plates.[200]

[198] That is, the torques exerted by the hair or silk thread are proportional to the angle of twist. Let a rigid body with moment of inertia I be able to rotate through an angle θ with respect to an inertial frame of reference, this angle being measured with respect to a fixed line in this frame of reference. The second law of motion of mechanics for the case of rotations is given by $\tau = Id^2\theta/dt^2 = I\ddot{\theta}$, where τ is the torque acting on the body. The body will describe a periodic harmonic motion of rotation that does not depend on the initial angle θ_o if $\tau = -k\theta$, where k is a positive constant. In the case of Coulomb's experiment, this constant k is called the torsional elastic constant, the torsion elastic modulus, the torsion coefficient or simply the torsion constant. In this case, assuming that the body starts from rest at the initial angle θ_o, the solution of the equation of motion $I\ddot{\theta} + k\theta = 0$ is given by $\theta = \theta_o \cos(\omega t)$, with $\omega = \sqrt{k/I}$ being the angular frequency of oscillation (measured in radians per second). The period for a complete round-trip oscillation is given by $T = 2\pi/\omega = 2\pi\sqrt{I/k}$, which does not depend on the initial angle θ_o.

[199] In the original: *T étant le temps d'une oscillation.*

[200] The moment of inertia of a disk of mass M_1 and radius R about its axis of symmetry is given by $I_1 = M_1R^2/2$. Therefore, if we have N equal disks, the moment of inertia of this set is given by $I_N = NM_1R^2/2 = NI_1$. From footnote 198 on page 85 we obtain that the square of the period of oscillation for a single disk is given by $T_1^2 = 4\pi^2 I_1/k$. The square of the period of oscillation of N disks is given by $T_N^2 = 4\pi^2 I_N/k = N(4\pi^2 I_1/k) = NT_1^2$, if k does not depend of the weight of the body that is supported by the thread. That is, T_N^2 is proportional to the number N of disks. The period of oscillation of N disks must be proportional to the square root of N, that is, $T_N = (\sqrt{N}\,)T_1$. It was using this formula that Coulomb presented the expected theoretical results that can be found in the second column of the following Table. For example, since the period found for a single disk was 16/2 s, the expected period for two equal disks is given by $(\sqrt{2}\,)16/2$ s $= 22.6/2$ s. In the following Table, all the times in the second and third columns

	by theory	by experiment
The first trial, compared with the second, gives, for the time of the oscillations of two pieces	22.6”	22”
Trials I and III	27.7	27
Trials I and IV	32	30
Trials I and V	35.8	35
Trials I and VI	39	39
Trials I and VII	42.3	42

We see, by this Table, that experiment and theory have the greatest conformity, and thus that the mass of the bodies supported by the hair, or, what comes to the same thing, the tension of these hairs, has no influence on the force of torsion.

It should, however, be noted that when the weight of the body is greatly increased, and the hair or silken threads are threatening to break, the same law is not exactly observed; in that case the torsion force seems much reduced; the oscillations are no longer isochronous, the time of the large ones is much greater than that of the small ones: it happens in this case that the thread, under too much tension, loses its elasticity, almost like a blade which only keeps its elasticity, when it is bent up to a certain point.

Third Experiment

45. We sought to determine, in this experiment, according to which law the increase in length, in the hair, reduced the force of torsion.

First trial. A hair three inches in length [8.12 cm], charged with a piece of copper, similar to those in the preceding Articles, made each oscillation in $\frac{11}{2}''$.

Second trial. A hair six inches in length, charged with the same piece, made its oscillations in $\frac{16}{2}''$.

Third trial. A hair twelve inches in length, by suspending the same piece on it, made its oscillations in $\frac{22}{2}''$.

Result of This Experiment

As the hair is lengthened, the copper plate can make a greater number of revolutions, without increasing the twist of this hair. If, for example, we compare the twist of each part of the hair, when the piece of copper makes one revolution with a hair of three inches in length, with the twist, when the plate makes one revolution with a hair of six inches; the twist of each part of the hair will be double in the first case, of what it will be in the second. It must therefore happen, according to all that we know of the action of springs, that the reaction of torsion must also be double in the first case; thus, the torsion forces must, at equal revolutions, be in inverse proportion to the lengths.[201] But the formulas of isochronous oscillatory motion say that the forces are[202] in inverse proportion to the square of the times

should have been divided by 2.

[201] That is, the torques exerted by the threads must be inversely proportional to their lengths.

[202] The forces of torsion, that is, the torques.

of the oscillations; thus, the squares of the times must be in direct proportion to the length of the hair. Let us compare this theory with experiment.[203]

	by theory	by experiment
Trials I and II give	$\frac{15}{2}'' + \frac{1}{4}$	$\frac{16}{2}''$
Trials I and III	$\frac{22}{2}''$	$\frac{22}{2}''$

The experiment and the theory thus agree again here, proving that the forces of torsion are, with equal revolutions, inversely proportional to the lengths of the hair.

Fourth Experiment

46. Finally, we tried to determine how much the diameter of the hair, or of homogeneous silk threads, influenced the force of torsion. I will not report here the details of the experiments that I have been able to make on this subject; because the difficulty of measuring the diameter of a hair, or of a very fine silk thread, and of making sure that it is homogeneous throughout its length, has caused the results to vary; but it has been generally found, by comparing a large number of experiments, that for homogeneous silk threads of the same length, the forces of torsion were, at equal revolutions, in the triple ratio of the diameters.[204]

These same experiments have been repeated with silken threads, preferring their use for suspending the compass needles; because we have recognized that with equal forces, they are infinitely more flexible than the first,[205] and we have found the same laws as in the preceding experiments.[206]

[203] It was seen in footnote 198 on page 85 that the period for a harmonic oscillation is given by $T = 2\pi/\omega = 2\pi\sqrt{I/k}$, where I is the moment of inertia of the body and k is the torsion coefficient of the suspension thread. Therefore, T^2 is inversely proportional to the coefficient k. If k is inversely proportional to the length ℓ of the thread, we will have T^2 proportional to this length ℓ. Thus T must be proportional to $\sqrt{\ell}$. Therefore, if a thread of length ℓ_1 performs its oscillations in a period T_1, a thread with a length $\ell_N = N\ell_1$ will perform its oscillations in a period $T_N = (\sqrt{N})T_1$. The theoretical results presented in the next Table were obtained using this formula. For example, since a 3-inch thread performed its oscillations in $11/2$ s, a 6-inch thread must perform its oscillations in $(\sqrt{2}) \cdot 11/2\ s = 15.556/2\ s \approx (2/15 + 4/1)\ s$. A 12-inch thread must perform its oscillations in $(\sqrt{4}) \cdot 11/2\ s = 22/2\ s$.

[204] That is, the torques are proportional to the cube of the diameters. Later on Coulomb corrected himself, arriving in 1784 at the result that the torques (or twisting forces) are proportional to the fourth power of the diameters of the wires, [Coulomb, 1787]. This work of 1784 is translated in Chapter 7.

[205] That is, silk threads are more flexible than hair.

[206] That is, Coulomb found that the torques exerted by silk threads follow the same laws as hairs. The torque τ exerted by these two types of thread as they rotate through an angle θ with respect to the relaxed orientation can be written as $\tau = -k\theta$, which is valid even for large angles of up to seven turns (that is, with an initial angle θ_o going up to $\theta_o = 7 \times 360^\circ = 2520^\circ = 14\pi\ rad$). In this equation k is a positive constant that Coulomb found to be independent of the weight of the body that is suspended on the thread, being inversely proportional to the length of the thread and proportional to the third power of the diameter of the thread. Later on Coulomb will correct himself, arriving at the result that the torques (or twisting forces) are proportional to the fourth power of the wire diameters, see Chapter 7.

5.3.2 Comparison of the *Moment* of the Magnetic Forces, with the *Moment* of the Torsional Force of Silk Threads

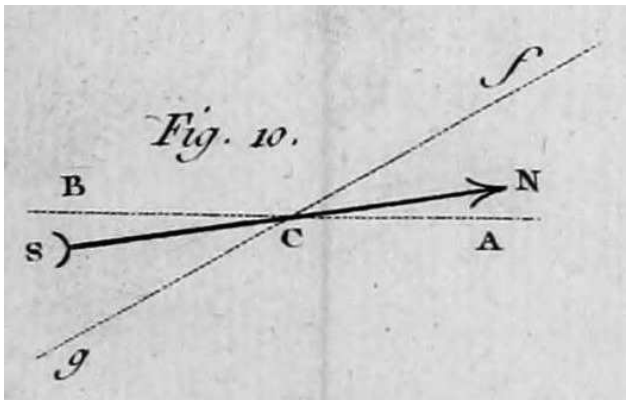
47. We have seen, in Articles 6 and following, that when a declination compass is moved away from its true [magnetic] meridian by a small angle C, the *moment*[207] of all magnetic forces, to bring it back to its meridian, is expressed by the quantity $C\int(\varphi\mu r)$, and that the time of the oscillations is given by the equation

$$\frac{\int \varphi\mu r}{\int \mu r^2} = \left(\frac{\pi}{T}\right)^2 .$$

But we have just seen, by experiments on torsion; that if a body is supported by a silk thread, whose angle of twist is C', we will have aC' for the *moment* of the torsion force; a being a constant quantity,[208] and thus we will have for the time of one oscillation,

$$\frac{a}{\int \mu' r'^2} = \left(\frac{\pi}{T'}\right)^2 .$$

Thus, in either case, the time of the oscillations being given by experiment, as well as the quantities $(\int \mu r^2)$ and $(\int \mu' r'^2)$, it will always be easy to determine, for a given angle C, the ratio between the *moment* of the torsion force a and [the *moment* of] the magnetic force $\int(\varphi\mu r)$, and to find, therefore, by how much a given angle of torsion can move a needle away from its true magnetic meridian.



If we suppose *(Figure 10)*, that the needle SCN suspended by a silk thread, and balanced horizontally, is distant from its true [magnetic] meridian BA by the angle $NCA = C$, and that the angle of torsion of the silk which supports this needle is $fCN = C'$; this needle, arriving at its state of rest, is solicited by two forces, namely, the magnetic force, whose *moment* to bring it towards $A = C\int \varphi\mu r$, and the force of torsion, whose *moment* is aC'; and as there is equilibrium, we find the equation

$$C\int \varphi\mu r = aC' , \qquad \text{or} \qquad C = \frac{aC'}{\int \varphi\mu r} ;$$

whence it follows that the error of the needle,[209] expressed by C, will increase as the product of the torsion force by the angle of torsion, and will decrease as the magnetic force increases.

[207] In the original: *momentum*. See footnote 150 on page 60.

[208] That is, the torque is given by aC'.

[209] In the original: *erreur de l'aiguille*. That is, the angle of deviation of the needle from its true magnetic meridian. This deviation can also be called the compass error since, because of the resistance of the silk thread, it will not be pointing at rest along the true local magnetic meridian.

It is therefore easy to suspend a needle in such a way that the torsion of the silk [thread] has very little influence on its orientation, and produces only insensible errors. Here is how we can do it: we will first suspend from the silk thread that we want to use, a silver or copper needle, and we will make sure that, when the silk will have arrived in its natural state, the direction of the copper needle coincides with the magnetic meridian: we will then substitute a magnetized needle, of the same weight, for the copper needle, and we will be sure that the torsion of the silk only insensibly influences the direction of the needle; since the torsion angle roughly coincides with the magnetic meridian.

48. But, to give to these principles the attention they appear to deserve, considering their utility in what follows, and relevance that they can have for the arts, we are going to prove that in suspending magnetic needles from very fine, untwisted silk threads, sufficient to support the weight without breaking, even if the angle of torsion were supposed to be more than 100 degrees with the magnetic meridian; the torsion force would still be so insignificant, compared to the magnetic force, that it would still only produce insensible errors.

Fifth Experiment

49. A silk thread, such as it emerges from the cocoon, supports, without breaking, a weight of 200 grains (10.6 g). To determine the *moment* of its force of torsion, we suspended, horizontally from this thread, a small cylindrical copper needle of one inch length (2.71 cm), and six grains of weight (0.32 g). The silken thread, from its attachment to the point of suspension, was only an inch in length; the copper wire[210] was rotated horizontally around its center of gravity: left to itself, it made its substantially isochronous oscillations in 40".[211,212]

Result of This Experiment

We find, for a body that oscillates due to torsion, the equation

$$a = \frac{\int \mu' r'^2}{T'^2}\pi^2 \ .$$

By naming P' the weight of the needle, and l' half of its length, we will have

[210] That is, the copper needle.

[211] [Note by Potier] That is, a torsion couple of 0.0012 for an angle = 1.

[212] The cylindrical needle has a length $\ell = 2.71\ cm = 0.0271\ m$ and a mass $M = 0.32\ g = 0.00032\ kg$. The moment of inertia of this needle about an axis orthogonal to it passing through its center is given by

$$I = \sum_i m_i r_i^2 = \int_{-\ell/2}^{\ell/2} x^2 dm = \int_{-\ell/2}^{\ell/2} x^2 \frac{M dx}{\ell} = \frac{M}{\ell}\int_{-\ell/2}^{\ell/2} x^2 dx = \frac{M\ell^2}{12} \ .$$

That is, in this case $I = 1.96 \times 10^{-8}\ kgm^2$. The reaction torque τ exerted by the wire when it is twisted through a small angle θ with respect to an inertial frame of reference is given by $\tau = -k\theta$, where k is a positive constant.

As shown in footnote 198 on page 85, the period T for the needle to leave and return to the same point when it is released from rest removed by a small angle θ relative to its natural orientation is given by $T = 2\pi\sqrt{I/k}$. In this case Coulomb obtained $T = 2 \times 40\ s = 80\ s$. So $k = 4\pi^2 I/T^2 = 1.2 \times 10^{-10}\ kgm^2/(s^2 rad) = 1.2 \times 10^{-3}\ dyn \cdot m/rad$.

Therefore the torque exerted by the thread when it is twisted through an angle $\theta = 1\ radian$ is given by $\tau = k\theta = 1.2 \times 10^{-10}\ Nm = 1.2 \times 10^{-3}\ dyn \cdot cm$, as calculated by Potier.

$$\int \mu' r'^2 = \frac{P' l'^2}{3g} \ ,$$

from which

$$a = \left(\frac{\pi}{3g}\right)^2 \frac{P' l'^2}{T'^2} \ .$$

We find, for the *moment* of the magnetic force,

$$\int (\varphi \mu r) = \left(\frac{\pi}{3g}\right)^2 \frac{P l^2}{T^2} \ .$$

By comparing these two equations, it follows

$$\frac{\int \varphi \mu r}{a} = \frac{P l^2 T'^2}{P' l'^2 T^2} \ .$$

Example

Thus, if we want to compare the magnetic force of a blade 4 inches long (10.83 cm), 3 lines wide (0.677 cm) and with 100 grains of weight (5.3 g), with the torsion force, which results from this experiment, we will find that this blade, magnetized to saturation, makes 20 oscillations in 80", so we have the time for one oscillation = 4″, [and]

$$\frac{P}{P'} = \frac{100}{6} \ , \qquad \frac{l^2}{l'^2} = \frac{4^2}{1} \ , \qquad \frac{T'^2}{T^2} = \left(\frac{40}{4}\right)^2 = (10)^2 \ ,$$

from where we find

$$\frac{\int \varphi \mu r}{a} = \frac{26670}{1} \ .$$

Thus, a torsion angle (Article 47) of 26670', or 444°, would produce, with this silk thread, only one minute of error in the orientation of a needle four inches in length, weighing 100 grains.[213,214]

[213] One degree has 60 minutes, that is, $1^\circ = 60'$. The magnetic torque τ_{mag} exerted by the Earth on a magnetized needle deflected by a small angle θ with respect to the local magnetic meridian, assuming $\theta \ll 1$ radian, is given by $\tau_{mag} = -k_{mag}\theta$, where $k_{mag} = \int \varphi \mu r$. The torque τ_{thread} due to twisting the thread at an angle θ with respect to the relaxed orientation of the thread is given by $\tau_{thread} = -k_{thread}\theta = -a\theta$, where the torsion constant of the thread is given by $k_{thread} = a$. The relationship Coulomb just found is that $k_{mag}/k_{thread} = 26670/1$, that is, $k_{mag} \gg k_{thread}$. That is, in practice we can neglect the torque exerted by the suspension thread on the needle in comparison with the magnetic torque exerted by the Earth on the magnetized needle. Only in the case of high precision experiments, such as those carried out later by Carl Friedrich Gauss (1777-1855) and Wilhelm Eduard Weber (1804-1891), will it be necessary to take into account both the magnetic torque exerted by the Earth on the magnetized needle, and the torque exerted by the torsion of the suspension thread. The works of Gauss and Weber related to this topic are already translated into English in the book *Wilhelm Weber's Main Works on Electrodynamics Translated into English*. Volume 1: Gauss and Weber's Absolute System of Units, [Assis (editor), 2021a]; Volume 2: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday, [Assis (editor), 2021b]; Volume 3: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, [Assis (editor), 2021c]; and Volume 4: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, [Assis (editor), 2021d].

[214] [Note by Potier] If we suppose the horizontal component of terrestrial magnetism equal to 0.185, the magnetic moment of the needle would be, per unit of volume, approximately 250. In the first experiment of Article 27, this moment would be 453 for a blade four times longer.

Sixth Experiment

50. We took a silk thread 20 inches in length (54.14 cm), and composed of 12 brins,[215] such as they come out of the cocoon or the spinneret of the silkworm.[216] These 12 threads were glued together without being twisted, and could support, without breaking, a weight of 1800 grains (95.6 g). The same copper needle was suspended from this thread horizontally as in the preceding experiment: it made its substantially isochronous oscillations in 29".

Result of This Experiment

The quantity $\int \mu' r'^2 = P' l'^2/(3g)$, is the same as in the previous experiment, since it is the same copper wire[217] that the torsion force causes to oscillate: thus, comparing this experiment with the previous one, we must find the *moments* of the forces of torsion, inversely proportional to the square of the times: thus, the force of torsion is here $a(40/29)^2 = 1.90a$ or double, approximately, the force of torsion calculated in the first experiment: thus, an angle of torsion, equal to 222°, would produce only one minute of error, in the orientation of the needle described in the previous Article.[218]

We are therefore certain that the torsion of the threads can only influence in an insensible manner the orientation of the magnetic needles which will be suspended there. It remains to be determined whether the cohesion of the air can produce errors.

5.3.3 Air Resistance in Very Slow Movements

51. Some famous authors have thought that the component of the resistance of the air, which is constant and independent of the speed, was of such magnitude that it should not be neglected in the formulas of the motion of bodies in this fluid. I will prove, I believe, that the *moment* of this constant resistance, is only a very small part of the *moment* of the magnetic force [acting] on a blade; that it can only produce insensible errors in the orientation of the compass, and that there is scarcely any research in which it cannot be neglected. This is what we can first, it seems to me, conclude from the following.

If we suspend horizontally from the silk thread of the previous experiment, a copper blade, it will always stop, within a few degrees, in the same direction: but as there is only the torsion force here which acts, and that we have found this force very small for a rather considerable angle; it follows that, since this blade is always brought back nearly to the same direction, the constant component of the resistance of the air can only be an insensible quantity.[219] But here is something more specific.

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[215] [Note by Bucciarelli] A brin is one of the two circular fibers that issue from the silkworm. It consists of two parts, the inner of fibroin, or true silk, and the outer of sericin, or silk-glue.

[216] A spinneret is a silk-spinning organ of a spider or the larva of an insect.

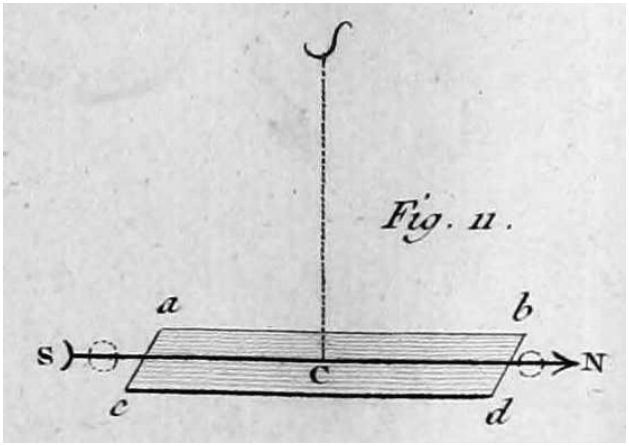
[217] That is, the same copper needle.

[218] Let us compare this result with the one presented in footnote 213 on page 90. We now have $k_{mag}/k_{thread} = 222°/1' = 13320/1$ such that again $k_{mag} \gg k_{thread}$.

[219] [Note by Bucciarelli] The copper blade is not magnetized. So the only torque acting is due to the stiffness of the silk thread. Coulomb seems to be thinking that the resistive medium of the air is like a constant frictional force (torque) which, after a forced rotation, the blade will return to a different orientation depending upon the initial displacement. But, in even a viscous fluid, is this possible? Will not the blade, given enough time, eventually return to the same initial orientation? But if the blade is in motion through the air, as in the experiments which follow, then the moving air (relative to the blade) will have a resisting effect. But as Coulomb's experiments show, this is negligible.

Seventh Experiment

52. An iron wire *NS (Figure 11)*, 9 inches in length [24.36 cm], and weighing 24 grains [1.27 g], was weakly magnetized. It was suspended, by its center C, with a silk thread of a single brin six inches in length [16.24 cm], and whose angle of twist was nil: its magnetic force made it perform 4 oscillations in 62". As it was only a question of determining the constant part of the resistance of the air, an attempt was made to further reduce the speed of the oscillations: this was easily [accomplished] by attaching to each end of this needle, a small weight of 50 grains [2.66 g]: we then glued to the iron wire a rectangle *abcd* of paper, one inch wide, and eight inches long.



In the first trial the plane of the paper was horizontal; in the second, it was vertical: the whole system made very small movements, very slowly; 4 oscillations in 155". Making the needle oscillate in the two tests, we observed how much the angle described diminished at each oscillation, from the beginning of the movement until the oscillations were insensible.

First trial. The plane of the paper being horizontal, the needle displaced 2° of its true magnetic meridian:

It arrived at	in
1° 45′	2 oscillations
1° 30′	2 oscillations
1° 15′	2 oscillations
1°	2 oscillations
45′	2 oscillations
30′	4 oscillations
15′	4 oscillations
0′	6 or 8 oscillations

Second trial. Vertical paper plane:

[it arrived at]	in
from 2° 5′ to 1° 50′	2 vibrations
1° 20′	2 vibrations
50′	4 vibrations
20′	4 vibrations
10′	4 vibrations
0′	4 or 6 vibrations

Result of This Experiment

The iron wire, which was used in this experiment, oscillates by virtue of the magnetic force. The twist of the silk is zero, the arcs described at each oscillation decrease by the resistance that the air opposes to the movement: now we have found (Article 9), that when a needle oscillated under the magnetic force, if it experienced a resistance whose *moment* was a constant quantity A, we would have at each vibration, for the difference of the described angles $(B - B') = 2A : \int \varphi\mu r$; thus, if we assume that the iron wire when oscillating does not move more than 30' away from its meridian, it then experiences a constant resistance. We will see that since, in the first trial, we still distinguish 15 oscillations up to the point of rest, we have 2 minutes of loss at each oscillation: thus, $A : \int \varphi\mu r = 1'$; a quantity which expresses (Article 6) the error that the quantity A can produce.

If we now compare the magnetic *moment* of this needle, which we have magnetized very weakly, with the magnetic *moment* of a blade 4 inches long, 3 lines wide, 100 grains of weight, we shall find that the quantity A could hardly produce, in the orientation of this needle, an error of 5 to 6", a quantity which may be neglected.

If we want to obtain the resistance that the plane of the paper experiences when it is vertical, we will find, by reasonings similar to those which precede, that, since it makes 11 or 12 oscillations when it began to vibrate at 30' from its meridian, the resistance of the air is still insensible in this case: it seems to follow that, in whatever way the plane is placed, in relation to the direction of its movement, the constant resistance is about the same, and that the difference found between the first and the second trial is due to the low speed in these two tests.

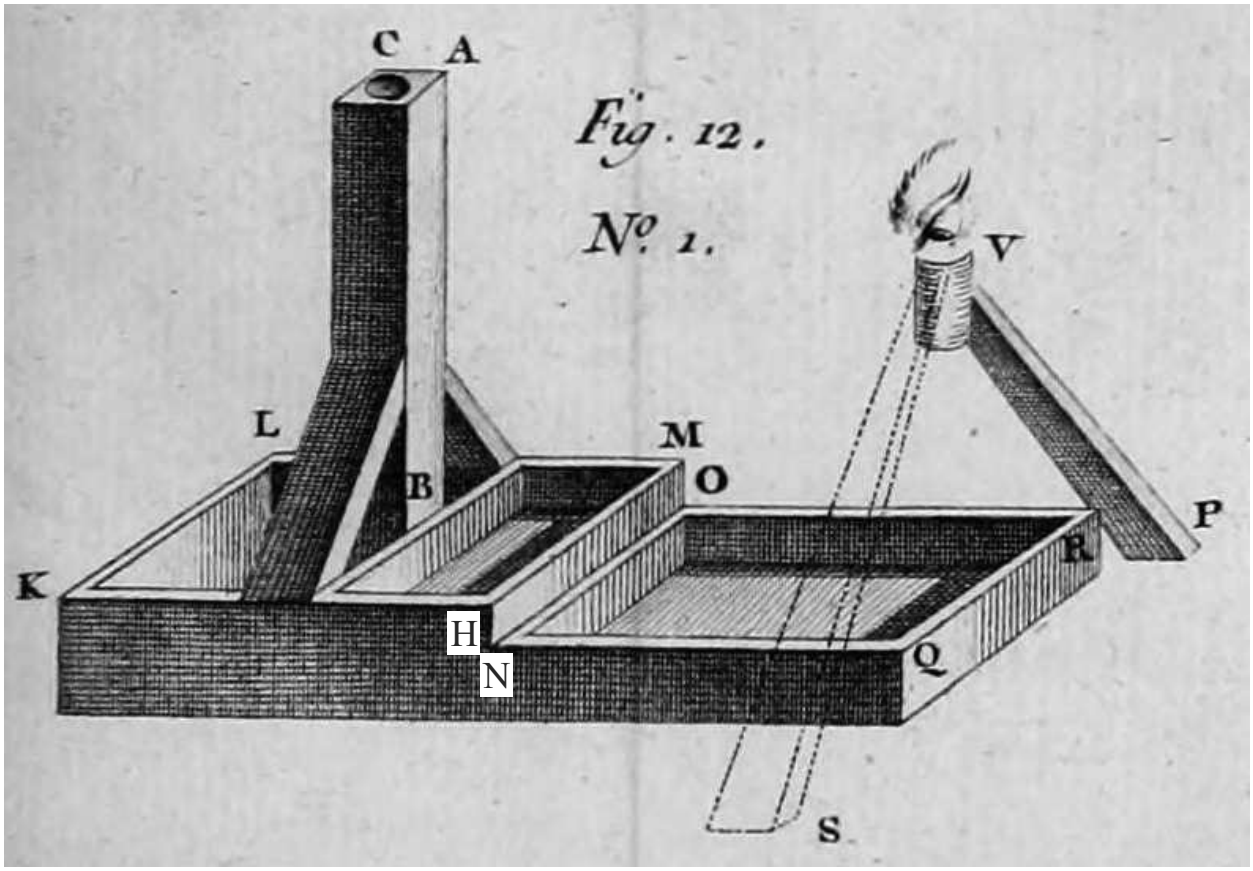
53. By oscillating different needles with small planes of paper, as in the previous Article, and by extending the oscillatory movement up to 10 or 12 degrees from the magnetic meridian, we have made observations which, compared using the formulas of oscillatory motion, appeared to us appropriate for the theory of the resistance of the air, when the bodies move very slowly; but this work has no connection with this Memoir.

These experiments, moreover, are very delicate, and demand the greatest attention. The needle and the thread which suspends it, must be enclosed in a box where the air cannot penetrate: We make the needles oscillate by presenting to them the [magnetic] pole of another needle outside the box; the small oscillations are observed with a magnifying glass.

5.3.4 Construction of a Compass Suitable for Observing Diurnal Variations

54. Knowing that the cohesion of the air and that the torsion of the silk threads could only influence the orientation of the magnetized needles in an imperceptible way, I had a compass made, almost without the help of any artisan,[220] with which I have been observing, for five months, the diurnal variation [of terrestrial magnetism] with a precision that we could never hope for with capped needles suspended on pivots.[221]

Figure 12, No. 1, represents in perspective all the parts of the box where the needle is enclosed.[222]



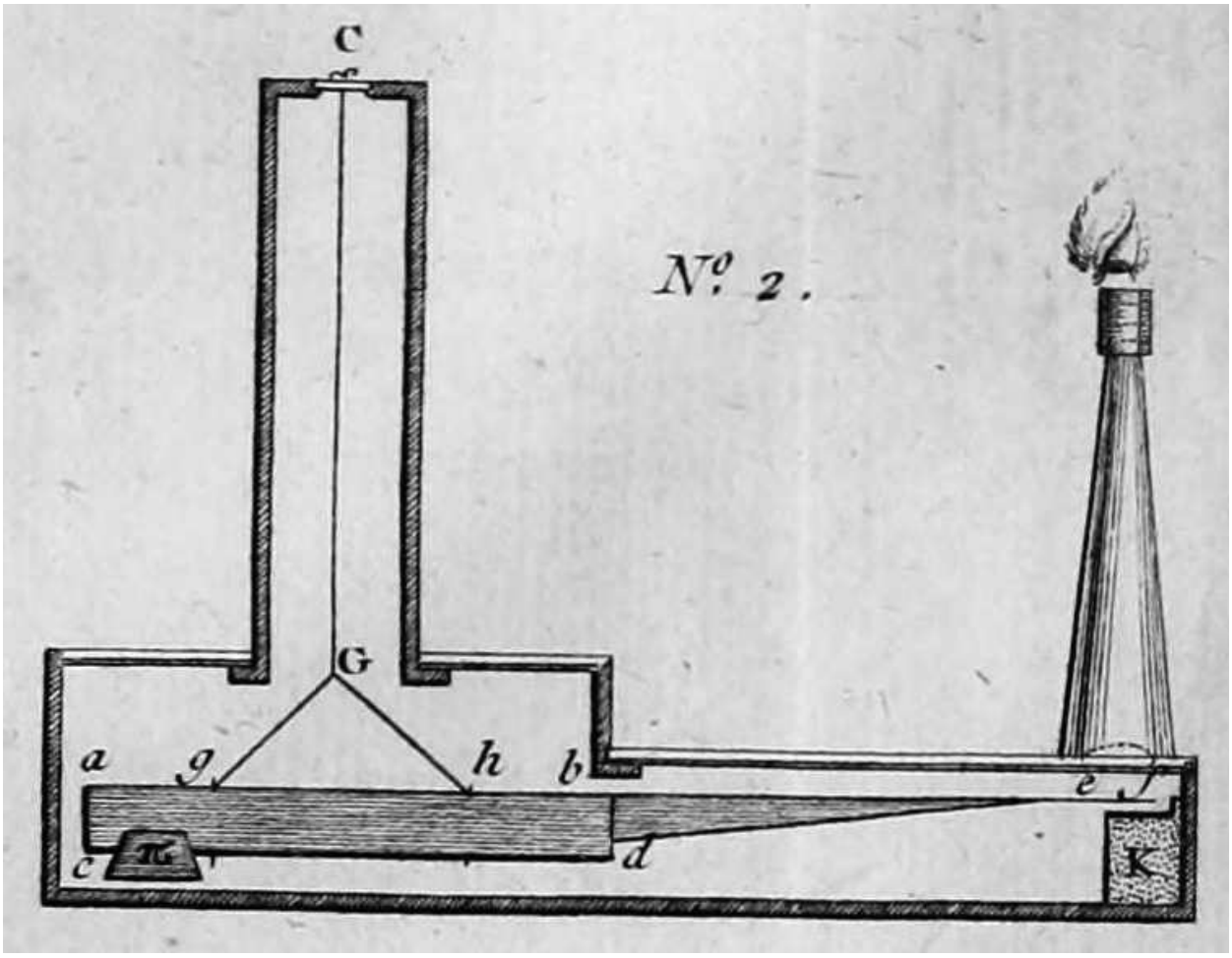
Part AB is a hollow rod, which rises 20 inches [54.14 cm] above the box $HKLM$, in the middle of which it is fixed, by means of a crosspiece and two small struts which support it. At the end of this rod, we put, at C, a circular copper plate, mobile and pierced in its center, to receive there the end of a silk thread which supports the needle. Part $ONQR$ is an extension of the large box $HKLM$ on a lower height. These boxes are closed by frames lined with glasses, which allow seeing everything that happens inside.

SVP is a wooden support, fixed to the table upon which the compass box is placed. This support carries at its summit V a small hollow cylinder, or a small telescope[223] with a very

[220] A discussion of this new compass and its later improvements can be found in [Heering and Osewold, 2005]. See also [Licoppe, 1995].
[221] See footnote 141 on page 58.
[222] Since the letters H and N were not showing up clearly in this Figure, I emphasized these letters.
[223] In the original: *lunette*. This word can also be translated as lunette or spyglass.

wide field of view, so that the observer always places his eye at the same point.

Figure 12, No. 2, represents a vertical section of the box, made along its greatest length, which we take care to place approximately parallel to the magnetic meridian.



abcd represents the steel blade, or magnetic needle suspended sideways.[224] It is ten inches long (27.07 cm), three and a half lines wide (0.795 cm), and weighs 250 grains (13.3 g).[225] At its boreal extremity b is welded a small, very light copper blade $bdef$, which ends in an extraordinarily fine tip. At the austral extremity is a small counterweight which embraces the blade, and is supported there by friction;[226] it serves to set the needle in a horizontal position. CB is a silk thread of 12 brins,[227] similar to that which we have worked in the preceding experiments: it has been untwisted, or brought back to its natural direction, by a copper needle which was first suspended by it; and as the attachment C is fixed to a circle movable around its center, it was easy to make the copper needle coincide, when the silk was in its natural state, with the magnetic meridian, which was more or less well known.

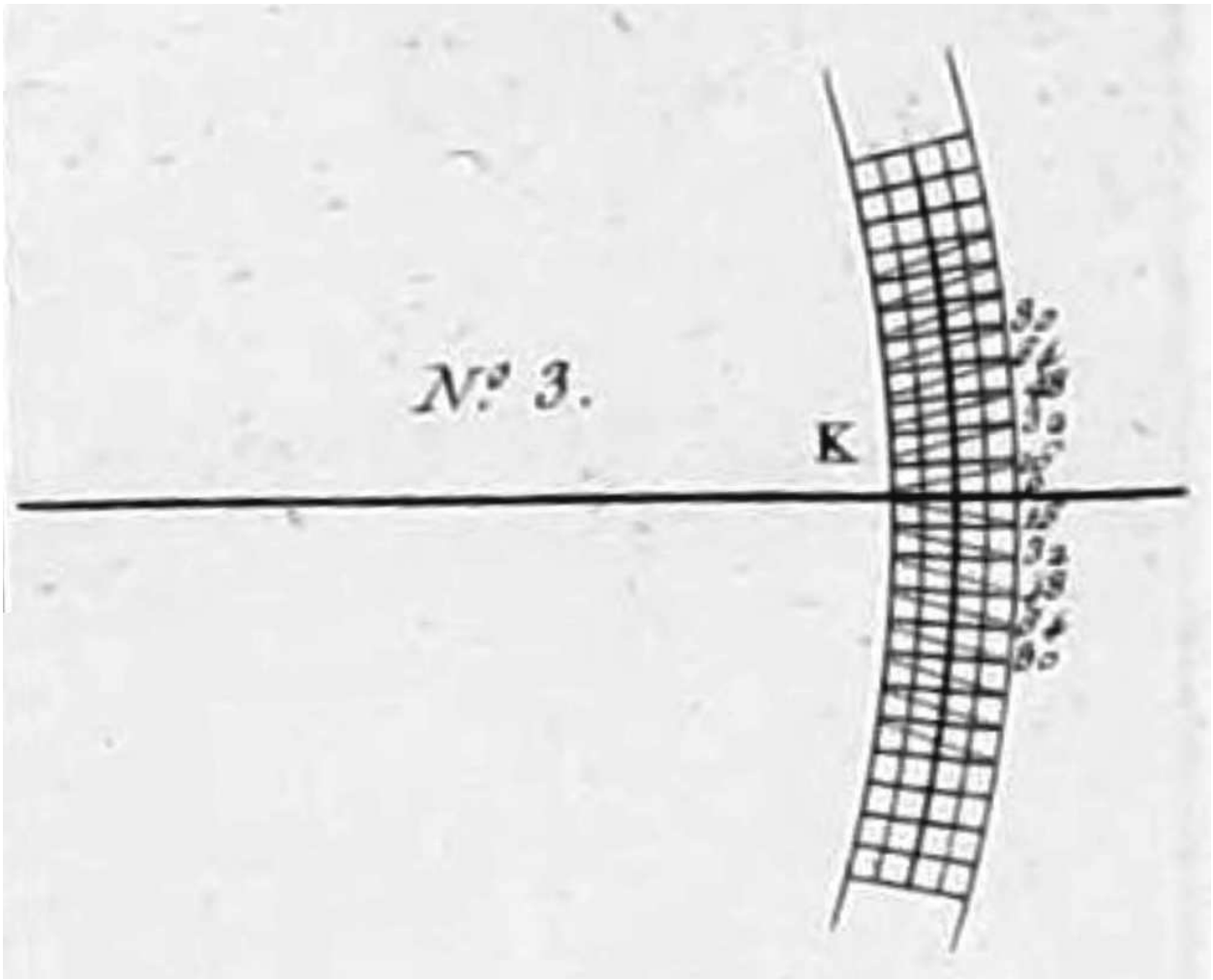
[224] [Note by Coulomb] We used, without choice, the first steel blade that presented itself: we could have determined the dimensions of this blade by the equations of Article 37 and following; but the resistance which the kind of suspension that we employ here, is so slight, that this degree of perfection appears unnecessary.

[225] Assuming a steel blade with a mass density of $8 \times 10^3\ kg/m^3$, its thickness would be 0.077 cm.

[226] This counterweight is represented in the Figure by the letter π.

[227] In Figure 12, Number 2, this wire is represented by the letters CG. The letters C and B appear in Figure 12, Number 1.

In K, is the arc of a circle which has a radius of 15 inches [40.61 cm], and whose center is on the vertical CG. This circle is divided every 16', or rather every 4', by means of the diagonals which cross the arc, as shown [in Figure 12,] No. 3.[228]



The distance from the extremity ef of the needle to the limb of the circle, was so small that it could produce, for the observer, in a variation of one or two degrees, only insensible errors; but that it is easy to calculate, because this distance is known, and the eye is always in the same position. We will give, in the last Chapter of this Memoir, the extract of the observations made with this needle.

55. This kind of suspension does not, it seems to me, entail any of the faults which it is perhaps impossible to correct when a needle with a cap is supported on a pivot: all the vertical forces are counterbalanced here necessarily; and their resultant passes through the vertical direction CG which is invariable: all the magnetic forces, which solicit the compass, being decomposed along a horizontal line, lie, because of the thinness of the plate which we suspend from the side, in the same vertical plane, and consequently this plane will be

[228]The numbers that appear in this Figure 12, No. 3, from bottom to top, are: 80, 64, 48, 32, 16, 0, 16, 32, 48, 64 and 80.

directed along the magnetic meridian. If we want more precision, it will be easy to suspend this same blade by the other side of its surface,[229] so that the surface is always vertical. It will be observed whether the surface of the blade retains the same direction; and in case there should be a difference, half the angle observed will give, as we shall see in the following Chapter, the true magnetic meridian.[230]

56. The facility of constructing compasses of the kind we have just proposed, and of making the arms longer without difficulty; the accuracy of observations of variations of declination, must, it seems to me, make them preferable to using needles suspended on pivot tips in making observations of all related physical phenomena.

But, on the other hand, as it would be very difficult to adapt such compasses to naval service, not only because of the movement of the vessels but because, in addition, the flexibility of suspension would allow them to oscillate for a very long time whenever displaced from their [magnetic] meridian; which cannot be suitable for the operations of navigators, which must almost always be done with celerity: we are obliged, for the usefulness of navigation, to try to discover where the inconveniences of the caps and pivots may come from, and what are the means of knowing the errors which result from it.

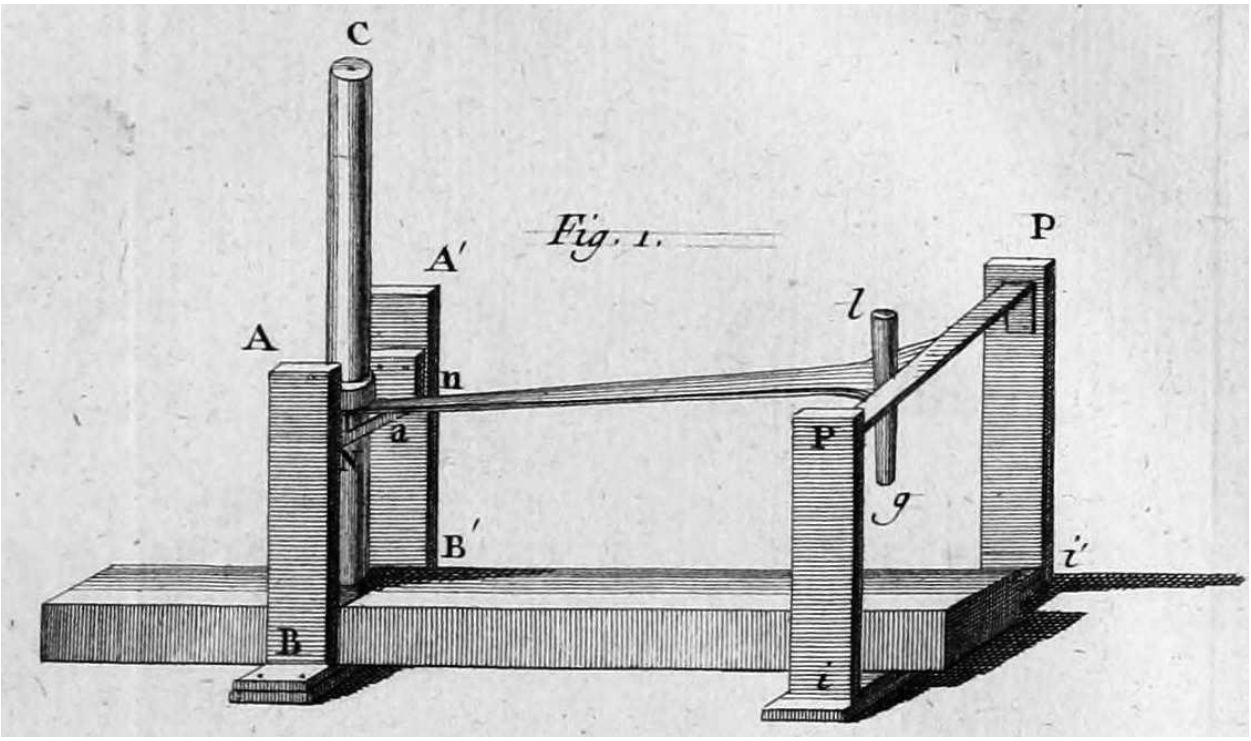
5.4 Supplement

Although[231,232] the compass of which we have just spoken gives observations in a more exact manner than any we have hitherto had, it will nevertheless be susceptible of a greater degree of perfection, when we are able to procure more skilled workers than those I had at my disposal, during the time that I was working on my Memoir: I was then in charge of some repairs to the Fortifications of La Hougue, a small Fort in Lower Normandy, on the Côtes of the Presqu'Isle du Cotentin, and I found absolutely no help there.

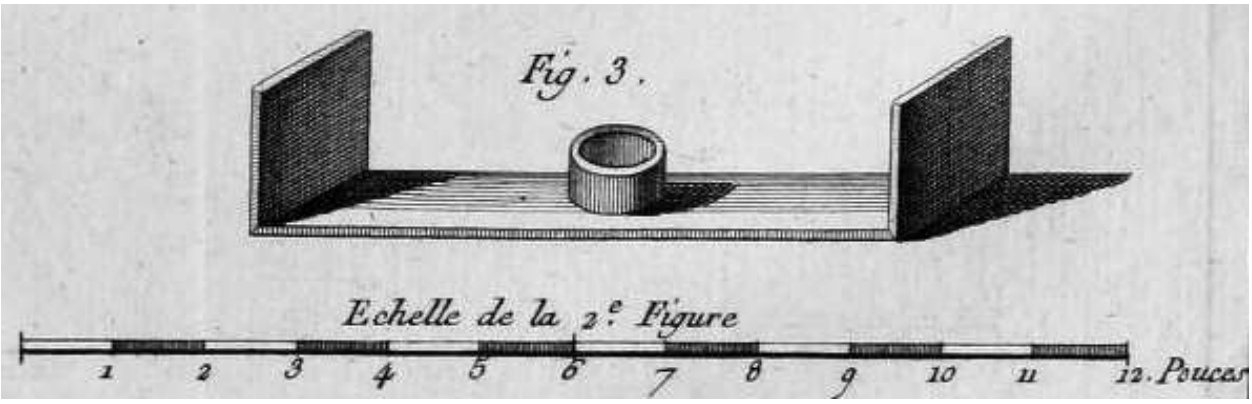
The compass, which I will describe in this Supplement, will give the variations [of terrestrial magnetism], within seconds [of arc].

The first Figure [of this Supplement], represents externally, in perspective, all the parts of the new compass.

[229] In the original: *par l'autre côté de son champ*. See also footnote 184 on page 73.
[230] This will be seen especially in Article 63 on page 104.
[231] [Note by Coulomb] This Supplement was made only after the judgment of the Academy.
[232] Coulomb is referring to the 1777 Academy of Sciences Award judgment. He was one of the winners of this contest with this Memoir. It should be noted that this Supplement was not included in the partial reprint of this Memoir contained in Potier's edition of Coulomb's works, [Potier, 1884].



It is formed of a box two or three feet in length [65 or 97 cm], on which rises a vertical hollow rod, intended to enclose the thread which supports the needle: this box is placed on two copper crosspieces BB', ii';[233] at the end of these crosspieces, and without touching the box, rise vertically the pillars AB, $A'B'$, and iP, $i'P'$; the first two are linked, in their upper part, by a crosspiece Nn, pierced towards the middle by a circular hole, which enfolds, without touching it, the hollow rod CF: on this same crosspiece, and corresponding to the same hole, one fixes a circular ring, such as one sees it represented in the third Figure [of this Supplement].[234]



This ring, which has its center in the vertical thread which supports the needle, serves as a circle of rotation with a horizontal alidade al;[235] the alidade carries, at its extremity l,

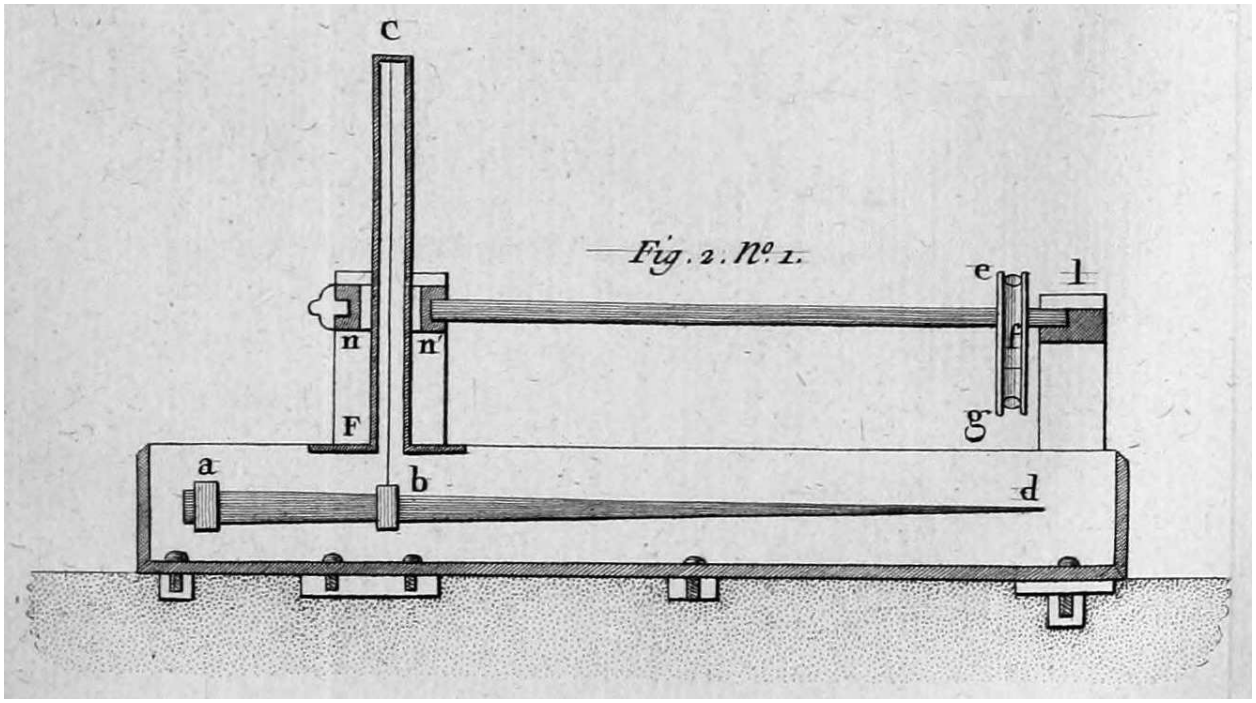
[233] In the original text we have here ii.

[234] Below the crosspiece of Figure 3 of the Supplement we see the scale (*Echelle*) of the Figure, in inches (*Pouces*), each inch corresponding to 2.7 cm.

[235] This alidade al appears in Figure 1 of the Supplement. An alidade is a device intended to measure angles

a small microscopic telescope, placed vertically, to observe the movements of the end of the magnetized needle.

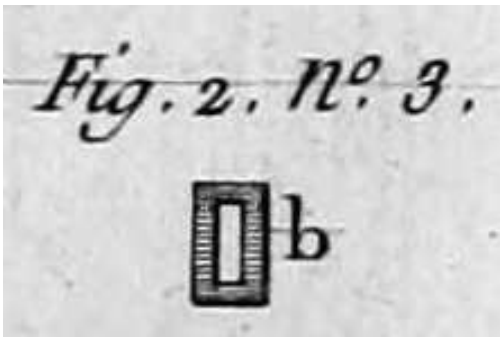
The second Figure, No. 1 [of this Supplement], represents a section of this same compass, along its length.

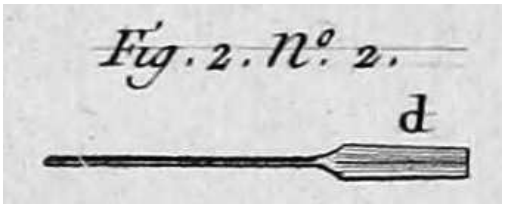


The needle *ad* is supported sideways by means of a slider *b*, attached at its upper part to the vertical silk thread *cb*:[236] at one end *a* of the compass, there is a second slider *a*, which serves as a counterweight to set the needle in a horizontal position: at the other end *d*, we weld a small copper plate, shown in [Figure 2,] No. 2, on which we draw a very fine line, following the length of the needle, which indicates the magnetic meridian.

by means of optical alignment. It can be a wooden or metal ruler that rotates around one of its points and of which one of the ends moves on a scale.

[236]This slider or ring *b*, *coulant* in French, appears in Figure 2, Number 3, of this Supplement:





At the focus of the telescope eg, we have placed, at f, a small silk thread, directed along the radius of the alidade, or whose center of rotation, in the movements of the alidade, is the same as that of the needle: at l, is the limb of a circle, which the end of the alidade travels, divided according to the method of *Nonius*:[237] the alidade executes the small movements of rotation, by means of a micrometer screw,[238] as is used in all instruments intended to give the angles with precision.

It is useless to go into greater detail on the construction of this compass: the box is placed so that its length corresponds to the magnetic meridian, and the variations of the needle are measured by means of the alidade, in matching the thread of the telescope, with the line which divides the copper plate, soldered to the end of the needle: instead of the line that we draw on this plate, it would perhaps be more convenient to pierce this plate, along its length, and to substitute for the line, a silk thread, in order to be able to check the direction of the needle, by reversing it.[239]

If we wanted to use this compass to determine the absolute declination, we would have to place horizontally on the alidade, and parallel to its radius, an ordinary telescope, with which we would observe a point on the horizon.

The box, as well as the crosspieces, are fixed to some solid bodies, by several copper screws, so that the whole is unshakable: the pillars are separated from the box; and, between the ring which serves as the center of rotation of the alidade, and the vertical rod, which contains the plumb line,[240] there is enough play, so that the movements of the alidade cannot cause any shaking, neither to the rod nor to the box.

[237] Nonius is a measuring tool used in navigation and astronomy named in honor of its inventor, the Portuguese mathematician and navigator Pedro Nunes (1502-1578) (Latin: Petrus Nonius). The nonius was created in 1542 as a system for taking finer measurements on circular instruments such as the astrolabe.

[238] In the original: *une vis de rappel*.

[239] [Note by Bucciarelli] I think what Coulomb is suggesting is to make a slot all the way through this plate so that one could see the line, now a silk thread, when one turns the needle over (reverses it). This would be a check for errors due to the needle not being exactly straight.

[240] That is, the silk thread.

5.5 Chapter IV. General Principles on the State of Equilibrium of Bodies. Their Application to Magnetized Plates, Placed on Horizontally Balanced Planes. What Results for the Suspension Point, and for Tracing on the Blades the True Magnetic Meridian. On the Friction of Pivots and Caps. Applications of All These Principles to the Construction of Marine Compasses

57.[241] When a free body, solicited by any powers whatsoever, is supposed to have reached its state of rest; if all the forces which act on this body are decomposed along two directions, one perpendicular to any plane, and the other parallel to this plane, the sum of the forces decomposed along the direction perpendicular to the plane, will be zero. If we then decompose all the forces parallel to the plane into two others still parallel to this same plane, but perpendicular to each other, the sum of the forces decomposed along each of these directions, will also be zero.

58. On the other hand, when a free body has arrived at its state of rest, any line whatsoever which crosses the body or not, can be regarded as a fixed axis, around which all the forces, which solicit the body, act to make it turn. Now the body being at rest, it is necessary that the *moments* of all the forces are balanced around this axis: thus, if we pass, in the plane assumed in the preceding Article [57], two lines perpendicular to each other, and that at their meeting point we draw a perpendicular to this plane, these three lines can be regarded as three fixed axes of rotation, and it is necessary, for the equilibrium to subsist, that the *moment*, of all the forces around these three axes, is zero.

These three conditions of equilibrium, joined together with the three other conditions explained in the preceding Article, form six conditions to be fulfilled, so that a body solicited by any powers, persists in its state of rest.

As these propositions for statics are developed in all books on mechanics, I will not stop there.

59. If we now suppose that this body, solicited by any powers whatsoever, is further pressed at one of the points on its surface by the tip of a pivot; then imagining a plane which touches the surface of the body at its point of pressure, and raising at the point of contact a perpendicular to this plane, we will have in consequence of the two preceding propositions:

1. That the sum of the forces decomposed along two directions perpendicular to each other, and parallel to the plane of contact, will be zero.

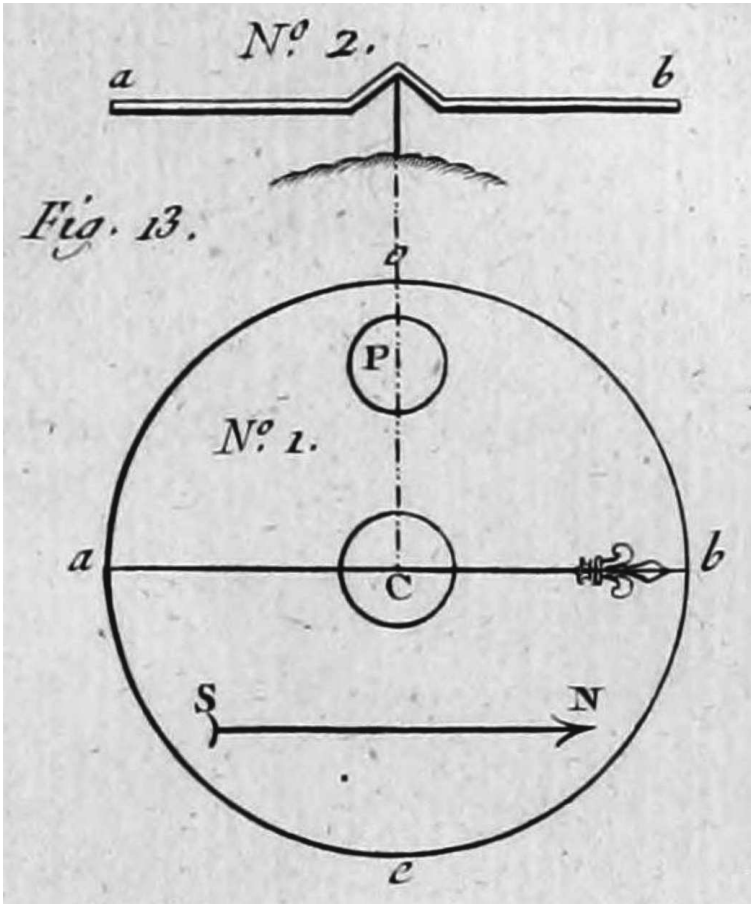
2. That the sum of the forces decomposed perpendicular to this plane, will be equal to the pressure experienced by the point of contact.

If we were to consider the friction caused by the pressure at the point of contact, equilibrium would exist whenever the friction provided a resistance greater than the resultant of all the forces which act parallel to the plane of contact.

[241] Articles 57 to 73 were not included in the partial reprint of this Memoir in Potier's edition of Coulomb's works, [Potier, 1884].

The first three conditions of equilibrium fulfilled, we will easily satisfy the three others for the *moment*, by passing three fixed axes through the point of contact, and friction will only be able to influence the conditions of equilibrium of the *moments* of all the forces around these three axes, if the tip of the pivot pressed on one surface or several points at the same time.

60. Let us apply these general propositions to the state of equilibrium of compasses, namely *(Figure 13)*, a heavy plane *aobe*, balanced horizontally by means of a cap, whose axis is vertical,[242] so that that the bottom of the cap, which bears on the tip of a pivot, can be regarded as a very small horizontal plane.



Let a magnetized needle be placed on this plane,[243] considered as a line, and balanced by the small counterweight P, so that the plane always remains horizontal; No. 1 [of Figure 13] represents the plane, and No. 2 its section along the magnetic meridian.[244]

If we here apply the foregoing principles, we shall find that all parts of the plane are acted upon by the forces of gravity. We will find in the second place that the needle is solicited by its gravity and by its magnetic force, which acts on each point [of the needle], along a constant direction. We will find, by decomposing the magnetic force, along a horizontal direction, and

[242]The vertical axis passes through the point C in Figure 13, Number 1, the cap being represented by the small circle around the point C. This cap has the shape of a cone. The section of this cone appears in the center of Figure 13, Number 2.

[243]This magnetized needle is represented in Figure 13 by the letters SN.

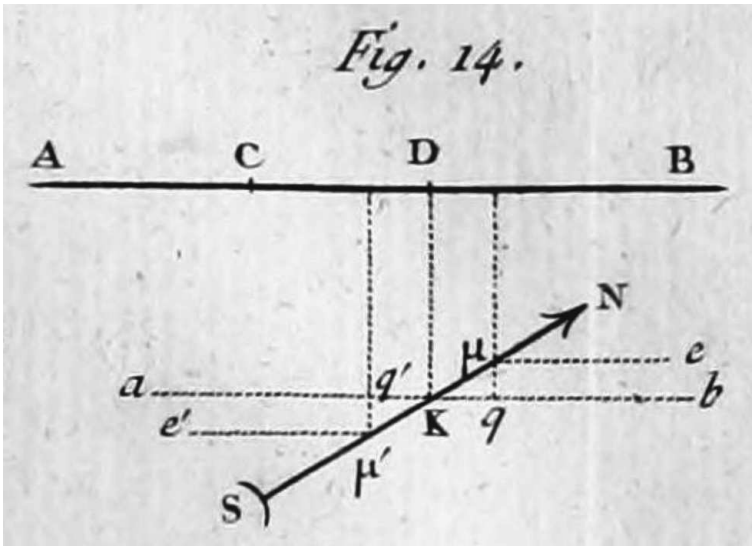
[244]That is, Figure 13, No. 2, represents the section of the heavy plane *aobe* as seen in a vertical plane along the magnetic meridian.

along a vertical direction, that the vertical force increases the weight of the boreal part [of the needle], and acts in a contrary direction, or decreases the gravity of the austral part.[245] We shall further find, in consequence of the second principle, that the increase in weight of the boreal part is exactly equal to the diminution of weight of the austral part; and that thus the pressure experienced by the tip of the pivot is equal to the weight of the whole system.

We will then notice that the action of the terrestrial globe on the needle, having, for all the points of this needle, a constant direction; if we decompose this action, there results, for each point, a horizontal force, along the magnetic meridian, and that, because of the equality of the sums of the forces, in the two opposite directions, the needle, like we have already explained in the second principle, will be directed along the magnetic meridian.

61. It also follows from the preceding principles, that in any position, and at any distance from the center of rotation, [for] this needle located on the horizontal plane, the force that will bring it back to its true [magnetic] meridian, when it is moved away from it by any angle b, will always experience the same *moment*.

To demonstrate this let C, *(Figure 14)*,[246] be the center of rotation of a needle SN, posed as in the previous Article, on a horizontally balanced plane; let the magnetic center of this needle be assumed at K;[247] through this center is drawn the line aKb, which represents the magnetic meridian; and through the point of rotation C, let the line AB be drawn parallel to this meridian; let the angle NKb, between the direction of the needle and its meridian, be assumed [equal to] b.



According to the first principle, each point μ of this needle is driven by a force μe,[248] parallel to the magnetic meridian. All the forces of the KN part, being directed towards the North; all the forces of the KS side, being directed towards the South, let φ be the force

[245] That is, the Earth's magnetic force acting on the boreal or Northern part of the needle has a downward component in Paris, which would be equivalent to an increase in weight on that side. In the austral or Southern part of the needle, the magnetic force has an upward component in Paris, equivalent to a decrease in weight on that side. See Section 4.3 on page 40 and the footnotes 127 and 135 on pages 56 and 57.

[246] All points and lines in Figure 14 must be understood as being in the same horizontal plane.

[247] In the original text the letter K appears in lower case, k. I substituted the uppercase letter K because the lowercase letter k does not appear in Figure 14.

[248] In the original text appears here μE. I replaced the uppercase letter E with the lowercase letter e so that the text matches Figure 14.

acting on the boreal point μ, and φ' the force which acts on the austral point μ', on the other side of the magnetic center;[249] we will have (when drawing from the points μ, K, μ' the perpendiculars on the meridian AB) for the *moment* of the force which acts on the point μ, around the center of rotation C,

$$(\varphi\mu) \cdot (DK - q\mu) \ ;$$

and, for the *moment* of the force acting on the austral point μ',[250]

$$\varphi'\mu'(DK + q'\mu') \ .$$

As the forces are directed in two opposite directions, it follows, for the total *moment* around the center of suspension C, the formula

$$\left(\int \varphi'\mu' - \int \varphi\mu\right) DK + \int \varphi'\mu' \cdot q'\mu' + \int \varphi\mu \cdot q\mu \ ;$$

and, as the sum of the austral forces, is equal to the sum of the boreal forces, it follows that the first term vanishes, and that the formula is reduced to

$$\int \varphi\mu r \, (\sin b) \ \ ,$$

integrated for the whole of the needle, r representing here the distance $(K\mu)$.

Thus, a magnetized needle, placed on a horizontally balanced plane, will always return to the direction of its magnetic meridian; and when it is moved away from this direction by any angle b, its *moment* about the center of rotation, will always be proportional to

$$\sin b \ ,$$

and independent of the distance of the needle, from the center of rotation.

62. *Remark.* If the sum of the austral forces were not equal to the sum of the boreal forces, then we would have the position of the needle, with respect to its meridian, in the state of equilibrium, assuming the preceding formula,[251]

$$\left(\int \varphi'\mu' - \int \varphi\mu\right) DK + \left(\int \varphi\mu \cdot K\mu + \int \varphi'\mu' \cdot K\mu'\right) \sin b = 0 \ ;$$

but, as we have shown that the sum of the austral forces is equal to the sum of the boreal forces, we will not stop to discuss these new results.

63. We come to see that there is never any difficulty in determining the magnetic meridian when the magnetized needle can be regarded as a line; since it necessarily aligns with the direction of the magnetic meridian. But let us now suppose that this needle is a magnetized blade, which has width; and see what angle the direction of the magnetic meridian will take

[249] In Figure 14, I replaced the letter μ near the S pole with the letter μ'.

[250] In the original text appears here $\mu'\mu$ instead of μ'.

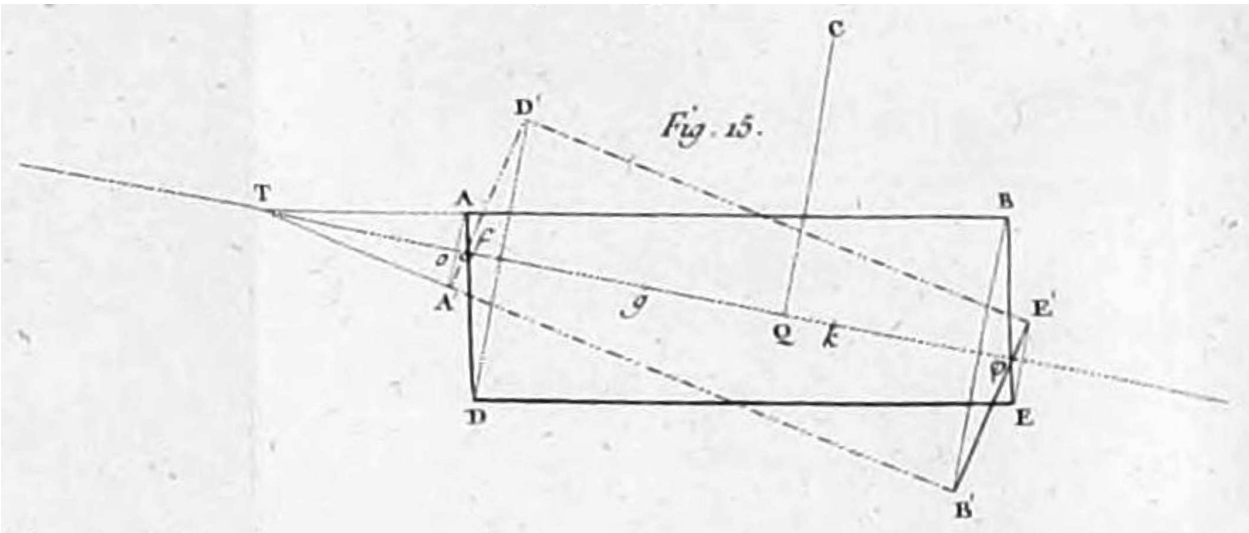
[251] In the original text the next equation appears with the lowercase K, namely, k:

$$\left(\int \varphi'\mu' - \int \varphi\mu\right) Dk + \left(\int \varphi\mu \cdot k\mu + \int \varphi'\mu' \cdot k\mu'\right) \frac{\sin b}{rayon} = 0 \ .$$

I replaced it with the capital letter K.

with one of the sides of the blade, when the plane, balanced horizontally, will have arrived at its state of rest.

On a horizontal plane, similar to that of the previous Article, let a magnetized blade ($ABDE$) *(Figure 15)* be placed;[252] let the extremity BE of this blade have a boreal force,[253] and the extremity AD an austral force.



If we suppose that the resultant of all the boreal forces is represented by $k\varphi$, and the resultant of all the austral forces by gf, the forces of the boreal part, as of the austral part, being all directed by virtue of the first principle, along the magnetic meridian, will consequently be all parallel to each other, and their resultants will be parallel to the magnetic meridian: thus, if when the compass has reached its state of rest, we draw, from the point of rotation C, a perpendicular to the magnetic meridian, it will be at the same time perpendicular to the resultant of the magnetic forces. Now, as the sum of the boreal forces equals the sum of the austral forces, the lever arm must be the same for the two resultants, which will consequently be in a straight line.[254]

We easily see that in whatever location of the horizontally balanced plane the blade is placed, it will always take the same direction, provided that it is supported on the same face.

If we now suppose that, for the blade $ABDE$, the line $f\varphi$ of the resultants remaining motionless, the whole blade is made to rotate about this line, until it is placed on its other face, it is obvious that when this blade has made a half-revolution around the axis $f\varphi$; if the blade was in equilibrium on its first face, due to the forces lying in the horizontal plane, it will still be so when it has made a half-revolution around the axis $f\varphi$, when it is placed on the other side, represented here by the dashed lines $A'B'D'E'$; where DD' represents the diameter or the projection of the semi-circle, traversed by the point D, which is cut into two equal parts by the resultant $f\varphi$; and AA' represents the diameter of the circle traversed by the point A.

If we now extend the corresponding lines AB, $A'B'$, as well as the resultant φf, we will see that these three lines must meet at the same point T, and that the angle ATf, formed between the direction of the side AB of the blade, and the resultant $f\varphi$, will be half of the

[252] All points in Figure 15 must be considered in the same horizontal plane.

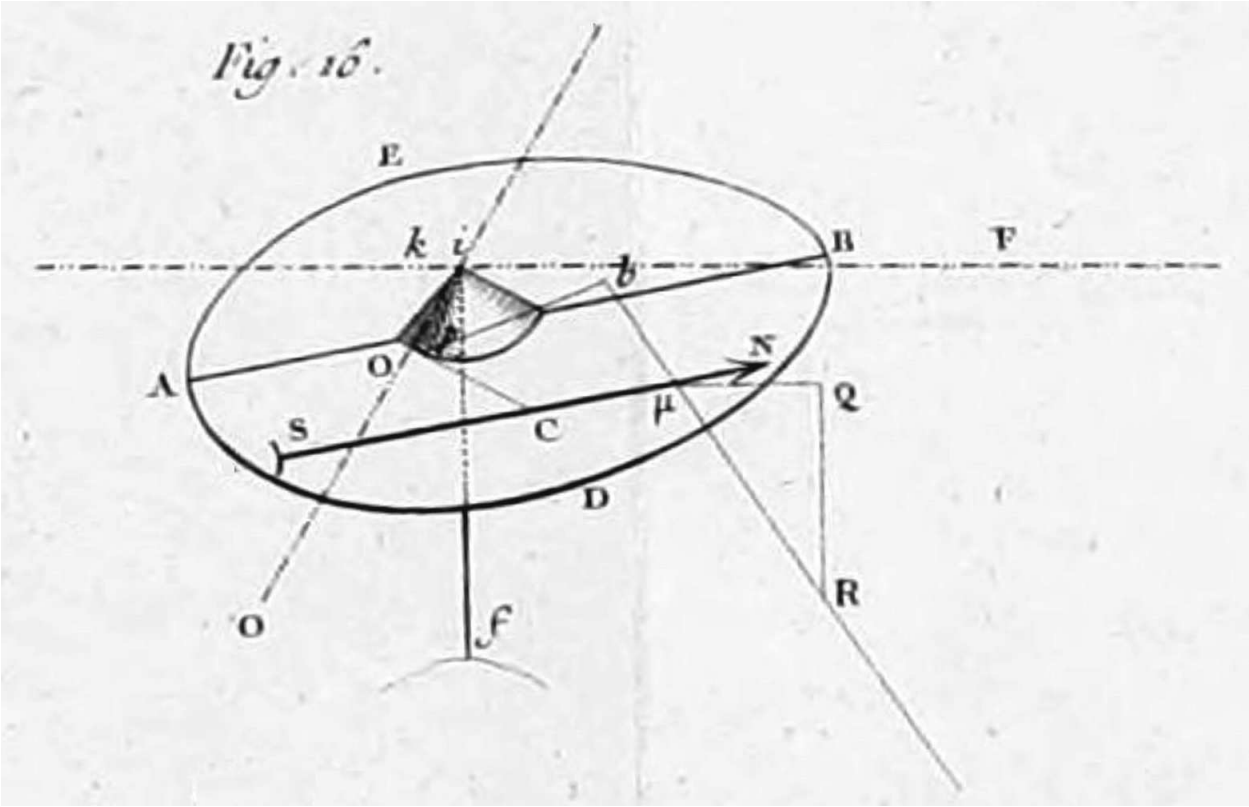
[253] That is, let the end BE be of the boreal type, namely, a North pole, see footnote 127 on page 56.

[254] That is, at equilibrium the two resultants will lie along the same straight line.

angle ATA'.

It will now be very easy for us to trace, on a blade, a line which corresponds to the magnetic meridian: we will place this blade on the horizontally balanced plane, and we will observe the direction of its side AB; we will reverse this blade on its other face: we will again balance the plane horizontally, and we will observe the new direction on the side $A'B'$. Half of the angle, formed by these two directions, will give us the magnetic meridian: instead of observing the direction of one of the sides of the blade, we can draw a line on the middle of this blade, and observe its direction, which will come to the same thing.

64. *Remark.* When the [magnetized] needle SN is placed on a plane $ABDE$ *(Figure 16)*, inclined to the horizon, suspended and mobile around the point of suspension i, it will still be easy to determine all the equilibrium conditions.



Through the point of suspension i, let us pass three axes: iF, horizontal axis in the plane of the magnetic meridian; iO, horizontal axis perpendicular to the magnetic meridian; if, vertical axis: all the forces of gravity and magnetism must be in equilibrium around these three axes.

If we decompose all the magnetic forces μR, which act on each point μ of the needle, into a horizontal force μQ, and a vertical force QR, located one and the other in the plane of the magnetic meridian, we will see that all the vertical forces have no *moment* around the axis if, since they are parallel to it. So only the horizontal forces remain: but because of the equality between the austral forces and the boreal forces, and because the horizontal forces are parallel with the magnetic meridian, the needle will necessarily take a direction parallel to the plane of this meridian.

On the other hand, the horizontal axis iF, being supposed along the magnetic meridian, and the needle SN being parallel to this meridian, the horizontal forces will have no *moment*

around the axis iF, since they are parallel to it; and the magnetic vertical forces, will all have the same lever arm around this axis. From there it follows, due to the equality between the sums of the magnetic forces, in the opposite directions, that these forces, decomposed vertically, are counterbalanced around the axis iF; thus, the forces of gravity of the whole system must also counterbalance each other; and if we cause a vertical pk to pass through the center of gravity p, it will necessarily meet the axis iF at an arbitrary point k, distant from the point i by the quantity ki, which we are going to determine.

To this end, all that remains is to find the *moment* of all these forces, around the horizontal axis iO, perpendicular to the magnetic meridian. We have just seen that the needle always aligns itself along the plane of the magnetic meridian: thus, the magnetic forces μR of each point μ, which we have found, by the first principle, parallel to each other, are in a plane perpendicular to the axis oi. Thus, if this plane meets this axis at the point O, and if C is the magnetic center of the needle SN, it follows, from Article 61, that the *moment*, of all the magnetic forces around this axis iO, will be the same as the *moment*, of all these forces, around the magnetic center C. Thus, by naming H the angle $N\mu Q$, inclination of the needle on the horizontal line μQ, and I the angle $Q\mu R$, formed by the magnetic direction μR, with this same horizontal; we will have, as in Article 61, by naming also $\mu R = \varphi$, and $C\mu = r$,[255] the equation

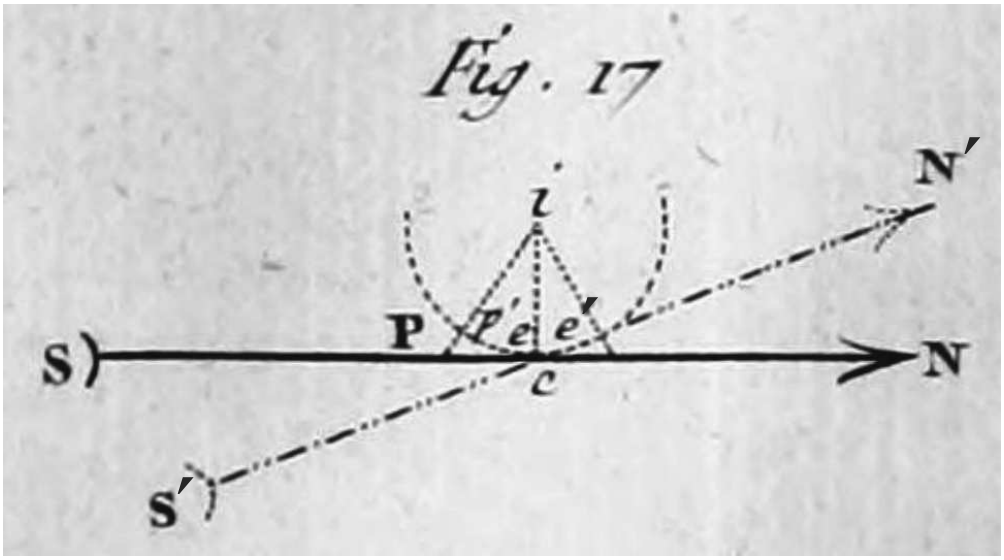
$$\int \varphi \mu r \cdot \sin(H + I) = \text{magnetic } \textit{moment} \text{ around the axis } iO \ .$$

If a is the distance from the center of gravity p, to the suspension point i; if, moreover, we suppose the angle $pif = G$, and the total weight $= P$, we will have, for the *moment* of the weight of the system, around the axis io, the quantity $aP \cdot \sin G$; and, because of the state of equilibrium, where the system is supposed to have arrived, we will have, in general,

$$Pa \cdot \sin G = \int (\varphi \mu \cdot c\mu) \cdot \sin(H + I) \ .$$

65. Here are several conclusions that can be drawn from this remark. When we have balanced horizontally, at any place, a compass rose,[256] with its magnetized needles, the magnetic force, as well as the direction, changing as we change latitude and longitude, the plane of the rose will revolve around its center of suspension, bowing to the horizon; but this plane will always remain perpendicular to the plane of the magnetic meridian of the place where we stop. The center of gravity of the whole system, will revolve around the point of suspension i, approaching or moving away from the perpendicular if, without ever leaving the plane of the magnetic meridian; and the angle traversed by this center of gravity will be equal to the inclination of the rose on the horizontal plane. This is shown in *Figure 17*, where the center of gravity is at P when the rose is horizontal, point i is the point of suspension, and ie' a perpendicular drawn from the point of suspension on the plane of the rose.[257]

[255] In the original text, the letter C appeared in lower case, namely: $c\mu = r$.
[256] In the original: *rose de boussole*.
[257] I replaced the letters SN in the dashed needle of Figure 17 with the letters $S'N'$.



When point P arrives at P', the vertical line ei will arrive at ie', and the compass NS at $N'S'$, the angle $NCN' = eie' = PiP'$; thus, in the equation of the previous Article,

$$a \sin G \cdot P = \int (\varphi \mu c \mu) \cdot \sin(H + I) \ .$$

If $\sin G = A$, when the rose is horizontal, we will have $A - G = H$, and consequently[258]

$$aP \cdot \sin(A - H) = \int \varphi \mu c \mu \cdot \sin(H + I) \ .$$

If the magnetic force and its direction were given, we would easily obtain the inclination H of the compass.

66. All that remains, to complete this theory, is to determine the coercive forces, which depend either on the friction of the pivot at the bottom of the cap,[259] or on the imperfection of these caps: but we notice in advance that the friction, necessarily increasing with the weight, and the magnetic force being, proportionally, smaller for a heavy blade than for a light blade, and moreover, the *moment* of a blade, tending to return it to its magnetic meridian, being independent of its position in the plane of the rose; we should always prefer compasses formed with several light blades, whose magnetic meridian lines are parallel to each other, to a single thick and heavy blade. But, to put us in a position to say something more precise, let us see what experiment and theory tell us about the friction of pivots.

5.5.1 Pivot Friction

67. All the authors who have written about friction, all the experiments that have been made on this subject, have all together proved that the surface plays a small part in the

[258] In the original text this equation appeared as follows:

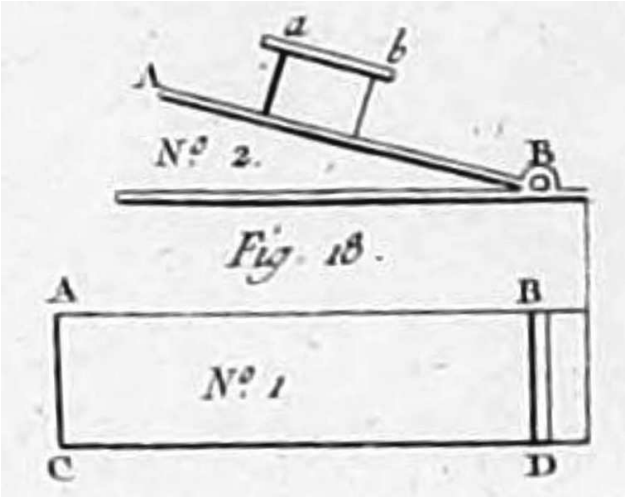
$$aP \cdot \sin(A - H) = \int \varphi \mu c \mu \cdot (\sin H + I) \ .$$

[259] See footnote 141 on page 58.

friction of light bodies, and that it follows more or less the ratio of pressures.[260] We have tried to determine if this same law holds when very hard surfaces are made to slide on needle tips.

5.5.2 Experiment

ABCD (Figure 18, No. 1 and 2), is a small board, on which three needles have been planted pointing upwards. This board is mobile around the axis BD, fixed to a horizontal table. By these means we gave the board AB, [Figure 18,] No. 2, the necessary inclination to cause a small blade ab of glass or copper placed on the tip of the three needles to begin to slip. Here are the results of some experiments.



Using a glass plate weighing 1/2 ounce [15.3 g], it was found that in order for it to begin to slide on the steel tips, the sine of inclination must be one-seventh of the radius.[261] We successively loaded this plate with 1, 2, 3 and 4 ounces [30.6 to 122.4 g], and we always found that it began to slide under the same inclination. The glass plate, reduced to 100 grains [5.3 g], gave substantially the same result.

For a blade of yellow copper,[262] flattened with a hammer, and very well polished, we had results analogous to the preceding ones: only the friction was more considerable between the steel tips and the copper, than between steel and glass. The copper plate only starts to slide on the steel tips, when the sine of inclination is to the radius :: 1 : $5\frac{1}{2}$.[263]

It follows from these two experiments that the friction of pivot tips is approximately proportional to the pressures.

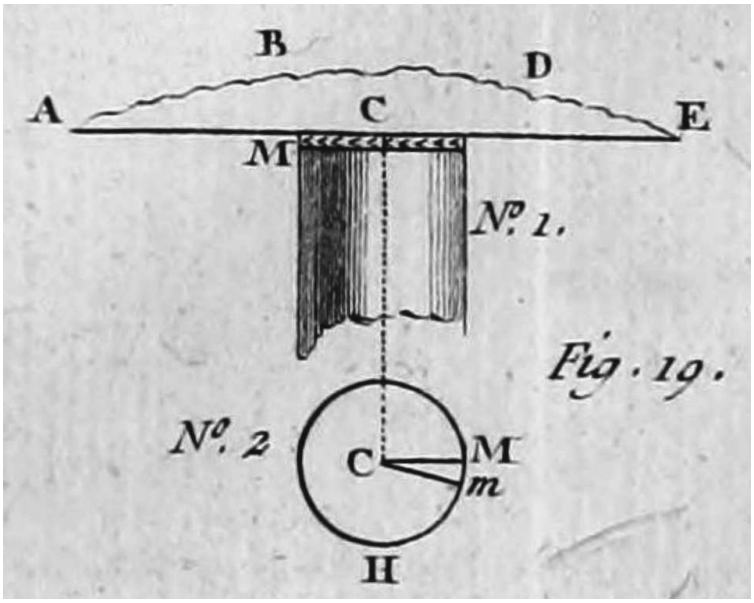
[260] That is, frictional forces are approximately proportional to the pressures or normal forces acting on the surfaces, being independent of the contact area. In 1699 Guillaume Amontons (1663-1705) arrived at the laws of friction, that is, the resistance to linear motion when bodies make contact, [Amontons, 1732], [Amontons, 1742], [Gillmor, 1971a, p. 119], [Heyman, 1997, p. 76] and [Oliveira, 2004]. He concluded that the frictional force is proportional to the normal force (applied load) and does not depend on the area of contact between the surfaces. He stated that the coefficient of friction was approximately the same for all tested materials and that the friction force was approximately 1/3 of the normal force.

[261] That is, the sine of the angle of inclination of the plate with respect to the horizon was 1/7.

[262] In the original: *cuivre jaune*. This expression can be translated as "brass", "bronze", "copper", "yellow brass" or "yellow copper". In cases where Coulomb uses the word *laiton*, we chose to translate it as *brass*. When he uses the word *cuivre*, we chose to translate it as *copper*. See also [Goodway and Savage, 1992].

[263] That is, when the sine of inclination of the copper plate with the horizon was 1/(5.5).

68. We now suppose that a body, *ABDE (Figure 19, No. 1)*, is supported on the horizontal section of a column, the circular plane of which is shown in [Figure 19,] No. 2.



If we suppose, moreover, that the center of gravity of this body corresponds to the point C, and that horizontal forces act to make it turn around this center, it is obvious that this body will be retained by the friction of its plane AE on the circle CMH; friction which will depend on the pressure experienced by all the points of the circle. But the pressure of all the points of this circle must here be a constant quantity;[264] this is what appears at least very probable, supposing the whole surface of this circle covered with an infinity of small equal springs, which, because of the center of gravity placed at C, will be compressed equally between the two contact planes. Thus, since we have found the friction proportional to the pressure, the *moment*, due to the resistance of each point of the circle, will be proportional to the distance of this point from the center C.

Let δ/n express the friction of a point of the circle, δ being the pressure of this point,[265] the total *moment* of friction around the point C, will be equal to[266] $\frac{\delta(CM)^3}{3\cdot n}\cdot 2\pi$; and, as the weight of the body equals the total pressure of the circle of contact, we will have, by naming P the weight of the body, $P=\frac{\delta(CM)^2}{2}\cdot 2\pi$; thus, the total *moment* of the friction around the center C, will be $\frac{2P}{3n}CM$, where $1/n$ expresses the ratio of the pressure to the friction: thus, the *moment* of the friction of a body, which rotates on a circle, is in ratio composed of its weight, and of the diameter of the circle of contact.[267]

69. *Remark.* If the vertical, passing through the center of gravity of the body, met the contact circle $ABDE$ in a point P *(Figure 20)*, other than its center of figure C, we would

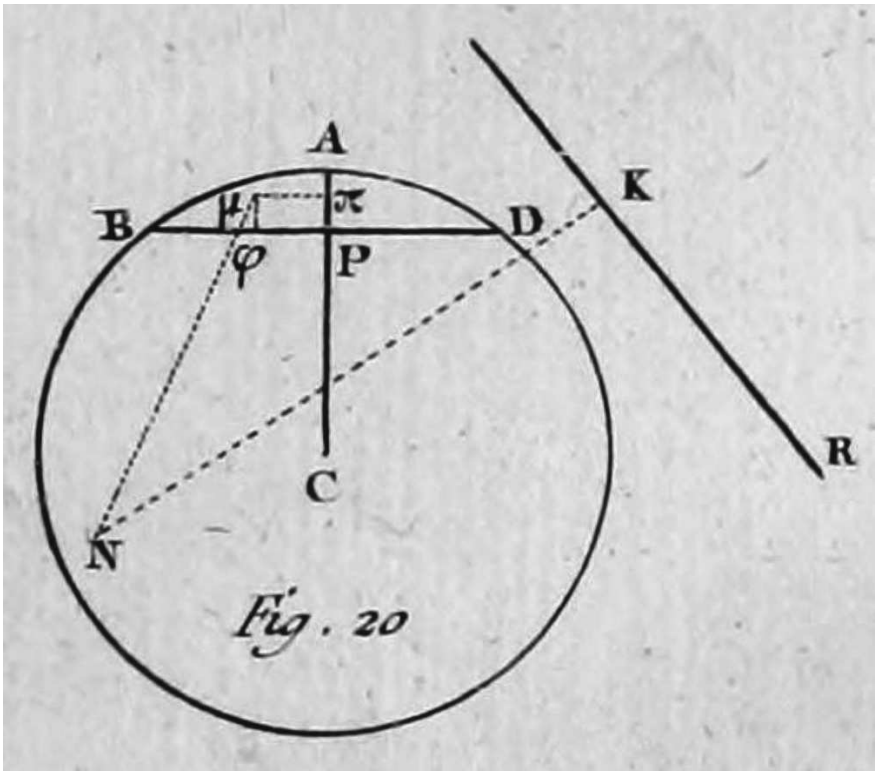
[264] [Note by Bucciarelli] This assumption is not realistic but other assumptions for the pressure distribution will give the same result. For example if you assume a linear distribution with maximum p_o at the center and going to zero at the edge, the moment of the friction is still proportional to the product of the weight and the diameter. Only the constant out front changes.

[265] That is, if the normal force on this point is represented by δ, then the force of static friction is given by δ/n, where $1/n$ is the coefficient of static friction.

[266] In the original text this formula was written as $\frac{\delta(CM)^3}{3\cdot n}\cdot 360^\circ$.

[267] That is, in this case the torque due to friction is proportional to the weight and diameter of the contact circle.

thus find the equilibrium conditions:



1. The sum of the pressures of all the points of the circle, must be equal to the weight of the body.

2. If we pass two axes BD, AC perpendicular to each other, through the point P, in the plane of the circle of contact, we will see that, since the point P is assumed in the vertical which corresponds to the center of gravity, the *moments* of the weight of all parts of the body, are in equilibrium around these two axes; and that consequently the *moments* of the reaction of the pressure experienced by all the points μ must be in equilibrium around the same two axes.

Thus, if we draw from all the points μ of the circle, two perpendiculars $\mu\varphi$ and $\mu\pi$, to these two axes; if δ represents the pressure of the point μ, and P the weight of the body, the three equations which express the state of equilibrium, will give,

First: $\int \delta \cdot \mu = P$.

Second: $\int \delta \cdot \mu\varphi = 0$.

[And third:] $\int \delta \cdot \mu\pi = 0$.

If we now wanted to determine the resistance that friction would oppose to a horizontal force KR,[268] suppose for a moment that the body can only rotate around a fixed vertical axis, passing through the point N; from the point N is drawn the perpendicular NK in the direction of the force KR, the frictional resistance to a horizontal rotation around the vertical axis N, will be $\int \frac{\delta}{n} \cdot N\mu$, and the total *moment*, around this same axis, will be, naming k the force RK,

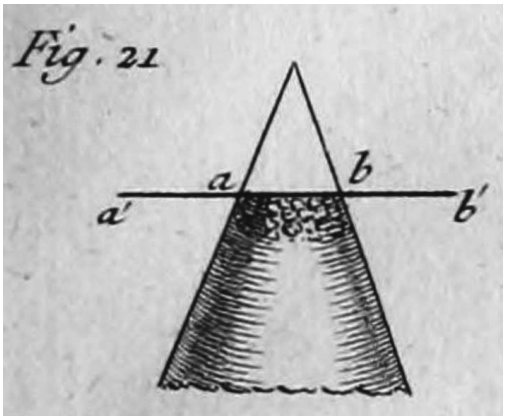
[268] [Note by Coulomb] This force KR is a resultant, which is only considered here relative to its *moment*; it must be supposed to be formed of different equal and opposite forces, which, apart from friction, would cause the body to turn horizontally around its center of gravity, which corresponds to the point P.

$$\left(k \cdot NK - \int \frac{\delta}{n} \cdot N\mu\right) \ ;$$

it is clear that, as soon as this quantity is positive, the body will turn around its axis N; thus, if the body is free, and there is no fixed axis, to obtain the true axis, around which the body will begin to turn, it is necessary, by varying the point N, to determine its position, assuming the quantity $(k \cdot NK - \int \frac{\delta}{n} N\mu)$, is a *maximum*; because, by this operation, we will easily see that, for this point N, thus found, the quantity $k \cdot NK - \int \frac{\delta}{n} N\mu$, will begin to have a positive value, while it will still be null, or negative for all the other points; so, once the value of δ is assumed to be given, the rest will be a fairly simple problem of geometry.

We will not dwell any longer on this remark; because, in the cases we need [to consider], either the tip of a pivot is enclosed in the conical bottom of a [compass] cap, or penetrates naturally forcing itself a little into the interior of a horizontal plane, the axis of the pivot necessarily becomes the axis of rotation; especially since the contact plane is always smaller than all sensible measures, as we will see, Article 81.

70. If nature provided us with bodies that were perfectly hard, or whose parts were united by an infinite cohesion, these would be, without a doubt, those that we should choose for making [compass] caps and pivots; since such pivots would have the diameters of their tips infinitely small without breaking, and without penetrating into the interior of the caps. But all the means that we can use, the firmest and best tempered steel, all vitrifications, even diamond, have only a certain degree of cohesion, which yields for a given surface at a given weight. Thus, when a heavy surface is balanced on the tip of a pivot, the surface of contact must be sufficiently great enough for the cohesion of all the parts of this surface to be able to resist without rupture the weight which presses it. Consequently, for finite weight, this surface must be finite. To better understand where the friction of a pivot can come from, suppose that solid bodies are formed of an infinity of small hollow globules, filled with an elastic fluid matter; that all these globules are joined together, as well as the solid parts which form the surface of these globules, by an adhesion of which the cause is unknown to us; it will be seen that each of these small globules can only support a certain small weight without breaking. Thus, as soon as the tip of a pivot is loaded by a heavy plane, *(Figure 21)*, the surface of the contact circle, whose diameter is ab, must be large enough for the number of globules, which form this surface, to be able to support a given weight; whence it follows that when the tip of a pivot is loaded with different weights, the surface of contact will be proportional to the weight.

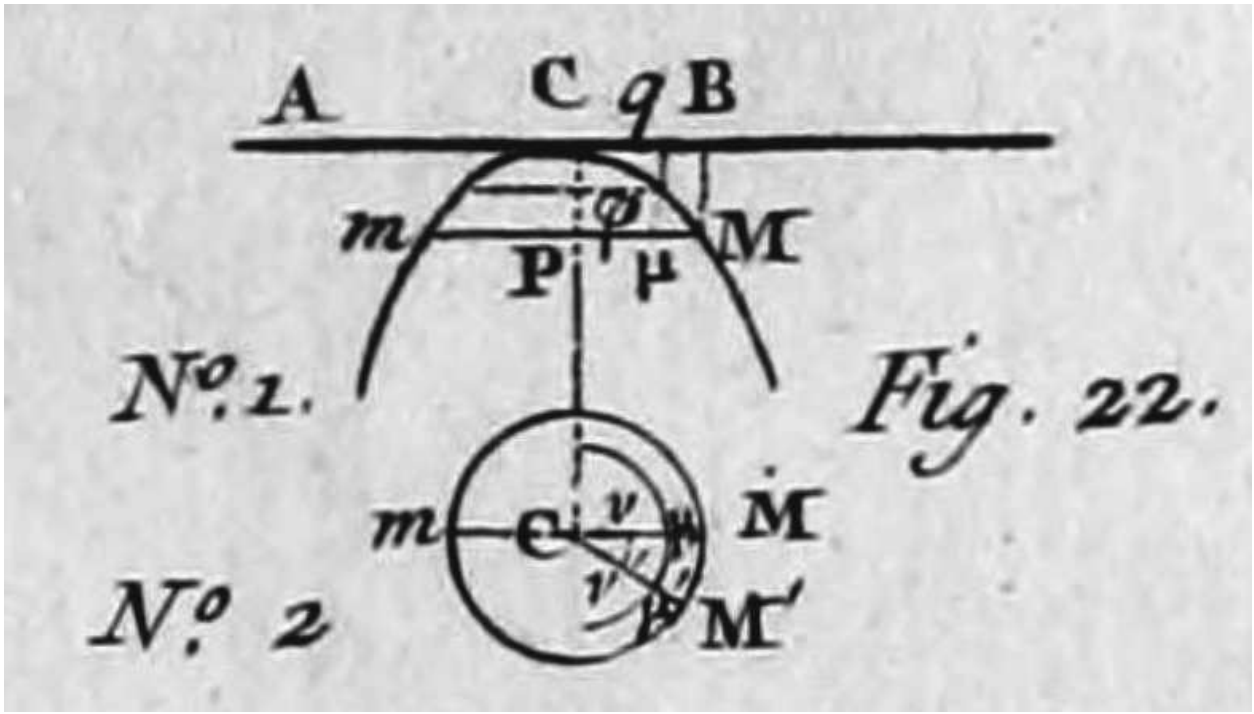


If, instead of supposing the bodies to be made up of an infinite number of small globules, we would suppose them to be made up of an infinite number of small springs, we would also find that each of these small springs could, without breaking, support only a small weight, and we would still conclude from this [assumption] that the contact surface is proportional to the pressure.

It therefore appears probable that when a very hard surface is supported on the tip of a pivot, the surface of contact is proportional to the total pressure: thus, this pressure is like the square of the diameter.

But we have found, Article 68, that the *moment* of friction of a heavy plane, on a circular surface, is in a compound ratio of the diameter and the pressure. Thus, as this diameter is here as the [square] root of the weight, we will find the *moment* of the friction of the pivots proportional to $P^{3/2}$.[269]

71. If we want to obtain a formula for the *moment* of friction, based on even more general hypotheses, suppose that the tip of the pivot at C, on which is supported the surface AB, which we consider as inflexible, is a solid of revolution, *(Figure 22, No. 1)*, if the tip of this pivot Mcm is compressed to Mm, which is the diameter of the contact circle, each point φ of the surface of the pivot will arrive at μ, and the pressure that the point μ will experience, will be like a function of the ordinate $\varphi\mu$, distance from the point φ in its natural situation to its forced situation in the state of compression. [Figure 22,] No. 2, represents the contact circle, whose diameter is Mm.



Let [Figure 22,] No. 2, the angle $MCM' = b$ [and] $c\mu = x$, we will have, for the pressure of the small elementary surface, $\mu\mu'\nu\nu'$,

$$function\ (\varphi\mu) \cdot dxbx = function\ (CP - q\varphi)bxdx\ ;$$

[269] Coulomb found that in the case of needles resting on pivots, the frictional torque (or resistance to circular motion) was proportional to the 3/2 power of the weight (or normal force), [Potier, 1884, p. 53] and [Gillmor, 1971a, p. 119].

but, since CM is only a small portion of the surface,[270] we will have, without noticeable error, φq proportional to $(Cq)^2$, and CP proportional to $(CB)^2 = (CM)^2$; thus, we will have, for the elementary pressure,

$$function\ (n'((CM)^2 - x^2))bxdx\ ,$$

where n' is a constant quantity, depending on the curvature of the point C: if, for simplicity, we suppose that the pressure of the point μ can be represented by a power m, of $(\varphi\mu)$, we will have, for the elementary pressure, $n'^m(cM^2 - x^2)^m bxdx$; this quantity, integrated in such a way that it vanishes, when $x = 0$, will give, for the whole of the circle,

$$\frac{n'^m 2\pi}{2(m+1)} CM^{2(m+1)}\ ;$$

a quantity which must be equal to the total pressure P.

Likewise, for the *moment* of the elemental pressure, $n'^m(CM^2 - x^2)^m bx^2dx$; quantity which will be integrated, either exactly, or by quadratures, according to the value of m; but, without going into the detail of this integration, it follows, from the operation of the integral calculus, and from the law of homogeneity, which must be observed between the integral quantities and their differentials, that we will always have the *moment* of the pressure of the entire circle, proportional to[271] $(CM)^{2(m+1)+1}$, and P proportional to $(CM)^{2(m+1)}$; thus, the *moment* of friction will be proportional to $PP^{\frac{1}{2(m+1)}}$. If $m = 0$, then the pressure will be constant for all points μ, and the *moment* of friction will be proportional to $P^{3/2}$.

If $m = 1$, we will have the *moment* of the pressure proportional to $P^{5/4}$.

5.5.3 General Formula for the Friction of Pivots, and of Flexible Caps

72. Finally this theory will extend over all, and it will become applicable in all parts of the arts, if we continue to suppose that the tip of the pivot is formed by a solid of revolution, and that this pivot carries a cap, the interior surface of which is also a surface of revolution; if we further assume that the bottom of the cap is compressible, as is the tip of the pivot, and that the weight with which the cap is loaded is balanced so that the axis of the pivot and the axis of the cap are on a vertical line, which passes through the center of gravity of the weight.

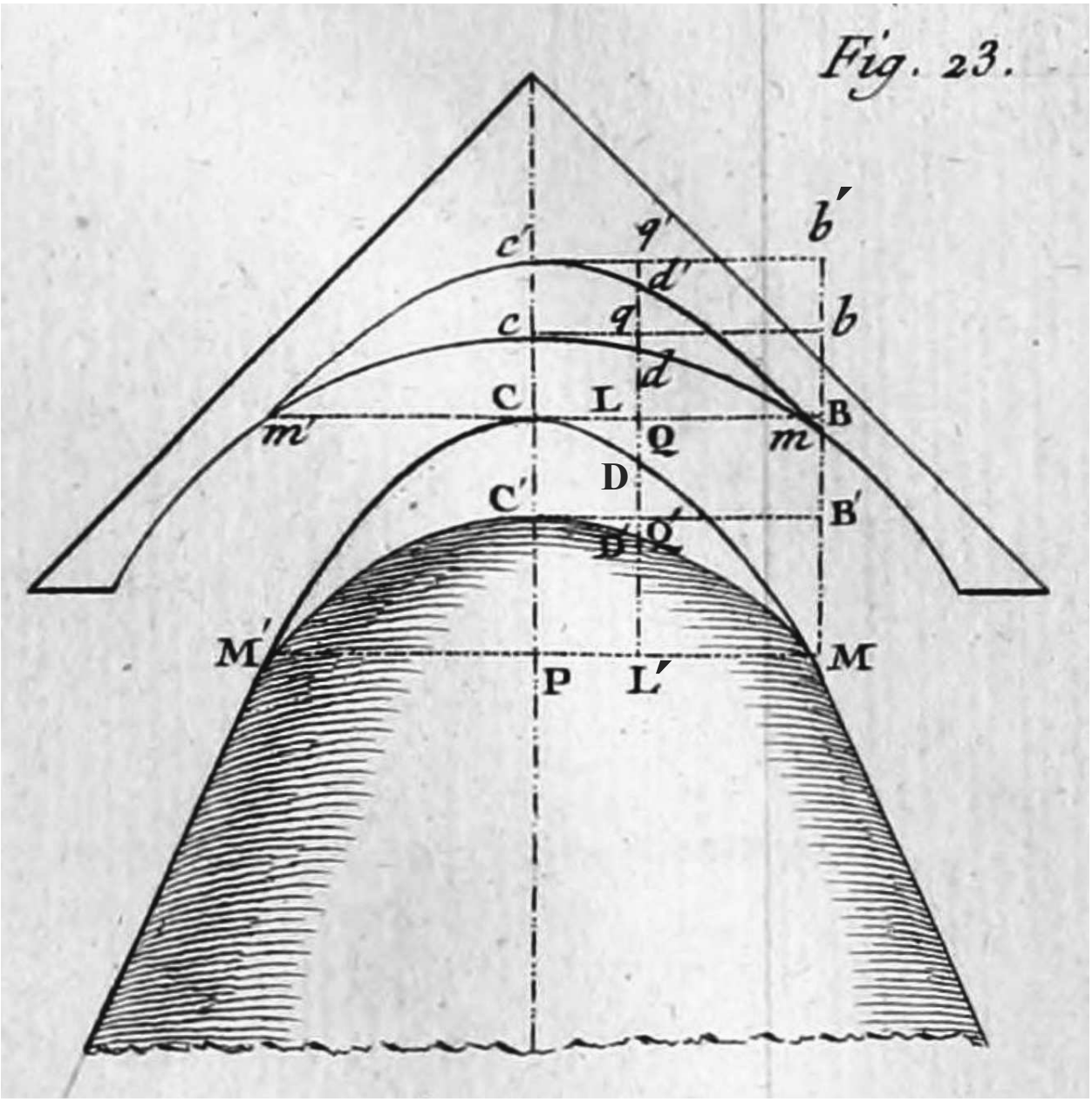
In *Figure 23*, MCM' represents the pivot head before its compression. $MC'M'$ represents the head of this same pivot after compression; mcm' represents the bottom of the [compass] cap before the compression; $mc'm'$ represents the bottom of this same cap after compression.

[270] [Note by Coulomb] See Article 81.

[271] [Note by Coulomb] The remark which we make here on the homogeneity of the integral quantities, and of their differentials, often avoids much calculation in the problems which relate to physics; thus, the integral of

$$(x^m dx(a^\mu + na^{\mu-1}x + n'a^{\mu-2}x^2 + x)^q)\ ,$$

taken when $x = a$, gives us $Ga^{m+1+\mu q}$, where G is a constant quantity, independent of the value of a, and where n and n' are zero-dimensional constant coefficients.



The tip of the pivot, as well as the bottom of the cap, must be of a very hard material, so that the problem can be simplified, and [it is also necessary to] suppose that the compressed parts $CC'M$ and $cc'm$ are terminated by small curved lines, which can, without appreciable error, be taken for arcs of a circle.

We draw the tangents CB and $C'B'$, cb and $c'b'$ at the top of all the curved surfaces, and we take the line Lq' parallel to the axis Pc', which intersects the tangents at the points Q, Q', q, q', and the curves at the corresponding points D, D', d , d'.[272]

It is clear that CM, $C'M'$, as well as cm and $c'm'$, being supposed, because of the hardness of the pivot and the cap, of very small portions of curve, CC' and DD', cc' and dd' will represent the compressions of the corresponding points C and D, c and d.

Now let us make $PM = a$, $PL = x$, the radius of curvature at C, of the pivot, before

[272]I inserted in Figure 23 the letter D mentioned by Coulomb. At the top right I replaced the letter b with b'. At the bottom horizontal line I replaced the letter L with L'.

its compression, $= D/2$, the radius of curvature at C', of the pivot, after its compression, $= D'/2$; the radius of curvature of the bottom of the cap, at C, before its compression, $= d/2$; the radius of curvature of the same cap, after its compression, $= d'/2$.[273]

We first see that after the compression, the bottom of the cap having to touch the end of the pivot at all the points of a part of its surface, the curve $M'C'M$ is exactly the same than the curve $m'c'm$, and thus $D' = d'$.

We also see that, as the small portions of curves, which touch after compression, can, because of the hardness of the pivot and the cap, be taken for a start of a curve, we will have, without sensible error[274] $BM = a^2/D$, $B'M = a^2/D'$, $b'm = a^2/D'$, $bm = a^2/d$, $QD = x^2/D$, $Q'D' = x^2/D'$, $q'd' = x^2/D'$, $qd = x^2/d$; but the compression of point D is

$$DD' = BM - QD - D'L = \left(\frac{1}{D} - \frac{1}{D'}\right)(a^2 - x^2) \ .$$

The compression of the corresponding point d, of the bottom of the cap, is

$$dd' = b'm - q'd' - dL' = \left(\frac{1}{D'} - \frac{1}{d}\right)(a^2 - x^2) \ .$$

At present, the pressure experienced by any point D', depends on the state of compression, and therefore can be expressed by a function of the compression: to simplify this hypothesis, suppose the pressure of the pivot as a power m of the compression, and the pressure of a point of the cap, as a power m' of the compression, we will have, for the pressure of the point D', the quantity

$$\left(\left(\frac{1}{D} - \frac{1}{D'}\right)\left(a^2 - x^2\right)\right)^m \ ;$$

and, for the pressure of the corresponding point d' of the cap, we will have

$$g^{m'}\left(\left(\frac{1}{D'} - \frac{1}{d}\right)\left(a^2 - x^2\right)\right)^{m'} \ ,$$

where g is a quantity, which depends on the ratio of the flexibility of the cap to that of the pivot.

Now, because the pressure is common to the corresponding points D' and d', these two quantities must be equal, and will give the equation

$$\left(\left(\frac{1}{D} - \frac{1}{D'}\right)\left(a^2 - x^2\right)\right)^m = g^{m'}\left(\left(\frac{1}{D'} - \frac{1}{d}\right)\left(a^2 - x^2\right)\right)^{m'} \ .$$

This first equality teaches us an interesting truth, which is that for it to subsist, it is necessary that $m = m'$, since it must take place for all the points of contact; thus, when the pressure is not expressed, for the cap and for the pivot, by the same power m, the cap will only touch the pivot in parts, and the contact will not be continuous.

But, as soon as we have used two homogeneous materials, then $m = m'$, and we immediately have

[273] [Note by Bucciarelli] Here is a problem. D is a label for a point. Coulomb is now changing its meaning to be twice the radius of curvature of the cap/pivot. The same change of meaning happens for D', d and d'.

[274] [Note by Coulomb] In Figure 23, the cap is shown detached vertically above the head of the pivot, so that the line mm', end of the contact, which must merge with the line MM', when the cap is supported on the head of the pivot, is here to make the same line with the tangent CB of the pivot, before its compression.

$$\left(\frac{1}{D}-\frac{1}{D'}\right)=g\left(\frac{1}{D'}-\frac{1}{d}\right)\ ,$$

from where we get

$$\frac{1}{D'}=\frac{\left(\frac{1}{D}+\frac{g}{d}\right)}{g+1}\ ;$$

whence it follows that the curvature produced by the compression is independent of the power m.

If we substitute this value $1/D'$, in the formula

$$\left(\left(\frac{1}{D}-\frac{1}{D'}\right)\left(a^2-x^2\right)\right)^m\ ,$$

which expresses the pressure of the point D', we will have

$$\left(\frac{\left(\frac{g}{D}-\frac{g}{d}\right)}{g+1}\left(a^2-x^2\right)\right)^m\ .$$

We are now in a position to determine the total pressure, and the *moment* of friction, which is the interesting part of our research.

As we assumed here that the compressed part $M'C'M$, is only a small portion of curve, we can, without significant error, suppose that the surface generated by the revolution of CDM, around the axis CP, or, what comes to the same thing, that the surface of the pivot, is equal to the surface of the circle, of which MM' is the diameter: according to this supposition we will have, for the pressure of a zone, of which $x=CQ$ is the radius, and dx the width, the quantity

$$2\pi\left(\frac{\left(\frac{g}{D}-\frac{g}{d}\right)}{g+1}\left(a^2-x^2\right)\right)^m x dx\ ;$$

by integrating, we will have, for the entire surface of the circle, of which $PM=a$ is the radius,

$$2\pi\left(\frac{\left(\frac{g}{D}-\frac{g}{d}\right)}{g+1}\right)^m\frac{a^{2(m+1)}}{2(m+1)}\ ;$$

a quantity which must be equal to the entire pressure, or to the weight P, of which the cap is supposed to be charged: to have the *moment* of the pressure, it will be necessary to multiply the elementary pressure by x; which will give

$$2\pi\left(\frac{\frac{g}{D}-\frac{g}{d}}{g+1}\right)^m\left(a^2-x^2\right)^m x^2dx\ .$$

This equation is exactly integrable whenever m is an integer; thus, in the case of perfect elasticity, where we would suppose the pressure proportional to the compression, we would have $m=1$; which would give, for the *moment* of the total pressure,

$$\frac{2}{15}(2\pi)\left(\frac{g}{g+1}\right)\left(\frac{1}{D}-\frac{1}{d}\right)a^5\ ;$$

but, on this assumption,

$$P=\frac{1}{4}(2\pi)\frac{g}{g+1}\left(\frac{1}{D}-\frac{1}{d}\right)a^4\ ,$$

from which we deduce the *moment* of the pressure

$$=\frac{2}{15}\frac{(4P)^{1/4}}{\left(2\pi\frac{g}{g+1}\left(\frac{1}{D}-\frac{1}{d}\right)\right)^{1/4}}\ ;$$

and if $1/f$ expresses the ratio of friction to pressure, it will suffice to divide this quantity by f, to obtain the *moment* of friction. By following the path of the previous Article, we will find, without going into the details of the integration, by naming

$$N=2\pi\left(\frac{g}{g+1}\right)^m\left(\frac{1}{D}-\frac{1}{d}\right)^m\ ,$$

that since the pressure of the zone is $N(a^2-x^2)^m x dx$, and the elementary *moment* of the pressure of this same zone is $N(a^2-x^2)^m x^2 dx$, we will have, for the integral of the pressure, when $x=a$, the quantity $GNa^{2(m+1)}$; and, for the integral of the *moment*, $G'Na^{2(m+1)+1}$, where G and G' are functions of m, independent of a and N; thus, since $GNa^{2(m+1)}=P$, we will have $\frac{G'}{G}Pa$ for the total *moment* of the pressure, or

$$\frac{G'}{G}P\left(\frac{P}{GN}\right)^{\frac{1}{2(m+1)}}\ ;$$

thus, by substituting the value of N, we will have the *moment* of the pressure, and consequently the *moment* of the friction, proportional to

$$\frac{P^{\frac{(2m+3)}{(2m+2)}}}{\left(\left(\frac{g}{g+1}\right)\left(\frac{1}{D}-\frac{1}{d}\right)\right)^{\frac{m}{2(m+1)}}}\ .$$

If $m=0$, we find this quantity $=P^{3/2}$, as in Article 70.

When using the same caps and the same pivots, we must find the *moment* of friction increasing with the weight, and proportional to $P^{\frac{2m+3}{2m+2}}$.

This quantity will decrease, everything else being equal, as $\left(\frac{1}{D}-\frac{1}{d}\right)$ will increase; that is to say, as we increase the curvature of the extremity of the pivot, and as one flattens, or decreases the curvature of the bottom of the cap; whence results this usual practice, when we need to make turn very large masses around a vertical axis, to present a convex surface to the convex extremity of the pivot, because then $\left(\frac{1}{D}-\frac{1}{d}\right)$ becomes $\left(\frac{1}{D}+\frac{1}{d}\right)$.

Finally the *moment* of friction will decrease as the quantity g increases; that is to say, as the hardness of the bottom of the cap increases, that of the pivot being given: if, instead of assuming the pressure of a point D of the pivot, $=(DD')^m$, we had assumed this pressure $=(\delta\cdot DD')^m\delta$ increasing with the hardness of the pivot, we would have, in the formula which

expresses the *moment* of friction, $\left(\frac{g\delta}{\delta+g}\right)$, instead of $g : (g+1)$. But, as the *moment* of friction decreases as this quantity increases, and as this quantity increases as g and δ are greater, it follows that the greater the hardness, or the inflexibility of the pivot and the cap, the more the *moment* of the friction will decrease; it will become zero, when g and δ will be infinite.

73. *Remark.* When the cap is supported by a very fine needle, if the tip of this needle is strongly tempered, and if it is much harder than the bottom of the cap, then it may happen that it will penetrate into the interior of the cap, of which it will divide the parts; the previous theory cannot be applied to this case.

In Figure 24, MCm represents the tip of the pivot that split the parts of the cap; let the small angle $mCM = 2A$; let $PM = a$ be the distance of the point M, from its natural situation, when it is in P, to its compressed situation, when it has reached m; let $P'D = x$, which expresses the compression of the point D.

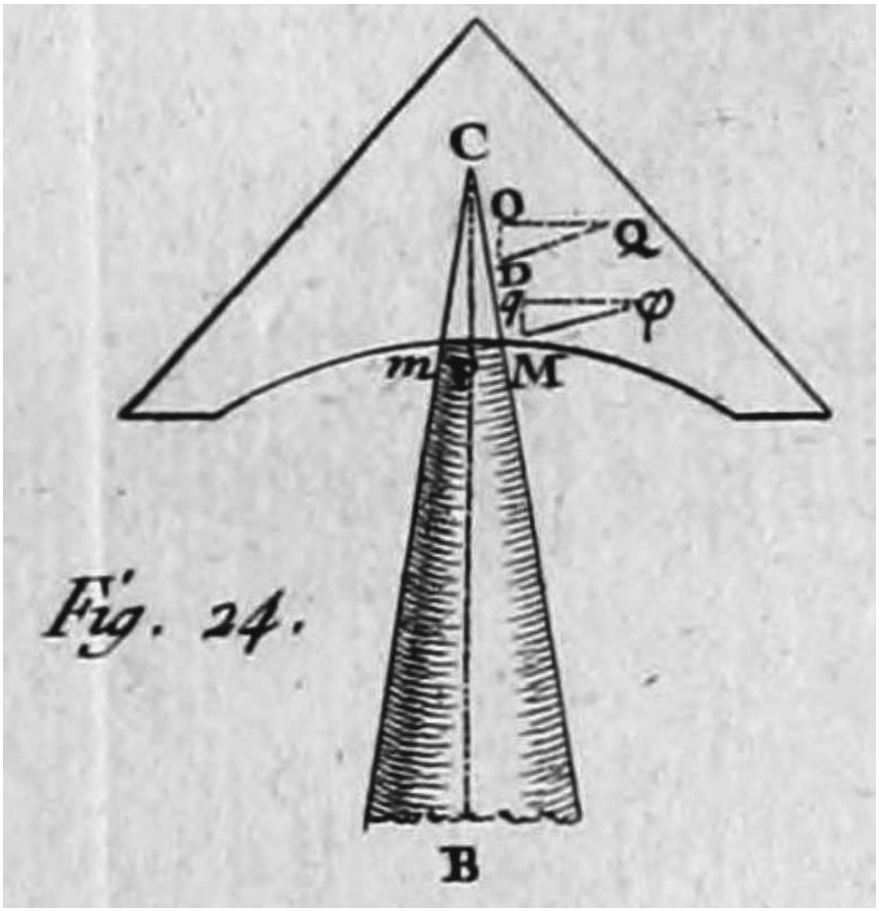


Fig. 24.

The pressure on the part CM, will depend on the compressions PM and pD, of the points D and M; suppose that this pressure is like a power m of the compression, we will have, for the pressure DQ, that the point D experiences perpendicular to CM, its value $(gP'D)^m$, or $(gx)^m$: let us decompose this force into two others, one vertical DO, and the other horizontal OQ, the similarity of the triangles will give us the vertical force

$$DO = (gx)^m \sin A \ .$$

If the tip of the pivot could penetrate the interior of the cap, without experiencing friction, it would be necessary, in consequence of the principles of statics, which we reported at the

beginning of this Chapter, that the sum of the vertical forces $(gx)^m \sin A$ was equal to the weight of the cap, and the weight with which it would be loaded.

But the pressure DQ generates a friction, and the pivot cannot pierce the solid cap, without overcoming this friction, which will be proportional to DQ, and will prevent the pivot from sliding along CM.

If $(1/f)$ expresses the ratio of pressure to friction, $\frac{(gx)^m}{f}$ will be the friction which acts, in D, along CM, to prevent the tip of the pivot from penetrating the solidity of the cap; let us decompose this new force into a vertical force and a horizontal force, we will have, for the vertical force,

$$\left(\frac{(gx)^m}{f} \cdot \cos A\right) .$$

Thus, the reaction of friction and pressure, to support the weight of the cap, will be, in D,

$$(gx)^m \left(\sin A + \frac{\cos A}{f}\right) .$$

This reaction, being multiplied by the small circular area of the surface of the pivot, of which pD or x is the radius, and $\frac{dx}{\sin A}$ the width, gives, for the vertical pressure of this small area,

$$g^m 2\pi \frac{1}{\sin A}\left(\sin A + \frac{\cos A}{f}\right) x^{m+1} dx .$$

This quantity, integrated for the whole surface of the pivot, gives, by making $x = a$,

$$g^m (2\pi)\frac{1}{\sin A}\left(\sin A + \frac{\cos A}{f}\right)\frac{a^{m+2}}{m+2} = P .$$

On the other hand, if the pressure of the point D, which we found [equal to] $(gx)^m$, is multiplied by $\frac{dx}{\sin A}$, width of the zone, and by its circumference $2\pi x$,[275] we will have

$$\frac{g^m}{f} 2\pi \frac{1}{\sin A} x^{m+2} dx$$

for the *moment* of the friction of this zone; integrating this quantity, assuming $x = a$, we will still have

$$\frac{g^m}{f} 2\pi \frac{1}{\sin A} \cdot \frac{a^{m+3}}{m+3} ;$$

and, since we have just seen that

$$P = g^m 2\pi \frac{1}{\sin A}\left(\sin A + \frac{\cos A}{f}\right)\frac{a^{m+2}}{m+2} ,$$

it results, for the total *moment* of the friction,

[275] In the original, [Coulomb, 1780, p. 239]: "*par sa circonférence* 360°x." We are here writing the circumference of the circle of radius x as being given by $2\pi x$. This substitution is also made in the other formulas of this work.

$$\frac{1}{f(m+3)}\left(\frac{\sin A}{2\pi g^m}\right)^{1/(m+2)}\left(\frac{(m+2)P}{\sin A+\frac{\cos A}{f}}\right)^{\frac{m+3}{m+2}}.$$

When the pivot is very acute, then the angle A being very small, we will have, for the *moment* of friction,

$$\frac{1}{m+3}\left(\frac{\sin A\cdot f}{2\pi g^m}\right)^{1/(m+2)}((m+2)\,P)^{\frac{m+3}{m+2}}.$$

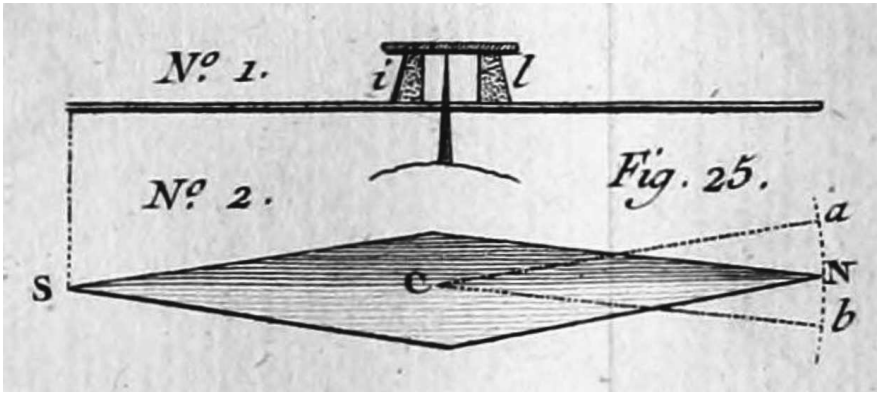
This quantity decreasing as $\sin A$ decreases, it follows that the most acute pivots would be the best; which would indeed be true, if we could give them sufficient strength to support the weight of the cap, without bending.

It is easy to foresee all the inconveniences which result, in practice, from a very fine pivot, which pierces the solid part of the cap: because, although we first suppose that this cap is supported by the lateral pressure of its divided parts, the slightest oscillatory movement will soon have worn away, by friction, the compressed lateral parts, and at that time the head of the pivot must either be rounded or bent in order to support all the weight alone: the case where the head of the pivot comes to be rounded, returns in the situation of the preceding Article: that [case] where the extremities of the pivot bend, give a significant friction, because the cap is then supported, not on a point, but on a small line, and the friction will therefore increase in proportion as the length of this line increases. If there are any inequalities in the bottom of the cap, the curved extremity of the pivot will penetrate there, either obliquely or horizontally, and we can no longer absolutely rely on the direction of the compass.

But, before entering into discussions relative to the practice, on the force and on the shape of the pivots, we are going to relate some experiments, which we will compare with the theory.

5.5.4 Pivot Friction Experiment

74. We took, *Figure 25, Nos. 1 and 2*, a compass needle, pierced at its center of gravity with a hole C, [Figure 25,] No. 2; we glued (No. 1), 3 or 4 lines [0.68 or 0.90 cm] above this hole, a small highly polished glass plate; this plate was separated from the needle by means of two small wooden posts i and l, glued to the compass and the plate. This compass weighed, all inclusive, 150 grains (8.97 g), the glass plate and the small pieces of wood weighed together 9 grains; the needle was ten inches long (27.07 cm) and made 10 oscillations in 60".



This needle was placed horizontally on the tip of a pivot of very hard steel ([Figure 25,] No. 1),[276] it was necessary to search to find the point of equilibrium, but, as the center of gravity is much lower than the point of suspension, this was easily done: when the needle was a little inclined, it was soon restored to the horizontal position with sand spread on the lighter end.

This needle being completely closed in a box, we sought the limits of its region of indifference[277] by presenting to it at a distance the pole of another [magnetized] needle, or the angle formed between all the directions *ca*, that it could take, without its magnetic and directing force bringing it back to its true [magnetic] meridian: it is obvious that the angle of indifference *acb* was proportional to the friction.

First trial. The compass suspended freely on its pivot gave the angle *acb* of 8 or 10′.

Second trial. The compass loaded with two small copper plates, weighing together 300 grains [15.93 g], gave the angle *acb* of 30′.

Third trial. The compass loaded with 600 grains, gave the angle *acb* of 60′.

Fourth trial. The compass loaded with 1200 grains, gave the angle *acb* of 3° 15′.

Fifth trial. The compass loaded with 1800 grains, gave the angle *acb* of 5°.

Result of This Experiment

In all these tests, the needle is always suspended horizontally placed on the tip of the same pivot; supported by a very polished plane, and which, on account of its great hardness, may be regarded as impenetrable to steel. The bottom of a cap,[278] its inequalities and its curvatures, could not influence here the increase in friction: thus, the errors of the compass measured by the region of indifference, could only be caused by the horizontal friction of the glass plate on the tip of a pivot: if we now suppose that the *moment* of friction is like a power n of the weight, or rather of the compression, we will find, neglecting the first trial, because of the difficulty of obtaining a correct measurement because of the small magnitude of the angle *acb*, and then comparing the second trial with all the others, that it results from the second and third trials,

$$(450)^n : (750)^n :: 30' : 60' \ ,$$

from which

$$n = 1.357 \ .$$

The second and fourth trial give

$$n = 1.703 \ .$$

The second and fifth trial give

[276] The pivot passed through the hole in the center of the magnetized needle and the glass plated rested against its tip. The pivot supported the combined weight of the glass plate, the two small wooden sticks and the magnetized needle.

[277] In the original: *champ d'indifférence*. This expression can also be translated as field of indifference, zone of indifference or sphere of indifference.

[278] See footnote 141 on page 58.

$$n = 1.571 \ .$$

By taking an average value, we will find

$$n = 1.544 \ ,$$

from which it seems to result, that the *moment* of the friction is approximately proportional to $P^{3/2}$, as the theory (of Article 70) seems to indicate; whence it follows, therefore, that when the tip of a pivot is compressed by an impenetrable plane, all the points of the circle of contact experience nearly an equal pressure.

We have made a very large number of experiments, suspending, as in the preceding tests, the magnetized needles by means of plates of glass, agate, yellow copper, and of different compositions, and we always have found similar results to those just described.

When the pivots had been in use for a long time, and their tip was worn, it was found quite exactly that the *moment* of the friction was proportional to the pressures.

The best caps we have been able to obtain have given us friction values proportional to $P^{3/2}$; but the slightest inclination in the position of the compass, and the small curvatures, which are found at the bottom of these caps, most often produce, in the result of the experiments, inequalities for which no hypothesis can account.

5.5.5 Comparison of the *Moment* of Magnetic Forces, with the *Moment* of Pivot Friction

75. We have hitherto endeavored to develop all the elements which produce the direction of the needles; we have also endeavored to determine the coercive forces, which can produce errors in this direction. By comparing now the coercive forces with the magnetic force, it will be easy to decide on the choice of blades that we must use to form compasses, according to the different uses to which they can be put.

We found (Article 6), that $(B - S)$ or the angle of error of a magnetic needle, could be represented by $R/(\int \varphi\mu r)$, and therefore, to reduce this error as much as possible, this quantity had to be a *minimum*.

But we found, Article 37,[279]

$$\int \varphi\mu r = \frac{KLEl^3}{(mL^{1/2}E + nl)^2} \ .$$

We found, by the preceding Articles, that the *moment* of the friction of a pivot, must be proportional to a power of the pressure, and if we call this power λ, experiment has taught us that it was approximately equal to $3/2$.

Thus, if we suppose, keeping the same letters, that a marine compass rose, of which the weight is $(2gA)$ is balanced horizontally on the tip of a pivot, and directed by a magnetized blade of uniform thickness and width throughout its length, we will find, naming $LEl = M$,[280] that

[279] Coulomb is considering here a magnetized needle of length l, width L and thickness E.

[280] That is, M is the volume of the needle.

$$\frac{R}{\int \varphi\mu r} = B\frac{2g(A+M)^\lambda}{kMl^2}\left(mL^{1/2}E+nl\right)^2 \ .$$

B being a constant coefficient, by substituting, in place of $L^{1/2}E$, its value $M : L^{1/2}l$, we will have, for the *minimum*, ...,[281]

$$d\left[\frac{(A+M)^\lambda}{M}\left(\frac{mM}{L^{1/2}l^2}+n\right)^2\right] = 0 \ .$$

which gives, by varying M,

$$\frac{\lambda dM}{A+M} - \frac{dM}{M} + \frac{\frac{2m}{L^{1/2}l^2}dM}{\frac{m}{L^{1/2}l^2}M+n} = 0 \ ,$$

a quadratic equation, from which it is easy to deduce the value of M.

It is useless to vary L and l, because we see immediately, that M remaining constant, it is necessary to increase these quantities to infinity, or at least as much as nature and the solidity of steel can allow it.

The quantities L and l being given, the previous equation will give the thickness of the blade.

76. We will easily determine, by the same process, the length of a blade whose other dimensions would be given, in the equation[282]

$$\frac{(A+M)^\lambda}{M}\left(\frac{mM}{L^{1/2}l^2}+n\right) \ ;$$

substituting, in the place of M, its value δl, or $\delta = LE$, a quantity that is constant here, by hypothesis,[283] and we will have

$$d\frac{(A+\delta l)^\lambda}{\delta l^3}\left(\frac{m\delta}{L^{1/2}}+nl\right)^2 = 0 \ ,$$

or[284]

$$\frac{\lambda\delta}{A+\delta l} - \frac{3}{l} + \frac{2n}{\frac{m\delta}{L^{1/2}}+nl} = 0 \ ,$$

a quadratic equation, from which we will deduce the value of l.

[281] That is, the minimum of this quantity is obtained when its derivative vanishes, as expressed in the next equation.

[282] The next equation did not appear in Potier's reprint of Coulomb's works, [Potier, 1884, p. 57]. Probably the second factor should be raised to the second power, namely:

$$\frac{(A+M)^\lambda}{M}\left(\frac{mM}{L^{1/2}l^2}+n\right)^2 \ .$$

[283] The quantity $\delta = LE$ represents the cross-sectional area of the needle with length l.

[284] [Note by Bucciarelli] Differentiation is here with respect to l.

77. If we suppose, in the formula of the preceding Article, that the needle is charged with no weight, then we will have $A = 0$, and the equation is reduced to

$$(\lambda - 3) + \frac{2nl}{\frac{m\delta}{L^{1/2}} + nl} = 0 \ ;$$

from which

$$l = \frac{3 - \lambda}{\lambda - 1} \frac{mEL^{1/2}}{n} \ .$$

5.5.6 Example

We found, first experiment, Article 26, that a blade 12 inches in length and weighing 288 grains, made, when reduced to 4 inches in length, 20 oscillations in 80". We have seen that a reduction of 4 inches in the length of this blade produces a reduction of 49" in the time of the oscillations: now, as $T = (mEL^{1/2} + nl)$, we will have, for a needle 4 inches in length, $nl = 49''$; and, as $T = 80''$, we will have

$$mEL^{1/2} = 31'' \ ;$$

substituting these values into the formula $l = \frac{3-\lambda}{\lambda-1}\frac{m}{n}EL^{1/2}$, we will find

$$l = \frac{3 - \lambda}{\lambda - 1} \frac{31''}{49''} \cdot 4 \text{ inches}$$

and, if we assume $\lambda = 3/2$, as we have learned from experience, we will have

$$l = \frac{31''}{49''} \times 3 \cdot 4 \text{ inches} = 7.59 \text{ inches} \qquad (20.57 \ cm) \ .$$

Remark. From the formula $l = \frac{3-\lambda}{\lambda-1}\frac{m}{n}EL^{1/2}$, we conclude that l will decrease as EL decreases, that is to say, that the length of the needles must be decreased as they become lighter; this is what practice had already indicated.

78. Questions[285] similar in kind [addressed] in the two preceding Articles that one might face are too easy to resolve for it to appear necessary to dwell on them any longer. We are going to end this theory with two small problems, which will often be of use in the construction of compasses formed with several magnetized needles.

We have seen, in the theory of magnetism, that the lightest blades were those which, proportionally speaking, were most strongly magnetized. We have seen (Articles 61 and 62), that a needle balanced on a horizontal plane, always had the same *moment*, to reestablish itself in the direction of its magnetic meridian: whence it is easy to see that a compass, formed of several parallel and separate blades, has more force to direct itself along its meridian, than

[285] In the original work, this Article came out with the number 77, as was the previous Article. We changed the numbering to 78, following this number onwards in the next Articles of this work.

a single blade which would have the same weight as all the blades united: these considerations present these two problems to us.

79. Problem. *The weight of the marine compass rose, being given, as well as all the dimensions of the magnetized blades which we wish to employ, of how many blades must the compass be composed, so that it comes as close as possible to its magnetic meridian.*

Let $2gA$ be, as above, the weight of the [compass] rose, and $2gM$, the weight of one of the given needles, let k be the number of needles, the *moment* of the pressure, and consequently of the friction, will be like $(A + kM)^\lambda$: but the *moment* of the magnetic force expressed for each lamina by $\int \varphi\mu r$, will give, because of the equality of the laminae, for the *moment* of the magnetic force, $k \int \varphi\mu r$: thus, the angle of error will be

$$(A + kM)^\lambda : k \int \varphi\mu r \ ;$$

a quantity to be differentiated, making only k variable; which gives, for the condition of the problem,

$$k = \frac{A}{(\lambda - 1)M} \ ;$$

and, if $\lambda = 3/2$, as experiment has taught us, $k = 2A/M$: thus, it would be necessary, for example, 4 blades of 100 grains, for a rose which would weigh 200 grains.

80. Problem. *The number k of the blades being given, as well as their length and their width, determine the thickness or the weight of these blades.*

We still have here the general equation

$$(A + kM)^\lambda : k \int \varphi\mu r$$

or

$$\frac{(A + kM)^\lambda}{kM} \left(\frac{m}{L^{1/2} l^2} M + n \right)^2 \ ;$$

this differentiated equation, by making only M variable, gives

$$\frac{\lambda k}{A + kM} - \frac{1}{M} + \frac{\frac{2m}{L^{1/2} l^2}}{\frac{m}{L^{1/2} l^2} M + n} = 0 \ ;$$

a quadratic equation, from which it results, by letting

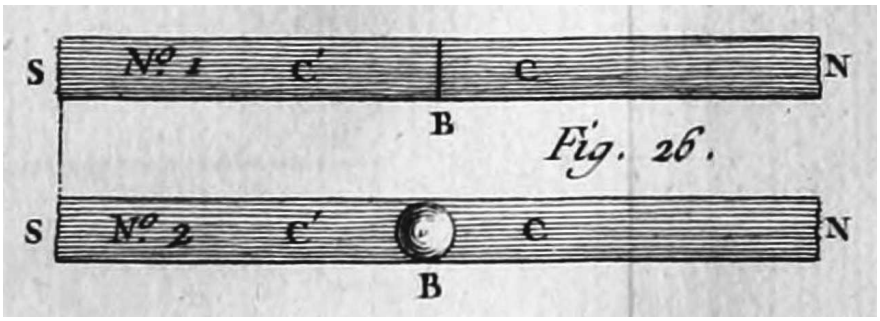
$$\frac{2m}{L^{1/2} l^2} = m' \ ,$$

the reduced formula

$$M = -\left[\frac{(\lambda-1)nk+m'A}{2(\lambda+1)m'k}\right] + \left[\left(\frac{(\lambda-1)nk+m'A}{2(\lambda+1)m'k}\right)^2 + \frac{nA}{(\lambda-1)m'k}\right]^{1/2}.$$

We believe that we have collected in this Chapter the greater part of the principles which can guide us in the construction of compasses appropriate to the service of the navy. We are going to end with a few remarks relating either to theory or to practice, which have not yet been mentioned.

81. *First remark.* If we cut in two parts, at point B, a blade NS *(Figure 26, No. 1)*, magnetized to saturation, of which N is the boreal extremity, and S the austral extremity, and whose magnetic center is placed nearly in the middle of the blade; after the separation, the extremity B of the part NB will be the austral pole, and the extremity N will retain its boreal force: the extremity B of the part SB will have a boreal force, the extremity S will retain its austral strength; each of these parts will take a magnetic center in C and C'.



If, instead of being divided into two parts, this same blade is only pierced with a hole B *(Figure 26, No. 2)*, then the two ends are in separate parts, and this blade must have two magnetic centers, like the preceding one.

This multiplication of poles led to the belief that a blade thus pierced must partly lose its directing force,[286] and that it was not very suitable for indicating the declinations. Here is what experiment gives on this subject.

When a blade was pierced in its center, with a hole whose diameter did not exceed half the width of the blade, it had sensibly the same directing force[287] as before being pierced; this is what it is easy to convince oneself of, by making this blade, magnetized to saturation and suspended horizontally, oscillate. It will be found that in both cases it gives substantially the same number of oscillations for the same time.

When the hole of the blade is almost equal to its width, we then find that the magnetic *moment* of this blade, is equal to the sum of the magnetic *moments* of two other blades which would have only half the length of the first: this is what also conforms to the theory that we have explained (Article 61 and 62); thus, when the blade is very light, as in this case its magnetic *moment* is approximately equal to a constant quantity, multiplied by its length; whether the blade, in this case, is pierced, or whether it is not, we will always have approximately the same *moment*.

[286] In the original: *force directrice*. This expression can also be translated as directive force, directed force, force of direction, directional force, driving force or guiding force.

[287] In the original: *force de direction*.

82. *Second remark.* After all that we have said on the communication of magnetism, we did not believe that it was necessary to make researches on the various shapes, either rectilinear, or curved, which we can give to the magnetized plates, it is easy to predict all that can be expected from these variations.

Arrow-shaped needles, of the form shown in Figure 25, give, at equal weight and thickness, the same ratio between the magnetic *moment* and the frictional *moment*, and therefore produce about the same errors as blades of uniform width; it is observed, however, that light blades of uniform width have an advantage over arrow-shaped blades, and that when the blades are heavy, the latter have an advantage over the former: the theory predicts this result, the experiment confirms it.

The needles, such as in Figure 25, which we are quite in the habit of using for the observations that have been made on Earth, are most often thicker towards their extremities than in the other parts: this practice appears disadvantageous; we will easily see that it is better, by preserving the same weight, to widen the end, and to decrease the thickness, so that the parts exert, the ones on the others, their magnetic action at a greater distance, and therefore retain a greater degree of magnetism.

83. *Third remark.*[288] We have said before that there is always an advantage in constructing compasses of several light blades: the theory of magnetism leaves no doubt about this. The more the blades are multiplied, making them light in proportion, the more the directing force of the compass will have an advantage over the friction. In the construction of navigational compasses, where magnetized needles can scarcely be given more than six inches in length [16.2 cm], blades of 50 to 60 grains [2.7 to 3.2 g] fulfill quite well all the uses for which these compasses are intended: we determine the number of blades by the formula of Article 81.[289] These blades calibrated exactly according to the same dimensions, are placed sideways, at equal distances from the point of suspension, spaced four or five lines [0.9 or 1.1 cm] apart from each other, so that their reciprocal action does not destroy the magnetism.[290] They must be quite straight and solidly fixed to the [compass] rose: the rose and the [compass] cap will be as light as possible. The [compass] caps that are well centered and turned with care must form in their concavity a cone rather obtuse than acute: most of the caps, although they appear to the eye fairly well centered and exactly polished, have, in their concavity, inequalities and small depressions which mesh with the tip of the pivot; so that there are positions where the center of gravity of the load is lower than in the others. In this situation, the cap cannot turn horizontally, its axis remaining vertical, without these inequalities being released, the center of gravity of this cap and of the mass which is fixed there, does not rise.

Thus, a compass carried by such a cap and balanced horizontally on the tip of a pivot, finds itself at the same time solicited by the magnetic force, and by its own weight, which,

[288] The text henceforth was not included in the partial reprint of this paper in Potier's work, [Potier, 1884].

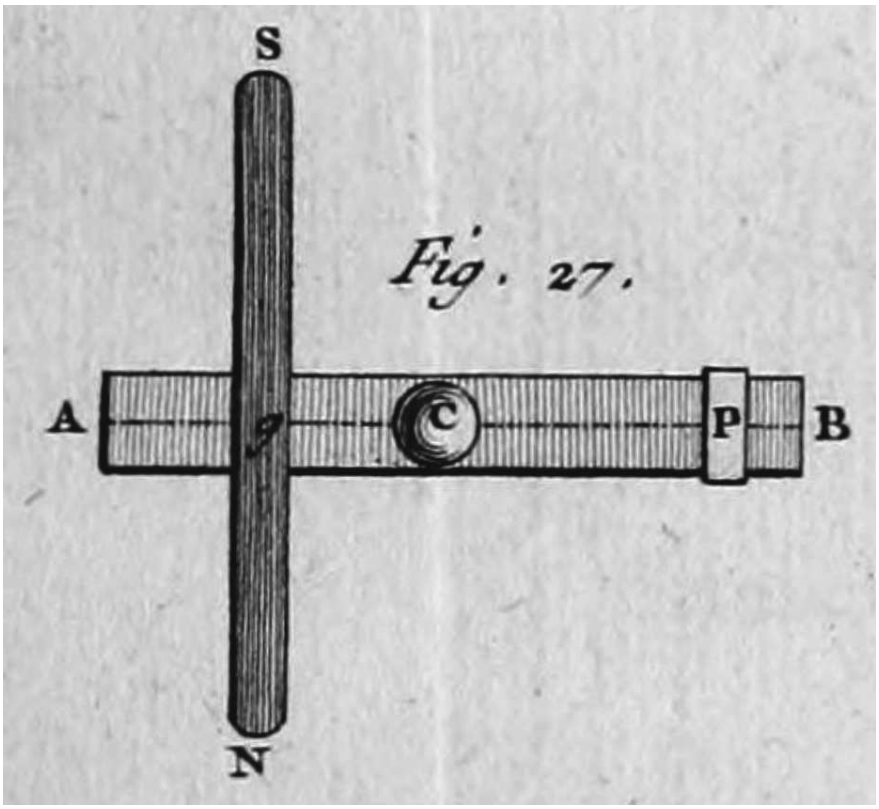
[289] In the original text appears here "Article 78". We changed this expression to "Article 81" due to the renumbering introduced earlier, see footnote 285 on page 125.

[290] [Note by Coulomb] When several blades are fixed to the plane of the [compass] rose, their reciprocal action cannot influence the position of the plane of this rose, because of the equality of action and reaction; however, this reciprocal action being able to diminish more or less the magnetism of the parts of each blade, there can be a change of position in the resultant of the magnetic forces of each blade; but it will always be easy to find the common resultant of all the needles attached to the rose, by reversing this rose, and practicing the same operations that we have indicated (Article 63), for a single blade.

by combining the inequalities of the bottom of the cap with those of the tip of the pivot, tends to make it take the position where the center of gravity is lowest: the patience and the skill of the artisan, can only provide us with exact caps; but the observer can also, by different trials, recognize its faults. Here is a way that worked pretty well for me.

A magnetized blade is suspended sideways with silk threads, as we have indicated before; in this way we easily determine the magnetic meridian. This meridian is also traced across the width of the blade, by suspending it so that its width is in a horizontal plane, and then turning it over on the other side, as we have explained (Article 63).

We now take (Figure 27), a small, very light wooden ruler AB, equipped in C with the [compass] cap we wish to test.



We place at any point g, a blade SN magnetized to saturation, whose magnetic meridian we know: we balance the whole thing horizontally, by means of a counterweight P, which can slide along CB, and with a little sand that we spread lightly on the parts that seem to rise.

If the cap is perfectly centered, if its axis is vertical, the small plane of contact between the pivot and the bottom of the cap will be a small horizontal circle, and the direction of the needle will be determined solely by the magnetic force: we will make the error due to friction vanish by knocking lightly and quickly on the table where the pivot is fixed; this produces, in the elastic parts of this table, a rapid oscillating movement, which causes the pivot to rise and fall. During the time that the pivot descends, it is conceivable that it is partly detached from the bottom of the cap, and that during this time the pressure and consequently the friction is insignificant. It actually follows from all that we have said previously, that in whatever position the magnetic needle is placed in relation to its center of rotation, it must take the direction of its magnetic meridian: now, as this direction is known to us, the angle

which it will form with this direction, will be the error due to the defects of the cap and the pivot.[291]

By successively posing this blade, so that its magnetic meridian SN, make different angles with the punctuated line AB of the balance; we will recognize all the imperfections of the cap, whose different defective points will be located differently in relation to the magnetic direction at each trial and will give different errors.[292]

Several other means can be used. If, for example, we suspend, with this cap, a needle pierced in its center, and that, through the point of suspension, we make pass the magnetic meridian; this needle must not only be placed on its magnetic meridian; but when it is put into oscillation, it must make equal excursions to the right and to the left of this meridian, or at least the excursions will decrease an equal amount.

If we don't have the help of a good artisan, and if we can get an obtuse hole punch, well polished and slightly rounded at its tip, we will form a fairly good cap, by driving this punch with a small hammer blow perpendicular at the center of a small copper plate in a quarter of a line [0.06 cm].

The pivots must be more or less acute, according to the loads they are intended to support: in compasses, for the service of ships, the pivots wear out a lot, not only because of the weight of the cardboard rose where the [magnetized] needles are fixed, but also because of the continual movement in the place where the compasses are located. It is customary, and with good reason, to make these pivots more reinforced, less sharp than those which must support needles intended to carry out operations in a fixed place.

The hardness of the caps and pivots is the most essential condition for the perfection of these instruments; the pivot should never be sharp enough to penetrate the solid part of the cap, nor to be able to bend under its weight.

84. *Fourth remark.* The point of suspension or the contact of the end of the pivot and the bottom of the cap, is a small circular surface, as we have seen (Article 70): it seems interesting for the topic we are dealing with, and for the arts in general, to seek to determine the diameter of this circle. Here, according to the theory and the experiments which precede, is the way to do it.

We have seen (Article 74,) that it resulted from experiment, that the small circle of contact, formed by the compression of the extremity of the pivot, was equally pressed in all its points: we found (Article 68), that the *moment* of friction on the small contact circle, was expressed by the quantity $\frac{2P}{3n} \cdot CM$, where CM is the radius of the contact circle, and P/n the friction of the weight P, which would slide along a surface; a quantity that we found for [the contact between] glass and steel, equal to $P/7$ (Article 67).

We saw, Article 6, that the angle of error of a magnetized needle[293] was $(B - S) = R/(\int \varphi\mu c\mu)$, and we found (Article 12), that

$$\int \varphi\mu c\mu = \frac{g \cdot \int \mu r^2}{\lambda} ,$$

[291] That is, the angle that the needle will make with the magnetic meridian will be the deviation of the declination due only to the defects of the cap and the pivot. This angle of deviation from the magnetic meridian is caused by the friction between the cap and the pivot.

[292] That is, with each defect producing a different angular deviation of the compass from the true magnetic meridian.

[293] That is, the angle of deviation of the needle from the magnetic meridian.

where λ is the length of a pendulum which beats oscillations isochronous to those of the needle: thus, we have

$$(B - S) = \frac{2\lambda P}{3gn \int \mu r^2} CM \ ;$$

and $B - S$ being given by experiment, as well as λ, we will easily find, for a given needle, the value of CM.

5.5.7 Example

We found *(Article 74, Experiment, First Trial)*, that a needle in [the shape of an] arrow, 10 inches in length and 150 grains of weight, made 10 oscillations in 60", and that its angle of error due to friction, was 5': now we have for a needle of this shape, $\int \mu r^2 = \frac{P \cdot l^2}{6g}$, where l here equals 5 inches: thus, $CM = \frac{(B-S)nl^2}{4\lambda}$; and, substituting the numerical values, $n = 7$, $\lambda = 1321$ inches, $(B - S) = 5'$, $l = 5$ inches we will have the diameter of the circle of contact, $2CM = 1/862$ of a line.[294]

We can conclude, it seems to me, from the smallness of the diameter of the circle of contact, found in this example, that the plane of contact can be regarded as a fixed point.

5.5.8 Dip Circles

85. *Fifth remark.* Dip circles have always presented very great difficulties in their execution, either because it is difficult to bring the center of gravity down into the axis of the trunnions,[295] or because the curvature of these needles, changing according to their inclination, makes the position of this center of gravity variable, with respect to the axis of suspension: nothing can be added to the scholarly research that Mr. Daniel Bernoulli has given on this subject:[296] but when we cannot have an artisan as skilful and as exact as those who appear necessary, to fulfill the views of this author, here is how it seemed to me that we could make up for it.

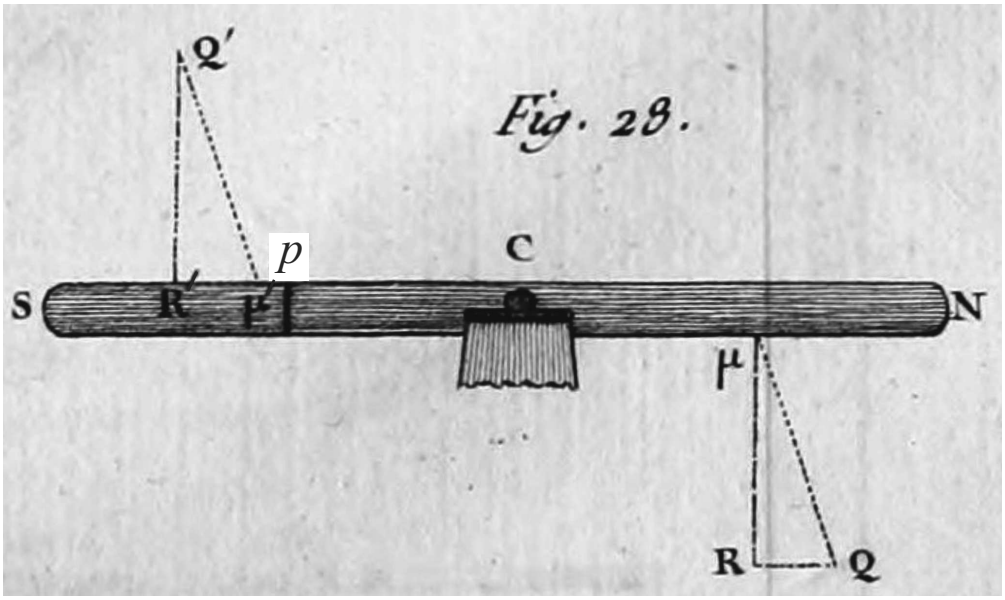
The needle SN *(Figure 28)*, is balanced on its trunnion C, which bear on two glass blades; a copper wire p weighing one or two grains [0.05 or 0.10 g], but whose weight is exactly determined, can slide along the austral part, from C to S, its *moment* is measured by the distance (Cp).[297]

[294] That is, diameter $= (1\ line)/862 = (0.226\ cm)/862 = 2.6 \times 10^{-4}\ cm$.

[295] In the original: *tourillons*. A trunnion is a pin or pivot on which something can be rotated or tilted.

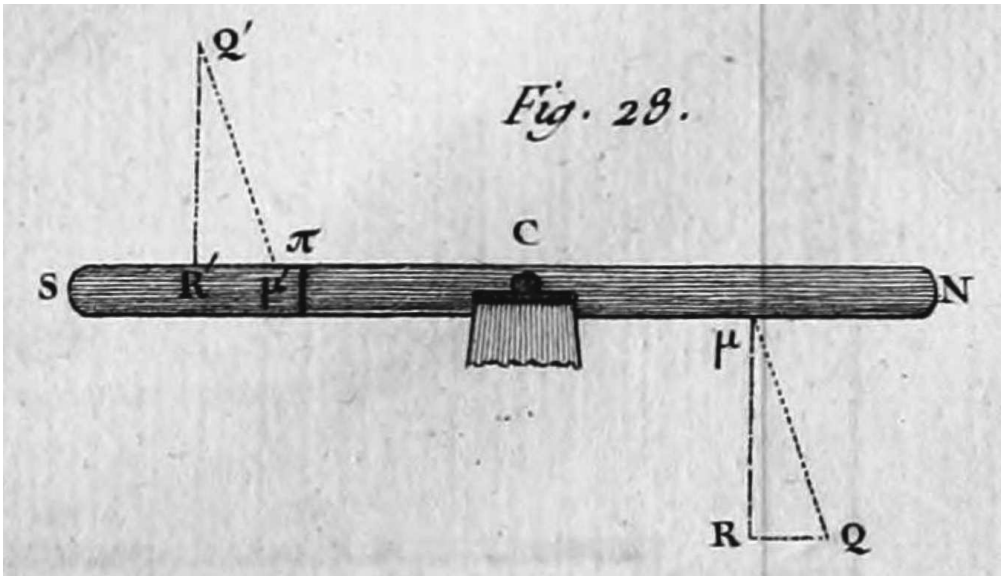
[296] Daniel Bernoulli (1700-1782) was a Swiss mathematician and physicist. See, in particular, [Bernoulli, 1752].

[297] In the original article Coulomb represented the copper wire by the letter π, see [Coulomb, 1787, p. 254] and the Figure in this footnote:



Before magnetizing this blade, now subject to no *moment*, it will be balanced very exactly on these trunnions, in a horizontal position, the small weight p then placed at C on the axis of the trunnions. The blade will then be magnetized to saturation: if it is of a very pure steel, the magnetizing center will fall approximately at C, the forces μQ which will act on each point μ of the boreal part CN, will be parallel to each other and to the magnetic direction: the forces $\mu' Q'$ which will act on the austral part, will have the same direction [but pointing] in an opposite sense; it will therefore be necessary, to keep the needle in the horizontal position, to bring the small weight p to the distance Cp from the point C.[298]

If we now decompose all the magnetic forces μQ into two other forces, one horizontal RQ, and the other vertical $R\mu$, and if we name φ the force following μQ, and B the angle



We are here following Potier and representing the copper wire by the letter p in the Figure 28 that was placed in the middle of the text, to avoid confusion with the number $\pi = 3.14159...$, see [Potier, 1884, p. x] and the page 32 of this English translation. This copper wire can be thought of as having the shape of an inverted U letter. It can slide along the axis of the magnetized needle.

[298] See footnote 135 on page 57.

μQR, inclination of the magnetic direction with the horizon, we will have, for the magnetic *moment* of all the vertical forces

$$\int \varphi \mu c \mu \sin B \ ,$$

which must be equal to $p \cdot pC$.[299]

If we then suspend this needle horizontally, by means of a silk thread, as we have done before, and make it oscillate,[300] the time of the oscillations will give us, in accordance with Article 13, a weight Q, which, multiplied by half the length of the needle, would have a *moment* equal to all the magnetic forces RQ, which act in a horizontal direction; from which we will we get

$$Q \cdot CN = \int \varphi \mu c \mu \cos B \ .$$

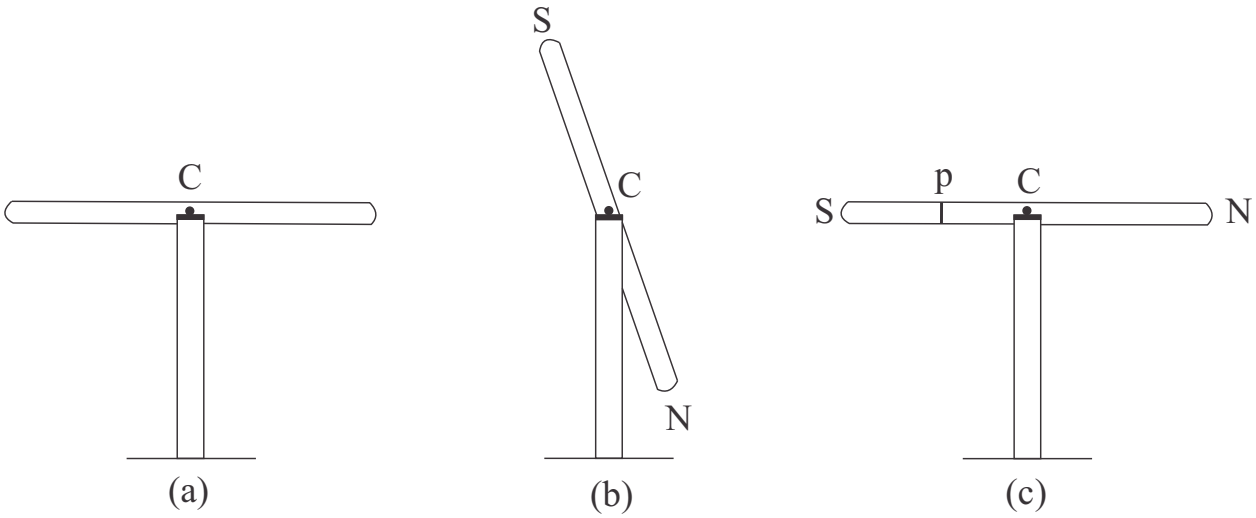
Now divide the *moment* of the vertical forces, by the *moment* of the horizontal forces, and we will have the equation

$$\frac{p \cdot pC}{Q \cdot CN} = \frac{\sin B}{\cos B} = \tan B \ ;$$

which will give the angle B that we are looking for.

By substituting the value of B, thus determined in the equation

[299] An illustration of this experiment appears in the Figure in this footnote:



We have a non-magnetized needle that can rotate in a vertical plane around a horizontal axis that passes through its center of gravity. This needle is initially placed in a horizontal orientation, as shown in letter (a) of the Figure of this footnote. When it is magnetized, the North pole of the needle will be below the horizon if the needle is located in Paris, while the South pole will be above the horizon, as shown in letter (b). If there is little friction on the axis and if this axis passes exactly through the center of gravity of the needle, the needle will orient along the direction of the Earth's magnetic force, see Section 4.3. Coulomb then places a copper wire of weight p that can slide around the side CS of the needle. He slides this weight from the center C of the needle towards the end S, until he finds a certain point at which the needle becomes horizontal again, as illustrated in letter (c). In this situation, the magnetic torque exerted by the Earth on the needle must be equal to the gravitational torque. This gravitational torque is given by the weight p of the copper wire multiplied by the lever arm, that is, multiplied by the distance pC.

[300] That is, the needle will oscillate in a horizontal plane around the vertical axis given by the silk thread.

$$\frac{Q \cdot CN}{\cos B} = \int \varphi \mu c \mu \ ,$$

we will deduce the *moment* of the magnetic forces, something of interest at different points on the Earth.

We note that, for this method to be feasible, it is necessary to take care to bring the axis of the trunnions very close to the center of gravity of the blade, because the slightest variation in the position of the weight p will result in an appreciable angle of inclination.

5.6 Chapter V. Regular Diurnal Variations of the Declination of the Needles

86. Up to now we have always preceded theory by experiments, and we have not made any assumptions about the nature of the magnetic fluid: but as it is a question, in this Chapter, of determining a cause, and as we lack observations made over a series of years, and on whose accuracy we can rely, we will not be able to follow such a sure course.

The general system of physicists has held for a long time that the cause of magnetism was due to a vortex of fluid matter, which made its revolution around magnets, whether artificial or natural, entering through a pole and leaving through the other. This fluid acted, it was said, on iron, because of the configuration of its pores, but it did not exert any action on any other species of body. As difficulties in this hypothesis were encountered in explaining new magnetic phenomena, or in the variations of the declination of the needles, so too some new suppositions were made, either by imagining several vortices or several magnetic poles, or by imagining a magnet in the center of the Earth, to which a particular movement was given.

On these principles were based the three Memoirs on the cause of magnetism, which were crowned in 1746.[301] However, the difficulty of explaining all magnetic phenomena with vortices, has made several physicists suspect, for some years, that the cause of magnetism might come from attraction.

I believe I have proved, in the beginning of this Memoir, that the causes of the orientation of the magnetic needle, could not be explained by impulse.[302] I would add here, that when we magnetize a steel bar with an artificial magnet, it does not appear that the bar, which is used to magnetize, has lost its magnetism after the operation: however, in the hypothesis of impulse, the vortex of the magnetizing bar produced a movement in the whole mass of the

[301] Coulomb is referring here to the works of Leonhard Euler (1707-1783); Étienne François Dutour de Salvert (1711-1789); Daniel Bernoulli (1700-1782) and Jean Bernoulli (see Section 4.4): [Euler, 1752], [Du Tour, 1752] and [Bernoulli and Bernoulli, 1752]. These authors followed the ideas of René Descartes (1596-1650) that magnetism would be due to a vortex of a material fluid that would circulate around the magnets, entering the magnets at one end and exiting at the other. The interaction between a magnet and a piece of iron, for example, would be caused by the pressure caused by this fluid when pushing or colliding with the pores or channels that would exist in the iron. The interaction between two magnets causing their mutual orientation would be due to the action of the vortices of material fluid of one magnet when interacting with the pores or channels of the other magnet, and vice versa. On the other hand, non-magnetic materials would not have these pores and, therefore, would not suffer the action of nearby magnets. For a discussion of this topic see, for example, Section 4.4 and [Gillmor, 1971a, p. 176].

[302] That is, the orientation of compasses could not be explained by the impulse of a fluid colliding with the parts of the magnetized needle.

magnetic fluid of the magnetized bar; this movement could not have been produced without an impulse, which, by its reaction, must have destroyed part of the movement, or changed the direction of the magnetic fluid of the bar which was used to magnetize; and consequently, if the magnetism were due to impulse, a magnet should lose its force while magnetizing, which is contrary to the experiment.

87. Among the different authors who have had recourse to attraction to explain magnetic phenomena, most, like Messrs. Brugman and Wilke,[303] used two elastic fluids; they supposed that, when a blade of steel was in its natural state, these two fluids were joined together, and spread uniformly throughout the blade; but that, when it was magnetized, the two fluids were divided: according to these authors, the two fluids exert an attractive action on each other; but they exert on their own parts a repulsive force,[304] like the air and all the elastic fluids: they named one of these fluids, positive, and the other, negative.

Mr. OEpinus has adapted Mr. Franklin's system of electricity to magnetism:[305] he thinks that, for magnetism, there is only one elastic fluid, which acts on its own parts by a repulsive force, and on parts of steel, by an attractive force. This fluid, once engaged in the pores of the steel, only emerges with difficulty. This system leads to a singular conclusion; it is that it results from magnetic phenomena, that the solid parts of the steel exert on each other a repulsive force. Mr. OEpinus calls *positive pole*, the part of the steel blade where the fluid abounds, and *negative pole*, the part which has been emptied, or which does not retain its natural portion of magnetic fluid.

These two hypotheses explain equally well, and in the same way, all the magnetic phenomena: there remain however some difficulties to be solved; here is one of the main ones. Suppose that a steel blade is magnetized to saturation, if we consider the hypothesis of the two magnetic fluids, these two fluids will be separated, the boreal fluid[306] will be carried to the boreal part; the austral fluid[307] will be carried to the austral part of the blade; the magnetic center, if the blade is homogeneous, will be almost in the middle [of the blade]; suppose that we cut this blade into two equal parts, if the two fluids are separated, each part of the blade will have only one species of fluid, and consequently will not be susceptible of the same degree of magnetism, as a blade of the same size, which would be in its natural state: however experiment proves the contrary. This same difficulty arises against the system of Mr. OEpinus. How, for example, can the part of the blade, empty of magnetic fluid, be susceptible to the same degree of magnetism as such a blade in its natural state?

It seems, according to this experiment, that it is necessary to admit that the quantity of fluid, transported by magnetism, from one end of a steel blade to the other, is much less considerable than the total quantity of fluid that each part of this blade contains. Perhaps, moreover, the greater part of the magnetic elastic fluid is found in the blades in a state of fixity, and without any kind of action, as experiment has taught us that the fixed air was diffused in all bodies in much greater quantity than the elastic air which these bodies can contain?[308] It seems, in fact, probable that it exists, in all bodies and in their atmospheres,

[303] Anton Brugmans (1732-1789) and Johan Carl Wilcke (1732-1796). See [Brugmans, 1765], [Wilcke, 1766], [Gillmor, 1971a, pp. 180 and 215-216] and [Aepinus, 1979, pp. 200, 209-210 and 216-217].

[304] That is, two fluids of the same type repel each other, while fluids of opposite types attract each other.

[305] See Section 4.4 on page 42. See also footnotes 80 and 182 on pages 36 and 73, respectively.

[306] In the original: *fluide boréal*.

[307] In the original: *fluide austral*.

[308] In the original: "*comme l'expérience nous a appris, que l'air fixe étoit répandu dans tous les corps en*

parts which exert attractive forces, and others which exert repulsive forces, of which different combinations with other bodies, change and develop the action: evaporation of fluids, elasticity and the cohesion of solids, electricity, in short all chemical analyses, are perhaps only the result of different properties analogous to magnetism: but we are still very far from having lifted the veil which hides this part of physics from us.

88. After explaining these different hypotheses, we will present some probabilities on the cause of the diurnal variations:[309] let us gather some facts.

We have seen, in the theory of magnetism, that the action of each point of a magnetized blade tends to destroy the magnetism of the neighboring parts: hence it appears that the magnetic state is a forced state, and that the magnetic fluid makes an effort to spread evenly; this is what experiment proves, since we are obliged to renew from time to time the magnetism of the needles.

The globe of the Earth is a natural magnet, which, left to itself, seems to preserve its magnetic force: however, the variation of the declination tells us that the magnetic matter is there in a continual movement: thus, reasoning by analogy, the magnetic matter should have spread uniformly, and the magnetism of the Earth should have long since been annihilated.

There is therefore some cause which preserves or renews the magnetism of the Earth.

It is probable that the same cause which maintains the magnetism of the Earth, produces the movements of magnetic matter, [and] produces in the declinations, the annual variations and the diurnal variations.

The diurnal variation is nearly regular; the needle is currently in our climates in its greatest declination at one o'clock after noon: this declination decreases until seven or eight o'clock in the evening; it is almost stationary until eight o'clock in the morning; it grows more rapidly when the Sun approaches its meridian... These variations are not always equal. I found, in 1776, from the month of March until the end of July, that they were sometimes 18 or 20 minutes, but more often between 8 and 12'. I did not find the daily excursions regularly greater during one month than during the other: the greatest variations were observed during the equinox, and during the heat of the month of July: there were during these five months, three noticeable irregular variations; the first, on March 28, began to be seen around six o'clock in the evening; at ten o'clock the needle was in its greatest excursion, and its declination of 61' less than at one o'clock in the afternoon. The second was noticeable on April 5; we began to observe the direction of the needle at seven o'clock in the morning; the declination was 41' larger than usual; at 1 o'clock it had diminished by 30', and the needle was almost in the same direction in which it is usually observed at the same hour. The third variation was noticeable on April 8, it began to be observed at 5 o'clock in the evening, it marked its greatest excursion at 9 o'clock and at that time the declination was 44' less than at 1 o'clock in the afternoon; at midnight the needle appeared in its usual direction; on the same day we could distinguish an aurora borealis in the northwestern part. During the first two observations, the weather was overcast and stormy. In the irregular variations, the needle is continually in motion, its oscillations are sometimes of half an hour, and of a quarter of a degree.

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beaucoup plus grande quantité, que l'air élastique que ces corps peuvent contenir?" Joseph Black (1728-1799) was a Scottish physicist and chemist. He discovered carbon dioxide, which he called "fixed air", because it can be absorbed or fixed by strong bases.

[309] Diurnal variations of the magnetic declination of a compass.

The influence of the aurora borealis on the direction of the needle has long been observed; it is even very probable that the fluid, which forms this meteor, is the same as the magnetic fluid:[310] we find, on this subject, the most ingenious ideas in the [work] *Loix du Magnétisme* of Mr. le Monnier;[311] it will be easy to adapt the following theory to it.

89. If we examine the regular diurnal variations of the needle, we will see that they have a revolution adjusted like that of the Sun. It is therefore an action due to this celestial body which produces these periodic variations. Some authors have claimed that these variations were the effect of the solar heat, which would destroy the magnetism of the part of the Earth which was opposite to it. They based themselves on the fact that a magnetized blade loses its magnetic virtue in the fire: but, even if this opinion would explain the diurnal variation, it would not be admissible; because a cause which tends continually to destroy the magnetism of the Earth, would not have left to it for a long time any magnetic quality.[312]

If it is not the heat of the Sun which produces the diurnal variations; if, however, this effect is due to this celestial body, the Sun must act on the terrestrial globe, as a magnet acts on another magnet.

Here is, according to this idea, how it seems that all magnetic phenomena can be explained: the solar atmosphere, known under the name of zodiacal light, will be nothing else than a magnetic fluid; this fluid, admitting the system of Mr. OEpinus, and it will be easy to adapt the same reasonings to any other system, will act on the parts of the Earth to drive out the magnetic fluid which is contained therein, like the positive pole of a magnet, tends to expel the magnetic fluid from the point of a steel blade where it is applied.

But the action of this fluid will be all the greater, as its density will be greater and its distance less; now this density decreasing as one moves away from the Sun, it follows that this fluid will act more strongly on the part of the hemisphere illuminated by the Sun, than on the opposite hemisphere; that its action will be greater in the perigee than in the apogee: thus, during the winter, the southern part of the Earth must be emptied of magnetic matter: during the summer, the solar atmosphere will produce an opposite effect; but the Sun being then at its apogee, this action will be smaller than during winter: whence it must result that the Earth will be magnetized positively in the boreal part, and negatively in the austral part: as the apogee will change, there will be a revolution in the position of the magnetic fluid; but this movement is too slow for the effect to be very perceptible, since the declination of the magnetic needle is observed with exactness.

It is now easy to explain the diurnal variations: the magnetic fluid, spread over the surface of the Earth, acts by its attractive force on the points of the needle negatively magnetized, and by a repulsive force on the parts of the needle positively magnetized. Thus, the direction of the needle will be determined by all these forces, and this action will follow a law of the density of the fluid in each point of the Earth,[313] and of its distance from the needle, on the different points of which it acts.

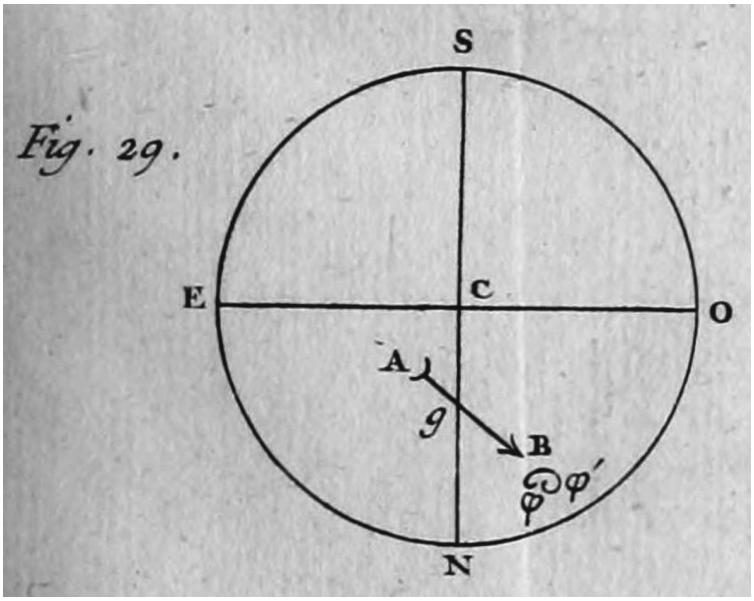
In *Figure 29*, the circle $ESON$ represents the globe of the Earth, EO the equator, and SN the meridian of the place, where the observation is made at g.

[310] In the original: *fluide aimantaire*.

[311] See footnote 172 on page 68.

[312] In other words, the Earth would have long ago lost its magnetic properties due to this cause.

[313] That is, the magnetic force exerted by each point on the Earth's surface is proportional to the density of magnetic fluid at that point.



The direction of the needle AB, placed here on the boreal hemisphere, will be determined by the magnetic action of all the parts of the Earth: thus, if the united action of all these forces, determines the compass to form an angle Bgn with the meridian, it will be because the magnetic fluid will be denser in the part OCN, than in the quarter hemisphere ECN: we can do *vice versa* a similar reasoning with respect to the austral part ESO of the Earth.

If E represents the East, and O the West, as the Sun approaches the meridian of the place where the compass is placed, it will expel the magnetic fluid from the part ECN, into the part OCN: thus, the density of the fluid will increase in the western part, and will decrease in the eastern part: so, if φ was the center of all the forces which solicited the needle when the Sun was at E, this center φ will move towards the West at φ', and the declination will increase, until the Sun is placed in the same meridian as this center φ'; and as the declination at present in our climates carries the needle towards the West, it must happen that the declination will still increase some time after the passage of the Sun in the meridian of the place where the observation is made. When the Sun will be to the West of the meridian where the center φ is placed, it is clear that in this case the declination must decrease; whence it must follow the periodic movement of the diurnal variations.

If the magnetic fluid were spread symmetrically around the pole of the world, if the Sun always traversed the same circle, and at the same distance from the Earth, the diurnal movement of the declination of the needle would be perfectly regular, and the center φ would run through an exactly closed oval. But [due to] the different density of this fluid in the eastern and western parts; the change of position of the Sun must entail, either that this center will not come to the point φ after a diurnal revolution, or that it will be carried further; so that this center will travel every day, not in an exactly closed oval, but in a spiral line; which will produce the annual movement, observed in the declination for more than a century.

Chapter 6

Remarks on Coulomb's 1777 Work

A. K. T. Assis

I discuss here in more detail some aspects of this fundamental work that Coulomb presented in 1777 to the French Academy of Sciences and that was published in 1780.

6.1 Coulomb's First Fundamental Principle

Coulomb described in Subsection 5.0.2 his first fundamental principle as follows:

> If, after having suspended a needle by its center of gravity, we move it away from the direction that it takes naturally, it is always brought back by forces which act parallel to this direction and which are different for different points along the needle, but which are the same for each of these points in particular, in whichever orientation the needle is placed in relation to its natural direction; so that a magnetized needle always experiences the same action, in any position, due to the magnetic forces of the Earth.

I illustrate this principle in Figure 6.1 assuming a particular case in which the magnetized needle can rotate in a horizontal plane when supported by a vertical axis passing through its center. Let NS be a uniformly magnetized needle of length ℓ that can rotate about a vertical axis passing through its fulcrum C in the middle of the needle. The dashed line AB indicates the local magnetic meridian, that is, the direction naturally pointed by the needle at that location, Figure 6.1 (a). Let us assume that the North pole of the needle points to B. Point m indicates an arbitrary point on the needle.

In Figure 6.1 (b) we see the needle rotated around point C at an angle θ_1 with respect to the magnetic meridian. By the first fundamental principle, the magnetic force exerted by the Earth on the point m of the needle is indicated by the arrow F_m, parallel to the direction AB.

Also by the first fundamental principle, as shown in Figure 6.1 (c), when the needle is rotated through another angle θ_2 relative to the magnetic meridian, the force acting on point m continues with the same intensity and direction as in the case of Figure 6.1 (b).

Let us now assume another point q of the needle, as shown in Figure 6.2 (a). When the needle is displaced by an angle θ_1 from the magnetic meridian, the force F_q acting on point q is still parallel to the direction AB of the magnetic meridian, but its magnitude may be

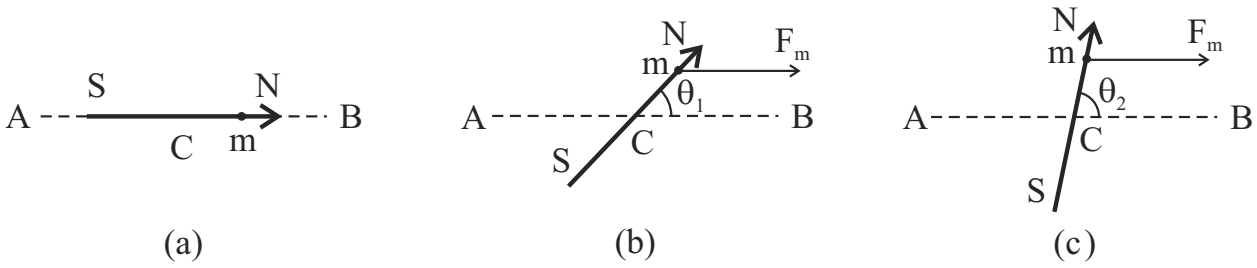


Figure 6.1: (a) Direction AB indicates the magnetic meridian. (b) The force F_m acting on a point m of the magnetized needle is parallel to the direction AB. (c) The intensity of this force F_m does not depend on the angle θ of the needle with respect to the meridian.

different from the force acting on the point m. This can be seen by comparing Figures 6.1 (b) and 6.2 (b).

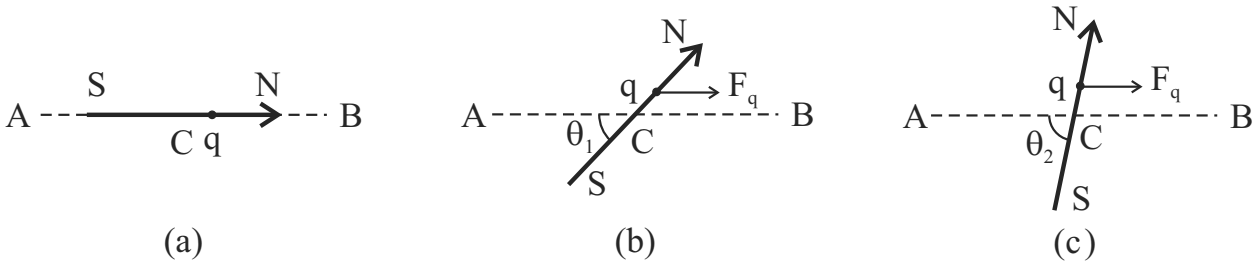


Figure 6.2: (a) Another point q on the needle. (b) The force F_q is also parallel to the magnetic meridian, but can have an intensity which is different from the force F_m. (c) The force F_q also does not depend on the angle θ.

When the needle is displaced by another angle θ_2 with respect to the magnetic meridian, the intensity and direction of F_q remain the same, as can be seen by comparing cases (b) and (c) of Figure 6.2.

6.2 Coulomb's Second Fundamental Principle

Coulomb's second fundamental principle was presented in Subsection 5.0.3:

> The magnetic forces of the terrestrial globe that attract the different points of a compass needle act in two opposite ways. The North part of the needle is attracted towards the North pole of the magnetic meridian. The South part of the needle is attracted in the opposite direction. Whatever may be the law according to which these forces act, the sum of the forces which attract the needle towards the North pole is exactly equal to the sum of the forces which attract the South pole of the needle in the opposite direction.

An illustration of this second principle can be found in Figure 6.3 assuming a particular case in which the needle can rotate in a horizontal plane when supported by a vertical axis passing through its center. In (a) we have a uniformly magnetized needle NS pointing along the magnetic meridian AB. Points 1 and 2 are equally distant from its center C, with point 1 being on the austral or southern part of the needle, while point 2 is on the boreal or northern part of the needle.

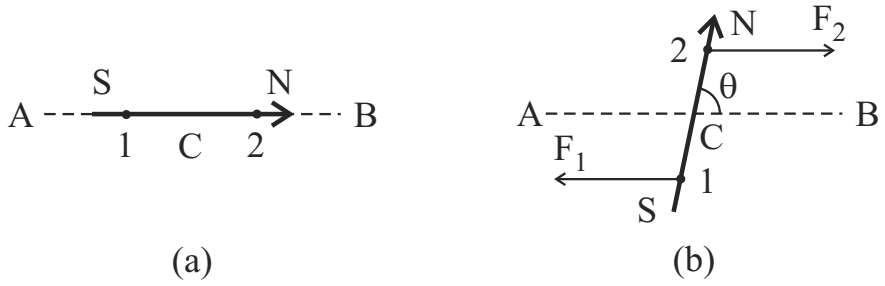


Figure 6.3: (a) Direction AB represents the magnetic meridian. Points 1 and 2 are equally distant from the center C of the uniformly magnetized needle. (b) Forces F_1 and F_2 acting on points 1 and 2.

When the needle is displaced in the horizontal plane by an angle θ with respect to the magnetic meridian, point 1 experiences a force F_1 parallel to the meridian (by the first principle) pointing to the South side of the magnetic meridian (that is, from B to A), while point 2 experiences a force F_2 also parallel to the magnetic meridian, but pointing to the North side of the magnetic meridian (that is, pointing from A to B), as shown in Figure 6.3. Furthermore, these two forces have the same intensity, although they act in opposite directions.

It is also possible to think of the force F_1 as the resultant force acting on the southern part CS of the needle, while F_2 would be the resultant force acting on the northern part CN of the needle. These forces are parallel to the magnetic meridian AB, have the same intensity and opposite directions, acting along parallel lines separated from each other. The two together do not exert a net force on the needle, they exert only a net torque on the needle, as they constitute a couple. That is, these two forces reduce to a couple, two equal and opposite forces whose lines of action are parallel but not congruent.

A couple is a pair of forces, equal in magnitude, oppositely directed, and displaced by a perpendicular distance d. The simplest kind of couple consists of two equal and opposite forces of magnitude F, whose lines of action do not coincide. The forces have a turning effect or moment called a torque about an axis which is normal (perpendicular) to the plane of the forces. The SI unit for the torque of the couple is Newton-meter. If the two forces are F and $-F$, then the magnitude of the torque τ is given by $\tau = Fd$.

6.3 Meaning Given by Coulomb to the Time Interval of a Total Oscillation

Article 11 of this work of 1777 appears on page 64 of this English translation. To understand this Article, I begin by quoting it in full in the original French language:[314]

> 11. Lorsque, dans l'équation précédente, l'on suppose $R = 0$, l'on a l'équation approchée $uu = \frac{\int \phi\mu r}{\int \mu r^2}(2BS - SS)$; d'où $\left(\frac{\int \phi\mu r}{\int \mu r^2}\right)^{1/2} dt = dS : (2BS - SS)^{1/2}$; or $\int dS : (2BS - SS)^{1/2}$ est l'angle dont le rayon est B; & S le sinus-verse; quantité égale à 90°, lorsque $S = B$; ainsi, en nommant T le temps d'une oscillation totale, l'on aura $T\left(\frac{\int \phi\mu r}{\int \mu r^2}\right)^{1/2} = (180)°$.

[314] [Coulomb, 1780, p. 176].

I will put here some comments that help to understand this sentence. I also show how to get the integrated result of the function presented by Coulomb.

First, what Coulomb here calls $(180)^\circ$ is the ratio of the circumference to the diameter of a circle.[315] Nowadays this ratio is represented by the letter $\pi = 3.14159....$ So I am going to replace 180° with π, while 90° will be replaced with $\pi/2$.

The letter ϕ used by Coulomb was replaced by Potier in the 1884 reprint of Coulomb's works with the letter φ. In this English translation I am adopting Potier's nomenclature.

Here is the translation we adopted for that paragraph:

> 11. When, in the previous equation, we suppose $R = 0$, we have the approximate equation
>
> $$u^2 = \frac{\int \varphi\mu r}{\int \mu r^2}\left(2BS - S^2\right) \ ;$$
>
> from where
>
> $$\left(\frac{\int \varphi\mu r}{\int \mu r^2}\right)^{1/2} dt = \frac{dS}{\sqrt{2BS - S^2}} \ ;$$
>
> now
>
> $$\int \frac{dS}{\sqrt{2BS - S^2}}$$
>
> is the angle whose radius is B, and S the sine-verse; quantity equal to $\pi/2$, when $S = B$; thus, by naming T the time of a total oscillation, we will have
>
> $$T\left(\frac{\int \varphi\mu r}{\int \mu r^2}\right)^{1/2} = \pi \ .$$

In this Article 11 Coulomb called T the "time of a total oscillation". What he called the time of a total oscillation is when the pole n of the magnetized needle ns goes from N to N' in Figure 6.4:

Line ACB in Figure 6.4 represents the magnetic meridian. The pole n of the magnetized needle ns is moved to point N and released from rest. It then oscillates between points N and N'. Coulomb denominates the angle NCn in this Figure by the capital letter S, while letter B denotes the angle NCB. We will call $T_{1/4}$ the time interval for the needle to leave its initial angle $S = 0$ at rest, until it arrives along the magnetic meridian with $S = B$. That is, $T_{1/4}$ is the time interval for the extremity n of the needle to start at point N and arrive at point B in this Figure. The equation we are going to integrate was provided by Coulomb in this Article 11, namely (replacing Coulomb's letter ϕ with the letter φ):

$$\left(\frac{\int \varphi\mu r}{\int \mu r^2}\right)^{1/2} dt = \frac{dS}{\sqrt{2BS - S^2}} \ . \tag{6.1}$$

[315] See Alfred Potier's Introduction to Coulomb's works, [Potier, 1884, pp. x and 13], as well as page 32 of this English translation.

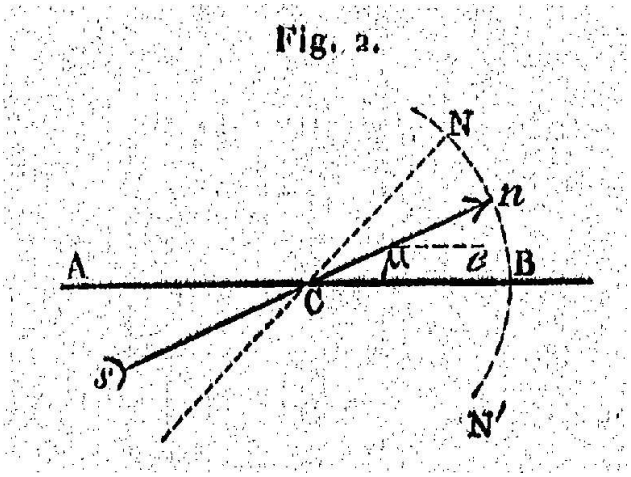


Figure 6.4: Figure 2 from Coulomb's 1777 work.

Integrating this equation with time t going from 0 to $T_{1/4}$ and with angle S going from 0 to B we get:

$$\int_{t=0}^{T_{1/4}} \left(\frac{\int \varphi\mu r}{\int \mu r^2}\right)^{1/2} dt = \int_{S=0}^{B} \frac{dS}{\sqrt{2BS - S^2}} \,. \tag{6.2}$$

The left side of this equation is given by:

$$\int_{t=0}^{T_{1/4}} \left(\frac{\int \varphi\mu r}{\int \mu r^2}\right)^{1/2} dt = \left(\frac{\int \varphi\mu r}{\int \mu r^2}\right)^{1/2} \int_{t=0}^{T_{1/4}} dt = \left(\frac{\int \varphi\mu r}{\int \mu r^2}\right)^{1/2} T_{1/4} \,. \tag{6.3}$$

The right side is given by:

$$\int_{S=0}^{B} \frac{dS}{\sqrt{2BS - S^2}} = \int_{S=0}^{B} \frac{dS}{\sqrt{B^2 - (S-B)^2}} = \int_{S=0}^{B} \frac{\frac{dS}{B}}{\sqrt{1 - \left(\frac{S-B}{B}\right)^2}} \,. \tag{6.4}$$

Defining the magnitude $\gamma = (S - B)/B$ we get $d\gamma = dS/B$ and:

$$\begin{aligned} \int_{S=0}^{B} \frac{dS}{\sqrt{B^2 - (B-S)^2}} = \int_{S=0}^{B} \frac{d\gamma}{\sqrt{1-\gamma^2}} = [\arcsin\gamma]_{S=0}^{B} = \left[\arcsin\frac{S-B}{B}\right]_{S=0}^{B} \\ = \; [\arcsin 0 - \arcsin(-1)] = \left(0 - \frac{-\pi}{2}\right) = \frac{\pi}{2} \,. \end{aligned} \tag{6.5}$$

From Equations (6.3) and (6.5) we get:

$$\left(\frac{\int \varphi\mu r}{\int \mu r^2}\right)^{1/2} T_{1/4} = \frac{\pi}{2} \,. \tag{6.6}$$

Twice $T_{1/4}$ gives the time interval for what Coulomb calls the "time of a total oscillation of the needle" to occur, which he will later on represent by the letter T:

$$T_{Coulomb} = 2T_{1/4} = \pi\sqrt{\frac{\int \mu r^2}{\int \varphi\mu r}} \,. \tag{6.7}$$

This result of the integration of Equation (6.1) was written by Coulomb in Article 11 as follows:

$$T\left(\frac{\int \phi\mu r}{\int \mu r^2}\right)^{1/2} = (180)^\circ \ . \tag{6.8}$$

Nowadays, the time interval between departure and return to the same point is called the complete period of oscillation. This is the time interval for the pole n of the needle to start from point N, arrive at point N', and return to point N. This time interval is given by the quadruple of $T_{1/4}$. This complete period of oscillation is also represented today by the letter T:

$$T_{modern} = 4T_{1/4} = 2\pi\sqrt{\frac{\int \mu r^2}{\int \varphi\mu r}} \ . \tag{6.9}$$

Gillmor mentioned the following related to this topic:[316]

> Coulomb employs the French definition of the period equal to π radians (that is, equal to one-half the English period).

Elsewhere in his book Gillmor stated that:[317]

> Note that in the French system utilized by Coulomb, the period of an oscillation is defined as π radians or 180°, rather than 360°.

Despite these words by Gillmor, it should be emphasized that in his works Coulomb does not use the word *period* (French: *période*). The terms Coulomb uses in his works are *temps d'une oscillation* (time of one oscillation), *temps d'une oscillation totale* (time of a total oscillation), *temps d'une oscillation entière* (time of a complete oscillation), etc.[318]

[316] [Gillmor, 1971a, p. 154].
[317] [Gillmor, 1971a, Note on p. 180].
[318] See, for instance, footnotes 160, 199 and 332 on pages 64, 85 and 149, respectively.

Chapter 7

Theoretical and Experimental Research on the Force of Torsion, and on the Elasticity of Metal Wires

Coulomb[319]

Application of this theory to the use of metals in the Arts and in various physics experiments: Construction of different kinds of torsion balances, for measuring the smallest force levels. Observations on the laws of elasticity and of coherence.

Read in 1784.

7.1 I

This Memoir has two objectives: the first, to determine the elastic force of torsion of filaments of iron and of brass[320] as a function of their length, their thickness, and their degree of tension. I have already had need, in a Memoir on magnetized needles printed in the *neuvième volume des Savans étrangers*,[321] to determine the force of torsion of hair and of silk; but I have never occupied myself with filaments of metal, because the nature of my research led me to choose the most flexible suspensions for the same force, and I have found that the filaments of silk had incomparably more flexibility than filaments of metal. The second objective of this Memoir is to evaluate the imperfection of the elastic reaction [inelastic behavior] of filaments of metal, and to examine the consequences that we can deduce about the laws of coherence[322] and elasticity of bodies.

[319] [Coulomb, 1787], with English translation in [Coulomb, 2012a] and [Coulomb, 2012b], and Portuguese translation in [Assis, 2022]. This work was read in 1784 to the French Academy of Sciences and published in 1787. Recently Heering replicated some of Coulomb's experiments on the torsion of metal wires, [Heering, 2006b] and [Heering, 2006a]. See also [Oliveira and Pisano, 2022].

[320] In the original: *laiton*. Brass is an alloy of copper and zinc.

[321] [Coulomb, 1780]. This Memoir is translated in Chapter 5.

[322] In the original: *cohérence*. This word can be translated as cohesion or coherence, [Gillmor, 1971a, pp. 126 and 150].

7.2 II

The method to determine the force of torsion, via experiment, consists of suspending a cylindrical weight by a filament of metal in a manner such that its axis is vertical, in the direction of the filament of suspension. As long as the filament of suspension is not twisted, the weight will remain at rest; but if one turns the weight about its axis, the filament twists, and will attempt to re-establish itself in its natural situation; if one lets go of the weight, it will oscillate for a longer or shorter length of time, accordingly as the elastic reaction in torsion is more or less perfect. If in this type of test, one carefully observes the duration of a fixed number of oscillations, it will be easy to determine, from the formulae of oscillatory movement, the force of reaction of torsion which produces these oscillations. Thus, in varying the weight[323] of the suspended load,[324] the length and the thickness of the filament of suspension, we can expect to determine the laws of reaction of torsion with respect to the tension, the length, the thickness, and the nature of the filaments.

7.3 III

If the filament of metal be perfectly elastic, and the resistance of the air does not alter the amplitude of oscillations, the weight supported by the filament of metal, once set in motion, will oscillate until one [forcibly] stops it. The diminution of the amplitudes of oscillations can be attributed to air resistance and to the imperfection of the elasticity of torsion; thus, in observing the successive diminution of the amplitude of each oscillation, and in taking out the part of the alteration that it is necessary to attribute to air resistance, we could, by means of the formulas of oscillatory movement, applied to these tests, determine according to which laws this force of elasticity of torsion is altered.

7.4 IV

This Memoir is divided into two Sections; in the first, we will determine the law of the force of torsion, in supposing the forces of torsion are proportional to the angle of twist, a supposition conforming to experiment when we do not give too great an amplitude to the angle of twist: we will give several applications of this theory to practice.

In the second Section, we will explore, by experiment, how the laws of elastic force of torsion is altered in large oscillations: we will make use of this research to determine the laws of coherence and of elasticity of metals and of all solid bodies.

[323] In the original: *pesanteur*.
[324] In the original: *poids*. This word can also be translated as weight.

7.5 V. First Section. Formulas of Oscillatory Movement, in Supposing the Reaction of the Force of Torsion Proportional to the Angle of Twist, or Altered by a Very Small Term

A cylindrical body B (Figure 1, Number 1) is supported by a filament RC, in a manner such that the axis of the cylinder is vertical, in line with the prolongation of the filament of suspension; we turn this cylinder about its axis, without disturbing this axis from the vertical; it is necessary to determine, in assuming the force of torsion proportional to the angle of twist, the formulas of oscillatory motion.

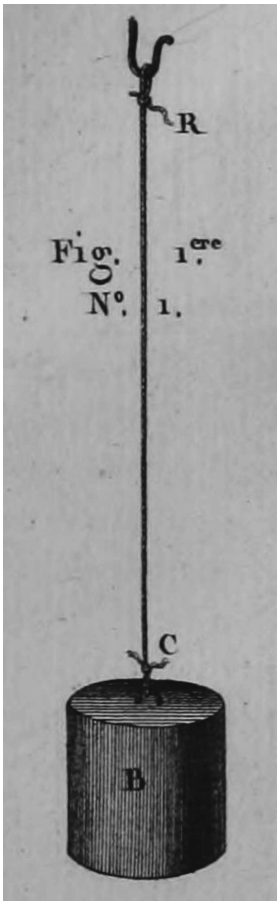
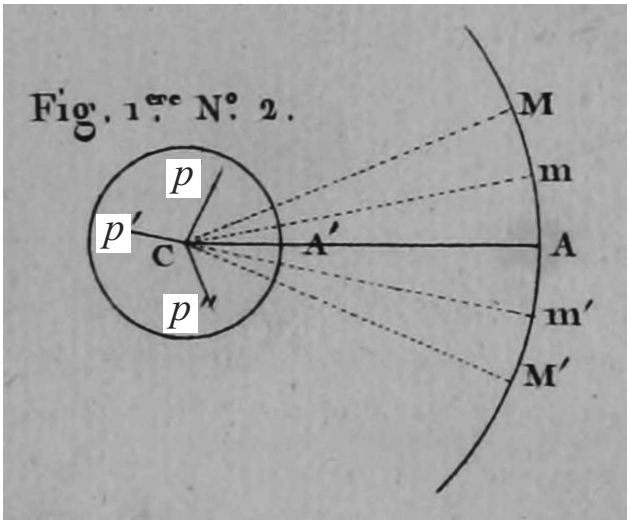


Figure 1, Number 2, shows a horizontal section of the cylinder; all the [mass] elements of the cylinder are projected on this circular section at p, p', p'', etc.[325]

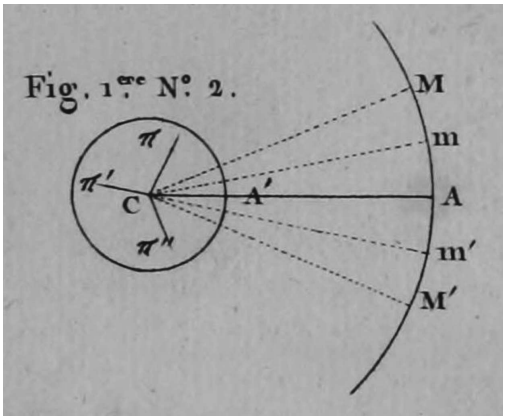
[325] In the original article Coulomb represented these material elements by the letters π, π' and π'', see [Coulomb, 1787, p. 231] and the Figure in the present footnote:



We assume that the starting angle of twist[326] is $ACM = A$, and that after time t, this angle is ACm, or that it is diminished by the angle $MCm = S$, so that

$$ACm = (A - S) \ .$$

Since we suppose the force of torsion is proportional to the angle of twist, the *moment*[327] of this force will be represented by $n(A-S)$, n being a constant coefficient, whose value will depend on the nature of the filament of metal, on its length and on its thickness.[328] If we call v the velocity of any point p, after a time t, when the angle of twist is ACm, we will have, by the principles of dynamics,[329]



I am here following Potier and representing these points by p, p' and p'' to avoid confusion with the angle of π radians, see [Potier, 1884, p. x] and the page 32 of this English translation.

[326]The body is released from rest at this initial angle.

[327]In the original: *momentum*. This word can be translated as moment, torque or moment of force. See footnote 150 on page 60.

[328]The magnitude n is the torsional stiffness of the wire. It has dimensions $[force] \cdot [length]/[angle]$, so that its SI units are $N \cdot m/rad$. When the wire is attached by the upper end and its lower end is rotated relative to an inertial frame of reference at an angle θ around the vertical axis, it exerts a reactive torque $\tau = n\theta$, where $\theta = 0$ is the situation where the wire is not twisted.

[329]In the original paper the next equation appears as follows, [Coulomb, 1787, p. 232]:

$$n(A - S)dt = \int \pi r dv \ .$$

$$n(A-S)dt = \int prdv \ ,$$

where r is the distance Cp from point p to the axes of rotation G.[330]

But if the radius CA' of the cylindrical weight $= a$, and the velocity of point A' on the circumference of the cylinder, be at the end of time t, represented by u, we will have

$$v = \frac{ru}{a} \ ,$$

from which it results

$$n(A-S)dt = du\frac{\int pr^2}{a} \ ;$$

and as $dt = adS/u$, we will have for the integrated equation

$$a(2AS - S^2) = \frac{u^2}{a^2}\int pr^2 \ ,$$

from which we draw

$$dt = dS\sqrt{\frac{\int pr^2}{n(2AS-S^2)}} \ .$$

But $dS/\sqrt{(2AS-S^2)}$ represents an angle of which A is the radius and S the versed sine,[331] which vanishes when $S = 0$, and which becomes equal to 90 degrees when $S = A$.

Thus the time of a complete oscillation will be[332]

$$T = \pi\sqrt{\frac{\int pr^2}{n}} \ .$$

In this English translation we wrote this equation as

$$n(A-S)dt = \int prdv \ .$$

The letter p represents the infinitesimal element of mass located at a distance r from the axis of rotation. Let δ be the volumetric density of mass and dV the infinitesimal volume occupied by this element of mass. In this case Coulomb's letter p can be replaced by $dm = \delta dV$, where dm is the infinitesimal mass of that element. This integration should be understood as being a three-dimensional integration throughout the volume of the cylinder.

[330] Probably the axis of rotation should be represented by the letter C instead of G, since point G does not appear in Figure 1.

[331] Original sentence, [Coulomb, 1787, p. 232]: *"Mais* $\frac{dS}{\sqrt{(2AS-SS)}}$ *représente un angle dont A est le rayon & S le sinus verse"*.

Potier, [Potier, 1884, p. 69], wrote this sentence as: *"Mais* $\frac{dS}{\sqrt{(2AS-S)^2}}$ *représente un angle dont* $\frac{S}{A}$ *est le sinus verse*". That is: "But $\frac{dS}{sqrt(2AS-S)^2}$ represents an angle of which $\frac{S}{A}$ is the versed sine".

It seems that due to a typographical error, the integral symbol was missing before $dS/\sqrt{(2AS-S^2)}$. The sentence should read: "But $\int dS/\sqrt{(2AS-S^2)}$ represents an angle of which A is the radius and S the versed sine". A discussion of how to arrive at this result and its integrated value can be found in Section 9.1 on page 187.

[332] In the original: *Ainsi le temps d'une oscillation entière sera*. A complete oscillation here is but one half of what we today call a full cycle.

7.6 VI

In order to compare the force of torsion with the force of gravity in a [simple] pendulum, it is necessary to remember that in the pendulum the time T of a complete oscillation

$$T = \pi\sqrt{\frac{\lambda}{g}} ,$$

where λ is the length of the pendulum and g the force of gravity.[333] Thus a pendulum which is isochronous to the oscillations of the cylinder gives

$$\frac{\int pr^2}{n} = \frac{\lambda}{g} .$$

From this formula we will easily draw the value of n from the experiment, since the dimensions of the cylinder or of the weight are given, and so too the time of one oscillation, which determines the value of λ.

If we wish then to search for a weight Q which, acting at the extremity of the lever b, would have a moment equal to the moment of the force of torsion, when the angle of twist is $(A - S)$, it requires setting $Qb = n(A - S)$.

7.7 VII

It is necessary now to search for a cylinder such that the value of $\int pr^2$, we will find equal to $\pi\delta La^4/2$, where π is the ratio of the circumference to the diameter, δ is the [volume] density of the cylinder, [L its length] and a its radius.[334]

But as the mass M of the cylinder is $= \pi\delta La^2$, we have

$$\int pr^2 = \frac{Ma^2}{2} ,$$

[333] In the original: *où g est la force de gravité.* Today we call g the gravitational force per unit mass, that is, the free fall acceleration due to the Earth's gravity. As will be seen in Section 9.1 on page 187, what Coulomb calls the time for a complete oscillation is half of what is now called the period of one complete swing of the pendulum.

[334] In the original, [Coulomb, 1787, p. 233]:

> Il faut actuellement chercher, pour un cylindre, la valeur de $\int \pi r^2$, que l'on trouvera égale à $\frac{\varphi\delta L\cdot a^4}{4}$, où φ est le rapport de la circonférence au rayon, δ est la densité du cylindre & a son rayon.

Here I am following Potier, [Potier, 1884, p. 70], in making several substitutions in this sentence. First of all Coulomb's letter π has been replaced by p, with this letter p representing an element of mass. The magnitude φ, which for Coulomb indicates the ratio of the circumference to the radius, is being replaced here by 2π with $\pi = 3.14159....$ The same substitutions are made in the other formulas in this work.

Let p be a mass element located at a distance r from the axis of the cylinder of mass M, length L, radius a and volume mass density $\delta = M/(\pi a^2 L)$. I will use here cylindrical coordinates (r, φ, z) with the polar angle φ going from 0 to 2π. The mass element p can be written as $p = dm = \delta dV$, where $dV = rd\varphi drdz$ is a volume element. The moment of inertia I of this cylinder with respect to its axis of symmetry is given by:

$$I = \sum_i m_i r_i^2 = \delta \int_{r=0}^{a}\int_{\varphi=0}^{2\pi}\int_{z=0}^{L} r^3 d\varphi drdz = \pi\delta L\frac{a^4}{2} ,$$

as obtained by Coulomb.

and consequently

$$T = \pi\sqrt{\frac{Ma^2}{2n}} \ ;$$

in comparing this, as in the *preceding Section*, with the isochronous pendulum, there results[335]

$$\frac{\lambda}{g} = \frac{Ma^2}{2n} \ ,$$

and as gM is the weight P of the cylinder, we will have

$$n = \frac{Pa^2}{2\lambda} \ ,$$

which gives a very simple formula for determining n from the experiment.[336]

7.8 VIII

If the force of torsion, which we have taken equal to $n(A-S)$, be altered by a quantity R,[337] the formula of oscillatory motion would give as a law

$$[n(A-S) - R]dt = du\frac{1}{a}\int pr^2 \ ;$$

and putting as before, in place of dt, its value adS/u, we will have for the integration

$$n(2AS - S^2) - 2\int RdS = \frac{u^2}{a^2}\int pr^2 \ .$$

If we wish to extend this integration to a complete oscillation, it requires dividing it into two parts, the first from M until A, where the force of torsion accelerates the velocity u, while the force of retardation diminishes [the velocity]; the second from A unto M',[338] where all the forces together retard the motion.

EXAMPLE I. Suppose $R = \mu(A-S)^m$, we will have, for the state of movement in the first portion MA,

$$n(2AS - S^2) + \frac{2\mu(A-S)^{m+1}}{m+1} - \frac{2\mu A^{m+1}}{m+1} = \frac{u^2}{a^2}\int pr^2 \ ;$$

thus when the angle of twist will be null, or that $(A-S) = 0$, we will have

$$nA^2 - \frac{2\mu A^{m+1}}{m+1} = \frac{U^2}{a^2}\int pr^2 \ .$$

[335] That is, by comparing this magnitude T with an isochronous pendulum that performs its oscillations in the same time.

[336] See footnote 328 on page 148.

[337] Coulomb will consider this torque R to be a frictional torque.

[338] Due to a typo, in the original text the letter M appears here instead of M'. I have made the correction and used M' instead of M.

Let us consider now the other part of the movement from A to M', and suppose the angle $AGm' = S'$, we will find, in calling U the velocity of point A;

$$\frac{nS'^2}{2} + \frac{\mu S'^{m+1}}{m+1} = \frac{U^2 - u^2}{2a^2} \int pr^2 \ .$$

Substituting in place of U^2 its value

$$\frac{a^2}{\int pr^2} \left(nA^2 - \frac{2\mu A^{m+1}}{m+1} \right) \ ,$$

we will have for the total integral, when the velocity becomes null, or when the oscillation will be completed,

$$(A - S') = \frac{2\mu}{n(m+1)} \frac{A^{m+1} + S'^{m+1}}{A + S'} \ ;$$

and if the retarding forces are such that at each oscillation, the amplitude be a little bit reduced,[339] we will have approximately for the value of $(A - S')$,

$$(A - S') = \frac{2\mu A^m}{n(m+1)} \ ;$$

and if this quantity $(A - S')$ be so small so that it can be treated as an ordinary differential, we would have then, for a number Z of oscillations,

$$\frac{2\mu}{n(m+1)} Z = \frac{1}{m-1} \left(\frac{1}{S^{m-1}} - \frac{1}{A^{m-1}} \right) \ ,$$

where S represents this that becomes A after a number of oscillations Z. Thus we will have

$$S = \frac{1}{\left[\frac{2\mu(m-1)}{n(m+1)} Z + \frac{1}{A^{m-1}} \right]^{\frac{1}{m-1}}} \ ,$$

which determines the value S, after any number of oscillations Z.

EXAMPLE II. If

$$R = \mu(A - S)^m + \mu'(A - S)^{m'} \ ,$$

with μ' and m' having other values than μ and m, we will have, following the procedure of the last example

$$n(A - S) = \frac{2\mu}{m+1} \frac{A^{m+1} + S^{m+1}}{A + S} + \frac{2\mu'}{m'+1} \frac{A^{m'+1} + S^{m'+1}}{A + S} \ ;$$

and if the retarding force is much less than the force of torsion, we will have for the value approached,

$$n(A - S) = 2\mu \frac{A^m}{m+1} + \frac{2\mu' A^{m'}}{m'+1} \ .$$

In general, if

$$R = \mu(A - S)^m + \mu'(A - S)^{m'} + \mu''(A - S)^{m''} + \ldots \ ,$$

we will always have for an oscillation, in supposing R much smaller than the force of torsion,

$$n(A - S) = \frac{2\mu A^m}{m+1} + \frac{2\mu' A^{m'}}{m'+1} + \frac{2\mu'' A^{m''}}{m''+1} + \ldots \ .$$

[339] That is, such that $S' = A - \varepsilon$, with $\varepsilon \ll A$.

7.9 IX. Experiments to Determine the Laws of the Force of Torsion. Preparation

On a small, flat board KA, supported upon four feet, [see Figure 2,] raise a post ABD: mount on the post AB, at four feet high [130 cm], the horizontal traverse DE, slid up and down on the post and fixed to it by means of a screw E; the cylinder or the weight P, carries at its top, along the prolongation of its axes, an end of a needle b, fixed to this cylinder. This needle is fixed by the lower part of a double clasp[340] a, which is tightened by some screws; the upper part of this clasp holds the lower extremity of the filament of suspension; the lower part of this same clasp holds the extremity of the needle fixed to the cylinder. The top end of the filament of suspension is held by another clasp g, attached to the traverse DE. On the surface AK, which serves as a base for the apparatus, we place a circle divided into degrees, whose center C should be located along the prolongation of the axes of the cylinder: we attach at the bottom of the cylinder an index eo, whose extremity o points to the divisions of the circle.

[340] In the original: *double pince*. The word "pince" can be translated as clasp, clamp, pincer, chuck or gripper. In the case of a double clasp, we have a piece composed of two articulated parts intended to hold objects.

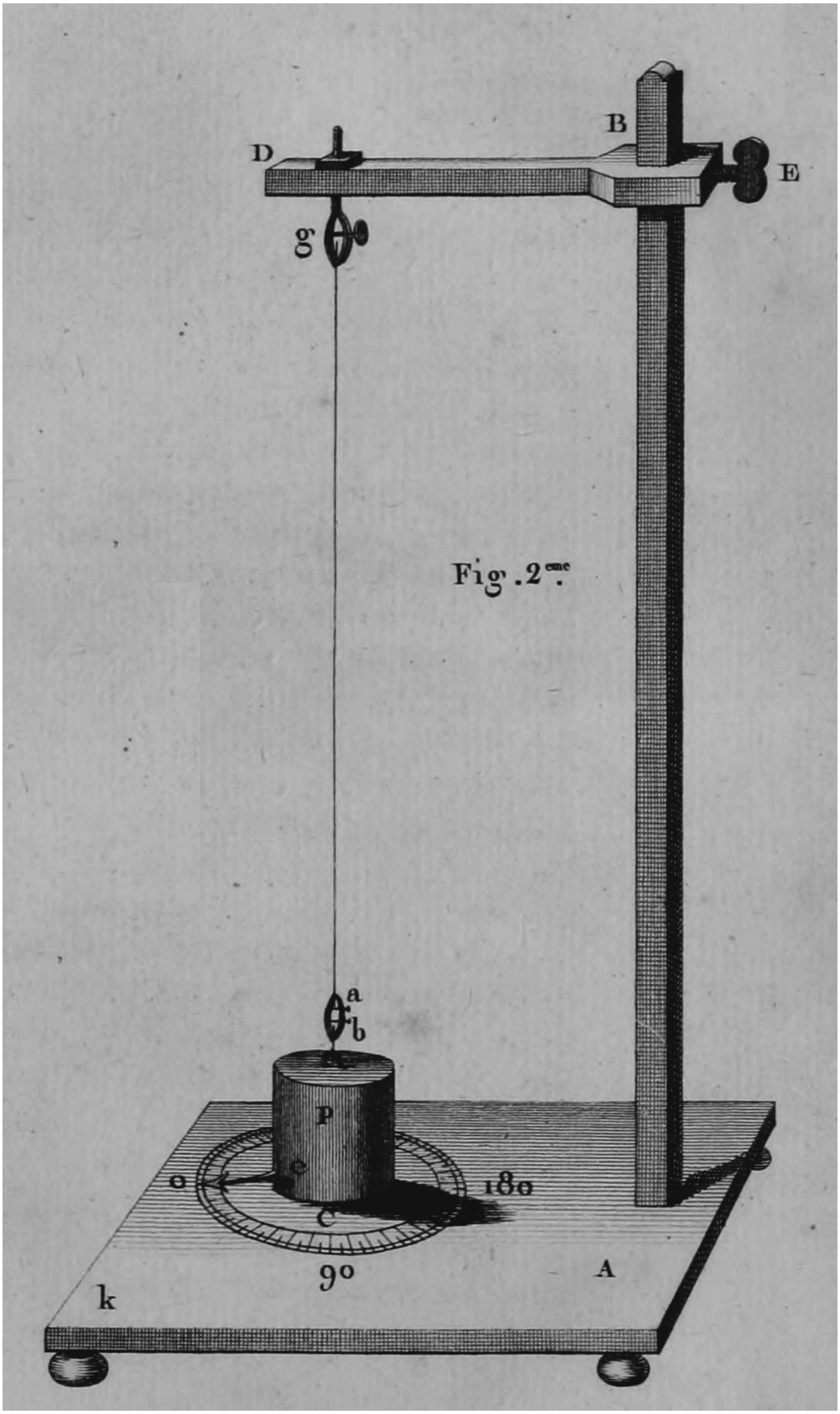


Fig. 2ème.

7.10 X. Experiments on the Torsion of Filaments of Iron

We have obtained three filaments of the harpsichord,[341] such as we find in commerce, rolled on bobbins, and numbered.[342]

The No. 12 filament of iron supported, before breaking, 3 pounds, 12 ounces (1836 g); its six feet of length weighed 5 grains (0.1365 gram per meter).[343]

The No. 7 filament of iron supported, before breaking, a weight of 10 pounds (4895 g); its six feet of length weighed 14 grains (0.1381 gram per meter).

The No. 1 filament of iron broke under a tension of 33 pounds (16154 g); its six feet of length weighed 56 grains (1.525 gram per meter).

7.10.1 First Experiment. Filament of Iron, No. 12, the Cylinder Weighed Half a Pound

We have taken a cylinder of lead weighing half a pound [244.75 g], which we have suspended by the filament of iron No. 12; this cylinder had a diameter of 19 lines and $6\frac{1}{2}$ lines of height ($D = 4.294\ cm$, $H = 1.466\ cm$); the filament of suspension had a length of 9 lines.[344] We rotated the cylinder about its axes, without disturbing this axis from the vertical, and we obtained the following results:

First test. When we turned the cylinder about its axes through an angle smaller than 180 degrees, it made twenty, sensibly isochronous, oscillations in 120".[345]

Second test. But in twisting three circles,[346] the ten first oscillations have been of 2 to 3 seconds longer than the ten of the first test; and after the ten first oscillations, the amplitude

[341] In the original: *j'ai pris trois fils de clavecin.* This word can also be translated as clavichord. It is a musical instrument played by means of a keyboard. The word "fil" can be translated as filament, cable, thread, string or wire. Probably Coulomb chose harpsichord wire because of its consistently high quality. See also [Goodway and Savage, 1992] and [Birkett and Poletti, s d].

[342] Gauge number or gauge size.

[343] Coulomb writes that his # 12 gauge wire with a length of *6 pieds* (6 feet = 195 cm) weights *5 grains* (5 grains = 0.266 g). The wire mass per unit length, m/L, is then $1.36 \times 10^{-3}\ g/cm = 0.136\ g/m$. The mass per unit length equals the product of the volumetric mass density, δ, and the cross-sectional area, A, that is, $m/L = \delta \cdot A$. For the density: Coulomb, in a final paragraph of Section 7.15, reports that "*le pied cube de fer, pesant à peu-prés 540 livres, ...*", that is, a cubic foot of iron (34265 cm^3) weighs approximately 540 pounds (264330 g). This gives, after conversion, $\delta = 7.70\ g/cm^3$ for the density. So $A = 1.77 \times 10^{-4}\ cm^2$; and the diameter $D = 1.50 \times 10^{-2}\ cm = 1.50 \times 10^{-4}\ m$. In that same paragraph 7.15, Coulomb goes on to state that "the diameter of a filament of iron, # 12, 6 feet long, weighing 5 grains, is approximately a fifteenth of a line", $(0.226\ cm)/15 = 0.0151\ cm$.

Moreover, Coulomb writes that his # 12 gauge wire supports, before breaking, *3 livres 12 onces* (3 pounds 12 ounces = 1836 g), that is, a weight $P = (1.836\ kg) \times (9.8\ m/s^2) = 18\ N$. So its rupture stress, tensile strength or fracture strength is approximately equal to $(18\ N)/(1.77 \times 10^{-8}\ m^2) = 1.0 \times 10^9\ N/m^2 = 1\ GPa$.

[344] In the original: *9 lignes de longueur.* This was a typographical error by the author. The correct should be 9 *pouces* or 9 inches (24.363 cm). Section 8.2 presents a discussion of this experiment.

[345] That is, each oscillation was performed in 6 seconds. Remember that each oscillation for Coulomb is the time taken to go from one end of the motion to the other end. For example, if the cylinder is rotated by $+90°$ and released from rest, it takes 3 seconds to reach the origin with an angle of $0°$ and maximum speed, taking another 3 seconds until it stopped at the opposite end at an angle of $-90°$.

Today we call the total time interval to go to and return to the same initial condition the period. In this example the total period of the cylinder is 12 seconds.

[346] That is, from an initial angle of $1080°$.

of oscillations, which was at the start three circles, was reduced to five fourths of a circle.

7.10.2 Second Experiment. Filament of Iron, No. 12, the Cylinder Weighed 2 Pounds

First test. In suspending a cylinder weighing 2 pounds [979 g], having the same diameter as the preceding but 26 lines of height [5.876 cm], from the same No. 12 filament of iron, we had, for an angle of torsion of 180 degrees or less, twenty oscillations sensibly isochronous in 242".

7.10.3 Third Experiment. Filament of Iron, No. 7, Cylinder Weighing Half a Pound

First test. In suspending the cylinder of half a pound by the No. 7 wire of iron, we obtained, for a torsion of 180 degrees or less, 200 oscillations sensibly isochronous in 42".

7.10.4 Fourth Experiment. Filament of Iron, No. 7, Cylinder Weighing 2 Pounds

Test. In suspending from the same filament a weight of 2 pounds, the twenty oscillations were achieved in 85".

7.10.5 Fifth Experiment. Filament of Iron, No. 1, Cylinder Weighing Half a Pound

Test. When we suspend a weight of half a pound by this filament of iron of 9 inches in length, its stiffness is so considerable that this weight is not sufficient to straighten it out; thus the oscillations are very irregular because they depend, not only on the angle of torsion, but also on the curvature that the filament of iron retains when uncoiled from the bobbin, even though it is stretched by the half-pound weight.

7.10.6 Sixth Experiment. Filament of Iron, No. 1, Cylinder Weighing 2 Pounds

Test. But in suspending a weight of two pounds from this filament of iron of 9 inches in length, the filament is visibly straightened and we have, for an angle of torsion of 45 degrees or less, 20 oscillation sensibly isochronous in 23".

Continuation of Experiments. Filaments of Brass.

Taking three filaments of brass, corresponding in number and approximately in thickness, to the three filaments of iron that were subject to experiment.

The No. 12 filament of brass carries, at the moment of its rupture, 2 pounds 3 ounces (1070 g): its six feet of length weighs 5 grains (0.136 gram per meter).

The No. 7 filament of brass carries, at the moment of its rupture, 14 pounds (6853 g):[347] its six feet of length weighs $18\frac{1}{2}$ grains (0.504 gram per meter).

The No. 1 filament of brass breaks under a tension of 22 pounds (10769 g): its six feet of length weighs 66 grains (1.797 gram per meter).

7.10.7 Seventh Experiment. Brass Filament No. 12, Cylinder Weighing Half a Pound

Test. The length of the filament of suspension was 9 inches, as in the preceding tests; we suspended a cylinder weighing half a pound from it and obtained, for an angle of twist of 360 degrees or less, twenty oscillations sensibly isochronous in 220".

But with an initial angle of twist of three full circles, the first twenty oscillations took 225 seconds; and after these initial twenty oscillations, the angle of twist was still approximately two full circles.

7.10.8 Eighth Experiment. Brass Filament No. 12, Cylinder Weighing Two Pounds

Test. The filament of suspension being 9 inches, and the cylinder weighing 2 pounds, we obtained, for an angle of 360 degrees or less, twenty oscillations sensibly isochronous in 442".

With an initial angle of twist of three full circles, the first twenty oscillation took approximately 444 seconds, and the initial angle of twist was found to be reduced to two and one quarter full circles [810°].

7.10.9 Ninth Experiment. Brass Filament No. 7, Cylinder Weighing Half a Pound

Test. The length of the filament of suspension always being 9 inches, the initial angle of twist being 360 degrees or less, one obtained twenty oscillations sensibly isochronous in 57".

7.10.10 Tenth Experiment. Brass Filament No. 7, Cylinder Weighing Two Pounds

Test. The length of the filament of suspension again of 9 inches, the initial angle of torsion being 360 degrees or less, one obtained twenty oscillations sensibly isochronous in 110".

But the initial angle of twist being two full circles, it took 111 seconds for the first twenty oscillation and the initial angle of twist, originally two circumferences, was reduced to one and a half circumference.

7.10.11 Eleventh Experiment. Brass Filament No. 1, Cylinder Weighing Half a Pound

Test. Under a tension of half a pound, the filament of suspension was not entirely straightened and the duration of the oscillation, depending in part on its initial curvature, is uncer-

[347] According to Goodway and Savage, this value of 14 *libres* for the breaking load was a misprint, [Goodway and Savage, 1992, p. 26].

tain.

7.10.12 Twelfth Experiment. Brass Filament No. 1, Cylinder Weighing Two Pounds

Test. The length of the filament of suspension, being, as always, 9 inches, the initial angle of torsion being 50 degrees or less, we obtained twenty oscillations sensibly isochronous in 32".

But the initial angle of twist being five-fourths of a circle, we observed the first twenty oscillations in $33\frac{1}{2}$ seconds; and at the end of these oscillations, the initial angle had been reduced to a quarter of a circle.

7.10.13 Thirteenth Experiment. Brass Filament No. 7, Cylinder Weighing Two Pounds

Test. The length of the filaments of suspension in all the preceding experiments being 9 inches; needing to determine the force of torsion relative to the length of the filaments, we have given 36 inches of length [97.45 cm] of suspension to this experiment and having had up to three circles of torsion or less, we obtained twenty oscillation sensibly isochronous in 222".

7.11 XI. Results of the Preceding Experiments

The force or reaction of the torsion of the filaments of metal ought to depend upon their length, their thickness, and their tension. In order to determine in general the law of this reaction, we have been obliged, in the preceding experiments, to suspend different weights from filaments of iron and brass, of different thicknesses and different lengths: Here are the results that these experiments present.

If we turn the cylinder about its axis, without disturbing this axis from the vertical, the filament twists: when we release the cylinder, the filament, by its force of reaction, will try to return to its natural situation; the cylinder will oscillate about this axis for a longer or shorter length of time accordingly as the elastic force is more or less perfect.

But we find, in all of the preceding experiments, that when the angle of twist is not very large, the period of oscillations is sensibly isochronous; thus we can regard as a first law, that for all the filaments of metal, when the angle of twist is not very great, the force of torsion is sensibly proportional to the angle of twist.

Having found from experiment that the force of reaction in torsion is proportional to the angle of twist, it follows that all the oscillatory formulae that we have given, *Sections 7.4 and following*, based upon the supposition that a force of torsion proportional to the angle of twist, or altered by a very small term, can be applied to these experiments.

Thus, as we have obtained, *Section 7.7*, by means of these formulas

$$T = \sqrt{\frac{Ma^2}{2n}}\pi \ ,$$

and that in all the preceding experiments, the cylinders of half a pound and of 2 pounds having the same diameter, it follows that n ought to be always proportional to (M/T^2).

Thus, if the tension in the filament, varying in magnitude, has no influence on the force of torsion, then the quantity n for the same filament will be the same for the case of a tension of half a pound and a tension of 2 pounds, and consequently we will have T proportional to $M^{1/2}$. Let us compare our experiments made with the two weights, one of half a pound, the other of 2 pounds, of which the [square] roots are as 1 is to 2.[348]

First experiment. The filament of iron, No. 12, stretched by the half-pound weight, makes 20 oscillation in 120".

Second experiment. The same filament, stretched by a weight of 2 pounds, makes 20 oscillation in 242".

Third experiment. The filament of iron, No. 7, stretched by the half-pound weight, makes 20 oscillation in 43".

Fourth experiment. The filament of iron, No. 7, stretched by a weight of 2 pounds, makes 20 oscillation in 85".

The *fifth experiment* cannot be compared with the *sixth.*

Seventh experiment. The filament of brass, No. 12, stretched by the half-pound weight, makes 20 oscillation in 220".

Eighth experiment. The filament of brass, No. 12, stretched by the 2 pounds weight, makes 20 oscillation in 442".

Ninth experiment. The filament of brass, No. 7, loaded with the half-pound weight, makes 20 oscillations in 57".

Tenth experiment. The filament of brass, No. 7, loaded with the 2 pounds weight, makes 20 oscillations in 110".

The *eleventh* and the *twelfth* experiments cannot be compared.

It thus results from all of these experiments, that for the same filament of metal, a weight of two pounds makes its oscillations in a time double of this of a weight of half a pound; consequently the period of oscillations is as the [square] root of the weights;[349] thus the tension, of varying magnitude, has no sensible influence on the force of reaction of torsion.

However, from many tests made with very great tensions relative to the force of the metal, it appears that the large tensions diminish or alter the force of torsion a small amount. We can see in fact, that as the tension increases, the filament elongates and its diameter diminishes, which ought to reduce the period of oscillation.

We have not been able to compare the filaments of iron or of brass No. 1, under the tensions of half a pound and of two pounds because, as we have said in the details of the experiments, the tension of half a pound is not sufficient to straighten the filament.

7.12 XII. On the Force of Torsion Relative to the Lengths of the Filaments

We have found, in the preceding Section, that the variable tension in the filaments only influences the force of torsion in a negligible way. We seek now to determine, from these same experiments, how much, for equal angles of torsion, the length of the filament of suspension increases or diminishes this force. But it is clear that to the extent that one

[348] Let m_1 and m_2 be the masses of the two bodies. So we have $m_1/m_2 = 0.5/2 = 1/4$. Therefore $\sqrt{m_1/m_2} = 1/2$.

[349] That is, the period of the oscillations is proportional to the square root of the weights.

increases the length of the filament of metal, we can make, in the same proportion, a greater number of revolutions of the cylinder, without changing the degree of torsion; thus the force of reaction of torsion ought to be, for the same number of revolutions, inversely proportional to the length of the filament. Let us see if this reasoning is in accord with experiment.

The formula, of Section 7.7, gives us

$$T = \sqrt{\frac{Ma^2}{2n}}\pi \ ,$$

or for the same weight, T proportional to $1/\sqrt{n}$. Thus, if n is inversely proportional to the length, as the theory claims, T will be as the [square] roots of the lengths of the filaments of suspension;[350] let us compare with experiment.

We find, *tenth experiment*, that the filament of brass, No. 7, of 9 inches of length, being stretched by the weight of half a pound, makes 20 oscillations in 110".

We find, *thirteenth experiment*, that the same filament of brass, No. 7, of 36 inches of length, stretched by the 2 pounds weight, makes 20 oscillations in 222".

Thus the lengths of filaments make between them :: 1 : 4, while the time of oscillations of the filaments make :: 1 : 2; thus the test proves that the times of the same number of oscillation, make, for the same filaments stretched by the same weights, as the [square] root of the length of these filaments, in accord with the claims of theory.[351]

We have made many tests of the same kind as the preceding, which have all very exactly confirmed this law. We have not believed it necessary to fatten this Memoir with them.

7.13 XIII. On the Force of Torsion Relative to the Thickness of the Filaments

We have determined the laws of the force of torsion relative to the tension and to the length of the filaments; it remains for us only to determine them relative to the thickness of the same filaments.[352]

We have, in the first six experiments, three filaments of iron of different thicknesses and of the same length; and in the following six experiments, three filaments of brass of the same length and of different thicknesses: but as we have the weights of one length of 6 feet of each of these filaments, it is easy from them to fix the ratio of their diameters.[353] Here is our reasoning and consequent prediction; the *moment*[354] of the reaction of torsion ought to increase, with the thickness of the filaments, in three ways. Take for example two filaments of the same material and the same length, where the diameter of one is double that of the other, it is clear that for the one whose diameter is double, there are four times more parts stretched by the torsion, than in those which have a simple diameter; and that the mean

[350] That is, if n is inversely proportional to the length of the suspension wire, T will be proportional to the square root of that length, since T is proportional to $1/\sqrt{n}$.

[351] Let L_1 and L_2 be the lengths of the two wires, while T_1 and T_2 are their periods of oscillation, respectively. So we have $L_1/L_2 = 1/4$, while $T_1/T_2 = 1/2$. That is, $T_1/T_2 = \sqrt{L_1/L_2}$.

[352] See Section 8.3 on page 185 for a discussion of this topic.

[353] The weight φ of a cylindrical wire of length L and diameter D is given by $\varphi = \delta\pi(D/2)^2 Lg$, where δ is the volumetric density of mass and g the acceleration due to gravity. Therefore, for two wires of the same material with the same length L and the same density δ, the ratio of their weights is given by $\varphi_1/\varphi_2 = (D_1/D_2)^2$.

[354] In the original: *momentum*.

extension[355] of all these parts will be proportional to the diameter of the filament, just as the mean arm of the lever relative to the axis of rotation. Thus we are led to believe, from theory, that the force of torsion of two filaments of metal of the same material and of the same length but of different thickness, is proportional to the fourth power of their diameter, or for the same length, to the square of their weights. Let us compare this with the experiments.[356]

We take here only the tests where the tension is 2 pounds, in order to compare all the numbers, the filaments of No. 1 not being as exactly stretched by the weights of half a pound: we have

Filaments of iron:

Second experiment. The filament of iron, No. 12, whose 6 feet of length weighs 5 grains, gives 20 oscillations in 242".

Fourth experiment. The filament of iron, No. 7, whose 6 feet of length weighs 14 grains, gives 20 oscillations in 85".

Sixth experiment. The filament of iron, No. 1, whose 6 feet of length weighs 56 grains, gives 20 oscillations in 23".

Filaments of brass:

Eighth experiment. The filament of brass, No. 12, whose 6 feet of length weighs 5 grains, gives 20 oscillations in 442".

Tenth experiment. The filament of brass, No. 7, whose 6 feet of length weighs $18\frac{1}{2}$ grains, gives 20 oscillations in 110".

Twelfth experiment. The filament of brass, No. 1, whose 6 feet of length weighs 66 grains, gives 20 oscillations in 32".

In order to determine, from these experiments, the law of reaction of the force of torsion, relative to the diameter of the filament of suspension, let us suppose that[357]

$$T : T' :: D^m : D'^m :: \varphi^{m/2} : \varphi'^{m/2} ,$$

where one supposes that T and T' represent the time of a certain number of oscillations for a filament of metal, whose diameter is D and D', and the weight for the same length is φ and φ'; m being the power that one searches to determine. From this proportion, we deduce

$$m = \frac{2(\log T - \log T')}{\log \varphi - \log \varphi'} ,$$

with which it is necessary to compare with the experiment.

The *second test*, compared with the *fourth*, gives $m = -1.82$.

The *second test*, compared with the *sixth*, gives $m = -1.95$.

The *eighth test*, compared with the *tenth*, gives $m = -2.04$.

[355] In the original: *l'extension moyenne.*

[356] Section 9.2 on page 189 explains how Coulomb concluded theoretically that the torque exerted by a metal filament must be proportional to the fourth power of its diameter.

[357] That is,

$$\frac{T}{T'} = \left(\frac{D}{D'}\right)^m = \left(\frac{\varphi}{\varphi'}\right)^{m/2} .$$

The *eight test*, compared with the *twelfth*, gives $m = -2.02$.
From which it results that[358]

$$T : T' :: \frac{1}{D^2} : \frac{1}{D'^2} :: \frac{1}{\varphi} : \frac{1}{\varphi'} .$$

But the formula of oscillatory movement

$$T = \left(\frac{Ma^2}{2n}\right)^{1/2} \pi$$

gives, in the preceding experiments, because of the equality of the tensile loading,[359] n proportional to $1/T^2$; thus the force of torsion, for the filaments of the same nature, of the same length, but of different thicknesses, is as the fourth power of the diameter, thus as the theory had predicted.[360]

7.14 XIV. General Results

It results thus from all the preceding experiments, that the *moment* of the force of torsion is, for filaments of the same metal, proportional to the angle of twist, the fourth power of the diameter, and inversely proportional to the length of the filament; so that if we let l be the length of the filament, D its diameter, B the angle of twist, we will have for the expression which represents the torque,

$$\frac{\mu B D^4}{l} ,$$

where μ is a constant coefficient which depends on the natural stiffness[361] of each metal: this quantity μ, a constant for filaments of the same metal, can be easily determined from experiment, as we see in the following Section.

7.15 XV. Effective Values of the Quantities n and μ

We have seen, in Section 7.7, that

[358] That is, as $m = -2$:

$$\frac{T}{T'} = \left(\frac{D'}{D}\right)^2 = \frac{\varphi'}{\varphi} .$$

[359] In the original: *poids de tension*. That is, due to the equality of the weights P of the cylinders hanging from the suspension wire.

[360] Coulomb had experimentally found that

$$\frac{T}{T'} = \left(\frac{D'}{D}\right)^2 = \frac{\varphi'}{\varphi} .$$

However, n is proportional to $1/T^2$ due to the equality of the weights of the cylinders supported by the threads. Since T is also proportional to $1/D^2$ by the formula just written, n will be proportional to $1/(1/D^2)^2 = D^4$, as he had theorized.

[361] In the original: *roideur*.

$$n = \frac{Pa^2}{2\lambda} \ ,$$

where P is the weight of the cylinder, a its radius, λ the length of a pendulum which is isochronous with the oscillations of the cylinder produced by the force of torsion.

Let us apply this formula to the *second experiment*, where the filament of iron, No. 12, is stretched by a 2 pounds weight, which has a radius of $9\frac{1}{2}$ lines, and makes 20 oscillations in 242".

As the length of a pendulum which completes one full swing in one second at Paris[362] is $440\frac{1}{2}$ lines (99.37 cm),[363] the length of a pendulum, isochronous with the oscillations of the cylinder, will be[364]

$$440\frac{1}{2}\left(\frac{242}{20}\right)^2 \ ;$$

thus

$$n = \frac{2 \text{ pounds } \left(9\frac{1}{2}\right)^2}{2 \cdot 440\frac{1}{2}\left(\frac{242}{20}\right)^2} = \frac{1 \text{ pound}}{715} \ ,$$

therefore the *moment* nB of the No. 12 filament of iron, 9 inches in length, is equal to $\frac{1}{715}$ pounds, multiplied by the angle of torsion B, acting at the extremity of a lever of one line in length.[365,366]

We have seen, that for the same metal, it follows from the theory and the experiments of the preceding Sections that the torque is inversely proportional to the length of the filament

[362] In the original: *qui bat les seconds à Paris*.

[363] A seconds pendulum is a pendulum whose period is precisely two seconds; one second for a swing in one direction and one second for the return swing, a frequency of 0.5 Hz. With $T_{modern} = 2\pi\sqrt{\ell/g}$ we obtain $\ell = T^2 g/(4\pi^2)$. With $T_{modern} = 2\ s$ and $g = 9.81\ m/s^2$ this yields $\ell = 0.994\ m$. The measure given by Coulomb is $\ell = 440.5$ lines $= 0.995\ m$.

[364] Using that $T_{Coulomb} = \pi\sqrt{\ell/g}$ we obtain $T_1/T_2 = \sqrt{\ell_1/\ell_2}$. If $T_1 = 1\ s$, $\ell_1 = 440.5$ lines and $T_2 = (242\ s)/20$, then we have $\ell_2 = \ell_1(T_2/T_1)^2 = 440.5(242/20)^2 = 64\,493.6$ lines $= 145.75\ m$.

[365] [Note by Potier] That is 151.50 (C.G.S.) for this wire, and 3691 for a wire number 12, 0.01 m long, and twisted by an angle equal to 1; or again a couple 64.42 (C.G.S.) is necessary to twist this wire of length 1, by *one* degree.

[366] In Section VII (Section 7.7) Coulomb showed that:

$$n = \frac{Pa^2}{2\lambda} \ ,$$

where n is what we call today the "torsional stiffness" of the wire (Coulomb never ascribes a label to n), see footnote 328. P is the weight of the suspended cylinder, a its radius and λ is the length of a pendulum isochronous with the period of oscillations of the cylinder. He applies this formula to the Second Experiment where the # 12 iron wire (of length 9 *pouces* = 9 *inches* = 24.363 *cm*) is loaded with a cylinder of mass $m = 2$ *livres* $= 2$ *pounds* $= 979\ g$ (that is, with a weight $P = mg = 0.979\ kg \cdot 9.8\ m/s^2 = 9.5942\ N$), whose radius a is 9.5 *lines* $= 0.02147\ m$, and which makes 20 oscillations in 20 seconds — which, he shows, is isochronous with a pendulum of length $\lambda = 6.45 \times 10^4$ *lines* $= 1.4577 \times 10^4\ cm = 146\ m$. The formula then gives $n = 1.52 \times 10^{-5}\ Nm/rad = 152\ dyn \cdot cm/rad$ which is equivalent to (1/715) *livre* · *line*, in the units Coulomb used. This is the torque $\tau = n\theta$ required to twist a # 12 iron wire, of length 9 *pouces*, by an angle $\theta = 1$ *radian*. Potier obtained the same result in footnote 365. As n is inversely proportional to the length, if the suspension wire were 1 *cm* long, we would have $n = 3.698 \times 10^{-4}\ Nm/rad$. Since 1 $rad = 57.30°$, the torque required to twist this 1 *cm* long wire by 1° is given by $6.454 \times 10^{-6}\ Nm$.

of suspension and proportional to the fourth power of the diameter. Thus it is easy to determine the value of the torque in a filament of iron, of any length and thickness; here is the calculation.

Since a cubic foot of iron weighs approximately 540 pounds,[367] the No. 12 filament of iron, weighing 5 grains and 6 feet in length, has a diameter very nearly equal to a fifteenth of a line;[368] thus the *moment* of torsion[369] of a filament of iron, of a fifteenth of a line in diameter, is equal to 1/715 pound acting at the extremity of a lever of one line in length, multiplied by the angle of twist.[370,371]

7.16 XVI. Comparison of the Stiffness of Torsion of Two Different Metals

We can easily deduce, from the preceding theory and experiments, the ratio of the stiffness in torsion of two different metals, for example, iron and yellow copper: we take the No. 12 filament of iron to compare with the No. 12 filament of brass.

In the *preceding Section*, we calculated the quantity n, for the filament of iron, which we found $= \frac{1}{715}$ pound, multiplied by a lever of one line. But as the filament of brass, loaded with a weight of 2 pounds, makes 20 oscillation in 442", we will have, by the same formula for the filament of brass,

$$n' = \frac{1 \text{ pound} \left(9\frac{1}{2}\right)^2}{440\frac{1}{2}\left(\frac{442}{20}\right)^2} \; ;$$

thus

$$\frac{n}{n'} = \left(\frac{442}{242}\right)^2 = 3.34 \; ;$$

therefore the stiffness of the filament of iron, No. 12, is to the stiffness of the filament of brass, No. 12, approximately in the ratio $3\frac{1}{3} : 1$.

[367] The volumetric mass density δ of this piece of iron is then given by $\delta = (540 \times 489.5\ g)/(32.48\ cm)^3 = 7.7\ g/cm^3 = 7.7 \times 10^3\ kg/m^3$. See also footnote 343 on page 155.

[368] That is, $D = 1\ line/15 = 0.226\ cm/15 = 0.015\ cm$. See footnote 343 on page 155.

[369] In the original: "le *momentum* de torsion". This expression can also be translated as "torque" or "moment of the force of torsion", see footnote 150 on page 60.

[370] [Note by Potier] We can also deduce from Coulomb's experiments Lamé's coefficient μ (called *rigidity* by the English authors). Assuming 7.8 for the specific weight of this wire, we should have

$$\mu = n \times \frac{2L}{\pi R^4} = 7.628 \times 10^{11}\ [dyn/cm^2] \; .$$

[371] As a final note (and check) on Coulomb's calculation of the effective value of n for the Second Experiment, Section XV (Section 7.15), we can draw out an estimate of the shear modulus, G, as it appears in today's expression for the torque, namely, $Torque = \tau = (GJ/L)B$, where $J = \pi D^4/32$ is the polar moment of inertia of the wire cross-section for a cylindrical wire of diameter D, for an angle of twist B. For our # 12 iron wire of length, L, equal to 9 *pouces* = 9 *inches* (0.244 m), a twist, B, of 1 radian, and diameter $D = 1.50 \times 10^{-4}\ m$ (see footnote 343) a torque of $1.52 \times 10^{-5}\ Nm$ would produce 1 radian of twist according to Coulomb, see footnote 366. Solving for G gives $G = 74.6 \times 10^9\ N/m^2 = 74.6 \times 10^{10}\ dyn/cm^2$, a value at the low end of the range $75 \times 10^9\ N/m^2$ to $80 \times 10^9\ N/m^2$ cited by Gillmor for the shear modulus of wrought iron, [Gillmor, 1971a, p. 155].

But as there is little difference between the specific weight of iron and of copper, which according to Mr. Musschembroek,[372] are in the ratio 77 : 83, we can suppose that the No. 12 filament of iron and that of copper *of the same number* have approximately the same diameter; thus for filaments of iron and of copper of the same diameter, every thing otherwise equal, the stiffnesses in torsion are in the ration $3\frac{1}{3} : 1$, which means that in twisting the filament of iron one circle, one would have the same torsional reaction, in twisting the filament of copper $3\frac{1}{3}$ circles.[373,374]

If we wish subsequently to compare the stiffness of torsion with the force of cohesion, we note that our filament of iron carries, at the instant of its rupture, 60 ounces [1835 g], while that of copper only carries 35 ounces; thus since they are approximately the same diameter, the ratio of their force of cohesion approaches 60 : 35, while their force of torsion is found to be [in the ratio] $3\frac{1}{3} : 1$.

This last result, however, ought to be regarded as a special case and not as a general result. We will see, in the second Section of this Memoir, that the force of metals varies following the degree of cold-working and heat treatment,[375] and that all the experiments which we have carried out until now aimed at determining the force of metals can only be regarded as some particular cases.

But what this last observation seems to indicate, and what practice confirms, is that if we wish to support a moving body on a pivot point, there is an advantage to using a pivot of steel or of iron to a pivot of copper, since under the same degree of pressure the iron bends much less than the copper; thus the circle of contact formed by the pivot point, pressed by the body that it supports, will be less for iron than for copper, this which, all else being otherwise equal, reduces the *moment* of friction that it is necessary to overcome in order to rotate a body about a pivot point: We will have occasion in the following to return to this Section.

From some other experiments and by means of calculations similar to the preceding, we have found that a filament of silk, formed of several brins[376] joined by boiling water and strong enough to carry up to 60 ounces [1835.4 g in tension], has 18 to 20 times less torsional stiffness than the filament of iron which carries the same weight at its moment of rupture.

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[372] Pieter van Musschenbroek (1692-1761) was a Dutch scientist.

[373] [Note by Potier] We conclude from this number, by assuming the specific gravity of brass to be 8.6, that the coefficient μ' of brass

$$= \frac{1}{3.34} \cdot \frac{8.6^2}{7.8^2} \cdot \mu = 2.78 \times 10^{11} \ .$$

This number is much lower than that indicated by Sir W. Thomson, Wertheim and Mr. Everett, which indicate from 3.4 to 4×10^{11}.

[374] See also [Goodway and Savage, 1992, p. 25].

[375] In the original: *degré d'écrouissement et de recuit.* Work hardening in materials science is the strengthening of a metal by plastic deformation. Cold working, cold forming processes or metalworking processes intentionally induce plastic deformation to exact a shape change. They are characterized by shaping the workpiece at a temperature below its recrystallization temperature, usually at ambient temperature. In metallurgy and materials science, annealing is a heat treatment that alters the physical properties of a material to increase its ductility and reduce its hardness, making it more workable. It involves heating a material above its recrystallization temperature, maintaining a suitable temperature for an appropriate amount of time and then cooling.

[376] See footnote 215 on page 91.

7.17 XVII. Use of the Experiments and of the Preceding Theory

Using the theory which precedes, and the experiments on which it is based, we are able to measure very small forces, with a precision that ordinary means cannot supply: we present an example.

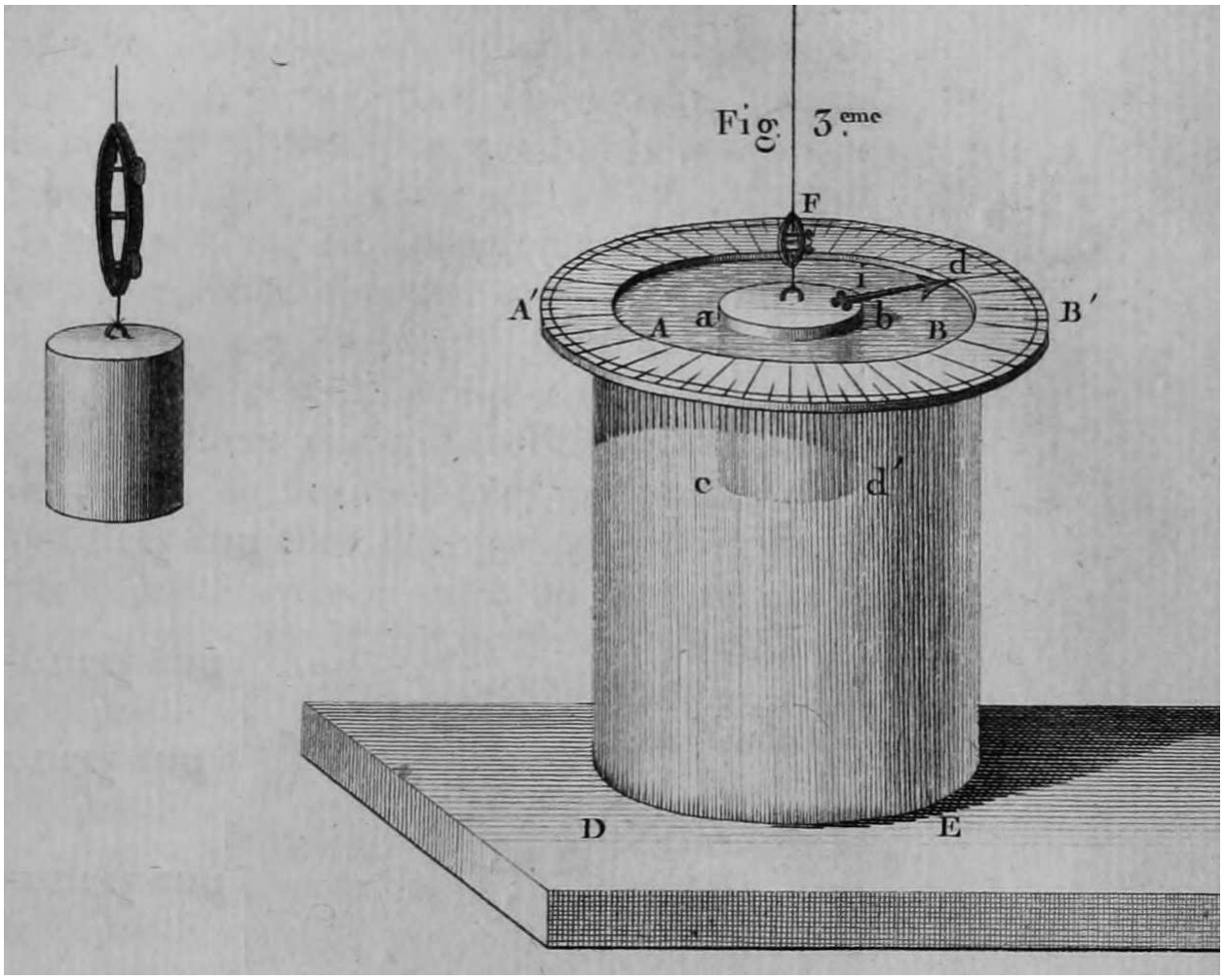
7.18 XVIII. Balance to Measure the Friction of Fluids Against Solids

The formula that expresses the resistance of fluids against a body in motion, appears composed of several terms, some of which depend on the impact of the fluids against the body, and others which are due to the friction of the fluid: among the terms due to friction, there is one which depends on adhesion,[377] and which is believed to be constant; but this term is so small, that confounded in the experiments with the other quantities which depend on impact, it is very difficult to evaluate: we can see in the experiments that M. Newton has made in order to discover this constant quantity. (*Livre II des Principes mathématiques de la Philosophie naturelle, Scholie du vingt-cinquième théorème.*)[378]

The force of torsion provides an easy means to determine the [friction due to] adhesion from experiment.

In a vase $ADBE$, *Figure 3*, filled with fluid of which we wish to determine the adhesion, we suspend, by means of a filament of copper, a cylinder $abcd$, of copper or of lead; we place above the vase a circle $A'FB'$, divided in degrees; the circle is located at the level of the end d of an index id attached to the cylinder.

[377] In the original: *adhérence*.
[378] Book II, Scholium to Theorem 25 of [Newton, 1934], [Newton, 1999] and [Newton, 2008, pp. 98-108].



When one turns the cylinder about its vertical axis, without disturbing it from its verticality, we can observe, by means of the small index, how much each oscillation is altered; and as the force of torsion of the filament which produces these oscillations, is known from the preceding experiments; thus one knows the alteration due to the imperfection of elasticity [of the filament], in making the cylinder oscillate in the void or even in the air; we can expect, by this means, to find the constant quantity due to adhesion.

Example and Experiment

We have suspended the cylinder of lead weighing two pounds [979 g], which we used in the preceding experiments, from a filament of copper, No. 12, of twenty-nine lines in length [78.5 cm], in a vase filled with water: The circle AB, on which we observed the oscillations, had a diameter of forty-four lines [9.9 cm]; we waited, before beginning our observations, until the amplitudes of oscillations diminished to the point at which the extremity d of the index only traveled an arc of one and one half line[379] on the circle, corresponding to approximately $3°55'$; and observing the displacement of the index through a lens,[380] we have distinctly counted fourteen oscillations before the movement ceased.

[379]That is, 1.5 line = 0.34 cm.
[380]In the original: *loupe*. That is, a convex magnifying lens.

If the successive diminution of each oscillation is supposed constant, and can be entirely attributed to the adhesion of the fluid to the surface of the lead cylinder, we will have, [from] Section 7.8:[381]

$$(A - S') = \left(\frac{2\mu}{n}\right) \ ,$$

where $(A - S')$ is the diminution in each oscillation, $n(A - S)$ the *moment* of the force due to torsion, and μ the *moment* of the retarding force due to adhesion.

But as, after observing the oscillations, the arc traveled diminishes one and a half line in fourteen oscillations, and given that the radius of the circle on which we observe this reduction is twenty-two lines; in supposing this diminution constant, we obtain that the angle $(A - S')$ by which the amplitude diminishes each oscillation $= \frac{3}{2 \cdot 22 \cdot 14}$.

But we found, Section 7.16, that for a filament of brass of nine inches in length, No. 12,

$$n = \frac{1 \text{ inch} \cdot \left(9\frac{1}{2}\right)^2}{440\frac{1}{2} \cdot \left(\frac{442}{20}\right)^2} \ ;$$

and as we have also found that the forces of torsion are proportional to the length of the filaments of suspension, we will have for our filament of twenty-nine inches in length

$$\mu = \frac{1}{3155000} \text{pound} \times 1 \text{ line} \ ,$$

which is to say that the *moment* of the constant retarding force, μ, is approximately equal to three millionths of a pound suspended at a lever arm of one line: a quantity which would have been impossible to measure by any other means than this that we have come to employ.[382]

In order to now deduce the value of the adhesion from this experiment, it is necessary to note that the height of the cylinder of lead, submerged in the water in the vase, is twenty-four lines, and that the diameter of this cylinder is nineteen lines. Thus, in taking 22/7 for the ratio of the circumference to the diameter, the surface of the submerged cylinder, is equal to $\frac{22}{7} \cdot 19 \cdot 24$; and as the movement is about the axis of the cylinder, whose radius is $9\frac{1}{2}$ lines, if δ is the adhesion, the *moment* of the adhesion about the axis of rotation, will be

$$\delta \frac{22}{7} (19)^2 \cdot 12 \ .$$

It is then necessary to add to this quantity the *moment* of the adhesion of the circle which forms the base of the cylinder submerged in the water, of which the *moment* is

$$\delta \frac{22}{7} 19^l \frac{19^l}{4} \frac{2}{3} \frac{19}{2} \ ,$$

[381] Coulomb is assuming here $m = 0$ in Section 7.8, such that the retarding force R is a constant equal to μ.

[382] These calculations show that Coulomb's torsion balance can measure extremely small torques on the order of

$$\mu = \frac{1 \text{ pound}}{3155000} \times 1 \text{ line} = \frac{4.802 \ N}{3155000} \times (2.26 \times 10^{-3} \ m) = 3.4 \times 10^{-9} \ Nm \ .$$

so that the total *moment* of the resistance of the fluid against the cylinder will be

$$\delta\frac{22}{7}(19)^2\left(12+\frac{19}{12}\right)=\delta\frac{22}{7}(19)^2\left(\frac{163}{12}\right)\ .$$

But the experiment has shown us that this same *moment* is equal to

$$\frac{1\text{ pound}}{3155000}\cdot 1\text{ line}$$

for a square inch; thus

$$\delta=\frac{1\text{ pound}}{3155000}\cdot\frac{7\cdot 12}{22\cdot 163\cdot(19)^2}\ ,$$

and for a square *foot* the adhesion will be

$$\delta(144)^2=\frac{1\text{ pound}}{2345000}\ ,^{383}$$

so that the constant resistance due to the adhesion of the water for a surface of 255 feet, can not be more than a grain; thus there are few cases where this constant alteration, if it takes place, cannot be neglected in the evaluation of the friction of water. We have not made any tests on other fluids.

In giving the cylinder oscillations of two or three full circles of amplitude, and comparing the successive diminutions of amplitudes of oscillations with the formulas of changing oscillatory movement,[384] I have believed to have seen that for very small velocities, the friction goes as the velocity, and for large velocities, as the square; but these experiments require special attention and ought to be made in different fluids.[385]

7.19 XIX

Since the reading of this Memoir, I have constructed, according to the theory of the reaction of torsion that I have put forward, an electric balance and a magnetic balance; but as these two instruments, as well as the results bearing on the electric and magnetic laws that they have given, will be described in the volumes following our Memoirs, I believe it suffices here to simply announce them.

7.20 XX. Second Section. On the Alteration of the Elastic Force in the Torsion of Filaments of Metal. Theory of the Coherence and of Elasticity

When we torque the filaments of iron or of brass, stretched, as in the preceding experiments, by a weight, we observe two things; if the angle of torsion is not so great, relative to the length of the filament of suspension, at the moment when one releases the weight, it returns

[383] [Note by Potier] That is, 1.94×10^{-4} dynes per square centimeter.
[384] Changing due to resistance.
[385] Later on Coulomb himself carried out these researches, [Coulomb, 1801b].

approximately to the position that it had before twisting, that is to say, the filament of suspension untwists completely by the quantity by which it had been torqued; but if the angle of torsion given the suspending filament, is very large, then the filament only unwinds a certain amount, and the center of the reaction of torsion will advance the whole quantity by which the filament failed to unwind. It follows from these two considerations, that two suites of experiments are required; the first to determine, by the diminution of oscillations, how much the elastic force of torsion is altered in oscillatory movement under conditions in which the center of reaction of torsion is not displaced; the second to determine the displacement of this center of reaction, when the angle of torsion is sufficiently large for this displacement to take place.

7.21 XXI. First Experiment

Filament of Iron, No. 1, Length, six inches six lines (17.96 cm).

We have taken a filament of iron of six inches six lines in length, that has been loaded with a weight of two pounds, the same as has served us in the experiments in the preceding Section. In turning the cylinder about its axis in order to twist the filament of suspension, we have sought to determine how many degrees the amplitude diminishes with each oscillation, and we have found:

	Angle of torsion	Loss of 10° in
First test	90°	$3\frac{1}{2}$ oscillations
Second test	45	$10\frac{1}{2}$
Third test	$22\frac{1}{2}$	23
Fourth test	$11\frac{1}{4}$	46

Remarks on This Experiment.

The reductions in amplitudes of oscillations have been very uncertain (irregular), when the initial angle of torsion was more than 90 degrees; we have even observed that in this case, in twisting the cylinder about its axis, it did not return to its initial position, and the respective position of the constitutive parts[386] of the filament have been altered, and consequently, its center of reaction of torsion has remained displaced: here is what the experiments gave for this displacement.

7.22 XXII. Follow on to the First Experiment

In this part of the first experiment, we have searched to determine the displacement of the center of torsion, due to the degree of torsion that we have given to the filament of suspension.

[386] In the original: *parties constitutives*. This expression can also be translated as constitutive particles or constitutive elements. Other possible translations: components, component parts, parts of the system, constituent parts, constituent particles or constituent elements. See also [Gillmor, 1971a, pp. 159, 201 and 217].

	In twisting the filament	The index or the center of torsion has been displaced
First test	$\frac{1}{2}C$ [that is, 180°]	8°
Second test	1 [that is, $1C = 360°$]	50
Third test	2	310
Fourth test	3	$1C + 300$ [that is, 660°]
Fifth test	4	2 + 290
Sixth test	5	3 + 280
Seventh test	6	4 + 260
Eighth test	10	8 + 240

Ninth test. Having wished to continue to twist the filament some 15 new circles, always in the same sense, it broke at the fourteenth. After this experiment, this filament was straight and very rigid, it had separated along its length into two parts; examining it with a magnifying glass, this separation was very evident and it had exactly the shape of a cord formed of two helices.[387]

7.23 XXIII. Remarks Concerning This Experiment

This first experiment and its sequel appears to show that below 45 degrees, the alterations made are approximately proportional to the amplitudes of the angles of twist, as one sees from the second, third and fourth tests of the first experiment;[388] that above 45°, the alterations augment in a ratio much greater; that the center of reaction of torsion only begins to displace when the angle of torsion is approximately a half circumference; that this displacement increases as the torsion of the filament increases; that it is very irregular up to 1 circle 10 degrees [370°]; and that, passing this level of torsion, the reaction of torsion remains approximately the same for all the angles of twist: Thus, for example, in the fourth test, in twisting the filament three circles, the center of reaction of torsion displaces one circle +300 degrees, so the reaction of torsion has only led the cylinder back one circle 60 degrees. In the seventh test, we see that after having already experienced in the previous tests a [total] displacement of more than eight circles, that six new circles of torsion displace the center of reaction of torsion by $4C + 260$ degrees, so that for more than fourteen circles of torsion, the reaction of torsion is still only one circle plus 100 degrees; thus it only differs by a tenth from the reaction of torsion for the fourth test which gave us one circle +60 degrees: the experiments which follow clarify this remark.

7.24 XXIV. Second Experiment

Filament of iron, No. 7, length, 6 inches 6 lines.

[387] In the original: *torons*.

[388] Let $A(t)$ be the amplitude of oscillation of the cylinder at the time t after the suspension wire has been twisted and released from rest at $t = 0$ with the initial amplitude A_o. Coulomb is concluding from his experiments that for $\theta < 45°$ the changes in amplitudes with respect to time are proportional to the amplitudes themselves. Mathematically this can be expressed as: $dA/dt = -kA$, where k is a positive constant, as he observed the amplitudes decreasing with the passage of time. Integrating this equation gives $A(t) = A_o e^{-kt}$.

We have searched, in the first part of this experiment, how much the amplitudes of oscillations diminish at each oscillation, when the center of torsion is not yet displaced.

	Angle of torsion	Loss of 10° in
First test	180°	$3\frac{1}{2}$ oscillations
Second test	90	12
Third test	45	27
Fourth test	$22\frac{1}{2}$	57

Follow on to This Second Experiment.

In this second part of the same experiment, we have sought the displacement of the center of torsion.

	In twisting the filament	The index or the center of torsion has been displaced
First test	3 circles	300°
Second test	4	$1C + 180$ [that is, 540°]
Third test	6	3 + 90 [that is, 1170°]
Fourth test	8	5 + 90
Fifth test	12	9 + 40
Sixth test	20	16 + 310
Seventh test	30	26 + 180
Eighth test	50	46 + 20
Ninth test	At the seventeenth circle of torsion, the filament broke.	

7.25 XXV. Third Experiment

Filament of iron, No. 12, length, 6 inches 6 lines.

The first part of this experiment has been made in accord with the first part of the two preceding experiments.

	Angle of torsion	Loss of 10° in
First test	360°	1 oscillation
Second test	180	2
Third test	90	5
Fourth test	45	11
Fifth test	$22\frac{1}{2}$	25

Follow on to This Third Experiment.
Displacement of the Center of Torsion.

	In twisting the filament	The index or the center of torsion has been displaced
First test	4 circles	300°
Second test	6	$2C + 40$
Third test	after six other turns the filament broke.	

The preceding experiments have been continued with the filaments of brass used in the experiments of the first Section.

7.26 XXVI. Fourth Experiment

Filament of brass, No. 1, length, 6 inches 6 lines.

	In twisting	Loss of	in
First test	180°	12°	2 oscillations
Second test	90	10°	6
Third test	45	10°	16
Fourth test	$22\frac{1}{2}$	10°	40
Fifth test	$11\frac{3}{4}$	10°	80

Follow on to the fourth experiment.
Displacement of the center of torsion.

	In twisting the filament	The index or the center of torsion has been displaced
First test	2 circles	160°
Second test	4	$2C + 0$
Third test	6	$3C + 300$
Fourth test	10	$7C + 300$
Fifth test	20	$17C + 340$
Sixth test	at the twenty-eighth circle of torsion the filament broke	

Fifth Experiment.
Filament of brass, No. 7, length, 6 inches 6 lines.
Decrease of the amplitudes in the oscillations.

	In twisting	Loss of 10° in
First test	360°	$2\frac{1}{2}$ oscillations
Second test	180	6
Third test	90	13
Fourth test	45	31
Fifth test	$22\frac{1}{2}$	72

Follow on to the fifth experiment.
Displacement of the center of torsion.

In twisting the filament four circles, the center is displaced 220 degrees; but in wishing to torque it six circles, the filament broke.

7.27 XXVII

In the filament employed in this last experiment, the torsion altered the oscillations, and hence the elastic force, less than in all the other experiments; it is this which occasions the great number of oscillations before the oscillatory movement dies out; it is this likewise which results in the sudden rupture of this filament, without being able to displace its center of reaction one circle. I have found in general that the filaments of brass, those available in commerce, between the numbers 5 and 8, were those whose elasticity in torsion was the least imperfect: in comparing the filaments of iron and of brass with the *same numbers*, we have similarly found that the filaments of brass have an amplitude of elasticity much more extensive than the filaments of iron.

For the rest, the experiment presents many irregularities in the results: two bobbins of the same filament and of the same number, do not always give the same displacement for the same angle of torsion, this which can only be attributed to the way in which the filaments are manufactured — to the more or less great pressure that they experience in passing under the die,[389] to the heat treatment given them in order to successively reduce the diameter from one number to the next, from large to small.

7.28 XXVIII. First Remark

Despite the uncertainty which reigns in the experiments of oscillations for the range of amplitudes,[390] it appears that below certain limits, these alterations are approximately proportional to the amplitude of oscillation, as we have announced in the remarks on the first experiment, and as all the other experiments confirm.[391] The resistance of the air can only alter the amplitude of oscillations very little in our experiments. I am assured of this by the following. The weight of two pounds, which has served us in the experiments of this Section, was 26 lines in height and 19 lines in diameter. I have formed with a very light paper, a cylindrical surface of the same diameter as this weight, but which had 70 lines of height: I put a part of the cylinder of lead into my envelope of paper, and formed thus a cylinder of 78 lines of height, or three times longer than the first, which should have tripled, in the oscillatory movement, the alterations due to the air resistance; but I have never found that these alterations were a tenth more considerable in the second case as in the first; most often

[389] In the original: *lèvre de la filière*. Drawing is a metalworking process that uses tensile forces to stretch (elongate) metal, glass, or plastic. As the metal is drawn (pulled), it stretches to become thinner, to achieve a desired shape and thickness. A die is a specialized machine tool used in manufacturing industries to cut and/or form material to a desired shape or profile. A draw plate is a type of die consisting of a hardened steel plate with one or more holes through which wire is drawn to make it thinner. See also [Birkett and Poletti, s d].

[390] In the original: *pour les amplitudes des étendues*.

[391] See footnote 388 on page 171.

they are equal; thus the resistance of air enters into our experiments only as quantities that we can neglect.

7.29 XXIX. Second Remark

In order to make a torsion balance, it is always necessary to choose the filaments which have the least imperfect elasticity; the filaments of brass are much more preferable to those of iron: the choice of the thickness depends on the forces which we wish to measure. I have a magnetic balance which will be described in our Memoirs, where I alternatively made use of a filament of brass of 3 feet in length, numbers 12 and 7; the elastic force of torsion is such that in twisting the filaments eight circles, over the course of thirty hours, there is not one degree [1°] of alteration or displacement in the center of torsion.

7.30 XXX. Third Remark

In all the filaments of metal, the behavior is elastic only up to a certain point:[392] The isochrony of the oscillations teaches us that in the first degrees of torsion, the elastic force is almost perfect; but beyond the angle of torsion which serves, for thus to say, as a measure of the elastic force, the center of reaction of torsion displaces nearly the whole of all the angle of torsion which exceeds this of the elastic reaction. However, as we can note in the preceding experiments, the amplitude of the elastic reaction is not a constant quantity for all angles of twist, it increases as the torsion increases; the less the initial elasticity, in the filament subject to test, has of extent, the greater this increase is. A filament of brass, No. 1, of 6 and one-half inches in length, made red in a fire, in order to make it loose, by heat treatment, the greatest part of its elasticity, only gives, after this operation, for the first circle of torsion, 50 degrees of reaction of elasticity; but it has acquired, after 90 circles of torsion, an elastic extension of nearly 500 degrees in this interval; from the 2nd to the 3rd circle of torsion, the reaction of elasticity increases 12 degrees; from the 40th to the 41st circle of torsion, the same reaction increases 6 degrees; and from the 90th to the 91st circle of torsion, almost a degree, such that the increase of the elastic reaction, after the center of reaction has been displaced a certain angle, is nearly inversely proportional to the angle of displacement. It is necessary to point out that after these 90 circles of torsion, I wished to twist the same filament another 50 circles, but it broke at the 49th, so this filament, before breaking, could be twisted to 140 circles. If we compare this result with that which followed from the first experiment, where the same filament, No. 1, had not been heat treated, we found that after 25 circles of torsion, the reaction of elasticity was 480 degrees and that in twisting 15 new circles, the filament fractured; this last filament can thus only take, without breaking, 40 circles of torsion. In following in this experiment the path of the elastic reaction, we deduce from it that at the point of rupture, this reaction is almost equal to that of the heat treated filament in the same point of rupture; from which it would appear that we are justified in concluding that by torsion alone we can give to a heat treated filament all the elasticity of which it is susceptible and that the plastic deformation[393] adds nothing more to it; such that reciprocally, if in passing it through the die[394] or by any other means, we have been able

[392] That is, the behavior of the filaments is elastic only up to a certain characteristic angle.
[393] In the original: *écrouissement*. See footnote 375 on page 165.
[394] In the original: *si en passant à la filière*. See footnote 389 on page 174.

to give to our filament of brass a cold working such that its elastic reaction had been 520 degrees, which appears to me to be this of our two filaments at the moment of rupture, in this case the elastic reaction had been carried to its *maximum* by this first operation: There would not have been any more possible displacement in the center of the reaction of torsion; but all the time that we would have made to test this filament to a torsion of more than 520 degrees, it would break.

7.31 XXXI. Fourth Remark

From the preceding experiments, this, it appears, is how we can explain the elasticity and coherence of metals. The integral parts[395] of the filament of iron or of brass, or of any metal, have an elasticity that we can regard as perfect, that is to say, that the forces necessary to compress or dilate these integral parts are proportional to the dilatation or compressions they experience; but they are only tied together by the coherence, a constant quantity and absolutely different from the elasticity. In the first stages of torsion, the integral parts change their shape, elongating or compressing, without the points by where they adhere together changing position because the force required to produce these first stages of torsion is considerably less than the force of adhesion; but when the angle of torsion becomes such, that the force with which these parts are compressed or dilated is equal to the coherence which unites these integral parts, then they ought to separate or slide one on the other. This sliding of parts takes place in all ductile bodies but if by this sliding of parts, the ones on the others, the bodies compress, the extent of the points of contact increases and the extent of the domain of elasticity becomes greater. However as these integral parts have a determined figure, the extent of the points of contact can only increase up to a certain degree, beyond which the body breaks; it is this which explains the detailed facts of the preceding Section. This which proves again that it is necessary to distinguish the cause of elasticity from the adhesion, is that we can vary the coherence at will by the degree of heat treatment without altering in any way the elasticity. It is thus the case when I heated to white[396] my No. 1 filament of copper in the preceding experiments, it lost a great part of its force of coherence: before heat treatment, it could carry up to the point of rupture 22 pounds and after the heat treatment it only carried 12 to 14 pounds; but while the adhesion was diminished nearly by half by the heat treatment and the amplitude of elasticity was nearly diminished in the same proportion, however in all the extent[397] of the elastic reaction that remained to the annealed wire, the elasticity was the same, at equal angle of torsion, as in the same filament not heat treated, since in suspending to one and the others the same weights, the time of the same number of oscillations was exactly equal in the two cases.

[395] In the original: *Les parties intégrantes*. This expression can also be translated as individual particles or integral particles, [Gillmor, 1971a, p. 159]. In Section 7.33 of this work Coulomb will refer to the *molécules intégrantes* of the metal, see footnote 414 on page 180. See also footnote 181 on page 72. The integral parts here can mean any part of the wire, small or large, which can be compressed or stretched. They can also be the parts which contribute to the integrity of the whole.

Definition of the *Dictionnaire de l'Académie Française*, [de l'Académie Française, 1798]: "Les parties intégrantes. On appelle ainsi en Philosophie, les parties qui contribuent à l'intégrité d'un tout, à la différence des parties qui en constituent l'essence. Les bras, les jambes sont des parties intégrantes du corps humain."

[396] In the original: *je faisais recuire à blanc*.

[397] In the original: *toute l'étendue*.

7.32 XXXII

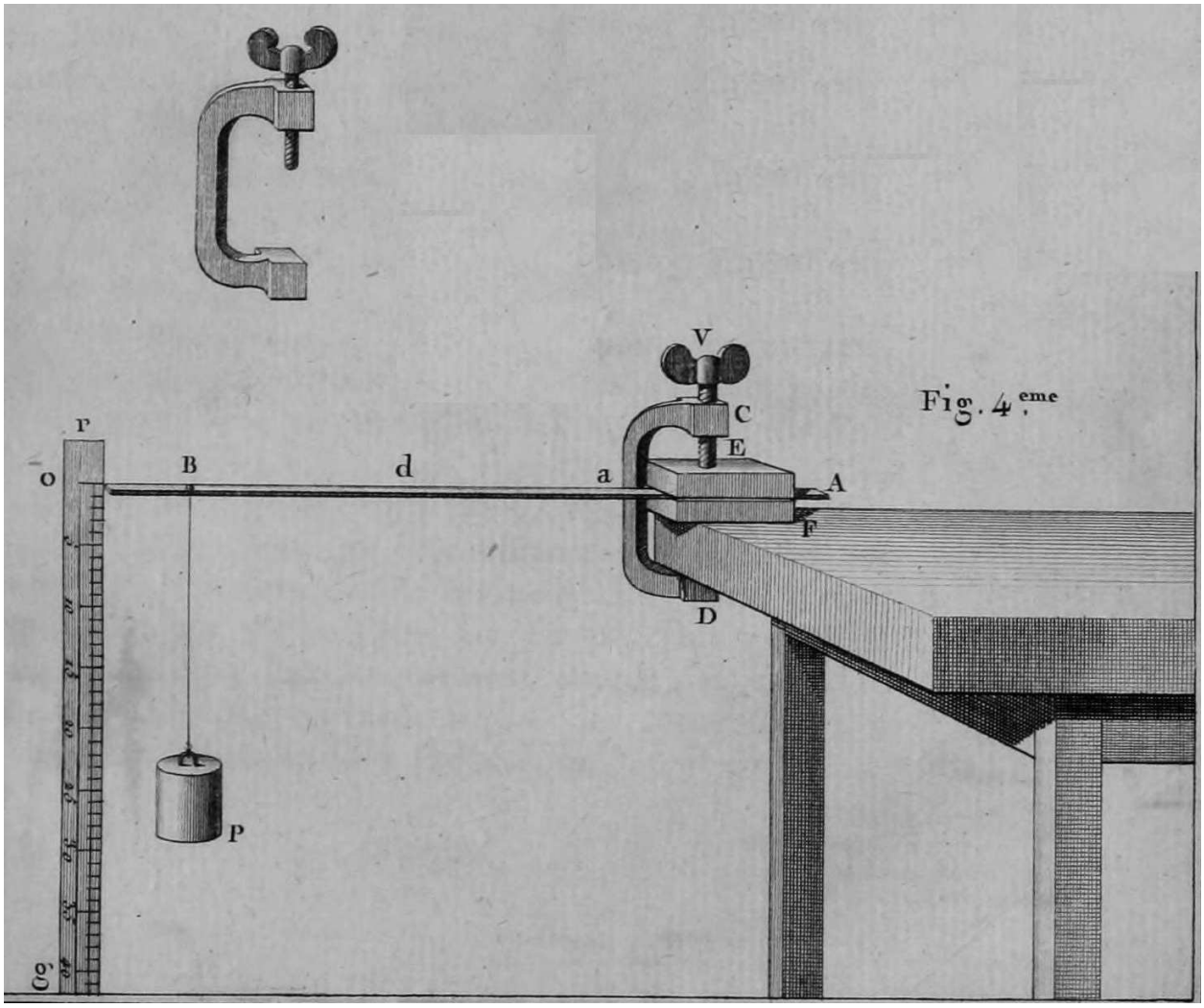
An equally interesting effect due to the approximation[398] of parts in torsion of filaments of metals is this which takes place when we twist a filament of iron, which by this operation alone acquires through the approximation of parts, the quality of taking the magnetism to a higher degree than it had before. Here is the experiment which revealed this to me; I have taken a filament of iron, such as we find them throughout the world of commerce, of the thickness of those which serve for the small sounding bars;[399] a length of six inches [16.24 cm], weighing 57 grains (3.13 g); this filament of six inches, magnetized and suspended horizontally by a filament of silk, untwisted[400] and very fine, makes an oscillation in 18 seconds: this same filament of six inches in length, twisted up to the point of rupture and magnetized as in the first case to saturation by the method of double touch,[401] makes an oscillation in 6 seconds; such that the *moment* of the directive force[402] for the two needles equal and similar, being as the inverse of the square of the times for the same number of oscillations, the magnetic *moment* of the twisted needle, was nine times more considerable than that of the needle not twisted: I will have the occasion to return to this Section in another Memoir.

7.33 XXXIII

To confirm all the preceding theory regarding the coherence and elasticity, I have made the following experiments.

We have fixed, *Figure 4*, by means of a clamp CD[403] with a vise V, a lamina of steel AB on the edge of a very solid table; this bundle being pressed and held[404] in its part Aa, between two plates of iron E and F, by the vise V: this lamina was 11 lines wide (2.48 cm) and half a line thick [0.11 cm] from point a to point B, where was suspended the weight P, there was seven inches of length (18.95 cm): we measured on the vertical rule rg, how much the weight P made lower the lamina AB at its extremity B. Here are the details of the results which took place following the different weights with which the lamina was loaded.

[398] In the original: *rapprochement*.
[399] In the original: *sonnettes*.
[400] In the original: *détordu*.
[401] See Section 4.6.
[402] See footnote 286 on page 127.
[403] In the original: *au moyen d'une agrafe CD*.
[404] In the original: *serrée*.



We heated the lamina to white and we have given it a quenching;[405] then we have attached at B at seven inches from point a, different weights.

With a weight of	The extremity B has deflected
1/2 pound (245 g)	8 lines (1.80 cm)
1 pound (489 g)	$15\frac{1}{2}$ lines (3.49 cm)
$1\frac{1}{2}$ pound (734 g)	23 lines (5.19 cm)

We have taken this same lamina and we have heated it until it took on a violet color and it returned to the consistency of an excellent spring; and we have found equally, that in loading it as in the first case,

With a weight of	The extremity B has deflected
1/2 pound (245 g)	8 lines (1.80 cm)
1 pound (489 g)	$15\frac{1}{2}$ lines (3.49 cm)
$1\frac{1}{2}$ pound (734 g)	23 lines (5.19 cm)

[405] In the original: *On a fait rougir la lame à blanc et on lui a donné une trempe très raide.* In materials science, quenching is the rapid cooling of a workpiece in water, oil or air to obtain certain material properties. A type of heat treating, quenching prevents undesired low-temperature processes, such as phase transformations, from occurring. Quenching can reduce the crystal grain size of both metallic and plastic materials, increasing their hardness.

Finally we heated this same lamina to white and let it cool[406] very slowly; and we have had, in loading the extremity B [with a weight P], exactly the same results as in the two preceding experiments.

It appears to us that these three experiments prove in an incontestable manner, that whatever the state of the lamina, the first degrees of its elastic force are in no way altered; since in taking account of the lever arm, which diminishes as the lamina is loaded, the same weights deflect it in the three states equally and proportionally to the load; [and] when one removes[407] the weights, it retakes exactly its original horizontal position.

I have wished to see subsequently what be the force of this lamina in these three different states; and in the case where the center of flexure would begin to displace, what would be the degree of flexure where the lamina would begin to be deformed without returning to its original position. Here is the result of this experiment.

I have cut from a sheet[408] of English steel, three lamina exactly similar to this of the preceding experiment: one of these lamina have been quenched, the second had been returned to the consistency of an excellent spring, and the third had been heat treated to white and slowly cooled. I attached, *Figure 4*, a spring scale[409] d at two and a half inches distant from point a and I had carefully exerted a pull always perpendicular to the direction of the lamina. Here is what I observed.

The lamina which had been rapidly quenched broke under a pull of six pounds; but under whatever angle at which it was deflected below this of rupture, it returned exactly to its original position. The lamina returned to a violet color, forming an excellent spring, broke only under a pull of eighteen pounds; it bent[410] up to the point of rupture, with an angle nearly proportional to the angle of torsion, and under any angle that it was bent before rupture, when we freed it,[411] it retook its original position. The lamina heat treated to white and slowly cooled, bent up to a pull of five to six pounds, proportionally to this force of pull, and with an angle absolutely equal under the same force that in the state of quenching

[406] In the original: *refroidir*.
[407] In the original: *ôtoit*.
[408] In the original: *planche de tôle*.
[409] In the original: *peson*. This word can also be translated as spring balance, Newton meter or spring dynamometer. In this case, this dynamometer is applied at point d, exerting a force perpendicular to the plate AB, directed upwards. It is held at the top, having a hook at its lower end that is connected at point d of the lamina. The Figure of this footnote shows a typical spring scale:

[410] In the original: *se plioit*.
[411] In the original: *on la lachoit*.

and of spring;[412] but in pulling always subsequently perpendicular to the direction of the lamina, in order to conserve the same lever, with a force of seven pounds, we have bent it under all the angles, without that it was necessary to augment this force: in letting go, it raised itself back up only by the quantity of which it had been originally deflected by a pull of six pounds; such that the angle of reaction of flexure, found itself changed from all the angle which we had bent it with a force greater than seven pounds.

These last experiments lead us back to the same results as those which went before. It is clear that in order to have an idea of what happens in the flexure of metals, it is necessary to distinguish the elastic force of the integral parts from the force of adhesion which ties these parts together: the elastic force depends, as we have already said, on the compression or dilation that the integral parts experience and is always proportional to the tractions. These integral parts are not altered, neither by the quenching nor by the heating, since we see that in theses different states, the elasticity is the same under the same degrees of flexure; but these integral parts, are only tied among themselves by a certain degree of adhesion which probably depends on their shape and on the respective portion of the different fluids with which their pores are filled, this which varies according to the quenching and the heating. In the quenched steel[413] and in the good springs the integral molecules[414] can neither slide one on the other nor experience the least displacement without the body breaking; but in the ductile bodies, in the heat treated metals, these parts can slide one on the other and displace themselves, without the adhesion being sensibly altered.

This that we have come to explain for metals appears to be able to be applied to all bodies; their parts are always of a perfect elasticity, but the bodies are hard, soft or fluid, according to the adhesion of their integral parts. If in the hard bodies, they can slide one upon the other, without their distance being sensibly altered, the body will be ductile or malleable; but if they cannot slide one on the other, without their respective distances being sensibly altered, the bodies break when the force with which the bodies will be pulled or compressed, will be equal to the adhesion.

[412] In the original: *dans l'état de trempe et de ressort*. See also footnote 405 on page 178.

[413] In the original: *dans l'acier trempé roide*.

[414] In the original: *molécules intégrantes*. See footnote 395 on page 176. This expression can also be translated as integral particles, [Gillmor, 1971a, p. 159].

Chapter 8

Bucciarelli's Remarks on Coulomb's 1784 Paper

L. L. Bucciarelli[415]

8.1 Weight and Length Conversion Factors

Source: Kisch, B., *Scales and Weights: A Historical Outline*, New Haven: Yale University Press.[416]

Table 1. Weights, Mass:

	livre	*marc*	*onces*	*gros*	*deniers*	*grain*	*grams*
livre	1						489.506
marc	2	1					244.753
onces	16	8	1				30.594
gros	128	64	8	1			3.823
deniers	384	192	24	3	1		1.274
grain	9,216	4,608	576	72	24	1	0.0531

Table 2. Lengths:

	toise	*pied*	*pouce*	*ligne*	*mm*
toise	1				
pied	6	1			324.8
pouce	72	12	1		27.07
ligne	864	144	12	1	2.26

8.2 Coulomb's Torsion Experiments. Iron Suspension Wire Characteristics

Figure 8.1 shows the experimental setup for the *Première Essai* of the *PREMIÈRE EXPÉRIENCE*, Section 7.10.

[415] See also [Bucciarelli, 2001].
[416] [Kisch, 1976].

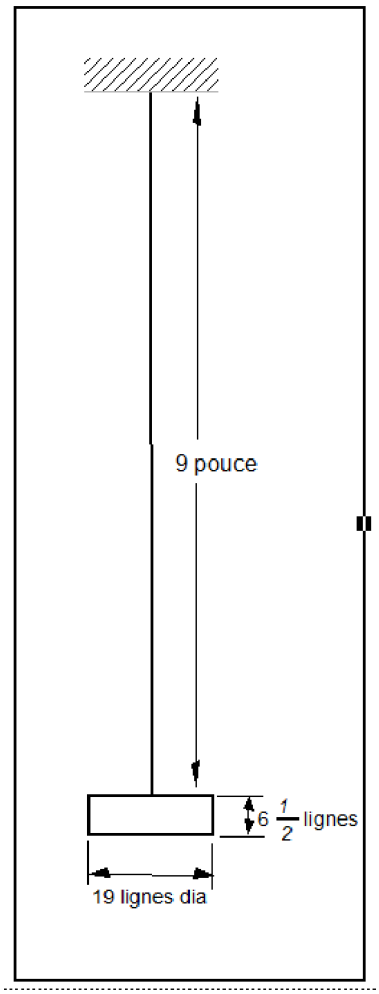


Figure 8.1: Setup for the *Première Essai* of the *Première Expérience*.

The lead solid cylinder, with diameter of 19 lignes and height of 6.5 lignes, is suspended by a steel wire, # 12 gauge, and of length, according to Coulomb's original text, of "9 lignes". As we will show, this last dimension is incorrect, a typo no doubt; we take it as 9 pouces (= 108 lignes) as shown in Figure 8.1.[417]

As a check on this, and on Coulomb's results in general, we first determine the wire diameter from the information given in his original text, then, from the expression for the fundamental frequency of the torsional pendulum in terms of the torsional stiffness of the wire and the polar mass moment of inertia of the lead cylinder, deduce the length of the suspension wire. First, the wire diameter:

Coulomb writes that his # 12 gauge wire supports, before breaking, *3 livres 12 onces* and a length of *6 pieds* (195 cm) weights *5 grains* (0.266 g). Leaving aside for the moment consideration of the tensile strength, we have that the wire mass per unit length (m/L) is then $1.36 \times 10^{-3}\ g/cm$. The mass per unit length equals the product of the density, δ, and the cross-sectional area, A:

[417] In the THIRTEENTH EXPERIMENT, Coulomb writes "The length of the filaments of suspension in all the preceding experiments being 9 pouces".

$$\frac{m}{L} = \delta \cdot A \ . \tag{8.1}$$

For the density: Coulomb, in a final paragraph of section XV, reports that

Le pied cube de fer, pesant à peu-près 540 livres,...

This gives, after conversion, $\delta = 7.70\ g/cm^3$ for the density. So $A = 1.77 \times 10^{-4}\ cm^2$; and the diameter $d = 1.50 \times 10^{-2}\ cm^2$.[418]

We can compare the properties of Coulomb's gauge # 12 wire with those of standard steel wire sizes of today. From a steel wire gauge chart posted by a maker of precision wire drawing dies,[419] we find a gauge # 44 has a diameter of 0.01472 cm and a weight per 1000 feet of 0.0908 pounds. The diameter is within a few percent of the wire diameter Coulomb apparently used; 0.0908 pounds per 1000 feet is equivalent to $1.35 \times 10^{-3}\ g/cm$. This is a very fine music wires size, at the limit of wires used in the treble range of a harpsichord.

Coulomb states that the # 12 gauge wire supports, before breaking, *3 livres 12 onces*. The latter is equivalent to 18 N, neglecting the weight of the wire itself. The stress at rupture, the ultimate tensile stress, is then the latter divided by the wire cross-sectional area which gives:

ultimate stress $= 1.02 \times 10^9\ N/m^2 = 1020\ MPa$ (mega pascals).

Our estimation of the ultimate strength of the # 12 gauge wire appears high. To be more precise, we would need to know more about the strings of the harpsichord of "fer" (and of "laiton"). Coulomb used in his experiments — "...strings of the clavecord such as one finds throughout commerce, wound on bobbins and numbered...". To explore further, we consider experiments done with wire of gauge # 7 and # 1.

Iron Suspension Wire Data					
Material	Rupture Load livre (g)	Mass/Length grain/pied (g/cm)	Diameter cm	Area cm^2	Rupture stress Mpa
Iron # 12	3.75 (1836)	5/6 (1.36×10^{-3})	0.0150	1.77×10^{-4}	1020
Iron # 7	10 (4895)	14/6 (3.81×10^{-3})	0.0251	4.95×10^{-4}	969
Iron # 1	33 (10769)	56/6 (1.52×10^{-2})	0.0502	1.98×10^{-3}	799

With decrease in wire diameter, moving up in the Table, values for the rupture stress increase significantly — a result of the cold working of the material drawn through a succession of dies.[420] Still these values are high when compared to data obtained from one online

[418] In that same paragraph, section XV, Coulomb goes on to state that "the diameter of a filament of iron, # 12, 6 feet long, weighing 5 grains, is approximately a fifteenth of a ligne", $(0.226/15 = 0.00151)$!

[419] https://www.fwwd.com/wp-content/uploads/2016/02/FWWD_Steel_wire_guage_chart.pdf published online by Fort Wayne Wire Die, https://www.fwwd.com/.

[420] *Reproduction of Authentic Historical Soft Iron Wire for Musical Instruments*, Stephen Birkett and Paul Poletti, https://www.researchgate.net/publication/228376665_Reproduction_of_Authentic_Historical_Soft_Iron_Wire_for_Musical_Instruments, [Birkett and Poletti, s d]. Also: https://fiskalloy.com/products/wire-facts/tensile-yield-and-elongation/.

source.[421] There the ultimate tensile strength of wrought iron is given as within the range 34 to 54 kpsi which converts to 234 to 372 MPa.

***

To check Coulomb's results for the period of oscillation of his torsion pendulum of varying suspension wire and suspended mass characteristics, we deduce the length of the suspension wire from the equation for the *frequency of oscillation* of the torsional pendulum, f, in terms of these system's characteristics. We have

$$f = \frac{1}{2\pi} \cdot \sqrt{\frac{k}{I}} \ , \tag{8.2}$$

where k is the *torsional stiffness of the wire* and I is the *polar mass moment of inertia* of the suspended mass. Noting that *Coulomb's time period* T of an oscillation is one half that of a full a full cycle we have

$$T = \pi \cdot \sqrt{\frac{I}{k}} \ , \tag{8.3}$$

where

$$I = \frac{1}{2} \cdot M \cdot R^2 \qquad \text{and} \qquad k = \frac{GJ}{L} \ , \tag{8.4}$$

M is the *mass of the cylinder*, R its *radius*; L is the *length* of the wire, G is the *shear modulus* of the wire ($G = 77 \times 10^9 \ N/m^2$),[422] and J is the *polar moment of inertia* of the wire cross section about the axis of the wire,

$$J = \frac{\pi \cdot r^4}{2} \ , \tag{8.5}$$

where r is the *wire radius*. Putting this all together and manipulating, we obtain the following expression for L, the *length of the wire*

$$L = \frac{T^2 G r^4}{\pi M R^2} \ . \tag{8.6}$$

For the *Première Essai* of the *PREMIÈRE EXPÉRIENCE*, the wire radius $r = 7.50 \times 10^{-5} \ m$. The cylinder makes 20 oscillations in 120 seconds, so $T = 6 \ sec$.

The suspended lead cylinder has a diameter of 19 *lignes* (4.29 cm) and height of 6.5 *lignes* (1.47 cm). With the *density of lead* $= 11.34 \ g/cm^3$, M, the mass of the cylinder is 0.241 kg. (Coulomb's weight of the cylinder is given as 1/2 *livre* which converts to 244 kg.).

Summarizing: $T = 6 \ sec$, $R = 4.29/2 \ cm = 2.145 \times 10^{-2} \ m$, $M = 0.241 \ kg$, $r = 7.50 \times 10^{-5} \ m$ and $G = 77 \times 10^9 \ N/m^2$.

This gives, for the wire length

$$L = 0.251 \ m = 9.3 \ pouce \ . \tag{8.7}$$

[421] https://engineersedge.com/manufacturing_spec/properties_of_metals_strength.htm.

[422] We take the shear modulus as 40% of the elastic modulus of wrought iron. The latter is given as 193 GPa in https://en.wikipedia.org/wiki/Wrought_iron (40% assumes the material is isotropic which is probably not justified but a more exact value for the shear modulus is difficult to define or find.)

Recall that Coulomb gave the wire length as 9 lignes, which we read as a typographical error. This calculation of the wire length using Coulomb's stated wire and mass characteristics confirms our reading.

8.3 Section XIII, *De la Force de Torsion Relativement à la Grosseur des Fils*

In Section XIII, *De la force de torsion relativement à la grosseur des fils*, Coulomb, first presents a "theory" from which he concludes that

> ...we are led to believe, from theory, that the force of torsion of two filaments of metal of the same material and of the same length but of different thickness, *is proportional to the fourth power of their diameter*...(emphasis mine).

Then, from an analysis of the results of his experiments suspension wires of brass and of iron, show that this is indeed the case (evidently within an acceptable limit of accuracy). It is his theory which concerns us here, to wit:

> Voici ce que le raisonnement doit faire prévoir; le *momentum* de la réaction de torsion doit augmenter, avec la grosseur des fils, de trois manières. Prenons pour exemple deux fils de même nature & de même longueur, que le diamètre de l'un soit double de celui de l'autre, il est clair que dans celui qui a un diamètre double, il y a quatre foîs plus de parties tendues par la torsion, que dans celui qui a un diamètre simple; & que l'extension moyenne de toutes ces parties sera proportionnelle au diamètre du fil, de meme que le bras moyen du levier relativement à l'axe de rotation. Ansi nous sommes portés à croire...,

Here is my conjecture of his reasoning. First he considers two wires of the same material and length, the diameter of one being double that of the other.

> Prenons pour exemple deux fils de même nature & de même longueur, que le diamètre de l'un soit double de celui de l'autre, il est clair que dans celui qui a un diamètre double, il y a quatre foîs plus de parties tendues par la torsion,

Since both wires have a uniform circular cross-section along their length, we need only consider a cross section and its "parts". Then think of each part as a differential element of area and since the area is proportional to the square of the diameter, the wire of double diameter will have four times more parts *stretched* by the torque of the wire.

Now we have to consider what is happening to each of the "parts". He writes

> l'extension moyenne de toutes ces parties sera proportionnelle au diamètre du fil,

Consider his analysis of V, First Section where he writes that the velocity of any point in a cross-section is a proportional to the radius, r, at the point, i.e., $v = u \cdot r/a$ where u is the velocity when $r = a = D/2$. Now here is a problem: If the velocity of each point in a cross-section moves in this way, it means that the cross section moves like a rigid disc. There is no relative motion of one point in the plane relative to any other point. So what is this *extension* he refers to?

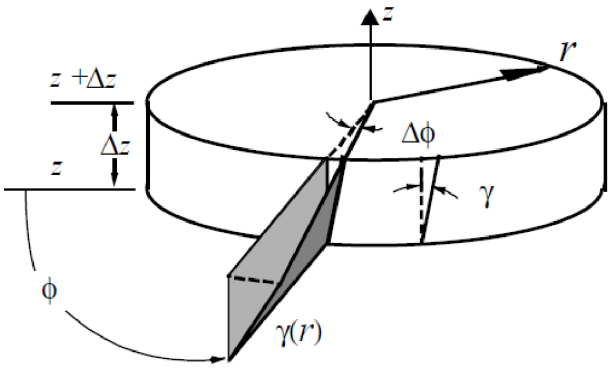


Figure 8.2: Shear strain.

In today's analysis, we consider the shear strain due to the rotation of one cross section relative to another as shown in Figure 8.2. The shear strain γ is a linear function of the radius and is given by $\gamma = r \cdot \Delta\phi/\Delta z$.

Did Coulomb "see" this shear strain as his *extension*? We can only conjecture.

The main point is that the extension varies linearly with radius so its mean is proportional to the diameter of the wire.

de même que le bras moyen du levier relativement à l'axe de rotation

just as the mean of the lever arm of the part is proportional to the diameter of the wire since the lever arm is by definition the radius, so it varies linearly with the radius.

So what! How does this all together justify the claim that the stiffness is proportional to the diameter to the fourth power?

Forget for the moment all talk about "means". Consider a part at radius r in the cross section. If we take the contribution to the torque as the product of the extension and the radius, with *the extension proportional to the radius*, then the contribution of the part to the torque will be proportional to the square of the radius. The sum of all these contributions of all the parts will then be proportional to the radius to the fourth power since we sum over the area which is proportional to the square of the radius.

The key ingredient of this "theory" is the claim that the extension is proportional to the radius and just as important, we interpret this extension as a force. This last reading reminds us that Coulomb does not hesitate to interpret an angular displacement as a reactive torque.

Chapter 9

Assis' Remarks on Coulomb's 1784 Paper

A. K. T. Assis

9.1 Period of Oscillation of a Cylinder

I present here some comments that help to understand how Coulomb obtained the period of oscillation of the cylinder in Section 7.5. The case being studied is illustrated in Figure 9.1.

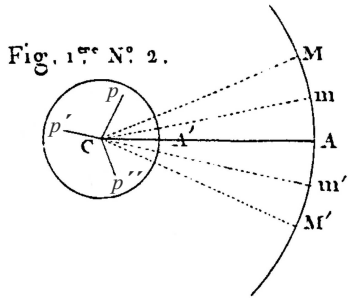


Figure 9.1: Figure 1, Number 2.

The direction $CA'A$ indicates the case in which the torsion wire is not twisted, where A' is a point on the circumference of the cylinder. The cylinder along with the lower part of the suspension wire is rotated through an angle ACM (this angle is represented by Coulomb by the capital letter A) and released from rest. The line CA' then oscillates between points M and M'. Coulomb represents the angle MCm by the capital letter S.

At the end of Section 7.5 Coulomb presented the equation of motion to be integrated:

$$dt = dS\sqrt{\frac{\int pr^2}{n(2AS - S^2)}} \ . \tag{9.1}$$

In this equation t indicates the time interval for the line CA' to start from M and reach point m, p represents an element of mass of the cylinder located at a distance r from the vertical axis of rotation passing through C, $\int pr^2$ is the moment of inertia of the cylinder, while n is a constant coefficient whose value will depend on the nature of the metal wire, its

length and its thickness. This quantity n is called the torsion coefficient of the wire. When the wire is attached by the upper end and its lower end is rotated through an angle θ about the vertical axis, the wire exerts a reaction torque τ on the cylinder given by $\tau = -n\theta$, where $\theta = 0$ is the situation where the wire is not twisted.

Then Coulomb described the result of the integration as follows:[423]

> Mais $\frac{dS}{\sqrt{2AS-SS}}$ représente un angle dont A est le rayon & S le sinus verse, qui s'évanouit lorsque $S = 0$, & qui devient égal à 90 degrés lorsque $S = A$.
>
> Ainsi le temps d'une oscillation entière sera

$$T = \left(\int \frac{\pi r^2}{n} \right)^{1/2} 180^d \, .$$

What Coulomb here calls 180^d is the ratio of the circumference to the diameter of a circle.[424] Today this ratio is represented by the letter $\pi = 3.14159....$ So I am going to replace 180^d with π, just as I am going to replace the expression "90 *degrees*" with $\pi/2$.

The letter π used by Coulomb to represent an element of mass in the expression of the moment of inertia $\int \pi r^2$ was replaced in the reprint of Coulomb's works in 1884 by the letter p, such that the moment of inertia is written as $\int pr^2$. In this English translation we are using this letter p adopted by Potier.

Here is the translation of that paragraph (see page 149 of the present work):

> But $dS/\sqrt{(2AS - S^2)}$ represents an angle of which A is the radius and S the versed sine,[425] which vanishes when $S = 0$, and which becomes equal to 90 degrees when $S = A$.
>
> Thus the time of a complete oscillation will be[426]

$$T = \pi \sqrt{\frac{\int pr^2}{n}} \, .$$

In that Section 7.5 Coulomb called T the "time of an entire oscillation". What he called the time of an entire oscillation is when the line CA' goes from M to M' in Figure 9.1.

I will call $T_{1/4}$ the time interval for the straight line CA' of the cylinder to start from rest from its initial angle $S = 0$ until it reaches the situation where the wire is not twisted with $S = A$, that is, for the straight line CA' to start from point M until it reaches point A in this Figure 9.1. Integrating Equation (9.1) with time t going from 0 to $T_{1/4}$ and with angle $MCm = S$ going from $S = 0$ to $S = MCA = A$ gives:

$$\int_{t=0}^{T_{1/4}} \sqrt{\frac{n}{\int pr^2}} dt = \int_{S=0}^{A} \frac{dS}{\sqrt{2AS - S^2}} \, . \tag{9.2}$$

[423] [Coulomb, 1787, p. 232] and [Potier, 1884, p. 69].

[424] See Alfred Potier's Introduction to Coulomb's works, [Potier, 1884, pp. x and 13], as well as the page 32 of this English translation.

[425] It seems that by a typographical error the integral symbol was missing before $dS/\sqrt{(2AS - S^2)}$. The sentence should read: "But $\int dS/\sqrt{(2AS - S^2)}$ represents an angle of which A is the radius and S the versed sine".

[426] In the original: *le temps d'une oscillation entière*. A complete oscillation here is but one half of what we today call a full cycle.

The left-hand side is given by:

$$\int_{t=0}^{T_{1/4}} \sqrt{\frac{n}{\int pr^2}} dt = \sqrt{\frac{n}{\int pr^2}} \int_{t=0}^{T_{1/4}} dt = \sqrt{\frac{n}{\int pr^2}} T_{1/4} \ . \tag{9.3}$$

The right-hand side is given by:

$$\int_{S=0}^{A} \frac{dS}{\sqrt{2AS - S^2}} = \int_{S=0}^{A} \frac{dS}{\sqrt{A^2 - (S-A)^2}} = \int_{S=0}^{A} \frac{\frac{dS}{A}}{\sqrt{1 - \left(\frac{S-A}{A}\right)^2}} \ . \tag{9.4}$$

Defining the magnitude $\gamma = (S - A)/A$ gives $d\gamma = dS/A$. Then:

$$\begin{aligned} \int_{S=0}^{A} \frac{dS}{\sqrt{2AS - S^2}} = \int_{S=0}^{A} \frac{d\gamma}{\sqrt{1-\gamma^2}} = [\arcsin\gamma]_{S=0}^{A} = \left[\arcsin\frac{S-A}{A}\right]_{S=0}^{A} \\ = \ [\arcsin 0 - \arcsin(-1)] = \left(0 - \frac{-\pi}{2}\right) = \frac{\pi}{2} \ . \end{aligned} \tag{9.5}$$

Equating the expressions (9.3) and (9.5) yields:

$$\sqrt{\frac{n}{\int pr^2}} T_{1/4} = \frac{\pi}{2} \ . \tag{9.6}$$

Twice $T_{1/4}$ gives the time interval for what Coulomb called the "time of a complete oscillation" (*une oscillation entière*) of the cylinder to occur, that is, the time T for the line CA' go from M to M' in Figure 9.1:

$$T_{Coulomb} = 2T_{1/4} = \pi \sqrt{\frac{\int pr^2}{n}} \ . \tag{9.7}$$

This result of the integration of Equation (9.1) was expressed by Coulomb as:

$$T = \left(\int \frac{\pi r^2}{n} \right)^{1/2} 180^d \ . \tag{9.8}$$

Nowadays, the time interval between the departure and the return to the same point is called the complete period of oscillation. That is, the time interval for the line CA' to start from point M, reach point M' and return to point M. This time interval is given by the quadruple of $T_{1/4}$. This complete period of oscillation is also represented today by the letter T:

$$T_{modern} = 4T_{1/4} = 2\pi \sqrt{\frac{\int pr^2}{n}} \ . \tag{9.9}$$

9.2 Torque Proportional to the Fourth Power of the Wire Diameter

I will detail here how Coulomb theoretically concluded that the torque exerted by the wire must be proportional to the fourth power of its diameter.[427]

[427] [Gillmor, 1971a, p. 156].

In Section 7.13 Coulomb gave three independent reasons why, assuming wires of the same material and of the same length, the torque should increase with increasing diameter D of the wire, namely:

1. The amount of matter (that is, the number of stretched parts, in Coulomb's terminology) is proportional to the cross-sectional area of the wire. It is then proportional to D^2.

2. The mean extension of the stretched parts of the wire[428] is proportional to the wire diameter D.

3. The mean lever arm with respect to the axis of rotation is also proportional to the diameter D.

Combining these three independent reasons, Coulomb concluded that the torque τ exerted by the wire must be proportional to D^4.

The first item is clear since the mass M of a cylindrical wire of length L, diameter D and volumetric density of mass δ is given by $M = \delta\pi(D/2)^2 L$. That is, M is proportional to D^2.

Regarding the second item, when Coulomb mentions the mean extension of the wire parts, he is referring to the horizontal distance of displacement or extension of the wire when it is twisted at an angle ψ with respect to the vertical axis. An element of mass dm at a distance ρ from the axis of rotation moves a distance $s = \rho\psi$ when the wire rotates through an angle ψ. Let us imagine a homogeneous disk of thickness dz, radius R, volumetric density of mass δ and mass $M = \delta\pi R^2 dz$. A mass element dm located at the cylindrical coordinates $(\rho,\ \varphi,\ z)$ can be written as $dm = \delta\rho d\varphi d\rho dz$. The average displacement $\bar{s}$ of all elements of mass when this disk rotates through an angle ψ is then given by:

$$\bar{s} = \frac{\int\int s dm}{M} = \frac{\int_{\varphi=0}^{2\pi}\int_{\rho=0}^{R}(\rho\psi\cdot\delta\rho d\varphi d\rho dz)}{\delta\pi R^2 dz} = \frac{2\psi\int_{\rho=0}^{R}\rho^2 d\rho}{R^2} = \frac{2R\psi}{3} = \frac{D\psi}{3}\ . \tag{9.10}$$

That is, the mean displacement is proportional to the diameter D, as stated by Coulomb.

Regarding the third item, it is concluded in the same way that the average arm of the lever $\bar{\rho}$ in relation to the vertical axis of rotation is given by:

$$\bar{\rho} = \frac{\int\int \rho dm}{M} = \frac{\int_{\varphi=0}^{2\pi}\int_{\rho=0}^{R}(\rho\cdot\delta\rho d\varphi d\rho dz)}{\delta\pi R^2 dz} = \frac{2\int_{\rho=0}^{R}\rho^2 d\rho}{R^2} = \frac{2R}{3} = \frac{D}{3}\ . \tag{9.11}$$

Again, this mean lever arm is proportional to the diameter D of the wire, as stated by Coulomb.

Suppose we have two wires of the same material, of the same length, twisted at the same angle, but with diameters D_1 and D_2. In this case, the ratio between the torques τ_1 and τ_2 exerted by them is given by:

$$\frac{\tau_1}{\tau_2} = \left(\frac{D_1}{D_2}\right)^4\ . \tag{9.12}$$

[428] Or the mean extension of the wire particles, according to Gillmor, [Gillmor, 1971a, p. 156].

Chapter 10

Description of a Compass Needle Suspended by a Silk Thread

Coulomb[429]

I described in a Supplement to the researches on magnetized needles printed in Volume XI of the *Savans étrangers*, on *page* 215, a compass needle suspended by a silk thread for the determination of the diurnal variations [of the terrestrial magnetism].[430] That presented today was built according to the same principles; but it has a simpler construction and it is more convenient in its uses.

Recall that it was proved on *page 205 and the following*[431] of the work just quoted that, when we take into account the precautions that were indicated in that work, the force of torsion of a silk thread can only have a negligible influence on the direction of a magnetized needle suspended by this thread.

It was also proved on *page 209 and the following*,[432] that air resistance has no influence upon the direction of the needle, or at least has a certain amount of influence that we can neglect.

Finally it was demonstrated on *page 221 and the following* of the same Memoir,[433] that when a magnetized lamina[434] is suspended horizontally, if we draw an arbitrary line on this lamina and observe the direction of this line, before and after the inversion of the lamina,[435] the direction of the meridian line will divide in two equal parts the angle formed between the two observed directions.

The *first Figure* shows in perspective all parts of the new compass.

[429] [Coulomb, 1788a], with Portuguese translation in [Assis, 2022]. This work was presented in 1785 to the French Academy of Sciences and published in 1788. It was not included by Potier in the reprint of Coulomb's works, [Potier, 1884].

[430] [Coulomb, 1780, p. 215]. This work was published in Volume IX and not in Volume XI as mentioned here by Coulomb. It is translated in Chapter 5. See, in particular, Section 5.4 on page 97.

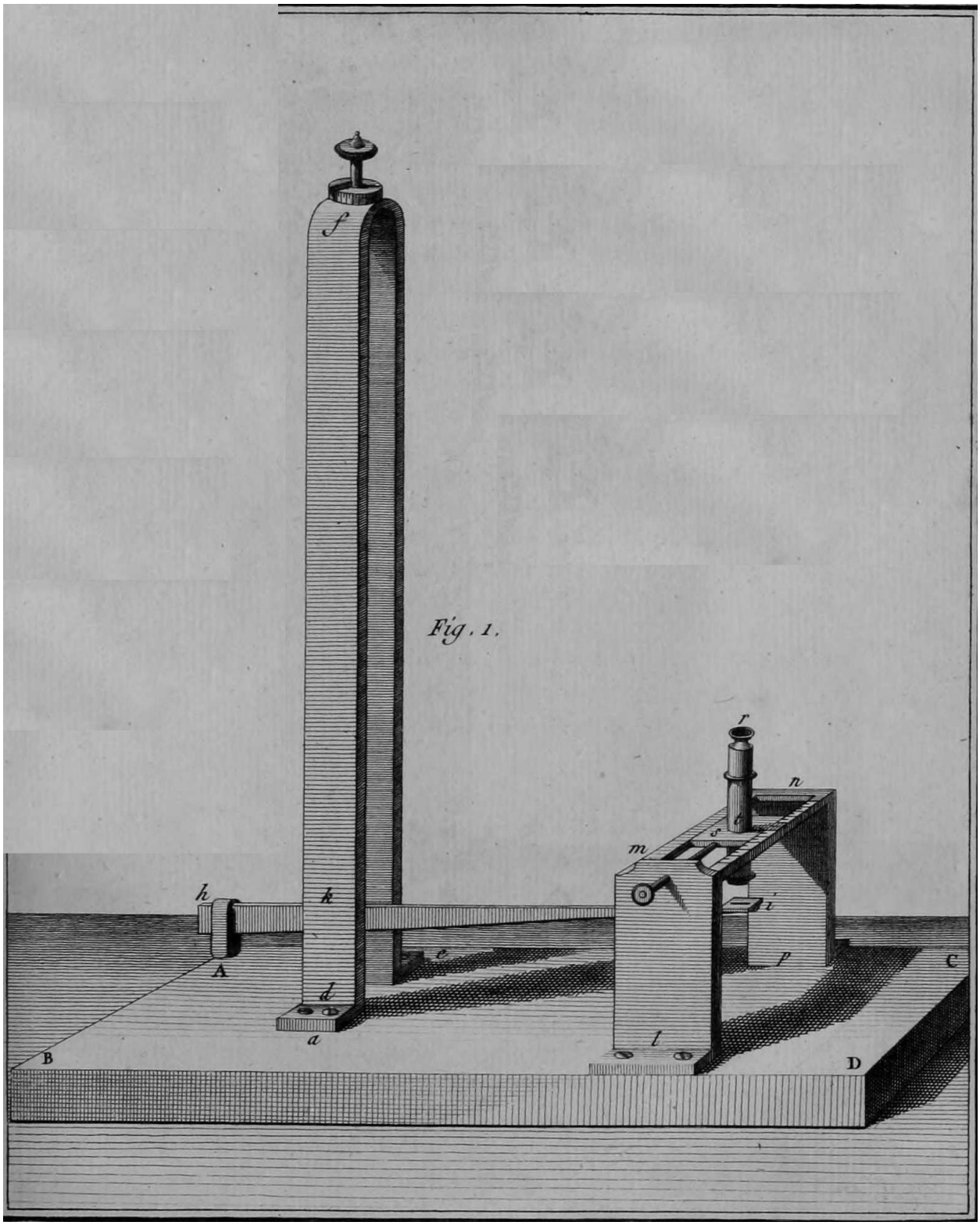
[431] Corresponding to pages 88 and the following of this English translation.

[432] Corresponding to pages 91 and the following of this English translation.

[433] Corresponding to pages 103 and the following of this translation. See, in particular, Article 63 on page 104.

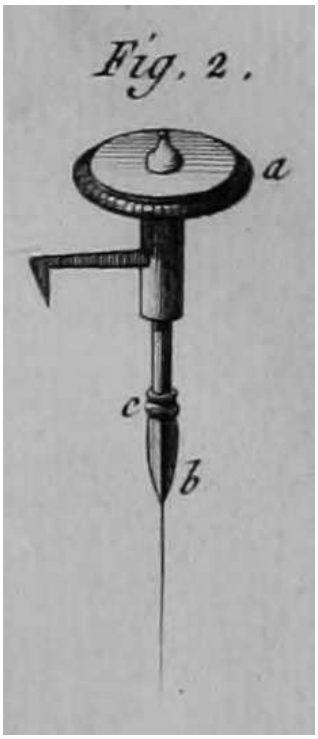
[434] In the original: *une lame aimantée*.

[435] In this inversion the lamina turns upside down, see Figure 15 on page 105.



ABCD is a block of stone cut at right angles and utilized to support the compass. This stone is twenty four inches long [65.0 cm], nine inches wide [24.4 cm] and four to five inches thick [10.8 to 13.5 cm]. The long side *BD* is orientated approximately in the direction of the magnetic meridian of the local where the observations will be made. One fixes in *ae*

parallel to the side AB, at ten inches [27.1 cm] from this side AB, a lamina of red copper[436] eighteen lines wide [4.1 cm], five inches long [13.5 cm] and two to three lines thick [0.45 to 0.68 cm]. This lamina is embedded and cemented into a notch made in the stone. On this plate rises perpendicularly a fork df,[437] fixed by its heels da on the first copper plate by means of the screws that we see in the Figure. On the upper portion of this fork, in f, there is a suspension clamp[438] seen in detail in the second Figure.[439]



Button a is utilized to turn this clamp. There is a slit in b which holds the suspension thread. In c we have the ring which tightens the clamp. We place in f, *Figure 1*, at the top of the fork, a small horizontal circle with its center aligned with the center of the clamp hole. This circle divided in degrees will serve, if we wish, to know how much a given angle of torsion is able to deflect the needle from its magnetic meridian.

Figure 3 presents a second clamp suspended at the silk thread by its upper part a and which, by its lower part b, seizes the magnetized needle which, in this way, is suspended sideways.[440]

[436] In the original: *une lame de cuivre rouge*. This expression can also be translated as a lamina of reddish copper.

[437] In the original: *fourchette*. This word can also be translated as bracket or wishbone.

[438] In the original: *pince de suspension*. This expression can also be translated as "clasp of suspension". See also footnote 340 on page 153.

[439] This suspension clamp seen in Figure 2 works as a micrometer. This micrometer is represented by button a. There is a horizontal index connected to it. At the lower part of the micrometer there is a slitted clamp b on which one fixes the upper end of the vertical thread utilizing ring c which tightens the clamp. When one turns the micrometer, the upper portion of the thread turns with it and the horizontal index moves around a scale graduated in degrees. This angular scale is fixed in the laboratory, going from zero degrees up to 360°. In this way it is possible to measure the torsion angle of the upper part of the thread connected to the micrometer.

[440] In the original: *suspendue de champ*. Consider a parallelepiped with larger side L, second side W and shorter side T such that $L > W > T$. It is suspended on the side when the length L and the thickness T

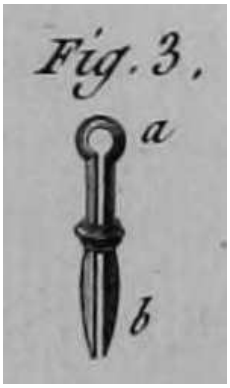


Figure 1 presents the magnetized needle in hi. It measures six inches [16.2 cm] from h to k, where its point of suspension is located, and twelve inches [32.5 cm] from this point k up to its other end i; in such a way that it has a total length of eighteen inches [48.7 cm]. In h we have a mobile ring working as a counterweight to keep the needle in a horizontal position; a small silver plate is welded horizontally above the needle in i, we draw a line on this plate along the direction hi, in the middle of the needle thickness. This needle ought to be of a good steel, well drawn, first quenched to be very hard and then brought to the consistency of a spring; then magnetized by the double touch method,[441] it can be given other dimensions than those I have just indicated, provided that the strength of the suspension wire is proportioned to its weight.[442] The one I use is ten lines wide [2.26 cm] at the extremity h, three lines [0.68 cm] at the end i and three quarters of a line [0.17 cm] of thickness uniformly [along its length].

To observe the variation of this needle,[443] one uses, *Figure 1*, the micrometer $lmnpq$ of red copper. It is composed of a supporting pad lp; of two uprights lm and pn; of a horizontal frame nm; and of a cursor s, which carries in its center a microscope[444] with two lenses rt. The focal point of this microscope is placed at twelve inches [32.5 cm] from the suspension wire fk. The frame mn of the micrometer is represented in detail and seen from above in *Figure 4*.

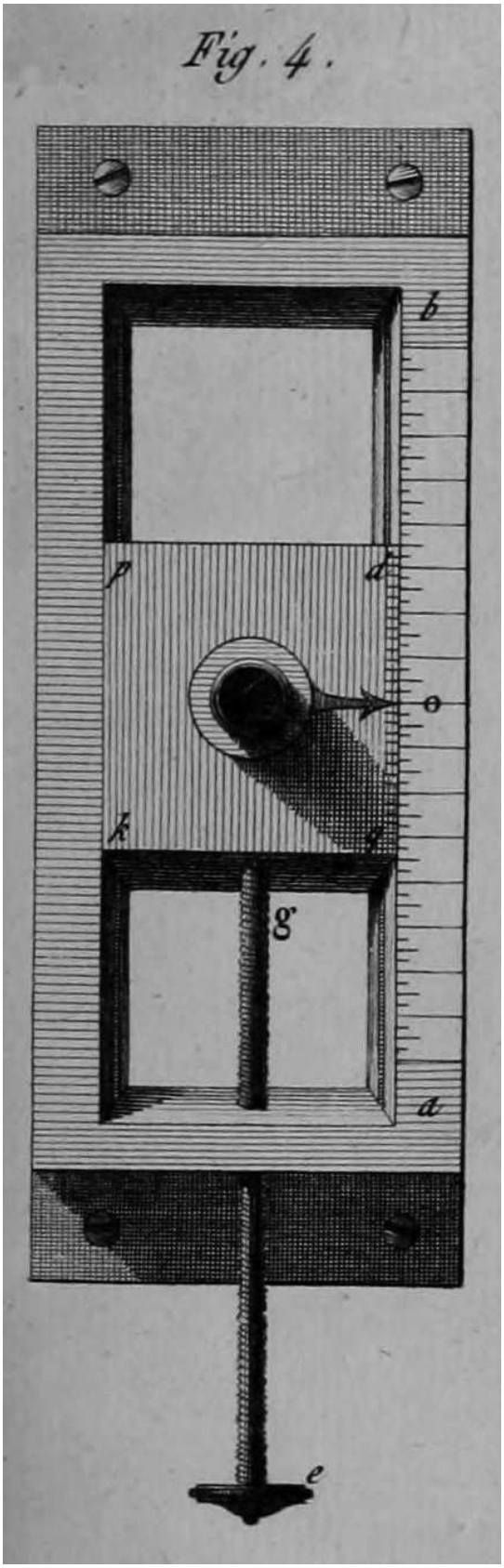
are horizontal, while the width W is vertical.

[441] See Section 4.6.

[442] That is, the strength of the suspension thread should increase proportional to the weight of the needle.

[443] That is, the diurnal change in the orientation of this needle due to the variation of terrestrial magnetism.

[444] In the original: *une lunette microscopique*.



We see that the side *ab* of this frame is divided on each side, from its middle *o*, in eight equal parts, of which the two or three initial [parts] of each side represent degrees, because

the arc, the sine and the tangent are nearly the same in the first degrees.[445] Each one of these degrees is divided into four parts, which are therefore equal to fifteen minutes.

Each side *od* and *oq* of the cursor corresponds to three degrees and a half, and is divided into fifteen parts, each of which, therefore, equals fourteen minutes, or differs of one minute from each division of the frame, which forms a Nonius[446] which measures the minutes. As during the diurnal variations the needle rarely moves more than thirty minutes we can, without perceptible error in observation, take the divisions for degrees. If, however, the variations were considerable, or if in the variations that the needle would have experienced, from the beginning of the observations, it had arrived at two or three degrees away from the first division *o*, we could, if we wanted greater precision, calculate the angular variations according to the measures given by the tangent divided here in equal parts.

The microscope has two very thin silk threads crossed at its focal point. It is necessary to turn this microscope utilized to observe the line of the needle at *i*, such that this line appears at the focal point of the microscope, in line with one of the threads. One makes the cursor (*Figure 4*) *qkpd* follow the motions of the needle by means of a screw *eg*, and of a bevelled groove on the interior sides of the frame, in which groove the cursor slides.

I will not carry this description further since the proportions of most of the parts of this compass, modified in implementation, do not alter its function provided that the mobility of the needle remains the same.

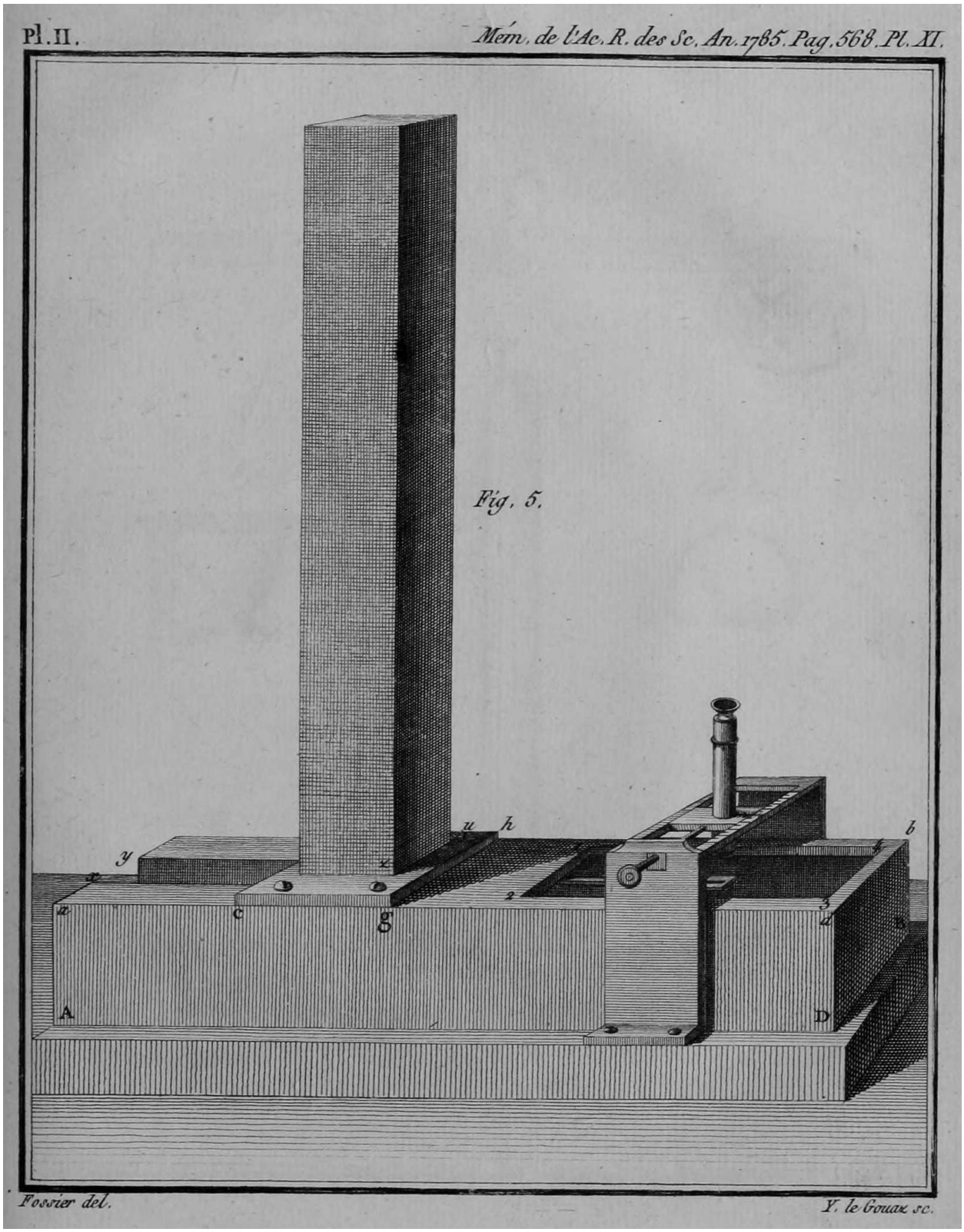
The silk threads do not require any preparation; we put them together by placing several cocoons in boiling water, unwinding them to the crusade, as usually done in the manufactures, or simply by pressing them between two fingers and fusing them together when they leave the boiling water. Although the strength of each thread of silk varies a lot, it is rarely less than 80 grains;[447] but, in practice, after joining them together, we can only count on each carrying 50 grains [2.7 g]. If we wish, we can replace the boiling water, uniting several silk threads by dipping them in gummed water. Although they become a little more rigid in this way than with the first procedure, this stiffness does not suffice to sensibly alter the displacement of the needle.

Before placing our [magnetized] needle in the suspension clamp *ab*, *Figure 3*, it is initially necessary, as we said in the aforementioned Memoir, to suspend in this clamp a lead or copper lamina, which can turn in the suspension fork, and which has exactly the same weight as the needle which will replace it after the first suspended body, having stopped naturally approximately along the magnetic meridian, will indicate that there is no longer any force of torsion in the suspension.

The fork, *Figure 1*, will be eighteen inches high [48.7 cm]. This height allows the suspension wire to be twelve to fifteen inches long [35.5 to 40.6 cm].

All the parts that we have described are those which essentially constitute the compass. However, the great sensitivity of the needle so suspended, would render any observation impracticable, if the needle is uncovered and exposed to all air movements in the room where the observations are made. The box [represented in] *Figure 5* seemed very convenient to us in order to cover the compass.

[445] Consider an angle θ in radians. We have $\theta \approx \sin\theta \approx \tan\theta$ when $\theta \ll 1$ radian, that is, when $\theta \ll 57.3°$.
[446] See footnote 237 on page 100.
[447] That is, a silk thread usually supports at least 80 grains = 4.25 grams without breaking up.



It is constructed in such a way that, when there is anything that needs to be replaced, we can disassemble and reassemble it without touching any of the parts of the compass. This box is formed, *Figure 5*, of two pieces. The first part $AadDBbC$ is twenty two inches long, five and a half inches wide and four inches high [59.6 cm, 14.9 cm and 10.8 cm, respectively]. This box is completely open at its bottom, its cover is open in part bd in order to put there

a glass 1234 through which we should observe the needle. The other end *ac* of the box is indented in *xy* up to *zu*, approximately nine inches in length [24.3 cm]. The fork *df* of *Figure 4* is located in this slot when one slides this box under the micrometer, in such a way that the extremity *ac*, *Figure 5*, exceeds by one inch [2.7 cm] the extremity *h* of the needle, *Figure 1*.

The upper part of the fork is covered by a second box, *Figure 5*, which is three and a half inches wide [9.5 cm] in all directions, eighteen to twenty inches high [48.7 to 54.1 cm], and which by means of the framework *cgh*, which finishes its lower portion, is connected by four screws on the cover *ab*. When the compass is so covered by the box, we block the slot with two small rulers, then we glue paper on all joints to prevent air from penetrating into the interior of the box.

We see, as shown in the Memoir already cited, that the needle, suspended in the way we have chosen, is very sensitive to displacement, it readily responds to the least disturbance, the smallest vibration starts it oscillating so that precautions must be taken on the part of the observer. However, we could decrease a great portion of this mobility by the following means: we weld, *Figure 1*, under the needle and in its plane, a very light copper or silver lamina, three or four inches high [8.1 or 10.8 cm], four lines thick [0.057 cm]; its length might be only eight or ten lines [1.8 or 2.3 cm] at its upper part, where it is fastened to the needle, but it would increase in its lower part, where it would be three or four inches long. This lamina would be plunged into a vase full of water, placed below the needle, but which allows its full freedom of motion: by means of a small tube, we can keep the water in the vase always at the same level. We saw on the *volume of 1784 of the Académie*,[448] from experiments that appear decisive, that the cohesion of water against the bodies cannot have an influence upon their position, when they reach a state of rest. Therefore, it results from these experiments and from the previous construction, that a copper lamina, plunged into water, will quickly damp out the oscillations of the needle, without altering the direction of the [needle relative to the] magnetic meridian. We could weld vertically a second copper plane, orthogonal to the first one, or to the plane of the needle. These two planes would stop very quickly in all directions the different motions of the needle. However, although the previous method should appreciably reduce the problems due to the extreme mobility of our suspension, we must not neglect any of the other precautions that can facilitate the observations. This is what has made me decide to set all parts of my compass firmly on a very heavy stone; [and] to separate the micrometer completely from the suspension fork, so that slowly turning the micrometer screw would not cause the needle to move. We should place, when possible, this compass at the ground floor, or at least on a vault of a very solid building, in such a way that the different parts of the box cannot acquire different degrees of temperature; otherwise circular air currents will be formed which will move the needle and put it into oscillation.

Finally, when making an observation, it is essential that one approaches the head of the compass slowly and, by means of the micrometer screw, carefully bring the focus of the microscope onto the line of the needle.

If, despite all these precautions, at the moment of making an observation, the needle moves — movement due to the particular electric state, the position, the temperature of different bodies or even of the masses of air surrounding the needle — it is necessary, in this case, to discount the reading since the magnetic force was probably altered by other [kinds of] forces. Without these perturbations, it would be very easy to determine the true position

[448] [Coulomb, 1787]. This work is translated in Chapter 7.

of the needle in oscillation, by measuring by means of the micrometer the total amplitude of oscillation; the midpoint of this amplitude would yield the real orientation of the needle, when it stops oscillating.[449]

After these remarks which inform that the needles are sometimes agitated by forces alien to magnetism, I proposed, in 1778, to always observe two absolutely similar needles, but whose magnetic forces were very different, for instance, in the ratio of 4 to 1, which can be easily determined by the number of oscillations. When the forces alien to magnetism act on these needles, that one which is more weakly magnetized, will be displaced in the inverse ratio of the directive force.[450]

I will not compare this new suspension with that due to caps on pivots,[451] of which I detailed a part of the defects, in 1777, in the IXth *volume des Savans étrangers*.[452,453,454]

10.1 Determination of the Magnetic Meridian

The compass used for the determination of the magnetic meridian is built according to the same principles just described; but its needle which is eighteen inches long [48.7 cm], is of the same width and thickness throughout this length. It is suspended by its center, as can be seen on *Figure 6*.

[449] Consider the directions of the needle at its two extreme orientations. The straight line that divides this angle into two equal parts yields the local magnetic meridian.

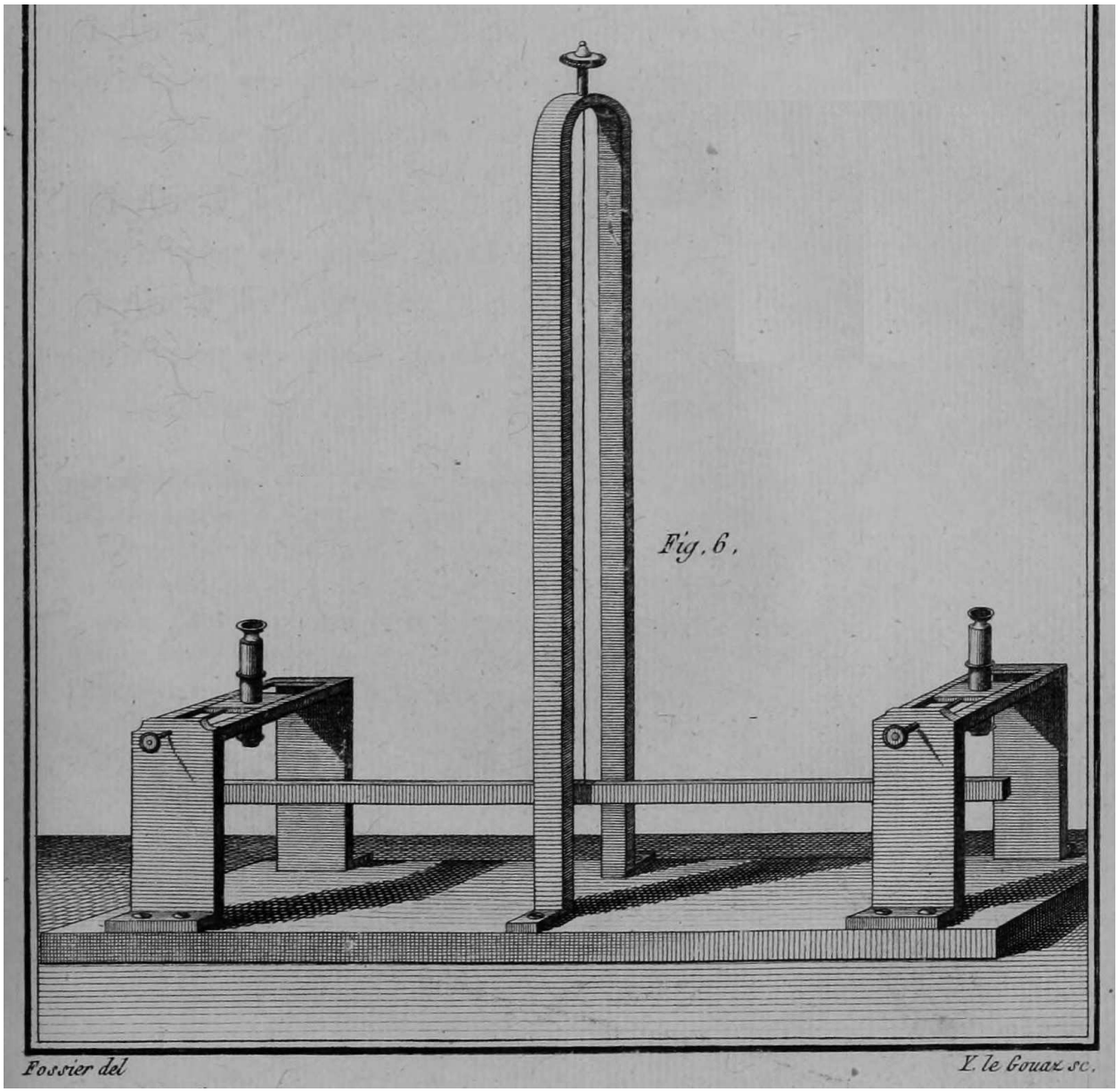
[450] That is, the greater the magnetic intensity of the needle, less it will be displaced relative to the magnetic meridian due to non magnetic forces.

[451] That is, suspension poising or supporting of an object on a sharp pivot. This is used for the needle in the ordinary compass. A cavity or inverted cup, which may be made of agate, is attached to the middle of the needle which has a hole for its reception. The center of gravity of the needle comes below the bottom of the cup. See footnote 141 on page 58.

[452] [Coulomb, 1780]. This work is translated in Chapter 5. The defects of pivot suspension are mainly related to friction.

[453] [Note by Coulomb] Mr. de Cassini has been utilizing for several years, in order to observe the diurnal variations, compasses built according to these principles. Every year he presents the results of his observations and the precautions he has taken to be sure of their exactness. I believed it was necessary to suspend the publication of this Memoir, until the moment where the work of a so brilliant observer had ensured the success of the compass just described, and had chosen the best way to utilize it.

[454] Jean-Dominique Cassini (1748-1845). See also [Licoppe, 1995].

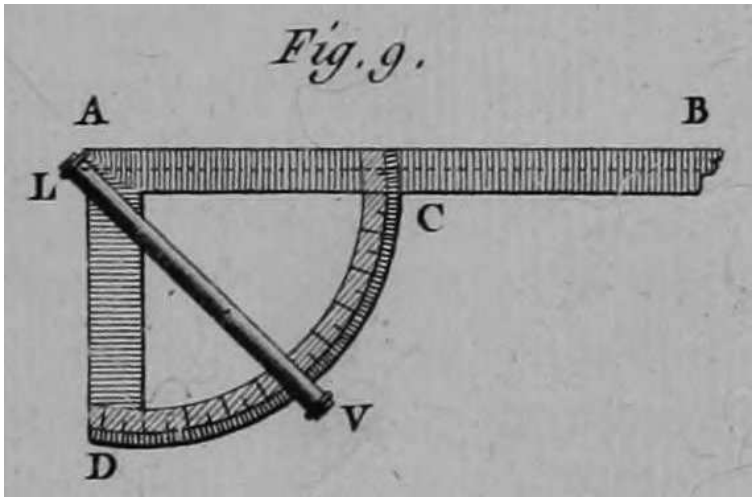


It must be perfectly prepared and suspended with its side set vertically: along the middle of its thickness, we draw a line from one extremity to the other, and we observe the two extremities of this line, by means of the two micrometers, as shown in *Figure 6*.

As our needle is everywhere of equal and very small thickness, as it is assumed to be well prepared, that it is suspended on the side, the plane which divides its thickness, seen vertically by the line drawn on its side, will be very nearly along the magnetic meridian. So the two focal points of the microscope will be found, after the observation, in this meridian line. Therefore, by stretching a silver wire under these two foci, after having removed the needle, and extending this silver wire to a [geographic] meridian line traced in the place of observation, it will be easy to determine the angle that the extended silver wire will form with this meridian; and therefore it will be easy to have the angle of the meridian with the magnetic meridian. Instead of this graphic procedure, we could also utilize advantageously a quadrant[455] $BACD$, *Figure 9*, on which the line of the first division will divide into two equal

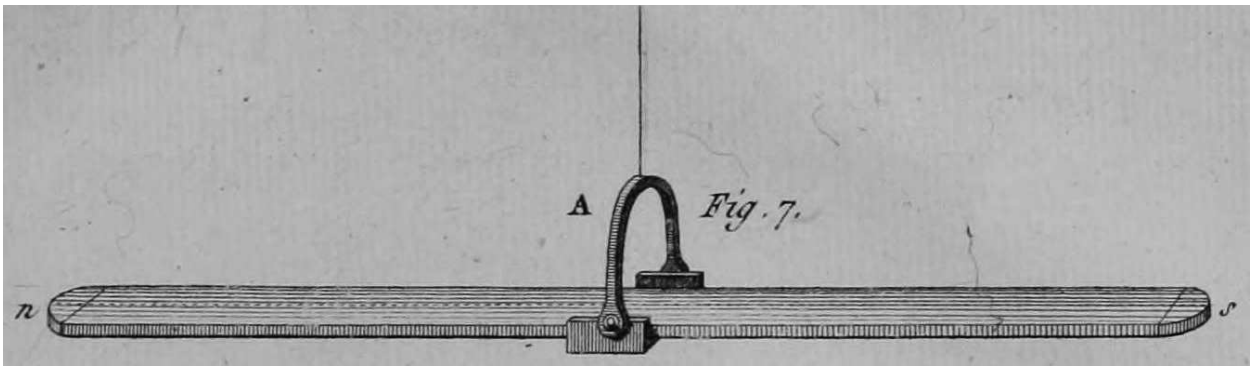
[455]In the original: *secteur*. A quadrant is an instrument that is used to measure angles up to 90°.

parts the branch AB; this branch extending beyond the limb of the quadrant of a length CB of two to three feet [65.0 or 97.4 cm]; we will slide this branch under the microscopes, so that the line AB aligns with the two foci: the quadrant being placed horizontally by means of the telescope LV, we will note some point on the horizon, of which the bearing,[456] relative to the place where the observation is made, will be determined.

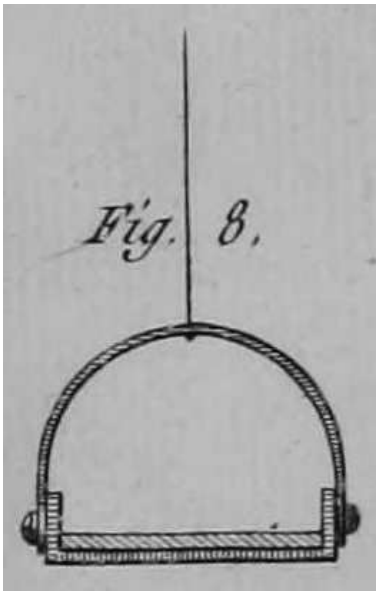


10.2 Comment

As, in practice, it is rather difficult to obtain a thin steel lamina, which is perfectly straight, we can, if we wish, use a needle (*Figure 7*) suspended horizontally in a case A, the profile of which is shown in *Figure 8*.



[456] In the original: *gisement*. Bearing is the horizontal angle between the direction of an object and that of true North.



Two small rings n and s, of silver or of copper, are welded to the ends of the lamina; one stretches a very fine wire of silk or silver from n to s, whose direction is observed, by means of the two micrometers, before and after the inversion of the needle: the half of the difference of the two directions observed, will determine the magnetic meridian.[457]

[457]To determine the magnetic meridian, one first observes the direction of the silk thread with the magnetized lamina placed as shown in Figure 7 and observes the direction of the silk thread. Then this lamina is turned upside down. One waits until it is at rest and observes once again the direction of the silk thread. The straight line drawn along the middle of the two previous directions will indicate the magnetic meridian.

Chapter 11

First Memoir on Electricity and Magnetism: Construction and Use of an Electric Balance Based on the Property that Filaments of Metal Produce a Reactive Force in Torsion Proportional to the Angle of Twist

Coulomb[458]

Experimental determination of the law according to which the elements of bodies, electrified with the same kind of electricity, are mutually repelled.

In a Memoir presented to the Academy, in 1784,[459] I have determined from experiments the laws governing the force of torsion[460] of a filament of metal and I have found that this force is proportional to the angle of torsion, to the fourth power of the diameter of the suspended filament and inversely proportional to its length — all multiplied by a constant coefficient which depends on the nature of the metal and is easily determined by experiment.[461]

I have shown in the same Memoir that by means of this force of torsion, it was possible to precisely measure extremely small forces as, for example, one ten thousandths of a grain (0.005 dyn).[462] In the same Memoir I described a first application of this theory, seeking to

[458] [Coulomb, 1788b] with partial English translation in [Coulomb, 1935a]. There are complete English, German and Portuguese translations in [Coulomb, 2012c], [Coulomb, 1890b] and [Assis, 2022], respectively. This work was presented in 1785 to the French Academy of Sciences and published in 1788.

[459] [Coulomb, 1787]. This Memoir is translated in Chapter 7.

[460] In the original: *force de torsion*. This expression can also be translated as "torque" or "torsional resistance".

[461] Coulomb also found that this torsional force does not depend on the tension to which the wire is subjected due to the supported weight.

[462] 1 grain $= 0.05311\ g = 5.311 \times 10^{-5}\ kg$. So this balance achieves an accuracy, in terms of a mass m, of the order $m = (1\ grain)/(10\,000) = 5.3 \times 10^{-9}\ kg$. Assuming the acceleration of gravity given by $g = 9.81\ m/s^2$, it follows that this balance achieves an accuracy, in terms of a force F, of the order $F = mg = 5.2 \times 10^{-8}\ N$, or 0.005 dyn, as calculated by Potier.

evaluate the constant force attributed to adhesion in the formula for the surface friction of a solid body moving through a fluid.

Today, I set before the eyes of the Academy, an electric balance constructed according to the same principles. It measures with the greatest precision the state and the electric force of a body, however weak the degree of electricity.[463]

11.1 Construction of the Balance

While practice has taught me that, in order to execute several electric experiments in a convenient way, it is necessary to correct some defaults in the first balance of this kind that I put to use, still, as this has been up until now the only one which I have employed, I am going to give its description, noting how its form and dimensions can and ought to be changed according to the nature of the experiments one plans to carry out. The first Figure presents the balance in perspective. Here are the details.

[463] That is, the balance can measure very small forces even with a low electrification of the interacting bodies.

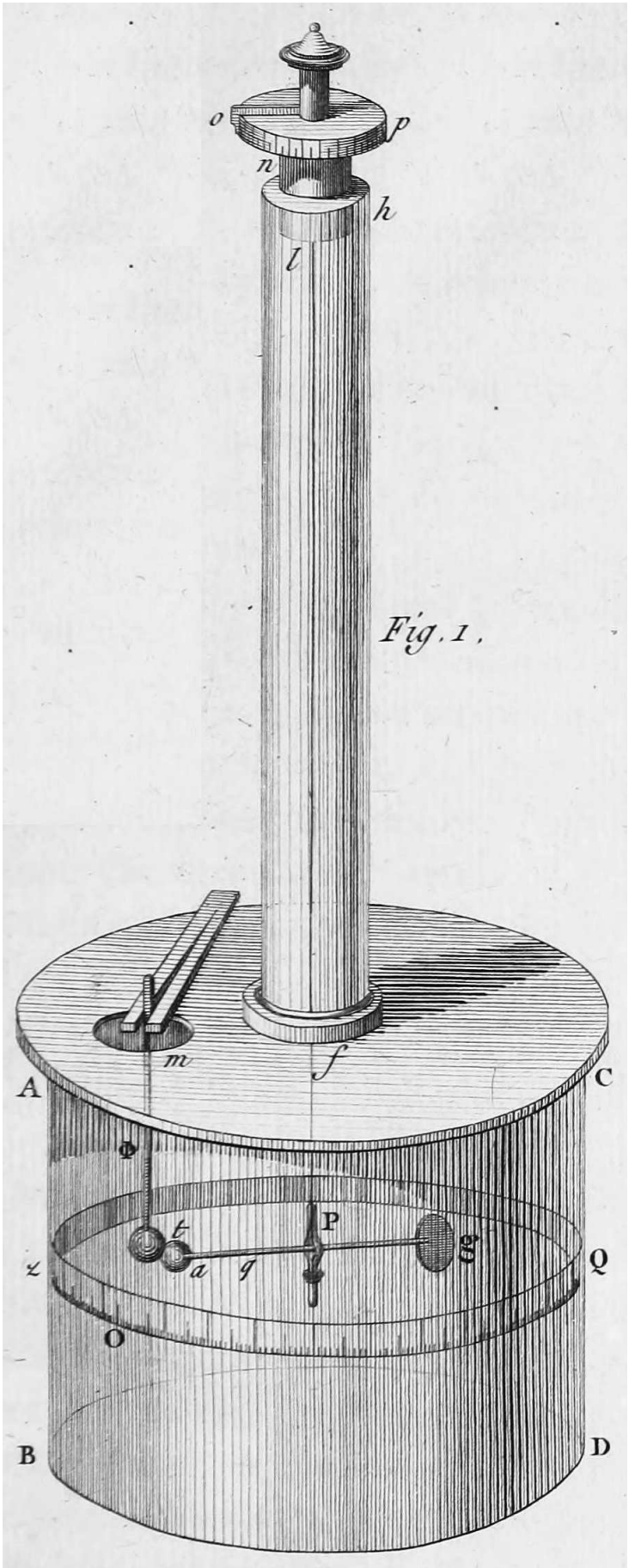
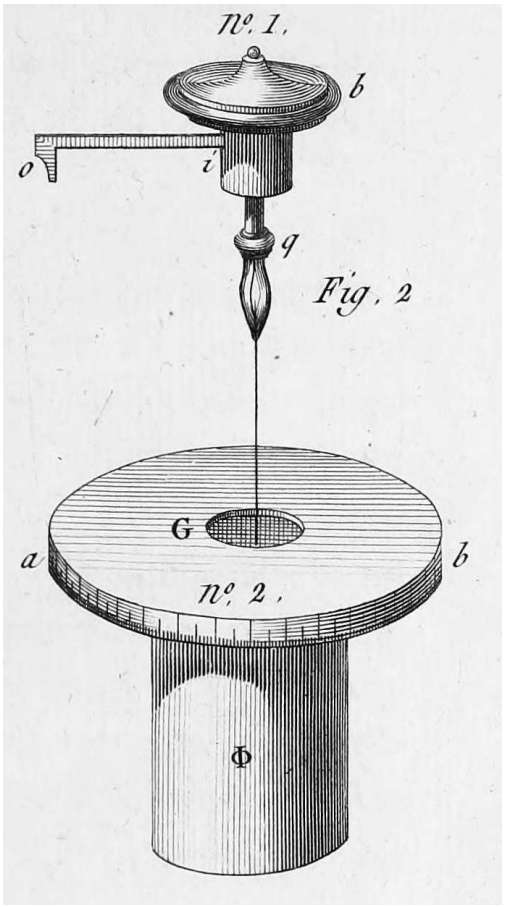


Fig. 1.

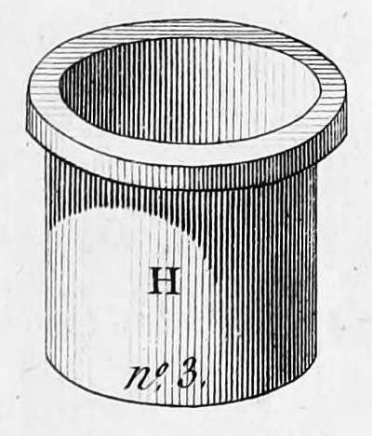
On a cylinder of glass $ABCD$, of 12 inches in diameter (32.48 cm) and 12 inches high, we place a plate of glass of 13 inches in diameter, which covers the whole vessel of glass. This cover is pierced with two holes of approximately 20 lines in diameter (4.51 cm), one in the middle, at f, on which is elevated a tube of glass of 24 inches in height. This tube is cemented over the hole f, with the cement used in electric apparatus: at the highest extremity of the tube at h, is placed a torsion micrometer the details of which are shown in Figure 2.[464]



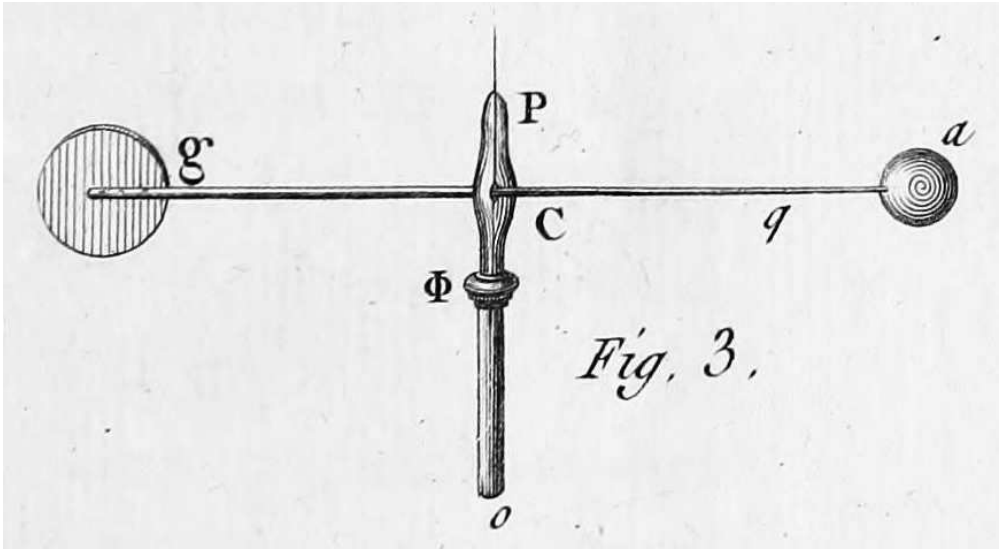
The top [of Figure 2], No. 1, bears a knob b, the pointer io, and the clasp of suspension, q.[465] This piece goes into the hole G of part No. 2 [of Figure 2]. Part No. 2 is formed of a circle ab divided on its edge into 360 degrees, and of a copper tube Φ which fits into the tube H, [Figure 2,] No. 3, sealed at the interior at its highest extremity of the tube or of the glass stem fh of Figure 1.

[464] See footnote 439 on page 193.

[465] In the original: *pince de suspension*. This expression can also be translated as clamp or pincer. See also footnote 340 on page 153.



The clasp q (Figure 2, No. 1) has approximately the shape of the extremity of a solid mechanical pencil clamp,[466] which can be tightened by means of the annulus q. The clasp of this pencil clamp holds the end of a filament of very thin silver. The other end of the filament of silver is fixed (Figure 3) at P, by the clasp of a cylinder Po of copper or of iron, whose diameter is but a line (0.22 cm), and whose end P is split and forms a clasp which is tightened by means of the collar Φ.



This small cylinder has a hole in C, in order to allow the needle ag to slide through (Figure 1). It is necessary that the weight of this small cylinder be of sufficient magnitude in order to put the filament of silver in tension without breaking it. The needle that one sees (Figure 1) at ag, suspended horizontally at approximately the midpoint of the height of the big vase which encloses it, is formed, either of a filament of silk plastered with sealing wax,[467] or of a straw likewise covered with Spanish wax, and finished off from q to a, a distance of 18

[466] In the original: *porte-crayon solide*. This expression can also be translated as "solid crayon holder" or "solid ruling pen". A ruling pen is an instrument for drawing with ink or with other drawing fluids. It contains ink in a slot between two flexible metal jaws, which are tapered to a point. The line width can be adjusted by an adjustment screw connecting the jaws.

[467] In the original: *cire d'Espagne*. This expression can also be translated as "Spanish wax". Sealing wax is a wax material which, after melting, hardens quickly forming a bond that is difficult to separate without noticeable tampering. It was also used as an electrical insulator.

lines (4.06 cm), by a cylindrical filament of shellac.[468] At the end a of this needle, is a small ball of pith[469] of two to three lines diameter [0.45 to 0.68 cm]. At g, is a small vertical paper disk coated with turpentine which serves as a counter-weight to ball a and which dampens the oscillations.

We have said that the cover AC is pierced with a second hole at m. It is in this second hole that one introduces a small cylinder $m\Phi t$, of which the lower part Φt is shellac. At t is a ball likewise made of pith. Around the vase, at the height of the needle, one scribes a circle zQ divided into 360 degrees: for simplicity, I have made use of a band of paper divided into 360 degrees, which I glue around the vase, at the height of the needle.

To begin to operate with this instrument, in placing the cover (atop the vase), I position the hole m approximately at the first division or at point O of the circle zOQ traced on the vase. I place the pointer oi of the micrometer on point o, at the first division of this micrometer. I then turn the micrometer within the vertical tube fh until, keeping in view the vertical filament which suspends the needle and the center of the ball, the needle ag is directed towards the first division of the circle zOQ.[470] I then introduce through the hole m the other ball t suspended by the filament $m\Phi t$ such that it touches ball a and that, in keeping in view the center of the filament of suspension and ball t, we encounter the first division o of the circle zOQ.[471] The balance is now in a state ready for all operations; we go on to give as an example, the means by which we are able to determine the fundamental law according to which electrified bodies repel themselves.

11.2 The Fundamental Law of Electricity

The repulsive force of two small globes electrified with the same kind of electricity, is inversely proportional to the square of the distance between the centers of the two globes.

11.2.1 The Experiment

One electrifies, Figure 4, a small conductor, which is nothing but a pin with a large head, which is insulated by forcing its point into the end of a rod of Spanish wax.[472] One introduces

[468] In the original: *gomme-laque*. Shellac is a resin secreted by the female lac bug on trees in the forests of India and Thailand. It is processed and sold as dry flakes and dissolved in alcohol to make liquid shellac. Shellac was once used in electrical applications as it possesses good insulation qualities and it seals out moisture.

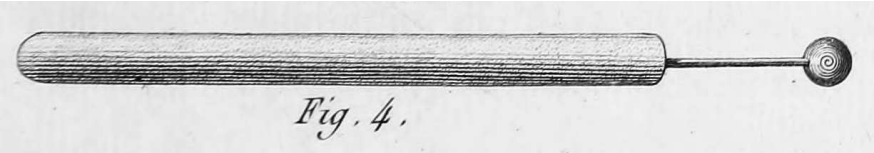
[469] In the original: *une petite balle de sureau*. This expression can also be translated as elderberry ball, elder pith ball, elderwood pith ball or elder ball, [Gillmor, 1971a, p. 184] and [Heilbron, 1999, pp. 451 and 471]. The various species of sambacus (*sureau*) are commonly called elder or elderberry. The pith is the marrow of the limb of the elderberry. These pith balls were often used in electric pendulums and electroscopes, behaving as conductors for the usual electrostatic experiments, [Assis, 2010b, p. 173], [Assis, 2010a], [Assis, 2011], [Assis, 2015b] and [Assis, 2017].

[470] That is, when the thread is not twisted, the needle is directed toward point O on the graduated scale zOQ.

[471] Point C is the center of needle ag, as shown in Figure 3. This point C coincides with the vertical line passing through the suspension thread. The line passing through C and the center of the fixed ball t passes through point O on the circle zOQ. Coulomb measures the angles from that point O. A representation of this situation can be found in Figure 12.1 in Section 12.1 of Chapter 12, page 215.

[472] Coulomb charged the insulated pin by use of a Leyden jar or an electrostatic machine, [Gillmor, 1971a, p. 184]. See also Chapter 12 (The Leyden Jar and Capacitors) of the Volume 2 of the book *The Experimental and Historical Foundations of Electricity*, [Assis, 2018b], [Assis, 2018a] and [Assis, 2019]. See also

this pin into the hole m, bringing it in contact with ball t, [which is] in contact with ball a.



In retracting the pin, the two balls find themselves electrified with the same kind of electricity and they repel themselves mutually to a distance that we measure by looking past the filament of suspension and the center of ball a, the division corresponding to the circle zOQ. Turning then the pointer of the micrometer in the direction pno, we torque the filament of suspension lP, and we produce a force proportional to the angle of torsion, which tends to make ball a approach ball t. We observe, by this means, the distance to which different angles of torsion bring ball a back toward ball t and in comparing the forces of torsion with the corresponding distances of the two balls, one determines the law of repulsion.

I will only present here, some tests which are easy to repeat and which will immediately reveal the law of repulsion.

First Test. Having electrified the two balls with the head of the pin, with the pointer of the micrometer positioned at o, ball a of the needle is displaced from the ball t by 36 degrees.

Second Test. Having torqued the filament of suspension by means of the knob o of the micrometer by 126 degrees, the two balls approach each other and stop at 18 degrees distance the one from the other.

Third Test. Having torqued the filament of suspension by 567 degrees, the two balls approach until 8 and one-half degrees.[473]

11.3 Explication and Result of This Experiment

Before the balls are electrified, yet touching, the center of ball a, fixed to the needle, is at a distance equal to the diameter of the balls from the point where the torsion of the filament of suspension is null. It is necessary to be warned that the filament of silver lP, 28 inches long (75.80 cm), which forms the suspension is so fine that the weight of one foot of length is but 1/16 of a grain (0.01 g per meter).[474] In calculating the force required to twist this filament, in acting at point a elongated some four inches (10.83 cm) from the filament lP or from the center of suspension, I have found, using the formulas derived in a Memoir on the laws of the force of torsion of filaments of metal, printed in the *volume of the Académie*

[Benguigui, 1995].

[473] I present a detailed discussion of Coulomb's procedure and measurements in Chapter 12, Section 12.1, page 215.

[474] Since 1 grain = 0.05311 g, the mass M of this silver wire was $M = 3.319 \times 10^{-6}\ kg$. Since 1 foot (*pied*) = 32.48 cm, the linear mass density λ of this silver wire is

$$\lambda = \frac{3.319 \times 10^{-6}\ kg}{3.248 \times 10^{-1}\ m} = 1.022 \times 10^{-5}\ kg/m = 0.010\ g/m\ .$$

Assuming a volumetric density of mass δ for silver of $\delta = 10.5\ g/cm^3 = 10.5 \times 10^3\ kg/m^3$, then the diameter D of this cylindrical wire can be obtained by $\delta = M/(\pi L D^2/4)$, that is, $D = 3.5 \times 10^{-5}\ m = 0.0035\ cm$. This value is the same order of magnitude as the diameter of a single strand of hair.

for 1784,[475] that to torque this filament 360 degrees, requires at point a, in acting with the lever aP of four inches of length (10.83 cm), a force of 1/340 of a grain.[476,477] Thus as the forces of torsion are, as proved in the Memoir [presented in 1784], proportional to the angles of torsion, the least repulsive force between the two balls, displaces them sensibly one from the other.

We find in our first test, where the pointer of the micrometer is at point o, that the balls are displaced 36 degrees, which produces in the same time a force of torsion of[478]

$$36^\circ = \frac{1}{3400} \text{ grain} .$$

In the second test, the distance of the balls is 18 degrees, but as we have torqued the micrometer 126 degrees, it results that at the distance [between the balls] of 18 degrees, the repulsive force is 144 degrees:[479] thus at half of the first distance, the repulsion of the balls is quadrupled.[480]

In the third test, where we have twisted the filament of suspension 567 degrees, the two balls find themselves no further apart than 8 and one-half degrees. The total torsion, is consequently, 576 degrees, quadruple the one of the second test, and it is only off by one-half a degree that the distance of the two balls in this third test was reduced to half of the distance it was in the second [test].[481] It results thus from these three tests, that the repulsive action of the two balls electrified with the same kind of electricity exert on each other was the inverse ratio of the square of the distances.[482,483]

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[475] [Coulomb, 1787]. This Memoir is translated in Chapter 7.

[476] [Note by Potier] 0.153 dyn.

[477] 1 grain = 0.05311 g = 5.311×10^{-5} kg. Then, in terms of a mass m, we have m = (1 grain)/340 = 1.562×10^{-7} kg. With $g = 9.81$ m/s^2, it follows that it would be necessary to employ a force $F = mg$ of $F = 1.53 \times 10^{-6}$ $N = 0.153$ dyn, as calculated by Potier.

With a lever arm $aP = 4$ inches = 1.083×10^{-1} m, the torque τ needed to twist this wire by an angle of 360° is $\tau = 1.66 \times 10^{-7}$ Nm.

[478] [Note by Potier] 0.0153 dyn.

[479] That is, the total twist of the wire in this case was $18^\circ + 126^\circ = 144^\circ$, see the Figures in Section 12.1 of Chapter 12 on page 215.

[480] That is, to halve the angular distance, from 36° to 18°, it was necessary to quadruple the twist of the wire, that is, to go from a twist of 36° to $4 \times 36^\circ = 144^\circ$. The torque exerted by the wire is proportional to its angle of torsion. Therefore, in the second case where the angular separation between the balls has halved, a torque four times as large is required to keep them at a fixed distance in equilibrium compared to the initial torque.

[481] In this last case the angular separation between the two balls was 8.5°. The total twist of the filament, on the other hand, is given by its counterclockwise bottom twist of 8.5°, added with its clockwise top twist of 567°. That is, a total twist of $575.5^\circ \approx 576^\circ$. This value is four times the previous twist of 144°, that is, $4 \times 144^\circ = 576^\circ$. On the other hand, the angular separation between the balls has again dropped by almost half, from 18° to $8.5^\circ \approx 9^\circ$. In other words, again it follows that the total angle of torsion of the wire is inversely proportional to the square of the angular separation between the balls that are repelling each other.

[482] [Note by Potier] The repulsion of the two balls, in the first test, is:

$$\frac{0.0153 \; dyn}{\cos 18^\circ} = 0.016 \; dyn ,$$

at a distance of $10.83 \times 2 \sin 18^\circ = 6.67$ cm. Therefore, the charge on each one of them is $6.67 \times \sqrt{0.016} = 0.84$ absolute units (C.G.S.).

[483] I present a detailed calculation of the amount of charge on each of the balls in Chapter 12, Section 12.2, page 217.

11.4 First Remark

In repeating the preceding tests, we will observe that in making use of a filament of silver, as thin as the one we have employed (which only requires a torsional force of approximately 24 thousands of a grain to twist it through an angle of 5 degrees),[484] that however calm be the air, and whatever precautions that we can take, we could not answer for the natural position of the needle when the torsion is zero, to within 2 or 3 degrees. Thus, in order to have a first test to compare with the following ones, it requires, after having electrified the two balls, to torque the filament of suspension some 30 to 40 degrees, this which will give a force of torsion strong enough so that the 2 or 3 degrees of uncertainty in the initial position of the needle, when the torsion is zero, does not produce any sensible error in the results. It is necessary furthermore to be warned that the filament of silver, which I used in this test, is so fine that it breaks with the least disturbance. I have found in the following that it would be more useful to employ in these tests a filament of suspension of nearly double the diameter, although its flexibility to torsion be 14 to 15 times smaller than that of the first. It is necessary to take care, before making use of this filament of silver, over the course of two or three days, to tension it by a weight which is approximately half this that might break it. It is necessary still yet to warn the reader, that in using this last filament of silver, never to torque it beyond 300 degrees, because in exceeding this degree of torsion it begins to strain-harden and reacts, as we have proven in the Memoir already cited, printed in 1784, with a force smaller than the one corresponding to the torsion angle.

11.5 Second Remark

The electricity of the two balls diminishes somewhat over the duration of the experiment. I noticed that, the day where I have made the preceding tests, the electrified balls, finding themselves repulsed to 30 degrees one from the other, under an angle of torsion of 50 degrees, they come back toward each other about one degree in three minutes. But as I have only used two minutes to make the three preceding tests, we can, in these tests, neglect the error which results from the loss of electricity. If one desires greater precision, as when the air is humid, and the electricity dissipates rapidly, one ought, by a preliminary observation, determine the law of diminution of the electric action of the two balls in each minute, and then, on the basis of this preliminary observation, use it to correct the results of tests that we wish to make that day.

11.6 Third Remark

The distance of the two balls, when they are displaced one from the other by their reciprocal repulsive action, is not precisely measured by the angle they make, but by the chord of the arc which joins their centers. In the same way that the lever at whose extremities the action

[484] That is, a mass m given by:

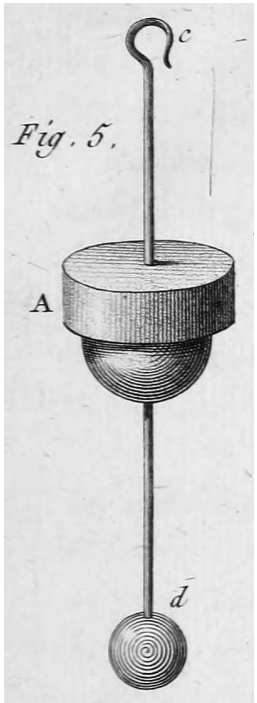
$$m = \frac{1 \text{ grain}}{24\,000} = \frac{5.311 \times 10^{-5}\ kg}{24\,000} = 2.2 \times 10^{-9}\ kg\ .$$

With $g = 9.81\ m/s^2$ it follows that a force of the order of $2.2 \times 10^{-8}\ N = 0.002\ dyn$ is needed to rotate it through an angle of 5°.

is exerted, is not measured by half the length of the needle, or by the radius, but by the sine of the half of the angle formed by the distance of the two balls. These two quantities, of which one is smaller than the arc and diminishes consequently the distance measured by this arc, while the other reduces the lever arm, compensate themselves in some way; and in the tests of this sort with which we are concerned, we can without sensible error, hold to the evaluation that we have given, if the distance of the two balls does not exceed 25 to 30 degrees; in the other cases, we must make the rigorous calculation.[485]

11.7 Fourth Remark

As experiment has shown, in a well closed chamber, we can determine with the first filament of silver, to within 2 or 3 degrees, the position of the needle, when the torsion is null, this which gives, after the calculation of the forces of torsion, proportional to the angle of torsion, a force more or less of a 40 thousandth of a grain (0.0013 dyn), the weakest degree of electricity will be measurable easily with this balance. For this operation, one makes pass, Figure 5, across a cork of Spanish wax, a small filament of copper cd, terminating at c by a crochet and in d, by a small ball of gilded pith, and we put the cork A in the hole m of the balance, Figure 1, in such a way that the center of the ball d, viewed by the suspension wire, retakes to point o of the circle zOQ.



In approaching then an electrified body of the crochet c, however weak be the electricity of this body, ball a separates from ball d, giving signs of electricity,[486] and the distance of the two balls measures the force between them, according to the principle of the inverse ratio of the square of the distances.

[485] See Section 12.3 of Chapter 12 on page 218.

[486] These small degrees of electrification are perceived both in the electrified body approaching the hook c, and in these two balls a and d.

But I ought to warn you that, since these first tests, I have had different small electrometers made according to the same principles of the force of torsion, using a filament of silk for the suspension, such as it leaves the cocoon, or a thread of the goat of Angora.[487] One of these electrometers which has almost the same shape as the electric balance, described in this Memoir, is much smaller. It is only 5 to 6 inches in diameter [13.5 or 16.2 cm], a stem of one inch (2.71 cm); the needle is a small filament of shellac of 12 lines of length (2.71 cm), terminated at a by a small very light disk of tinsel.[488]

The needle and the tinsel weigh a little more than a quarter of a grain (0.013 g); the filament of suspension, such as it leaves from the cocoon, is 4 inches long [10.8 cm], having a flexibility such that in acting with a lever arm of one inch (2.71 cm), it only requires a [force of a] sixtieth thousands of a grain to twist it an entire circle or 360 degrees.[489] In presenting in this electrometer at the crochet C of Figure 5, an ordinary rod of Spanish wax, electrified by friction, at a 3 feet distance (0.97 m) from this crochet, the needle is chased to more than 90 degrees. We will describe in more detail in the following this electrometer, when we will determine the nature and the degree of electricity of different bodies which through rubbing each other, take on a very weak degree of electricity.[490,491]

[487] In the original: *un poil de chèvre d'Angora*. Mohair is a fabric or yarn made from the hair of the Angora goat.

[488] In the original: *clinquant*.

[489] That is, a mass m given by:

$$m = \frac{1 \text{ grain}}{60\,000} = \frac{5.311 \times 10^{-5}\ kg}{60\,000} = 8.85 \times 10^{-10}\ kg\ .$$

With $g = 9.81\ m/s^2$ it takes a force of the order of $8.7 \times 10^{-9}\ N = 0.0009\ dyn$ to twist it an entire circle.

[490] [Note by Potier] Under the conditions in which Coulomb worked, if we neglect the action of the charges induced on the glass of the container, which was located around 0.04 m from the center of the balls, the influence of the distribution of electricity on the surface of the balls is small. Indeed, the reciprocal action between two equal spheres charged with equal amounts of electricity is given by

$$\frac{e^2}{c^2}\left(1 - 4\frac{a^3}{c^3}\right)\ ,$$

if a represents their radii, c the distance between their centers, [where e is the total charge on each ball]. This formula gives exact results to an approximation of 2/1000, as long as the distance between the [surfaces of the] spheres is equal to their radius. In the experiments cited by Coulomb, a/c is always less than 1/6.

[491] Potier is considering the influence of charge distribution on the surfaces of two electrified conducting spheres when they are close together. He considers equal spheres of radius a, equally electrified with a charge e on each sphere, their centers being separated by a distance c. If the charges on the two spheres were concentrated at their centers, the force between them would be proportional to e^2/c^2. If $c = 6a$, the expression provided by Potier for the force between these conducting spheres would be proportional to $(e^2/c^2)(1 - 0.0185)$. That is, the percentage difference between these forces would be on the order of 2%. However, in Coulomb's experiments we have $c > 6a$, so the difference is even smaller. That is, in these experiments it is not necessary to take into account the redistribution of charges on the conducting spheres, it is sufficient to consider these charges as being concentrated at the centers of the spheres, as done by Coulomb.

For a deduction of the formula given by Potier in footnote 490 see, for example, [Maxwell, 1954, Vol. I, §§171-174, pp. 266-273].

An estimate of the force exerted by the induced charges located on the glass of the container and acting on the charges located inside the torsion balance, assuming the container to be spherical and made of metal, was given by Maxwell: [Maxwell, 1954, Vol. I, §215, pp. 327-329].

Chapter 12

Remarks on Coulomb's First Memoir

A. K. T. Assis

12.1 Coulomb's Experimental Procedure

Here I give a detailed discussion of the procedure and measurements that Coulomb presented in Section 11.2.

In Figure 12.1 (a) we have a top-down view of the needle with center C along the projection of the suspension thread, ball a and paper disk g that acts as a counterweight and as a damper for the oscillations. In the situation where the thread is not twisted, the needle is directed toward point O fixed on the graduated scale of the circle zOQ attached to the glass container around the needle.

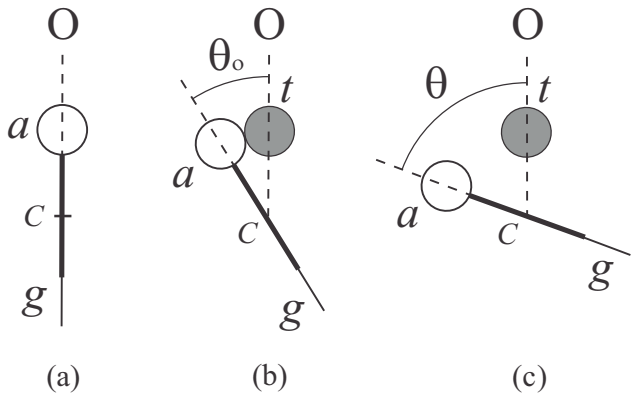


Figure 12.1: (a) Untwisted wire with the needle pointing toward O. (b) Beginning of the experiment with discharged balls a and t. (c) Final equilibrium configuration of the experiment with the balls electrified with charges of the same sign.

Before starting the experiments, Coulomb displaces ball a a little away from its original position by placing ball t against it, Figure 12.1 (b). Ball t always stays fixed in the laboratory. The straight line passing through C and through the center of the fixed ball t always is directed toward point O. Coulomb measures the angles from this point O. The horizontal needle ag can rotate around its center C attached to the vertical suspension thread. The initial twist of the wire is represented by the angle θ_o.

Both balls become electrified when an electrified pin touches ball t. By doing so they acquire charges of the same sign and repel each other. The needle rotates in the horizontal

plane, counterclockwise, around point C by which it is suspended by the vertical thread. Ball a moves away from ball t. At equilibrium the thread is twisted at an angle θ, Figure 12.1 (c). The torque exerted on the needle by the electric force of repulsion between the electrified balls is balanced by the counter-torque exerted by the wire twisted at angle θ.

Figure 12.2 shows the sequence of Coulomb's experimental tests. Point O indicates from where Coulomb measures the twist of the lower part of the wire in the graduated circle zOQ attached to the glass container around the needle. Point S, on the other hand, indicates from where Coulomb measures the twist of the top of the wire in the small graduated circle ab attached to the top of the tube from which the wire is suspended. The arrow Co is the index of the micrometer attached to the top of the wire. I will assume that initially the straight line Co of the micrometer index is directed toward point S. I am assuming that initially the straight line through C and through the center of the fixed ball t, pointing to O, is along the same vertical plane as the straight line Co pointing to S.

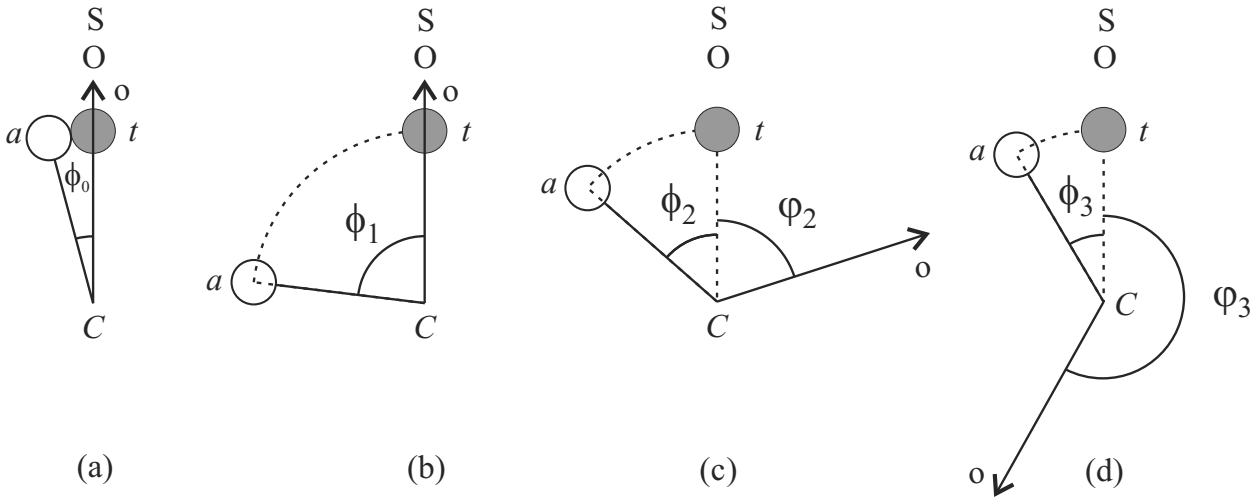


Figure 12.2: (a) Beginning of the experiment with discharged balls. (b) Test 1. (c) Test 2. (d) Test 3.

In Figure 12.2 (a) we have the initial situation with the discharged balls. The thread is twisted from an initial angle $\theta_o = \phi_o$. Ball a was 2 to 3 lines in diameter. Let us assume here two spheres a and t of equal diameters given by $D_1 = D_2 = 2.5$ lines $= 0.565\ cm$. When they are touching each other, this value also represents the distance between their centers. Coulomb states that the distance between the center of ball a and the center C of the needle is $R = 4\ cm$. From these values we obtain the initial angle of twist of the thread as given by $\theta_o = \phi_o \approx 0.565/10.83 = 0.052\ rad = 3.0°$.

In the first test the balls are electrified with charges of the same sign, Figure 12.2 (b). Ball a moves away from ball t until it stops at the position ϕ_1 due to the counterclockwise twisting of the lower part of the wire. Coulomb measures the angle ϕ_1 of the twist of the lower part of the wire, this angle representing the total twist of the wire. That is, the angle θ in Figure 12.1 (b) is given by $\theta_1 = \phi_1$.

Next Coulomb performs the second test in which he twists the micrometer together with the top of the wire clockwise from an angle φ_2, Figure 12.2 (c). With this rotation of the micrometer, ball a approaches t until it stops at angle ϕ_2. Coulomb measures the equilibrium angles, namely ϕ_2 and φ_2 on the lower and upper graduated scales, respectively. The total angle of twist of the wire in Figure 12.1 (b), θ, corresponds here to the sum of these angles,

that is, $\theta_2 = \phi_2 + \varphi_2$.

Then Coulomb twists clockwise even further the micrometer attached to the top of the wire, bringing ball a once again closer to ball t, Figure 12.2 (d). The lower part of the wire gets twisted from an angle ϕ_3, while the upper part of the wire gets twisted from an angle φ_3. The total twist of the wire is given by the sum of these angles, $\theta_3 = \phi_3 + \varphi_3$.

Coulomb's three experimental tests are given in Table 12.1 with the angles shown in Figures 12.1 and 12.2, [Gillmor, 1971a, p. 185].

	Test No. 1	Test No. 2	Test No. 3
Angular separation of the balls $= \phi =$	36°	18°	8.5°
Micrometer angle $= \varphi =$	0°	126°	567°
Total wire twist angle $= \theta = \phi + \varphi =$	36°	144°	575.5°

Table 12.1: Angles measured by Coulomb.

Blondel and Wolff presented a video showing a reproduction of Coulomb's balance in 2007.[492]

12.2 Estimated Value of the Electrical Charge Used by Coulomb

Here I will estimate the value of the amount of charge on the electrified balls in Coulomb's experiment. As seen in Section 11.3 on page 209 and in footnotes 476 and 477, Coulomb calculated that a force $F = 1.53 \times 10^{-6}\ N$ acting on a lever arm of 4 inches $= 1.083 \times 10^{-1}\ m$ was needed to twist his wire at an angle of 360°. In this first test the wire was twisted 36°. Therefore, in this case a force $F = 1.53 \times 10^{-7}\ N$ acting on an arm of length $1.08 \times 10^{-1}\ m$ was needed, that is, exerting a torque τ given by the following value:

$$\tau = 1.65 \times 10^{-8}\ Nm\ . \tag{12.1}$$

In Figure 12.3 (a) balls a and t are separated by a distance d.

Point C is the center of the needle, which coincides with the vertical projection of the suspension thread. R is the distance between C and the center of ball a, namely, $R =$ 4 inches $= 1.083 \times 10^{-1}\ m$, as given by Coulomb. The angle of separation between the centers of the two balls is represented by θ, Figure 12.1 (c). In this first test we have $\theta = 36°$.

The torque exerted by the force F acting on the center of the sphere a, relative to point C, is given by

$$\tau = FR\cos\frac{36°}{2} = F(0.1083\ m)(0.951) = 0.103F\ . \tag{12.2}$$

Comparing this equation with Equation (12.1) gives:

$$F = 1.60 \times 10^{-7}\ N\ . \tag{12.3}$$

[492] [Blondel and Wolff, 2007].

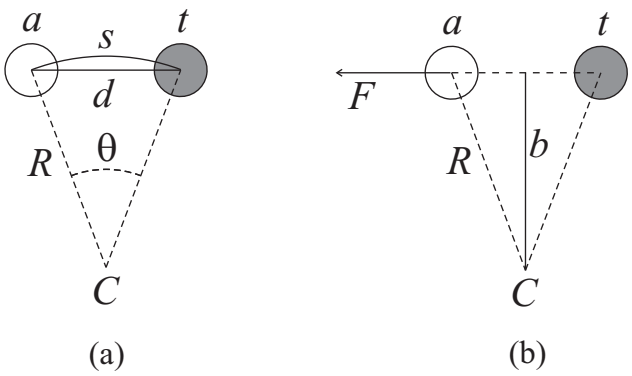


Figure 12.3: (a) Balls a and t separated by a distance d. (b) Repulsive force F between the electrified balls and arm b of the lever.

I will assume here two balls of the same material and of the same size equally electrified with charges $q_1 = q_2 = q$. I will also use Coulomb's force in the International System of Units expressed by

$$F = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{1}{r^2} \ , \tag{12.4}$$

in which $\varepsilon_o = 8.85 \times 10^{-12}\ C/(Vm)$ is a constant called vacuum permittivity or the permittivity of free space.

The distance $d = r$ between the centers of the two balls is given by $d = 2R\sin(36°/2) = 2 \cdot 0.1083 \cdot 0.309 = 0.0669\ m$. Using this value in Equations (12.3) and (12.4) we obtain

$$q \approx 2.8 \times 10^{-10}\ C \ . \tag{12.5}$$

This charge is equivalent to a value of 0.85 $cm^{3/2}g^{3/2}s^{-1}$, that is, 0.85 units of charge in the C.G.S. system of units, as calculated by Potier in footnote 482 on page 210. See also [Devons, 1975, p. 40].

12.3 Comparison of the Distance Between the Balls with the Circular Arc Between Them, Together with Comparison of the Actual Lever Arm to Half the Needle's Length

Here I will discuss Coulomb's Third Remark, Section 11.6. From his experimental data Coulomb concluded that the torque τ exerted on the wire due to the repulsion between the spheres was inversely proportional to the square of the angle of twist of the wire. From this fact he inferred that the force of repulsion between the electrified spheres was inversely proportional to the square of the distance between the centers of the spheres. On the one hand we have torque, torsion angle, and lever arm. On the other hand we have the force of repulsion between the spheres and the distance between their centers. So it is not immediately clear how one obtains that the force is inversely proportional to the square of the distance between the centers of the spheres from the observation that the torque is inversely proportional to the square of the angle of torsion.

From Figure 12.3 (a), the arc s is related to the radius R and the angle θ of separation between the spheres (expressed in radians) by:

$$\theta = \frac{s}{R} \ . \tag{12.6}$$

The distance d between the balls is given by:

$$d = 2R \sin \frac{\theta}{2} \ . \tag{12.7}$$

If we consider the angle $\theta \ll 1 \ rad$, we can expand this function and approximate this distance by:

$$d \approx 2R \left(\frac{\theta}{2} - \frac{1}{6} \frac{\theta^3}{8} \right) = R \left(\theta - \frac{\theta^3}{24} \right) \ . \tag{12.8}$$

Combining this Equation (12.8) with Equation (12.6) we get:

$$d \approx s \left(1 - \frac{s^2}{24R^2} \right) \ . \tag{12.9}$$

Let us assume the force F between the electrified balls to be inversely proportional to the square of the distance d between their centers, Equation (12.4). In this case we can write

$$F = \frac{K}{d^2} \ , \tag{12.10}$$

in which K is a constant for each specific experimental situation with a certain electrification of balls a and t.

In Figure 12.3 (b) we have the force F of repulsion between the spheres along the straight line joining their centers. The arm b of the lever is related to the radius R and the angle θ of separation between them (expressed in radians) by:

$$b = R \cos \frac{\theta}{2} \ . \tag{12.11}$$

If we consider angles $\theta \ll 1 \ rad$, we can approximate the value of this arm b by:

$$b \approx R \left(1 - \frac{\theta^2}{8} \right) \ . \tag{12.12}$$

Combining this equation with Equation (12.6) gives:

$$b \approx R \left(1 - \frac{s^2}{8R^2} \right) \ . \tag{12.13}$$

Using Equation (12.3) we obtain that the torque τ exerted by the force F on the needle calculated with respect to its center C is given by:

$$\tau = Fb = K \frac{b}{d^2} \ . \tag{12.14}$$

Combining this result with Equations (12.13) and (12.9) we obtain

$$\tau = K\frac{b}{d^2} = KR\left(1-\frac{s^2}{8R^2}\right)\frac{1}{s^2\left(1-\frac{s^2}{24R^2}\right)^2} \approx K\left[R\left(1-\frac{s^2}{8R^2}\right)\right]\left[\frac{1}{s^2}\left(1+\frac{s^2}{12R^2}\right)\right] \ . \tag{12.15}$$

This equation justifies Coulomb's statement on the Third Remark that the two quantities he considered compensate themselves in some way, since the lever arm b gets smaller than R as the angle $\theta = s/R$ of the torsion of the wire increases, while $1/s^2$ gets larger than $1/d^2$ as the angle of the torsion of the wire increases.

However, this compensation is not complete, not even at second order s/R. That is, expanding Equation (12.15) to the order s^2/R^2 gives:

$$\tau \approx K\frac{R}{s^2}\left(1-\frac{s^2}{24R^2}\right) \ . \tag{12.16}$$

For a torsion angle of $\theta = 30° = 0.524\,rad$ we have $\theta^2/24 = s^2/(24R^2) = 0.011$. Therefore, Coulomb's conclusion is justified for angles of this order of magnitude or smaller than this, as was the case in his experiments. This equation also shows that for torsion angles larger than some 30° or 40°, we would have to do the calculations and necessary corrections rigorously, as pointed out by Coulomb.

12.4 Low-Cost Torsion Balance

I would like to point out here that it is possible to do some activities with simple and easily found materials that resemble some of Coulomb's original experiments. These experiments with low-cost material are highly recommended especially in teaching science and physics.

Some activities along these lines were developed by Norberto Ferreira at the University of São Paulo, USP, in Brazil.[493]

[493] See, in particular, [Ferreira, 1978, Section 4.10.4 (Coulomb's law)]; the activities "Coulomb's balance", "the torsion balance" and "the electrostatic balance" of Project RIPE, [Ferreira, s da] and [Ferreira, s db]; and the activities "double pendulum — determining the electric charge" of [Ferreira and Ramos, 2008, pp. 44-48].

Chapter 13

Did Coulomb Experimentally Obtain the Results He Described in His Articles?

A. K. T. Assis

13.1 Introductory Remarks

In 1992 Peter Heering published a paper presenting a replication of Coulomb's setup of the First Memoir on electricity and magnetism.[494] Heering was unable to reproduce Coulomb's experimental results.[495]

In Coulomb's experiment a ball a at the tip of a horizontal needle which can rotate around a vertical twisting thread passing through the center of the needle stays at a certain angle away from another ball t fixed in the laboratory when the two balls become electrified with charges of the same kind, as illustrated in Coulomb's Figure 1, see page 205 of this translation. In particular, in Heering's reproduction the electrified ball a at the end of the needle did not reach an equilibrium position in which it would remain stationary after being repelled by the electrified ball t, despite having a counterweight that damped the needle's oscillations. Heering noticed needle oscillations, mentioning that it was impossible to measure the exact position of the electrified ball a due to these oscillations. He concluded that the main reason for these oscillations was the electrical charges acquired by the experimenter himself, and that these charges constantly affected the position of the mobile electrified ball, causing it to oscillate. Only when he placed a Faraday cage around the torsion balance was he able to make the electrified ball attain equilibrium positions at rest for different torsions of the suspension wire. But Coulomb did not use a Faraday cage in his experiment, since this instrument was not described by Faraday until 1838.[496] Heering presented the following conclusion:[497]

[494] This Memoir is translated in Chapter 11.

[495] [Heering, 1992], [Heering, 1994], [Heering, 1995], [Heering, 2009] and [Heering, 2022]. See also [Dickman, 1993], [Heilbron, 1994], [Chevalier, 1995] and the articles presented in [Blondel and Dörries (Editors), 1994].

[496] [Faraday, 1838]. See also Section 8.4 (Faraday Cage) of [Assis, 2018b], [Assis, 2018a] and [Assis, 2019].

[497] [Heering, 1992, pp. 991 and 993].

> Because of these arguments it seems reasonable to assume that Coulomb did not get the data he published in his memoir by measurement. [...] From our work in replicating Coulomb's experiment it seems quite plausible that Coulomb did not find the inverse square law by the doubtful measurements from his torsion balance experiments but by theoretical considerations.

Coulomb, according to Heering, would have already implicitly assumed that the force between electrified bodies should behave like Newton's law of gravitation of 1687, that is, varying inversely with the square of the distance between the bodies. He would then have presented numerical values in his paper that would fit this law, although these values were not obtained by the experiments he described.

Heering's paper had a strong impact and many authors agreed with him, repeating his arguments. I myself have followed in that vein.[498]

I have now totally changed my mind after studying Coulomb's original papers. In this Section I present several arguments with which I question Heering's conclusions. I will now defend the opposite idea, that is, that Coulomb really obtained that the electric force is inversely proportional to the square of the distance between the electrified bodies from the experimental measurements he made with his torsion balance. The reasons that made me change my mind are related to a much deeper knowledge of Coulomb's original articles, and to several recent works that have analyzed this controversy. Previously I was not aware of these recent works.

13.2 Recent Replications of Coulomb's Experiments Arriving at Opposite Conclusions

In addition to Heering's work published in 1992, I am aware of two other replications of Coulomb's work: Martinez in 2006, as well as Shech and Hatleback in 2014.[499]

Shech and Hatleback were unable to reobtain the results presented by Coulomb in his First Memoir and considered Coulomb's results to be atypical. Furthermore, they realized that torsion balance experiments degenerate rapidly when small changes are made in some experimental parameters.

Martinez, on the other hand, was able to reproduce Coulomb's experimental results with his replication of the torsion balance. Martinez did not need to use a Faraday cage around the torsion balance to obtain results analogous to Coulomb's. Martinez's evaluation of this issue:[500]

> In sum, these results all converge to one conclusion: Coulomb obtained his reported numbers from experiment. His results were not unusual, they were almost certainly typical. Therefore, he was justified in his claim that he had experimentally demonstrated what he confidently called the "Fundamental Law of Electricity".

[498] [Assis, 1994, p. 26], [Assis, 1995, p. 20], [Assis, 1998a, p. 26], [Assis, 1999a, p. 33], [Assis, 1999b, p. 18], [Assis, 2013, p. 29], [Assis, 2014, p. 32], [Assis, 2015a, p. 14], [Assis and Chaib, 2015, p. 21], [Assis, 2018b, p. 268], [Assis, 2018a, p. 262] and [Assis, 2019, p. 281].

[499] [Martinez, 2006] and [Shech and Hatleback, 2014]. See also [Blondel and Wolff, 2013e].

[500] [Martinez, 2006, p. 547].

Therefore, there is divergence between authors who were able to reproduce Coulomb's results by replicating his experiments using materials, equipment, and procedures as close as possible to Coulomb's, and authors who were not able to reproduce Coulomb's experimental values.

13.3 Faraday Cage Around the Torsion Balance

By reproducing Coulomb's experiments with his replication of the torsion balance, Heering observed that the needle with the ball electrified at its tip hardly ever reached a resting state of equilibrium.[501] He concluded that the constant oscillations of the needle were caused by the electrification of the person performing the experiments. The oscillations ended only when Heering surrounded the torsion balance with a Faraday cage.

Coulomb did not explicitly use a Faraday cage in his experiment, since this instrument was only described by Faraday in 1838. Despite this fact, some researchers have hypothesized that the very glass used by Coulomb in his balance may have behaved as a conductor, functioning as a Faraday cage.[502]

This effect may well have happened, although it was not intended by Coulomb. As I discussed in the book *The Experimental and Historical Foundations of Electricity*, there are several types of glass depending on the materials they are made from, the manufacturing processes, the impurities they acquire over time, the environment in which they are located, etc. Some of these glasses behave as insulators for the usual electrostatics experiments, although most of them behave as conductors for these experiments.[503]

All this shows that the hypothesis presented by Wolff and Blondel that the glass cylinder around the torsion balance may well have behaved as a conductor in Coulomb's original experiments, that is, as a Faraday cage, is reasonable. The glass around Heering's torsion balance, on the other hand, may have behaved as an insulator. This hypothesis would justify the results obtained by Coulomb which could not be replicated by Heering. The latter author was only able to reproduce results similar to Coulomb's by placing a Faraday cage around his balance. Unfortunately, Coulomb's original torsion balances have not survived to this day. That is, they no longer exist, and we cannot test the properties of the glass he used.

13.4 Coulomb and External Electrostatic Disturbances Acting on his Torsion Balances

It should be noted that Coulomb himself was aware at least as early as 1782 about possible influences exerted by external electric charges disturbing the operation of his torsion balances, both those used for electrical and magnetic research.[504]

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[501] [Heering, 1992].

[502] [Wolff and Blondel, 2009]. See also [Blondel, 1994, pp. 106-107 and 116-117].

[503] See, in particular, Section 6.3.2 (Bodies which Behave as Conductors and Insulators in the Usual Experiments of Electrostatics) from [Assis, 2010b], [Assis, 2010a], [Assis, 2011], [Assis, 2015b] and [Assis, 2017]; as well as Section 4.3 (Differences between Old and Modern Glasses) from [Assis, 2018b], [Assis, 2018a] and [Assis, 2019].

[504] [Gillmor, 1971a, pp. 146-150], [Licoppe, 1995], [Heilbron, 1999, pp. 469-470], [Heering and Osewold, 2005] and [Martinez, 2006].

Coulomb began working on magnetism in the 1770s. In particular, he was one of the winners of the 1777 prize of the Paris Academy of Sciences related to research on the best method of making compasses and on an explanation of the diurnal variations of terrestrial magnetism. The needles supported on a pivot, when deviated from their usual orientation by the variation in the direction of the Earth's magnetic force, were under the action of frictional forces generated by the pivot, which produced torques of the same order of magnitude as those due to the Earth's magnetism. To reduce these frictional torques so that he could measure the diurnal variations of the Earth's magnetism, Coulomb changed the suspension of the needles. He adopted needles suspended by fine silk threads. This method had already been adopted by other researchers such as Francesco Lana de Terzi (1631-1687) in 1686, Lous in 1773 etc.[505] The problem is that this instrument became extremely sensitive, being affected by any disturbance that occurred in the room where the suspended needle was located. In particular, Coulomb assisted Jean-Dominique Cassini de Thury (1748-1845) with such a suspension in Cassini's research into the diurnal variations of terrestrial magnetism. In 1782 it was noticed that when small electrical discharges occurred around the instrument, the magnetic needle oscillated simultaneously. To avoid this problem, Coulomb followed the suggestion of Ettiene François Gattey (1756-1819) to ground the magnetized needle. In order to ground the needle, he replaced the silk thread suspension with a suspension made of a thin metal wire, keeping the rest of the instrument grounded.[506] It was this metal wire that he went on to use in his future electric balances.

Because he worked for many years building and developing various torsion balances for his magnetic and electrical research, Coulomb certainly gained a great deal of knowledge of how they worked and what needed to be done to obtain valid and reliable experimental results with them. Moreover, he did not necessarily present all the details of his experimental procedures and practices in the accounts he presented in his Memoirs.[507]

13.5 Coulomb's Experimental Results that Could Not Be Compared with Previous Theoretical Predictions

There is another argument that strongly suggests that Coulomb did in fact perform the electrical experiments and measurements he described with his torsion balance. If he had only obtained that the force between two small electrified bodies was inversely proportional to the square of the distance between them, it could be argued that he was being influenced by Newton's law of universal gravitation.

However, in the Third Memoir published in 1788, Coulomb experimentally studied the loss of charge by an electrified sphere using his torsion balance.[508] From these experiments he arrived at the following equation:

$$-\frac{d\delta}{\delta} = mdt \ , \tag{13.1}$$

[505] [Gillmor, 1971a, pp. 141-142 and 162-165] and [Heilbron, 1999, pp. 185-192 and 469].
[506] [Gillmor, 1971a, pp. 146-150], [Heilbron, 1999, pp. 469-470], [Heering and Osewold, 2005] and [Martinez, 2006].
[507] [Heilbron, 1994].
[508] [Coulomb, 1788e]. This Memoir is translated in Chapter 17.

in which δ represents the surface density of electricity on the sphere (proportional to its total amount of charge), $d\delta$ the loss of electricity in time dt, while m was a constant. Integration of this equation leads to an exponential decay of electricity on the sphere as a function of time t:

$$\delta = \delta_0 e^{-mt} \ , \tag{13.2}$$

where δ_0 represents the initial surface density of electricity on the sphere.

None of this was previously known. Equations of this type had not been predicted for electricity by other authors. In Gillmor's words:[509]

> There were a number of factors unknown to Coulomb that affect charge leakage on dielectrics.[510] Nevertheless, the laws that he determined in the Third Memoir are exact, especially his exponential law of charge leakage. The work of this memoir provided him with an accurate law (if not an accurate theory) for use in his Fifth and Sixth Memoirs, dealing with charge distribution.

In addition, in the Fifth and Sixth Memoirs published in 1789 and 1791, respectively, he studied the distribution of charge on the surface of electrified conducting spheres in contact.[511] These spheres could have the same radius or different radii. They could also be of the same or different materials. In particular, he used a proof plane[512] to collect some charge at different points on these spheres and measured this collected charge with his electric balance. These measurements were certainly much more difficult and delicate than his experiments of the First Memoir in which he obtained the resultant force between two electrified spheres. After all, besides the effects now being smaller, there were also the difficulties involved in collecting charges by the proof plane and introducing this electrified proof plane into his torsion balance.

The calculations that Coulomb made to compare his experimental results with possible theoretical values were extremely simple and approximate, since there was not yet in his time a proper mathematical theory to predict what the distribution of surface charges at different points of conducting spheres in contact should look like. These detailed calculations were first made by Siméon Denis Poisson (1781-1840) in 1811-1812 using the electric potential function introduced by Pierre-Simon de Laplace (1749-1827) around 1783.[513] The differences found between the experimental measurements of Coulomb and the exact theoretical calculations made by Poisson were only a few percentage points.[514]

William Thomson (1824-1907), also known as Lord Kelvin, made the following assessment:[515]

> In the papers of Poisson on electricity we find the analytical solution of the problems that are combined with the most important parts of Coulomb's experimental

[509] [Gillmor, 1971a, p. 198].
[510] [Note by Gillmor] One factor, for example, is the ionization of the air.
[511] These Memoirs are translated in Chapters 22 and 24.
[512] See footnote 53 on page 30.
[513] [Poisson, 1812a], [Poisson, 1812b], [Poisson, 1813] and [Poisson, 1814].
[514] [Gillmor, 1971a, pp. 202-210], [Potier, 1884, pp. 192-193, 198, 204 and 218-219], [Heilbron, 1999, pp. 494-500] and [Blondel and Wolff, 2011c]. See also footnotes 820, 772 and 778 on pages 361, 339 and 340, respectively. See, moreover, Sections 23.2, 23.3 and 23.5 on pages 373, 374 and 377, respectively.
[515] [Thomson, 1845, pp. 209-210], [Heering, 1992] and [Falconer, 2004].

> researches; the correspondence of the results is very satisfactory, and the strength and beauty of the analysis are placing the theory of electricity next to the theory of gravitation, through mathematical correspondence at the first place of natural science.

Blondel expressed herself as follows:[516]

> Among the numerous measurements taken by Coulomb, a certain number of them could not be compared with numerical results deduced from mathematical laws. This is the case for the distribution of electricity on the surface of two spheres of different diameters, brought into contact and then separated. *The results are complex and impossible to predict analytically.* It was only when Poisson succeeded in 1811 in carrying out the approximation calculations making it possible to calculate the theoretical results corresponding to these experiments by Coulomb that their precision could be evaluated at a few percent.

13.6 Conclusion

In conclusion, today I have no doubt that Coulomb carried out the experiments he described and that he obtained the experimental results he presented in his works, including those of his First Memoir.

It is important to present here another example in which the torsion balance played a crucial role in the precise determination of a fundamental magnitude of nature, namely, Weber and Kohlrausch's famous experiment to find the value of the constant c appearing in Weber's electrodynamics.[517]

Before performing this experiment in 1854-1856, Weber and Kohlrausch had no idea of the order of magnitude of c. It might be, for instance, $10^5\ m/s$ or $10^{11}\ m/s$. Moreover, at that time nobody had any idea that Weber's constant c might have any relation whatsoever with light velocity in vacuum. A crucial portion of their experiment involved the determination of the amount of electricity in two charged balls. To this end they utilized a torsion balance to measure the force they exerted on one another in Gauss and Weber's absolute system of units. In Appendix I of their 1857 paper they presented a detailed description of the torsion balance utilized in their extremely important and original experiment.[518]

It is worth while quoting here Kirchner's words related to this experiment:[519]

> [...] if we are to use the words of Weber and Kohlrausch, we have to formulate the obtained results in the following way.
>
> The ratio between the mechanical and the magnetic measurement of current intensity is as $1 : 3.1074 \times 10^{11}$ in the mm-mg-sec system or as $1 : 3.1074 \times 10^{10}$ in the cgs system.

[516] [Blondel, 1994, pp. 110-111] with my emphasis in italics

[517] [Weber, 1855] with English translation in [Weber, 2021]; [Weber and Kohlrausch, 1856] with Portuguese translation in [Weber and Kohlrausch, 2008] and English translation in [Weber and Kohlrausch, 2021]; and [Kohlrausch and Weber, 1857] with English translation in [Kohlrausch and Weber, 2021]. See also [Weber and Kohlrausch, 1968], [Assis, 1992], [Assis, 1994], [Assis, 2014] and [Assis, 2021].

[518] Appendix I (Description of the Torsion Balance) of [Kohlrausch and Weber, 1857] with English translation in [Kohlrausch and Weber, 2021].

[519] [Kirchner, 1956, p. 531] with English translation in [Kirchner, 1957, p. 625].

Considering that this ratio was then not even known as to its order of magnitude, that we deal therefore with a real pioneering effort, and if one realizes furthermore the primitive equipment they had to work with, one has to admire the work by Weber and Kohlrausch as a masterpiece in the art of experimentation, very few of which exist in the history of our science.

Chapter 14

Second Memoir on Electricity and Magnetism in Which We Determine the Laws According to Which the Magnetic Fluid as Also the Electric Fluid Act Either by Repulsion or by Attraction

Coulomb[520]

The electric balance which I presented to the Academy in the month of June 1785,[521] measuring with exactness, and in a simple and direct manner, the repulsion of two balls which have an electricity of the same nature, it was easy to prove, using this balance, that the repulsive action of two balls charged with the same nature of electricity, and placed at different distances, was very exactly in inverse proportion to the square of the distances; but when I wanted to use the same means to determine the attractive force of the two charged balls of an electricity of different nature, I met, in using this balance to measure the attraction of two balls, a disadvantage in practice, which does not take place in the operation to measure the repulsion. The practical difficulty is that when the two balls come closer by attracting each other, the growing force of attraction, as we will soon see, changing in the inverse ratio of the square of the distances, often increases in a greater ratio than the force of torsion which increases only as the angle of torsion; so that it is only after having failed at many attempts that we succeed in preventing the mutually attractive balls from touching, unless an insulating obstacle[522] is opposed to the needle movement; but as our balance is

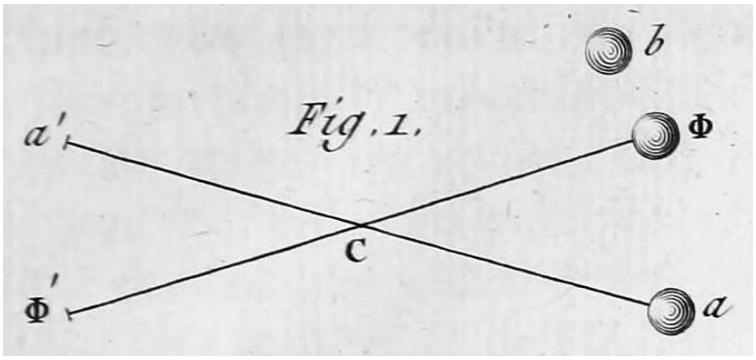
[520] [Coulomb, 1788d], with a complete German translation in [Coulomb, 1890e] and a complete Portuguese translation in [Assis, 2022]. There is a partial English translation in [Coulomb, 1935b]. This work was presented in 1785 to the French Academy of Sciences and published in 1788.

[521] Coulomb's work describing the electric balance is translated in Chapter 11.

[522] In the original: *obstacle idio-électrique*. This expression can also be translated as idio-electric, idio-electric or dielectric obstacle. It refers to substances that can be electrified by friction. That is, they are substances that behave as an insulator or bad conductor in the usual electrostatic experiments. For example, when you hold a piece of amber in your hand and rub it against a cloth, the amber becomes electrified. The term "idio" means "own", "private" or "peculiar". Originally the term "electric" was introduced by

often intended to measure actions of less than one thousandth grains,[523] the cohesion of the needle with this obstacle, disturbs the results, and requires trial and error during which some of the electricity is lost.[524]

Figure 1, and the following calculation, will show in what consist the difficulties of the operation, and will show at the same time the limits within which it is necessary to contain the experiments to ensure their success.



Let aca' be the natural position of the needle, when the suspension thread is not yet twisted; a represents the pith ball,[525] attached to the needle aa' of insulating nature; b is the ball hanging in the hole in the balance.[526] If we electrify the two balls, one with the electricity which we call *positive*, the other with the electricity which we call *negative*, they will attract each other mutually; ball a of the needle tending to approach globe b, will take the position $\Phi c\Phi'$; this position will be such that the torsional reaction force represented by $ac\Phi$, angle whose suspension wire has been twisted, will be equal to the attractive force of the two balls; and if this attractive force was proportional to the inverse ratio of the square of the distances, as we found for the repulsive force, in our First Memoir,[527] we would have, by setting $ab = a$, $a\Phi = x$, D equal to the product of electric mass of the two balls,[528] and the arcs a and x small enough that they can measure the distance of the two balls (otherwise we should take the chord of this arc for the distance, and the cosine of its half for the lever

Gilbert (1544-1603) to characterize substances that behave like amber, that is, that can be electrified by friction, [Assis, 2010b, Section 2.8: Gilbert's Nomenclature: Electric and Non-Electric Bodies], [Assis, 2010a], [Assis, 2011], [Assis, 2015b] and [Assis, 2017]. See also [Roller and Roller, 1953], [Roller and Roller, 1957], [Roller, 1959] and [Gillmor, 1971a, p. 194]. A dielectric substance, or an idioelectric substance, is then a substance that is electric by virtue of its own particular properties, that is, that behaves like an insulator due to its own nature or chemical composition.

[523] That is, forces less than $5.2 \times 10^{-4} N$.

[524] That is, this trial and error procedure lasts for a certain time. During this period of time a portion of the electrification of the balls is lost to the environment due to the conductivity of the air and the supports.

[525] See footnote 469 on page 208. In the original: *la balle de sureau*. This expression can also be translated as elderberry ball, elder pith ball, elderwood pith ball or elder ball, [Gillmor, 1971a, p. 184] and [Heilbron, 1999, pp. 451 and 471].

[526] This ball b remains always at rest relative to the ground.

[527] This 1st Memoir is translated in Chapter 11.

[528] In the original: D = *le produit de la masse électrique des deux balles*. That is, Coulomb is assuming here that the force is directly proportional to the product of the electric charges on the two balls. He called the amount of electric charge in each ball its "electric mass". Certainly this assumption that the force is proportional to the product of the charges and this denomination "electric mass" were influenced by Newton's law of universal gravitation. According to Newton in Book III of his masterpiece *Principia*, the gravitational force between two bodies is proportional to the product of their masses, [Newton, 1934], [Newton, 1999] and [Newton, 2008].

arm); we would have, based on these assumptions, for the balance between the attraction of the two balls and the reaction of the torsion, the formula

$$nx = \frac{D}{(a-x)^2} \ ,$$

or

$$D = nx(a-x)^2 \ .$$

Whence it follows that when $x = a$ or 0, the value of D will be null, that thus there is a point Φ between a and b, where the quantity D is a *maximum*: the calculation gives for this point $x = a/3$. By substituting that value of x in the formula that represents D in the equilibrium case, we will have $D = 4na^3/27$;[529] and therefore whenever D is greater than $4na^3/27$, there will be no position Φ between a and b, where the needle can remain in equilibrium, and the balls will necessarily touch each other: but it should be observed that in practice, although D is smaller than $4na^3/27$, the balls often join together, because the flexibility of the needle suspensions allows the needle to oscillate, and that, past $a/3$, the force of attraction increases in a greater ratio than the force of torsion; so that when the ball Φ arrives, by the amplitude of its oscillation, at a distance x, where D is greater than $nx \cdot (a-x)^2$, the two balls continue approaching each other until they touch.

By conducting myself according to this theory, I succeeded in putting in equilibrium, at different distances, the attractive force of the two electrified balls, with the force of torsion of my micrometer; then comparing the different experiments, I concluded that the attractive force of the two electrified balls, one with the electricity that one calls *positive*, the other with that which is called *negative*, was in inverse proportion to the square of the distances from the center of these two balls, the same relation already found for the repulsive force.

To confirm this result, I tried, for the case of attraction, another means which, although less simple and less direct than the first, requires less care and precautions to succeed; it has, moreover, the apparent advantage of providing experiments made with globes of a very large diameter, instead of being able to operate in the balance only with small globes; but this advantage is only apparent, and we will see thereafter, in the various Memoirs which I will present successively to the Academy, that with balls of two or three lines in diameter [0.45 or 0.68 cm], and by means of the balance, described in our First Memoir, we can not only measure the total mass of the electric fluid contained in a body of any shape, but also the electric density of each part of this body.[530]

[529] We have $D = nx^3 - 2nax^2 + na^2x$. So, $dD/dx = 3nx^2 - 4nax + na^2$. Setting $dD/dx = 0$ we obtain two roots, $x = a$ and $x = a/3$. The first root makes $D = 0$. The second root makes $D = 4na^3/27$, as stated by Coulomb.

[530] That is, Coulomb measures not only the total amount of electric charge on a body, but also the surface charge density at different points on a conductor.

14.1 Second Experimental Method to Determine the Law with which a Sphere One or Two Feet in Diameter Attracts a Small Body Charged with Electricity of a Different Sort from Its Own

The method which we shall follow is analogous to that which we have used in the seventh volume of the *Savans Étrangers* to determine the magnetic force of a steel lamina in relation to its length, its thickness, and its width.[531] It consists in suspending a needle horizontally, of which the end only is electrified and which, when brought to a certain distance from a sphere, electrified with the other sort of electricity,[532] is attracted and oscillates because of the action of the sphere: we determine then by calculation from the number of oscillations in a given time the attractive force at different distances, just as we determine the force of gravity by the oscillations of an ordinary pendulum.[533]

We first consider some observations which have guided us in the experiments which follow. A fiber of silk, taken from a cocoon, and which can carry up to 80 grains without breaking,[534] has a torsional flexibility such that if we suspend a small circular plate, of known weight and diameter, horizontally and in a vacuum from such a fiber of 3 inches in length (8.12 cm), we shall find from the period of oscillation of this little plate, using the formulas explained in a Memoir on the force of torsion printed in the Volume of the Academy for 1784,[535] that when we use a lever [with an arm] of 7 or 8 lines long [1.6 to 1.8 cm] to twist the fiber about its axis of suspension we shall need to apply, for a complete rotation, usually not more than the force of a sixty thousandth of a grain (0.0009 dyn);[536] and if the suspended fiber is twice as long there will be needed only [a force of] a hundred and twenty-thousandth of a grain. Therefore if we suspend a needle horizontally on this fiber, when the needle has come to rest and the fiber is entirely untwisted, and if by means of any force we set the needle in oscillations whose amplitude does not depart from the line in which the torsion is zero by more than 20 or 30 degrees, the force of torsion will have no sensible effect on the period of the oscillation, even when the force that produces the oscillations is not more than a hundredth of a grain (0.52 dyn).[537] Premising this much, let us see how we proceed to determine the law of electrical attraction.

We suspend, (Figure 2) a needle lg made of shellac by a silk thread sc 7 to 8 inches long [18.9 to 21.7 cm] of a single fiber such as is drawn from the cocoon.

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[531] Coulomb is probably referring to the work published in Volume 9 of that periodical which was published in 1780 and which had received the 1777 prize of the French Academy of Sciences, [Coulomb, 1780] and [Coulomb, 1890c, Note 6, p. 84]. This work is translated in Chapter 5.

[532] That is, the tip of the needle and the globe are electrified with charges of opposite sign.

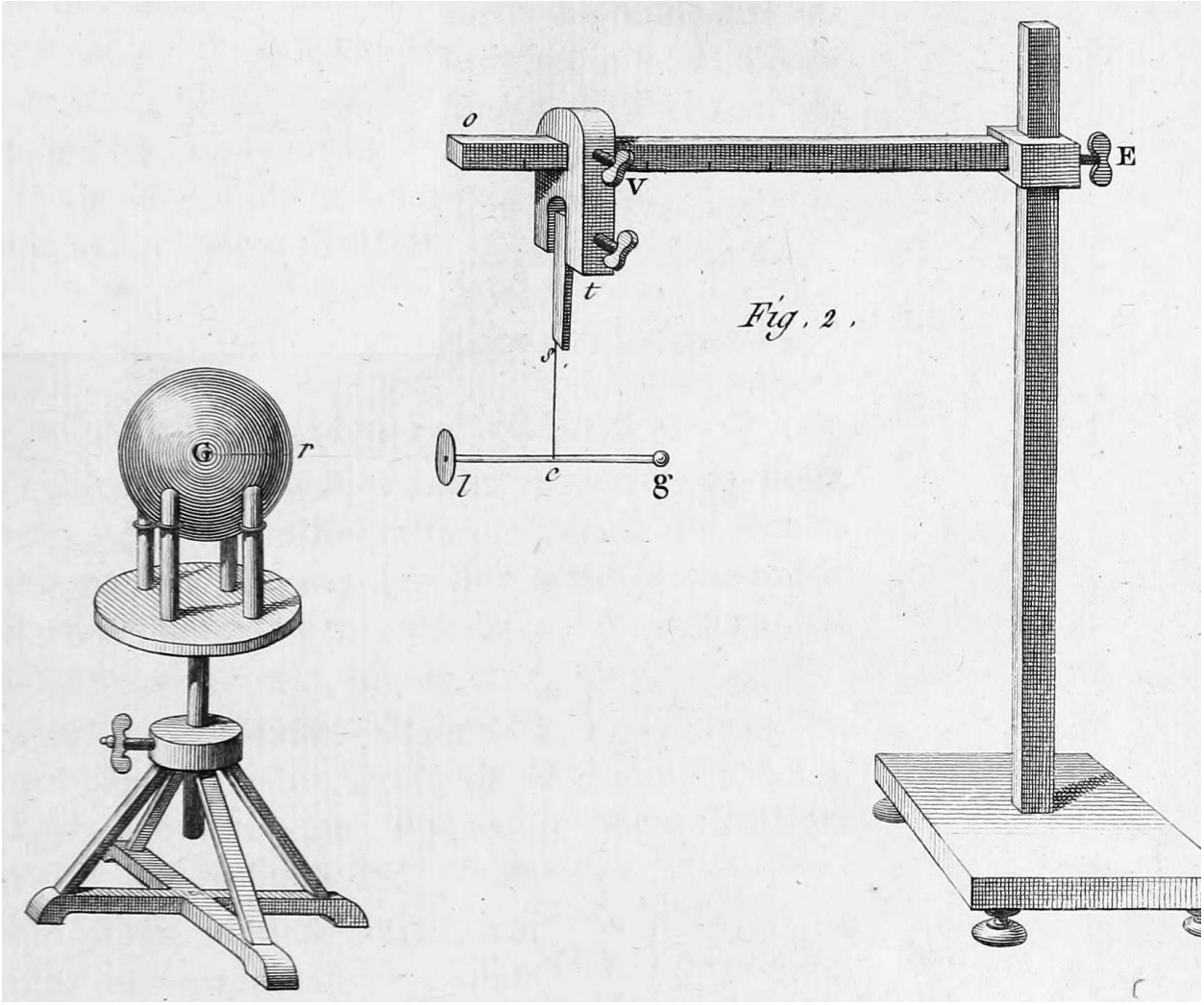
[533] See Section 16.1 on page 267.

[534] Potier said here that 80 grains would be equivalent to 0.424 g, [Potier, 1884, p. 120]. That was a lapse by Potier. As 1 grain = 0.053 g, 80 grains = 4.24 g.

[535] [Coulomb, 1787] with English translation of Sections I and II in [Coulomb, 2012a] and [Coulomb, 2012b]. This Memoir is translated in Chapter 7.

[536] That is, a mass $m = (1\ grain)/(60\,000) = 8.85 \times 10^{-10}\ kg$. With $g = 9.8\ m/s^2$ this mass has a weight $P = mg \approx 9 \times 10^{-9}\ N = 9 \times 10^{-4}\ dyn$. See also footnote 489 on page 213.

[537] That is, in these cases Coulomb will be able to neglect the force of torsion of the wire compared to the electric force between the electrified balls. The period of the oscillations of the needle will depend only on the electric force between the electrified balls, as the force of torsion of the suspension wire can be neglected.



At the end l we fix perpendicularly to the needle[538] a little disk 8 or 10 lines in diameter (1.8 to 2.2 cm), made very light and cut from a sheet of gilt paper;[539] the silk thread is attached at s to the lower end of a little rod st dried in a furnace and coated with shellac or with Spanish wax;[540] this rod is held at t by a clamp which slides along a ruled rod oE and can be placed anywhere we desire by means of the screw V.

G is a globe of copper or of cardboard covered with tin. It is carried on four uprights of glass coated with Spanish wax, and terminated, in order to make the insulation more perfect, by four rods of Spanish wax three or four inches long [8.1 to 10.8 cm]. The lower ends of these four uprights are set in a base which is placed on a little movable table that, as the Figure shows, can be set at the height which is most convenient for the experiment; the rod Eo may also, by means of the screw E, be set at a convenient height.

When everything is ready we adjust globe G in such a way that its horizontal diameter Gr is opposite the center of the plate l, which is some inches away from it. We give an

[538] In the original: *l'on fixe perpendiculairement à ce fil.* Surely this was an oversight. From Figure 2 we can see that Coulomb meant *perpendicular to the needle lg*, [Coulomb, 1890c, Note 7, p. 84].

[539] The shellac needle lg and the silk thread sc behave as insulators. The gilt paper disk l behaves as a conductor.

[540] See footnotes 467 and 468 on page 207.

electric spark to the sphere from a Leyden jar,[541] we then touch the plate l with a conductor and the action of the electrified sphere on the electric fluid of the unelectrified plate gives to the plate a charge of the other sort from that of the sphere; so that when the conducting body is removed the sphere and the plate act on each other by attraction.[542]

14.1.1 Experiment

The sphere G was a foot in diameter (32.48 cm); the plate l was 7 lines in diameter (1.58 cm); the shellac needle lg was 15 lines long (3.38 cm); the suspension fiber sc was a silk fiber taken from the cocoon and 8 lines long:[543] when the slider was at point o the plate l touched the sphere at r, and as the slider was moved toward E the plate was removed from the center of the sphere by the quantity given by the divisions 0, 3, 6, 9, 12 inches [0 to 32.5 cm], and when the sphere was electrified with what is called *positive electricity* and the plate with negative electricity by the method which has been described, we had:

Trial 1. The plate l being at 3 inches (8.12 cm) from the surface of the sphere or 9 inches (24.36 cm) from its center gave 15 oscillations in 20".[544]

Trial 2. The plate l distant by 18 inches (48.72 cm) from the center of the sphere gave 15 oscillations in 40".

Trial 3. The plate l distant by 24 inches (64.97 cm) from the center of the sphere gave 15 oscillations in 60".

14.1.2 Explanation of This Experiment and Its Result

When all the points of a spherical surface act by an attractive or repulsive force which varies inversely as the square of the distance on a point placed at any distance from this surface, it is known that the action is the same as if all the spherical surface were concentrated at the center of the sphere.[545]

[541] See Chapter 12 (The Leyden Jar and Capacitors) of [Assis, 2018b], [Assis, 2018a] and [Assis, 2019].

[542] That is, the plate and the globe become electrified with charges of opposite sign. Initially the conducting globe G and the conducting plate l were discharged. The globe was electrified by the spark coming from a Leyden jar. As a result, the plate l that was insulated from the Earth becomes polarized, with its part closest to the globe becoming electrified with a charge of sign opposite to the charge on the globe, and with its part farthest from the globe becoming electrified with a charge of the same sign as the charge of the globe. When the plate was grounded in the presence of the globe, the charge on the farthest part of the plate was neutralized. Upon removing the grounding, the plate became electrified with a charge of opposite sign to the charge on the globe, and there was an attraction between them, see Section 7.5 (Utilizing Polarization to Charge an Electroscope) of [Assis, 2010b], [Assis, 2010a], [Assis, 2011], [Assis, 2015b] and [Assis, 2017].

[543] The dimensions in Figure 2 suggest that the silk thread sc was 8 inches long (21.656 cm), rather than the 8 lines long (1.80 cm) written here. This might be a lapse by Coulomb, since earlier in mentioning Figure 2 he had said that the silk thread sc was 7 or 8 inches long. On the other hand, from Figure 2 it appears that sc is a little greater than one-half lg. With lg stated as 15 lines, $sc = 8$ lines seems correct. Note it appears the globe and its stand are drawn to a different scale since, if they were the same scale, lg would be greater than the diameter of the globe scale, 1 foot. Note also that in the prior description where sc was given as 8 inches, the plate diameter was given as 8-10 lines whereas in the experiment it is 7 lines.

[544] The meaning of each oscillation for Coulomb corresponds to half the modern period of a complete oscillation, see Sections 6.3 and 9.1 on pages 141 and 187, respectively. So in this first test the modern round-trip period for each oscillation is given by $(2 \times 20s)/15 = 2.666\ s = 2.666$ seconds.

[545] See Section 16.2 on page 268.

As in our experiment the plate l was only 7 lines in diameter and as in the trials its least distance from the center of the sphere was 9 inches, we may, without sensible error, suppose that all the lines which are drawn from the center of the sphere to a point of the plate are parallel and equal; and in consequence the total action of the plate can be supposed to be united at its center just as in the case of the sphere; so that for the small oscillations of the needle, the action which makes the needle oscillate will be a constant quantity for a given distance and will act along the line which joins the two centers. Therefore if we call the force φ and the time of a certain number of oscillations T we shall have T proportional to $1/\sqrt{\varphi}$,[546] but if d is the distance Gl from the center of the sphere to the center of the plate and if the attractive forces are proportional to the reciprocal of the square of the distances or to $1/d^2$, it follows that T will be proportional to d or to the distance;[547] so that when we make our trials and change the distance, the time of the same number of oscillations ought to be proportional to the distance from the center of the plate to the center of the sphere: let us compare this theory with experiment.

	Distance between centers	Duration of 15 oscillations
Trial 1	9 inches	20”
Trial 2	18 inches	41”
Trial 3	24 inches	60”

The distances are as the numbers 3, 6, 8.[548]

The times of the same number of oscillations :: 20, 41, 60.[549]

By theory they ought to have been :: 20, 40, 54.

Thus in these three trials the difference between theory and experiment is 1/10 for the last trial compared with the first, and almost nothing for the second trial compared with the first; but it should be remarked that it took almost four minutes to make the three trials; that although the electricity held pretty well on the day this experiment was tried, it nevertheless lost 1/40 of its amount each minute. We shall see, in a Memoir which will follow the one which I am presenting today, that when the electric density is not very great, the electric action of two electrified bodies diminishes in a given time exactly as the electric density or as the intensity of the action; therefore, since our trials lasted four minutes and since the electric action lost 1/40 each minute from the first to the last trial, the action arising from the intensity of the electric density independently of the distance should be diminished by almost a tenth; consequently, to have the corrected time of the 15 oscillations in the last trial, we must set $\sqrt{10} : \sqrt{9} :: 60'' :$ the quantity required, which will be found to be 57 seconds,[550] which differs only by 1/20 from the 60 seconds found by experiment.

[546] See Equation (16.3) in Section 16.1 on page 267.

[547] I will use α as the proportionality symbol. It was shown in Section 16.1 on page 267 that the force φ which causes small oscillations of a simple pendulum is inversely proportional to the square root of the period T of oscillation of the pendulum, that is, $\varphi \alpha 1/T^2$ or $T \alpha 1/\sqrt{\varphi}$. Therefore, if $\varphi \alpha 1/d^2$, we will have

$$T \alpha \frac{1}{\sqrt{\varphi}} \alpha \frac{1}{\sqrt{(1/d^2)}} \alpha d .$$

That is, in this case T will be proportional to the distance d, as Coulomb said.

[548] That is, the distances are proportional to the numbers 3, 6 and 8.

[549] That is, times are proportional to the numbers 20, 41 and 60.

[550] That is, let x be the required quantity. We have to solve the following equation:

We have thus come, by a method absolutely different from the first,[551,552] to a similar result; we may therefore conclude that the mutual attraction of the electric fluid which is called *positive* on the electric fluid which is ordinarily called *negative* is in the inverse ratio of the square of the distances; just as we have found in our First Memoir, that the mutual action of the electric fluid of the same sort is in the inverse ratio of the square of the distance.[553,554]

14.1.3 First Observation

We feel that it is very easy, by employing the method which precedes, to obtain by means of the oscillations of the electric needle, the laws of repulsive force in the same way we have determined that of the attractive force. Indeed, if the plate is made to touch the electric globe, it will take an electricity of the same nature as that of the globe, and will be repelled; so that the needle will oscillate under this repulsion, in a position absolutely opposed to the first,[555] and the number of oscillations, in a given time, compared with the distance from the center of the plate to the center of the globe, will make known the repulsive force, by the same calculation that we have just followed to obtain the attractive force. However we must say that experiments on the repulsive force of the electric fluid can be carried out in a simpler, more exact and more convenient way by means of the balance we described in our First Memoir, as we shall see in what follows.

14.1.4 Second Observation

If we wanted to use the same method to determine the amount of electricity that is shared between an electrified globe and a conducting body of any shape, in contact with this globe, here is how it could be done: after having electrified the globe and determined, in this first

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$$\frac{\sqrt{10}}{\sqrt{9}} = \frac{60\ s}{x}\ .$$

From this equation it follows that $x = (180\ s)/\sqrt{10} = 56.92\ s \approx 57\ s$.

[551] In the first method described at the beginning of this paper Coulomb had found the variation of the force with distance by comparing the torsion angles of the suspension wire for different distances between the electrified spheres, measuring these angles with the spheres at rest relative to each other.

[552] [Note by Bucciarelli] But note that Coulomb does not give any experimental data for the first method. I conjecture his data was not good enough to satisfy any reader and so he felt compelled to develop a second method.

[553] [Note by Potier] Coulomb did not provide enough information for us to estimate the charges of the globe and the gilt paper disk. But it is clear that the experiment can only be successful under these conditions if the charge on the disk is very small relative to the charge on the globe. Considering the disk as a point, the attraction of the globe on the disk is given by

$$\frac{eM}{d^2}\left[1 + \frac{e}{M}\frac{R^3}{d}\frac{(2d^2 - R^2)}{(d^2 - R^2)^2}\right]\ .$$

The variations of the factor in square brackets will only be negligible if e (the disk charge) is a very small fraction of M (the globe charge).

[554] For a deduction of this result see, for example, [Maxwell, 1954, Vol. I, §160, pp. 250-252].

[555] In the case of Figure 2, if disk l and globe G are electrified with charges of the same sign, then the disk will be repelled by the globe and the silk thread cs will be twisted. At equilibrium, disk l will be as far away from G as possible, such that the order of the letters in this case would be $Grgcl$. The needle could then oscillate around this new equilibrium position.

state, its electric action on the plate of the needle set at a given distance by observing its oscillations, one would immediately touch the globe with the conducting body which must take up a portion of the electricity of the globe; and by separating this body from the globe, we would again determine, by the oscillations of the needle, the quantity of electricity which remains in the globe; and the difference of this quantity with that which the globe had before contact, will measure that taken by the body put in contact. It is unnecessary to warn [the reader] that such experiments can only succeed in very dry days, when the insulated bodies slowly lose their electricity; that it is necessary to take into account this decrease of electricity in the reduction of the experiments which follow one another; that it is necessary to avoid the formation of any air current in the room where one operates, and to move any conducting body at least three feet [97 cm] from the electrified globe, and even from the needle: but we repeat that when we determine in what follows, by experiment and theory, the manner in which the electric fluid is distributed in the different parts of the bodies, it will be seen that all these experiments are more successful when the electric balance is used rather than when the method of oscillations, which we have just explained, is used.

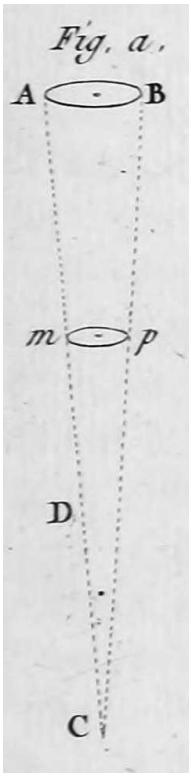
14.2 Experiments to Determine the Law According to Which the Magnetic Fluid Acts Whether by Attraction or by Repulsion

Magnetized bodies, as well as electrified bodies, act on each other by attraction and by repulsion at finite distances, the magnetic fluid seems to have, if not by its nature, at least by this property, an analogy with the electric fluid; and by this analogy, it can be presumed that these two fluids act according to the same laws: all the other attractive or repulsive phenomena with which Nature presents us; whether in the cohesion of bodies, or in their elasticity, or in chemical affinities, where the forces of attraction and repulsion appear to act only over very small distances; it would seem to result, that they do not follow the same laws as electricity and magnetism. In effect, the theory and calculation of the attraction and repulsion of the elements of bodies, teach us that whenever the elementary molecules of bodies attract or repel each other by forces which decrease in the ratio, or in a ratio less than the cube of the distances, for example, as [linearly] with distance, bodies can act on each other at finite distances; but that in the case when the action of molecules decreases in the ratio, or in a greater ratio than the cube of the distances, in this case the bodies can

only act on each other at infinitely small distances.[556,557]

[556][Note by Coulomb] *On the attractive and repulsive action of bodies, according to the law of distance.*

Figure *a* represents a very sharp cone or small pyramid, all the parts of which attract point C, according to the inverse ratio $(n+2)$ of the distances [that is, with the force being inversely proportional to the distance raised to the power $(n+2)$].



Let $x = Cp$, the action of the circular zone pm on the point C, will be $mdx \cdot x^2/x^{2+n}$, whose integral will be $\frac{m}{1-n}\left(k + x^{1-n}\right)$; to have k, it is necessary to suppose the truncated pyramid, or that the action vanishes in D when $x = CD = A$, which gives for the complete integration $\frac{m}{1-n}(-A^{1-n} + x^{1-n})$, where it is necessary to notice that when A is equal to 0, if n is greater than 1, [then] A^{1-n} will be equal to $1/0$, or infinity; if n is less than unity, [and still assuming $A = 0$,] in this case (A^{1-n}) will be equal to 0; or, if you will, the whole attractive force will be $mx^{1-n}/(1-n)$.

That is, in the case where n is greater than unity, or when the repulsion or attraction diminishes in a ratio equal to or greater than the cube of the distances, the value of the constant is infinite relative to the value of the variable that expresses the greater or lesser extent of the cone; and that thus the attraction or repulsion takes place only in the point of contact, and that [the action] of the distant parts is infinitely small relative to that [action] of the contact; but in the case where n is less than unity, that is, whenever the action decreases in a ratio less than the cube of the distances, then the action of the distant parts influences the total attraction, which is zero for an infinitely small pyramid and proportional to x^{1-n} for the pyramid whose length is x.

It seems to result from this calculation, that the cohesion, the elasticity and all the chemical affinities where the elements of the bodies do not appear to have any action except very near the point of contact, and where the elective attraction appears to depend on the figure of these elements, can act between them

We may have reason to return to this subject in the continuation of our Memoirs on electricity.

We employed in this new research two methods to determine by the experiment, by which law the magnetic fluid acts. The first of these methods, consists of suspending a magnetic needle, present to it in his magnetic meridian another magnetic needle, suitably placed, and to determine by calculation and observation, at different distances, with what force the magnetic fluid of one of the needles, acts on the magnetic fluid of the other. In the second method, we use a magnetic balance, approximately similar to our electric balance, described in the First Memoir; but before reporting the details of our experiments, it is necessary to recall some known properties of magnetized needles, which will be useful to us.

A needle, from 0 to 24 inches in length (0 to 64.97 cm), of good steel, strongly tempered, magnetized by the method of the double touch, as Mr. Aepinus has described and employed in accord with his excellent theory of magnetism and electricity,[558] acquires a pole at each end; its magnetic center[559] is placed approximately towards its middle.[560]

In two magnetized needles, the poles of the same name repel each other, and the poles of a different name attract each other. This attraction or repulsion increases as the distance where the ends of the needles are presented to the other, decreases.

If a magnetized needle is suspended horizontally, so that it can turn freely around its center, it will always place itself in the same direction, which is called its *magnetic meridian*; this meridian will form an angle with the meridian of the world,[561] this angle will vary a little during the day, according to the hour of the day, by a kind of periodic movement: it will vary every year, by another movement probably also periodic, but whose duration, for each point of the Earth, is still unknown to us.

If a needle, thus suspended horizontally, is placed in oscillation, it will be displaced equally on both sides of its magnetic meridian; and it will always be brought back there, by a force easy to determine, if we observe the duration of the oscillations, and if we know the figure and the weight of the needle. See the seventh volume of the *Savants étrangers, Mémoires de l'Académie*.[562]

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only in a relation very close to the inverse ratio of the cube of the distances. Perhaps, in addition, all the chemical affinities depend of two actions, one repulsive, the other attractive, analogous to those which we find in electricity and magnetism.

[557] In the German translation of this work by Coulomb, there appears a Note with some further clarification, [Coulomb, 1890e, Note 9, p. 85]. There it is stated that the case in which the forces decrease according to the cube of the distances is not immediately solved by the consideration made by Coulomb in this footnote, because for $n = 1$ and $A = 0$ we obtain the indefinite form 0^0. However, for $n = 1$ the integration of the differential expression established by Coulomb, namely, $mdx \cdot x^2/x^3 = mdx/x$, leads to the value $m(\ln x - \ln A)$. Therefore we see that also in this case, for $A = 0$, the value of the constant, that is, of $-\ln A$, is infinitely large in relation to the value of the variable $\ln x$. Therefore the presentation given by Coulomb in footnote 556 is also correct in this case where $n = 1$ and $A = 0$.

[558] See Section 4.6 and footnote 182 on pages 48 and 73, respectively.

[559] See footnote 146 on page 59.

[560] That is, the magnetic center will be approximately in the middle of the needle, at its central point.

[561] See footnote 126 on page 55. This angle is called the magnetic declination.

[562] See footnote 531 on page 232.

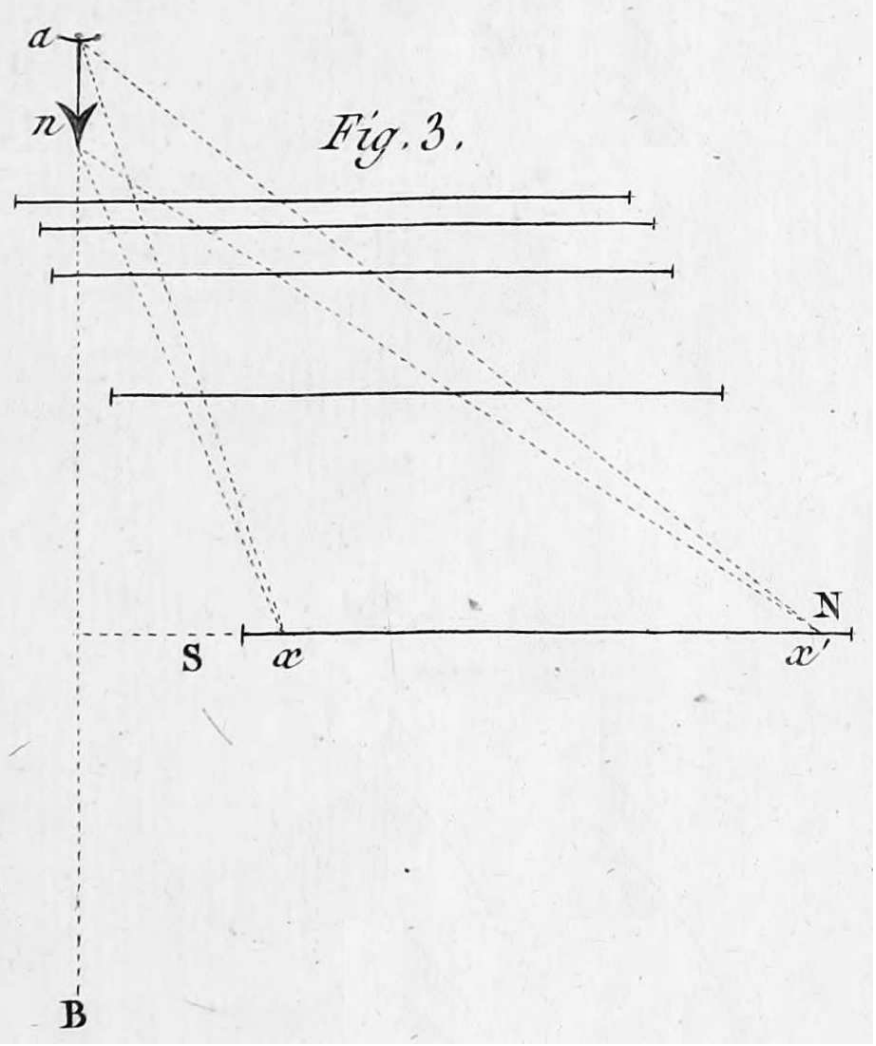
14.2.1 Preparation for the Experiments

I took a wire of excellent steel, pulled through the die,[563] it was 25 inches in length (67.68 cm), and $1\frac{1}{2}$ line in diameter (0.33 cm); I magnetized it by the double touch method, its magnetic center was found approximately at its middle. I then suspended, by means of a silk thread, such as it emerges from the cocoon, three lines in length,[564] a magnetized needle 3 inches long (8.12 cm); and when this needle stopped, I traced its magnetic meridian, which I extended up to two feet [65 cm] away from the center of suspension. I then raised (Figure 3), perpendiculars on this magnetic meridian.[565]

[563] In the original: *tiré à la filière*. See footnote 389 on page 174.

[564] Again there might be an oversight here, as described in footnote 543 on page 234. In this case the silk thread might be 3 inches long (8.12 cm) instead of 3 lines long (0.68 cm), although a suspension thread of 3 lines (0.68 cm) can not be ruled out.

[565] Those lines that were drawn perpendicular to the magnetic meridian were also horizontal. The magnetic meridian is represented by the line anB in Figure 3. The magnetized wire that will interact with the needle an is represented by SN. Its poles or centers of action are represented by the points x and x'. Coulomb will show that they are not located exactly at the ends of the wire. We can see from Figure 3 that the distance nx between the North pole of the needle and the South pole of the wire is always smaller than the distance nx' between the North pole of the needle and the North pole of the wire. That is, the greatest interaction between the wire and the needle occurs between their opposite poles. In order for the needle not to be deflected from its magnetic meridian by the magnetized wire, the net force on its poles must be along this meridian.



I placed my steel wire along these perpendiculars, and I slid it along until the needle *na* returned to the direction of its magnetic meridian, as it was placed there naturally before the steel wire was presented to it; and I observed, according as my magnetized wire was more or less close to the suspended needle, how much the end of this wire was beyond, or under, the magnetic meridian, when the needle stopped on its meridian.

First Experiment[566]

[566]The + sign in the next Table indicates that the end of the wire has passed the magnetic meridian of the needle. The sign − indicates that the end of the wire is before the magnetic meridian, as illustrated in Figure 3. These signs were not included by Potier in the reprint of Coulomb's Memoirs, [Potier, 1884, p. 128]. When Coulomb speaks of the distance from the wire to the end of the needle, we must understand the shortest distance between the end n of the needle and any point along an infinite straight line passing along the wire. In the case of Figure 3, this distance is measured along the line anB. It is measured between the end n of the needle and the intersection of this line anB with the line orthogonal to it passing along the wire SN.

	Distance of the wire from the tip of the needle	The extremity [of the magnetized wire] exceeds the magnetic meridian of
First trial	1 inch	+10 lines (+2.25 cm)
Second trial	2 inches	+9 lines (+2.03 cm)
Third trial	4 inches	+8 lines (+1.80 cm)
Fourth trial	8 inches	−4 lines (−0.90 cm)
Fifth trial	16 inches	−42 lines (−9.49 cm)

Second Experiment

A magnetized needle two inches in length [5.4 cm] was suspended horizontally at its center: free and solicited only by the magnetic force of the globe of the Earth, it made 34 oscillations in 60 seconds. We have again used the same magnetized wire of the previous experiment, which was 25 inches in length; but, instead of placing it horizontally and perpendicular to the magnetic meridian, as earlier, we placed it vertically in this meridian at a distance of 2 inches (5.4 cm) from the end of the hanging needle.[567] The South pole of the vertical wire matching the North pole of the needle, and then in lowering it vertically, always at the distance of 2 inches from the end of the needle, we counted the number of oscillations the needle made in 60 seconds, depending on whether the end of the steel wire was more or less lowered below the level of the needle: here is the result of this experiment.

	The extremity of the wire	Number of oscillations in 60 s
First trial	at the level of the needle	120
Second trial	lowered by 6 lines [1.356 cm]	122
Third trial	lowered by 1 inch [2.707 cm]	122
Fourth trial	lowered by 2 inches [5.414 cm]	115
Fifth trial	lowered by 3 inches [8.121 cm]	112
Sixth trial	lowered by 4 inches [10.828 cm]	98
Seventh trial	lowered by 8 inches [21.656 cm]	39

Third Experiment

We hung a needle 4 lines long (0.90 cm) instead of the first; the steel wire was placed at 3 inches (8.12 cm) from the end of this needle, vertically, as in the preceding experiment, of which we followed all the procedures. The free needle being solicited only by the magnetic force of the Earth makes 53 oscillations in 60".

	The extremity of the steel wire	Number of oscillation in 60 s
First trial	at the level of the needle	152
Second trial	lowered by 1 inch [2.707 cm]	152
Third trial	lowered by 2 inches [5.414 cm]	148
Fourth trial	lowered by 4 inches [10.828 cm]	120
Fifth trial	lowered by 8 inches [21.656 cm]	58

[567] See footnote 566 for the meaning of this distance between the wire and the extremity of the needle.

14.2.2 Explanation and Results of These Three Experiments

The three experiments above prove that the center of action[568] of each half of our wire is placed at a very short distance from the end of this wire; so that in our steel wire, 25 inches in length, we may, without appreciable error, suppose all the magnetic fluid condensed towards the end of this wire within 2 or 3 inches of length. Indeed, in the first experiment, the steel wire is placed horizontally and perpendicularly to the direction of the magnetic meridian where the suspended needle is located; this needle is solicited by two forces, the magnetic force of the globe of the Earth, which retains it in the meridian, and the magnetic force of the different points of the magnetized steel wire; but since in our first experiment, the needle is found, at all tests, placed on its magnetic meridian, it results that all the magnetic forces of the steel wire of 25 inches in length, acting on the needle, are in equilibrium with each other: thus, in the first three trials, where the distances are 1, 2 and 4 inches, the magnetic forces of the last eight to ten lines of the end of the wire, which lie beyond the meridian, are in equilibrium with the forces of all the rest of the wire; so that it seems that we can roughly assume that half of the magnetic fluid with which half of the wire is loaded, is concentrated towards the last ten lines from its end.[569]

The second and third experiments give the same result. In these two experiments, the steel wire is placed vertically in the magnetic meridian of the needle, therefore, the action of the upper part of the wire being very oblique to the suspended needle, and acting moreover at a great distance, should only slightly influence the needle oscillations; but we see in these two experiments, that the greater number of the oscillations of the suspended needle occurs when the end of the wire was placed a little less than an inch below the level of the suspended needle: thus the average force of the lower half of the steel wire, had its resultant at 8 or 10 lines above its end, as we have just found it by the first experiment, from which it follows that in the steel wire 25 inches in length which we employed, and which had been magnetized by the double touch method, we can, without appreciable error, assume that the magnetic fluid is concentrated at 10 lines from its end. This first result was necessary before attempting to determine the law according to which attraction and repulsion take place in relation to the distance: we will see in another Memoir, that the concentration of the magnetic fluid towards the end of the needles magnetized by the double touch method, is a necessary consequence of this way of magnetizing.

[568] In the original: *centre d'action*. That is, the magnetic pole.

[569] In the original (my emphasis in italics): "les forces magnétiques des 8 à 10 dernières lignes de l'extrémité de l'*aiguille*, qui dépassent le méridien, sont en équilibre avec les forces de tout le reste de l'*aiguille*; en sorte qu'il paraît que l'on peut à peu près supposer que la moitié du fluide magnétique, dont la moitié de l'*aiguille* est chargée, est concentrée vers les dix dernière lignes de son extrémité." It seems to me that by an oversight Coulomb wrote about the end of the needle and the rest of the needle, when in fact he was referring to the end of the wire and the rest of the wire (whose torques on the suspended needle balance each other out). Due to this oversight and to clarify the meaning of Coulomb's sentence, I wrote "wire" three times at the end of this sentence, instead of "needle".

14.3 The Magnetic Fluid Acts by Attraction or Repulsion in a Ratio Compounded Directly of the Density of the Fluid and Inversely of the Square of the Distance of Its Molecules

The first part of this proposition does not need to be proved;[570] let us pass to the second.

We have seen that the magnetic fluid in our steel wire 25 inches long was concentrated at its ends within a length of 2 or 3 inches; that the center of action of each half of this wire[571] was about 10 lines (2.25 cm) from its ends: therefore, by setting up some inches away from our steel wire a very short needle, in which, as we shall see in the sequel, the magnetic fluid may be supposed to be concentrated in 1 or 2 lines [0.226 or 0.452 cm] at its ends, we may calculate the mutual action of the wire on the needle and of the needle on the wire by supposing the magnetic fluid in the wire concentrated at a point 10 lines from its ends and in a needle an inch long [2.7 cm] at a point 1 or 2 lines from its end. These reflections have directed us in the experiment which follows:

Fourth Experiment

We suspended a steel wire weighing 70 grains (3.72 g) and an inch in length, magnetized by the method of double touch, by a silk thread 3 lines long[572] made of a single brin such as it comes from a cocoon;[573] we allowed it to come to rest in the magnetic meridian, we then placed vertically in the meridian at different distances a steel wire 25 inches long, in such a way that its end was always ten lines below the level of the suspended needle;[574] in each trial we changed the distance, and then by oscillating the suspended needle we counted the number of oscillations which it made in the same number of seconds. The following is the result of these experiments:

First trial. The free needle oscillating because of the action of the Earth [alone] makes 15 oscillations in 60".

Second trial. The wire placed at 4 inches [10.83 cm] from the center of the needle, the needle makes 41 oscillations in 60".

Third trial. The wire placed 8 inches from the center of the needle, the needle makes 24 oscillations in 60".

Fourth trial. The wire placed 16 inches from the center of the needle, the needle makes 17 oscillations in 60".

[570] That is, Coulomb considers that it is not necessary to prove that the magnetic force between two magnetic particles is proportional to the product of the quantities of magnetic fluid contained in these two particles. I discuss this statement by Coulomb in Section 16.6, page 279.

[571] By a lapse Coulomb wrote here on each half of this *needle* instead of each half of this *wire*, namely, *de chaque moitié de cette aiguille*. I replaced here needle with wire.

[572] Again there might be a lapse here, as in footnotes 543 and 564 on pages 234 and 240, respectively. The silk thread might be 3 inches long (8.12 cm) instead of 3 lines long (0.68 cm), although a silk thread of 3 lines might also work.

[573] [Note by Bucciarelli] "Brin" (a single silk fiber as obtained from a cocoon) and a "brave" composed of two brin. See the abstract of an article at https://researchportal.hw.ac.uk/en/publications/mechanical-properties-of-single-brin-silkworm-silk.

[574] In this way the lower magnetic pole of the vertical wire was at the same horizontal level as the suspended needle.

14.3.1 Explanation of This Experiment and Its Result

When a pendulum is freely suspended and acted on by forces in a given direction, which make it oscillate, the forces are measured by the inverse ratio of the square of the time of the same number of oscillations, or, what comes to the same thing, by the direct ratio of the square of the number of oscillations made in the same time.[575]

In the preceding experiment, the needle oscillates because of two different forces; the one is the magnetic force of the Earth, the other is the action of all the points of the wire on the points of the needle. In our experiment all the forces are in the plane of the magnetic meridian, and since the needle is suspended horizontally the true force which makes it oscillate depends on the horizontal components of all these forces.

Now we have seen from the three preceding experiments that since the magnetic fluid is concentrated at the ends of our wire, it may be supposed to be concentrated at a point 10 lines from the end of the wire. And, since the suspended needle is an inch long, that the boreal end[576] is attracted at a distance of 3 inches and a half [9.47 cm] and that the austral end is repelled by the lower pole of the wire,[577] which is distant from it $4\frac{1}{2}$ inches [12.18 cm]; it may be supposed without sensible error that the mean distance at which the lower end of the steel wire exerts its action on the two poles of the needle is 4 inches [10.83 cm]. Consequently, if the action of the magnetic fluid was in the inverse ratio of the square of the distances, the action of the lower pole of the steel wire on the needle should be proportional to $1/4^2$, $1/8^2$, $1/16^2$ or to 1, 1/4, 1/16.

Now since the horizontal forces which make the needle oscillate are proportional to the square of the number of oscillations made in the same time, and since because of the magnetic force of the Earth alone, the free needle makes 15 oscillations in 60", this force can be measured by the square of these 15 oscillations or by 15^2. In the second trial the combined forces of the Earth and of the steel wire make the needle perform 41 oscillations in 60"; therefore, these forces combined are measured by 41^2, and the force resulting from the action of the magnetized steel wire alone is consequently measured by the difference of these two squares; it is thus proportional to $\overline{41}^2 - \overline{15}^2$. We shall then have for the action of the wire on the needle:

	Distance	Force depending on the magnetic action of the steel wire
In the second trial	at 4 inches	$\overline{41}^2 - \overline{15}^2 = 1456$
In the third trial	at 8 inches	$\overline{24}^2 - \overline{15}^2 = 351$
In the fourth trial	at 16 inches	$\overline{17}^2 - \overline{15}^2 = 64$

The second and third trials, in which the distances are as 1 : 2, give very approximately for the forces the inverse ratio of the square of the distance.[578] The fourth trial gives a number which is a little too small; but it may be remarked that in this fourth trial the

[575] See Section 16.1 on page 267.

[576] That is, the end with the North pole. See footnote 127 on page 56.

[577] In the original we have here "by the lower pole of the *needle*", instead of "by the lower pole of the *wire*", (*par le pôle inférieur de l'aiguille*). However, certainly Coulomb was referring to the lower pole of the wire, S, which is repelling the austral (or southern) end of the needle.

[578] That is, the ratio between the squares of the distances is very approximately the ratio between the numbers of oscillations made at the same time, with these numbers being due only to the interaction between the wire and the needle (disregarding the influence of the terrestrial globe), namely:

distance of the lower pole of the steel wire from the center of the needle is 16 inches; and that the distance of the upper pole from the center of the needle is about $\sqrt{16^2+23^2}$: thus if we represent the action of the lower pole by $1/16^2$, the horizontal action of the upper pole will be $\frac{16}{(16^2+23^2)^{3/2}}$; so that the action of the lower pole is to that of the upper pole about as 100 : 19; from which it follows that since the oscillations of the needle are caused by the action of these two poles, and since the action of the upper pole is opposed in sense to that of the lower pole, the square of the oscillations which the action of the lower pole of the magnetized wire alone would produce is diminished by 19/100 by the opposite action of the upper end of the same wire; and so to have the action of the lower part of the wire alone, we must, if we represent the true value of this force by x, set $(x - 19x/100) = 64$, from which $x = 79$. If we substitute this quantity in the result of the fourth trial we shall find

	Distance	Force
Second trial	4 inches	1456
Third trial	8 inches	351
Fourth trial	16 inches	79

These forces are very approximately as the numbers 16, 4, 1, or are in the inverse ratio of the square of the distance.[579]

I have repeated this experiment several times by suspending needles two and three inches long [5.41 and 8.12 cm] and I have always found that when I have made the necessary corrections which I have just explained, the action of the magnetic fluid, whether repulsive or attractive, was inversely as the square of the distances.

14.3.2 First Observation

We can see, that in this experiment, we suppose that our wire is magnetized by the double-touch method; if one presents alternately at the same distance, its boreal pole and its austral pole, to the end of a needle magnetized by the method of the double touch, the boreal pole of the magnetized wire will attract the austral pole of the needle, with exactly the same force as the austral pole of this wire will repel the austral pole of the needle, and *vice versa* for the boreal pole of the needle. This property which, as we shall see later, is a necessary consequence of the theory of magnetism, will moreover be proved by experiment, using the magnetic balance, of which we will presently give the description and uses.

$$\frac{(1/4)^2}{(1/8)^2} \approx \frac{1456}{351}\ , \qquad \text{that is,} \qquad 4 \approx 4.15\ .$$

[579] Gillmor made the following comment regarding this ratio, [Gillmor, 1971a, p. 190, Note n]:

> If 1456 is chosen as unity, the ratios of
>
> $$\frac{1}{1456} : \frac{1}{351} : \frac{1}{79}$$
>
> are as $1 : 4.15 : 18.4$. If 79 is chosen as unity, the ratios of $79 : 351 : 1456$ are as $1 : 4.4 : 18.4$. For the inverse square law, the exact ratios should be as $1 : 4 : 16$.

14.3.3 Second Observation

Once we accept the law of the inverse ratio of the square of the distances, it would be easy — in the first experiment, where the magnetized wire is placed horizontal and perpendicular to the magnetic meridian, and where we find, in the last trial, that it is necessary to move the end of the wire about 42 lines from the meridian of the needle — to assess if the calculation [in accord with the inverse square law] would give, for the direction of the resultant of all the actions of each half of the wire, a line which would pass [through a point] nine or ten lines from the extremity of this wire. We are going to present the calculation which will determine this direction according to the last trial of the first experiment, where the needle is three inches in length, and where the magnetized steel wire, 25 inches in length, is placed horizontally and perpendicularly to the magnetic meridian, 16 inches away from the tip of the needle.

Let in Figure 3,[580] x, be the point where this resultant passes, for the pole which is placed closest to the meridian line of the needle; x', the point at the other end of the wire where one supposes all the magnetic fluid concentrated: as regards the magnetic fluid of the suspended needle, although its center of action is at two or three lines from its ends, we can suppose it [to be] at its extremities, because each pole of the wire acts on the two poles of this needle; and that if, by this supposition, one makes the pole n of the needle, too close to the pole S of the steel wire by two or three lines, one makes at the same time the pole a of the needle too far from the S pole by the same amount; thus the error of the supposition is found to be almost compensated.

But we find by experiment, that the distance from the end of the wire to the meridian line of the needle, is in the last trial of $3\frac{1}{2}$ inches.[581] Thus by doing $x = Sx = Nx'$, distance from the end of the wire to the center of action, we will have the following formulas, for the force that the centers of action of the wire exert on each end of the needle, in a direction perpendicular to the needle.

Action of pole S on pole n:

$$\frac{3.5 + x}{[(16)^2 + (3.5 + x)^2]^{3/2}} .$$

Action of pole S on pole a:

$$\frac{3.5 + x}{[(19)^2 + (3.5 + x)^2]^{3/2}} .$$

Action of pole N on pole n:

$$\frac{28.5 - x}{[(16)^2 + (28.5 - x)^2]^{3/2}} .$$

Action of pole N on pole a:

$$\frac{28.5 - x}{[(19)^2 + (28.5 - x)^2]^{3/2}} .$$

But, since in this [first] experiment, the steel needle lies along it's magnetic meridian, and in that each of the preceding forces acting perpendicular to this needle have the same lever arm tending to rotate it about its point of suspension, it follows that all the forces are in equilibrium;[582] from which we draw the equation:

[580] This Figure 3 appears on page 241.

[581] The value of 3.5 inches is equivalent to 42 lines, both of which are equivalent to 9.5 cm.

[582] That is, the resulting torque to make the needle rotate around the suspension thread is zero.

$$\frac{3.5+x}{[(16)^2+(3.5+x)^2]^{3/2}}+\frac{3.5+x}{[(19)^2+(3.5+x)^2]^{3/2}}$$
$$=\frac{28.5-x}{[(16)^2+(28.5-x)^2]^{3/2}}+\frac{28.5-x}{[(19)^2+(28.5-x)^2]^{3/2}}\,.$$

But, as we have already seen that x must be less than an inch, we can, as a first approximation, neglect it in the denominator of our equation, the numbers of which are very large, relative to x, or make x equal to 1/2 inch, which more closely approximates its true value.

Thus, it will result from the calculation of the formula, for the value $Sx = x = 56/75$ inches, about 9 lines (2.03 cm), as in the first two trials.

By a similar calculation, it will be found that, when the extremity of the steel wire was 8 inches distant from the extremity of the suspended needle, the distance from the point x to the meridian, was approximately $12\frac{1}{2}$ lines; but as the experiment then gives 4 lines of distance from the meridian to the end of the needle, it follows that, in this test, it is necessary to subtract 4 lines to obtain the distance from the center of action to the end of the needle.[583] So the calculation here again gives $8\frac{1}{2}$ lines for the distance from the center of action to the ends of the needle.[584]

In the third trial, where the distance from the end of the needle to the steel wire is 4 inches, the calculation will give approximately 2 lines for the distance from the center of action to the meridian: but we find by experiment that, in this trial, the end of the wire exceeded the meridian, by 8 lines; thus, in this test, the calculation gives the center of action of the ends of the steel wire, 10 lines from its ends.

Thus, it results from experiment and calculation, that whenever steel wires, 25 inches in length, act on each other, we can suppose the centers of action, or what amounts to the same thing, all the magnetic fluid united at 9 or 10 lines from the extremities of these wires, and calculate, according to this supposition. In very short needles, the center of action is nearer to the extremities; we shall need in the following to determine the law of this reduction, relative to the length of the needles, when we give the most advantageous way to magnetize the needles, and to form artificial magnets.

We will determine at the same time the curve which, in a magnetized steel wire, describes the density of the magnetic fluid from its extremity to its middle where its magnetic center is located; but it is easy to foresee in advance, from the preceding experiments, that the locus of this density cannot be a straight line, as some authors have believed.[585]

14.4 Second Method of Determining the Law of Attraction and Repulsion of Magnetic Fluid

After having found by the preceding experiments, that in a needle 25 inches in length,[586] and a fortiori in shorter needles, the magnetic fluid can be supposed to be concentrated in the last two or three inches, towards their extremities, and that in needles 20 to 25 inches

[583] End of the wire?

[584] Ends of the wire?

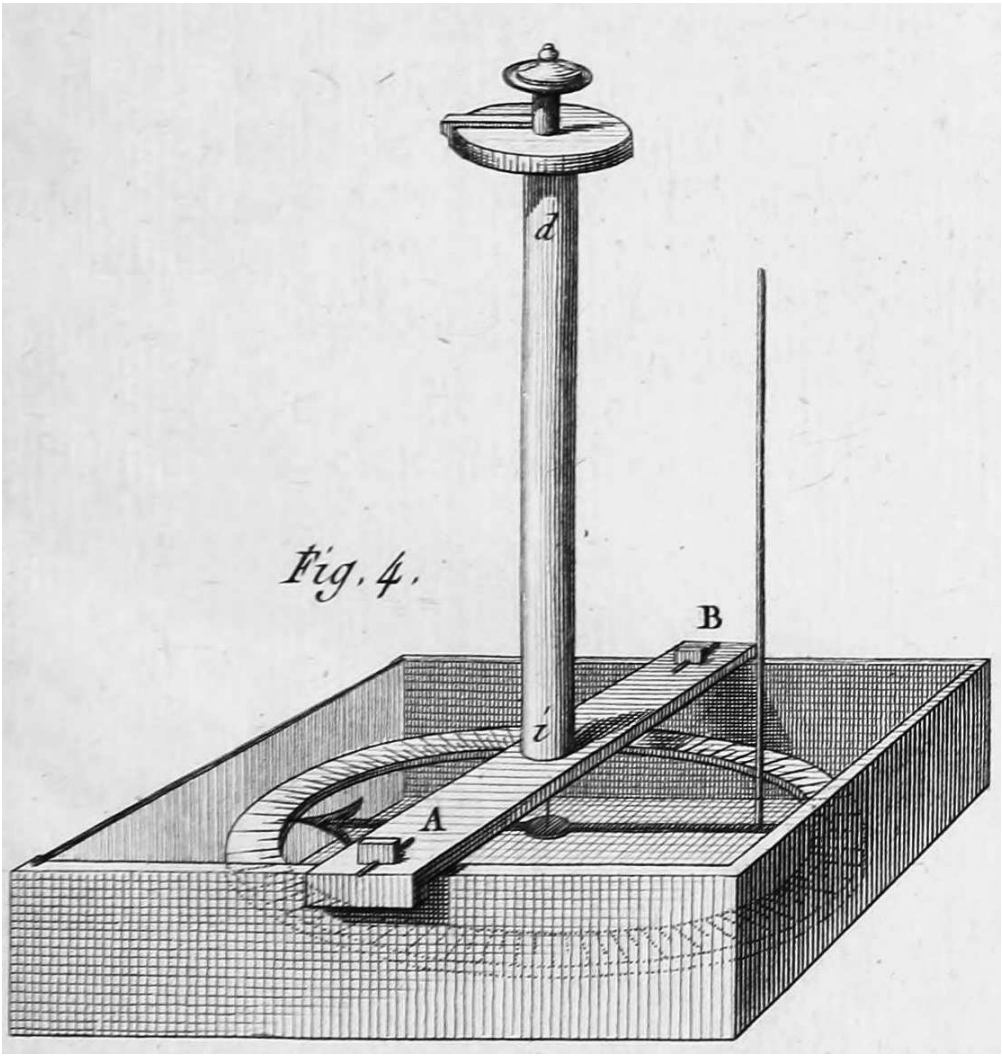
[585] A discussion of Coulomb's calculations presented in this Section can be found in Chapter 15.

[586] In Potier's edition of Coulomb's works it appears here, by a typo, 23 inches instead of the 25 inches of the original text, which corresponds to 67.675 cm, [Potier, 1884, p. 136] and [Coulomb, 1788d, p. 600].

the center of action may be assumed [as concentrated at a point] 9 or 10 lines from each end; it was easy to construct a magnetic balance, according to the same principles which I used to construct the electric balance, which I described in my First Memoir. But I should note that the form and dimensions of the magnetic balance that I am going to present, can and ought to be changed as practice suggests. I only sought, in this first attempt, to give this balance a simple inexpensive form, and which was however more or less sufficient for the experiments which I intended to make.

14.5 Description of the Magnetic Balance

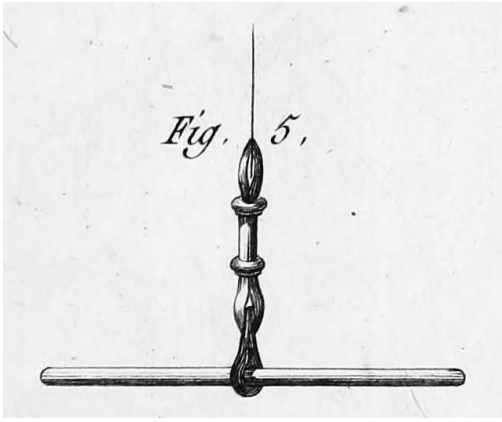
I had a square box made, 3 feet on a side (97.45 cm), and 18 inches high (48.73 cm), Figure 4.[587]



The boards are only fixed together with tenons, mortises and wooden pegs. Nine inches [24.36 cm] above the bottom, is placed a horizontal circle, of very seasoned wood, or red

[587]The vertical wire located on the right in Figure 4 is magnetized. It will be used in the experiments to determine the force law between magnetic poles.

copper,[588] 2 feet 10 inches in diameter (92.03 cm), divided in the ordinary way into 360 degrees. On this box is placed a crosspiece AB which carries in its middle a hollow shaft id, 30 inches in length (81.21 cm), terminated at d, by a torsion micrometer, similar to that which we have described for the electric balance. The clamp of this micrometer grasps the end of a yellow copper wire,[589] numbered 12 in commerce, whose six feet weighs 5 grains (0.1365 gram per meter), and whose force we have determined,[590] in the Memoir on the forces of torsion of metal wires, printed in the *Volume of the Academy for 1784*.[591] The lower part of this wire is grasped by a double clamp,[592] having the shape of a ruling pen,[593] represented in Figure 5.



This double clamp is split, as shown in the Figure, in almost its entire length, to form a clamp at its two ends, which open and close by means of two sliders. The lower end grips a ring of lead or copper; this ring is intended to carry the magnetized steel needle, with which we wish to experiment.

Before starting the experiments with this balance, it is necessary that, when the torsion is null, the magnetized needle is placed naturally on its magnetic meridian; this is easily accomplished by first placing in the ring suspended from the ruling pen, a red copper wire, of the same dimensions as the magnetized steel wire, which one intends to submit to experiment;[594] then keeping the index of the micrometer fixed on the first division of this micrometer, the whole micrometer is rotated (whose stem, as we saw for the electric balance, can slip and rotate atop the hollow shaft id, Figure 4), until the copper needle stops naturally on the direction of the magnetic meridian, which has been drawn in advance.

The box should be placed on this magnetic meridian, so that the direction of this meridian is aligned with the divisions 0, 180 of the horizontal circle, which we have said is placed in the box, 9 inches above its bottom.

After this preparation, we replace the copper needle with the magnetized steel needle, and we are ready to begin the operations.

[588] See footnote 436 on page 193.
[589] See footnote 262 on page 109.
[590] [Note by Bucciarelli] "Force" here is, I suggest, the force of resistance to rotation, that is, the wire's "stiffness".
[591] [Coulomb, 1787] with English translation of Sections I and II in [Coulomb, 2012a] and [Coulomb, 2012b]. This Memoir is translated in Chapter 7.
[592] In the original: *double pince*. See footnote 340 on page 153.
[593] In the original: *porte-crayon*. See footnote 466 on page 207.
[594] This copper wire is not magnetized.

We will give here only the experiments and the results which are absolutely necessary for us to determine the law according to which the magnetic fluid acts, when the magnetic molecules[595] are placed at different distances one from the other.

14.5.1 First Result. The resultant force of all the magnetic forces which the globe of the Earth exerts on each point of a magnetized needle, is a constant quantity, the direction of which, parallel to the magnetic meridian, always passes through the same point of the needle, in whatever situation this needle is placed in relation to this meridian

I had already tried to prove this principle in a Memoir on magnetized needles, printed in the seventh volume of the *Savants étrangers*;[596] but the experiments which I have reported so far might be subject to some dispute; that which follows is direct, and seems to me decisive.

Experiment

I have suspended horizontally in the balance a magnetized steel wire, 22 inches long (59.56 cm), and $1\frac{1}{4}$ line in diameter (0.27 cm). In accord with the setting of our balance, this needle is positioned in its magnetic direction,[597] its northern end corresponding to point 0 of the great circle of 2 feet 10 inches in diameter; the torsion of the wire filament being zero, and the index of the micrometer being on the point 0, or on the first division of this micrometer.

By means of the button which carries the index of the micrometer, we twisted the copper wire of suspension through various angles, which forced the needle to be displaced from its magnetic meridian: with each operation, we observed the angle from which it was displaced from the meridian, and the force of torsion which was required to produce this angle, and we obtained the following results.[598]

	Torsion of the suspension thread	The needle stoped at
First trial	1 circle = 360°	10.5° from its meridian
Second trial	2 circles	21.25° from its meridian
Third trial	3 circles	33° from its meridian
Fourth trial	4 circles	46° from its meridian
Fifth trial	5 circles	63.5° from its meridian
Sixth trial	5.5 circles	85° from its meridian

[595] In the original: *molécules aimantaires*. See also [Gillmor, 1971a, p. 217].

[596] See footnote 531 on page 232. In particular, we draw attention to the First Fundamental Principle, Subsection 5.0.2 on page 56. A discussion and illustration of this first fundamental principle can be found in Section 6.1.

[597] That is, along the magnetic meridian.

[598] I present an illustration of this experiment in Section 16.3 on page 272.

14.5.2 Result and Explanation of This Experiment

Our magnetic needle is suspended here by a copper wire, numbered 12 in commerce; we saw, in a Memoir printed in the volume of 1784,[599] that, for the same wire of suspension, the force of torsion is proportional to the angle of torsion; thus, in the first trial, the force of torsion is 1 circle $-10\frac{1}{2}$ degrees; in the second trial, it is 2 circles $-21\frac{1}{4}$ degrees. If we compare, in this experiment, the force of torsion with the angle which the needle is displaced from its meridian for each trial, we will find very exactly that in the successive trials the sines of the angle formed by the magnetic meridian and the direction of the needle are proportional to the angle of torsion;[600] whence it follows, as we have seen, in the seventh volume of the *Savants étrangers*,[601] that the force resulting from the magnetic action of the globe of the Earth, is a constant force directed parallel to the magnetic meridian, and always passing [through a point lying] at a fixed distance from the extremity of the needle, in whatever position this needle is placed, relatively to its meridian: here is the calculation comparing [the results] of the experiment.[602]

Let A be the angle of torsion of any trial, which will serve as a term of comparison.

B, the angle at which the needle moves away from its meridian on this trial.

A', the torsion angle found in another trial.

B', the angle by which the needle moves away from its meridian on this trial; we will generally have, according to the theory

$$A : A' :: \sin B : \sin B' \ .$$

From where

$$\log A' = \log A + \log \sin B' - \log \sin B \ .$$

Take the second trial for comparison term; correcting the angle of torsion, from the angle the needle moves away from its meridian, this angle will be 699 degrees, and its logarithm will be

$$2.8444 \ ;$$

the angle B being of $21°15'$, $\log \sin B$ will be

$$9.5592 \ .$$

By comparing these two quantities, according to the formula, with the angle by which the needle is displaced from its meridian in the other trials, we will find that:

The 2nd and 3rd *Trials* compared by the theory,	
give for the force of torsion of the *3rd Trial*	1052°
The experiment gives for the force of torsion of the *3rd Trial*	1047°
Difference:	5°
Error of the experiment:	$-\frac{1}{210}$

[599] [Coulomb, 1787] with English translation of Sections I and II in [Coulomb, 2012a] and [Coulomb, 2012b]. This Memoir is translated in Chapter 7.

[600] I show this proportionality in detail in Section 16.3 on page 272.

[601] See footnote 531 on page 232.

[602] A discussion of Coulomb's calculations presented in this Section can be found in Section 15.2.

The 2nd and 4th *Trials* compared by the theory,	
give for the force of torsion	1388°
The experiment gives for the force of torsion of the *4th Trial*[a]	1394°
Difference:	−6°
Error of the experiment:	$+\frac{1}{232}$
a. For a lapse in the original text,	
it appears written here third trial instead of fourth trial.	

The 2nd and 5th *Trials* compared by the theory,	
give for the force of torsion	1726°
The experiment gives for the force of torsion of the *5th Trial*	$1736\frac{1}{2}°$
Difference:	$-10\frac{1}{2}°$
Error of the experiment:	$+\frac{1}{169}$

The 2nd and 6th *Trials* compared by the theory,	
give for the force of torsion	1921°
The experiment gives for the force of torsion of the *6th Trial*[b]	1895°
Difference:	66°
Error of the experiment:	$-\frac{1}{75}$
b. For a lapse in the original text,	
it appears written here fifth trial instead of sixth trial.	

We therefore find the greatest agreement between theory and experiment, which proves at the same time the truth of the theory and the exactness of the method; an accuracy which can only be attributed to the simplicity of the instrument, for the box and all the parts which form the balance were executed without much care.

14.5.3 First Observation

This property established in a way that seems to me indisputable, it will be easy, by means of our balance, to compare immediately and without calculation, the force of different magnetized needles,[603] either among themselves, or with the *moment*[604] of a weight which would act at the end of a given lever.[605]

For this operation, it is only necessary to suspend horizontally in our balance, one after the other, the different needles we wish to compare in a way such that they lie freely along the magnetic meridian when the torsion in the suspending wire is nul; one then twists the wire by means of the micrometer so that the suspended needles take up the same angle with respect to the magnetic meridian in all tests and we will conclude from this experiment that, since the angle formed with the magnetic meridian is constant, the moment of the force which tends to bring the needle back to its meridian due to the magnetic action of the Earth is proportional to the angle of torsion applied in the experiment.

[603] In the original: *la force de diffèrentes aiguilles aimantées*. Coulomb is referring here to the degree of magnetization of different needles.

[604] In the original: *momentum*. This word can also be translated as torque. See footnote 150 on page 60.

[605] That is, this procedure allows the magnetic moments of different magnetized needles to be compared with each other. It also allows the magnetic torque exerted by the Earth on a specific compass needle to be compared with the torque exerted by a given weight acting on a lever at a given distance from its fulcrum. That is, the magnetic torque exerted by the Earth on the magnetized needle can easily be compared with a known gravitational torque.

We shall have occasion, in another Memoir, to return in detail to this subject, as well as to many others relating to magnetism.

14.6 Use of the Magnetic Balance, to Determine the Law According to Which the Magnetic Parts Act on Each Other at Different Distances

We magnetized[606] a wire of good steel, pulled through the die,[607] 24 inches in length (64.97 cm), and $1\frac{1}{2}$ line in diameter (0.34 cm), we suspended it horizontally in our magnetic balance; we first sought, with what force the magnetism of the Earth brought this needle back to its meridian, and we found that by twisting the suspension wire by two circles minus 20 degrees, the needle stopped at 20 degrees from its magnetic meridian, so that for angles from 20 to 24 degrees and below, the sines being roughly proportional to the arcs, it required a force of torsion very close to 35 degrees in order to move the needle one degree away from its magnetic meridian.[608]

We then placed another magnetized wire of the same dimensions, vertically in the magnetic meridian, 11 inches 2 lines (30.22 cm) from the center of suspension of the first needle, by lowering the end of this wire, approximately one inch below the level of the needle hanging horizontally;[609] so that, if the two needles, one suspended horizontally, the other placed fixedly vertically in the meridian of the first, had touched each other, they would have met at 1 inch from their extremities; but as it was the North poles, or [the poles] of the same name of each needle, which were opposite, they mutually repelled each other, and the horizontal needle, suspended in the balance, was repelled from the direction of its meridian, and stopped only when the repulsive force of poles of the same type[610] was in equilibrium with the directing force[611] of the globe of the Earth. Here are the results of the various trials.[612]

Experiment

First trial. The needle suspended horizontally without twisting the suspension wire, it was repelled, and stopped at 24 degrees from its magnetic meridian.

Second trial. Having twisted [the upper part of the suspension wire] by three circles, the needle stopped at 17 degrees from its magnetic meridian.

[606] In the original text the title of this Section appears as: *Usage de la balance magnétique, pour déterminer la loi suivant laquelle les parties aimantaires agissent l'une sur l'autre à différentes distances.* The expression "the parties aimantaires" (*the magnetic parts*) was translated in the German version as "*the magnetic particles*" (*die magnetischen Theilchen*), [Coulomb, 1890e, p. 37]. See also [Gillmor, 1971a, pp. 159, 201 and 217] and footnotes 181 and 909 on pages 72 and 419, respectively. By writing "magnetic parts" Coulomb might be pointing to the areas of the needle where the magnetic fluid is concentrated.

[607] In the original: *tiré à la filière.* See footnotes 389 and 563 on pages 174 and 240.

[608] That is, $(720° - 20°)/20 = 35°$.

[609] This vertical magnetized wire appears on the right side of Figure 4 on page 249. When its lower end is 1 inch below the level of the horizontal needle, the lower magnetic pole of the wire is at the same horizontal level as the hanging needle.

[610] In the original text it appeared here "*la force de répulsion des pôles opposés*". This was certainly an oversight, since Coulomb is referring to the force of repulsion between magnetic poles of the same type.

[611] See footnote 286 on page 127.

[612] This experimental procedure is illustrated and discussed in detail in Section 16.4 on page 273.

Third trial. Having twisted [the upper part of the suspension wire] by eight circles, the needle stopped at 12 degrees from its magnetic meridian.

14.6.1 Explanation and Result of This Experiment

We have said that the needle, free and only subjected to the magnetic action of the globe of the Earth, was held at 20 degrees from its meridian by a force of torsion of two circles minus 20 degrees; thus, when the needle formed an angle of 20 degrees with its magnetic meridian, the force tending to take it back towards this meridian was 700°; and consequently, as in the first trial it stopped at 24° from its meridian, it was drawn back to the meridian with a force of 840°;[613] but, as by the repulsion of the needles, the suspension thread was twisted by 24°, the total repulsion was 864°.[614]

In the *2nd trial*, the needle stopped at 17° from its magnetic meridian; thus, it was brought back to this meridian by the magnetic action of the Earth,[615] with a force of 595°.[616] But the torsion [of the suspension wire] which held it at this distance was 3 circles + 17°. Thus, as this force of torsion acted in the same direction as the magnetic force of the Earth, the action of the two poles of the needle was measured by 1692°.[617]

In the *3rd trial*, the needle is only at 12° from its magnetic meridian. Thus, the action of the terrestrial globe is only measured by a force of 420°.[618] But we find in this trial, that to bring the needle back to this distance of 12°, it was necessary to twist the suspension thread by 8 circles + 12° = 2890°. Thus, the repulsive force of the two needles placed 12° apart is measured in this last trial, by a twist of

$$2892° + 420° = 3312° \ .$$

Thus, in our experiments, where the distances are 24, 17, 12, the inverse ratio of the square of the distances is measured by the numbers 1/576, 1/289, 1/144, which is very close to the numbers 1/4, 1/2, 1. But the experiments give for the corresponding repulsive forces 864, 1692, 3312, which are also very close, like the numbers 1/4, 1/2, 1.[619] Thus, in assuming that all the magnetic fluid is concentrated at 10 lines from the extremity of our 24 inch long needles, as we have seen it was permissible in what came before, if results that the repulsive action of the magnetic fluid is inversely proportional to the square of the distances.

We were able to neglect in this operation, the action of the other poles of the needles; for, since the action is in inverse proportion to the square of the distances, since the needles are two feet in length, these other poles always being at a distance at least four times greater than the first, and acting moreover very obliquely to the length of the needles, their

[613] Due to a typographical error, in the original text we have here 849°, [Coulomb, 1788d, p. 608]. Potier has corrected this error and put the correct value of 840°, [Potier, 1884, p. 143]. This value is obtained as follows. It was seen in footnote 608 that a force of torsion of 35° is required for every 1° the needle moves away from its magnetic meridian. Since the needle in this trial was 24° away, the force of torsion required to cause this deviation was $24 \times 35° = 840°$.

[614] The way to obtain this value is detailed in Section 16.4 on page 273.

[615] In the original: *par l'action aimantaire de la Terre.*

[616] That is, $17 \times 35° = 595°$.

[617] That is, the repulsive action between the North poles of the two magnetized needles was measured by 1692°. The way to obtain this value is detailed in Section 16.4 on page 273.

[618] That is, $12 \times 35° = 420°$.

[619] See Section 16.4 on page 273.

action cannot alter our result in a very perceptible way.[620] But if there were less difference between the distance of the different poles of the needle than in the preceding experiment, it would be necessary, in the calculation, to have regard to the reciprocal action of all the poles, and to the length of the [arm of the] lever on which each of these actions is exercised. This calculation would have no more difficulty than that which we have made previously to determine the center of action of the extremities of the needles, or the point, towards these extremities, in which it is permitted to suppose the magnetic fluid concentrated.

Furthermore, by means of the magnetic balance which we have just described, we can prove in an indisputable way, that the magnetic fluid in the steel wires magnetized by the method of the double touch, is concentrated towards the ends of these wires.

Here is the summary of the operation that leads to this result. Having placed in the magnetic meridian of our balance, a vertical ruler two lines thick (0.451 cm), near the end of the suspended needle, we slide vertically, along this ruler, the magnetized steel wire, so that the poles of the same name respond to each other, the ruler being in-between. As the two extremities, or the two poles, one of the steel wire and the other of the needle, repel each other, we twist, by means of the micrometer, the wire of suspension, until we have brought the horizontal needle back in contact with the ruler, so that only the thickness of the ruler, or two lines of distance, remains between the nearest points of the two needles; but as the steel wire that we place behind the ruler is vertical, all the points of the two needles which are four or five lines [0.90 or 1.13 cm] away from the intersection mutually repel each other with but a very weak force, on account of their distance and the obliqueness of their action; so that the force of torsion which must be employed to hold the needle suspended horizontally in contact with the ruler, is proportional to the density of the two or three lines of length of the magnetic fluid which are close to the points of the two needles, which are only two lines apart. Thus, by sliding our steel wire vertically along the ruler, we will present at this small distance of two lines from the needle, all the points of this wire, and the force of torsion of the suspension [thread] to hold the needle suspended horizontally in contact with the ruler, will be proportional to the density of the magnetic fluid of the point of the vertical wire, which, in each trial, will be two lines away from the needle. If this experiment be attempted, it will be found that if a twist of eight circles is required when the point of intersection is at two lines from the end of the wire, only two or three circles of twist are required at one inch, and at most a semicircle of twist at two inches; and that when the vertical steel wire has its end placed at three inches below the end of the horizontally hanging needle, the repulsion is almost nil. The same thing will be found for the attraction of the poles of different name;[621] but it is necessary to warn [the reader] that to count on the result of such an experiment, it is necessary to employ only strongly hardened needles of excellent steel, and not to give them a too strong degree of magnetism; otherwise, as in this operation the point of intersection of the two needles is only two lines apart, if the force of the magnetic fluid is such that the fluid can move into the parts of the needles which adjoin each other, the results will no longer be comparable. We will see, in another Memoir, that the coercive force, which prevents the magnetic fluid once concentrated by the operation of

[620] Heilbron estimated that the influence of the far poles was less than 1% of the action between the near poles, [Heilbron, 1999, p. 96].

[621] By an oversight, the attraction of the poles "of the same name" appears in the original text, *pour l'attraction des poles du même nom*. We changed the phrase for attraction of the poles of different names, that is, attraction between the North pole of the suspended needle and the South pole of the vertical wire, or attraction between the South pole of the suspended needle and the North pole of the vertical wire.

the double touch, from moving, is a constant quantity, which varies according to the nature and the hardness of steel; but that, when a point of a needle is magnetized to saturation, this coercive force, which we can compare to friction in mechanics, balances with the resultant of all the forces, either repulsive, or attractive of all the magnetic fluid diffused in the needle, the force of each point being directly proportional to the densities [of magnetic fluids] and inversely proportional to the square of the distances.

14.7 Summary of the Subjects Contained in This Memoir

From the above research, it will result:

1. The electric action, whether repulsive or attractive, of two electrified spheres, and therefore of two electrified molecules,[622] is in the ratio compounded of the densities of the electric fluid of the two electrified molecules and inversely as the square of the distances.[623]

2. In a needle 20 to 25 inches in length, magnetized by the double-touch method, the magnetic fluid can be supposed to be concentrated at 10 lines from the ends of the needle.

3. When a needle is magnetized, in whatever position it is placed on a horizontal plane, with respect to its magnetic meridian, it is always attracted back to this meridian by a force, constant and parallel to the meridian, of which the resultant passes always through the same point of the suspended needle.

4. The attractive and repulsive force of the magnetic fluid, as of the electric fluid, is exactly in the ratio directly of the densities [of the fluid] and inversely of the square of the distances between the magnetic molecules.[624]

[622] In the original: *molécules électriques*. That is, between two electrified particles.

[623] What Coulomb is calling here the "density of the electric fluid" (*densités du fluide électrique*) can be understood as the electric charge of each electrified molecule. He is then claiming that the force between two electrified particles is directly proportional to the product of their electric charges and inversely proportional to the square of the distance between them. I discuss the first part of this statement in Section 16.5 on page 276.

[624] In the original: *molécules magnétiques*. I discuss the first part of this statement in Section 16.6 on page 279.

Chapter 15

Bucciarelli's Remarks on Coulomb's Second Memoir

L. L. Bucciarelli

15.1 Note on Coulomb's Second Section

In Section 14.2, Coulomb, (after the lengthy footnote 556 on attraction and repulsion of bodies when the law is other than inverse square), writes that he will use two methods to determine the law according to which the magnetic fluid acts: The **first method** observes the interaction of a suspended magnetized needle of 3 inches in length and a longer (25 inches) magnetized steel wire to first determine the position of the north and south poles of this wire relative to its ends, then uses these same elements to verify the inverse square law of attraction by comparing the frequency of oscillations of the suspended needle in the presence of the wire placed at different distances from the needle. The **second method** makes use of a balance, like that used in Coulomb's experiments of electricity to determine the law. I consider only the first of four experiments conducted in accord with the **first method**.

In this first of four experiments, Coulomb suspends a magnetized needle, an, directed along the magnetic meridian, and positions a longer wire, SN, perpendicular to the meridian at five different distances from the north pole of the suspended needle and in the same plane, sliding it from left to right until the needle returns to the meridian, Figure 15.1.

First Experiment		
	Distance from needle	End displacement from meridian
1st Trial	1 inch	+10 lines
2nd Trial	2	+9
3rd Trial	4	+8
4th Trial	8	−4
5th Trial	16	−42

At this point, he only considers the first three trials of this First Experiment. He follows with two other experiments with the same objective as this first — to determine the position of the north and south poles of the magnetized wire relative to its ends; in an *Explanation*

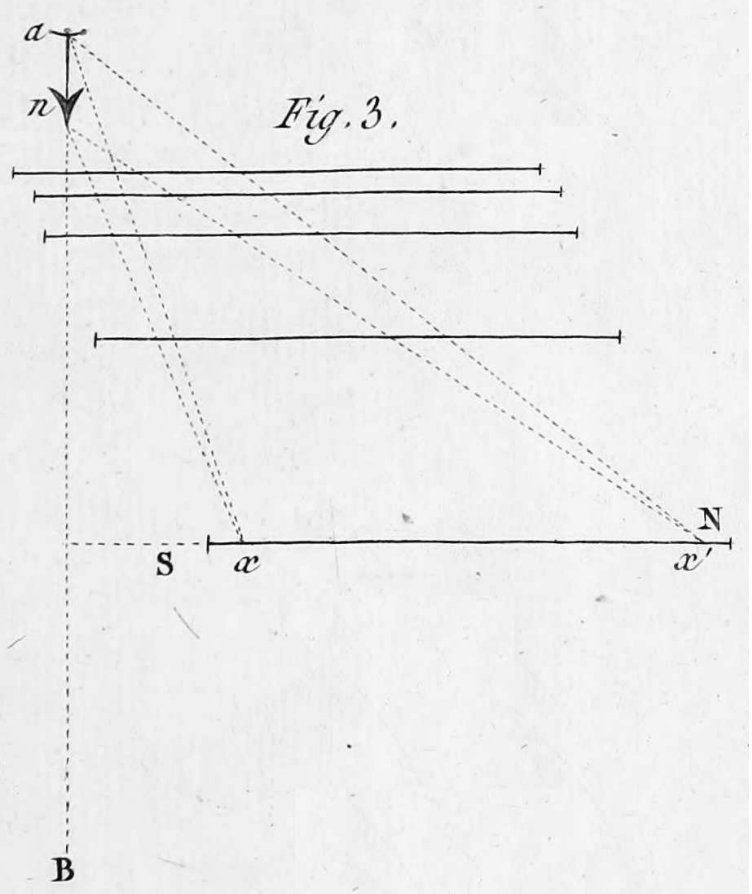


Figure 15.1: Figure 3 of Coulomb's Second Memoir.

and results of these three experiments, Subsection 14.2.2, he claims that they prove that the center of action of each half of the wire is placed at a very short distance from the end of the wire, e.g., from the first three trials:

> thus, in the first three trials, where the distances [of the wire from the needles] are 1, 2 and 4 inches, the magnetic forces of the last eight to ten lines of the end of the needle, which lie beyond the meridian, are in equilibrium with the forces of all the rest of the needle; so that it seems that we can roughly assume that half of the magnetic fluid with which half of the needle is loaded, is concentrated towards the last ten lines from its end.

The approximate nature of this result is evident in the data and the Figure: If the wire went beyond the meridian the same distance in these first three trials, say 9 lines, then, in the Figure, they would all be positioned identically, as the wire of the second trial.

Coulomb comes back to trials four and five of the first experiment after describing a fourth experiment in a next Section titled *The magnetic fluid acts by attraction or repulsion in a ratio compounded directly of the density of the fluid and inversely of the square of the distance of its molecules*, Section 14.3. Crucial to his fourth experiment (not discussed here) is knowing the (approximate) location of the poles relative to the ends of the wire — hence, why he did the three experiments.

In a *Second Observation* following a description of results of the fourth experiment, Subsection 14.3.3, Coulomb explains how he will use the data of trial 5 of the first experiment to test out the inverse square law which he has obtained from the fourth experiment.

> Once we accept the law of the inverse ratio of the square of the distances, it would be easy — in the first experiment, where the magnetized wire is placed horizontal and perpendicular to the magnetic meridian, and where we find, in the last trial, that it is necessary to move the end of the wire about 42 lines from the meridian of the needle — to assess if the calculation [in accord with the inverse square law] would give, for the direction of the resultant of all the actions of each half of the wire, a line which would pass [through a point] nine or ten lines from the extremity of this wire.

In his "calculation" Coulomb lets $x = Sx = Nx'$ be the distance from the ends of the wire to either "center of action" and writes four expressions for the horizontal components of the action of these two centers of action on the needle, components that cause the needle to rotate off the meridian. I show at Figure 15.2 the horizontal component of the action of the S pole of the wire on the north pole of the needle which tends to rotate the needle counterclockwise.

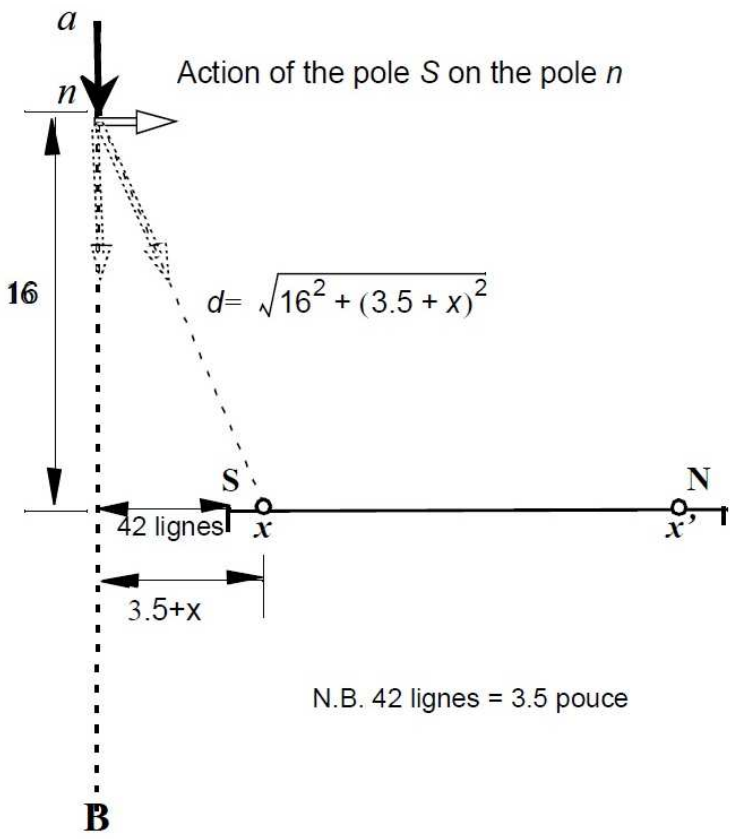


Figure 15.2: Action of the pole S on the pole n. See also Equation (15.2).

In terms of the distance measures shown in this Figure 15.2, this component is

$$\frac{3\frac{1}{2}+x}{\left[(16)^2+\left(3\frac{1}{2}+x\right)^2\right]^{3/2}}\ . \tag{15.1}$$

Coulomb writes out a like expression for the other horizontal component of the action of pole S on the south pole, s, of the needle — which also tends to rotate the needle *counterclockwise* — and expressions for the two horizontal components of the wire's pole N on the north and south poles of the needle — both of which tend to rotate the needle *clockwise*.

With the wire positioned in trial 5 such that the needle lies along the magnetic meridian, the resultant moment of all four components about the suspension point at the center of the needle must be zero. Since all four components have the same lever arm about the suspension point, this is equivalent to making the sum of all the components vanish. In Coulomb's words:

> But, since in this [first] experiment, the steel needle lies along it's magnetic meridian, and in that each of the preceding forces acting perpendicular to this needle have the same lever arm tending to rotate it about its point of suspension, it follows that all the forces are in equilibrium; from which we draw the equation:

$$\begin{aligned}&\frac{3\frac{1}{2}+x}{\left[(16)^2+\left(3\frac{1}{2}+x\right)^2\right]^{3/2}}+\frac{3\frac{1}{2}+x}{\left[(19)^2+\left(3\frac{1}{2}+x\right)^2\right]^{3/2}}\\ =\ &\frac{28\frac{1}{2}-x}{\left[(16)^2+\left(28\frac{1}{2}-x\right)^2\right]^{3/2}}+\frac{28\frac{1}{2}-x}{\left[(19)^2+\left(28\frac{1}{2}-x\right)^2\right]^{3/2}}\ .\end{aligned} \tag{15.2}$$

To solve for x, Coulomb notes that, since x is small, as a first approximation, we can neglect x in the denominator. A better approximation is to take x equal to $1/2$ (which, we note, makes all numerical entries whole numbers). Using this, he obtains for the value $Sx = x = 56/75$ (0.747 inch) which is approximately 9 lines.

For the fourth trial of the First Experiment, where the distance of the wire from the needle is 8 inches, he obtains 8 1/2 lines for the distance from the end of the wire to the center of action. For the third trial where the distance of the wire from the needle is 4 inches, he obtains 10 lines. Summarizing in a Table (4th column):

First Experiment					
	Distance from needle	Displacement from meridian	Distance, wire end to center of action from calculation	Distance, wire end to center of action from computation[a]	Distance end to center[b]
1st Trial	1 inch	+10 lines	no calculation	.837 inches $\sim$ 10 lines	.835 inch
2nd Trial	2	+9	no calculation	.776 inch $\sim$ 9 lines	.774 inch
3rd Trial	4	+8	10 lines (.83 inch)	.839 inch $\sim$ 10 lines	.845 inch
4th Trial	8	−4	$8\frac{1}{2}$ lines (.71 inch)	.650 inch $\sim$ 8 lines	.675 inch
5th Trial	16	−42	9 lines (56/75 inch)	.788 inch $\sim 9\frac{1}{2}$ lines	1.04 inch
a. Computation done taking $x = 1/2$ in the denominator (as did Coulomb).					
b. Computation done making no approximation for x in denominator (solution requires iteration).					

The fifth column of the Table shows the results of computation of x for all trials taking, as did Coulomb, x *equal to* $1/2$ in the denominator of all terms of the equation solved. The sixth column, results of "exact" solution to the equilibrium equation, no approximation for x in denominator.

15.2 Note on Coulomb's Fifth Section

Coulomb's balance used to study the magnetic force, like the balance used to study the interaction of electrically charged objects, relies on knowing the torsional properties of an elastic filament — of a silk thread, a steel wire, etc. He knows, from his exemplary Memoir of 1784, that the torque of a filament is linearly related to the rotation of one end relative to the other and that if the filament is sufficiently long, the relative angle of twist can be large; the relative twist increasing linearly with length for the same torque. (He also knows that the stiffness of a filament is inversely proportional to its length, but although that knowledge was no doubt of use in the design and making of the balance, it is not relevant to the conduct and the analysis of results of the experiment.)

In the experiment of Section 14.5, Subsection 14.5.1, Coulomb suspends a magnetized steel needle from a copper wire of approximately 0.14 mm diameter[625] so that initially, the needle aligns with the magnetic meridian. He then rotates the micrometer atop the wire to displace the needle from north-south. To counter the Earth's restoring force at any displaced position, he must turn the micrometer until the torque is sufficient to hold the needle in place. He takes data at 6 positions of the needle. In his words:

> By means of the button which carries the index of the micrometer, we twisted the copper wire of suspension through various angles, which forced the needle to be displaced from its magnetic meridian: with each operation, we observed the angle from which it was displaced from the meridian, and the force of torsion which was required to produce this angle, and we obtained the following results.

	Torsion of the suspension thread	The needle stoped at
First trial	1 circle = 360°	10.5° from its meridian
Second trial	2 circles	21.25° from its meridian
Third trial	3 circles	33° from its meridian
Fourth trial	4 circles	46° from its meridian
Fifth trial	5 circles	63.5° from its meridian
Sixth trial	5.5 circles	85° from its meridian

From this data, he will show that the resultant force of all the magnetic forces that the Earth exerts on each point of a magnetized needle is a constant quantity whose direction, parallel to the magnetic meridian, always passes through the same point of the needle, e.g, his *First Result*. In his words:[626]

[625] Six feet weighs 5 grains (0.1365 g/m). With density = 8.92 g/cm^3 weight per length divided by density gives the area, then the diameter (keeping track of units). This is similar to AWG gauge copper wire gauge 35. See http://www.nessengr.com/technical-data/bare-copper-wire.

[626] My emphasis.

> If we compare, in accord with this experiment, the force of torsion with the angle which the needle is displaced from its meridian for each trial, we will find very exactly that in the successive trials the sines of the angle formed by the magnetic meridian and the direction of the needle are proportional to the angle of torsion; whence it follows, as we have seen [...] that *the force resulting from the magnetic action of the globe of the Earth, is a constant force directed parallel to the magnetic meridian, and always passing at an equal distance from the extremity of the needle,* [...]

Figure 15.3 shows the magnetic needle displaced an angle, ϕ, from the meridian — ten and one half degrees in the first trial.

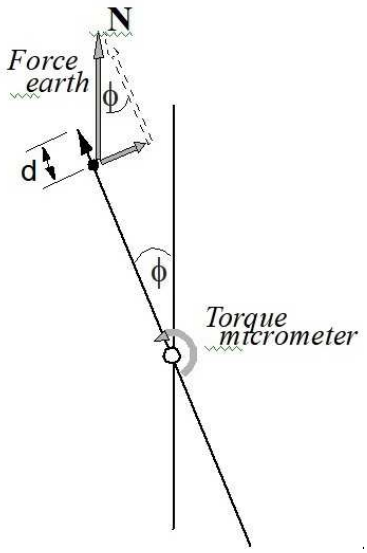


Figure 15.3: The magnetic needle displaced an angle, ϕ, from the meridian.

For equilibrium of the needle, the torque due to torsion of the suspension wire must equal the restoring torque due to Earth's magnetic force. The torque in the suspension wire required to hold the needle at this angle is proportional to the angle of torsion — in the first trial, one full circle less ϕ. The torque tending to return the needle to the meridian is proportional to the component of the Earth's force that is perpendicular to the displaced needle, i.e., proportional to the $\sin\phi$.

But this last is only true if "the force resulting from the magnetic action of the globe of the Earth is a constant and directed parallel to the magnetic meridian, and always passing at an equal distance [d in the Figure] from the extremity of the needle". (If d is a constant, then the moment arm of the component of the Earth's force is also constant.)

Coulomb proceeds to do the calculation, testing the proportionality of the angle of torsion to the $\sin\phi$. Letting A be the angle of torsion and B be the needle displacement of any trial (our ϕ of the Figure), he will determine if the proportionality $A : A' :: \sin B : \sin B'$ holds where the primed variables are the corresponding values for another trial. As was customary in times prior to the availability of calculating machinery, Coulomb relies on the addition and subtraction of logarithms to avoid the error prone operations of multiplication and division; he writes:

$$\log A' = \log A + \log\sin B' - \log\sin B \ . \tag{15.3}$$

He takes the unprimed values for A and B from *the second trial* as the reference trial, computes the right hand side using the observed value for B' of any other trial, calculates A' and compares this with the observed value for the angle of torsion for the other trial.

For the second trial, A, the angle of torsion equals 2 full circles, 720 degrees less the displacement of the needle, 21.25 degrees. He rounds this up to 699°. Then $\log A = 2.8444$; (my calculator gives 2.8445). With $B = 21.25°$, he evaluates the $\log \sin B$ and expresses this as 9.5592. Evidently this was the custom for writing the log of a number ($\sin B = 0.3624$) less than 1.0 for we would write $\log \sin B = -0.4408$ which is $9.5592 - 10$.

For the other trial, he considers trials three through six. *For the third trial*, the needle displacement is 33° hence, $\log \sin B = -0.2639$. His proportional relationship in logarithmic form then gives, (according to my calculation), $\log A' = 3.021$ which gives $A' = 1050°$. Coulomb obtains 1052°.

The measure angle of torsion in trial 3 is three full circles (1800°) less the needle displacement of 33 degrees so A' observed is $1800 - 33 = 1047°$, a difference of 5 according to his calculations. He expresses the error as $-1/210$.[627] If we take 1050° as the correct value for the result of theory, the difference is reduced to 3 and the error to 1/350.

Here are the results of the comparison of trials three through six with the results of the second trial:

> The 2nd and 3rd *Trials* compared by the theory, give for the force of torsion of the 3rd *Trial*, 1052°. The experiment gives for the force of torsion of the 3rd *Trial* 1047°. Difference: 5°. Error of the experiment: $-\frac{1}{210}$.
>
> The 2nd and 4th *Trials* compared by the theory, give for the force of torsion, 1388°. The experiment gives for the force of torsion of the 4th *Trial* 1394°. Difference: −6°. Error of the experiment: $+\frac{1}{232}$.
>
> The 2nd and 5th *Trials* compared by the theory, give for the force of torsion, 1726°. The experiment gives for the force of torsion of the 5th *Trial* $1736\frac{1}{2}°$. Difference: $-10\frac{1}{2}°$. Error of the experiment: $+\frac{1}{169}$.
>
> The 2nd and 6th *Trials* compared by the theory, give for the force of torsion, 1921°. The experiment gives for the force of torsion of the 6th *Trial* 1895°. Difference: 66°. Error of the experiment: $-\frac{1}{75}$.

Why did he not consider the first trial either as the reference case or as a trial to compare with the second trial? Doing the comparison with the second trial gives an angle of torsion of 351°. The experimental value is $360 - 10.5 = 349.5°$, a difference of 1.5. The error, is $-1/234$ which is not out of line with the error of the other trials. Nor does taking the first trial as the reference case give appreciably different error values.

[627] Note that $5 * 210 = 1050$ and $1052/5 = 210.4$ so Coulomb evidently rounded off the error.

Chapter 16

Assis' Remarks on Coulomb's Second Memoir

A. K. T. Assis

16.1 Determination of the Force of Gravity by the Oscillations of a Simple Pendulum

The period T for the round-trip oscillations in a vertical plane of a simple pendulum with length ℓ and mass m, for small amplitudes, due only to the gravitational action of the Earth is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}}\ , \tag{16.1}$$

where g is the value of the free-fall acceleration at that location. The body of mass m experiences a gravitational force F at this location given by $F = mg$. By Equation (16.1) we get

$$F = mg = \frac{4\pi^2 \ell m}{T^2}\ . \tag{16.2}$$

We then see that the gravitational force F acting downwards on the pendulum is inversely proportional to the square of the period of oscillation T of the pendulum, that is, F is inversely proportional to T^2. Therefore, this force F is also inversely proportional to the square of the time t_n spent by the pendulum to make the same number n of oscillations, with $t_n = nT$. That is, F is inversely proportional to t_n^2:

$$F = mg = \frac{4\pi^2 \ell m}{T^2} = \frac{4\pi^2 \ell m n^2}{t_n^2}\ . \tag{16.3}$$

For example, suppose that $T_A = 1\ s$ for a pendulum A. In a time interval $t_A = 6\ s$ this pendulum will make 6 round-trip oscillations. On the other hand, if $T_B = 3\ s$ for another pendulum B, then in the same time interval of 6 s this pendulum B will make 2 round-trip oscillations. This pendulum B will perform 6 oscillations in a time interval of $t_B = 6 \times 3\ s = 18\ s$, that is, at three times the time interval that the pendulum A spends to perform the same number of 6 oscillations. That is, suppose that $T_B = 3T_A$. In this case

pendulum A will perform in a time interval t_A the same number of round-trip oscillations that pendulum B will perform in a time interval t_B such that $t_B = 3t_A$.

The frequency $f = 1/T$ represents the number of oscillations per unit time. The number n of oscillations performed in a time interval t_n is then given by $n = ft_n = t_n/T$. We have by Equation (16.3):

$$F = mg = \frac{4\pi^2 \ell m}{T^2} = 4\pi^2 \ell m f^2 = \frac{4\pi^2 \ell m}{t_n^2} n^2 \ . \tag{16.4}$$

We then see that the gravitational force F is proportional to the square of the number of oscillations performed by the simple pendulum per unit time, that is, F is proportional to f^2. This force F is also proportional to the square of the number n of oscillations performed in a given time interval, that is, F is proportional to n^2.

16.2 The Attractive Forces of Spherical Bodies

In Subsection 14.1.2 Coulomb mentioned the following:

> When all the points of a spherical surface act by an attractive or repulsive force which varies inversely as the square of the distance on a point placed at any distance from this surface, it is known that the action is the same as if all the spherical surface were concentrated at the center of the sphere.

The point considered here by Coulomb on which the force of the spherical surface is acting is located outside this surface. This theorem was proved by Newton in the *Principia*. In Section 12 of Book I of the *Principia*, Newton proved two extremely important theorems related with the force exerted by a spherical shell acting on internal and external point particles. He supposed forces which vary inversely with the square of the distance between the interacting particles, as is the case with his gravitational force. In the first theorem Newton proved the following result:[628]

> *Section 12: The attractive forces of spherical bodies.*
>
> *Proposition 70. Theorem 30: If to every point of a spherical surface there tend equal centripetal forces decreasing as the square of the distances from these points, I say, that a corpuscle placed within that surface will not be attracted by those forces any way.*

That is, if a body is placed anywhere inside the spherical shell (not only on its center), the resultant force exerted by the shell on the body is zero. This situation is represented in Figure 16.1, in which there is a spherical shell of gravitational mass M_g, radius R and center C, with a corpuscle of gravitational mass m_g located in an arbitrary location inside the shell, at a distance $r < R$ from the center of the shell.

Newton's result can be expressed mathematically as follows:

[628] [Newton, 1934, p. 193] and [Newton, 1990, p. 221]. See also [Assis and Karam, 2018] and Section 1.4 (The Forces Exerted by Spherical Shells) of the book *Relational Mechanics and Implementation of Mach's Principle with Weber's Gravitational Force*, [Assis, 2014] and [Assis, 2013].

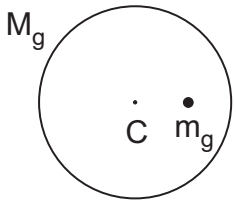


Figure 16.1: The spherical shell exerts no resultant force on a particle located anywhere inside the shell.

$$\vec{F} = \vec{0}, \quad \text{if} \quad r < R \ . \tag{16.5}$$

By symmetry it might have been concluded that the net force would be zero if the corpuscle were located exactly at the center of the shell. If it were not on its center, as represented in Figure 16.1, the only conclusion that could be drawn based upon arguments of symmetry, is that the net force acting on the particle must be along the line connecting it to the center of the shell. No argument of symmetry would lead to the conclusion that this force must be zero. It is possible to show that this net force is zero only when the force between the particles is inversely proportional to the square of the distance between them. If the force between the particles had another behavior (if it varied as the inverse of the distance or as the inverse of the distance cubed, for instance), then the result given by equation (16.5) would no longer be valid.

With theorem 31 Newton proved the following result:[629]

> *Proposition 71. Theorem 31: The same things supposed as above, I say, that a corpuscle placed without the spherical surface is attracted towards the centre of the sphere with a force inversely proportional to the square of its distance from that centre.*

That is, a particle placed outside the spherical shell is attracted as if the shell were concentrated at its center. This is represented in Figure 16.2, in which there is a spherical shell of gravitational mass M_g, radius R and center C, with a corpuscle of gravitational mass m_g located outside it in an arbitrary location, at a distance $r > R$ from the center of the shell. The net force on this particle is directed towards the center of the shell. Its magnitude varies inversely as the square of the distance between the corpuscle and the center of the shell.

Newton's law of universal gravitation can be written in modern vector notation and in the International System of Units MKSA as given by:

$$\vec{F}_{21} = -G\frac{m_{g1}m_{g2}}{r^2}\hat{r} = -\vec{F}_{12} \ . \tag{16.6}$$

In this equation $\vec{F}_{21}$ is the force exerted by the gravitational mass m_{g2} on the gravitational mass m_{g1}, G is a constant of proportionality, r is the distance between the point bodies, $\hat{r}$ is the unit vector pointing from 2 to 1, while $\vec{F}_{12}$ is the force exerted by m_{g1} on m_{g2}, Figures 16.3 and 16.4.

[629] [Newton, 1934, p. 193] and [Newton, 1990, p. 222]. See also [Assis and Karam, 2018] and Section 1.4 (The Forces Exerted by Spherical Shells) of the book *Relational Mechanics and Implementation of Mach's Principle with Weber's Gravitational Force*, [Assis, 2014] and [Assis, 2013].

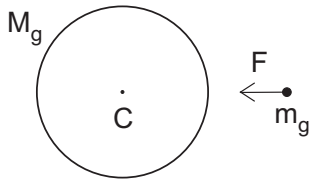


Figure 16.2: The spherical shell exerts an attractive force on an external corpuscle. The force is directed towards the center of the shell. Its magnitude is inversely proportional to the square of the distance between this center and the particle.

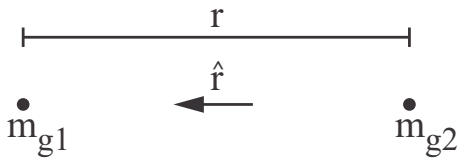


Figure 16.3: Two bodies separated by a distance r.

In the International System of Units the constant G, usually called the constant of universal gravitation, is given by:

$$G = 6.67 \times 10^{-11} \, \frac{m^3}{kgs^2} \, . \tag{16.7}$$

Utilizing Equation (16.6) together with Newton's Proposition 71, Theorem 31 of the *Principia*, we obtain that a spherical shell of gravitational mass M_g and radius R exerts a force $\vec{F}$ on a particle of gravitational mass m_g located at a distance $r > R$ from the center of the shell given by:

$$\vec{F} = -G \frac{M_g m_g}{r^2} \hat{r} \, , \quad \text{if} \quad r > R \, . \tag{16.8}$$

Here $\hat{r}$ represents a unit vector pointing radially outwards from the center of the shell towards the location of m_g, that is, pointing from C towards the particle.

Let us now consider the situation when the corpuscle of gravitational mass m_g is exactly over the surface of the spherical shell of radius R and gravitational mass M_g, as in Figure 16.5.

Integration of Newton's law, Equation (16.6), yields the following net force $\vec{F}$ exerted by the shell on the particle:

$$\vec{F} = -\frac{G}{2} \frac{M_g m_g}{R^2} \hat{r} \, , \quad \text{if} \quad r = R \, . \tag{16.9}$$

Propositions 70 and 71 of Book I of the *Principia* are nowadays presented as follows. Suppose we have a spherical shell of gravitational mass M_g and radius R centered on the point O, Figure 16.6. Let us suppose a reference frame at rest relative to the spherical shell,

Figure 16.4: Force $\vec{F}_{21}$ exerted by 2 on 1 and force $\vec{F}_{12}$ exerted by 1 on 2.

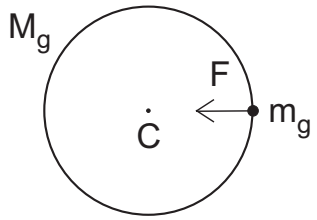


Figure 16.5: The spherical shell exerts an attractive force on a corpuscle located exactly over the surface of the shell. This force points towards the center of the shell.

with its origin located at the center of the shell. Let $\vec{r}$ represent the position vector pointing from the center of the shell towards an arbitrary material point.

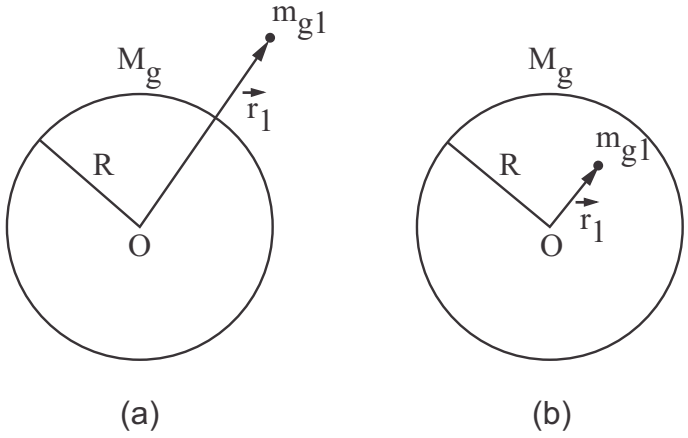


Figure 16.6: Spherical shell.

An element of gravitational mass dm_{g2} located at $\vec{r}_2$ over the surface of the shell is given by $dm_{g2} = \sigma_{g2} da_2 = \sigma_{g2} R^2 d\Omega_2 = \sigma_{g2} R^2 \sin\theta_2 d\theta_2 d\varphi_2$, where $\sigma_{g2} = M_g/4\pi R^2$ is the surface density of gravitational mass distributed uniformly over the surface of the shell, $d\Omega_2$ is the element of spherical angle, θ_2 and φ_2 are the polar and azimuth angles of spherical coordinates, θ_2 varying from 0 to π rad, and φ_2 varying from 0 to 2π rad. The gravitational force exerted by this element of gravitational mass on a test particle m_{g1} located at $\vec{r}_1$ is given by equation (16.6):

$$d\vec{F}_{21}(\vec{r}_1) = -G\frac{m_{g1} dm_{g2}}{r_{12}^2}\hat{r}_{12} \ , \tag{16.10}$$

where $\vec{r}_{12} = \vec{r} = \vec{r}_1 - \vec{r}_2$ is the vector pointing from dm_{g2} to m_{g1}, $r_{12} = |\vec{r}_{12}| = r$ is the distance between dm_{g2} and m_{g1}, while $\hat{r}_{12} = \vec{r}_{12}/r_{12} = \hat{r}$ represents the unit vector pointing from dm_{g2} to m_{g1}. After integration over the surface of the shell, the net force exerted by the shell on m_{g1} is given by (utilizing that $r_1 \equiv |\vec{r}_1|$ and $\hat{r}_1 \equiv \vec{r}_1/r_1$):

$$\vec{F}(\vec{r}_1) = \left\{ \begin{array}{ll} -GM_g m_{g1}\hat{r}_1/r_1^2 \ , & \text{if } r_1 > R \\ -GM_g m_{g1}\hat{r}_1/(2R^2) \ , & \text{if } r_1 = R \\ \vec{0} \ , & \text{if } r_1 < R \end{array} \right\} \ . \tag{16.11}$$

That is, if the particle is outside the shell, it will be attracted as if the shell were concentrated at O. If the particle is anywhere inside the shell, it will not feel any net force exerted by the shell. And if the particle is located exactly at the surface of the shell, it will be

attracted towards the center of the shell with a force which is the arithmetic mean between the values of the force when the particle is slightly outside and slightly inside the shell.

16.3 On the Second Experimental Method Used to Determine the Law of Interaction for Magnetic Fluids

The next Figure represents the experimental method discussed in Subsection 14.5.1:

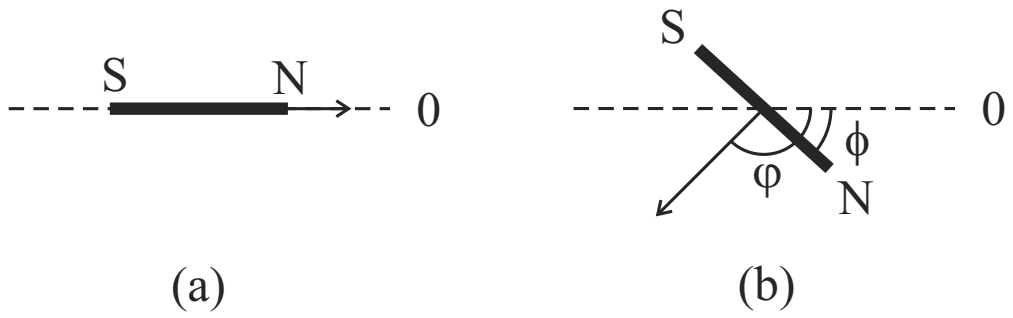


Figure 16.7: Coulomb's experimental procedure.

The dashed line indicates the direction of the magnetic meridian. I present in Figure 16.7 (a) the initial situation of the experiment in which the compass needle SN (or the magnetized wire SN) is along the magnetic meridian, with its end N pointing towards point 0 on the graduated circle. This magnetized needle is attached to the bottom of the suspension wire. The arrow is the pointer of the micrometer attached to the top of the suspension wire. I assume that initially this arrow also points towards point 0.

The top of the wire is then rotated clockwise through an angle φ. It can be seen that the magnetized needle also rotates clockwise, until it stops at an angle ϕ in relation to the magnetic meridian, as shown in Figure 16.7 (b). In this equilibrium situation, the torque exerted by the torsion $\varphi - \phi$ of the suspension wire is balanced by the magnetic torque exerted by the Earth on the magnetized needle SN. These two torques act in opposite directions, with the suspension wire tending to rotate the magnetized needle clockwise and the Earth tending to rotate it counterclockwise.

Coulomb varies the angle φ and observes the angle ϕ acquired by the magnetized needle for each value of φ.

From these experimental results, Coulomb compares in Subsection 14.5.2 the torque exerted by the suspension wire, measured by $\varphi - \phi$, with the angle ϕ with which the magnetized needle moves away from its meridian. He concludes from this comparison that the sines of ϕ in the successive tests are proportional to the angles of torsion $\varphi - \phi$. I present here a Table with these values:

	$\sin\phi$	Angle $(\varphi-\phi)$ of torsion of the suspension wire
First trial	$\sin 10.5^\circ = 0.182$	$360^\circ - 10.5^\circ = 349.5^\circ$
Second trial	$\sin 21.25^\circ = 0.362$	$720^\circ - 21.25^\circ = 698.75^\circ$
Third trial	$\sin 33^\circ = 0.545$	$1080^\circ - 33^\circ = 1047^\circ$
Fourth trial	$\sin 46^\circ = 0.719$	$1440^\circ - 46^\circ = 1394^\circ$
Fifth trial	$\sin 63.5^\circ = 0.895$	$1800^\circ - 63.5^\circ = 1736.5^\circ$
Sixth trial	$\sin 85^\circ = 0.996$	$1980^\circ - 85^\circ = 1895^\circ$

Choosing the values from the first test as a standard of comparison, we obtain the following Table:

	$\sin\phi_1/\sin\phi_j$	$(\varphi_1-\phi_1)/(\varphi_j-\phi_j)$
1st and 2nd trials, $j=2$	$0.182/0.362 = 0.503$	$349.5/698.75 = 0.500$
1st and 3rd trials, $j=3$	$0.182/0.545 = 0.334$	$349.5/1047 = 0.334$
1st and 4th trials, $j=4$	$0.182/0.719 = 0.253$	$349.5/1394 = 0.251$
1st and 5th trials, $j=5$	$0.182/0.895 = 0.203$	$349.5/1736.5 = 0.201$
1st and 6th trials, $j=6$	$0.182/0.996 = 0.183$	$349.5/1895 = 0.184$

From this Table it can be seen that $\sin\phi$ is proportional to $\varphi-\phi$, as stated by Coulomb.

16.4 Using the Magnetic Balance to Determine the Force between the Magnetic Poles

Coulomb's experimental procedure in Section 14.6 is illustrated in Figure 16.8.

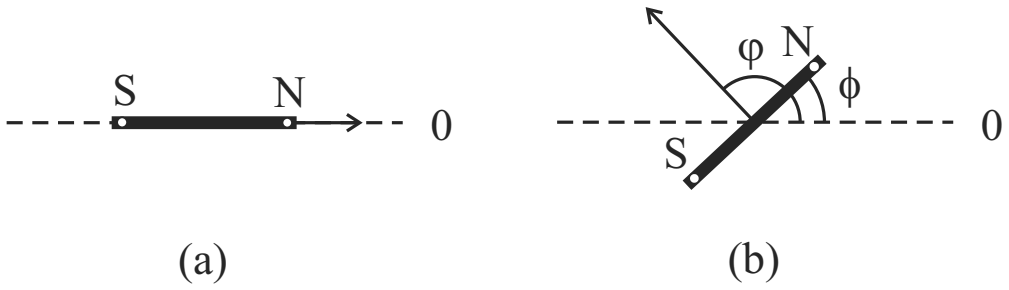


Figure 16.8: Using the magnetic balance to determine the magnetic torque exerted by the Earth on the magnetized needle.

The dashed line indicates the direction of the magnetic meridian. Figure 16.8 (a) presents the initial situation of the experiment in which the magnetized needle SN (or the magnetized wire SN), 24 inches long, is in horizontal equilibrium along the meridian with its end N pointing towards point 0 on the graduated circle. The white circles indicate the positions of the centers of action of this needle, that is, the location of its magnetic poles, about 1 inch from each end. The center of the magnetized needle is attached to the bottom of the suspension thread. The arrow is the micrometer pointer attached to the top of the suspension wire. I assume that initially this arrow is also directed toward point 0.

In Figure 16.8 (b) Coulomb twists the upper part of the suspension wire counterclockwise at an angle φ, causing the magnetized needle to rotate in that direction until it stops at an

angle ϕ with respect to the magnetic meridian. The counterclockwise torque due to the torsion of the suspension wire is proportional to the angle $\varphi - \phi$. At equilibrium, this torque is balanced by the clockwise magnetic torque exerted by the Earth, which causes the magnetized needle to tend to return to the magnetic meridian. This magnetic torque due to the Earth is proportional to the sine of the angle ϕ. For small angles (that is, with $\phi \ll 1$ *radian* or $\phi \ll 57.3°$), we can assume $\sin\phi \approx \phi$, with ϕ expressed in radians. Coulomb observed that by turning the top of the suspension thread with $\varphi = 720°$, the magnetized needle stopped at an angle of $\phi = 20°$. The resulting twist of the wire was then $720° - 20° = 700°$. He concluded that a resultant torsional force of $700°/20 = 35°$ is required to move the horizontal needle 1 degree away from its magnetic meridian. From this force of torsion, he estimated the torque exerted by terrestrial magnetism when acting on this magnetized needle.

We now return to the situation in which the suspension wire is not twisted, with the horizontal magnetized needle pointing along the magnetic meridian, Figure 16.9 (a).

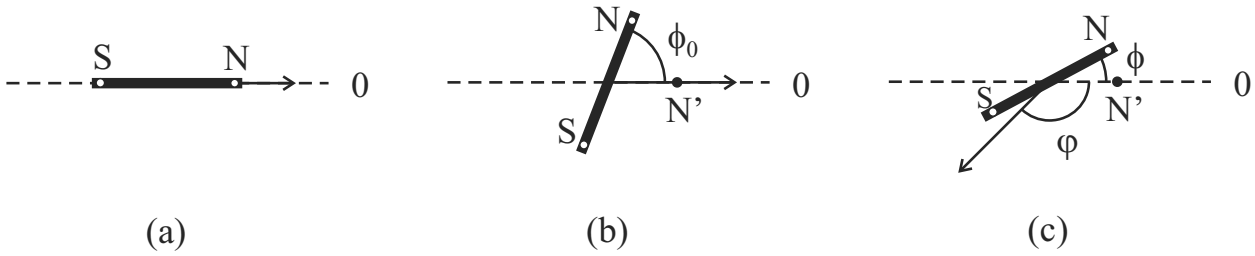


Figure 16.9: Using the magnetic balance to determine the force between two magnetic poles.

A vertical magnetized needle $S'N'$ (or a vertical magnetized wire $S'N'$) is placed along the magnetic meridian of the horizontal needle, with the poles of the same type close together. There is a repulsion between these poles that causes the suspension wire to be twisted until the horizontally suspended needle stops at a new equilibrium situation. In Figure 16.9 (b) we have the North pole N' of the vertical magnetized needle represented by the black ball. This vertical needle is also 24 inches long, as is the horizontally suspended magnetized needle. The pole N' is placed in the same horizontal plane of the suspended needle, so that if the pole N' and the horizontal needle were to touch each other, the poles of the same name N and N' would coincide at the same point. The South pole of the vertical needle, S', is not shown in this Figure. It is about 22 inches above the plane of the horizontal needle. Because of the repulsion between the North poles of the two needles, the bottom of the suspension wire is twisted by placing the N' pole of the vertical needle in this position, until the horizontal needle stops at an orientation deviated by an angle ϕ_0 with respect to the magnetic meridian, Figure 16.9 (b). I am assuming here that this deviation was counterclockwise. In this situation the repulsive torque between the poles of the same name tends to rotate the horizontal needle counterclockwise. At equilibrium this torque is counterbalanced by two other torques that tend to rotate the horizontal needle clockwise, namely, the torque due to the torsion of the suspension wire and the magnetic torque due to the action of the Earth on the magnetized needle.

In Figure 16.9 (c) the needle of the micrometer attached to the top of the suspension wire is rotated clockwise through an angle φ, causing the horizontal needle to rotate clockwise, approaching the magnetic meridian until it stops at an angle ϕ. In this case the total torsion of the suspension wire is given by the angle $\varphi + \phi$.

The next Table presents the values of these angles in the case of Figure 16.9 (c), according to the experiments described in Section 14.6:

	φ	ϕ	$\varphi+\phi$	$35\times\phi$	$\alpha=(\varphi+\phi)+(35\times\phi)$
First trial	0°	24°	24°	840°	864°
Second trial	1080°	17°	1097°	595°	1692°
Third trial	2880°	12°	2892°	420°	3312°

I will represent by the angle α the repulsive torque exerted by the North pole N' of the vertical needle when acting on the horizontal needle. At equilibrium this repulsive torque is counterbalanced by two other torques acting in the opposite direction, namely, the torque due to the total torsion of the suspension wire (represented by $\varphi+\phi$) and the torque due to the magnetic action of the terrestrial globe on the horizontal needle (represented by $35\times\phi$).[630] Coulomb obtains the value of α from the following equation describing equilibrium between these three torques in Figure 16.9 (c):

$$\alpha=(\varphi+\phi)+(35\times\phi)\ . \tag{16.12}$$

These values of α are given in the last column of the previous Table.

From the values of ϕ and α, Coulomb concluded that the repulsion between the North pole N of the horizontally suspended needle and the pole N' of the vertical wire is inversely proportional to the square of the distance between these poles. To reach this conclusion, he disregarded the actions of the other poles among themselves, since they are much further away than the closest poles N and N'. Furthermore, he estimated the distances between these poles N and N' by the angle ϕ between them. In the next Table I present (using the degree as a unit of measurement) the values of ϕ, $1/\phi^2$ and α:

	ϕ	$1/\phi^2$	α
First trial	24	0.00174	864
Second trial	17	0.00346	1692
Third trial	12	0.00694	3312

Choosing the values of the first test as a standard of comparison, we obtain the following Table:

	$(1/\phi_1^2)/(1/\phi_j^2)$	α_1/α_j
1st trial, $j=1$	$0.00174/0.00174=1$	$864/864=1$
1st and 2nd trials, $j=2$	$0.00174/0.00346=0.503$	$864/1692=0.511$
1st and 3rd trials, $j=3$	$0.00174/0.00694=0.251$	$864/3312=0.261$

The numbers in the second and third columns are very close together, being approximately given by 1, 1/2 and 1/4.

This proximity between the values in the second and third columns justifies Coulomb's conclusion that the forces between the magnetic poles are inversely proportional to the squares of the distances between these poles.

[630] Because Coulomb is assuming for these small angles that $\sin\phi\approx\phi$. In addition, Coulomb estimated that a force of torsion represented by 35° was required for every 1 degree of displacement of the horizontal needle from the direction of the magnetic meridian.

16.5 The Proportionality of Electric Force in Relation to the Product of Charges: Definition of the Amount of Charge or Experimental Law?

Nowadays Coulomb's law is presented saying that the force between two particles electrified with charges q_1 and q_2, separated by a distance d much greater than their sizes, is proportional to

$$\frac{q_1 q_2}{d^2} \ . \tag{16.13}$$

Gillmor, Blondel, Wolff and Gliozzi presented very important discussions concerning the proportionality of the electric force with respect to the product of charges.[631] There are two main possibilities to consider: (a) The amount of electrical charge can be defined by postulating or assuming this proportionality. (b) This proportionality may come from an experimental result. These two possibilities are mutually exclusive.

In his First Memoir Coulomb showed experimentally that the force between two small balls electrified with charges of the same sign was inversely proportional to the square of the distance between their centers. To obtain this result, he compared the electric force with the force of torsion of the suspension wire of his balance. In the Second Memoir he showed that the same law is valid in the attractive case with charges of opposite signs. In this case he studied the period of oscillation of a small electrified ball placed in front of a large and stationary electrified ball. At the end of this Second Memoir he then presented the conclusion of his experiments with the following words, see page 257:

> From the foregoing researches the following results: That the action, whether repulsive or attractive between two electrified globes, and consequently between two electric molecules, is directly proportional to the densities of electric fluid of the two electrified molecules and inversely proportional to the square of the distances.

That is, the force F between two electrified particles is proportional to the product of the total amounts of electric fluid in the two particles and inversely proportional to the square of the distance between them.

Although he said that these results are a consequence of his previous research, at no point did Coulomb experimentally test the proportionality of the force in relation to the product of the amounts of charge of the bodies that were interacting. In the next Memoirs he used this proportionality to interpret various experiments. It seems to have been evident to Coulomb that the force between two small electrified bodies had to be proportional to the product of the electric charges of those bodies.

Blondel and Wolff stated that Coulomb in fact *implicitly defined* the amount of charge on an electrified particle from the proportionality with respect to the electric force that this particle exerts on another electrified particle.[632] That is, the greater the observed force, the greater the amount of charge on the particle. Let us assume that in the initial situation an electrified particle with a certain charge q_1 exerts a force F_1 on another electrified particle 2. If the measured force on that same electrified particle 2 doubles or triples, then the amount

[631] [Gillmor, 1971b], [Gillmor, 1971a, pp. 190-192], [Blondel and Wolff, 2013d] and [Gliozzi, 2022, p. 425]. See also Section 14.1 (electrostatic force or Coulomb force) of [Assis, 2018b], [Assis, 2018a] and [Assis, 2019].
[632] [Blondel and Wolff, 2013d].

of charge on the particle that is exerting the force will be, by definition, two or three times greater than q_1.

The analogy with Newton's law of universal gravitation, according to which the gravitational force between two particles is proportional to the product of their masses and inversely proportional to the square of the distance between them, underlies all of Coulomb's work. This analogy would be represented, for example, by his choice of the expression "electric mass" to designate what is called today an "electric charge". As early as the Second Memoir, Coulomb explicitly used that the force was proportional to the product of the charges by introducing the following expression:[633]

> D equal to the product of the electric mass of the two balls.

As quoted by Peter Heering, the following words by H. G. Hammon show a similar conclusion:[634]

> Coulomb had not described any experiment to establish the dependence of the force on the quantities of charge on the spheres. His statement of the charge-dependence of the force was clearly by analogy with Newton's law of gravitation. This points out, perhaps more strongly than any other aspect of the history, the character of Coulomb's law as a *definition* of the quantity of charge. His choice of the simple product of the quantities of charge is certainly the simplest choice, but it is *not* the only possible choice.

Another example of the importance of Newton's ideas in Coulomb's work is contained in Coulomb's assumption when considering the action of an electrified spherical shell on another electrified body as if the entire spherical shell were concentrated at its center. See, for instance, Subsection 14.1.2 and Section 16.2 on pages 234 and 268.

Gliozzi believed that Coulomb's approach to postulate that the force between two electric charges is proportional to the product of their charges was the most intuitive one:[635]

> Coulomb assumed, as Aepinus had done before him, the postulate that the force between two electric charges is proportional to their product. The attempts of some later scholars to demonstrate this postulate were all illusory; they serve only to confuse the ideas of inexperienced students, though they aimed to clarify, because they deduced Coulomb's postulate from other, much less intuitive ones.

Let us quote here some words of Aepinus with this assumption:[636]

> 10) Let there be a body A, Fig. I, constituted first in the natural state, that is it contains the natural quantity of fluid, which we shall call Q; and if we suppose that B is a particle of either electric or magnetic fluid clinging round the surface of the body, the particle B will be attracted by the whole body A, but will be repelled by the fluid with which its pores have been filled. Let us call this attractive force $= a$, and the repulsive force which is exercised on $B = r$, and the force by which the particle B is

[633] See footnote 528 on page 230 and also [Gillmor, 1971a, p. 191].
[634] [Heering, 1992, p. 993] and [Heering, 1994, p. 64].
[635] [Gliozzi, 2022, p. 423].
[636] [Aepinus, 1979, § 10, pp. 246-247].

attracted toward the body A will be $= a - r$. But since the natural state is supposed to exist, the body A contains precisely as much fluid as its attractive force can retain, and so the whole action on the particle, or $a - r$, $= 0$. Let us suppose that, however this happens, there is added to the fluid contained in the body a definite quantity of it, uniformly distributed through the whole body, which is to the natural quantity of the fluid as α is to Q. Then the repulsion[637] of the particle $B = \frac{(Q+\alpha)r}{Q}$, but the attraction is the same as before, so the particle B will be attracted toward the body A by a force $= a - r - \frac{\alpha r}{Q}$. But since $a - r = 0$, the force which draws the particle B towards $A = -\frac{\alpha r}{Q}$, or the particle B will be repelled from the body A by a force $\frac{\alpha r}{Q}$.

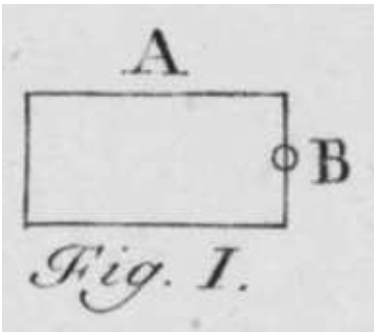


Figure 16.10: Figure I of Aepinus' *Essay*.

The beginning of § 30 of Aepinus' work where this assumption is made explicit reads as follows:[638]

30) From this we can immediately deduce various noteworthy conclusions. It is clear first that $A = a$. To make this evident, let the mass of the body $A = M$, the natural quantity of fluid contained in $A = Q$, and similarly the mass of the body $B = m$, and the natural quantity of fluid pertaining to that body $= q$. Since corporeal actions always happen in the ratio of the masses, a will be to A, in the composite ratio of $M : m$ and $q : Q$, whence is obtained the analogy $a : A = Mq : mQ$, and $A = \frac{amQ}{Mq}$.

According to Gillmor, Blondel and Wolff, it would have been possible for Coulomb to give an experimental foundation in relation to the proportionality of electric force to the product of charges. To this end, he would have had to find a way to quantify the concept of electric charge without using the concept of force. Once this was done, he could then use his torsion balance to experimentally verify the relationship of force as a function of the amount of charge on the interacting spheres.

In his Fourth Memoir Coulomb investigates whether or not the distribution of charge between two conductors depends on their chemical compositions. To do this, he introduces an electrified copper ball into his electric balance, leaving it in a fixed position relative to the ground. This ball comes into contact with another initially discharged conducting ball. This latter ball is at the end of a horizontal insulating needle attached at its center to a vertical wire that can be twisted. After contact, the two balls repel each other over a large distance.

[637] [Note by Home in [Aepinus, 1979]] Aepinus here, as elsewhere, makes the unstated assumption that the force exerted is proportional to the quantity of fluid involved. The assumption is made explicit in § 30 below.
[638] [Aepinus, 1979, § 30, p. 257].

He brings the second ball closer to the first ball by a certain distance d by means of the torsion he can provide to the upper part of the suspension wire. The measure of the total angle of twist gives him a measure of the repulsive force between the two electrified balls at this distance d. Let us call this force F_1. He then puts a third ball in contact with the first copper ball. This third ball is initially discharged, conductive, has the same diameter as the copper ball, but is made of a different material. The electricity from the copper ball is divided between it and this third ball by the contact between them. Then the third ball is removed from the balance. Since the copper ball has lost some of its charge, the needle ball moves closer to it. Coulomb then decreases the twist of the wire until the needle ball and the copper ball return to the same distance d as before. Again the repulsive force between these balls is measured by the new angle of torsion. Let us call it F_2. He observed that this force was essentially half the previous force, that is, $F_2 = F_1/2$. Since Coulomb is *postulating or assuming* the proportionality between the electric force and the amount of charge, he then concludes that the third ball acquired exactly half the electric fluid of the first copper ball when it came into contact with it. Since this third ball has the same diameter as the copper ball, but is made of a different material, this means that the division of charge between two conducting balls upon contact between them does not depend on the material they are made of. This experimental result could not be predicted *a priori*, since it is not something trivial or obvious because in this experiment balls made of different materials were used. Symmetry arguments cannot be used in this case.

This procedure could have been reversed to experimentally arrive at the conclusion that the electric force is proportional to the product of the charges between the interacting bodies. This would require using the first and the third ball not only of the same size, but also of the same material (the two balls made of copper, for example). For reasons of symmetry we can postulate that the electric fluid will divide equally between the two balls when they come into contact, one of them being initially electrified and the other discharged, since they are both conductors, have the same shape, the same size, and are made of the same material. Then, following the same experimental procedure presented in the previous paragraph, we would conclude that $F_2 = F_1/2$. In this case it would come from experiment that, by assuming the initial force between two charges q_1 and q_2 being given F_1, when dividing only one of the charges by half, the force exerted between $q_1/2$ and q_2 also halves, being given by $F_1/2$. However, in this case this fact would be established experimentally through the torsion angles measured on the electric balance.

16.6 The Proportionality of Magnetic Force in Relation to the Product of Pole Intensities: Definition of the Amount of Magnetic Fluid or Experimental Law?

Nowadays Coulomb's law for magnetostatics is presented as saying that the force between two magnetic poles with intensities p_1 and p_2 separated by a distance d is proportional to

$$\frac{p_1 p_2}{d^2} \ . \tag{16.14}$$

Just as in the case of the electric force, we can ask an analogous question. Is this

proportionality of the force in relation to the product of the pole strengths a definition of pole intensity or an experimental law?

The title of Section 14.3 of Coulomb's Second Memoir is as follows:

> The Magnetic Fluid Acts by Attraction or Repulsion in a Ratio Composed Directly of the Density of the Fluid and Inversely of the Square of the Distance Between Its Molecules.

In other words, Coulomb is stating that the magnetic force between two particles is proportional to the product of the total amounts of magnetic fluid in the two particles and inversely proportional to the square of the distance between them.

Immediately following this title, Coulomb stated the following:[639]

> There is no need to prove the first part of this proposition; let us go to the second part.

Coulomb presented another formulation of this law at the end of the Second Memoir, namely:[640]

> The following results from previous research: That the attractive and repulsive force of the magnetic fluid is exactly, as in the case of the electric fluid, directly proportional to the densities [of the fluid] and inversely proportional to the square of the distances between the magnetic molecules.

To prove that the force between the magnetic molecules was inversely proportional to the square of the distance between them, he used long and thin steel needles of cylindrical shape homogeneously magnetized by the double-touch method. From his experiments he concluded that in a cylindrical needle having a length of 54 to 68 cm (20 to 25 inches), a diameter of 0.3 cm (1.5 line), magnetized by the double-touch method, the magnetic fluid can be supposed to be concentrated at a point located at 2.3 cm (10 lines) from the ends of the needle.[641]

He then used these needles on his torsion balance to determine the force between the magnetic poles of two magnetized needles. With this procedure he experimentally concluded the second part of the quotation above, namely, that the attractive and repulsive force of the magnetic fluid is inversely proportional to the square of the distances between these poles.

As seen in the quotes from that Section, Coulomb explicitly mentioned that there would be no need to prove that the force between the supposed magnetic molecules or particles was proportional to their fluid densities, that is, to the product of their amounts of magnetic fluid. He also did not make experiments to show that the force between the poles of two magnetized needles was proportional to the product of their magnetic pole intensities. He implies that this proportionality should be accepted without proof, that is, as a postulate or definition of the amount of pole intensities. As in the case of the electric force discussed in Section 16.5, it seemed to him that it was evident that the force between two magnetic particles had to be proportional to the product of their quantities of magnetic fluid. Likewise, the force between the centers of action of two magnetized needles had to be proportional

[639] See footnote 570 on page 244.
[640] See page 257.
[641] See page 257 of this English translation.

to the product of their magnetic pole intensities. He used this proportionality to interpret various experiments. He also used this proportionality to estimate the change in magnetic fluid density along the length of a magnetized needle.

Gillmor, Blondel, and Wolff also discussed the proportionality of force with respect to the product of magnetic pole intensities.[642] According to these authors, Coulomb would have considered it unnecessary to prove this proportionality due to the analogy he assumed between the magnetic force and the Newtonian gravitational force. This is evident from its magnetic terminology presented in Section 4.5. In particular, the amount of magnetic fluid in a particle is indicated by the expressions "mass intensity", "magnetic intensity" and "mass of magnetic fluid".[643] The term "mass" in this magnetic context is a clear indication of the influence of Newton's ideas on Coulomb. Another direct indication that Coulomb was being influenced by Newton's gravitational force in the case of magnetism is the following quote from his 1777 work:[644]

> Therefore, it appears from experiment that it is not the vortices that produce the different magnetic phenomena and that, to explain these magnetic phenomena, it is necessary to resort to attractive and repulsive forces of the same nature as those which we are obliged to use to explain the weight of bodies and celestial physics.

The reaction torque exerted by a metal wire is directly proportional to the angle of torsion of the wire. Therefore, this angle serves as a measure of the magnetic force. In this Second Memoir Coulomb used this fact to establish that the force between magnetic poles was inversely proportional to the square of the distance between the poles. Coulomb could use an analogous procedure to experimentally test the proportionality of the force in relation to the product of the magnetic pole intensities, that is, to verify whether or not the force is proportional to this product.

To this end, he could use the experimental procedure described in Sections 14.6 and 16.4. He could use a long magnetized needle suspended horizontally along the magnetic meridian by a vertical wire attached to the center of the needle and placed on his torsion balance. Then he would observe the twist angle ϕ_1 when a first vertical magnetized wire is placed along the magnetic meridian with the poles of the same name placed close together and in the same horizontal plane, as illustrated in Figure 16.9. When this vertical wire is removed, the horizontal needle returns along the magnetic meridian. He could then test multiple magnetized wires made of the same material as the first wire, also having the same shape and size as the first wire, all of them being magnetized by the same double-touch method. Let us call them wires 2, 3, 4, 5, ... By placing them one at a time vertically along the magnetic meridian of the first horizontal needle, at the same place and height as he had previously placed the first wire, with the poles of the same name in the same horizontal plane (for example, with the North pole N of the horizontal needle and with the North pole N' of the vertical wire in the same horizontal plane, with these two poles close to each other), he would check the displacement angles ϕ_2, ϕ_3, ϕ_4, ϕ_5, ..., of the horizontal needle from the magnetic meridian. He could then define that the vertical wire that displaces the horizontal needle from the same angle ϕ_1 caused by the first wire, would have the same magnetic pole intensity as the first wire, since they exert the same action on the horizontal needle when

[642] [Gillmor, 1971b], [Gillmor, 1971a, pp. 190-192], [Blondel and Wolff, 2013d].
[643] See footnotes 129, 998 and 1072 on pages 56, 454 and 478, respectively.
[644] See page 60 of this English translation.

placed at the same point. Let us assume that the fourth wire exerts the same action on the needle as the first wire, that is, $\phi_4 = \phi_1$. He could then postulate that by putting the first and fourth wires together, with the poles of the same type side by side, they would together have twice the pole strength as the first wire alone. He could then use a procedure similar to that used in the Second Memoir to verify whether the force exerted by the two North poles of these two wires together, when acting on the North pole of the horizontal needle, was or was not twice the value of the force exerted only by the North pole of wire 1 when acting on the North pole of the horizontal needle. The force in this case would be measured by the angle of twist of the horizontal needle with respect to its magnetic meridian.

That is, he could initially establish that two magnetic poles have the same intensity when, being at the same distance from a third magnetic pole, they exert separately the same force F on this third magnetic pole. He would then postulate that two poles of the same intensity, when placed side by side, would have twice the intensity of just one of these poles. Then he would check whether the force exerted by these two poles together, when acting on the third pole, has or not twice the intensity of the force F exerted by one of these poles when acting alone on the third pole. In this case, the intensity of the magnetic force would be estimated by the angle of torsion of his magnetic balance (assuming that in all these cases the estimation of the magnetic forces is done with the poles always at the same distance, this distance being much greater than the thickness of the needles). This would be a possibility to experimentally establish that the force between magnetic poles is proportional to the product of the magnetic intensities (that is, to establish that the magnetic force is proportional to the product of the pole strengths).

Chapter 17

Third Memoir on Electricity and Magnetism: On the Quantity of Electricity that an Insulated Body Loses in a Given Time, Either by Contact with More or Less Humid Air, or Along More or Less Insulating Supports

Coulomb[645]

When an electrified conducting body is isolated by insulating supports,[646] experiment shows that the electricity of this body decreases and is destroyed rather rapidly. The subject of this Memoir is to determine the law that this decrease follows: the knowledge of this law is absolutely necessary in order to be able to submit to calculation the other phenomena of electricity; because the experiments intended to evaluate these phenomena, not being able to be carried out at the same moment, cannot be compared with each other, without knowing the alteration which they experience in the time which elapses from one to the other.

Two causes seem to contribute principally to the loss of electricity in bodies: the first is that it is probable that there is no perfectly insulating support in nature, that is to say, that there is no body entirely impenetrable to electricity, when it is carried to a very great degree of intensity; furthermore, even if such a body existed, the humidity in the air — air being always humid to some degree — attaches itself to the surface of the insulating body to a greater or less extent depending upon whether the air is more or less humid and whether the insulating body by nature has a greater or less affinity with water than do the parts of air;[647]

[645] [Coulomb, 1788e], with complete translations into German and Portuguese in [Coulomb, 1890a] and [Assis, 2022], respectively. This work was presented in 1785 to the French Academy of Sciences and published in 1788.

[646] In the original: *soutiens idio-électriques*. See footnote 522 on page 229. This expression can also be translated as idio-electric supports or dielectric supports, [Gillmor, 1971a, p. 194].

[647] In the original: *les parties de l'air*. This expression was translated in German as the particles of air, *die*

as such it often happens that the aqueous parts[648] expanded on the surface of the insulating body, which serves to support an electrified body, are closer together one to another than they are in the surrounding air; and as these aqueous parts are conductors of electricity, in this case, when the insulating body which serve to support does not have sufficient length, the electricity is lost more easily along the surface of the insulating body which serves as support than by contact with the air.

The second cause is that the electrified body being surrounded by atmospheric air, this air composed of different elements, is more or less insulating, either by the nature of these elements, or by their affinity with aqueous molecules; an affinity which varies according to the degree of heat, so that the air may be regarded as composed of an infinity of elements partly insulating, partly conducting. But, as a conducting body is always charged with a part of the electricity of the body which touches it, and so charged with this electricity, it is repelled by this body; as a result, each molecule of the air which touches an electrified body, is charged with the electricity of this body more or less rapidly, according to whether the electric density of the body[649] is more or less intense, and whether the air is more or less charged with humidity or electrically conducting parts: as soon as a molecule of the air is charged with electricity, it is repelled by the electrified body, and replaced by another which becomes electrified, and is driven out in turn; each of these molecules carrying away a part of the electricity of the electrified body which they envelop, the electric density diminishes more or less rapidly, according to the state of the atmosphere. The explanation we have just given of how electricity is lost by contact with air, whose infinitely small molecules move with great facility, is not applicable, as experiment teaches us, to the way the electricity dissipates along the surfaces of supports which have become imperfect insulators by contact with humid air; because in this second case, the aqueous parts contract so great a degree of adherence with the surface of these supports that this adherence is sometimes much greater than the repulsive action that the electrified body exerts on the aqueous molecule, to which it has transmitted a part of its electricity; from which it results, and this is confirmed by experiment, that when the humid molecule, nearest to the electrified body, is charged with electricity, this electricity passes in part to the following molecule, without displacing this molecule, and thence from molecule to molecule until a certain distance of the body: thus the [electric] density of each molecule would diminish according to how far it would be from the electrified body, because these aqueous molecules being separated by a small insulating interval,[650] it requires a certain degree of force for electricity to pass from one molecule to another. The resistance which this small insulating interval opposes to the flow of the electric fluid appears to be able to be represented only by a constant quantity for a constant interval, and must consequently be proportional to the difference in the action of two consecutive molecules. We will see presently, that the calculation and the experiments which determine the law of the density of the electric fluid along the imperfect insulating

Luftheilchen, [Coulomb, 1890c, p. 44]. This German translation makes sense. At the last paragraph of the Fourth Memoir Coulomb will talk explicitly about the *particules idio-électriques de l'air*, see footnote 733 on page 322.

[648] In the original: *les parties aqueuses*. This expression was translated in German as aqueous particles or water particles, *die Wassertheilchen*, [Coulomb, 1890c, p. 44].

[649] In the original: *la densité électrique du corps*. That is, the electric density, or charge per unit area, on the surface of the body.

[650] In the original: *intervalle idio-électrique*. That is, every aqueous particle or molecule that can be electrified would behave like a conductor. According to Coulomb, these molecules would be separated from each other by a small insulating interval along the surface of the support.

supports, agree with the preceding reasoning.

The researches which are to follow must therefore have two objectives; the first, to determine the law according to which the electricity is lost by contact with air; the second, to determine the law according to which this same electricity is lost along the surface of the insulating supports: but as in all the experiments which we can make, the conducting bodies charged with electricity, are always supported by insulating bodies, these experiments must naturally always present a result composed of the loss of electricity by contact with the air, and the loss of electricity along the surface of the insulating support, unless we succeed in supporting the body by an insulating support whose surface is proportionally less charged with humidity or conducting parts than the molecules of the surrounding air; for then by greatly diminishing the surface of contact of the electrified body and of its support, the loss of electricity of the body would be entirely due to contact with the air. According to this reasoning, I tried several insulating materials to serve as a support for the electrified body, and I found that when the electric density of the supported body was not significant, a small cylinder of Spanish wax or shellac,[651] half a line in diameter [0.11 cm] and 18 to 20 lines in length [4.1 to 4.5 cm], was almost always sufficient to perfectly insulate a pith ball[652] five or six lines in diameter [1.1 to 1.4 cm]; I also found that when the air was dry, a very fine silk thread, passed through boiling Spanish wax, and then formed into a small cylinder at most a quarter of a line in diameter [0.06 cm], fulfilled the same object, provided that one made this thread five or six inches long [13.5 to 16.2 cm]. A thread of glass, drawn with an enameler's lamp,[653] five or six inches in length, only insulates the ball on very dry days, and when it is charged with a very feeble degree of electricity; it is the same with a hair or a silk [thread] which is not coated with Spanish wax, or what is still better, with pure shellac.

17.1 First Part: Experiment to Determine the Loss of Electricity by Contact with Air

I gave, in my First Memoir on electricity, the description of the balance which I use in all electrical experiments. One can recall, casting our eyes on the Figure of this balance,[654] that a horizontal needle formed by a silk thread coated with Spanish wax or even by a straw terminated by a small cylinder of shellac, carries a small pith ball four or five lines in diameter [0.90 to 1.1 cm] at its end; and that this needle is suspended horizontally by a silver wire 28 inches in length (75.80 cm), and that you only need to apply a force of 1/340 grain (0.153 dyn) over a lever arm of 4 inches (10.83 cm) to twist this supporting wire 360° about its axis;[655] and that the forces of torsion are generally proportional to the angle of torsion, so that, for example, to twist our thread 36 degrees or to vary the needle by 36°, only requires [a force of] 1/3400 of a grain. Furthermore, one ought to recall that the force of torsion of this suspension thread is measured in a very simple way, by means of a micrometer placed at the top of the rod of our balance, and that by presenting to the ball of the needle a second ball of the same size, insulated as that of the needle, their reciprocal action tends

[651] See footnotes 467 and 468 on page 207.

[652] In the original: *balle de sureau*. See footnote 469 on page 208.

[653] In the original: *lampe d'émailleur*. Enameler is a person who applies enamel. Vitreous enamel is a material made by fusing powdered glass to a substrate by firing. The powder melts, flows, and then hardens to a smooth, durable vitreous coating.

[654] See Figures 1 and 3 on pages 205 and 207, respectively.

[655] See footnote 477 on page 210.

to move them away from each other when they are charged with an electricity of the same nature; and that by twisting the wire of suspension by means of the micrometer, it is easy to measure this action, which we have found, in this Memoir, [varies] exactly as the inverse of the square of the distance [separating] the two balls.

To determine, by means of this same balance, the law according to which an electrified body loses its electricity in a given time, here is the method which seemed to me the simplest and most exact.

I suspend from a very fine silk thread, coated with Spanish wax and terminated by a small cylinder of shellac 18 to 20 lines long (4 to 4.5 cm), a small pith ball similar to that [supported by] the needle; I introduce it through the hole in the lid of my balance, as I did in my First Memoir, and place it in the same way.

By means of a large-headed pin which I charge with electricity and which is insulated as in the First Memoir, I equally electrify the two balls, which is very easy by making them touch one another; when these balls are electrified, they mutually repel each other and the needle only stops when the distance of the two balls is such that the force of torsion is equal to the repulsive force: an example will make the operation better understood than any other explanation.

I assume the needle ball is driven out 40 degrees; by twisting the suspension wire, I bring it back to a lesser distance, to 20 degrees for example, which I suppose I obtained by twisting the suspension wire by 140 degrees. I observe the moment when the ball reaches exactly 20 degrees: as the electricity is lost, the balls will come together a few minutes after this operation; so, in order to measure [the twist required] to keep the ball at the first distance of 20 degrees I untwist the suspending wire 30 degrees by means of the [micrometer] index and the force of torsion being diminished by these 30 degrees, the balls are repelled at just over 20 degrees. I wait for the moment when the ball of the needle reaches 20 degrees, and I take account very exactly of the time elapsed between the two operations; I assume that time is three minutes; it will result from this operation that at the first observation, the distance of the balls being 20 [degrees], the repulsive force measured 140 degrees plus 20 degrees; that three minutes after the repulsive force, at the same distance of 20 degrees, was only 110 degrees plus 20 degrees, that is to say, it was diminished by 30 degrees [in three minutes] or 10 degrees per minute: thus, as the average force between the two observations was measured by 145 degrees and it decreases by 30 degrees in three minutes or by 10 degrees per minute, the electric force of the two balls decreased by 10/145 per minute.

It is according to this method[656] that I made the first Table which represents the observations made on May 28, May 29, June 22 and July 2; I chose these four observations among an infinity of others, because the hygrometer showed considerable differences in degree of humidity of the air over these four days while the degree of heat was about the same.

17.1.1 Comments on the Following Table

In this Table, the first column represents the instant of time of the observation; the second, the distance of the two balls; the third, the degree of torsion given by the micrometer; the fourth, the duration of the time elapsed between two consecutive observations; the fifth, the loss of electric force in the time between two observations; the sixth, the average force of repulsion between two consecutive observations, measured by the average torsion, indicated

[656] An illustration of this method can be found in Figures 19.1 and 19.2 of Section 19.1.

by the micrometer, and by the distance of two balls; finally, the seventh column indicates the ratio of the electrical force lost in 1 minute to the total force.[657]

First Table for determining the quantity of electricity lost during one minute by contact with the air							
	Time of experiment (morning)	Distance of the balls	Torsion of the micrometer	Time elapsed between two consecutive observations	Electric force lost between two observations	Average force between two observations	Ratio of the electric force lost by the body during 1 minute, to the average force of the body
First experiment on May 28. Hygrometer, 75°; thermometer, 15.5°; barometer, $28^P\ 3^l$							
1st test	6h 32m 30s	30	120				
				5m 45s	20	140	1/40
2nd test	6h 38m 15s	30	100				
				6m 15s	20	120	1/38
3rd test	6h 44m 30s	30	80				
				8m 30s	20	100	1/42
4th test	6h 53m 0s	30	60				
				10m	20	80	1/40
5th test	7h 3m 0s	30	40				
				14m	20	60	1/42
6th test	7h 17m 0s	30	20				
Second experiment on May 29. Hygrometer, 69°; thermometer, 15.5°; barometer, $28^P\ 4^l$							
1st test	5h 45m 30s	30	130				
				7m 30s	20	150	1/56
2nd test	5h 53m 0s	30	110				
				9m 30s	20	130	1/61
3rd test	6h 2m 30s	30	90				
				9m 45s	20	110	1/54
4th test	6h 12m 15s	30	70				
				20m 45s	30	75	1/58
5th test	6h 33m 30s	30	40				
				18m	20	60	1/54
6th test	6h 51m 0s	30	20				
Third experiment on June 22. Hygrometer, 87°; thermometer, 15.75°; barometer, $27^P\ 11^l$							
1st test	11h 53m 45s	20	80				
				3m	20	90	1/13.5
2nd test	11h 56m 45s	20	60				
				3m	20	70	1/11
3rd test	11h 59m 45s	20	40				
				5m 15s	20	50	1/13.5
4th test	12h 5m 0s	20	20				
				11m 15s	25	28	1/13.5
5th test	12h 16m 15s	20	5				
Fourth experiment on July 2. Hygrometer, 80°; thermometer, 15.75°; barometer, $28^P\ 2^l$							
1st test	7h 43m 40s	20	80				
				5m 20s	20	90	1/14
2nd test	7h 49m 0s	20	60				
				8m 20s	20	70	1/19
3rd test	7h 57m 20s	20	40				
				12m	20	50	1/30
4th test	8h 9m 15s	20	20				
				8m 15s	10	35	1/19
5th test	8h 17m 30s	20	10				

We see, from this seventh column, that the ratio of the electric force lost to the total force was represented, on the same day or in the same state of the humidity of the air, by a constant quantity; that this ratio has varied only as the hygrometer has indicated a variation in the humidity of the air, from which it follows that, for the same air condition, the loss of electricity is always proportional to the electric density.

The law of the loss of the electric density being determined by the experiments which precede, it is easy to have by calculation the electric state of the two balls after a given time;

[657] A detailed example of the calculations related to this Table can be found in Section 19.1.

let us take for example the first experiment of our Table where we saw that the electric action of the two balls, whose initial electricity was the same, diminished by 1/41 part each minute. Since the electric density decreases, as we have just seen, in proportion to the densities, we have

$$-\left(\frac{d\delta}{\delta}\right) = mdt \ ,$$

where δ represents the density of each ball;[658] but, since this density decreases, as will be seen in the following Section, by 1/82 per minute, if $dt = 1$ minute, we will have

$$m = \left(\frac{1}{82}\right) \ .$$

So, in this experiment,

$$-\frac{d\delta}{\delta} = \left(\frac{dt}{82}\right) \ .$$

Multiplying by the modulus μ of the logarithmic system,[659] we will have

$$-\mu\frac{d\delta}{\delta} = \left(\frac{\mu dt}{82}\right) \ ,$$

whose integral gives

$$\frac{\mu t}{82} = \log\left(\frac{D}{\delta}\right) \ ,$$

[658] Let a sphere of radius r be uniformly electrified with a charge q. The symbol δ can represent either q or the surface charge density $q/(4\pi r^2)$. If δ_0 represents the value of δ at the initial time $t = 0$, the integration of this equation gives:

$$\delta = \delta_0 e^{-mt} \ . \tag{17.1}$$

[659] The modulus of a logarithm is the number by which a logarithm to one base is multiplied to give the corresponding logarithm to another base. The modulus for converting from a logarithm system with base a to a logarithm system with base b is the number $M = 1/\log_a b$. To obtain the logarithms of the numbers x to the base b, if the logarithms of these numbers are known to the base a, it is necessary to multiply the latter by the modulus: $\log_b x = M \log_a x$.

If we have the logarithm of a number N in base a and we want to obtain its logarithm in base b, we can use the relation

$$\log_a N = \log_a b \cdot \log_b N \ .$$

The factor $\log_a b$ is called the modulus of the logarithm system with base a with respect to the system with base b.

For conversion between natural and decimal logarithms we use

$$\log_{10} N = \log_{10} e \cdot \log_e N \qquad \text{or} \qquad \log N = 0.4343 \ln N \ .$$

That is,

$$\mu = \log e = \frac{1}{\ln 10} = \frac{1}{2.30258} = 0.4343.$$

D representing the initial density of the electric fluid of each ball and, therefore

$$\frac{2\mu t}{82} = \frac{\mu}{41} t = \log\left(\frac{D^2}{\delta^2}\right) \ ;$$

but the distance being constant, D^2 is proportional to the initial action and δ^2 is proportional to the action when the time $= t$; thus, using ordinary [logarithm] tables, since the modulus $\mu = 0.4343$, we will have

$$\frac{0.4343}{41} t = \log\left(\frac{D^2}{\delta^2}\right) \ .$$

If we seek, according to this formula, the value of δ in this first experiment, we will find that at the first trial $D^2 = 150$, and at the sixth trial $\delta^2 = 50$; thus,

$$\frac{0.4343}{41} t = \log\frac{150}{50} = \log 3 \ ,$$

and, consequently,

$$t = \left(\frac{41 \log 3}{0.4343}\right) = 45 \text{ minutes} \ .$$

The first test started at 6h 32m 30s; the sixth test only happened at 7h 17m; which gives 44m 30s, instead of 45 minutes found by experiment.

17.1.2 Second Remark

The ratio given in the seventh column of the Table represents exactly the portion of the force lost in one minute by the electrified body to the total force, but this ratio is double that of the loss of the density of each body to the total density; it is easy to convince oneself of this [fact] by the following considerations.

We have seen, in our first two Memoirs, that when two electrified globes acted one on the other, their reciprocal action was in the compound ratio of the electric densities and the inverse of the square of the distances of these two globes. Thus, since in our experiments the two balls are equal[660] and they have at the first moment received an equal dose of electricity, their reciprocal action, by naming δ the electric density and a the distance of the two balls, will be proportional to (δ^2/a^2); therefore the variation of this action during the time dt will be proportional to[661]

[660] The two balls have the same size, the same shape and are made of the same material.

[661] In the original we have here, [Coulomb, 1788e, p. 620]:

$$\text{“}\left(\frac{2\delta d\delta}{a^2} + d\delta^2\right) \ ;$$

ainsi le rapport de cette variation d'action, à l'action, sera en négligeant $(d\delta)^2$ égal à $\left(\frac{2d\delta}{\delta}\right)$.”

Potier, [Potier, 1884, p. 155], replaced this equation and this sentence with:

$$\text{“}\frac{2\delta d\delta}{a^2} \ ,$$

ainsi le rapport de cette variation d'action à l'action sera égal à $(2d\delta/\delta)$.”

$$\left(\frac{2\delta d\delta}{a^2} + (d\delta)^2\right) \ ;$$

so the ratio of this variation of action, to the action, will be, by neglecting $(d\delta)^2$, equal to $\left(\frac{2d\delta}{\delta}\right)$. But $(d\delta/\delta)$ is the ratio of the loss of the density of each ball to its density and, therefore, it has as a measure half the ratio given by the loss of the action to the action given in our experiments; thus, on June 28, our Table giving an average of 1/41 for the ratio of the electric force lost in one minute to the total force, it follows that, on that same day, the electric density of the balls decreased by 1/82 part per minute.[662]

By a series of experiments of the same kind, I also found that, although the balls had very different sizes, that the mass of electricity[663] and the electric density of each ball were very different, the ratio of the force lost in one minute to the total force always remained a constant amount; so, for example, that, although on June 28 I presented to the needle ball a ball double in size, and that I gave to this ball an electric density greater or less than that of the needle, the loss of electric force loss per minute was always 1/41 part of the total force. If we pay attention, we will see that, if in a given time the density decreases in proportion to its intensity, the result which the experiment gives is a necessary consequence of the theory; because the action of the two balls whose size and density are different being represented by $m(D\delta/a^2)$ where m is a constant coefficient depending on the surface of the balls, where D and δ represent the densities and a the distance, the variation of the repulsive force divided by this force will have as a measure

$$\left(\frac{dD}{D}\right) + \left(\frac{d\delta}{\delta}\right) \ ,$$

quantity which will always be a constant quantity, whatever the value of δ, D and m, provided that, for the same time interval dt,

$$\frac{dD}{D} = \frac{d\delta}{\delta} \ = \ \text{a constant quantity.}$$

But a remark provided by experiment, and which seems to me to deserve the greatest attention, is that, whatever shape an electrified body has and whatever its size, the decrease in the electric density, relatively to this density, has in any case for measure nearly a constant quantity when the air is dry and the degree of electricity is not significant.[664] I made this experiment with a globe 1 foot in diameter [32.5 cm], with cylinders of all sizes and lengths; I have substituted instead of balls, in my electric balance, disks of paper or metal; I even,

[662] According to footnote 658 on page 288, see Equation (17.1), if an electrified sphere has a charge q_0 at the initial time $t = 0$, its charge at time t is given by:

$$q = q_0 e^{-mt} \ . \tag{17.2}$$

In this specific case Coulomb found $m = 1/82$ parts per minute. Therefore, after a time interval of 1 minute has elapsed, the charge on the sphere will be:

$$q = q_0 e^{-1/82} = q_0 e^{-0.0122} = 0.988 q_0 \ . \tag{17.3}$$

This number means that in 1 minute this sphere will have lost 1.2% of its initial charge.

[663] In the original: *masse d'électricité*. That is, the total value of the electric charge on each ball.

[664] That is, when the body is not highly electrified.

one very dry day, armed one of the balls with a small copper wire 10 lines long [2.26 cm] and 1/4 line in diameter [0.0565 cm], and, observing the decrease of the electricity, I found, the day when I made this experiment, that the electric density decreased in all these bodies, whatever shapes they had, of 1/100 part per minute: but it is necessary to warn [the reader] that the bodies of different shapes give this equality of decrease in the electric density only when this density is diminished to a certain point; that in all angular shapes, when a very strong electricity is communicated to them, they rapidly lose a portion of this electricity, according to laws which we will determine when addressing the electricity of the points;[665] but when the electricity is diminished to a certain point, then, whatever be the electric density, its ratio to the decrease during the time interval dt will be a constant quantity.

A second observation that experiment has led me to make is that the nature of the body has no influence on the law of the decrease of electricity; thus, on June 28, when we see by our Table that the electricity decreased by 1/82 per minute for pith balls, it decreased by the same quantity for a copper ball and, what will appear more extraordinary, for a ball of insulating nature formed with Spanish wax and which had been charged with electricity, by making it touch a strongly electrified body. We shall have occasion later to return to all these results, when we shall have determined by experiment and calculation the laws of the other electrical phenomena.

17.1.3 Third Remark

If we now want to find, according to the Table which represents the decrease of electricity in one minute, the correspondence between the more or less humid state of the air and this decrease of electricity, we will form the following small Table:[666,667]

	Hygrometer	Amount of water a cubic foot of air holds in solution	Electricity lost every minute
On May 29	69	6.197 grains	1/60
On May 28	75	7.205 grains	1/41
On July 2	80	8.045 grains	1/29
On June 22	87	9.221 grains	1/14

In this Table, the first column marks the day on which the experiment was made; the second, the state of Mr. de Saussure's hygrometer;[668] the third, the quantity of water which the air holds in solution per cubic foot when the thermometer is between 15 and 16 degrees, evaluated according to a small table of Chapter X, page 173, of the *Hygrometry* of Mr. de Saussure,[669,670] which expresses for all the degrees of the thermometer the quantity of water

[665] That is, point discharge or electric discharge by sharp edges.

[666] [Note by Potier] That is, 9.68 g, 11.28 g, 12.42 g and 14.26 g per cubic meter.

[667] In the first case, for example, we have a volumetric density of mass given by 6.197 grains per cubic foot, that is, 6.197×0.05311 grams per $(0.3248\ m)^3$, that is, 9.6 g/m^3, as calculated by Potier.

[668] Horace-Bénédict de Saussure (1740-1799) was a Genevan geologist, meteorologist and physicist. He devised a hair hygrometer and used it for a series of investigations on atmospheric humidity and evaporation.

[669] [de Saussure, 1783] with German translation in [de Saussure, 1784].

[670] By a lapse Coulomb wrote 7.295 grains in the third line of this Table corresponding to the 28th of May. I corrected this number to the value given by de Saussure, 7.205 grains when the hygrometer reads 75°, [de Saussure, 1783, 172].

which the air holds in solution relatively to the degree marked by the hygrometer of this author.

If, according to this Table, we seek by calculation to determine a law relating the decrease of electricity and the quantity of water contained in 1 cubic foot of air, when the thermometer is between 15 and 16 degrees, [the] level found at the time of the four experiments, by naming m the power which expresses this ratio and by comparing the first experiment with the three others, we will have:[671]

First and second experiment:

$$\frac{60}{41} = \left(\frac{7.197}{6.180}\right)^m , \qquad \text{hence} \qquad m = 2.76 ;$$

First and third experiment:

$$\frac{60}{29} = \left(\frac{8.045}{6.180}\right)^m , \qquad \text{hence} \qquad m = 2.76 ;$$

First and fourth experiment:

$$\frac{60}{14} = \left(\frac{9.221}{6.180}\right)^m , \qquad \text{hence} \qquad m = 3.04 ,$$

and the average quantity gives $m = 3.04$.

So that it would appear that the decrease of the force or, what amounts to the same thing, of the electric density, is proportional to the cube of the weight of the water contained in 1 volume of air.

But this result, depending on several elements, which are perhaps not yet determined with sufficient certainty, needs to be confirmed by more direct research. It was with this in view that I had imagined, to complete my work, enclosing electrified bodies in different kinds of air, giving this air different degrees of density and humidity, then searching in each state of these airs the law of the decrease of electricity; but I soon realized that this operation required a lot of time, patience and instruments that I did not have, or which do not even exist yet, to measure with precision the degree of purity of each air and its degree of humidity: I have been obliged, with regret, to give up, at least for the moment, a work to which I wish to be able to return later.

[671] The numerical values Coulomb is using here for the amount of water that 1 cubic foot of air has in dissolution do not match the values in the previous Table, [Coulomb, 1890a, p. 86, Note 13]. Using these values we would obtain in the first and second experiment:

$$\frac{60}{41} = \left(\frac{7.205}{6.197}\right)^m , \qquad \text{hence} \qquad m = \frac{\log(60/41)}{\log(7.205/6.197)} = 2.53 ;$$

in the first and third experiment:

$$\frac{60}{29} = \left(\frac{8.045}{6.197}\right)^m , \qquad \text{hence} \qquad m = \frac{\log(60/29)}{\log(8.045/6.197)} = 2.79 ;$$

in the first and fourth experiment:

$$\frac{60}{14} = \left(\frac{9.221}{6.197}\right)^m , \qquad \text{hence} \qquad m = \frac{\log(60/14)}{\log(9.221/6.197)} = 3.66 .$$

The average of these 3 values gives: 2.99. This value is close to the value that Coulomb will calculate, namely 3.04.

17.1.4 Fourth Remark

In the different essays which form the general Table of our experiments, I made sure that the electricity was lost only by contact with the air and not along the insulating bodies which formed the supports, by the following method.

The balls contained in the electric balance being supported by a single thread of silk coated with Spanish wax, terminated by a shellac thread 18 lines long (4.06 cm), I sought the quantity of electricity which was lost in one minute and which is in the Table of experiments; I then caused the ball to be touched by four threads absolutely similar to that which served as support, and I determined in this state the decrease of the electricity in a minute which I found the same as if there had been only one support: it is clear that, having in this experiment four supports instead of only one, if a significant part of the electricity had been lost by the supports, the decrease would have been appreciably greater when the ball was touched by four threads coated with Spanish wax than when supported by one; and, since experiment has proved the contrary, it follows that the electricity was lost only by contact with the air, and not along the insulating bodies which formed the supports.

17.1.5 Fifth Remark

Although the hygrometer of M. de Saussure, which served for the comparison of our experiments, remains at the same degree, as the degree of heat indicated by the thermometer increases, nonetheless the quantity of water which a fixed volume of air holds in solution increases with this heat. But, as it appears that the more or less rapid decrease of electricity depends on the quantity of water or the number of conducting parts which are found in the same volume of air, it must result that, for the same hygrometric degree, electricity must be lost more quickly on hot days than on cold days. This is indeed what experiment always confirms; but it remains to be investigated whether at different degrees of heat the decrease of electricity depends solely on the quantity of water held in solution in a fixed volume of air.

Here we lack experiments: we find, in truth, in the excellent essay on hygrometry by Mr. de Saussure, *Chapter X, page 181*, a Table which represents the correspondence of the degrees of his hygrometer with the quantity of water that a cubic foot of air holds in solution at each degree of the thermometer, but Mr. de Saussure states that he does not answer for this Table, which he published only to present a model of the results of the experiments he intends to do next. Thus, all the results that we might draw by comparing, according to this Table, the electrical loss with the quantity of water held in solution in a cubic foot of air[672] at 1 degree of heat and of the observed hygrometer, would be only hypothetical. We can only say, in general, it appears that in calculating from this Table the quantity of water a cubic foot of air holds in solution, that as the degree of heat increases, the electricity is not lost as quickly as it ought to be lost; that is to say, by admitting as true the Table of Mr. de Saussure, a cubic foot of air holding, for example, 6 grains of water in solution is more insulating or less conductive of electricity the higher the heat.

[672] For a lapse, the following appears in the original: *dans un pied cube d'eau*, [Coulomb, 1788e, p. 626] and [Potier, 1884, p. 160].

17.1.6 Sixth Remark

Before finishing this First Part of my Memoir, I must still warn that, although the thermometer, the hygrometer and even the barometer measure the same degrees on different days, the decrease of electricity is not always the same: we cannot, it seems to me, explain these variations by any other cause than by the composition of the air composed of more or less different insulating elements whose density, the proportions of which vary nearly continuously, and which have different degrees of affinity with the aqueous vapors. The only observation which appears to me to suffice, in general, is that when the weather suddenly changes, and the hygrometer varies appreciably within a few hours from dampness to dryness, the loss of electricity relative to its density remains for some time greater than it should be based on that degree of dryness indicated by the hygrometer, and *vice versa*, when the hygrometer suddenly changes from dry to wet. Thus, for example, if in twelve or fifteen hours the hygrometer passes from wet to dry by 8 or 10 degrees and then settles at this degree of dryness for several days, it will often be observed that, if the electric density decreases by 1/50 per minute the first day after this step change in the hygrometer, several days later, though the dryness indicated by the hygrometer remains the same, the electric density decreases by only 1/100 parts per minute. Ought not the cause of this phenomenon be that the aqueous vapors, after having settled for a certain time in the air, contract among themselves a greater and greater adherence and the hair of the hygrometer only attracts the aqueous parts which are still free and which have a lower degree of adhesion with air than the former; from which it would result that, after sudden variations, the hygrometer would measure only the quantity of the free *aqueous* parts in the air and not the absolute quantity of these parts? What would appear to support this conjecture is that the state of electrical losses almost always settles after a few hours, relative to the hygrometer, when the rapid variation of dryness or humidity takes place with a violent wind and that it is only with calm weather that one sometimes experiences the opposite. It could be, however, that this phenomenon was produced solely by the humidity or the dryness of the bodies which are near the needle.

This remark, as well as the third, depending, as we have said, on several hygrometric elements which are still uncertain, the results are only hypothetical and they should not be confused with the main points regarding this Memoir which have as a basis a suite of experimental results.

17.2 Second Part: Of the Amount of Electricity that is Lost Along Imperfect Insulating Supports

We have seen, in the First Part of this Memoir, that when electricity is lost through contact with air, the momentary decrease of electricity was exactly proportional to the electric density of the electrified body. We recall that in order for us to conduct the individual experiments so as to lead to this result, we had to try to isolate the electrified body on a support as insulating as possible.

To follow the same method it would be necessary, in the current research, to support the body by insulators whose insulating capacity was so poor[673] that the ratio of the loss of

[673] In the original: *soutenir les corps par des isoloirs dont l'idio-électricité fût tellement imparfaite.* See footnote 522 on page 229.

electricity along the supports to the quantity of electricity that the body lost by contact with the air was very large. But we feel that the greater this ratio, the more rapidly the electricity of the electrified body will be lost. And since, in the conduct of the experiments, from the moment that, in our electric balance, the ball supported by the needle is electrified, the needle oscillates for a few minutes, and it also oscillates every time we touch at the micrometer, to increase or decrease the torsion of the wire of suspension, we see that if the electricity were lost very quickly, with each observation the electricity would be found almost entirely annihilated before the needle stopped and its position determined in a precise manner:[674] this practical inconvenience therefore obliged us to use supports which had enough insulating forces to be able, without electrifying the balls each time, to make several consecutive observations; it is then easy, by calculation, to determine, in these experiments, the part of the electricity lost by contact with the air, and that lost along the support.

The second Table was formed on the same model as the first, as the titles indicate: but the ball introduced into the hole of the balance, and which is intended to repel the ball from the needle, instead of being insulated, as in the experiments of this First Part, by a small cylinder of shellac 15 to 18 lines in length, is supported by a thread of silk of a single strand [brin], such as it comes out of the cocoon; this thread is 15 inches in length [40.6 cm].

Second Table for determining the loss of electricity along imperfect insulating supports

	Time of experiment	Distance of the balls	Torsion of the micrometer	Time elapsed between two consecutive observations	Electric force lost between two observations	Average force between two observations	Ratio of the electric force lost during 1 minute, to that remaining in the body
First experiment on May 28.							
1st test	10h 0m 0s	30	150				
				2m 30s	30	165	1/14
2nd test	10h 2m 30s	30	120				
				5m 30s	40	130	1/18
3rd test	10h 8m 0s	30	80				
				5m	20	100	1/25
4th test	10h 13m 0s	30	60				
				16m 30s	40	70	1/29
5th test	10h 29m 30s	30	20				
				21m	20	40	1/42
6th test	10h 50m 30s	30	0				
				16m 30s	10	25	1/41
7th test	11h 7m 0s	30	10				
Second experiment on May 29.							
1st test	7h 34m 0s	30	150				
				2m 40s	20	170	1/23
2nd test	7h 36m 40s	30	130				
				4m 50s	20	150	1/29
3rd test	7h 41m 30s	30	110				
				6m 50s	20	130	1/44
4th test	7h 48m 20s	30	90				
				7m 25s	20	110	1/43
5th test	7h 55m 45s	30	70				
				11m 45s	20	90	1/53
6th test	8h 7m 30s	30	50				
				17m 30s	20	70	1/61
7th test	8h 25m 0s	30	30				
				17m 30s	15	50	1/58
8th test	8h 42m 30s	30	15				
				22m 30s	14	38	1/56
9th test	9h 5m 0s	30	1				

[674] If the electricity is lost very quickly, the needle's position can not be determined in a precise manner. Hence sufficient insulating capacity is needed to make a reading.

The two experiments of this second Table, were made like that of the first, on May 28 and 29. The first Table determines the quantity of electricity which the contact with the air caused to be lost: thus, by comparing the result of this first Table with that of the second, it will be easy to determine the quantity of electricity lost at each instant along the supports.

But a very important remark prompted by this second Table is that the decrease of electricity, when the electric density is initially large,[675] is much more rapid than it would be if produced solely by contact with the air, becomes, when the electric density of the ball supported by the silk thread is reduced to a certain degree (as in both experiments of the second Table) precisely the same as when the insulating capacity of the insulator is perfect — or better said, when the loss of electricity is entirely due to contact with the air as in the first Table.

It certainly results from this observation, that our silk thread, fifteen inches in length, insulates perfectly, when the reciprocal action of the two balls is measured in the first experiment of our second Table, by a force of torsion of 40 degrees and below, since in this case the electrical loss is only 1/42 per minute, the same which had been found for the same day in the first Table, and which was, as is proved in the First Part of this Memoir, solely due to contact with air. It also results from this same observation, that in the second experiment of our second Table, the silk thread fifteen inches in length insulates perfectly, when the repulsive action of the two balls was 70 degrees and below, since then the loss of the electric action was only 1/60, as we found it the same day in the first Table. Since the repulsive forces are measured for a constant distance, by the product of the densities of the two equal balls,[676] now we will seek to uncover the relationship between the initial density, and the degrees of density of the ball supported by the silk thread, when that silk thread begins to insulate that ball perfectly.

17.2.1 Determination of the Electric Density of the Ball Supported by the Silk Thread, When This Thread Begins to Insulate Perfectly

An application of the calculation developed in the First Part of this Memoir and compared with the result of the first experiment of our second Table will suffice to show the method that we must follow in this research. In the first experiment of our second Table which began at 10 o'clock, we gave an equal quantity of electric fluid to the two balls, since these balls are equal and care was taken to make them touch after they had been electrified. That day, the ball supported by the needle being insulated by means of shellac, lost 1/82 part of its electric fluid per minute, and lost this fluid only by contact with the air. The ball supported by the silk thread lost its electricity by contact with the air and along its imperfect insulating support: it is only at about ten hours and forty minutes[677] that the thread of silk began to perfectly insulate this second ball, and then the repulsive action of the two balls

[675] That is, when the surface charge density on the electrified ball is large.

[676] That is, as the force is proportional to the product of the charges of the two balls of the same size.

[677] In the original it is written ten hours and forty minutes, [Coulomb, 1788e, p. 630]. Potier put here $10^h\ 50^m$, [Potier, 1884, p. 164]. I kept the time written by Coulomb as he obtained this value by averaging the time of the fifth and sixth tests in the first experiment of the second Table, that is, (10h 29m 30s + 10h 50m 30s)/2 = 10h 40m. It was around this time that the ratio between the electrical force lost per minute and the initial force reached the value of 1/42 part per minute, that is, the same value as that found in the first Table in which the loss of electricity occurred only through the air.

measured 40 degrees, but at 10^h, at the beginning of the experiment, the repulsive action of the two charged balls, each charged with an equal quantity of electric fluid, had for measure 180 degrees [30 + 150 degrees], as indicated by the first test of this experiment: thus the electric density of each ball was, at 10^h, proportional to $\sqrt{180}$, since the action, for a constant distance, is always proportional to the product of the densities and the densities, at the first attempt, were equal. But we saw in the First Part of this Memoir that the decrease of electricity, in contact with air, was expressed by the formula $\frac{d\delta}{\delta} = -mdt$,[678] where m, in our first experiment, $= (1/82)$; this integrated formula gives

$$\log\left(\frac{D}{\delta}\right) = \frac{0.4343}{82}t \ ,$$

where D is the initial density of the ball, δ its density after a time t, and 0.4343 the module of the decimal logarithmic system of the ordinary tables:[679] thus we will have

$$\log\delta = \log D - \frac{0.4343}{82}t \ ;$$

therefore, if we seek what has become of the density D, after 40 minutes,[680] when the silk thread begins to insulate perfectly, we find, for the ball of the needle supported by shellac, and perfectly insulated throughout the experiment, assuming $D = \sqrt{180}$,[681]

$$\log\delta = 1.1276 - 0.2648 = 0.8628 \ .$$

Thus δ or the density of the ball of the needle, at 10h 40m,[682] having been measured at the beginning of the experiment as $\sqrt{180} = 13.4$, was measured 40 minutes later, by the number 7.3;[683] but since the action of the two balls is always proportional to the product of the density, if we assume Z is the density of the ball supported by the silk thread, when this thread insulates perfectly, or when the action of the two balls measures 40 degrees; we will have

$$7.3 \times z = 40^\circ \qquad \text{or} \qquad z = 5.5 \ ;$$

[678] For a lapse in the original text this formula appeared here without the negative sign.

[679] That is, log to the base 10 of e.

[680] I put 40 minutes as in the original text, while Potier wrote 50 minutes here, see footnote 677.

[681] That is, $D = \sqrt{180} = 13.416$. Therefore, $\log D = 1.1276$. At $t = 40$ minutes, we have $0.4343 \cdot 40/82 = 0.2119$. Coulomb's calculation would be:

$$\log\delta = 1.1276 - 0.2119 = 0.9157 \ .$$

This gives $\delta = 10^{0.9157} = 8.2$.

For some lapse, although he wrote a time of 40 minutes, he did the following calculations with $t = 50$ minutes: $0.4343 \cdot 50/82 = 0.2648$. With that figure he found:

$$\log\delta = 1.1276 - 0.2648 = 0.8628 \ .$$

This gives $\delta = 10^{0.8628} = 7.3$.

Perhaps for this reason Potier has substituted in Coulomb's text several times the expression "forty minutes" for "fifty minutes".

[682] Again I followed here Coulomb's original text, 10h 40m, and not what was written by Potier, 10h 50m, see footnote 677.

[683] As $\log\delta = 0.8628$, it follows that $\delta = 10^{0.8628} = 7.3$.

from which we conclude that the electric density of the ball supported by the silk thread 10 inches in length has the value 5.5 when this thread begins to insulate perfectly, the two balls being 30 degrees apart one from the other. According to this calculation, by comparing several experiments, I found that a small cylinder of shellac, 18 lines in length [4.1 cm], ceased to insulate perfectly, only when the ball was charged with an electric density approximately three times that of our silk thread; that is to say, taking the number 5.5 for the electric density of the ball, supported by our silk thread 15 inches in length, when it begins to insulate perfectly, it would be necessary to almost triple this density in order to have a small cylinder of 18 lines begin to insulate perfectly — and it ceases to insulate when the density is greater. By this theory, it will be easy to determine by experiment the degree of insulating capacity desired of the different bodies which are commonly used to insulate electrified bodies. The attempts that I have made on this subject are not numerous enough to publish the results yet: we feel moreover that these results vary for the same body with the heat and the humidity of the air, and that each day gives a different ratio.

After having found that in imperfect insulating supports, there was always a certain degree of electric density below which these supports insulate perfectly, I sought, by the methods which I have just explained, the relationship between this electric density and the length of the supports; and experiment has taught me that the degree of electric density when a silk [thread], a hair, or any very fine cylindrical body whose insulating capability was imperfect, begins to insulate [perfectly], was for the same state of the air, proportional to the [square] root of the length; so that, for example, if a silk thread 1 foot in length begins to insulate the body perfectly when the density is D, a thread 4 feet in length will begin to insulate it when its density is $2D$.

What experiment teaches us here is found to be in conformity with the theory, supposing, as we proved in our first two Memoirs, that the action of the electric fluid follows the inverse ratio of the square of the distances, and that the imperfection in the insulation of the bodies depends on the insulating distance[684] between the conducting molecules which enter into the composition of the imperfect, insulating support, or which are distributed along its surface;[685] that, consequently, in order for the electric fluid to pass from one conducting molecule to another, it must cross a small insulating space, of size depending upon the nature of the body, that offers a constant resistance for the given body, because these conducting molecules are distributed uniformly, at the same distance one from another. These suppositions admitted, to apply the theory, it will be observed that, in a very fine conducting filament, the electric fluid would spread uniformly along all its length; if this filament has a certain degree of insulating capability, and the fluid is distributed according to any law, the action which each point would experience would only depend on the electric density of the molecule in contact with this point, and the action of the rest of the filament can be regarded as null. Here is the proof of these two propositions.

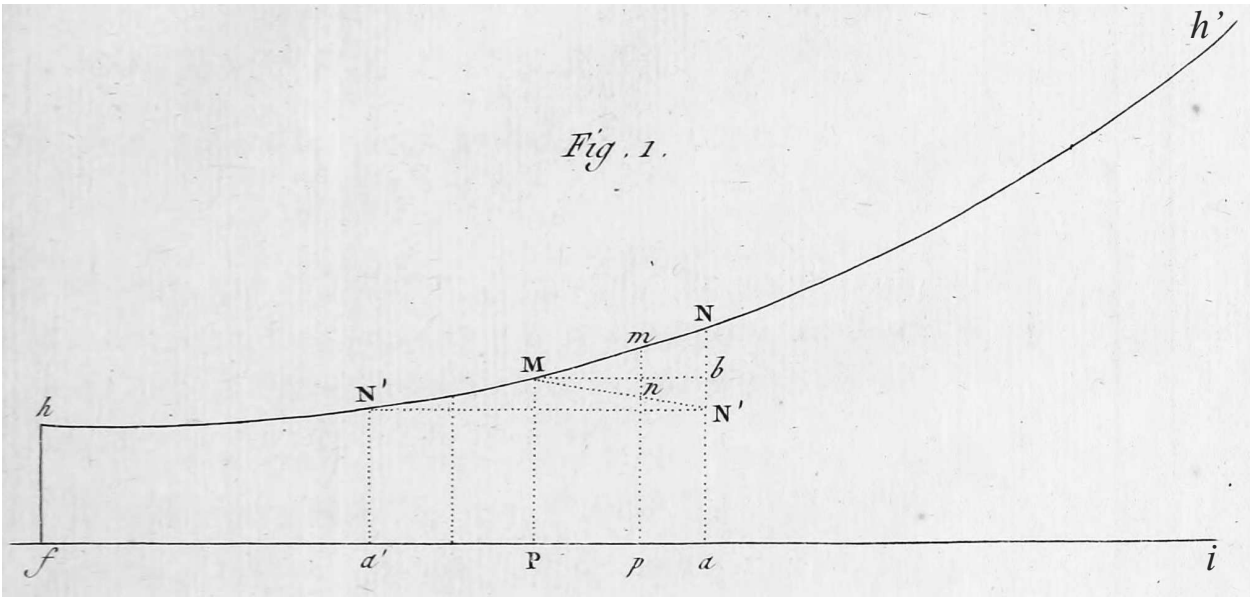
In Figure 1,[686] fi represents a filament whose parts act on each other, according to the inverse ratio of the square of the distances, the curve hMh' represents the electric density of each point of the filament; on the length of this thread, I take two portions Pa and Pa',

[684] In the original: *distance idio-électrique*. See footnote 650 on page 284.

[685] Coulomb is assuming here that there are conductive particles, "molecules", inside or on the surface of imperfect insulating supports.

[686] I have included in this Figure 1 the letter i at the right end of the lower line and the letter h' at the right end of the upper curved line. The letters on the upper curve were then, from left to right: $hN'MmNh'$, on the lower line: $fa'Ppai$. The letters on the right line, from top to bottom: $NbN'a$. The point n lies at the intersection of the horizontal line Mb and the vertical line mp.

equal, finite, but small enough that, in practice, MNb can be regarded as a triangle.



Let $Mn = Pp = x$, $bN/MN = a$, nm will be $= ax$,[687] and the action that point M whose density is D will experience, on the part of the small element dx placed at p, will be

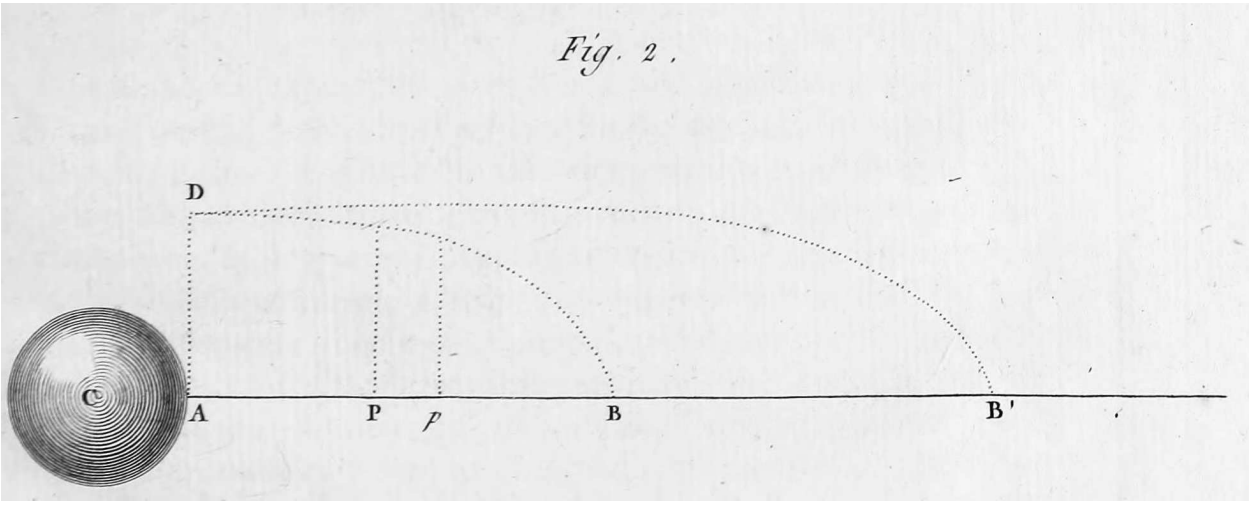
$$\frac{Dax \cdot dx}{x^2} = Da\left(\frac{dx}{x}\right) \ ;$$

integrating this quantity and supposing that it vanishes when $x = A$, we will have, for the action of the whole part Pp, [the value] $Da\log(x/A)$, quantity which will be finite as long as A is a finite quantity, but which will become infinite when $A = 0$: from which it follows that the action experienced by point P depends solely on the increment of the density in the element that touches point P and that the density of the rest of the line does not influence it; from which also results that if this action depends on a fluid which can move freely along the filament, or if this filament is a perfect conductor, the fluid which acts in inverse proportion to the square of the distances will spread uniformly all along the length of this filament: we will determine in the continuation the electric density at the end of this filament.

Let us apply the preceding result to the present question: the globe in C (Figure 2) is supported by means of the silk thread AB, whose insulating capacity is imperfect, that is to say, each element of which opposes a constant resistance A to the flow of this fluid.[688]

[687] These relationships $Mn = Pp = x$ and $bN/MN = a$ leading to the equation $nm = ax$ make no sense to each other. König, [Coulomb, 1890c, p. 86, Note 14], suggested replacing them with: $Mn = Pp = x$ and $bN/bM = a$ leading to the equation $nm = ax$. Even so, the next calculation presented by Coulomb is still confusing.

[688] That is, the silk thread being considered here is not a perfect insulator.



Let A' be the electric mass of the globe,[689] united at its center; let δ be the electric density at p, we will have for the total action with which point p is repelled by the electric fluid [the following value:][690]

$$\frac{A'\delta}{(R+x)^2} - \frac{\delta d\delta}{dx} ,$$

quantity equal to the insulating resistance B[691] of the filament which we have seen should be a constant amount. We take $d\delta$ negatively because δ decreases as x increases; but we will prove, in the Memoir which will follow this one, that the action of the small electrified globe C on point P is incomparably smaller than the action of the element dx multiplied by the increment of δ; thus we can, without appreciable error, neglect the first term $A'\delta/(R+x)^2$, and the equation will be reduced to

$$-\frac{\delta d\delta}{dx} = B ,$$

which, integrated, gives

$$K - \frac{\delta^2}{2} = Bx .$$

But, when $x = 0$, [the magnitude] δ becomes equal to the density D of the globe: thus we will have the general equation

$$D^2 - \delta^2 = 2Bx ;$$

and if, in this equation, we make $\delta = 0$, it will give the length x where the filament begins to insulate perfectly, and we will then have[692]

[689] That is, A' is the total electric charge of this globe.

[690] Coulomb did not specify in the next equation what the quantities R and x are. One possibility is that R is the radius of the globe, while x is the distance Ap.

[691] In the original: *résistance idio-électrique B*.

[692] The next equation should have been written as:

$$x = \frac{D^2}{2B} .$$

This lapse was not noticed by Coulomb and continues in the next equations.

$$x = \frac{D^2}{B} \ ;$$

thus the lengths of different threads of silk or of any imperfect insulating supports are between them like the square of the densities, when they begin to insulate perfectly, as we had found it by experiment; it is easy to see, from the formula, that the curve which represents in our figure the density of electricity for each point of the silk thread is a parabola whose axis is BA, whose vertex is at B, point where the density is zero and whose concavity is turned towards the side of the ball; because, since we have $(D^2 - \delta^2) = Bx$ and $AB = (D^2/B)$, we will have

$$Bp = \left(\frac{D^2}{B} - x\right) = z \qquad \text{or} \qquad x = \left(\frac{D^2}{B} - z\right) \ ;$$

substituting this value of x, in our equation, we will have

$$\delta^2 = Bz \ ,$$

the equation of a parabola, whose vertex is at B, the axis Bp, and whose parameter is B, quantity which increases with the insulating capacity of the support.[693]

Reflecting on the theory just presented, it is easy to see that the foregoing formula determines the disposition of the electric fluid along the imperfect insulating support, assuming we have communicated, as we have in our experiments, a certain dose of electric fluid to the globe supported by the silk; because then this fluid communicating itself step by step along the insulating support will spread up to point B, so that the repulsion of the fluid is in all the points exactly in balance with the *maximum* resistance that the coercive force of the insulating support can oppose to the flow of this fluid. But it should be noted that, as this *maximum* of resistance is a coercive and non-active force that can be compared to the resistance of a friction, any repulsive action of the electric fluid less than the *maximum* of this resistance will not disturb the state of stability of this fluid spread according to any law whatsoever along the support; so that, if the line AD which represents in the attached Figure the density of the globe remains constant, that we prolong by any quantity BB' the axis AB, and that we describe any density curve DB', provided that all points $\frac{\delta d\delta}{dx}$ be smaller than B, the electric fluid spread along the line AB' will keep its state of stability without flowing from one point to another; from which one concludes that there is always an infinity of density curves DB' which also satisfy the state of stability of the electric fluid spread along an imperfect insulating support and that the general search of the disposition of the electric fluid in an imperfect insulating body is an indeterminate problem which, to become determinate, needs to be subjected to some particular conditions. Thus, in the curve ADB that we found, in the previous Section, represented by the formula $(D^2 - \delta^2) = Bx$, we had the condition that the *maximum* of the insulating resistance was in all points equal to the electric repulsion; this curve is moreover the particular case of the general indeterminate problem where the axis AB is a minimum. Indeed, since in all the other density curves it is necessary that $\frac{\delta d\delta}{dx}$ is smaller than B, if in the curve DB we varied a single element, so that the state of stability was not disturbed by leaving $d\delta$ constant, it would necessarily be

[693] That is, the more insulating the support, the greater the value of B.

necessary, so that $\frac{\delta d\delta}{dx}$ was smaller than B, to increase the amount dx and lengthen the axis of the curve.

It follows again from the theory which we have just explained that, in all the conducting bodies where the electric fluid spreads freely, the determination of the density of the electric fluid, for any point whatever, is a determined problem; but that, for imperfect insulating bodies, the problem is indeterminate, one of its limits being however fixed by the state of the electric fluid when it is disposed in the imperfect insulating body, so that, at all points, the action of this fluid is exactly in balance with the *maximum* resistance which the insulating coercive force opposes,[694] to prevent the fluid from flowing from one point to another.

It is needless to warn [the reader] that, according to the theory and the experiments which precede, it is necessary in several cases to take many precautions when we want to obtain the electric force of a small body insulated by an imperfect insulating support, and that it often happens after several experiments, especially when the first have been made with a significant degree of electric density, the insulating support is charged with a certain quantity of electricity, of which it is stripped with difficulty, which then has a significant influence on the results; that with each experiment, it is necessary at the same time that one strips of its electricity the body carried on the support, to strip of it, as much as it is possible, the insulating support itself; that it is necessary to change support at each experiment, [especially] when the electric density which one communicates [to the body] is a little strong; that finally it must always *be sure* that the support has a force of insulating resistance large enough so that, in all the experiments, the quantity of electricity with which it will be charged is much smaller than that of the conductor body whose action we want to determine.

It is easy to see that the preceding theory may be applicable to magnetism; that in a steel needle, for example, the disposition of the magnetic fluid, for all states of stability, is an indeterminate problem, which only becomes determined by the conditions to be fulfilled. Thus, for example, if one asks the best way to magnetize a needle of inclination or declination, the problem to be solved consists in giving to the magnetic fluid of this needle, among all the provisions of which it is capable without disturbing its state of stability, that where the *moment*[695] of the magnetic directing force of the globe of the Earth on this needle is a *maximum*.[696,697]

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[694] In the original: *que la force coërcitive idio-électrique oppose*.

[695] In the original: *momentum*.

[696] [Note by Potier] Coulomb's conclusions [related to charge leakage] do not seem justified by more recent experiments; it would result from these that the loss [of electricity of charged bodies] due to the atmosphere itself is very low, whatever its degree of humidity.

By remaining in the general conditions where Coulomb placed himself, with weak charges, the law stated by him can be considered as evident; but the coefficient of loss [of electricity by the charged body] varies with the shape of the body studied and its position in relation to neighboring bodies. It would therefore be wrong to apply to a body shielded from all external influence the coefficient measured on the same day in the balance.

It is also evident that, in Coulomb's method, the variable way with the hygrometric state in which the induced charges on the cage [that is, on the glass container around his torsion balance,] are modified with time plays an important role and that this cause remains, independently of the supports, in the experiments where Coulomb believed he had eliminated everything that was not loss by air alone.

[697] Potier did not indicate what these more recent experiments are. Probably he was referring to the experiments of Biot, Gaugin, Matteucci, Riess and Warburg as quoted by Mascart in his book *Treatise on Static Electricity*, in the third Chapter on the loss of electricity by a charged body, [Mascart, 1876, Chapter 3: Déperdition de l'électricité]. In Section 19.2 I discuss the influence of air humidity on the loss of electricity from a charged body.

Chapter 18

Essay on Coulomb's Analysis of Losses Along an Insulator

L. L. Bucciarelli

In the *Second Part: Of the Quantity of Electricity which is Lost Along Imperfect Insulating Supports* of his *Third Memoir on Electricity and Magnetism*, Coulomb constructs an analysis meant to determine the length of an insulator required to ensure it keeps loss of charge to a minimum. He has shown that for a given length of the insulating filament, there is a level of charge below which the filament (of silk, hair or any very fine cylindrical body), acts as a perfect insulator, at least with respect to losses due to conduction. (Losses to the surrounding air may still occur.) In a few obscure final paragraphs, he strives to develop relationships defining how the density of charge along the filament varies with distance and, in particular, the length required to ensure the filament functions as desired.

First he sets out to prove,[698]

> in a very fine conducting filament, the electric fluid would spread uniformly along all its length; if this filament has a certain degree of insulating capability, and the fluid is distributed according to any law, the action which each point would experience would only depend on the electric density of the molecule in contact with this point, and the action of the rest of the filament can be regarded as null.

Recall that Coulomb has posited in the opening paragraphs of the Section, that[699]

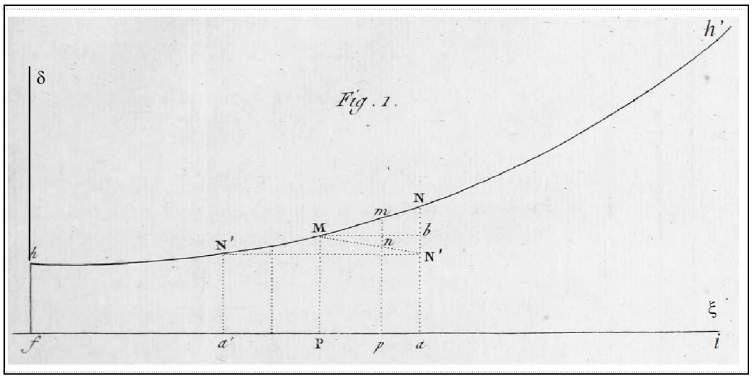
> ...the electricity dissipates along the surfaces of supports which have become imperfect insulators by contact with humid air; because in this second case, the aqueous parts contract so great a degree of adherence with the surface of these supports that this adherence is sometimes much greater than the repulsive action that the electrified body exerts on the aqueous molecule, to which it has transmitted a part of its electricity; from which it results, a result confirmed by experiment, that when the humid molecule nearest to the electrified body is charged with electricity, this electricity passes *in part* to the following molecule, without displacing this molecule, and thence from molecule to molecule until a certain distance of the body: thus the [electric] density of each molecule would diminish according to how far it would be

[698] [Coulomb, 1788e, p. 633] and page 298 of this English translation.
[699] [Coulomb, 1788e, p. 614] and page 284 of this English translation.

from the electrified body, because these aqueous molecules being separated by a small insulating interval, it requires a certain degree of force in order to push electricity from one molecule to another. (emphasis mine)

So along the insulating support of an electrified body, one has (infinitesimal) aqueous molecules adhering to the surface of the filament. The aqueous molecule closest to the electrified body is charged with electricity; if this molecule did not adhere to the filament, it would be repulsed, sent flying into the air. But the adhesion to the filament is greater than the repulsive force so instead the electricity, *in part*, passes to the following molecule without displacing the molecule but not without resistance due to small insulating intervals separating the molecules on the surface of the filament. Eventually, at some length along the imperfect conductor, the electric density of the molecule diminishes to zero; the loss of charge of the electrified body ceases, the filament functions as a perfect insulator.

Coulomb's Figure 1 shows an arbitrary distribution of charge density [*densité électrique*] along a filament. For our analysis that follows, we have labeled the axes, δ and ξ. Coulomb considers two finite, equal segments Pa and Pa' which are so small that one can consider, practically speaking, MNb as a triangle.[700]



Coulomb then writes

Let $Mn = Pp$, $\frac{bN}{MN} = a$, nm will be $a \cdot x$

Something is awry here: the second equality makes a the sine of the angle NMb whereas the last implies that a is the tangent of the angle NMB. But if angle NMb is small, as it appears in the figure, then the sine is approximately equal to the tangent. Let us assume this is the case, that Coulomb was thinking such. He goes on

and then the action that the point M, whose density is D, experiences due to the small element dx located at p will be

$$\frac{Dax \cdot dx}{x^2} = Da\left(\frac{dx}{x}\right) \ .$$

This is not obvious. We offer the following as conjecture of Coulomb's thinking: The "action" that point M experiences due to the element dx at p is given by the inverse square

[700] Note: The figure presents certain extraneous information: why does Coulomb speak of the equality of Pa' and Pa since this relationship enters not into his analysis that follows? Why are two points labeled N'? Neither point enters into his analysis.

law of repulsion of like charge. We interpret point M as a differential element like that at p. We let δ_P be the charge density at P so that the charge of point M, that is $D = \delta_P \cdot d\xi$.[701]

The charge at point p is then, accepting Coulomb's triangle representation,

$$(\delta_P + mn) \cdot d\xi = (\delta_P + ax) \cdot d\xi = D + ax \cdot d\xi \ .$$

The action is then[702]

$$\frac{D \cdot (D + ax \cdot dx)}{x^2} = \frac{D \cdot D}{x^2} + \frac{Dax \cdot dx}{x^2} \ .$$

To get rid of the first term in this last expression for "the action that the point M... experiences due to the small element dx located at p" we posit that it represents the interaction of any two points as a function of their separation when the charge is uniformly distributed along the insulator and the fluid able to flow freely along the "imperfect insulator"; it is the second term that represents the resistance to flow. We conclude the action of the small element dx at p on M is, as written by Coulomb

$$\frac{Dax \cdot dx}{x^2} = Da\left(\frac{dx}{x}\right) \ .$$

Coulomb continues

> integrating this quantity and supposing that it vanishes when $x = A$, we will have, for the action of the whole segment Pp (on point M),[703]
>
> $$Da \cdot \log_e\left(\frac{x}{A}\right) \ .$$

That is with,

$$Da \cdot \int \left(\frac{1}{x}\right) dx = Da \cdot \log_e x + C \ ,$$

then if this integral vanishes when $x = A$ and assuming x and C positive, then, $C = -\log_e A$ and so the action of the whole segment Pp (on point M) is as above as deduced by Coulomb.

Note: If we take the "action" of p on P as negative, as like charges repel so the action on P will be in the direction of negative x. Then we evaluate the integral within the limits x to A and obtain this same result,

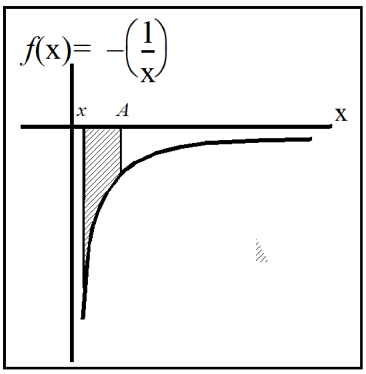
$$-Da\int_x^A \left(\frac{dx}{x}\right) = -Da \cdot [\log_e A - \log_e x] = +Da \cdot \log_e\left(\frac{x}{A}\right) \ .$$

He writes, having this expression for the quantity of action of the segment Pp upon M

[701] The product ax is a charge per unit length, which I take as a density; $axdx$ is then the charge of the element of length dx; D, which Coulomb calls a density, in my view must be a charge.

[702] Since $x = \xi - fP$, we can replace $d\xi$ with dx.

[703] Why *de l'action de toute la partie Pp?* Why not the whole segment Pd?



this quantity will be a finite quantity as long as A will be a finite quantity but will become infinite when $A = 0$; from which it results that the action which the point P experiences depends uniquely on the increment of the density in the segment which touches the point P and that the density of the rest of the line has no influence on it;

This is what he set out to prove. Now it is true that as A approaches zero (and so too x), this mathematical expression for the action of Pp on M grows infinitely large. What is not clear is what Coulomb means by "segment", its physical manifestation, for it too, if it is a length, must vanish when A (and x) approach 0. Coulomb sees the action not only vanishing at A but at every point greater than A along the filament. Perhaps he imagines the charge carrying molecules as discrete infinitesimally small "segments" then the repulsive action between the first, the one closest to the electrified body, and the electrified body will be infinitely greater than the action between the (lessor) charged molecules and the electrified body. Perhaps....

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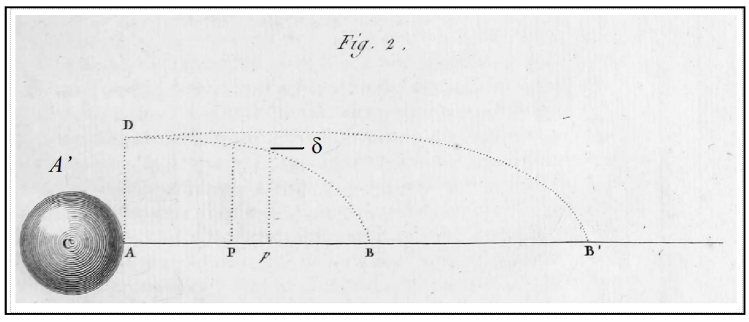
Coulomb goes on to determine the length at which imperfect insulator begins to insulate perfectly. For this he needs another Figure



He writes

Let A' be the electric mass of the globe concentrated at its center; let δ be the electric density at p, we will have for the total action with which point p is repelled by the electric fluid

$$\frac{A' \cdot \delta}{(R+x)^2} - \delta \cdot \frac{d\delta}{dx} \ .$$

He writes that he will prove, in a subsequent Memoir, that the first term is incomparably smaller than the second and so can be neglected. He identifies the second term in this expression as B, the insulating resistance of the filament which he "as we have seen, is a constant" along the filament.

How he constructed this expression is not evident, nor how it is that he can neglect the first term, the action of the globe, C, on point P (p?), with respect to the second, "the action of the element dx multiplied by the increment of δ".

Immediately following the introductory paragraph quoted above he describes the resistance to the flow of the electric fluid.

> ...because these aqueous molecules being separated by a small insulating interval, it requires a certain degree of force in order to push electricity passed one molecule to another.
>
> The resistance that this small insulating interval opposes to the flow of the electric fluid appears only able to be represented by a constant quantity for a constant interval, and consequently *to be proportional to the difference in the action of two consecutive molecules.* (emphasis mine).

While unable to deduce this expression knowing that this resistance is *proportional to the difference in the action of two consecutive molecules*, we can at least check the dimensions of these terms and the variables that appear. A' is the "electric mass" of the globe which we take to be its total *charge*; δ, the "electric density at p", a point along the insulator, we take as *charge/length*. The first term then has dimensions $charge^2/length^3$ and, in accord with Coulomb's inverse square law of repulsive action (force) between two charged particles, the first term in the expression then has the dimensions of *Force per unit length*. A similar walk through with the entries to the second term, shows that it too has the dimensions $charge^2/length^3$ or, as the first, *Force per unit length*.

This then is B the (constant) resistive force (per unit length) to the flow of the electric fluid.

$$B = -\delta \cdot \frac{d\delta}{dx} \ .$$

With B a constant, Coulomb integrates this to obtain

$$D^2 - \delta^2 = 2Bx \ ,$$

where D is the density of the globe not A' the total mass of electricity of the globe.[704]

He then obtains the length x at which the density d goes to zero, the length at which the filament begins to insulate perfectly, namely[705]

$$x|_{perfection} = \frac{D^2}{2B}$$

[704] Recall that Coulomb considers the total mass as concentrated at the center of the globe; hence at a point, just as δ is the mass density at a point along the filament.

[705] The notation is mine; also Coulomb erroneously drops the "2" here and in his subsequent analysis.

which, indeed, has the dimensions of length. This is the principal result of the section.

Coulomb ends with this reflection

> Reflecting on the theory just presented, it is easy to see that the foregoing formula determines the disposition of the electric fluid along the imperfect insulating support, assuming we have communicated, as we have in our experiments, a certain dose of electric fluid to the globe supported by the silk; because then this fluid communicating itself step by step along the insulating support will spread up to point B, (my $x|_{perfection}$)[706] so that the repulsion of the fluid is in all the points exactly in balance with the maximum resistance that the coercive force of the insulating support can oppose to the flow of this fluid. But it should be noted that, as this maximum of resistance is a coercive and non-active force that can be compared to the resistance of a friction, any repulsive action of the electric fluid less than the maximum of this resistance will not disturb the state of stability of this fluid spread according to any law whatsoever along the support; so that, if the line AD which represents in the attached Figure the density of the globe remains constant, that we prolong by any quantity BB' the axis AB, and that we describe any density curve DB', provided that all points $\delta d\delta dx$ be smaller than B, the electric fluid spread along the line AB' will keep its state of stability without flowing from one point to another; from which one concludes that there is always an infinity of density curves DB' which also satisfy the state of stability of the electric fluid spread along an imperfect insulating support and that the general search of the disposition of the electric fluid in an imperfect insulating body is an indeterminate problem which, to become determinate, needs to be subjected to some particular conditions.

***

More could be written in attempting to elucidate this claim of indeterminacy, to figure out the train of Coulomb's thought. Let it suffice to say that why this B is a maximum resistive force remains a mystery.

Indeed, much of these final paragraphs is a mystery. Generally it is the task of the historian of science, the historian generally, to not dismiss the obscure and focus solely on what we today consider the "truth" but to try to understand the mysterious within its own context. We quote Collingwood[707]

> The historian thinks it a wrong way; but wrong ways of thinking are just as much historical facts as right ones, and, no less than they, determine the situation (always a thought-situation) in which the man who shares them is placed.

And elsewhere[708]

> The important question concerning any statement contained in an historical source "is not whether it is true or false, but what it means." Ignoring this advice leads to writing "scissors-and-paste" histories where the sources are treated as worthwhile

[706] This B is a point on the filament, *not* the B representing the resistive force per unit length.
[707] [Collingwood, 1993, p. 317].
[708] [Collingwood, 1993, pp. 260 and 299].

> historical material and admitted into the historian's narrative only if they are deemed to be believable by the historian's own standards of evidence. To understand the past historically is to understand the "context of thought" of past agents, their frame of mind... To understand past agents is to understand the way in which they reasoned, the inferences that they drew, the conceptual connections which they made, the symbolic significance they attached to certain events.

In this brief essay I have tried to make Coulomb's obscure analysis, considered by at least one critic as not making sense, "reasonable" within the context of his own time. This is a very difficult task if said reasoning appears to not make sense according to our own, present day understanding of the loss of electricity along a so called insulator — none the less reasoning that appears suspect even within Coulomb's own time.

Perhaps it was the tentative nature of his argument and analysis of these last few paragraphs that hints at, is the source of, its incoherence, its obscurity. Recall that Coulomb is going to show in a subsequent Memoir that one can neglect the first term relative to the second in the expression

$$\frac{A' \cdot \delta}{(R+x)^2} - \delta \cdot \frac{d\delta}{dx} \ .$$

So his analysis here does not stand on its own. Did he have this argument fully worked out?

Consider too the extraneous information, the excess symbols in Figure 1. Including these suggests that Coulomb had been concerned with the action of the points to the left of the point P. This suggests these last few paragraphs were but a first work-out, a trial-run analysis, one of perhaps several, helpful to Coulomb in constructing a veritable, coherent picture in his own mind of losses along an insulator.

I leave it for others to do a proper history.

Chapter 19

Remarks on Coulomb's Third Memoir

A. K. T. Assis

19.1 Coulomb's Experimental Method

In Section 17.1 Coulomb presented a numerical example of his experimental method for determining the loss of electricity of an electrified sphere to its surroundings. The preparation of this method is illustrated in Figure 19.1.

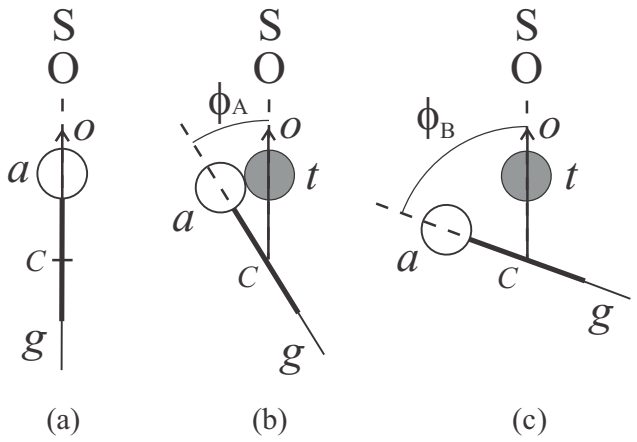


Figure 19.1: (a) Untwisted wire with the needle pointing toward O. (b) Beginning of the experiment with discharged balls. (b) Final equilibrium configuration of the experiment with electrified balls.

In Figure 19.1 (a) we have a top-down view of the needle with center C along the projection of the suspension thread, ball a and paper disk g which works as a counterweight and as an oscillation damper. In the situation where the thread is not twisted, the needle is directed toward point O fixed on the graduated scale of the circle zOQ attached to the glass container around the needle. This circle zOQ is at the same height as the horizontal needle and the lower end of the hanging thread. The arrow Co is the micrometer pointer attached to the top of the torsion wire. Initially it is directed toward the fixed point S on the graduated scale placed on top of the balance. I am assuming that in the beginning of the experiment the needle Ca and the indicator Co are along the same vertical plane passing through COS.

Before starting the experiments, Coulomb moved ball a slightly away from its original position by placing ball t against it, Figure 19.1 (b). Ball t is always fixed in the laboratory. The line passing through C and through the center of the fixed ball t is always directed to point O. Coulomb measures the angles from that point O. The horizontal needle ag can rotate around its center C attached to the vertical hanging thread. The initial twist of the wire is represented by the angle ϕ_A.

The two balls are charged when an electrified pin touches ball t, this pin being removed after contact. As a result, the balls acquire charges of the same sign and repel each other. The needle rotates counterclockwise in the horizontal plane around point C from which it is suspended by the vertical thread. Ball a moves away from ball t. At equilibrium the wire is twisted at an angle ϕ_B, Figure 19.1 (c). In Coulomb's hypothetical example, this angle was $\phi_B = 40°$. The torque exerted on the needle by the electrical force of repulsion between the electrified balls is balanced by the counter-torque exerted by the wire twisted at angle ϕ_B.

Now begins the main part of the experiment to determine the loss of charge of the electrified balls to the environment around them. This procedure is illustrated in Figure 19.2.

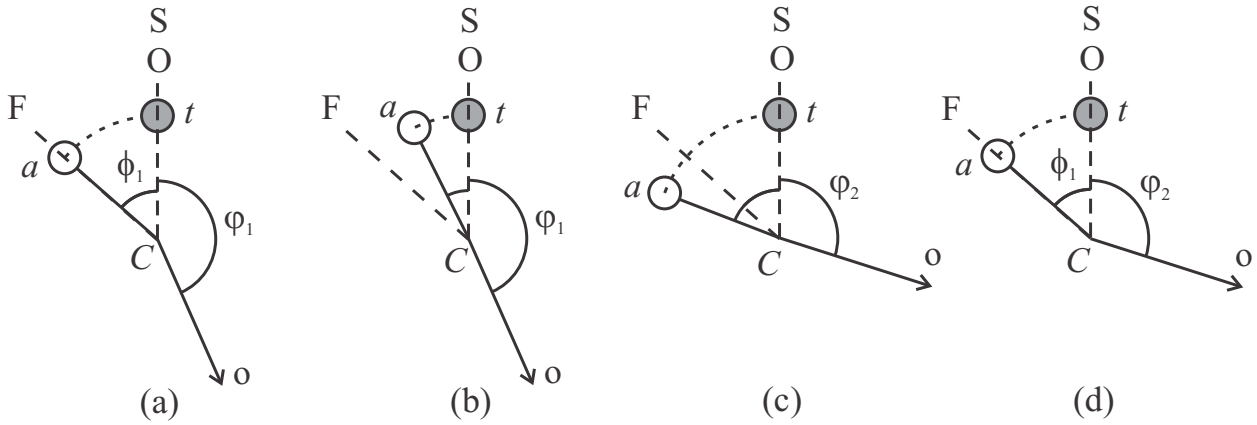


Figure 19.2: Coulomb's experimental procedure.

Initially Coulomb twists the micrometer attached to the top of the torsion wire clockwise through an angle φ_1. This reduces the initial angle ϕ_B represented in Figure 19.1, causing the electrified balls a and t to approach each other until they reach a specific angular separation ϕ_1 indicated by the line CF, this angle being $\phi_1 = FCO$. In Coulomb's hypothetical example, the separation between the balls changed from $\phi_B = 40°$ in the situation of Figure 19.1 (c), to $\phi_1 = 20°$ in the situation of Figure 19.2 (a) when the micrometer has been twisted with an angle $\varphi_1 = 140°$. At equilibrium the repulsion between the balls indicated by the angle ϕ_1 is counterbalanced by the total twist of the wire indicated by $\phi_1 + \varphi_1$. Coulomb begins to mark time from that moment of equilibrium.

As time passes, the angle ϕ_1 decreases due to the loss of charge of balls a and t. The needle Ca no longer points along the line CF. This situation is illustrated in Figure 19.2 (b).

Coulomb then decreases the twist of the upper end of the thread to a value of φ_2, such that the needle rotates counterclockwise to an angle slightly greater than ϕ_1. In Coulomb's example, this occurred when the micrometer angle decreased 30°, from $\varphi_1 = 140°$ to $\varphi_2 = 140° - 30° = 110°$, as indicated in Figure 19.2 (c).

As time passes, the angular separation between balls a and t decreases again due to the loss of their electricity. At a certain instant, the angular separation between them returns to the value ϕ_1 previously specified, as indicated in Figure 19.2 (d). Coulomb then observes the exact moment at which this occurs, measuring the time interval between situations (a) and (d) of this Figure 19.2.

Coulomb's conclusions are based on measurements of the angle ϕ_1 of separation between the balls, on the measurement of the angles φ_1 and φ_2 of torsion of the micrometer, as well as on the measurement of the time interval between the situations of the Figures 19.2 (a) and (d). The total twist of the wire in case (a) is represented by $\phi_1 + \varphi_1$, while in case (d) it is given by $\phi_1 + \varphi_2$.

Coulomb continued this procedure for other tests, each time decreasing the value of the angle φ and measuring the time intervals between consecutive tests in which the separation between the balls was always ϕ_1.

I will give here a detailed example of Coulomb's calculations in the case of the first and second tests of the first experiment contained in the Table located on page 287. The first test was performed at 6h 32m 30s and the second at 6h 38m 15s, such that the time elapsed between these two observations was 5m 45s = 5.75 minutes. The total force in the first test (indicated by the total angle of twist of the wire) was $30° + 120° = 150°$. In the second test it was $30° + 100° = 130°$. The average force between these two tests was then $(150° + 130°)/2 = 140°$. During those 5.75 minutes there was an average loss of $150° - 130° = 20°$. So the average loss per minute was $20°/5.75 = 3.478°$. Therefore, the ratio between the average loss per minute and the average force between balls was $3.478°/140° \approx 1/40$, as indicated in the upper value of the seventh column of this Table.

The same procedure was used to obtain the other values in this Table.

Let a sphere of radius r be uniformly electrified with a charge q. The symbol δ used by Coulomb in this Section can represent both q and the surface charge density $q/(4\pi r^2)$.

In the first experiment Coulomb found that the ratio $(d\delta/dt)/\delta$ had an essentially constant value from a total initial twist of the thread given by $30° + 120° = 150°$, up to a final value three times smaller, that is, with a final twist given by $30° + 20° = 50°$. On the basis of this experimental observation he found that

$$\frac{d\delta}{\delta} = -mdt \ , \tag{19.1}$$

where δ is the value of charge (or surface charge density) on each ball at time t, and m is a positive constant (due to the fact that the charge decreases with the passage of time). The unit of m is the inverse of time, that is, $time^{-1}$. For example, if time is measured in minutes, the constant m will be expressed in $minute^{-1}$. Integrating this equation (using that $\delta = D$ at the initial instant $t = 0$) yields:

$$\ln \frac{\delta}{D} = -mt \ . \tag{19.2}$$

We then have an exponential decay of the charge on each ball represented by the equation:

$$\delta = De^{-mt} \ . \tag{19.3}$$

Coulomb also found that the value of m depends on the humidity of the air.

19.2 Influence of Air Humidity

In this Third Memoir Coulomb studied the loss of electricity from a charged body. He distinguished two main causes, that due to the insulating support that prevents the body from coming into contact with the Earth and that due to the body's contact with the air around it.

Water is a conducting substance for electrostatic experiments.[709] Moisture in the air can cause water to bind to the surface of the supports making them more conductive. This factor is especially relevant in hydrophilic supports. That is, the greater the humidity in the air, the surfaces of the supports will become more conductive, facilitating the loss of electricity from the body by the transport of charge by these supports to the Earth. This fact seems to be well established experimentally.

Another question much more difficult to resolve is whether the conductivity of the air itself would be affected by its degree of humidity. Coulomb concluded that the loss of electricity from a charged sphere to air increases with air humidity. However, uncertainties due to changes in temperature, rapid variations in humidity, as well as the construction method of the hygrometers that measure this humidity made him state that this whole subject needed further research.[710]

This conclusion by Coulomb that the loss of electricity of a body to the air increases with the humidity of the air seems to be false, but this was only established at the end of the 19th century.[711] The main loss of electricity through the air does not appear to be due to dust or moisture it may contain, but rather seems to be due to its ionization. This ionization can increase in a variety of ways (by being near a flame, by natural radioactivity in the environment, etc.). Many of these factors that increase the ionization of the air are random phenomena.

However, it should be emphasized that this possible misinterpretation on the part of Coulomb does not affect the validity of his law of exponential decay of the electrification of a charged sphere. This law is in agreement with the experimental facts.

[709] See Section 7.11 (The Conductivity of Water) of [Assis, 2010b], [Assis, 2010a], [Assis, 2011], [Assis, 2015b] and [Assis, 2017]. See also Section 3.1 (Classifying Substances as Conductors or Insulators with the Electroscope) from [Assis, 2018b], [Assis, 2018a] and [Assis, 2019].

[710] See Sections 17.1.5 and 17.1.6, as well as [Gillmor, 1971a, p. 197].

[711] [Thomson, 1906, pp. 1-9], [Bauer, 1949], [Heilbron, 1999, pp. 297, 336 and 477] and [Blondel and Wolff, 2011e].

Chapter 20

Fourth Memoir on Electricity, where We Demonstrate Two Main Properties of the Electric Fluid: The First One, that This Fluid Does Not Spread in Any Body by Chemical Affinity or Elective Attraction, but that It is Shared Between Different Bodies Brought into Contact Exclusively by Its Repulsive Action; the Second One, that in Conducting Bodies the Fluid, Having Reached a State of Equilibrium, Is Spread Over the Surface of the Body and Does Not Penetrate Its Interior

Coulomb[712]

[712] [Coulomb, 1788c], with complete German and Portuguese translations in [Coulomb, 1890d] and [Assis, 2022], respectively. This work was presented in 1786 to the French Academy of Sciences and published in 1788.

20.1 I

We determined in the three previous Memoirs the law of repulsion of the electric fluid of the same nature, and the law of attraction of two electric fluids of different nature, and we proved by some very simple and apparently decisive experiments that this action was very exactly in the inverse ratio of the square of the distances. We also proved, by the same kind of experiments, that the action, either repulsive or attractive, of the magnetic fluid obeys the same law. In the Third Memoir we have determined the law according to which the electric density of an insulated body decreases,[713] either by the contact with more or less humid air, or along the insulating supports[714] when they are not of sufficient length; which mainly depends, as we have seen, from the more or less insulating capacity of these supports, from their more or less affinity with the aqueous vapors, the state of the air, the density of the electric fluid of the insulated body, and on the size of this body.

20.2 II

We will utilize here the balance described in our First Memoir, printed in the Volume of 1785.[715] The only change we have made is to substitute the paper strip glued around the cylinder which encloses the needle, and which, divided into degrees, is used to determine the [angular] distance of the two balls, by a wooden circle placed on four pillars, whose diameter is about double that of the cylinder: this circle is placed in such a way that its center is on the plumb line of the thread which suspends the needle, and that the first division of this circle is on line with the suspension thread and the center of the ball supported by the needle, when the needle naturally stops, and that the index of the micrometer also aligns with the first division of the micrometer circle.

We must, however, warn that since the reading of the Memoir which we quote,[716] and which contains the description of this balance, we have built several others of a different shape: the largest is square, it has thirty-two inches on a side (86.62 cm), twenty inches in height (54.14 cm), it is closed on the sides by four windows fixed by an insulating coating, in very light frames of wood heated on the oven, hot coated with a varnish formed of shellac and turpentine. Above the box is a crosspiece which carries a vertical cylinder of fifteen inches (40.60 cm) made of glass, surmounted by a micrometer; a circle placed outside this box is used to measure the [angular] distance between the balls. In this balance, we can make experiments with electrified globes from four to five inches in diameter [10.8 to 13.5 cm]: in the first balance whose cylinder is only one foot in diameter [32.5 cm], we could only use globes of at most one inch in diameter [2.7 cm]. But it should be noted that there are many cases where the small experiments are more decisive than the large ones, because the attraction or the repulsion of the electric fluid is, for each element, inversely proportional to the square of the distance, in order that the results be simple, it is almost always necessary that the distance between the bodies whose reciprocal action we wish to measure be much larger than the particular dimensions of these bodies.

[713] That is, the decrease in the electric charge of a body as a function of time.
[714] In the original: *soutiens idio-électriques*. See footnote 522 on page 229.
[715] This Memoir is translated in Chapter 11. See, in particular, Figure 1 on page 205.
[716] The First Memoir was read in 1785 and published in 1788.

20.3 III. First Principle. *The Electric Fluid Spreads in All Conducting Bodies According to Their Shape, Without This Fluid Seeming to Have Affinity or an Elective Attraction for One Body Preferably to Another*

20.3.1 First Experiment

I suspended in the hole of the balance, at the height of the ball of the needle, a small copper ball of eight lines in diameter (1.804 cm), supported by a small cylinder of shellac. The center of this ball was placed so that it fell on a line connecting the suspension thread and the first division of the circle placed outside of the balance. The ball of the needle which touched against the copper ball, was thus displaced from the position where the torsion is zero, by the sum of the half-diameters of the two balls in contact.

The two balls were electrified by the process described in the in the First Memoir; the needle was driven away to about 48 degrees. By means of the micrometer button the suspension wire was twisted by 120 degrees, in order to bring the ball of the needle back towards the copper one, and we waited until the needle stopped oscillating; it stopped at 28 degrees: in this state, I touched immediately the copper ball of eight lines in diameter with a pith ball,[717] exactly of the same size, supported by a small cylinder of shellac. By removing the pith ball, the needle came closer to the copper ball; and to bring it back to the first distance of 28 degrees, I was obliged to untwist the thread; so that the micrometer, which before the contact marked 120 degrees, after the contact marked only 44 degrees.[718]

20.3.2 Second Experiment

Instead of the copper ball, I suspended in the hole of the balance, by means of a small cylinder of shellac, an iron disk[719] of ten lines in diameter [2.3 cm], the vertical plane of which passed through the suspension wire of the needle and through the zero point of the circle located outside the balance which is used to measure the displacement of the balls. Having then, as in the preceding experiment, electrified the ball of the needle and the iron plane,[720] the ball of the needle was driven away; I twisted the suspension wire to bring the needle back towards the iron disk, and by means of 110 degrees of twist, the needle stopped at 30 degrees from this disk. Immediately I made the iron disk touch a small paper disk which had exactly the same diameter, and after having removed the paper disk, I found that for the needle to stop at 30 degrees, it was necessary to reduce the torsion to a little less than 40 degrees.

20.4 IV. Result of These Two Experiments

In the first experiment, the copper ball, before the touch of the pith ball, repels the needle at 28 degrees, the micrometer marking 120 degrees; so the strength of torsion was then 148

[717] In the original: *balle de sureau.* See footnote 469 on page 208.
[718] A representation of this experiment can be found in Section 21.1.
[719] In the original: *un cercle de fer.*
[720] In the original: *le plan de fer.*

degrees.[721] After the ball of elderberry had touched the copper ball, the latter repelled the needle at 28 degrees, the micrometer marking only 44 degrees; so that the total force of torsion, equal to the repulsive force of the two balls, was 72 degrees; but there was approximately a minute interval between the two observations, and the electric force decreased by one fiftieth per minute on the day of this experiment: thus the total force of torsion would have been about $73\frac{1}{2}^{\circ}$, if the electricity had not diminished by one fiftieth. This quantity differs only by half a degree, or 1/147 of 74 degrees, half of the first twisting force 148 [degrees] which measures electric repulsion before contact; thus, since in both observations, the distance of the two balls is exactly the same, and that the action is inversely proportional to the square of the distances and directly proportional to the densities of the electric fluid, it follows that the elderberry ball took exactly half of the electric fluid of the copper ball; thus the metal ball did not have an elective affinity or attraction for the electric fluid larger than that of elderberry.[722]

In the second experiment, where the iron disk was touched by a paper disk of exactly the same diameter, the electric fluid was again divided equally between the two disks. These experiments were carried out with balls of different materials, they were repeated in the large balance with globes of five or six inches, and the results were always the same.

20.5 V. First Observation

It should be observed that when two equal and similar bodies brought into contact are perfectly conductive as are all the metals, it takes but an imperceptible moment for the electricity to be shared equally between the two bodies. But when one of the two bodies is an imperfect conductor, such as our paper disk, for example, it often takes several seconds before the paper disk has taken exactly half of the electric fluid of the disk of metal, which depends not only on the particular conductive quality of the two bodies but also on their relative areas and the way they are brought into contact. In the previous Memoir, we have already attempted to explain how the coercive force of the imperfect insulating supports only allows the electric fluid to spread and penetrate [into the insulating support] to a certain distance from the electrically charged, conducting body.

20.6 VI. Second Observation

It is still necessary to observe, in repeating the second experiment, to place in contact the two disks symmetrically, so, for example, that the limb of one does not touch, by forming an angle, a point on the surface of the other, because then the electric fluid would be divided in an unequal way between the two disks: in the previous experiment, I touched the limb of one of the disks by the limb of the other, taking care to hold it in the same plane.

[721] See Coulomb's 1784 Memoir *Theoretical and Experimental Research on the Force of Torsion* translated in Chapter 7.

[722] Section 21.2 presents a discussion of Coulomb's calculations.

20.7 VII. Second Principle. *In a Conducting Body Charged with Electricity, the Electric Fluid Spreads Over the Surface of the Body, But Does Not Penetrate Inside the Body*

The experiments intended to prove this proposition require much more sensitive electrometers than all those that are in use. Here is the one I utilize: pull, by heating a candle, a shellac thread about the thickness of strong hair of ten to twelve lines in length [2.26 to 2.71 cm]; one of its ends was attached to the top of a small headless pin, suspended on a silk thread, such as a silkworm produces; at the other end of the shellac thread, we fix a small tinsel disk[723] about two lines in diameter [0.45 cm]: we suspend this small electrometer in a glass cylinder; its sensitivity is such that a force of one sixty thousandth of a grain (0.0009 dyn) repels the needle more than 90 degrees.[724] I give this electrometer a weak degree of electricity, of the nature of that which I want to communicate to the body which will be submitted to test, and I suspend it in a glass cylinder, to protect it from the air currents; this done, I place a solid body, of any figure, pierced with several shallow holes, on an insulating, idio-electric support. The body that I am going to submit to experiments is a solid wooden cylinder, four inches in diameter (10.83 cm), pierced with several holes four lines in diameter (0.90 cm) and four lines deep.

20.8 VIII. Experiment

I place this cylinder on an insulating support; by means of the Leyden jar, or the metal plate of an electrophorus,[725] I give it one or more electric sparks. I insulate at the end of a small cylinder of shellac of one line in diameter (0.226 cm), a small disk of gilded paper one line and a half in diameter (0.338 cm).[726]

First test. The tinsel [disk] of the electrometer being electrified, I touch the surface of the electrified cylinder by the small disk of gilded paper, I present it to the electrometer; the needle of this electrometer is driven out with force.

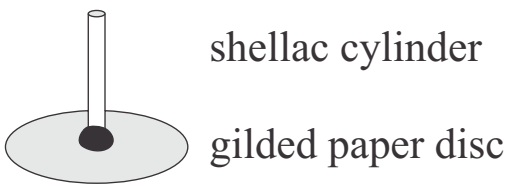
Second test. But if I introduce the small paper disk in one of the holes of the cylinder, and make it touch the bottom of one of these holes; then present it to the tinsel [disk] supported

[723] In the original: *cercle de clinquant*. See footnote 488 on page 213.

[724] See footnote 489 on page 213.

[725] See Chapters 6 (The Electrophorus) and 12 (The Leyden Jar and Capacitors) of [Assis, 2018b], [Assis, 2018a] and [Assis, 2019].

[726] This small disk of gilded paper connected to a shellac cylinder is the first example of Coulomb's famous proof plane. A detailed description of this instrument can be found in Section 7.2 (Charge Collectors) of the book *The Experimental and Historical Foundations of Electricity*, [Assis, 2010b], [Assis, 2010a], [Assis, 2011], [Assis, 2015b] and [Assis, 2017]; and also in Section 2.6 of Volume 2 of the same book, [Assis, 2018b], [Assis, 2018a] and [Assis, 2019]. The Figure of this footnote illustrates this proof plane. The shellac cylinder behaves as an insulator, while the gilded paper disk behaves as a conductor of electricity:



at the end of the needle of the electrometer, this needle will not give any sign of electricity.

20.9 IX. Explanation and Result of This Experiment

In the first test, I make the small disk of gilded paper touch the surface of the cylinder; as this disk is only one eighteenth of a line thick [0.0126 cm], it becomes a part of the surface of this cylinder, and consequently takes a quantity of electric fluid, equal to that which contains a part of the surface equal to this small disk.[727] In this test, the small disk is charged with a quantity of electricity which is not only sensitive to our little electrometer, but which can even be measured exactly by means of our electric balance.

In the second trial, we touch the small disk of gilded paper, at the bottom of one of the holes in the cylinder, about four lines below the surface, or twenty lines [4.52 cm] from its axis; carefully removing this small disk, without it touching the edge of the hole, we find, by presenting it to the needle of the electrometer, either that it gives no sign of electricity, or that it gives very weak signs of electricity contrary to that of the cylinder:[728] it is therefore clear that in this experiment there is no electric fluid in the interior of the body, even very near its surface.

The signs of contrary electricity, which are only sometimes seen, are due to the fact that when the small cylinder of shellac is introduced into the holes, the electric action of the surface of the electrified body gives, outside this body, to the shellac thread, a small electricity of a different nature from its own, because this small shellac wire is isolated in its sphere of activity. The proof that everything happens this way, that this small degree of electricity exists in the shellac wire, and not in the small disk of gilded paper which was put in contact with an interior point of the body; is that if we touch this disk,[729] we do not destroy this small electricity, which is always very weak when the shellac is pure, and the air is not very humid.

20.10 X

This property of the electric fluid to spread on the surface of conducting bodies, and not to penetrate into the interior of these bodies when this fluid has reached equilibrium, is a consequence of the law of repulsion of its elements, in inverse proportion to the square of the distances, a law that we found in our First Memoir: but since it is experiment, not theory that led us [to this property], we thought we should follow the same procedure in the presentation of our research; let us see now how theory generalizes the result announced by experiment.

[727] This is the correct assumption related to the charge acquired by a proof plane when in contact with an electrified conductor. Later on Coulomb will arrive at another wrong conclusion, see footnotes 920 and 927 on pages 421 and 423, respectively.

[728] That is, the gilded paper disk can be neutral, or it can contain a very small amount of electricity, of opposite sign to that in the cylinder. Moreover, the magnitude of this charge is much smaller than the magnitude of the charge obtained in the first test.

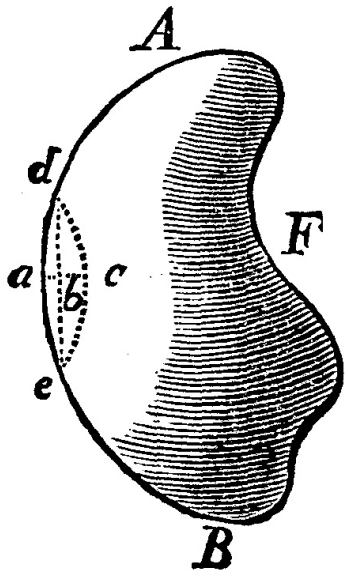
[729] That is, when we ground the gilded paper disk by touching it with our finger.

20.11 XI. Theorem

Whenever a fluid enclosed in a body where it can move freely, acts by repulsion among all its elementary parts,[730] with a force less than the inverse ratio of the cube of distances, such as, for example, [a force proportional to] the inverse of the fourth power, then the action of the masses of fluid located at a finite distance from one of these particles is null relative to the action of points in contact; this is what we have proved in a Note of our Second Memoir printed in the volume of the Academy, 1785.[731] Thus, the fluid which owes its elasticity[732] to this law of repulsion, will spread uniformly throughout the body; but whenever the repulsive action of the elements of the fluid which produces its elasticity is greater than the inverse of the cube, such as, for example, we have found for electricity, which is as the inverse square of the distances; then the action of the masses of the electric fluid placed at a finite distance from one of the elements of this fluid, not being infinitely small in relation to the elementary action of the points in contact, all the fluid must be carried to the surface of the body, and there must be no fluid left in its interior.

20.11.1 *Demonstration*

In a body of any shape AaB, which I suppose to be filled with fluid whose elementary parts act on each other in inverse proportion to the square of the distances, I raise at a point a an infinitely small normal ab; and through point b, I pass a plane perpendicular to this normal, which divides the body into two parts, one infinitely small $daeb$, the other finite $dAFBeb$.



[730] In the original: *parties élémentaires*. The word *parties* can be translated as parts, particles or elements, [Gillmor, 1971a, pp. 159, 201 and 217]. The word *élémentaires* can be translated as elementary, fundamental, primary, essential or basic. This expression was interpreted by Gillmor as "particles", [Gillmor, 1971a, penultimate line of p. 201]. The German translation of this expression was *Elementartheilchen*, that is, "elementary particles", [Coulomb, 1890d, pp. 76-77].

[731] See footnote 556 on page 238.

[732] In the original: *électricitè*. As pointed out by König, [Coulomb, 1890c, pp. 87-88, Note 16], this word was a lapse of Coulomb. It is clear in the sequence of this sentence that Coulomb was referring to the elasticity of the fluid, that is, to the repulsive force between its particles.

Thus, by decomposing according to ab, all the forces with which the infinitely small part $dabe$ acts on point b, it must balance the resultant action, according to ba, of all the mass of the fluid spread in the body $dAFBe$. Let us imagine at present on the plane dbe, on the other side of a, a small cap dce exactly equal to the dae cap, by prolonging ab to c, cb will be equal to ab. But if the fluid is spread throughout the body, for the law of continuity to exist, it is necessary, since ac can be decreased to infinity, that the density of the fluid at point c must be equal to that of point a, or at least differ from it only by an amount that can be decreased to infinity. Thus, the only small mass of electric fluid contained in the cap $dcbe$ must balance the one contained in the cap $daeb$; from which it results that the action of all the mass of fluid which would be contained in the rest of the body must be null; which cannot take place when the action of the masses placed at a finite distance from a point of the fluid, is not infinitely small in relation to the action of an element of the body in contact with this point, unless the density of these masses is zero. Hence it follows that in the stable state of the fluid, all this fluid will be carried to the surface of the body, and that there will be none in the interior.

The first part of the theorem, that the fluid must spread evenly throughout the body, when the action of elements in contact is infinite relatively to the action of the finite masses which are at a finite distance from these same elements, needs no demonstration.

20.12 XII

We will see in one of the Memoirs which will follow this one, what is the electric density of each point of the surface of a body, of a given figure, and what is the state of the insulating particles of the air[733] immediately in contact with these surfaces.

[733] In the original: *particules idio-électriques de l'air*. See footnote 522 on page 229.

Chapter 21

Remarks on Coulomb's Fourth Memoir

A. K. T. Assis

21.1 Coulomb's Experimental Method

Figure 21.1 illustrates Coulomb's experimental procedure described in Section 20.3.

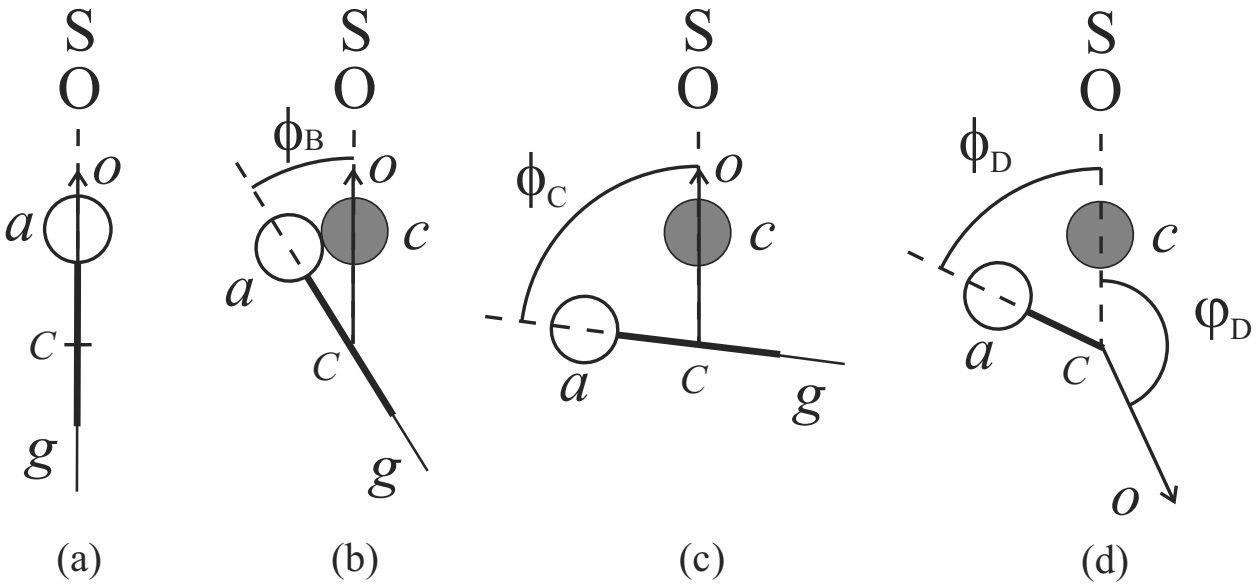


Figure 21.1: First part of the experimental procedure.

Figure 21.1 (a) shows a top-down view of the needle with center C along the projection of the suspension thread, ball a of the needle and paper disk g that works as a counterweight and as an oscillation damper. In the situation where the thread is not twisted, the needle points towards the fixed point O on the graduated scale which is on the outside of the balance. The twist angle ϕ of the bottom of the wire is measured from this point O. The micrometer pointer attached to the top of the suspension wire is indicated by the arrow Co. Initially it points towards the fixed point S on the small graduated circle attached around the micrometer. I am assuming that initially the needle and the micrometer pointer are in the same vertical plane.

Before starting the experiment, Coulomb moves ball a of the needle slightly away from its original position by placing the copper ball c against it, Figure 21.1 (b). The copper ball c remains fixed in this position. The line passing through C and through the center of the copper ball c is always directed toward point O. So in the situation where the balls are discharged, the bottom of the wire is twisted at an angle ϕ_B.

Coulomb then electrifies with charges of the same sign balls a and c that were touching each other. They repel each other until the needle comes to rest with the wire twisted at an angle ϕ_C, Figure 21.1 (c). In Coulomb's example this angle was 48°, which I am illustrating as having been counterclockwise.

Coulomb then twisted the micrometer clockwise at an angle φ_D, torquing the wire suspension and causing the needle ball a to approach the copper ball c at an angle φ_D, Figure 21.1 (d). In his example $\phi_D = 28°$ when $\varphi_D = 120°$.

Now comes the second part of this experiment illustrated in Figure 21.2.

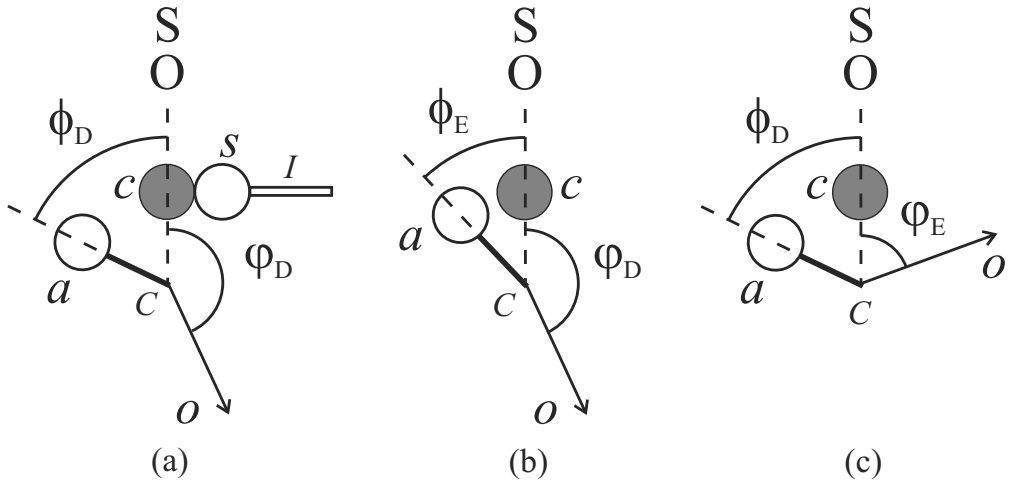


Figure 21.2: Second part of the experimental procedure.

While the needle ball a was at an angle $\phi_D = 28°$ from the copper ball c, Coulomb touched this copper ball with an elder ball s of the same size as ball c and which was supported by an insulating cylinder I, Figure 21.2 (a).

Upon removing the elder ball, he observed that the needle ball approached the copper ball, stopping at an angle ϕ_E, Figure 21.2 (b).

Coulomb then backed off the micrometer pointer so that the needle returned to the angle $\phi_D = 28°$. For this to occur, he observed that the top of the wire was twisted (relative to the bottom) an angle $\varphi_E = 44°$, Figure 21.2 (c).

21.2 Analysis of the Experiment

In Section 20.4 Coulomb analyzed this experiment. In the case of Figure 21.1 (d), the total repulsion between the needle ball a and the copper ball c was measured by the angle $\phi_D + \varphi_D = 28° + 120° = 148°$ which gives the total twist of the suspension wire. In the situation of Figure 21.2 (c), the repulsion between the same balls was $\phi_D + \varphi_E = 28° + 44° = 72°$. Taking into account the loss of charge to the environment during the 1 minute interval between these two observations (he must have timed the diminishing of the repulsive force previously), Coulomb calculated that the repulsion between them without this loss would be

73.5°. This value differs by only 0.5° from the value of 74°, and this difference is practically negligible. He then concludes that the force between a and c in the situation of the Figure 21.2 (c) was essentially half the force between a and c in the situation of the Figure 21.1 (d).

The angular distance between balls a and c in Figures 21.1 (d) and 21.2 (c) is the same, namely, $\phi_D = 28°$. Since between these two experiments the copper ball c had been touched by an elder ball s of the same size that was initially discharged, as illustrated in Figure 21.2 (a), Coulomb concluded that the elder ball acquired exactly half the electric charge that was on the copper ball c before contact. Since these two balls, despite being the same size, are made of different materials, he concluded that the copper ball does not possess an affinity or attraction for the electric charges greater than the affinity or attraction possessed by the elder ball.

He arrived at the same result in the second experiment in which he used iron and paper disks of the same size, instead of the copper and elderberry balls. He also obtained the same result with other materials and with large globes.

21.3 Volta Versus Coulomb

In Coulomb's Fourth Memoir he concluded that the electric fluid does not spread in any body by chemical affinity or elective attraction. In particular, he showed with his torsion balance that the electric fluid spreads in all conducting bodies according to their shape, without this fluid seeming to have affinity or an elective attraction for one body preferably to another made of a different substance.

In 1800 Alessandro Volta (1745-1827) described his invention of the electric pile.[734] According to Volta's theory, the seat of the electromotive force in his battery was located in the junction of two different metals. Volta's conclusion contrasted with Coulomb's experiment presented in this Fourth Memoir. Other scientists believed that the seat of the electromotive force in Volta's battery was located at the junction of each metal with the moist conductor placed between them, being due to chemical reactions. This controversy between the contact theory of Volta and the chemical theory of the pile lasted for several decades. As it goes much beyond Coulomb's works, we limit ourselves here to quoting a few references related to this important topic.[735]

[734] [Volta, 1800a] with English, Italian, Portuguese and Spanish translations in [Volta, 1800b] and [Volta, 1964], [Volta, 1923], [Magnaghi and Assis, 2008], and [Colombo, 2000], respectively.

[735] [Dibner, 1964], [Varney and Fisher, 1980], [Blondel, 1982, Chapter I], [Blondel and Chairopoulos, 1995], [Heilbron, 1999, pp. 491-494] and [Bevilacqua and Giannetto (Editors), 2003].

Chapter 22

Fifth Memoir on Electricity: The Manner in which the Electric Fluid is Divided Between Two Conducting Bodies Brought into Contact, and the Distribution of This Fluid on the Different Parts of the Surface of These Bodies

Coulomb[736]

22.1 I

We have seen in our Fourth Memoir on Electricity,[737] printed in the Volume of the Academy of 1786, that the electric fluid spreads equally in all bodies, provided that they were of a conductive nature: thus a globe of metal being touched by a wooden globe of equal diameter, the electricity is divided equally between the two globes; the experiment has given this result in an incontestable way.

We have also seen in the same Memoir that the electric fluid in the state of stability spreads only over the surface of bodies without penetrating in any sensible way into the interior of these bodies. Experiment has made this law known, and theory has proved that it was a consequence of the repulsive or attractive action of the molecules of the fluid in the inverse ratio of the square of the distances.

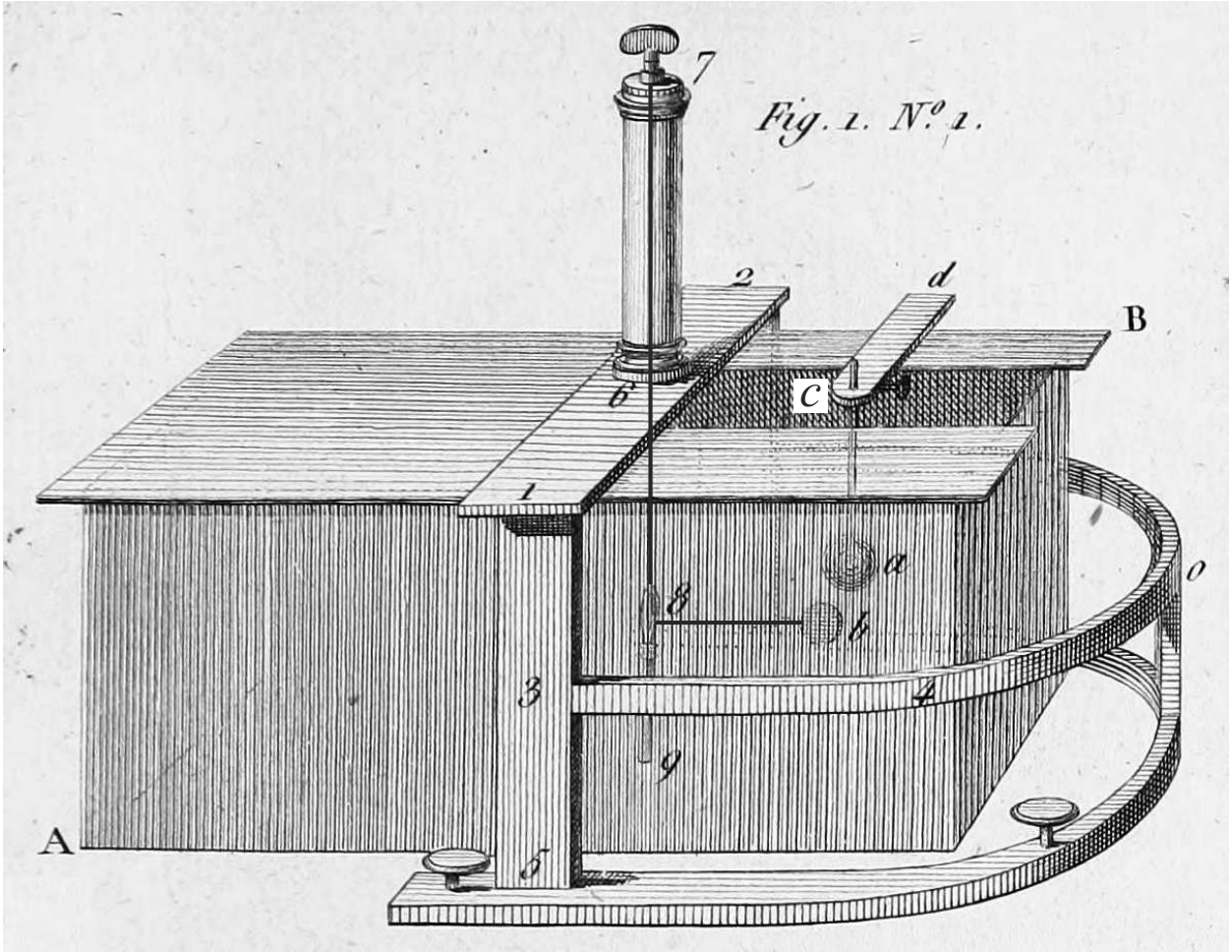
We are now going to seek in what ratios the electric fluid is divided between two unequal bodies of the same shape, or of a different shape, when these two bodies are brought into contact, and what is the density of this fluid on the different points of the surface of each of these bodies, density which varies for each point according to the figure of the body.

[736] [Coulomb, 1789] with Portuguese translation in [Assis, 2022]. This work was presented in 1787 to the French Academy of Sciences and published in 1789.

[737] This Memoir is translated in Chapter 20.

However, as we often made use of a torsion balance larger than that which was described in our First Memoir to measure the electricity, it is necessary to provide here a description and illustration [of this balance].[738]

Figure 1, No. 1, represents this new balance.[739]



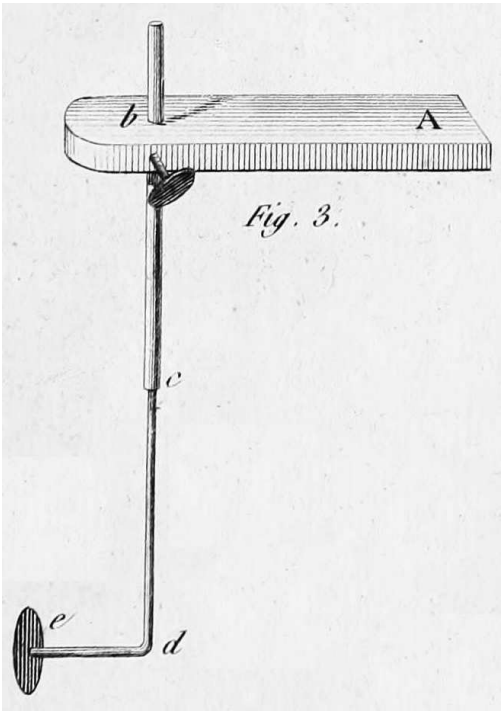
The square box AB is formed by four glasses 2 feet long [64.96 cm] by 15 to 16 inches high [40.61 to 43.31 cm]; it sits on a dried table coated with insulating varnish.[740] This box is covered by several pieces of movable glass with a cut-out at c so that one can insert the globe a therein supported by a small cylinder ac made of shellac; this cylinder is terminated by a small cylindrical stick dried in the oven and coated with shellac, which passes through a hole in the support cd, in which it is stopped by a screw; this support, intended to introduce globe a into the balance, is seen in more detail in Figure 3.[741]

[738]This Memoir is translated in Chapter 11.

[739]I have highlighted in Figure 1, Number 1, the letter c, the metal thread 78 and the needle 8b. The horizontal wire 8b is made of shellac and has the gilded paper disk at its extremity b.

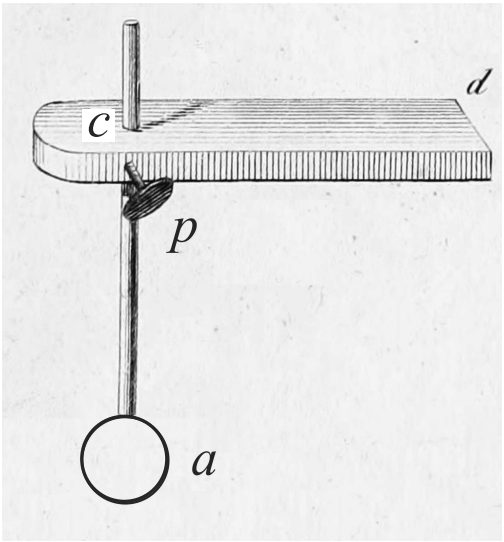
[740]In the original: *vernis idio-électrique*. See footnote 522 on page 229.

[741]Coulomb's support cd of Figure 1 appears in Figure 3 as being represented by the letters bA. It holds the proof plane $bcde$. I adapted this image in the Figure of this footnote:



The frame 1, 2, 3, 5 [of Figure 1, Number 1] is used to support the vertical tube 6, 7. This tube, 12 to 15 inches in height [32.48 to 40.61 cm], is made of glass; at the end of this tube at 7 is placed the torsion micrometer which can be seen in detail in Figure 2, Nos. 1 and 2.[742]

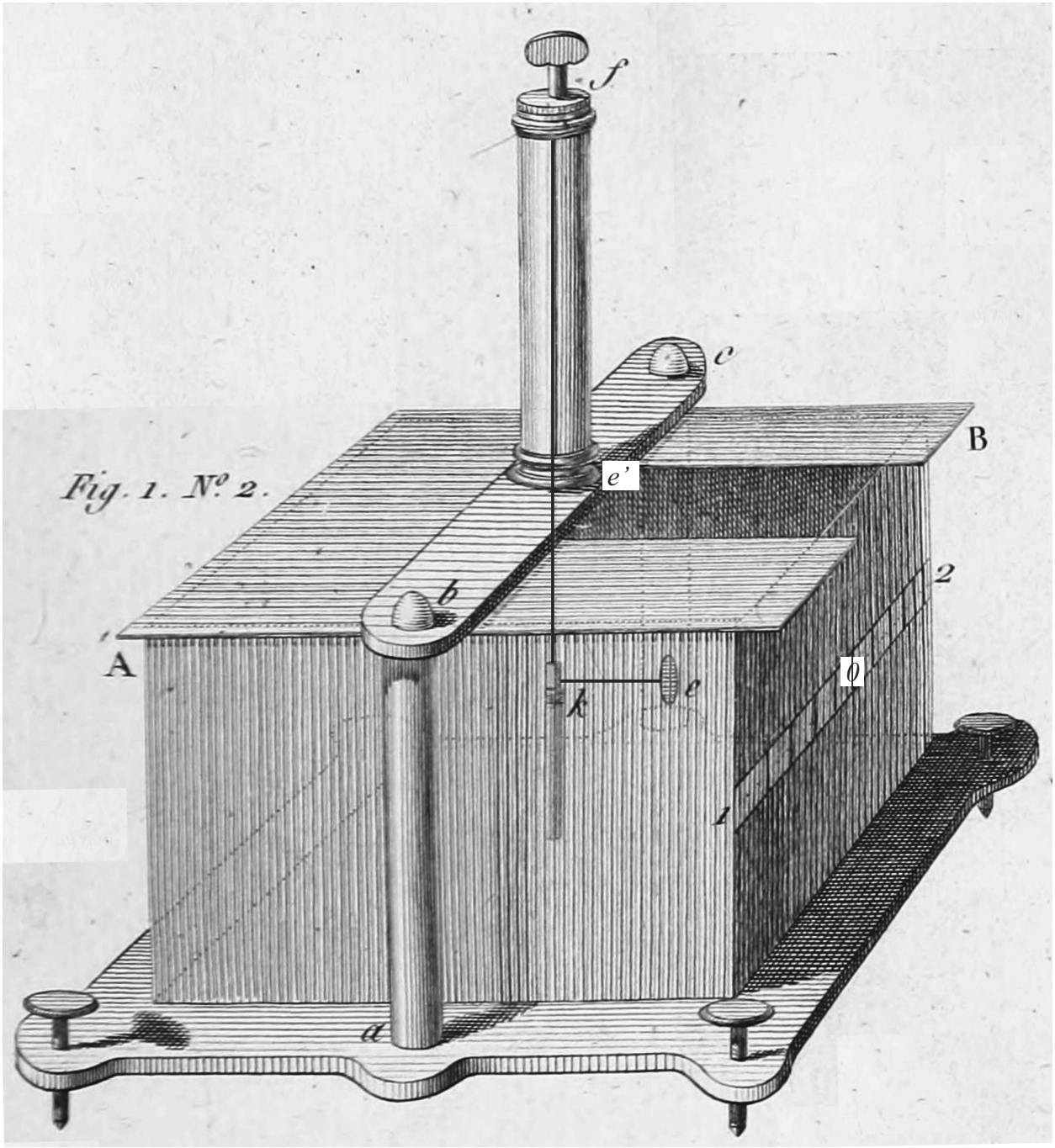
The circle, 3, 4, *o* [of Figure 1, Number 1], which holds to the frame, forms a half-circumference having about 4 feet in diameter [130 cm]; it is divided into 90 degrees from its middle *o*; its center responds to vertical thread 7, 8, which supports clamp 8, 9; to this clamp is attached horizontally a shellac thread 8*b*, terminated at *b* by a small disk of gilded paper.



I represented in the Figure of this footnote the support by the letters *cd*. Furthermore, I replaced the proof plane *bcde* of Coulomb's original Figure 3 by globe *a* which is attached to the cylinder *ac* coated with shellac. This cylinder is attached to the support *cd* by a screw *p*.

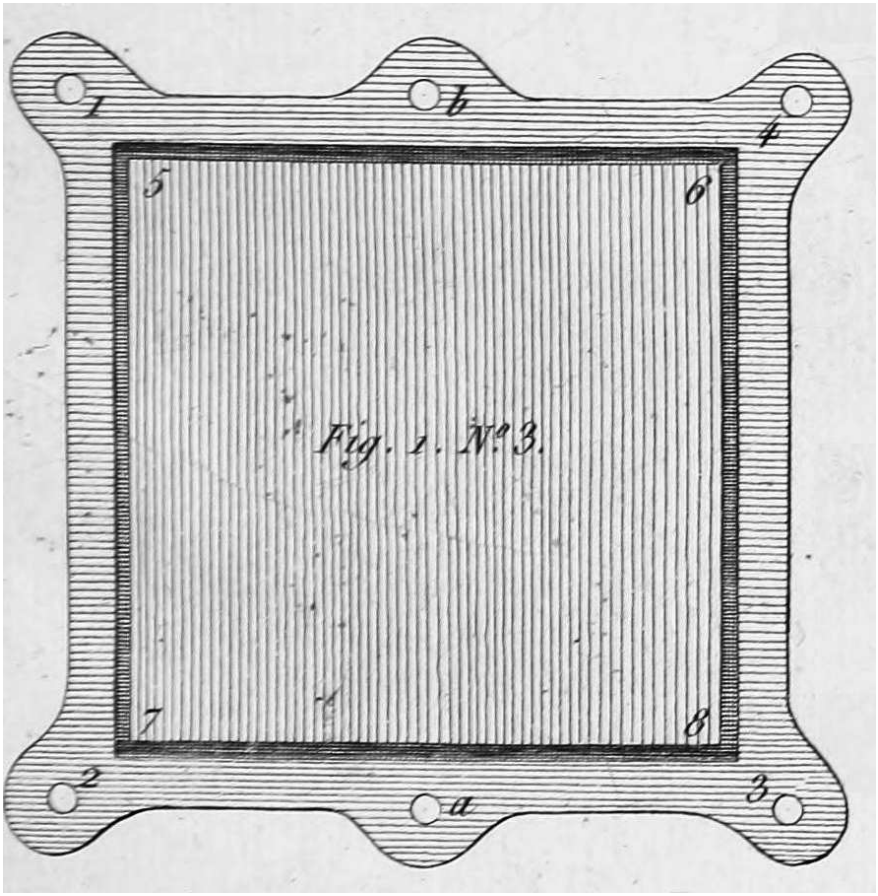
[742]Figure 2, Number 1, appears on page 332, while Figure 2, Number 2, is located on page 333.

Figure 1, No. 2, represents a balance of the same kind, but even simpler.[743]



Its base (Figure 1, No. 3) is a wooden frame dried in the oven, in which we see at a and b two mortises which support the vertical frame.

[743] I have emphasized in Figure 1, Number 2, the thread fk and the needle ke which has at its end e a gilded paper disc. I also highlighted point o in the paper band $1o2$. Moreover, I added the letter e' at the bottom of the glass tube $e'f$. This letter e' will be mentioned later, see footnote 746 on page 332.

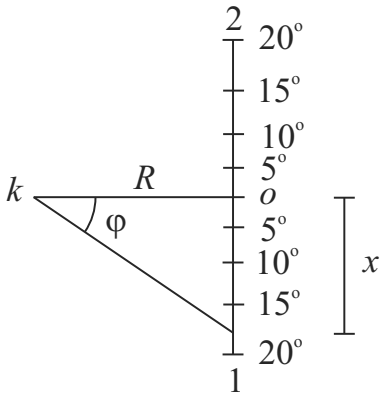


A groove 1, 2, 3, 4 has been traced on this frame which must receive the four vertical glasses which form the box of the torsion balance; 5, 6, 7, 8 represent the interior empty space within the frame which is closed either with a glass or with a small frame lined with taffeta coated with insulating varnish.[744]

Instead of the circle 3, 4, o of Figure 1, No. 1, we stick on one of the glasses (Figure 1, No. 2) a strip of paper 1, 2 divided from its midpoint o to its extremities, into degrees representing the tangent of a circle which has its center on the plumb line fk.[745] The four

[744] In the original: *vernis idio-électrique*. See footnote 522 on page 229.

[745] In the Figure of this footnote we have $x = R\tan\varphi$. We want angles of 5° marked along the strip of paper 1o2 of Figure 1, Number 2. So, given R, we can use this formula to calculate the corresponding value of x. For example, the first mark on both sides of the midpoint o is given by $x = R\tan 5° = 0.0875R$. The fourth markup is given by $x = R\tan 20° = 0.364R$:



glasses which form the box are lined all around with silk ribbons which are glued to them, and to which other small ribbons have been attached in order to be able to tie these glasses together and take them apart at will. The glass tube $e'f$,[746] furnished with a small wooden collar at e', is mounted by screws on the crosspiece bc, as well as all the parts of the machine.

Figure 2, No. 1, represents in perspective the different parts of the copper micrometer placed at the top of the tube.

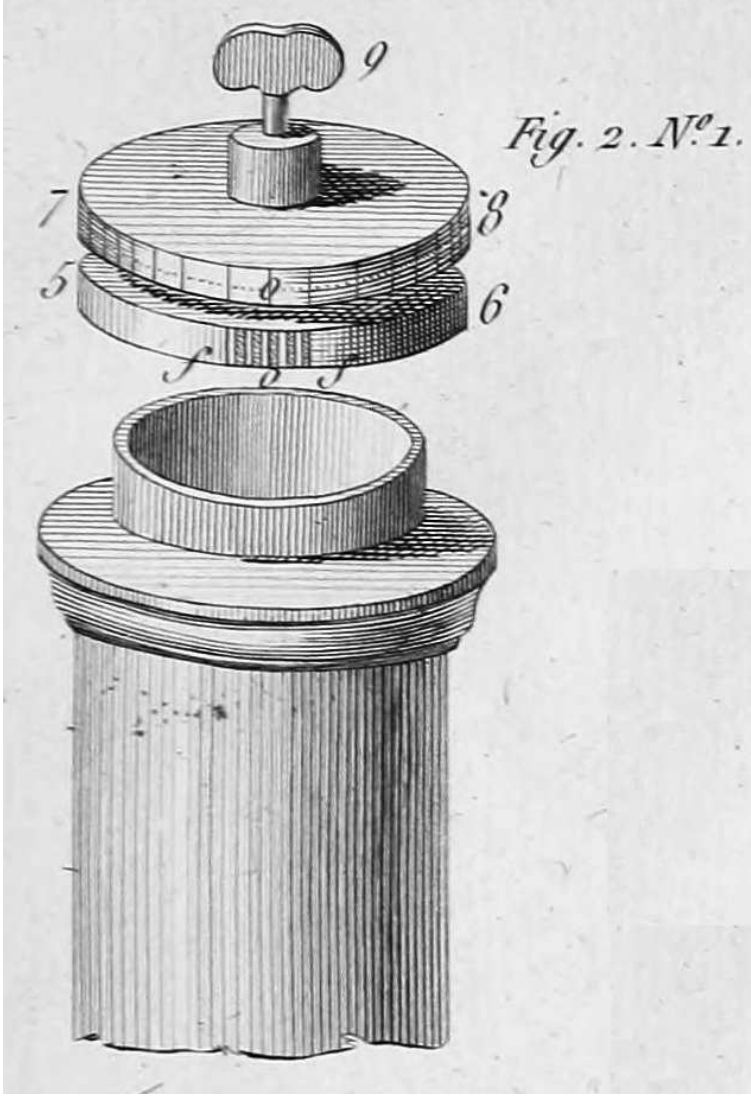


Figure 2, No. 2, represents a vertical section of this micrometer.[747]

[746]In the original: *le tuyau de verre ef*. Coulomb appears to be referring to Figure 1, Number 2. The letter e in this Figure indicates the gilded paper disk attached to the tip of the needle ke. In this Figure I added the letter e' at the base of the glass tube $e'f$. Furthermore, the expression *Le tuyau de verre ef* was translated as "the glass tube $e'f$", instead of "the glass tube ef", in order to avoid confusion with the letter e that appears in this Figure indicating the gilded paper disk.

[747]I have emphasized in Figure 2, Number 2, the numbers 1, 3 and 10.

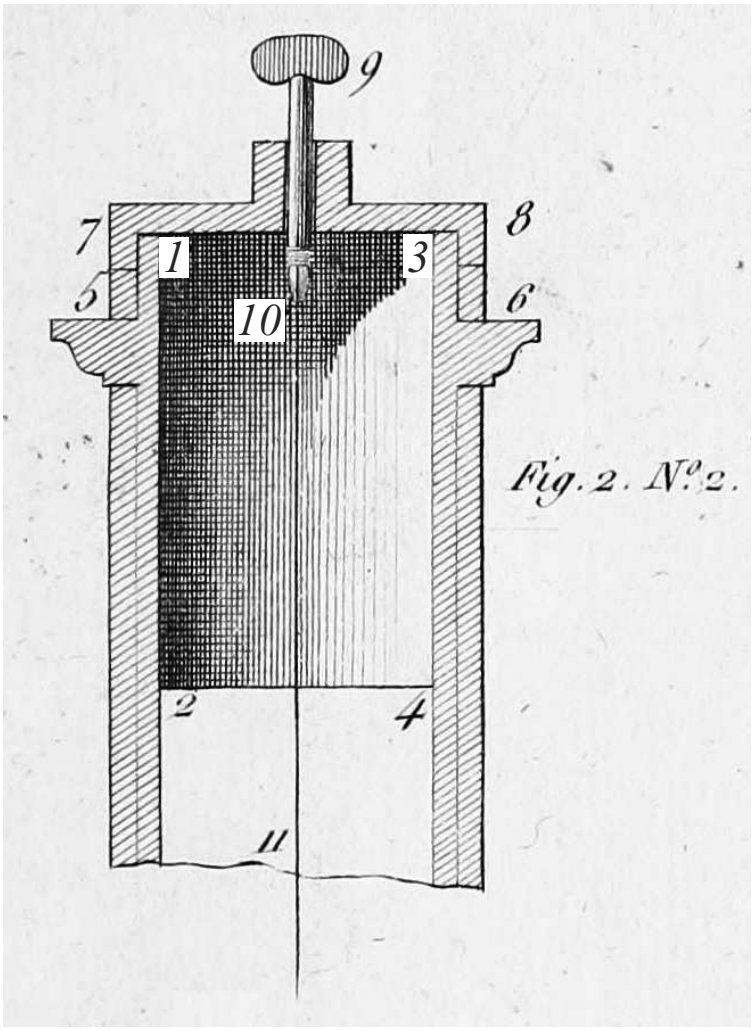


Fig. 2. N.º 2.

It is composed of several parts; in the first place, a copper tube 1, 2, 3, 4 into which first enters the ring 5, 6 which rests on a washer[748] of this tube; this ring has only a simple division of, answering and divided into 5 degrees.[749] The circle 7, 8, which forms the cap of the micrometer, is divided from 5 to 5 degrees around its entire circumference. In this cap enters (Figure 2, No. 2) the rod 9, 10 which clamps in 10 the suspension wire 10, 11; this clamp can turn with fairly strong friction in a ring of the cap and is used to direct the needle ke almost towards point o (Figure 1, No. 2). When we wish to put the balance in operation, we observe, by vertically aligning the suspension wire with the plane e of the gold paper attached, the position of the needle from point o; if it is positioned for example at 5 degrees, then in turning the cap 5 degrees, we will be sure to make the direction of the needle coincide with point o; we then bring point o (Figure 2, No. 1) of the circle 5, 6 on which we said that there was a division of of 5 degrees to point o of the cap 7, 8 divided from 5 to 5 degrees; for then the ring 5, 6 and cap 7, 8, being placed (Figure 2, No. 2) a very small distance apart — a distance just sufficient to ensure they do not touch — can be moved

[748] [Note by Bucciarelli] In French: *bourlet*. This is like a "washer" in English, here like a spacer, a buffer between two moving parts that don't move frequently or any great distance.

[749] From Figure 2, Number 1, it can be seen that the lower ring 56 has a central division at point o, in addition to a division on each side represented by the letter f. The angles fo and of are 5°. The three divisions of ring 56 coincide with the three central divisions of the disk 78, and the central division of this disk is also represented by the letter o.

independently of each other; thus point o remaining motionless in the circle 5, 6 while cap 7, 8 is turned, the angle of torsion of the suspension thread will be measured by the angle of rotation of the cap, plus by the [angular] distance from the needle to point o [of the strip of paper 1o2], when the plane b of this needle (Figure 1, No. 1) will be electrified and repelled by the equally electrified globe a.[750] We use in this balance a copper wire, numbered 12 in commerce, to suspend the needle. We showed in 1784, in the *Mémoires de l'Académie*,[751] that this kind of wire had a very great degree of elasticity, and it would be preferable to silver wire in small experiments if it could be drawn so fine.

22.2 II

We have used two methods to determine the way in which the electric fluid is divided between two bodies brought into contact.

The first consists in placing the electrified body in the electric balance, after having electrified the small disk of gilded paper placed at the end of the needle with the same kind of electricity. When we bring the needle, repulsed by the electric action, back to any distance [we choose] from the electrified body by means of the micrometer of torsion, the angle of torsion given by the micrometer, plus the [angular] distance of the needle from point o, would measure the repulsive action that the two bodies exercise one on the other at this distance. We then bring the electrified body mounted in the balance in contact with the body with which we want it[752] to share its electricity;[753] and by untwisting the suspension wire by means of the micrometer, the needle is brought back to the distance from the body mounted in the balance observed in the first operation. The angle of torsion measured with the micrometer, plus the [angular] distance from the needle to point o, will measure the quantity of electricity which has been left in the body placed in the balance by the body which has been brought into contact with it. Indeed, the distance is the same between the needle and the electrified globe in the first and second observations, but the action of each element of the electric fluid is, as we have proved in the Memoirs which precede, in inverse ratio of the square of the distances and direct ratio of the densities: thus, as here the distances [between the globe and the paper disk] are the same in the two operations, the repulsive action measured by the angle of torsion will be proportional to the quantity of electric fluid.[754]

In this operation, unless the weather is very dry, consideration must be given to the quantity of electricity which is lost in the [time] interval between observations.

22.3 III

The preceding method gives us as a whole the ratio of the quantities of electricity shared between the two bodies;[755] but when I want to obtain the electric density at every point of

[750] That is, the gilded paper disk b and globe a are electrified with charges of the same sign, repelling one another.

[751] See [Coulomb, 1787]. This Memoir is translated in Chapter 7.

[752] That is, the electrified body.

[753] Let g be the body that was initially discharged and that touched the electrified globe a that is in a fixed position inside the electric balance. After the contact between a and g, body g is immediately removed from the balance.

[754] Section 23.1 presents an illustration of this method.

[755] That is, this method gives the ratio of the total amounts of charge in the two bodies.

a conducting body, here is the method I follow.

We use the small balance described in 1785 in the *Mémoires de l'Académie*[756] or better yet we substitute a very fine silver wire in the balance, Figure 2, No. 1, for the copper wire which supports the needle ke. The force of torsion of the silver wire I use is only one-thirtieth that of commercially available number 12 copper wire.

A shellac thread cde (Figure 3) is then drawn by melting a small piece of very pure shellac with a candle; this shellac thread, about the thickness of a coarse horsehair, forms an angle at d; we attach in e vertically a disk e of gilded paper.[757]

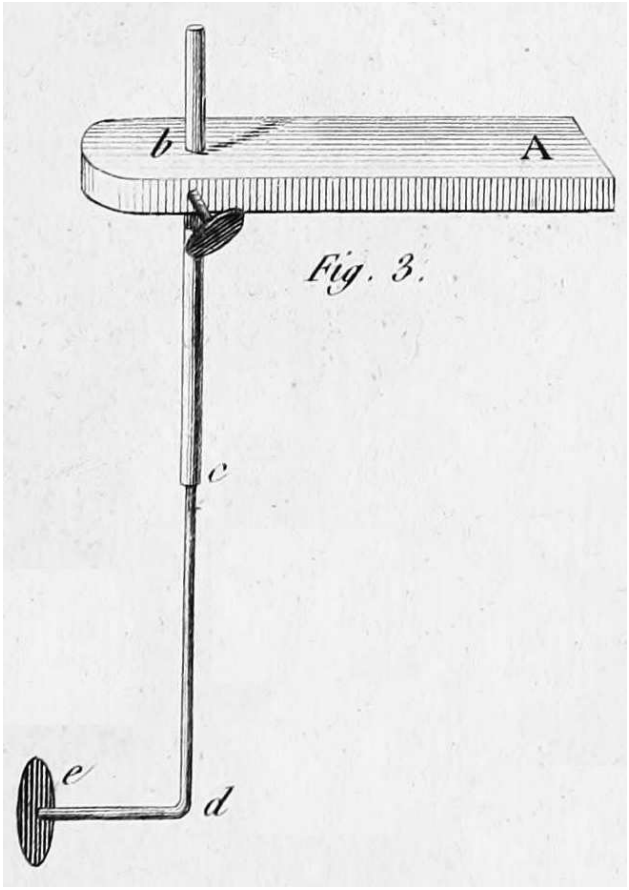


Fig. 3.

After having electrified the disk of the needle by the means described in 1785, we electrify the body, and then touch the paper disk e, which is supported by the shellac thread and the clamp bA, to the body at the point on the body whose density we want to obtain. We then place this small disk in the balance, taking care, in the observations that we want to compare, to always put it at the same point, which is easy, by setting reference points on the lid of the balance so as to always put bA in exactly the same place. As the small disk e is usually only 5 or 6 lines in diameter (1.13 to 1.35 cm) and only 1/18 of a line thick (0.0125

[756] See [Coulomb, 1788b], [Coulomb, 1788d] and [Coulomb, 1788e]. These Memoirs are translated in Chapters 11, 14 and 17, respectively.

[757] In the original: *un plan e de papier doré*. The word "plan" is usually translated as "plane". The alternative expression "disk" is being utilized in this translation to make the text easier to understand. Figure 3 represents another model of Coulomb's proof plane, see footnote 726 on page 319. It consists of a small conducting disk (denoted by the letter e) attached to the insulating support $bcde$. Cylinder bc is coated with an insulating varnish.

cm), it merges in contact with the surface it touches; thus, in the contact, it takes either the [electric] density of the point of the surface which it touches,[758] or at least a density proportional to that of this point; thus, by making it successively touch different points of the body, and presenting it after each contact with the needle, always bringing the needle back to the same point,[759] we will have the ratio of the densities at the different points touched.

In the comparison of the observations which succeed one another, consideration must be given to the loss of electricity by contact with the air; but we easily compensate for this correction, if we always compare two points by three operations carried out at approximately equal intervals of time; here is the method I use to compare [the electric densities of] two points. I first touch one of the points, and I determine its density by placing in the balance the small paper disk which has touched [one of the points]; I touch in the second operation the point whose density I want to compare with that of the first, I determine its density; I touch in the third operation the first point whose density I determined in the first operation; I again determine its density, which I find less than in the first operation, because the electricity has been diminished in the interval by contact with the air; but, taking an average quantity between the two [electric] densities found at the first and at the third observation, I have the measure of its value at the moment of the second observation, the moment when I determined the density of the second point that I want to compare to the first.

In this second method, which is, in general, the most convenient, the simplest and perhaps the most exact for comparing the electric density of the different points of the same or of different bodies, which moreover requires only small torsion balances, a practical difficulty sometimes arises which would disturb all the results, if we were not warned of it; it is that the threads of shellac are not perfectly impenetrable to electricity; they are less so on humid days than on dry days; and the extent more or less depends on the nature of the shellac: the less clear is generally more impenetrable to electricity than the other. This first, drawn in a thread the thickness of a coarse horsehair, must still be tested by making it touch an electrified body at its point e where we attach (Figure 3) the small disk e; it is then presented to the needle of the balance equally electrified.[760] If the extremity of this [shellac] thread seems to drive the needle sensibly away, it must be rejected and only shellac threads should be used which, after having touched an electrified body, have no sensible action on the needle of the balance. We suspect that the reason for this observation is that, when the electricity has penetrated the shellac thread, it is then very difficult to get rid of it: thus, in the comparison of two successive operations where the small disk of gilded paper will first have touched a strongly electrified point, if in a second operation we cause this disk to touch a weakly electrified point, the shellac thread will retain a part of the first electricity with which it will have been penetrated, and the action will be greater than that which would be due solely to the density imparted in the second contact to the gilded paper disk. Thus whenever large surfaces can be used to measure the density of the different points of a body, they should be preferred; we will see several examples of this later in this Memoir.

[758] See also footnote 727 on page 320.

[759] That is, with the needle always at the same angle to the center point o.

[760] That is, the disk attached to the needle of the balance is electrified with a charge of the same sign as the charge of the electrified body that was touched by the tip of the shellac wire.

22.4 IV. First Part. *Of the Manner in which the Electric Fluid is Shared Between Two Globes of Different Diameters Placed in Contact*

22.4.1 First Experiment

We placed in the large balance (Figure 1, No. 1), whose needle is supported by a copper wire numbered 12 in commerce, an electrified globe 6 inches 3 lines in circumference (16.92 cm);[761] the force of torsion that had to be used to bring the needle back to 30 degrees from this globe was observed; this first globe was immediately made to touch another globe 24 inches in circumference (64.97 cm)[762] and, by bringing the needle back to the same point,[763] the force of the torsion was again observed. Here is the result of this experiment:[764]

First test. The globe placed in the balance repelled the needle to 30 degrees before contact, with a twisting force of 145°.[765]

The same globe, after its contact with the large globe, repelled the needle at 30 degrees with a twisting force of 12°.[766]

Second test. The same globe, before contact, repelled the needle at a distance of 30 degrees with a force of 145°.

After contact, with a force of 12°.

Third test. Prior to contact the needle was pushed out 26° with a force of 259°.

After contact, with a force of 21°.

Fourth test. Prior to contact, the needle was driven 22° with a twisting force of 255°.

After contact, with a force of 21°.

Fifth test. Before contact, the globe was repelling the needle at 18° with a force of 231°.

After contact, with a force of 19°.

[761] That is, its diameter is given by $16.92/\pi = 5.39$ *cm*.
[762] That is, its diameter is given by $64.97/\pi = 20.68$ *cm*.
[763] That is, separated 30° from the first globe.
[764] Section 23.1 illustrates this experiment.
[765] This angle of 145° measures the total torsion of the suspension wire taking into account the 30° deviation of the needle attached to the bottom of the wire and the angular deviation of the micrometer attached to the top of the wire.
[766] This angle of 12° also measures the total twist of the wire taking into account the 30° deviation of the needle attached to the bottom of the wire and the deviation of the micrometer attached to the top of the wire.

22.4.2 Result of This Experiment

One of the two globes brought into contact had a circumference of $6\frac{1}{4}$ inches [16.92 cm], the other 24 inches [64.97 cm]; the ratio of their area is approximately :: 14.8 : 1.0;[767] but each test was carried out in less than 1 minute, and the electricity diminished only 1/40 [part] per minute on the day of this experiment. Now, in order to be able to compare the quantity of electricity which the small globe has retained after contact with that which it has lost or, which comes to the same thing, which it has communicated to the large globe due to the contact, it is necessary to observe, as we have already said before, that in each test the distance from the center of the globe placed in the balance to the disk of gilded paper fixed vertically at the end of the needle of the balance is the same in both observations before and after contact; because then, since the distance is the same in the two consecutive operations of each test, the action of the globe on a electrified point placed outside of this globe,[768] being measured by the inverse of the square of the distances of their center and in direct ratio of the quantities of electricity expanded on the surface of the globe, will be proportional to the quantities of electricity which the globe contains before and after contact. However, this action being proportional to the angle of torsion, it follows that the quantity of electricity which the small globe contains before and after contact is proportional to the angle of torsion.

In the first test, the twisting force for an [angular] distance of 30 degrees between the end of the needle and the center of the globe is, before contact, 145 degrees; it is reduced to 12 degrees after the contact: thus, to have the quantity of electricity which the large globe has taken on, it is necessary to subtract 12 degrees from 145 degrees. It will result that in the contact the globe of 24 inches in circumference has taken on a mass of electricity measured by 133 degrees and has left on the small globe only a [mass of electricity] measured by 12 degrees: thus the quantities of electricity shared between these two globes are very close :: 11.1 : 1.0.[769]

By following the same procedure, this ratio will be found almost exactly the same for all the other tests.

But the surfaces of the two globes placed in contact are in the ratio of 14.8 to 1.0; thus the two globes placed in contact are not charged with electric fluid in a ratio as great as [the ratio of] their surfaces. If, according to this experiment, we want to determine the ratio of the density of the electric fluid which spreads after contact uniformly over the surface of the two globes, without penetrating into the interior of the two globes, as we have proved in our Fourth Memoir,[770] we must divide the ratio of the surfaces of the two globes by the ratio

[767] The ratio of their surfaces is equal to the square of the ratio of their radii, or the square of the ratio of their circumferences. Therefore this ratio is given by

$$\frac{\textit{Larger area}}{\textit{Smaller area}} = \left(\frac{24}{6.25}\right)^2 = \frac{14.8}{1.0} \ .$$

[768] This portion of the original sentence, *sur un point électrique placé en dehors de ce globe*, does not appear in Potier's edition of Coulomb's works, [Coulomb, 1789, p. 429] and [Potier, 1884, p. 191].

[769] That is,

$$\frac{\textit{Electricity in the larger globe}}{\textit{Electricity in the smaller globe}} = \frac{133}{12} = \frac{11.1}{1.0} \ .$$

[770] This Memoir is translated in Chapter 20.

of the quantities of electricity they contain; thus the ratio of the surfaces being :: 14.8: 1.0 and that of the quantities of electric fluid :: 11.1 : 1.0, the mean density of the electric fluid spread after contact on the surface of the small globe will be to that of the large globe

$$:: 14.8 : 11.1 \ .$$

To prove it, let S be the surface of the large globe, Q the quantity of electric fluid spread over its surface after contact, D the [surface] density of this fluid.

Let S' be the surface of the small globe, Q' its quantity[771] of electric fluid, D' its [surface] density, we will have

$$D' = \frac{Q'}{S'} \qquad \text{and} \qquad D = \frac{Q}{S} \ ;$$

thus,

$$\frac{D'}{D} = \frac{Q'S}{S'Q} \ .$$

In our experiment,

$$\frac{Q'}{Q} = \frac{1.0}{11.1} \qquad \text{and} \qquad \frac{S}{S'} = \frac{14.8}{1.0} \ ;$$

thus

$$\frac{D'}{D} = \frac{14.8}{11.1} = 1.33 \ .$$

We neglect here the amount of electricity lost on each trial from one observation to the next; it was hardly more than 1/50 [part per minute] the day this experiment was made, because each observation lasted only fifty seconds and the electricity of the globes did not quite decrease by 1/40 [part] per minute.[772,773]

22.5 V. Second Experiment

We wanted to compare in this experiment the amount of electricity acquired by a globe of $11\frac{1}{2}$ inches in circumference[774] (31.13 cm), put in contact with a globe of $6\frac{1}{4}$ inches in circumference (16.92 cm), placed in the balance at the same point, before and after contact, as in the preceding experiment.

First test. The globe of $6\frac{1}{4}$ inches in circumference electrified and placed in the balance repels, before contact, the disk of the needle at 27 degrees, with a twisting force of 170°.

After contact, with a force of 42°.

Second test. Before contact, it repels the needle at 26 degrees, with a twisting force of 169°.

After contact, with a force of 41°.

[771] In the original: *quantité*, [Coulomb, 1789, p. 430]. By a lapse of Potier in his edition of the works of Coulomb, he substituted here the word "quantitè" for "densité" (density), [Potier, 1884, p. 192].

[772] [Note by Potier] From the tables calculated by Mr. Plana, in volume VII of the second series of the *Mémoires de l'Academie de Turin*, result, for the ratio of the densities, 1.31.

[773] See [Plana, 1845] and [Plana, 1854].

[774] In the original: *de tour*.

22.6 VI. Result of This Experiment

In this experiment, we compare two globes whose surfaces are in the ratio :: 3.36 : 1.[775]

By calculating the two tests, we find that after contact the mass of the electric fluid of the large globe is to that of the small one [in the following ratios:]

By the first test :: 3.05 : 1.00 .
By the second test :: 3.12 : 1.00 .
[Total:] 6.17 ,

which gives for the average ratio 3.08 : 1.00.[776]

Thus, by a calculation similar to that which ends the *previous Section*, we will find the [surface] density [of electricity] of the small globe to the density of the large globe :: 3.36 : 3.17 :: 1.06 : 1.[777]

Thus, in this experiment where the surfaces are approximately in the ratio of $3\frac{1}{3}$ to 1, the electric densities differ very little from each other.[778,779]

22.7 VII

When the globe which we wish to compare is very small relatively to that with which it is brought into contact, then the quantity of electric fluid which remains in the small globe after the contact is almost insensible; and, unless the air is very dry, the supports are very insulating, and the small globe, before contact, has been charged with a large quantity of electricity, then, using the preceding method, we can only approximately determine the ratios according to which the electric fluid is divided between the two globes. In this case, here is the means I used: I electrify the large globe placed (Figure 4) outside the balance, on an insulating support.

[775]That is,

$$\frac{Larger\ area}{Smaller\ area} = \left(\frac{11.5}{6.25}\right)^2 = \frac{3.38}{1.0} .$$

[776]That is, the ratio of the total amount of electricity on the large globe to the total amount of electricity on the small globe is given by 3.08 to 1.

[777]That is,

$$\frac{Surface\ density\ of\ charge\ on\ the\ smaller\ globe}{Surface\ density\ of\ charge\ on\ the\ larger\ globe} = \frac{3.36}{3.17} = \frac{1.06}{1.0} .$$

[778][Note by Potier] The surface ratio $(11.5/6.25)^2$ is given by $(1.84)^2 = 3.3856$; the ratio of the charges 3.085; the ratio of the densities would therefore be 1.10 and not 1.06. Plana's tables give 1.11.

[779]See [Plana, 1845] and [Plana, 1854].

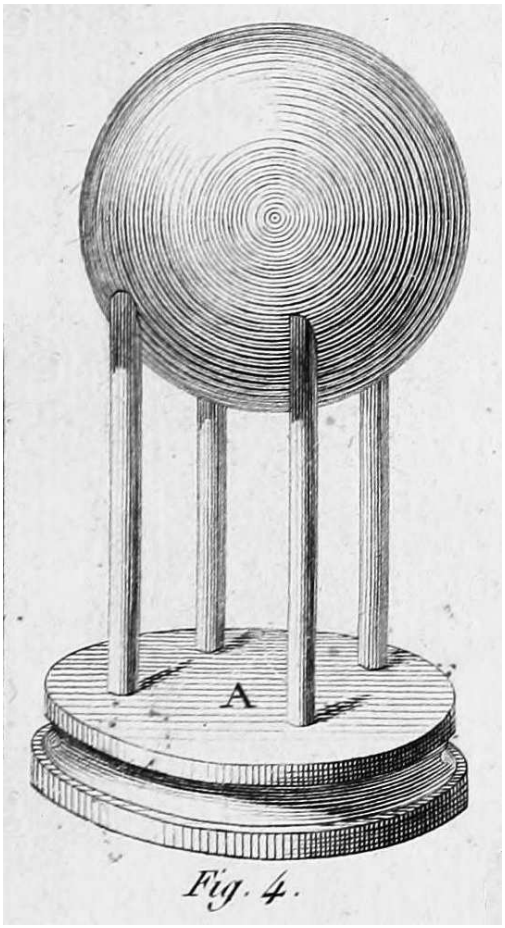


Fig. 4.

After having also electrified the disk of the needle,[780] I make the small globe touch the large electrified globe; I present this small globe to the needle of the balance and I bring this needle closer to the small globe by twisting the very fine and sensitive suspension thread. I determine in this first observation the [angular] distance of the needle to the small globe and the force of torsion which retains it at this distance. We then take the small globe out of the balance, we destroy its electricity by touching it with the finger; it is then made to touch twenty times in succession, more or less, the large globe, by destroying the electricity of the small globe after each contact, except at the twentieth, when the small globe is replaced in the balance, at the same point where it was at the first observation. The needle is then brought back by twisting the suspension wire at the same [angular] distance from the globe as in the first observation; we observe this angle of torsion and we reduce the observation by taking into account, in the result, the quantity of electricity which would have been naturally lost by the sole contact with the air, from one observation to another.

22.8 VIII. Third Experiment

The 8-inch (21.66 cm) globe, electrified and placed outside of the balance on the insulating support (Figure 4), was touched by a small globe about 1 inch.[781] The surfaces calculated

[780] That is, after having electrified the gilded paper disk of the needle with an electric charge of the same sign as the charge with which the great globe was electrified.

[781] In Section 22.9 it will be clear that Coulomb is referring here to a globe 8 inches in diameter and another approximately 1 inch in diameter.

from the most accurate measurements that could be made were approximately [in the ratio] :: 62 : 1; the needle of the balance was suspended with a silver wire, the twisting force of which at the same angle was hardly more than the sixtieth part of that of the copper wire numbered 12 in commerce.

Test. The large globe being electrified after a first contact, the small globe placed in the balance repelled the needle at a distance of 44 degrees from the center of this globe, with a force of torsion of 244 degrees.

After twenty contacts, at each of which the electricity had been destroyed, except for the last, the needle was driven out at 44 degrees with a twisting force of 126 degrees.

Continuing the same experiment, after twenty new contacts, the needle was driven out at 44 degrees with a force of 66 degrees.

22.8.1 Result of This Experiment

The force of torsion is proportional to the quantity of electric fluid with which the small globe was charged each time it was presented in the balance, since it was always, at the time of each observation, placed at the same distance of the needle: this force was initially 244 degrees, which was reduced to 126 degrees after twenty contacts; thus the diminution of electricity occasioned by these twenty contacts was 244 - 126 = 118.

Thus 118 degrees represent the loss occasioned by the twenty contacts; therefore, to determine the quantity of electricity that the small globe took in an average contact, it is necessary to divide 118 by 20, which will give approximately the quantity of electricity that the small globe took in an average contact, that is to say approximately around the tenth contact: but, in this case, the force of repulsion measured in the balance had to be approximately an average between those of the two observations, that is to say, it had to be equal to

$$\frac{244 + 126}{2} = 185 \ .$$

Thus the ratio between the quantity of electricity of the large globe and that of the small one, after one contact, will be

$$:: 185 : \frac{118}{20} = 31.4 \ ;$$

but it must be remarked that from one observation to another, in the time necessary for the twenty contacts, the electricity of the large globe diminished about 1/8 or 4/32; thus, since we have just found the diminution occasioned by each contact of $\frac{1.0}{31.4}$, it follows that the diminution 4/32 occasioned by the contact with the air was approximately equivalent to four contacts; thus, in the reduction of the observations, it is necessary to count on twenty-four contacts instead of twenty, which will give, for the corrected ratio between the quantities of electricity of the large globe and the small globe,

$$185 : \frac{118}{24} :: 37.6 : 1.00 \ .$$

But, since we have proved that electricity is spread only over the surface of bodies and that the ratio of the surfaces is :: 62 : 1, it follows from a calculation analogous to that at the end of Section 22.4 that the [surface] density [of electricity] of the small globe is to that of the large globe :: 62 : 37.6 :: 1.65 : 1.00.

22.9 IX

To complete this research, I had the globe 8 inches in diameter [21.66 cm] touched alternately by a globe 1 inch in diameter [2.71 cm], and by a small globe whose diameter, calculated from its weight, was only 2 lines [0.45 cm]. By successively placing these two globes in the balance, I found that the density of the electric fluid on the surface of the globe of 2 lines in diameter was greater than on the surface of the globe of 1 inch, but that it was not quite double that on the surface of the globe 8 inches in diameter. In this experiment, the diameter of the 8-inch globe is to that of 2 lines :: 48 : 1; the surfaces are therefore in the ratio :: 2304 : 1; the densities [of electricity] on the surfaces are [to one another] from small to large :: 2 : 1; thus this ratio 2 to 1 can be regarded as the limit of the ratio of the average electric density of two globes which are separated after having brought them into contact.

In the rest of this Memoir, we will see that, when the surface of a globe is touched by a very small [conducting] disk, this small disk, at the moment of its separation, takes a quantity of electricity double that of the surface of the globe it has just touched.[782] The gilded paper disk we use for this experiment is only 1/18 of a line thick [0.0126 cm]; it is easy to perceive, and theory will subsequently demonstrate, the analogy of these two effects.

22.10 X. Remark

We could rigorously calculate the quantity of electricity which is shared at each contact between the globe 8 inches in diameter and that of 1 inch which we have just determined (in Section 22.8) by approximation by taking average quantities.

Let A be the quantity of electricity contained on its surface by the globe 8 inches in diameter, let the globe of 1 inch, at the moment of contact, remove from it a portion A/n, the quantity of electric fluid which will remain in the large globe after the first contact will be

$$\left(A - \frac{A}{n}\right) = \frac{n-1}{n}A \ ;$$

at the second contact it will be

$$\left(\frac{n-1}{n}\right)^2 A \ ,$$

and at the twentieth contact it will be

$$\left(\frac{n-1}{n}\right)^{20} A \ .$$

But we have just seen that it was necessary to count on twenty-four contacts instead of twenty, because of the quantity of electricity lost by the contact with the air from one observation to another; thus, if we calculate immediately from the first to the third observation, we must count on twenty-four contacts from the first to the second, and on about twenty-five [contacts] from the second to the third [observation], because it lasted a little longer than the first. Thus, from the first until the end of the third observation, it is necessary to count

[782] This is an erroneous assumption by Coulomb. The small conducting disk acquires the same amount of electricity as there was in an area equal to that disk on the surface of the globe at the point it just touched.

on forty-nine contacts, and as we have at the first observation $A = 244$, which is reduced to 66 at the end of the third observation, we will have the equation

$$\left(\frac{n-1}{n}\right)^{49} \cdot 244 = 66$$

or

$$\frac{n}{n-1} = \left(\frac{244}{66}\right)^{1/49} = 1.027 \ ;$$

thus

$$n = \frac{1027}{27} = 38.04 \ .$$

But it should be noted that the [initial] quantity of electric fluid of the large globe being A, it has been reduced by the contact to $A\frac{n-1}{n}$ while the little globe took a portion A/n; thus the quantities of electric fluid shared between the two globes are between them

$$:: n-1 : 1 :: 37.04 : 1 \ ;$$

and the [ratio of the] surfaces being :: 62 : 1, the density of the electric fluid on the surface of the small globe will be to the density of the electric fluid on the surface of the large globe

$$:: \frac{62.00}{37.04} : 1 \ ,$$

[that is,] :: 1.67 : 1. We found by approximation for this ratio [the value] 1.65 : 1, which does not differ significantly [from the present result].

22.11 XI. General Result

By taking an average value between the results of many performed experiments or by the preceding method, or by touching the two globes alternately with a small circular disk of 5 lines,[783] as we have explained (in Section 22.3), we have formed the following Table, which represents the way in which the electric fluid is divided between two globes of different diameters:[784]

Ratio of radii	Ratio of surface areas	Ratio of the electric density between the small and the large globe
1	1	1
2	4	1.08
4	16	1.30
8	64	1.65
∞	∞	2.00

It should be observed that this Table only indicates the ratio of the densities of the electric fluid when, after having separated the two globes, the electric fluid spreads uniformly over their surface: we are going to see presently that during all the time the globes are in contact, the electric fluid is far from being spread uniformly [on their surfaces].

[783] This conducting disk is attached to an insulating handle, and the proof plane is used to collect charges. It has 5 lines in diameter, that is, 1.13 cm.

[784] Section 23.2 compares Coulomb's observations and Poisson's theoretical calculations made 20 years later.

22.12 XII. *The Density of the Electric Fluid on the Different Points of Two Globes in Contact*

After having compared two globes of different diameters with each other to determine the quantity of electricity which they acquire when they are brought into contact, I sought to determine according to which law the electric fluid is distributed, during the time of contact, on the different points of the globes; I used here the small electric balance where the needle is suspended by a very flexible silver wire: this balance is described in my First Memoir on electricity, printed in the Volume of the Academy for 1784.[785] We use, to determine the electric density of the different points of the globes, a small circular disk of gilded paper e (Figure 3), 4 to 5 lines in diameter [0.90 to 1.13 cm], supported by a shellac thread cde, fixed to a cylinder cb of glass or wood dried in the oven and coated with insulating varnish.[786] This cylinder enters and is fixed with a screw in the hole b of the clamp Ab, which is placed on the lid of the balance. The whole operation, when we want to compare [the amount of electricity contained at] two points, consists in making the plane e touch against the first point; we then present this disk [e] in the balance to the disk of the needle which we took care to electrify beforehand;[787] the needle is brought back to a given distance from this disk [e] by twisting the suspension wire; we carefully observe the point where the needle responds and the angle of torsion of the wire measured by the micrometer plus the [angular] distance of the needle to point o where the torsion is zero. We then touch the second point that we want to compare with the same disk e and, by placing it in the balance, we bring the needle back through the micrometer to the same distance as in the first observation: we take into account the angle of torsion; we then retouch the first point observed and, by always bringing the needle back to the same distance, we have the variation of the electricity from the first to the third experiment. Thus, if we take care to put between each observation the same duration of time, it suffices, to compare the density of the first point to the second, to take for the first point an average quantity between the forces of torsion found at the first and at the third observation; this average quantity will give the density of the first point at the time of the second observation: thus, by comparing it with the force of torsion found at the second observation, we will have the ratio of the electric density of the first and second points.[788]

22.13 XIII. Fourth Experiment

The two globes are equal and each 8 inches in diameter [21.66 cm]; the point placed at 90 degrees from the contact is compared with the points placed at 30 degrees, 60 degrees and 180 degrees.[789]

[785] This Memoir is translated in Chapter 11.

[786] Coulomb will use the proof plane shown in Figure 3 to collect the electric charge at different points on the surface of an electrified globe, see footnotes 726 and 757 on pages 319 and 335, respectively.

[787] Coulomb electrifies the needle disk with a charge of the same sign as that collected by the small paper disk e of the proof plane in such a way that these two disks repel each other.

[788] Section 23.3 illustrates this procedure.

[789] Let two globes with centers C and C' touch each other at just one point. The angles α indicated by Coulomb are illustrated in the Figure of this footnote, namely, $\alpha = 30°$, $60°$, $90°$ and $180°$:

First test. The point placed at 30 degrees from the contact of the two globes compared with the point placed at 90 degrees (Figure 5).[790]

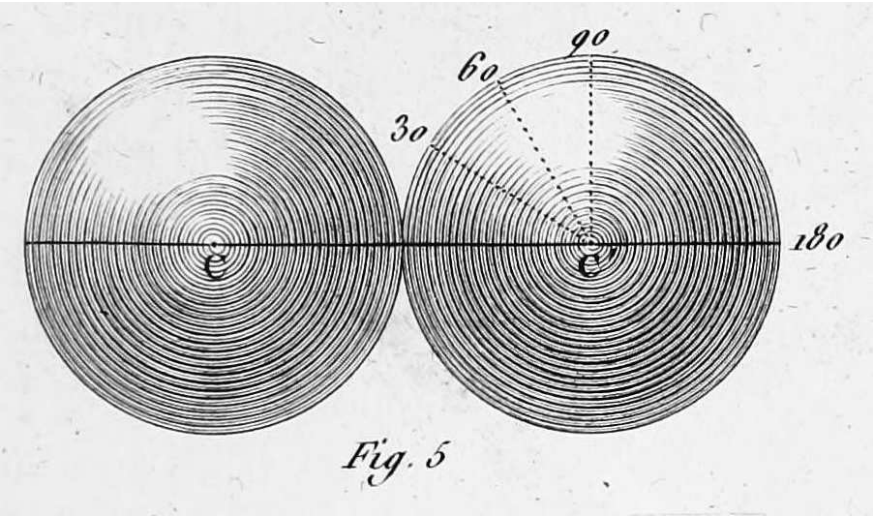


Fig. 5

Having touched one of the globes with the small plane of gilded paper, at 30 degrees from the contact, the needle was observed at a distance of 20 degrees from the small disk placed in the balance; the twisting force or repulsive force that drove the needle away was 7 degrees.

Having touched at 90 degrees, everything else, as in the preceding observation, the repulsive force was ..... 31°.

Having touched at 30 degrees, the repulsive force was ..... 6°.

Having touched at 90 degrees, the repulsive force was ..... 27°.

Second test. We compare the point placed at 60 degrees from the contact, with that which is at 90 degrees; the distance from the needle to the small disk e, when it is placed in the balance, is always 22 degrees.

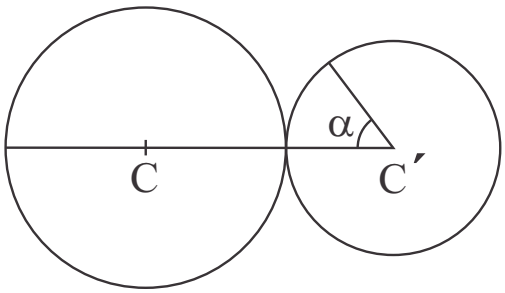
Having touched at 60°, the repulsive force was ..... 21°.

Having touched at 90°, the repulsive force was ..... 23°.

Having touched at 60°, the repulsive force was ..... 17°.

Having touched at 90°, the repulsive force was ..... 21°.

Third test. The point placed at 90 degrees from the contact is compared with that placed at 180 degrees. The needle and the disk e, placed in the balance, are at 25 degrees from each other.



[790]Figure 23.5 of Section 23.3 illustrates the proof plane touching the point placed at 90° from the contact point.

Having touched at 90°, the repulsive force was ..... 20°.
Having touched at 180°, the repulsive force was ..... 19°.
Having touched at 90°, the repulsive force was ..... 17°.
Having touched at 180°, the repulsive force was ..... 18°.

Fourth test. When we touch [with the gilded paper disk e] one of the globes at 20 degrees from the point of contact and below, and then present the small disk e which has touched [this point of the globe] in the balance, we notice that the action is nil, or at least insensible on the needle; so that we can in the two globes in contact, regard the electricity as zero from the point of contact up to 20 degrees from this point.

22.14 XIV. Fifth Experiment

We put two globes in contact, one of which is 8 inches in diameter, and the other 4 inches [21.66 and 10.83 cm], and we try to determine how the electric fluid is distributed on the surface of the two globes, by comparing, as in the preceding experiment, the point at 90 degrees from the contact, with all the others.

First test, small globe. By comparing on the small globe the point at 30 degrees from the contact, with that at 90 degrees, the density at the point 30 [degrees] was almost insensible, and we cannot evaluate it beyond the eighteenth part of that at 90 degrees.

Second test, small globe. The point placed at 90 degrees is compared with that placed at 60 degrees.
Touched at 60°, the repulsive force was ..... 18°.
Touched at 90°, the repulsive force was ..... 28°.
Touched at 60°, the repulsive force was ..... 18°.
Touched at 90°, the repulsive force was ..... 24°.

Third test, small globe. We compared the point at 90° with that at 180°.
Touched at 90°, the repulsive force was ..... 21°.
Touched at 180°, the repulsive force was ..... 28°.
Touched at 90°, the repulsive force was ..... 20°.
Touched at 180°, the repulsive force was ..... 26°.

Fourth test, large globe. Touching the large globe at 30° from the contact and at 90°.
Touched at 30°, the repulsive force was ..... 16°.
Touched at 90°, the repulsive force was ..... 18°.
Touched at 30°, the repulsive force was ..... 14°.
Touched at 90°, the repulsive force was ..... 15°.

Fifth test, large globe. The density is approximately the same at 90 degrees and at 180 degrees from the point of contact; it is almost insensible up to 6 or 7 degrees from this point. By touching alternately the point at 90 degrees of the two globes, we find the density of the small globe greater than that of the large globe in these points, in the ratio of 1.25 to 1.

22.15 XV. Sixth Experiment

An 8 inch globe and a 2 inch globe were brought into contact.

First test, small globe. Touched at 90° and at 180°.
Touched at 90°, the repulsive force was ..... 27°.
Touched at 180°, the repulsive force was ..... 35°.
Touched at 90°, the repulsive force was ..... 22°.
Touched at 180°, the repulsive force was ..... 29°.
Touched at 90°, the repulsive force was ..... 19°.

22.16 XVI. Result of the Three Previous Experiments

It will be easy, from these experiments, to determine by calculation the ratio of the densities at the different points of the globes in contact: let us take for example the fourth experiment, where the two globes are equal. We find (*first test*) that at the first observation, the repulsive force of the point at 30 degrees, is represented by 7 degrees; at the third observation, it is represented by 6 degrees; thus the average force at the moment of the second observation, when the density of the point at 90 degrees from the contact was determined, was 6°30′: but at the same moment, the density or the repulsive force of the point at 90 degrees from the contact, was found 31 degrees; thus, dividing 31 degrees by 6 and $\frac{1}{2}^{\circ}$ we find, to express the ratio of the density at 90 degrees, with that at 30 degrees, the number 4.77.

By comparing by the same method the second, third and fourth observation, we will have for the same ratio [the value of] 4.83.

If we take an average value between these two results, which however differ very little from each other, we will have for the average ratio [the value of] 4.80.

By the same method we will find, according to the second test, for the average ratio of the electric density of the points at 60 and at 90 degrees from the point of contact [the value of] 1.25.

In the third test of the same experiment for the points at 90 degrees and at 180 degrees, we will find the average ratio of the densities, measured by 0.95.

Thus the density is very small up to 30 degrees; it increases rapidly up to 60 degrees, little from 60 to 90 degrees, and it is almost uniform from 90 up to 180 degrees.

By the same calculation, it will be found (*fifth experiment*) that when one of the globes is only half the diameter of the other, the density is almost zero in the small globe up to 30 degrees.

[We also find] that the point at 90 degrees, compared with that at 60 [degrees], gives for the average ratio of the densities approximately [the value of] 1.70.

[We also find] that the point at 90°, compared with that at 180°, gives for the average ratio of the densities the quantity 0.75.

So that it increases from 60 to 90 degrees in the ratio of 10 to 17, and from 90 to 180 degrees, in that [ratio] of 75 to 100.

The same calculation will give (*sixth experiment*), that when two globes are put in contact, whose diameters are like 4 to 1, the density of the small globe from 90 degrees to 180, increases in the ratio of 100 to 1.43.[791]

[791] Section 23.3 presents a comparison of the values measured by Coulomb with the values calculated by Poisson.

22.17 XVII

It results from these three experiments, that the more the two globes are unequal [in size], the more the density varies on the small globe, from the point of contact up to 180 degrees from this point, and the more it approaches uniformity on the large globe; increasing rapidly from the point of contact where it is zero, up to 7 or 8 degrees from this point, being uniform over the rest of the globe. Thus, for example, when an 8-inch globe was brought into contact with a 2-inch globe, the density was found to be insensible in the small globe from the point of contact up to 30 degrees of this point; that at 45 degrees from the point of contact, it was about a quarter of that at 90 [degrees]; and that from 90 to 180 degrees, it increased in the ratio of 10 to 14: in the 8-inch globe, on the contrary, the density was zero up to 4 or 5 degrees from the point of contact; it then increased rapidly, and from 30 to 180 degrees it was almost uniform. We will see in the Second Part of this Memoir, that these results are indicated by the theory, by calculating the action, either repulsive or attractive of the electric fluid, according to the law of the inverse ratio of the square of the distances.

22.18 XVIII. Second Part. *Theoretical Analysis to Determine the Distribution of the Electric Fluid on the Surface of Two Globes in Contact, and to Determine Their Average Density, When the Two Globes Being Separated, They Cease to Act on Each Other*

The experiments reported in the First Part of this Memoir, were made before having attempted to calculate, according to the theory, the distribution of the electric fluid on the surface of the two globes in contact. When I wanted to try to calculate this distribution according to the law of the inverse ratio of the square of the distances, I saw that I lacked some facts to which the calculation could be applied directly; I have therefore been obliged to report in this Second Part, as far as I needed them, the result of several new experiments made according to the procedures indicated in the First Part

We saw in our Fourth Memoir (Volume of 1786),[792] that when a conducting body was charged with electricity, the electric fluid did not penetrate into the interior of the body, but that it was distributed only on its surface; from there it results that when we touch a solid body with a surface of the same shape as the body, whatever thickness this surface has, it will take on half of the electricity of the body when put in contact at homologous points.[793]

This last phenomenon had already been observed by several authors, using ordinary electrometers; we can verify it in an exact manner by placing, on a very dry day, a solid body in our large balance, on a highly insulating support; if this body is touched, after having electrified it, by a surface which has exactly the same shape, taking care to put the two bodies in contact in a homologous position,[794] and if we observe, by bringing back the

[792] This Memoir is translated in Chapter 20.

[793] Again, Coulomb made an error here, as was pointed out in footnote 782 on page 343, since this surface will acquire the same amount of electricity as there was in the same size area of the electrified body.

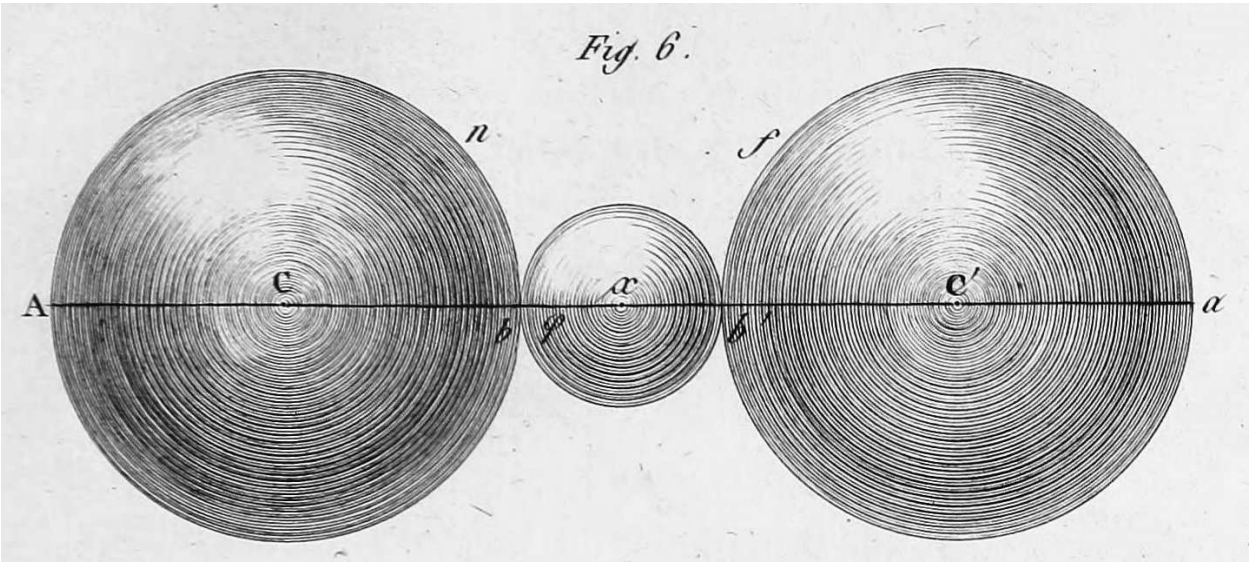
By "homologous points" or "similar points" (in the original: *points homologues*), Coulomb was referring to points located on equivalent parts.

[794] That is, in a similar, homologous or equivalent position.

needle to the same point, the twist of the micrometer before and after the contact, it will be found that the surface has taken from the solid body exactly half of its electricity. If the air were impenetrable to electricity, if the surface of the best polished body was not an assemblage of small irregularities of molecules and voids which are probably infinitely greater than the volume of small solids, the electric fluid would only have an infinitely thin thickness on the body, as the theory indicates. But as there is never in nature a perfect surface nor air impenetrable to electricity, the electric fluid, in its distribution, forms a layer of a certain thickness around the body, that we will search to determine in another Memoir; a thickness which varies according to the density of the electric fluid and according to the state of the air, but which in general is too small, especially on very dry days, for it to be necessary to have regard to it in all questions where we seek to determine the distribution of the electric fluid on the non-angular surfaces.

22.19 XIX

To have a first idea of the way in which the electric fluid is distributed between the different globes, let us place (Figure 6) three globes in contact in a straight line; the axis Aa passing through the points of contact, let us suppose that the two globes at the extremities are equal [in size].



No matter how the electric fluid is distributed between the three globes, since the two globes A and a are similar and similarly posed, relatively to globe x, it is clear that they will both contain an equal amount of electric fluid: this electric fluid, as the theory indicates, will be unevenly spread over the surface of the three-body system; it will be compressed towards the points of the surface which are near A and a, and null towards the points of contact b and b'.

But suppose that the electric fluid of each globe is uniformly spread over the surface of these globes, and that it can only escape through the point of contact; there must, in this supposition, be a ratio between the density of globes C and C' at the extremities and [that] of globe x at the center, such that there is equilibrium between the action of the electric fluid of globe C on the point of contact in the direction CB, and of the other two globes C'

and x in the opposite direction.

Let R be the radius of globes A and a; let r be the radius of the middle globe, whose center is at x; let D represent the [surface] density of the electric fluid which we suppose uniformly spread over the two globes A and a; let δ represent the density of the fluid uniformly spread over the surface of the middle globe, whose center is at x.[795]

[In this case,] the action of globe A on the point of contact b which is placed on the surface of this body, will be equal to D.[796]

The contrary action of globe a on the same point b, which is distant from its surface by the quantity $2r$, will be equal to[797]

$$2DR^2 : (R + 2r)^2 \ .$$

The action of globe x on point b which is on its surface, will be equal to δ.[798]

Thus, for the electric fluid not to pass from one globe to the other, and for there to be equilibrium at the point of contact, it is necessary that the action of globe C along Cb must be equal to the action of the two other globes on point b along the opposite direction; so we have the formula[799]

$$D\left[1 - \frac{2R^2}{(R + 2r)^2}\right] = \delta \ .$$

By examining this formula, we find that the density δ of the electric fluid of the central globe is negative if $2R^2/(R + 2r)^2$ is greater than unity; that it is zero when this quantity is equal to unity, that is, $\delta = 0$, when

$$R + 2r = R\sqrt{2} = 1.41R \ ,$$

or when $R = 5r$;[800] finally δ will be positive whenever R is smaller than $5r$.

Although this first formula is not based on a rigorous theory, but only approximate, it is good to see how far it goes from the truth, by comparing it with experiment.

[795] Let Q be the total amount of charge on the large globe of radius R on the far left with center at C and q be the total amount of charge on the small globe in the middle with radius r and center at x. Their surface charge densities D and δ, assuming they are uniformly distributed over each of the globes, are then given by, respectively: $D = Q/(4\pi R^2)$ and $\delta = q/(4\pi r^2)$.

[796] See Section 23.4 on page 376 for a discussion of the results presented in this Section. See, in particular, Equations (23.5) and (23.6).

[797] I will follow here the nomenclature of Section 23.4. In this case the force per unit charge, F/q_1, would be given by:

$$\frac{F}{q_1} = \frac{kQ}{r_1^2} = \frac{kQ}{(R + 2r)^2} = \frac{D4\pi kR^2}{(R + 2r)^2} = 2\pi \cdot \frac{2DkR^2}{(R + 2r)^2} \ . \tag{22.1}$$

Supposing $k = 1$, we can see once again that Coulomb is not including the constant factor 2π, as mentioned in Section 23.4.

[798] Due to a lapse in the original text, it appeared here d instead of δ. Moreover, just like in footnote 796, the force per unit charge here should be written by Coulomb as being given by $2\pi\delta$ instead of δ, assuming $k = 1$. In this Fifth Memoir Coulomb is not including the factor 2π in all equations, see Section 23.4.

[799] The coefficient 2π that was neglected in the previous expressions of the actions of the three globes is irrelevant in the next equation, as it is common to the three components of this equation and can be canceled out.

[800] The exact solution of equation $R + 2r = R\sqrt{2}R$ is $R = 2r/(\sqrt{2} - 1) = 4.828r$. Coulomb is approaching this value to $R \approx 5r$.

22.20 XX. Seventh Experiment

The details into which we have entered in giving an account of the preceding experiments, indicate sufficiently the corrections and the precautions which must be employed; in order not to unnecessarily lengthen this Memoir, we will suppress the details of the experiments in the following, unless we are obliged to perform some new operations not yet indicated.

When I placed (Figure 6) between the two electrified bodies A and a, a small globe whose diameter was less than the sixth part of the diameters of globes A and a, and that I then presented this small globe to a very sensitive torsion balance, the little globe gave me no sign of electricity; however, no matter how small this globe was, I did not find that it had acquired a negative electricity, as the theory indicated.

22.21 XXI. Explanation of This Experiment

The difference here between experiment and theory comes from the fact that when the intermediate globe is very small, the action of the large globes on each other is very significant; that at the point of contact, as well as in the parts which are close to this point, the electric density of the large globes is almost nil: thus, if to determine the action of globe C' on point b, we divide the surface in two parts, one formed of a small circle whose diameter is approximately $b'f$,[801] on which the density is nil or very small; the other [part formed] from the rest of the surface of the globe, where we shall suppose the density uniform and equal to D, the action of globe a on point b will no longer be measured by

$$\frac{2DR^2}{(R+2r)^2} ,$$

which represents the entire action of the surface of a globe covered with electric fluid, whose density would be D; but only by this quantity diminished of the action of the surface, the diameter of which is $b'f$, a surface which can be taken for a circular plane, if $b'f$ is not greatly extended. Now, if we determine (Figure 7) the action of a circular surface BC, all the points of which act on point a, in the direction Ca, with a force in inverse proportion to the square of the distances, we will find, by naming $CB = R'$, $Ca = a$, D the density of the surface, for the action of the circle on point a [the following value],[802,803]

$$D\left[1 - \frac{a}{(a^2 + R'^2)^{1/2}}\right] .$$

[801] Coulomb seems to be referring here to a small circle whose *radius* is approximately $b'f$, see Figure 6.

[802] [Note by Potier] Coulomb neglects here, as above, the factor 2π.

[803] Section 23.4 on page 376 presents a discussion of this factor 2π. The formula given in the sequence by Coulomb for the action of an electrified disk of radius R' acting on an electrified particle placed at a distance a from the center of the disk, along its axis, assumes that the disk is uniformly electrified with a surface charge density D that has the same value at all points on the disk.

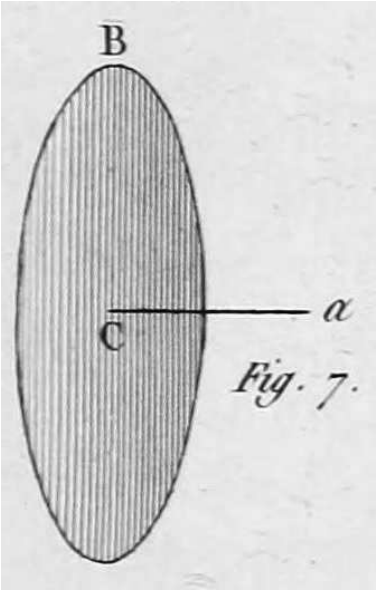


Fig. 7.

Thus the action of globe C' on point b, will be

$$\frac{2DR^2}{(R+2r)^2} - D + \frac{Da}{(a^2+R'^2)^{1/2}} \ ;$$

therefore, the equation which expresses the equilibrium of action for point b, will give

$$D\left[2 - \frac{2R^2}{(R+2r)^2} - \frac{a}{(a^2+R'^2)^{1/2}}\right] = \delta \ ,$$

in the case where the small globe has a very small diameter relatively to those of the extremities; as $a = 2r$, if $2r$ vanishes relatively R, we will have δ very small, and not negative; thus, no matter how small is globe x, placed between globes A and a, its electricity will be either null or insensible, but never negative, supposing the two globes A and a positively electrified; thus theory and experiment agree here.

22.22 XXII. *Three Equal Globes in Contact on a Straight Line*

22.22.1 Eighth Experiment

I put[804] in contact three equal globes two inches in diameter [5.41 cm], placed in a straight line, as in (Figure 6); one of these bodies, supported by the clamp (Figure 3), was positioned successively between the two bodies C and C', and at the extremity of these two bodies which were joined together; at each operation it was placed in the large balance, always bringing the needle back to the same distance from the globe;[805] it was found that when the

[804] In Coulomb's original article the Sections henceforth were wrongly numbered. That is, this Section appeared as Section XXI, as did the previous Section, [Coulomb, 1789, pp. 446 and 447]. I corrected this numbering, as was done by Potier, [Potier, 1884, pp. 208 and 210].

[805] The Figure in this footnote presents an illustration of this assembly. Globe a is attached to an insulating cylinder ac fixed to a support cd. In the left image this globe a is located between globes C and C' of the

globe was placed between the two others, it took a quantity of electricity less than that which it took when it was placed at the extremities in the ratio of 1.00 to 1.34. This result is an average value of more than twenty operations carried out successively at equal time intervals, in order to be able to take account of the quantity of electricity lost [to the environment] from one observation to another.

22.23 XXIII. *Explanation of This Experiment*

We can use the formula from Section 22.16,

$$D\left[1-\frac{2R^2}{(R+2r)^2}\right]=\delta\ ,$$

in which δ represents the [surface] density [of electricity] of the globe placed between the two others, and D that of the globe at the ends; since $R=r$, we will have

$$\delta=D\left(1-\frac{2}{9}\right)=\frac{7}{9}D\ ,$$

from which

$$D=1.29\delta\ ;$$

but experiment has just given us

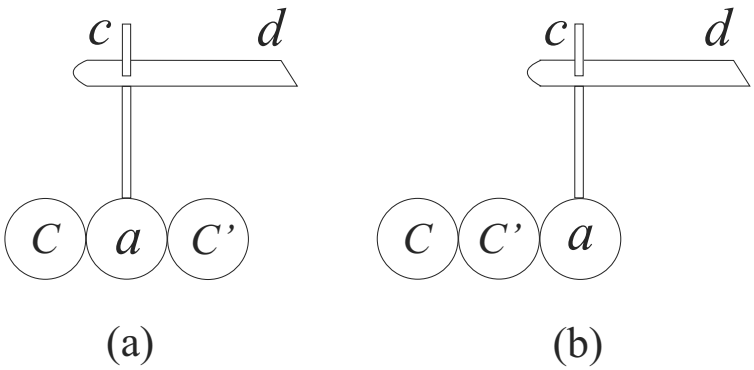
$$D=1.34\delta\ ,$$

which differs, as we see, only by one twenty-seventh from the ratio given by the theory. We see that here the action of globe C (Figure 6) on point b, is very close to

$$\frac{2R^2}{(R+2r)^2}\ ,$$

because the action of the small circle $b'f$, whose density is zero,[806] as we have seen in Section 22.21, is expressed by

same diameter as globe a, while in the right image we have a at the end. Clamp dca is brought to the balance after each of these contacts in order to measure the amount of charge of globe a:



[806] In the original: *dont la densité est nulle*, [Coulomb, 1789, pp. 448-449] and [Potier, 1884, p. 211]. Probably this was a misprint. The correct expression should had been *dont la densité est D*, that is, whose density is D.

$$D\left[1-\frac{2R}{(4R^2+(b'f)^2)^{1/2}}\right],$$

a quantity that vanishes here, because $b'f$ is much smaller than $2R$.

22.24 XXIV

All the preceding theory will be confirmed by an experiment which seems to me to shed light on this matter.

We have just seen in the preceding Sections; that when two globes were in contact, whatever the diameter of these two globes, the density at the point of contact and in the points which adjoin it, was null and not negative, if the two globes are positively electrified. But from the moment that we separate the two globes, if one of the globes is smaller than the other, and if the distance of the two globes is not considerable, we will find (Figure 8) that point a of the small globe, which has been in contact with point A of the large globe, becomes negative until these two globes are separated at a certain distance at which the electricity of point a is zero; that the same point a then becomes positive, when we continue to move the two globes apart.

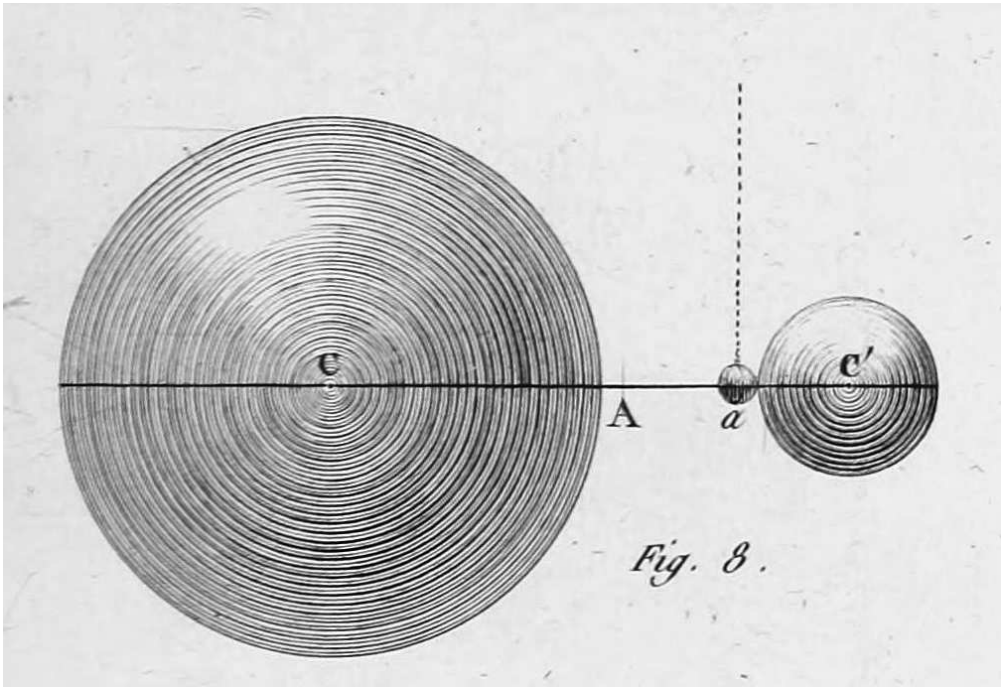


Fig. 8.

22.25 XXV. Ninth Experiment

A globe C eleven inches in diameter [29.78 cm] has been insulated (Figure 8); a C' globe of a smaller diameter was also insulated; these globes were electrified and made to touch; the small globe C' was then moved away little by little, and by means of a small lead pellet a, suspended from a shellac thread, or [by means of] a small circle of gilded paper, as in Figure 3, which was made to touch point a, and which was then presented in the small balance or

in a small electrometer with very sensitive silk thread, such as it was described in our Fourth Memoir,[807] we determined the nature of the electricity from point a at different distances Aa.

First test. Globe C being eleven inches in diameter, and globe C' eight inches, the two globes having been positively electrified and brought into contact, point A of the large globe has always given signs of positive electricity, whatever the distance Aa; but point a of globe C' gave signs of negative electricity up to an inch away; at an inch [2.71 cm], the electricity of this point a was null, it was positive beyond.[808]

Second test. Globe C always being eleven inches in diameter, and globe C' four inches [10.83 cm]: up to two inches away, point a of the small globe gave signs of negative electricity; at two inches, the electricity of this point was nil: the electricity of point A is always positive.

Third test. Globe C always being eleven inches in diameter, when the small globe C' was two inches, one inch and below, the electricity of point a was negative until the small globe was moved two inches five lines from the large globe [6.54 cm]; at this distance of two inches five lines it was zero, positive when the distance Aa was more than two inches five lines.

22.26 XXVI. Remark on This Experiment

When a spherical surface, uniformly covered with an electric fluid whose density is D, acts on a point placed at the surface of the globe, its action on this point is equal to D;[809] but when this same fluid acts on a point placed outside the same surface by the quantity a, its action on this point, if the radius of the globe is R, will be $2DR^2/(R+a)^2$.[810]

If we now suppose the small globe C' (Figure 8) in contact with the large globe C, if globe C' is very small relatively to globe C, the electric fluid of the large globe will always remain almost uniformly spread over the large globe, because the small globe will only have an action on the point of contact and on those which are near it; this is what is easy to perceive from theory; thus the action of the large globe on the point of contact will still be quite exactly represented by D: but although the average density of the small globe in contact is greater than that of the large globe, as there must be equilibrium at the point of contact when the large and the small globe touch each other, the action of the small globe on the point of contact, however, has as its measure the quantity D, like the large globe. But if we separate the small globe from the large one, and if we move it away by a small amount $Aa = a$, the action of the small globe on point A of the large globe, will be almost

[807] This Memoir is translated in Chapter 20.

[808] That is, when $aA < 1$ inch, point a was negatively charged; when $aA = 1$ inch, the electricity at point a was zero; when $aA > 1$ inch, point a was positively electrified.

[809] The force per unit charge acting on a particle located on the surface of an electrified spherical shell is given by $2\pi kD$. Coulomb is here neglecting the constant factor 2π (assuming $k = 1$), just as he had done earlier, see Section 23.4 on page 376 and footnote 796 on page 351.

[810] Let a spherical shell of radius R have a total charge Q uniformly distributed over it with a surface charge density $D = Q/(4\pi R^2)$. When acting on an electrified particle with charge q located at a distance $R + a$ from its center, the force F per unit charge is given by (with k being a constant of proportionality):

$$\frac{F}{q} = \frac{kQ}{(R+a)^2} = \frac{k(D4\pi R^2)}{(R+a)^2} = 2\pi k \cdot \frac{2DR^2}{(R+a)^2} \ .$$

Again Coulomb is neglecting the constant factor 2π, see Section 23.4 on page 376 and footnote 796 on page 351.

nil, while the action of the large globe C[811] on point a, will be

$$\frac{2R^2}{(R+a)^2} \ ;$$

thus the action of the small globe on point a remaining D as in the contact, we will have, to determine the density δ of point a, the equation

$$\frac{2DR^2}{(R+a)^2} + \delta = D \qquad \text{or} \qquad \delta = D\left[1 - \frac{2R^2}{(R+a)^2}\right] \ ;$$

therefore, if $2R^2/(R+a)^2$ is greater than 1, δ will be negative; if this quantity is equal to 1, δ will be zero, it will be positive if $2R^2/(R+a)^2$ is smaller than 1.

We can therefore determine the distance Aa when the density of point $a = 0$, by making $2R^2/(R+a)^2 = 1$; from which results

$$(R+a) = R\sqrt{2} = 1.415R \qquad \text{and} \qquad a = 0.415R \ .$$

But we have just seen in our experiment, that when a small one-inch globe, for example, has been brought into contact with our eleven-inch globe, it must be moved two inches five lines away from globe A, so that the electricity of point a ceases to be negative and is zero, that it is positive beyond this distance: here $R = 5$ inches 6 lines $= 66$ lines, $a = 2$ inches 5 lines $= 29$ lines; thus

$$R = 5 \text{ inches and } 6 \text{ lines } = 66 \text{ lines } [= \ 14.92 \ cm] \ ,$$

$$a = 2 \text{ inches and } 5 \text{ lines } = 29 \text{ lines } [= \ 6.55 \ cm] \ ;$$

thus

$$\frac{a}{R} = \frac{29}{66} = 0.439 \ ,$$

which differs very little, as we see, from what is indicated by the theory.

It is easy to see, from the reflections on which the preceding calculation is based, that as the two globes approach equality [in size], the distance Aa, where the density of point a is zero, must decrease, because in this case the action of the small globe on point A, at the distance Aa, leaves only little density of the electric fluid at point A and the points which are close to it; thus the action of the large globe A on point a, is then less than $2DR^2/(R+a)^2$; it is for the same reason that the electric fluid of the large globe is never negative at A, whatever the distance Aa.[812,813]

[811] For a lapse in the original text, C' appears here instead of C.

[812] [Note by Potier] Poisson applied the calculation to the experiment made on the 11-inch and 4-inch globes, and found that the minimum density on the small sphere was only 0.037 of its average density when it was 2 inches away from the larger [globe].

Mr. Plana calculated the ratio y of the distance where the minimum density is zero to the radius of the large sphere, as a function of the ratio x of the radii of the two spheres. When x decreases from 1 to 0, y increases rapidly at first, from 0 up to a maximum equal to 0.54 which it reaches for $x = 0.5$, then decreases while tending towards the limit 0.355, which it would reach for $x = 0$.

[813] [Poisson, 1812a], [Poisson, 1812b], [Poisson, 1813] and [Poisson, 1814]; [Plana, 1845] and [Plana, 1854].

22.27 XXVII

It seems that we can conclude from the experiments and observations which precede, that the electric fluid is almost entirely distributed on the surface of electrified conducting bodies, and that it does not form a very extended atmosphere around these bodies, as several authors have thought: this consequence can even be confirmed by an experiment which seems almost decisive; here it is. If we place a conducting globe in the [electric] balance, electrify it, and make it touch alternately by two copper wires, of the same thickness and length,[814] but one of which is wrapped over its entire length, except at the end intended to touch the globe, by a layer of very pure shellac, five or six lines thick [1.13 to 1.36 cm]; it will be found, by a process and a calculation analogous to those of the Sections of the First Part, that both copper wires, when placed in contact with the globe at their extremity, take an equal quantity of electricity.

But it is known that the electric fluid cannot penetrate through a layer of shellac; thus, when the wire covered with shellac is brought into contact, and presented by its extremity to the [electrified] globe, the electric fluid can only be distributed on the surface of this wire; consequently, since, whether the wire is covered with shellac or not, it takes the same quantity of electricity, half, for example, of that of the globe, it must exert in both cases, on any point whatever, the point of contact for example, the same action: whence it results that, whether the copper wire be surrounded by shellac, or not, the electric fluid is distributed therein in the same manner and in the same quantity .

However, it must be warned that as the air is not a perfect insulator,[815] as it is charged with humid conductive parts, the electric fluid of an electrified body must penetrate more or less into the layers of air which surround it; but on very dry days, the preceding experiments prove that this fluid does not penetrate the strata of air to a sufficiently great depth nor in sufficient quantity to make it necessary to take it into account in the greater part of the calculations. We will return to this subject in another Memoir intended to determine the state of an insulating body in contact with an electrified conducting body;[816] but we cannot occupy ourselves with this subject with any hope of success, until we shall have determined exactly by experiment the manner in which the electric fluid is distributed on surfaces, either flat or curved, and on bodies of different shapes; this research will form the Second Part of this Memoir.

22.28 XXVIII. *Determination of the Density of the Electric Fluid, from the Point of Contact, up to 180 [Degrees] from This Point, in Two Electrified Globes which Touch Each Other*

Let us suppose the two globes (*Figure 9*) in contact through point A, both electrified and supported by insulators, such as that of *Figure 4*.

[814] These wires are manipulated with an insulating handle to prevent grounding.

[815] In the original: *comme l'air n'est pas d'une parfaite idio-électricitè*. That is, as the air is not perfectly insulating. See footnote 522 on page 229.

[816] This Memoir was never published.

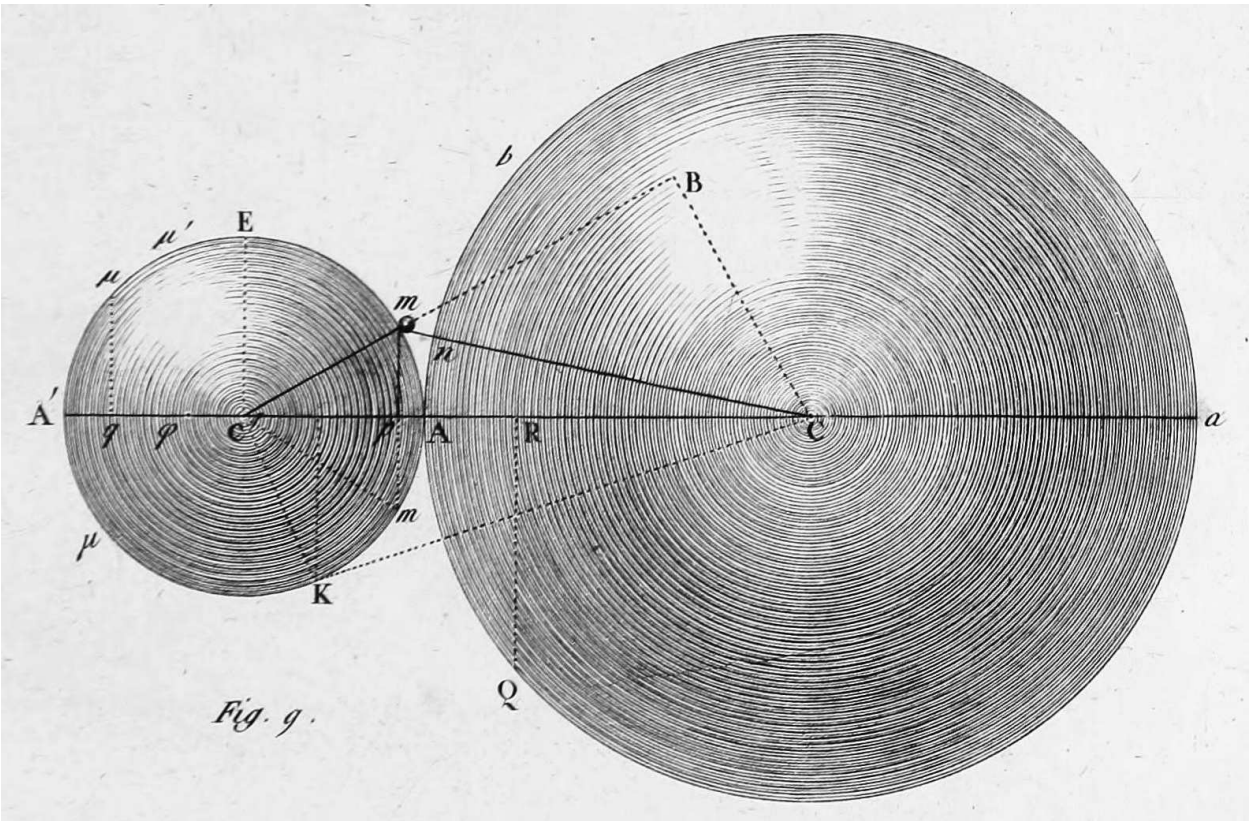


Fig. 9.

Since we demonstrated in our Fourth Memoir, that in conducting bodies, the electric fluid was only distributed on the surface, and did not penetrate into the interior of these bodies, we can suppose each globe covered with an infinity of small conducting globules charged with electricity; thus the electric action of each of these globules on the point of the globe where it is in contact, will be counterbalanced by the action of all the other globules which cover the two bodies.

If D were the [surface] density [of electric fluid] of the large globe whose center is at C, and if this density were uniformly spread over the whole globe, its action on a point m of the small globe C', would be expressed by[817]

$$2D\left(\frac{Ca}{Cm}\right)^2 ,$$

and this action decomposed in the direction mB, radius of the small globe, will be

$$\frac{2D(CA)^2 mB}{(Cm)^3} .$$

If the density of the electric fluid were likewise uniformly spread over the small globe, and equal to D', its action on point m would be D'. Thus, if we put in contact with the small globe whose center is C', a small globule m which is charged with electricity, the electric density of this small globule must be such that at its point of contact there is equilibrium between the action of the small globe C' acting along $C'm$, and that of globe C acting along Bm, joined to that of the globule m acting in the same direction, so if δ is the average density of the small globule in m, we will have

[817] Throughout this Section Coulomb will continue to neglect the factor 2π in the expressions of the actions exerted by electrified spherical shells, see Section 23.4 on page 376.

$$D' = \delta + \frac{2D(CA)^2 mB}{(Cm)^3} \qquad \text{or} \qquad \delta = D' - \frac{2DCA^2 mB}{(Cm)^3} \ .$$

If we make, to have this equation in an analytical form, $CA = R$, $C'A = r$, $AP = x$,[818] the two similar triangles $CC'B$, $C'pm$ will give

$$\frac{C'm}{C'p} = \frac{CC'}{BC'} \ .$$

Thus

$$Bm = BC' - C'A = R - \frac{R+r}{r}x \ ,$$
$$(Cm)^2 = R^2 + 2(R+r)x \ ;$$

by substituting, in the formula, the values of Bm and Cm, it becomes

$$\delta = D' - \frac{2DR^2\left(R - \frac{R+r}{r}x\right)}{[R^2 + 2(R+r)x]^{3/2}} \ .$$

If in this equation we make the angle $AC'm = \alpha$, we will have

$$x = r(1 - \cos\alpha) \ ,$$

and, consequently,

$$\delta = D' - \frac{2DR^2[R - (R+r)(1-\cos\alpha)]}{[R^2 + 2(R+r)r(1-\cos\alpha)]^{3/2}} \ .$$

22.29 XXIX

If the two globes are equal, then $D = D'$, $R = r$, and the previous formula reduces to

$$\delta = D\left[1 + \frac{2 - 4\cos\alpha}{(5 - 4\cos\alpha)^{3/2}}\right] \ .$$

We determined in the First Part of this Memoir (fourth experiment, Sections 22.13 and 22.15), the electric density of two equal globes in contact; from 30 degrees of the point of contact up to 180 degrees of this point; so we can compare our formula with this experiment and its result.

1. If we calculate the density δ according to our formula, we will find it negative up to around 23 degrees. Experiment gives it insensitive up to this point; we have given the reason for this difference, Section 22.20;

[818] By $AP = x$ Coulomb seems to be referring to the distance between points A and p shown in Figure 9. If this is the case, then we should have here $Ap = x$.

2. If we calculate for a point at 30 degrees from the contact of the two globes, we will find $\delta = 0.23D$.

3. At 60 degrees from the point of contact, $\delta = D$;

4. At 90 degrees from the point of contact, $\delta = 1.18D$;

5. At 180 degrees from the point of contact, $\delta = 1.22D$.

To have the quantity δ according to the experiment, we compared (Section 22.15) the density of the point at 90 degrees from the contact with that of all the other points; thus the same comparison must be made in the results given by the theory; experiment has given us, Section 22.15:

The density of the electric fluid at 90 degrees from the point of contact, is to that at 30 degrees ..... :: 4.80 : 1.00.

If we make the same comparison according to our formula, we will find it .:: 5.13 : 1.00.

The point at 90 degrees, compared to that at 60 degrees, experiment gives the [ratio of the] densities ..... :: 1.25 : 1.00.

Theoretical calculation [gives] ..... :: 1.18 : 1.00.

The point at 90 degrees, compared to that at 180 degrees, experiment gives the [ratio of the] densities ..... :: 0.95 : 1.00.

Theory [gives] ..... :: 0.97 : 1.00.

We find here a conformity between the results of the experiment and those of the theory, which we could hardly hope for.[819,820]

22.30 XXX. Tenth Experiment

To make the comparison of theory and experiment more direct and easier in determining the quantity of electricity acquired by two globes of different diameters brought into contact, here is the result of some new experiments which I thought useful to add to the preceding ones.

I put in contact (Figure 9) two globes of different diameters, I electrified them; I then touched point A' at 180 degrees from the point of contact, with a small circle of gilded paper, 5 lines in diameter (1.13 cm), insulated, as shown in Figure 3, by a shellac thread: I presented this little [paper] disk in the balance with a very fine silver wire; I then separated the two globes C and C', and I touched the large globe C with the same disk. I again presented this little disk in the balance; the comparison of the force with which the needle was driven in the first and second observations, gave the ratio of the densities, of point A' when the two globes are in contact, and of the average density on the large globe C, when these globes are separated.

[819] [Note by Potier] Poisson gives, for these three ratios, 5.86; 1.34; 0.88.

[820] [Poisson, 1812b, p. 76]. The percentage differences between Coulomb's experimental values and Poisson's theoretical values are given by:

$$\frac{5.86 - 4.80}{5.86} \cdot 100\% = 18.1\% , \qquad \frac{1.34 - 1.25}{1.34} \cdot 100\% = 6.7\% \qquad \text{and} \qquad \frac{0.88 - 0.95}{0.88} \cdot 100\% = -8.0\% .$$

That is, there is a very reasonable agreement between these values.

I formed, according to the different results that this experiment gave me, a Table for two globes of different diameters: in this table, R is the radius of the large globe, r that of the small one; D is the average [electric] density of the large globe separated from the small globe; δ is the density of point A', end of the axis of the small globe in contact with the large globe.[821]

If $R/r =$	we will have $\delta/D =$
1	1.27
2	1.55
4	2.35
8	3.18
∞	4.00

22.31 XXXI. *Remarks on the Previous Experiment*

If we want to determine, according to the theory, the quantity δ for point A', it is necessary, to have a first approximation, to suppose the electric fluid of each globe uniformly spread over this globe; by naming δ the density of point A' or of a small globule placed at A', D the average density of the large globe C, D' that of the small globe; we will have the equation

$$\delta = D' + \frac{2DR^2}{(R+2r)^2} .$$

In this equation, the average density D' of the small globe is necessarily greater than the density D of the large globe, as is easy to see from theory. But suppose as a first approximation $D = D'$, we will form, according to the formula

$$\delta = D\left[1 + \frac{2R^2}{(R+2r)^2}\right] ,$$

which expresses the value of the density at point A', compared with the average density of the large globe, the following Table:

If $R/r =$	we will have, according to the formula, $\delta/D =$
1	1.22
2	1.50
4	1.89
8	2.28
∞	3.00

Comparison of this result with that provided by the experiment in the *previous Section*, shows that it is only when R is greater than $2r$, that the theory and the calculation begin to differ, the theory giving an approximate value, when R is greater than $2r$, less than that provided by experiment. But if we notice that in our Table, calculated according to the formula, we have assumed the density D' of the small globe, equal to that of the large globe,

[821] Let Q be the electric charge on the large globe of radius R. When this globe is far away from the small globe, its surface density of electricity is given by $D = Q/(4\pi R^2)$. The magnitude δ, on the other hand, is the surface charge density at point A' in Figure 9 measured while the two globes are in contact. Section 23.5 presents a comparison between the experimental results of Coulomb and the theoretical values of Poisson.

and that by the action of the large globe, the fluid of the small globe must be concentrated at point A' of the small globe;[822] that however this fluid by its action in inverse proportion to the square of the distances, must balance at the point of contact with the action of the fluid spread almost uniformly over the large globe; it will be seen that the average density of the electric fluid must be greater on the small globe than on the large one; that thus D' is greater than D, and consequently that the result given in the Table by the calculation, requires a correction which increases the value of δ, which is in accordance with experiment. We will find in the Sections that follow methods to get closer to the true value of δ.

22.32 XXXII. *Determination by Approximation of the Ratio According to Which Electricity is Shared Between Two Globes of Different Diameters Placed in Contact*

22.32.1 First Example: When $R = \infty r$

As we can determine only by approximation the manner in which the electric fluid divides between two globes, it will be easier to grasp the spirit of the methods which we have followed, by applying them to particular examples, than by generalizing them. In this example, one of the globes is infinitely large relative to the other; but according to this supposition, it is easy to conceive that the formula which we used (Section 22.26) to determine the density at all the points of the small globe, must approach the truth; for supposing the small globe at a very short distance from the large globe, the electric fluid with which the large globe will be charged will be carried on the small globe until there is equilibrium at all the points of the surface between the action of the large globe and the action of all the electrified points on the surface of the small globe: the action of the small globe on the large globe being proportional to the average density multiplied by its surface, will be infinitely small for any point other than the point of contact; thus the action of the large globe on each point of the small one will be about the same for any other point than the point of contact, as if all the electric fluid of the large globe were [concentrated] at its center C. Let us now take D' for the average density of the small globe, a quantity which must be variable when we seek the action of the small globe on each of the points of its surface, but which we can suppose constant in a first approximation, provided that we determine its value according to the conditions of equilibrium at the point of contact: since we suppose in this example the radius r very small relatively to R, the formula

$$\delta = D' - \frac{2DR^2\left[R - \frac{(R+r)x}{r}\right]}{[R^2 + 2(R+r)x]^{3/2}} ,$$

reduces to

$$\delta = D' - \frac{2D(r-x)}{r} .$$

Now it is necessary that the action exerted by all the fluid of the large globe on the point of contact A (Figure 9), in the direction CA, is equal to the action on the same point of

[822] In the original text we have "petit globe" (small globe), [Coulomb, 1789, p. 458], while by a mistake Potier replaced this expression with "gros globe" (large globe), [Potier, 1884, p. 220].

all the fluid spread on the surface of the small globe: but as according to our formula, δ represents (Figure 9) the density of the fluid on point m; and the density δ is the same for all the points of the surface area mm, perpendicular to the axis Ap, the action of this zone, decomposed along the direction pA, will be on point A [given by:]

$$\frac{\delta dx}{2\sqrt{2r}\sqrt{x}} = \frac{dx}{2\sqrt{2r}\sqrt{x}}\left(D' - 2D + \frac{2Dx}{r}\right) .$$

Taking for D' the average density of the small globe on each point of its surface, and supposing it constant, the integral of this quantity will give for the action of the small globe on point A,

$$\frac{1}{2\sqrt{2r}}\left[(2D' - 4D)\sqrt{x} + \frac{4}{3}D\frac{x^{3/2}}{r}\right] ,$$

a quantity which must vanish when $x = 0$, and be completed when $x = 2r$, which will give for the entire action of the small globe on the point of contact A [the following value:]

$$D' - \frac{2}{3}D .$$

But it should be noted that in the contact of the two globes, the electric fluid being in a state of stability, there must be equilibrium at the point of contact between the action of the small and the action of the large globe. As the density of the large globe is almost uniform on all the points of its surface, the action of the large globe on the point of contact A, will be D; so we have the equation

$$D' - \frac{2}{3}D = D \qquad \text{or} \qquad D' = 1.67D ,$$

quantity smaller than that which was found by the experiment which gave us (Section 22.11), when $R = \infty r$, $D' = 2D$. Before looking for a more approximate value of D', let us determine by approximation the density in A', extremity of the axis. To achieve this, it should be noted that since D' represents the action exerted by the electric fluid on any point of its surface, this quantity D' cannot be constant, as we have just assumed for a first approximation, but it must vary increasing from the point of contact A, to the end of the axis in A'. In the point of contact, this action of the small globe must balance the action of the large globe; thus it must be equivalent to D; at point A', it must be determined by the action of the entire surface of the small globe on this point. To have an approximate action of the small globe on this point, it must be calculated according to the density $\delta = D' - 2D - \frac{2Dx}{r}$; by making [see Figure 9]:

$$A'q = z = (2r - x) ,$$

the action of the small zone $\mu\mu$ on point A' will be

$$\frac{\delta dz}{2\sqrt{2r}\sqrt{z}} = \frac{dz}{2\sqrt{2r}\sqrt{z}}\left(D' + 2D - \frac{2Dz}{r}\right) ,$$

which integrated and completed will give, for the entire action of the small globe on point A, [the value] $D' + \frac{2}{3}D$. Let us put in the place of D' its approximate value, which we have just

found, $1.666D$, and we will have for the approximate action of the small globe on point A, [the value] $2.33D$; thus the action of the small globe on all the points of its surface, increases from point A to point A', so that at the point of contact A, it is equal to D, and at the extremity of the axis in A',[823] it is $2.33D$.

To obtain the density of the electric fluid at point A' in accord with the quantity of action that the small globe exerts on this point, it is necessary to suppose that we touch with a small insulated [proof] plane alternately point A' and a point of the large globe C. It is clear that at point A' the density of the small globules must be such that there is equilibrium between the action of a small globule at A' and that of the two globes; thus, by naming δ the density of the small globule, we must have

$$\delta = 2D + 2.33D = 4.33D \ ;$$

the experiment did indeed seem to indicate to us (Section 22.30) that when r was infinitely smaller than R, the density of the small globe at the extremity of its axis in A', was a little greater than $4D$, D expressing the average density of the large globe.

Let us return to determine more exactly the average density D' of the small globe, which we found by a first approximation, equal to $1.67D$, and which experiment (Section 22.11) has taught us be equal to $2D$.

Since the action of the small globe on each point of its surface increases from point A to point A'; since at point A, it is approximately equal to D or the average quantity $D'/1.67$; [and] since at point A', it is $2.33D'/1.67$, when we wanted to determine the value of δ, we had to make D' variable, instead of making it constant. Thus, assuming that the action of the small globe is represented by

$$D' \left(a + \frac{bx}{2r} \right) \ ,$$

this action must be such that when $x = 0$,

$$D'a = \frac{D'}{1.67} \ ,$$

and that, when $x = 2r$,

$$D' \left(\frac{1.00}{1.67} + b \right) = \frac{2.33D}{1.67} \qquad \text{or} \qquad b = \frac{1.33}{1.67} \ ;$$

which will give, for the approximate density δ of each point m of the small globe, [the following value:]

$$\delta = D'a - 2D + \frac{bD' + 4D}{2r} x \ ;$$

and the action of a small surface area mm, on the point of contact A, will be[824]

[823] For a lapse in the original text we have there A instead of A'.

[824] The following equation appears in the original text:

$$\frac{Dx}{2\sqrt{2r}\sqrt{x}} \left[(D'a - 2D) + \left(\frac{D' + 4D}{2r} \right) x \right] \ .$$

I replaced Dx in the first numerator with dx. In the last numerator I put bD' instead of D'.

$$\frac{dx}{2\sqrt{2r}\sqrt{x}}\left[(D'a-2D)+\left(\frac{bD'+4D}{2r}\right)x\right]\ ,$$

of which the integral [is given by:][825]

$$\frac{1}{2\sqrt{2r}}\left[2x^{1/2}(D'a-2D)+\frac{2}{3}x^{3/2}\left(\frac{bD'+4D}{2r}\right)\right]\ ,$$

quantity which must vanish when $x=0$, and be completed when $x=2r$. Thus the entire action of the small globe on the point of contact A, will be

$$D'a-2D+\frac{bD'+4D}{3}\ ;$$

but as the action of the small globe must balance at the point of contact, with the action of the large globe which is equal to D, density of the fluid of this large globe, we will have

$$D'\left(a+\frac{b}{3}\right)=\frac{5}{3}D \qquad \text{or} \qquad D'=1.93D\ .$$

To obtain now the average density of the electric fluid of the small globe, when, by removing it from contact with the large globe, it will spread uniformly over the surface of this small globe, it is necessary to have the quantity of electric fluid spread over the small globe, and divide it by the surface of this globe; so we have to reconsider the equation

$$\delta=D'a-2D+(bD'+4D)\frac{x}{2r}\ ,$$

multiply it by dxr, which expresses the elementary surface of the globe, integrate this quantity for the entire surface, which will give

$$(D'a-2D)2r^2+(bD'+4D)\frac{r^2}{2}\ ,$$

and divide by the area of the small globe $2r^2$,[826] which will give for the average density [the following value:]

$$D'\left(\frac{2a+b}{2}\right)\ ,$$

from which

$$D'=1.93D\ ,$$

a quantity, as we see, which differs from $2.00D$ found by experiment only by an amount too small to be appreciated in researches of this kind.

[825] The following equation appears in the original text:

$$\frac{dx}{2\sqrt{2r}}\left[2x^{1/2}(D'a-2D)+\frac{2}{3}x^{3/2}\left(\frac{bD'+4D}{2r}\right)\right]\ .$$

I replaced in the first numerator dx with 1.

[826] The area of the small globe of radius r is given by $4\pi r^2$. In this Section Coulomb is not including the factor 2π in all formulas, see Section 23.4 on page 376. See also footnotes 796, 802 and 817 on pages 351, 352 and 359, respectively.

22.33 XXXIII. *Second Approximation Method*

We are going to use here an approximation method different from the previous one, but which can be applied to all values of R/r.

Let D (Figure 9) be the average [surface] density [of electricity] of the large globe; D' the average action of the small globe on each point of its surface that we want to determine. We have seen that when the two globes were in contact, the density was zero at the point of contact; if we now determine it for two other points [on the surface of the small globe], one at 90 degrees from the point of contact, the other at 180 degrees from this point, we will find, always naming R the radius of the large globe and r the radius of the small globe, that the density δ of the small globe at point E at 90 degrees from the contact is, according to our formula (Section 22.28),

$$\delta = D' + \frac{2DR^2r}{[(R+r)^2+r^2]^{3/2}} ,$$

and that the density δ at A', extremity of the axis, is

$$D' + \frac{2DR^2}{(R+2r)^2} .$$

Thus if we suppose the density δ which increases from point A to point A', represented by

$$D'\left[\frac{ax}{2r} + \frac{bx^2}{(2r)^2}\right] ,^{827}$$

this quantity must be 0 when $x = 0$, it must be [given by]

$$D' + \frac{2DR^2r}{[(R+r)^2+r^2]^{3/2}}$$

when $x = r$, and it must be [given by]

$$D' + \frac{2DR^2r}{(R+2r)^2}$$

when $x = 2r$.

Let us make, to simplify the calculation,

$$\frac{2R^2r}{[(R+r)^2+r^2]^{3/2}} = A ,$$

and

$$\frac{2R^2}{(R+2r)^2} = B .$$

We will determine a and b by the two equations

[827] As defined earlier by Coulomb, see footnote 818 on page 360, the magnitude x is defined by $x = AP$, with points A and P (or A and p) shown in Figure 9.

$$D'\left(\frac{a}{2}+\frac{b}{4}\right)=D'+AD\ ,$$

and

$$D'(a+b)=D'+BD\ ,$$

from which will result

$$a=3+(4A-B)\frac{D}{D'}\ ,$$

and

$$b=-2+2(B-2A)\frac{D}{D'}\ .$$

To determine D', consider the equation

$$\delta=D'\left[a\frac{x}{2r}+b\frac{x^2}{(2r)^2}\right]\ ,$$

and we will have, for the surface action of a zone mm on the contact point A,

$$\frac{D'dx}{2\sqrt{2r}\sqrt{x}}\left[\frac{ax}{2r}+\frac{bx^2}{(2r)^2}\right]\ ,$$

which, integrated and completed, will give for the entire action of the small globe on the point of contact A, the quantity

$$D'\left(\frac{a}{3}+\frac{b}{5}\right)\ ,$$

which must be equal to the action of the large globe on the same point of contact. If this globe is much larger than the small one, its action will be approximately equal to D; thus in this case, we will have, to determine the ratio D'/D, the equation

$$\frac{D}{D'}=\frac{a}{3}+\frac{b}{5}\ .$$

22.34 XXXIV

To obtain the average density of the fluid spread uniformly on the small globe after the contact, it is necessary to divide the quantity of electricity of the small globe by its surface, which will give for this density,

$$\int\frac{\delta dxr}{2r^2}\ .$$

By substituting, in place of δ, its value, and performing the operation, we will find

$$D'\left(\frac{a}{2}+\frac{b}{3}\right)\ ,$$

which expresses the average density, that is to say, the density of the electric fluid, when, after the contact, the small globe will be separated from the large globe, and the large globe ceasing to act on the small globe, the electric fluid will spread uniformly over the surface of the small globe.

22.35 XXXV. Second Example: $R = 4r$

Let us apply the preceding formulas to an example whose results we obtained by the experiments reported, in the form of a Table, at Section 22.11: by supposing the action of the large globe on the point of contact $= D$.[828]

As [we are now assuming] $R = 4r$, we find

$$A = 0.24 \qquad \text{and} \qquad B = 0.89 \ ;$$

thus

$$a = 3.00 + 0.07\frac{D}{D'} \qquad \text{and} \qquad b = -2.00 + 0.82\frac{D}{D'} \ ;$$

from which results

$$\frac{D'}{D} = 1.36 \ .$$

Substituting now the values of a, b and D, in the formula $D'(a/2 + b/3)$, which expresses the average density, we will find this density equal to $1.42D$, which is a little greater than the quantity 1.30, which was given, Section 22.11, by the experiments; but it should be noted that we have supposed the action D of the large globe C equal to its average density, as if the electric fluid were spread uniformly over this globe: but as it is a little repelled by the action of the small globe, its action on the point of contact will be smaller than its average density. Thus, in the comparison of the average density of the small and the large globe, we must have had, according to this observation, a ratio a little smaller than that which we have just found, which, as we see, conforms to experiment.

To have, according to the average density D', the density of point A' at the extremity of the axis of a small globe, it is necessary, as we have already said, that the action of a small globule which we would place in A', would balance at its point of contact the action of the two globes C and C', which would give

$$\delta = 1.42D + 0.89D = 2.31D \ ,$$

a quantity that we have found by experiment, Section 22.30, [as given by] $2.35D$, which is not significantly different from this [value].

If we wanted to obtain something more precise, it would be necessary to employ for the large globe a calculation similar to that one which we made for the small globe, to determine in this way the action on the point of contact A, and to equate the two actions.

[828] See Section 23.4 on page 376.

Chapter 23

Remarks on Coulomb's Fifth Memoir

A. K. T. Assis

23.1 Experimental Method to Determine the Division of Charges Between Two Conducting Globes of Different Sizes when They Touch One Another

In Section 22.2 of this Memoir, Coulomb electrifies a conducting globe a of radius R_a with a charge q, this globe being supported by the insulator I, as illustrated in Figure 23.1 (a). A second conducting globe g of radius R_g, also insulated from the Earth, is initially discharged and far away from globe a. He then brings the two globes into contact, Figure 23.1 (b). After moving the globes apart, globe a is left with a charge q_1 and globe g with a charge q_2, Figure 23.1 (c). Coulomb has two goals in this experiment. The first is to obtain the ratio q_2/q_1 as a function of R_2/R_1. The second is to obtain the ratio between the surface charge densities as a function of the ratio between the radii, that is, to determine $[q_2/(4\pi R_2^2)]/[q_1/(4\pi R_1^2)]$ as a function of R_2/R_1.

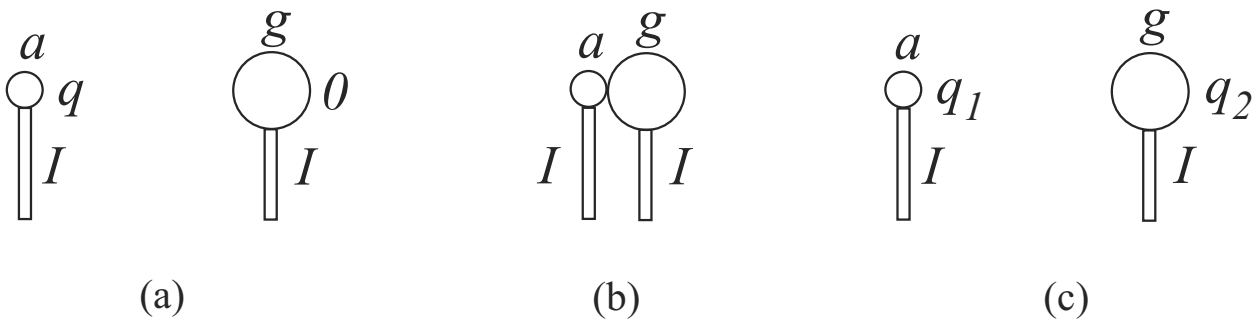


Figure 23.1: Overview of the experiment.

Figure 23.2 illustrates qualitatively Coulomb's procedure. He uses the large electric balance shown in Figure 1, Number 1, located on page 328. Point o in Figure 23.1 indicates from where he measures the twist of the lower part of the suspension wire attached to the needle 8b. Point S is the point on the micrometer from where he measures the twist of the upper part of the wire, this micrometer being located at point 7 shown in Figure 1, Number 1. I am assuming that the wire 78 and points o and S are initially in the same vertical plane

when the wire is not twisted. The needle $8b$ is an insulator having a gilded paper disk b at its end. This conducting disk b is attached vertically to the needle. The arrow of Figure 23.2 indicates the micrometer orientation.

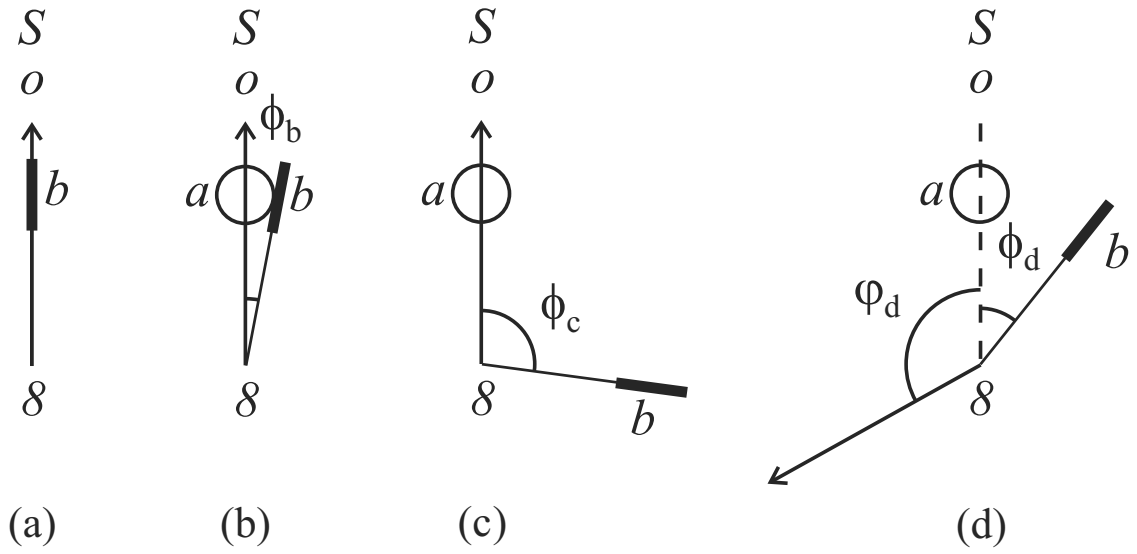


Figure 23.2: Preparation of the experiment.

In Figure 23.2 (a) the wire is untwisted. In (b) I am assuming that the conducting globe a has slightly displaced the paper disk b, causing the needle to be displaced by an angle ϕ_b with respect to its natural orientation. In this initial situation, globe a and the paper disk b are discharged. In (c) they are electrified with charges of the same sign, causing a repulsion between them. At equilibrium the needle $8b$ is then rotated clockwise through an angle ϕ_c due to this repulsion, Figure 23.2 (c). In Figure 23.2 (d) Coulomb rotates the micrometer attached to the top of the suspension wire counterclockwise through an angle φ_d such that the angular separation between the electrified globe a and the electrified paper disk b has a value ϕ_d which he had chosen previously. The total twist of the suspension wire is given by $\varphi_d + \phi_d$.

In this situation, the electrified globe a is touched by another conducting globe g, initially discharged, which is manipulated by the insulating handle I, as illustrated in Figure 23.3 (a). After globe g is removed from the balance, globe a and the needle are separated by an angle ϕ_e smaller than ϕ_d, while the micrometer remains twisted at an angle φ_d, Figure 23.3 (b). Coulomb then untwists the micrometer clockwise until globe a and the needle $8b$ are separated by the previously specified angle ϕ_d, as indicated in Figure 23.3 (c). This angle ϕ_d of Figure 23.3 (c) has the same value as the angular separation ϕ_d of Figure 23.2 (d). I will assume that at this moment the micrometer is twisted at an angle φ_e with respect to the line $8S$, Figure 23.3 (c). The total twist of the suspension wire in this case is given by $\phi_d + \varphi_e$.

Sometimes the micrometer has to be untwisted in a clockwise direction even going beyond the line $8S$, as indicated in Figure 23.4 (c). This occurs when there is very little charge left on globe a, such that, to bring the needle back to the separation ϕ_d of Figure 23.2 (d), it is necessary to greatly untwist the micrometer. The micrometer twist φ_e in this case is to the right of the $8S$ line. The total twist of the suspension wire in this case of Figure 23.4 (c) is given by $\phi_d - \varphi_e$, where we are only assuming here the positive magnitude of φ_e, regardless

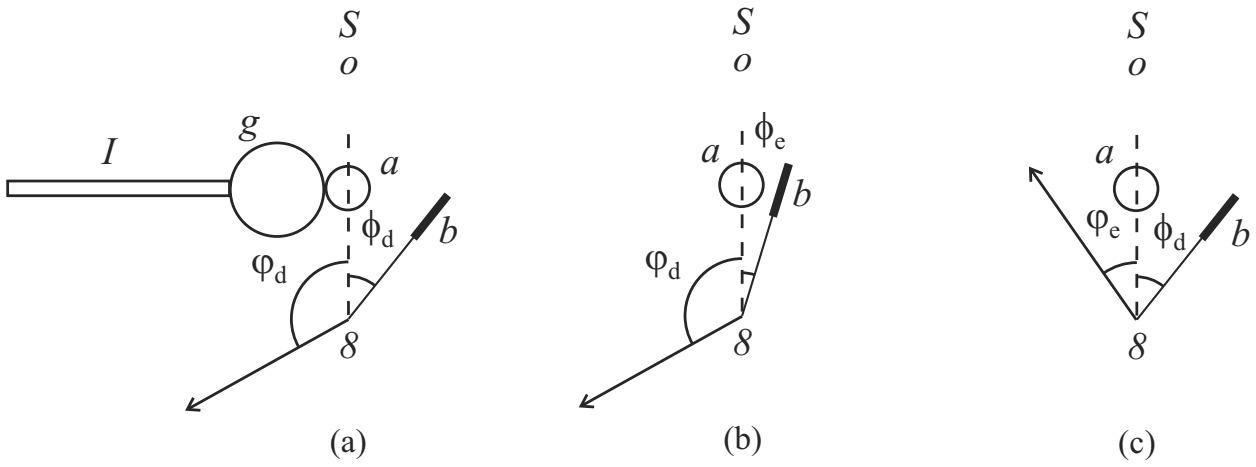


Figure 23.3: Final part of the first experiment.

of whether the micrometer is twisted clockwise or counterclockwise with respect to the line $8S$.

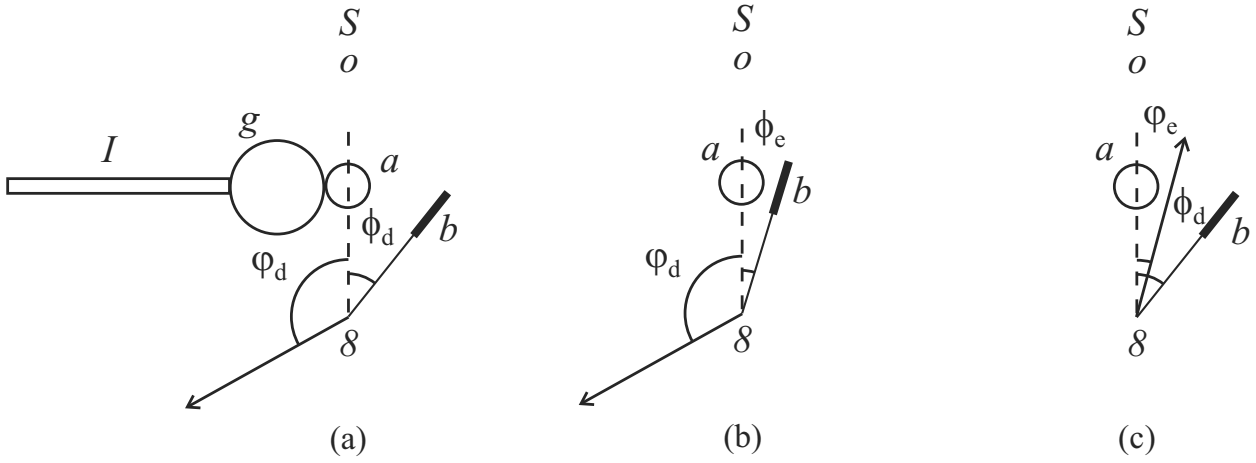


Figure 23.4: Final part of the first experiment when the micrometer pointer crosses the straight line $8S$.

23.2 Comparison of Coulomb's Experiment with the Theoretical Calculation of Poisson

I present in the next Table the experimental results of Coulomb given in Section 22.11 compared with Poisson's calculations published more than 20 years after Coulomb presented his measurements.[829] Let Q be the total amount of charge or electricity in a conducting globe of radius R, its surface charge density D is then given by $D = Q/(4\pi R^2)$. Initially we had globe a (or globe 1) with a charge q and globe g (or globe 2) was discharged. After contact and separation, globe a was left with charge Q_1 and globe 2 with charge Q_2. Suppose we have globes 1 and 2 of radii R_1 and R_2 with $R_2 \geq R_1$. After they have been brought

[829] [Poisson, 1812b, pp. 60-62], [Potier, 1884, p. 198] and [Gillmor, 1971a, p. 203].

into contact and separated, their charges are Q_1 and Q_2, respectively. The ratio of their radii is given by R_2/R_1, the ratio of their areas (or between their surfaces) is given by $(4\pi R_2^2)/(4\pi R_1^2) = R_2^2/R_1^2$. The ratio between the surface charge densities of the smaller globe to the larger globe is given by

$$\frac{D_1}{D_2} = \frac{Q_1/(4\pi R_1^2)}{Q_2/(4\pi R_2^2)} = \frac{Q_1/R_1^2}{Q_2/R_2^2} = \frac{Q_1 R_2^2}{Q_2 R_1^2} \ . \tag{23.1}$$

With $R_2 > R_1$ Coulomb experimentally obtained $Q_2 > Q_1$ and $D_2 < D_1$. The last column of this Table shows the percentage difference between the value V_c calculated by Poisson and the value V_m measured by Coulomb, that is, $[(V_c - V_m)/V_c] \cdot 100\%$.

Ratio between the globes		Ratio between the electric densities of the small and large globes, that is, $D_1/D_2 = (Q_1 R_2^2)/(Q_2 R_1^2)$		
Radii R_2/R_1	Surfaces $(R_2/R_1)^2$	Observed by Coulomb	Calculated by Poisson	Percentage difference
1	1	1	1	0%
2	4	1.08	1.16	+ 7%
4	16	1.30	1.32	+ 1%
8	64	1.65	1.44	−15%
∞	∞	2.00	$\pi^2/6 = 1.65$	$< -21\%$

Coulomb obtained the last line of this Table as a limiting case of Section 22.9 in which a globe 8 inches in diameter came into contact with another globe having 2 lines in diameter (that is, (1/6)th of an inch), such that the ratio of their radii was 48/1 and the ratio of their surfaces was 2304/1. The value 2 for the ratio between their surface charge densities is then an upper bound for the case where the ratio between their radii or between their surfaces tends to infinity.

It is amazing that Coulomb's measurements only deviate by a few percentage points from Poisson's theoretical values that were only calculated 20 years after Coulomb. Coulomb could not have made these exact theoretical calculations with the mathematical tools available to him. Therefore, he could not predict in advance the values he came to obtain experimentally. The small percentage differences between his measured values and the theoretical results of Poisson show the precision of his measurements and his manual dexterity, especially considering the difficulty and delicacy of these experiments. See also the discussion presented in Section 13.5.

23.3 Experimental Method for Determining the Surface Charge Distribution on Two Electrified Globes While They Are in Contact

In Section 22.12 Coulomb experimentally determined the surface charge density at different points of two electrified globes while they were in contact. He initially considered two conducting globes of the same radius insulated from the Earth. To determine the amount of charge at different points on any given globe, he used his proof plane shown in Figure 3 on

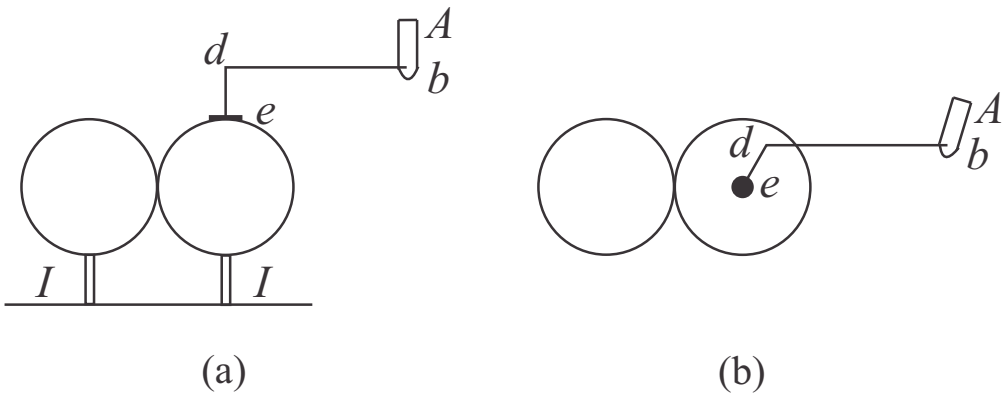


Figure 23.5: Two conducting globes of the same size, electrified, in contact and supported by insulating supports I. Conducting disk e of the proof plane placed at 90° from the point of contact. (a) Side view. (b) Top view.

page 335. Figure 23.5 illustrates placing the conducting disk e of the proof plane at a point on one of the globes located at 90° from the point of contact between the electrified globes.

In Coulomb's experiments we have two conducting globes with radii R_1 and R_2, with the ratio between their radii given by R_2/R_1. They are in contact, electrified and insulated from the Earth. In the fourth experiment, Section 22.13, we had $R_2/R_1 = 1$. In the fifth experiment, Section 22.14, we had $R_2/R_1 = 2$. In the sixth experiment, Section 22.15, we had $R_2/R_1 = 4$.

Coulomb measured the surface charge density (that is, charge per unit area) of a point on the smaller globe located at a certain angular separation θ from the point of contact. That is, he measured the surface density D_θ always on the smaller globe, D_θ being the amount of charge per unit area as a function of the angle θ, where this angle is measured relative to the point of contact. In particular, he made measurements at 30°, 60°, 90°, and 180° from the point of contact, Figure 23.6. In his calculations he normalized these measurements by the surface charge density at a point on that same small globe located at 90° from the point of contact.

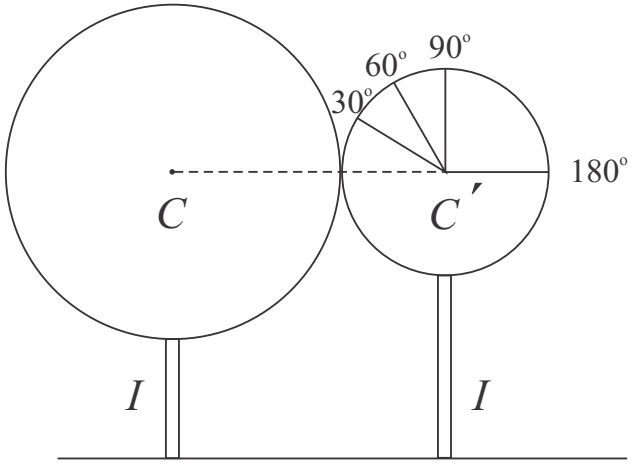


Figure 23.6: Two conducting globes of different sizes in contact, electrified and insulated from the ground. Coulomb compared the surface charge densities on the small globe at the points located at 30°, 60°, 90°, and 180° from the point of contact.

The next Table compares Coulomb's measurements presented in Section 22.16 with Poisson's theoretical calculations made 20 years later.[830] The last column of this Table shows the percentage difference between the value V_c calculated by Poisson and the value V_m measured by Coulomb, that is, $[(V_c - V_m)/V_c] \cdot 100\%$.

Ratio between the radii	Angular distance between the observed point and the contact point	Ratio between the electric densities of the observed point and the point located at 90° from the contact, that is, D_θ/D_{90°		
R_2/R_1	θ	Observed by Coulomb	Calculated by Poisson	Percentage difference
1	30°	$1/4.80 = 0.21$	$1/5.857 = 0.171$	−22.8%
1	60°	$1.25 = 0.80$	$1/1.342 = 0.746$	−7.2%
1	90°	1	1	0%
1	180°	$1/0.95 = 1.05$	$1/0.877 = 1.14$	+7.9%
2	60°	$1/1.70 = 0.59$	$1/1.80 = 0.556$	−6.1%
2	90°	1	1	0%
2	180°	$1/0.75 = 1.33$	$1/0.741 = 1.35$	+1.5%
4	90°	1	1	0%
4	180°	$1/0.70 = 1.43$	$1/0.599 = 1.67$	+14.4%

Once more there is a very good agreement between Coulomb's experimental measurements and Poisson's theoretical calculations, mainly taking into account the extremely small values of the electric forces acting on this experiment. Furthermore, no one before Coulomb had tried to estimate these quantities. Precise theoretical calculations came only 20 years after his measurements with the publication of Poisson's work. See also the discussion presented in Section 13.5.

23.4 The Attractive Forces of Electrified Spherical Shells

Let F be the force between two point charges q_1 and q_2 separated by a distance x. Nowadays this force F is written as

$$F = k\frac{q_1 q_2}{x^2} \ , \tag{23.2}$$

where k is a constant of proportionality. In the International System of Units we have $k = 1/(4\pi\varepsilon_0)$, where $\varepsilon_0 = 8.85 \times 10^{-12}\ F/m = 8.85 \times 10^{-12}\ C/(Vm)$ is a constant called vacuum permittivity. In the CGS system we have $k = 1$ dimensionless.

A spherical shell of radius R evenly electrified with a total charge Q has a surface charge density D given by

$$D = \frac{Q}{4\pi R^2} \ . \tag{23.3}$$

Following Newton in the *Principia*, we can calculate, according to Section 16.2 on page 268, the force F exerted by this spherical shell evenly electrified on a point charge

[830] [Poisson, 1812b, pp. 76-80] and [Potier, 1884, p. 204].

q_1 located at a distance r_1 from the center of the shell. This force is directed along the line joining q_1 to the center of the shell and is given by:

$$F = \left\{ \begin{array}{ll} 0\ , & \text{if } r_1 < R \\ kq_1Q/(2R^2) = kq_1 4\pi DR^2/(2R^2) = 2\pi kq_1 D\ , & \text{if } r_1 = R \\ kq_1Q/(r_1^2) = kq_1 4\pi DR^2/(r_1^2) = 4\pi kq_1 DR^2/(r_1^2)\ , & \text{if } r_1 > R \end{array} \right\}\ . \tag{23.4}$$

The force per unit charge is then given by

$$\frac{F}{q_1} = \left\{ \begin{array}{ll} 0\ , & \text{if } r_1 < R \\ 2\pi kD\ , & \text{if } r_1 = R \\ 4\pi kDR^2/(r_1^2)\ , & \text{if } r_1 > R \end{array} \right\}\ . \tag{23.5}$$

In Section 22.19 of this Fifth Memoir, Coulomb mentions an electrified spherical shell acting on an electrified point located at the surface or outside the shell. Let R be the radius of the shell and r_1 the distance of the test particle from the center of the shell. In this case the action of the shell, according to Coulomb, is measured by

$$\left\{ \begin{array}{ll} 0\ , & \text{if } r_1 < R \\ D\ , & \text{if } r_1 = R \\ 2DR^2/(r_1^2)\ , & \text{if } r_1 > R \end{array} \right\}\ . \tag{23.6}$$

By comparing Equations (23.5) and (23.6) we can see that Coulomb does not consider the factor $2\pi k$ in his equations. For instance, the force per unit charge when particle q_1 is located on the surface should be given for $2\pi kD$, instead of Coulomb's expression D. Likewise, the force per unit charge when particle q_1 is located at a distance $r_1 > R$ from the center of the shell should be given by $4\pi kDR^2/r_1^2$, instead of Coulomb's expression $2DR^2/r_1^2$.

In any event, it can be said that these forces are proportional to the surface charge density D of the spherical shell. Since the measures that Coulomb obtained in the First Part of this work were only ratios of densities, it is irrelevant whether we consider the factor $2\pi k$ in all expressions or if we neglect this constant factor.

In the Fifth Memoir Coulomb does not include this factor 2π, but it will appear in his Sixth Memoir. This factor 2π will be included by Coulomb in his Sixth Memoir when he will express the action of the spherical shell as given by (see footnote 849 on page 383, together with Section 25.2 on page 446):

$$\left\{ \begin{array}{ll} 0\ , & \text{if } r_1 < R \\ 2\pi D\ , & \text{if } r_1 = R \\ 4\pi DR^2/(r_1^2)\ , & \text{if } r_1 > R \end{array} \right\}\ . \tag{23.7}$$

23.5 Experimental Method for Determining the Distribution of Charges of Two Electrified Globes While They Are in Contact and After They Have Been Separated

In the tenth experiment of Section 22.30 Coulomb placed two conducting globes C' and C in contact while they were electrified and insulated from the Earth, Figure 23.7.

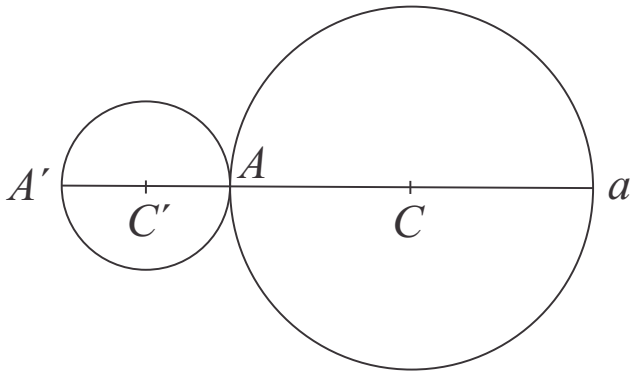


Figure 23.7: Two conducting globes with centers located at C' and C. The insulated globes touch one another at point A.

Let R be the radius of the large globe and r the radius of the small globe. Coulomb measured the surface charge density δ at point A' at the end of the smaller globe while they were in contact. He then separated the two electrified globes and measured the average surface charge density D of the large globe. If this large globe is electrified with a total charge Q, this surface density is given by $D = Q/(4\pi R^2)$. He then compared these two charge densities.

In the next Table I present Coulomb's experimental results compared with Poisson's theoretical calculations.[831] The last column of this Table shows the percentage difference between the value V_c calculated by Poisson and the value V_m measured by Coulomb, that is, $[(V_c - V_m)/V_c] \cdot 100\%$.

R/r	δ/D observed by Coulomb	δ/D calculated by Poisson	Percentage difference
1	1.27	1.32	+3.8%
2	1.55	1.83	+15.3%
4	2.35	2.48	+5.2%
8	3.18	3.09	−2.9%
∞	> 4.00	4.27	$< 6.3\%$

Once more there is a very good agreement between the values measured by Coulomb and the theoretical values calculated with great precision by Poisson 20 years after these experiments. See also the discussion presented in Section 13.5.

[831] [Poisson, 1812b, p. 66], [Potier, 1884, p. 219], [Gillmor, 1971a, p. 204] and [Heilbron, 1999, p. 497].

Chapter 24

Sixth Memoir on Electricity: Continuation of the Researches on the Distribution of the Electric Fluid Between Several Conducting Bodies. Determination of the Electric Density in the Different Points of the Surface of These Bodies

Coulomb[832]

24.1 I

In our *Fifth Memoir*,[833] of which this is a continuation, we tried to determine the manner in which the electric fluid is shared between two globes of different diameters brought into contact, and between three globes of the same diameter. At the same time we have determined by experiment, as well as by theory, the electric density[834] of each point of the surface of these globes when they are in contact. We are now going to seek:[835]

1. How electricity is distributed between any number of equal globes placed (in contact) in such a manner that all centers are on a straight line.

2. How the electric fluid is distributed on the different parts of an electrified cylinder.

3. How it is distributed between a large globe and a line of small globes in contact with this large globe.

[832] [Coulomb, 1791] with Portuguese translation in [Assis, 2022]. This work was presented in 1788 to the French Academy of Sciences and published in 1791.
[833] This Memoir is translated in Chapter 22.
[834] That is, the amount of charge per unit area.
[835] Always assuming electrified conducting bodies in contact and insulated from the Earth.

4. In what ratio the electric fluid is divided between a large globe and cylinders of different diameters and of different lengths, brought successively into contact with the globe.

24.2 II. *Determination of the Distribution of the Electric Fluid of Six Equal Globes in Contact*

I have formed a line of six globes two inches in diameter [5.4 cm], which can be separated at will, one of which, C, *Figure 1*, is supported by a small shellac cylinder,[836] and can be placed either in the electric balance or in the row of globes.

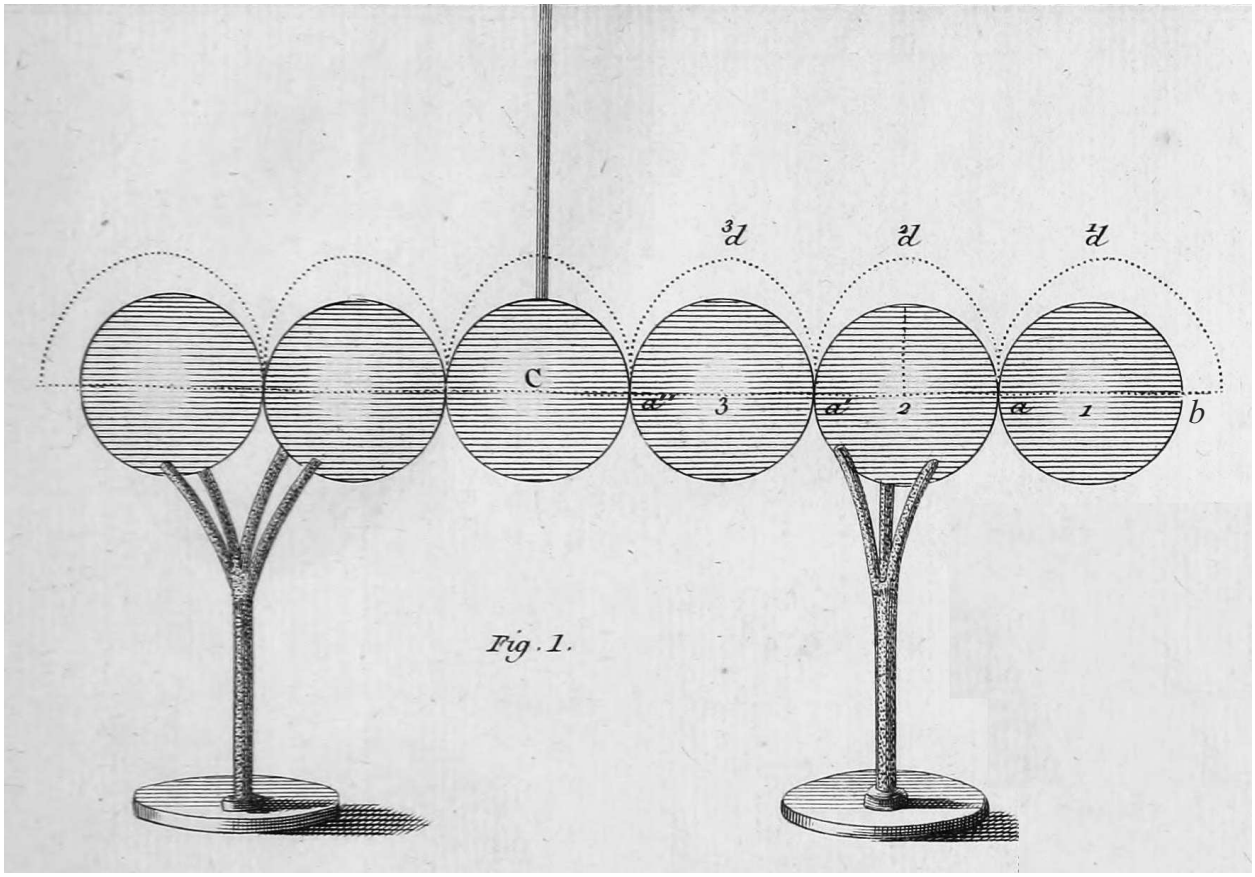


Fig. 1.

After having, according to the methods indicated in the *Volume of 1787, pages 421 and following*,[837] electrified the small paper disk at the end of the needle of the balance, I electrify the six globes[838] which are placed, *Figure 1*, on insulating supports:[839] I then alternately place the C globe first and second in line, and at each trial I present it in the balance to the needle that I have taken care to bring back to the same distance from the center of globe C: I then do the same operation by placing the globe [C] alternately first and the third in the line; by these two operations, I determine the ratios between the quantities of electricity

[836] See footnote 468 on page 208.
[837] See [Coulomb, 1789]. This Memoir is translated in Chapter 22.
[838] The six globes are electrified with a charge of the same sign as the charge on the needle of the electric balance.
[839] In the original: *supports idio-électriques*. See footnote 522 on page 229.

which the first, the second and the third globe in the line contain.

24.2.1 First Experiment. *Globe C Placed First in Line, Compared with the Same Globe Placed Second in Line*

In each test, when globe C, after having been removed from the line, was placed in the balance, the needle was brought back, by the force of torsion [of the suspension wire], to 30 degrees from the center of globe C.[840]

First trial. Globe C placed in 2, or the second in line, and then presented in the [electric] balance, repelled the needle, which was brought back to 30 degrees from the center of this globe, by a force of torsion, everything included,[841] of ..... 44°.
Second trial. Placed first in line ..... 64°.
Third trial. Placed second ..... 40°.
Fourth trial. Placed first ..... 54°.
Fifth trial. Placed second ..... 34°.

24.2.2 Second Experiment. *Globe C Placed First in Line, Compared with the Same Globe Placed Third in Line*

First trial. Globe C placed third in line; the remainder as in the preceding experiment, the force of torsion is ..... 81°.
Second trial. Placed first in line ..... 111°.
Third trial. Placed third ..... 61°.
Fourth trial. Placed first ..... 85°.
Fifth trial. Placed third ..... 51°.

24.3 III. *Result of the Two Preceding Experiments*

The five trials in each experiment were made at approximately equal time intervals, so that by taking an average between the first and the third trial, for example, this average could be compared with the second trial; the difference between the result given by the experiment between the first and third test, arises from the loss of electricity which is occasioned in this interval of time by the contact with the air, as we have already observed in the previous Memoirs.

In the first experiment, by taking an average value between the first and the third test compared to the second, it will be found that the quantity of electricity which the first globe contains, is to that which the second globe contains

$$:: 64 : 42 \qquad \text{or} \qquad :: 1.52 : 1.00 \ .$$[842]

[840] Section 25.1 illustrates Coulomb's experimental procedure.

[841] That is, taking into account the torsion of the needle attached to the lower part of the suspension wire and also the torsion of the micrometer attached to the upper part of the suspension wire, see Section 25.1.

[842] Let Q_1 and Q_2 be the total amounts of electricity contained in the first and second globes, respectively, where the first globe is the one at the far right of the row of six globes and the second globe is the one immediately next this one. Coulomb found that:

An average between the second and fourth test, compared to the third, will give this ratio :: 1.47 : 1.00.

An average between the third and fifth test, compared to the fourth, will give this ratio :: 1.46 : 1.00.

Thus, taking an average value between these three results, we will find that in our row of six globes, the quantity of electricity of the first globe is to that of the second as 1.48 is to 1.00.

A similar calculation between the first and the third globe, will give, according to the tests of the second experiment, that the quantity of electricity which the first globe contains in the line of the six globes, is to that which the third globe contains :: 1.56 : 1.00. So that the mass of the electric fluid[843] diminishes by nearly a third from the first to the second globe, and only by a fifteenth from the second to the third.

24.4 IV. *Application of the Theory to This Experiment*

It must be remembered in all the Sections of this Memoir, theory requires:

1. That the electric fluid acts in inverse proportion to the square of the distances of its parts.

2. That it is distributed on the surface of the bodies, but that it does not penetrate at least in a sensible way into the interior of the bodies.

We have proved the first proposition in our First Memoir, *Volume of 1785*; the second in the Fourth Memoir, printed in 1786.[844] It can be confirmed by a new experiment which appears decisive: here is what it consists of. We insulate a conducting body that we electrify; we then make an envelope cut into two parts which leaves a little clearance between it and the body.[845] Whether or not this envelope has the same shape as the body has little importance for the success of the experiment. If one electrifies the body, which is placed on an insulator, and if one covers it with the two parts of the envelope, supported by two insulating rods, upon withdrawing the two enveloping halves, we will find, by means of our small silk-suspension electrometers, that all electricity of the body has passed to the envelopes and that the body either retains none of it or retains only an imperceptible part.[846]

These two propositions being admitted, in order to determine by approximation the average quantity of electricity which each globe contains in our row of six globes, I assume,

$$\frac{Q_1}{Q_2} = \frac{64}{42} = \frac{1.52}{1.00} .$$

[843] In the original: *la masse du fluide électrique*. That is, the amount of electric charge contained in the globe.

[844] These Memoirs are translated in Chapters 11 and 20, respectively.

[845] This envelope is made of a conducting material and must touch the electrified body inside it in at least one point during the experiment.

[846] This experiment is sometimes referred to as the Cavendish hemisphere experiment. Although Henri Cavendish (1731-1810) did a similar experiment in 1771, before Coulomb, his manuscripts were only published by Maxwell in 1879, [Cavendish, 1879], [Gillmor, 1971a, pp. 206-210], [Blondel and Wolff, 2008a], [Blondel and Wolff, 2009] and [Falconer, 2017]. Coulomb, therefore, was the first to make known this very important experiment.

for a first approximation, that the electric mass of each of the globes is spread uniformly over the surface of these globes,[847] but that it is different for each globe, so that the electric action of all the globes on each point of contact is in equilibrium. In this supposition, the action of a spherical surface, all the points of which have the same density D,[848] acting on a point of the surface whose electric mass would be μ, would be represented by $2\pi D\mu$, being 2π the ratio of the circumference to the radius.[849]

But if the same spherical surface whose radius is R, acts on a point distant from the surface by the quantity a, the action on this point will be represented by

$$4\pi D\mu R^2 : (R+a)^2 \ .^{[850]}$$

Thus, by calculating in our experiment, *Figure 1*, the action of the six globes on the points of contact a and a',[851] we will have, by naming δ_1 the average [surface] density of the electric fluid on globe 1; δ_2 the average density on globe 2; δ_3 that on globe 3, the following two equations:[852]

First equation, equilibrium at [point] a:

$$\delta_1 = \delta_2 + \frac{2\delta_3}{3^2} + \frac{2\delta_3}{5^2} + \frac{2\delta_2}{7^2} + \frac{2\delta_1}{9^2} \ .$$

[847] That is, Coulomb is assuming that the total electric charge of each globe is evenly distributed over its surface.

[848] D is the surface density of electricity, that is, the amount of charge per unit area.

[849] In the original, [Coulomb, 1791, p. 621]:

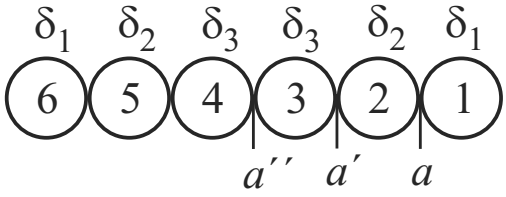
> Dans cette supposition, l'action d'une surface sphérique, dont tous les points ont la même densité D, agissant sur un point de la surface dont la masse électrique seroit μ, seroit représentée par $\Pi D\mu$; Π étant le rapport de la circonférence au rayon.

Coulomb was one of the authors who during the 18th century defined Π or π as the ratio of the circumference of a circle to its radius, that is, with $\Pi = \pi = 6.28318....$ To make Coulomb's text intelligible to the modern reader and following Potier's procedure, [Potier, 1884, p. 233], Coulomb's symbols Π or π were replaced with 2π, emphasizing the modern definition of π as the ratio of the circumference to the diameter, that is, with $\pi = 3.14159...$ I present a discussion of this expression $2\pi D\mu$ in Sections 23.4 and 25.2 on pages 376 and 446, respectively.

[850] See Section 23.4 on page 376. See also footnotes 796 and 797.

[851] From Figure 1, point a is located in the contact between globe 1 of the right end and globe 2, point a' is the point of contact between globes 2 and 3, while point a'' is the point of contact between globe 3 and globe C.

[852] These equations can be obtained using the results of Section 23.4 noting further that, by symmetry, $\delta_4 = \delta_3$, $\delta_5 = \delta_2$ and $\delta_6 = \delta_1$, as shown in the Figure of this footnote in which point a is the point of contact between globes 1 and 2, point a' is the point of contact between globes 2 and 3, while point a'' is the point of contact between globes 3 and 4:



That is, the force to the left exerted by globe 1 on a charge μ located at point a, contact between globes 1 and 2, is balanced by the force to the right exerted by globes 2 to 6 acting on the same charge μ.

On the other hand, the force to the left exerted by globes 1 and 2 on a charge μ located at point a', contact between globes 2 and 3, is balanced by the force to the right exerted by globes 3 to 6 acting on the same charge μ.

Second equation, equilibrium at a':

$$\frac{2}{3^2}\delta_1 = -\delta_2 + \delta_3 + \frac{2\delta_3}{3^2} + \frac{2\delta_2}{5^2} + \frac{2\delta_1}{7^2} \ ,$$

which are reduced to,
first equation:

$$0.98\delta_1 = 1.04\delta_2 + 0.29\delta_3 \ ,$$

second equation:

$$0.18\delta_1 = -0.92\delta_2 + 1.22\delta_3 \ ;$$

from which we derive $\delta_1 = 1.33\delta_2$ and $\delta_1 = 1.42\delta_3$.

We found by experiment, $\delta_1 = 1.48\delta_2$ and $\delta_2 = 1.56\delta_3$; thus the experiment gives the ratio of the average density of the electric fluid of the first globe to the two others, of a tenth or so greater than the theory. We had already obtained this result in the previous Memoir, for three equal globes placed in a straight line.[853]

It is easy to see to what the difference of the results between the calculation which we have just given and the experiment is in large part due; in the preceding calculation we have assumed that the electric density is uniformly spread over each globe; but in reality this density is nil, or at least insensible at all the points of contact of the globes, as we have proved, *volume of 1787, page 437 and the following.*[854] In globe 2, *Figure 1*, as well as in all the others, except the first and the last in the row, the electric density increases from the point of contact to [point] d_2, placed towards the top of the equator, where it is a *maximum.* In the first and the last globe of the row, this density increases from the point of contact to point b, the opposite pole:[855] the dotted lines in our Figure give the approximate shape of the curve of the densities.

If we now seek to determine the equilibrium at point a, assuming that all the mass of the electric fluid of globe 2 is gathered at point d_2 or at the equator,[856] and that [all the mass of electric fluid] of globe 1 is an average quantity between that gathered at point d_1 and that gathered at point b; where $4\pi\delta r^2$ is the quantity of electric fluid spread over the surface of each globe, 2π being the ratio of the circumference to the radius, the radius of the globe being r, and δ the average [surface] density of the electric fluid, or that which would exist if the electric fluid of each globe spread uniformly over the surface of this globe, we would have

$$\frac{4\pi\delta_2}{2^{3/2}} = 1.40\pi \cdot \delta_2$$

for the action of the electric fluid of globe 2 on point a, assuming that all this fluid would be concentrated at the equator; this action evaluated in the direction of the axis $a1$. The other

[853] See Sections 22.22 and 22.23.

[854] [Coulomb, 1789, page 437 and the following]. See Section 22.12.

[855] This point b mentioned by Coulomb was not represented in his original Figure. I indicated this point in Figure 1.

The passage from here on to the end of this Section was not included in Potier's reprint of Coulomb's works, [Potier, 1884].

[856] That is, Coulomb will assume that the entire electric charge of globe 2 is distributed along the equator line of globe 2, with the equator plane being orthogonal to the line passing through the centers of the aligned globes.

globes distant from the point of contact on which the equilibrium of action is calculated, may, without sensible error, be supposed to act as if the mass of their electric fluid were at the center of each globe.

As for globe 1, whose action we want to obtain relatively to point a, its average density being δ_1, if the mass of its fluid were concentrated at the point of the equator d_1, its action would be equal to $1.40\pi\delta_1$; if it were concentrated at point b, it would be $1.00\pi\delta_1$: taking the average between these two quantities, it will be $1.20\pi\delta_1$. We can now form two equations, in accord with this new supposition; the first, which expresses the equilibrium at point a, the second, which expresses the equilibrium at point a'.

First equation:

$$0.60\delta_1 = 0.70\delta_2 + 0.22\delta_3 + 0.08\delta_3 + 0.04\delta_2 + 0.02\delta_1 \ ;$$

second equation:

$$0.22\delta_1 = -0.70\delta_2 + 0.70\delta_3 + 0.22\delta_3 + 0.08\delta_2 + 0.04\delta_1 \ ;$$

which reduced give

$$\left(\frac{58}{74} + \frac{18}{62}\right)\delta_1 = \left(\frac{30}{74} + \frac{92}{62}\right)\delta_3 \ ,$$

from which $\delta_3 = 1.75\delta_1$, [while] experiment has given us $\delta_3 = 1.56\delta_1$. Thus, in accord with our new assumption, calculation gives δ_3 about one-eighth larger than the experiment: in our first assumption of the electric fluid uniformly spread over each globe, we found $\delta_3 = 1.42\delta_1$; but, as we have proved in our preceding Memoir, the electric fluid is distributed over the surface of the globes, as an average shape of those following from our two suppositions, the density being zero at the point of contact, and the mass of the fluid not wholly collected at the equator; thus, the ratio given by experiment should be approximately an average quantity between the results of the calculation of our two suppositions. We have, by the first supposition of the fluid uniformly spread over the surface of each globe $\delta_3 = 1.42\delta_1$, by the second supposition of the fluid concentrated at the equator $\delta_3 = 1.75\delta_1$, which gives for the mean quantity $\delta_3 = 1.58\delta_1$: experiment gives 1.56. The correction indicated by the calculation of this Section, will easily apply to all the theory of this Memoir.

24.5 V. Third Experiment. *Of the Manner in Which the Electric Fluid is Distributed Between Twelve Equal Globes of Two Inches in Diameter, Placed in Contact on the Same Line*

The details into which we have entered in explaining the preceding experiment suffice, I believe, to make clear the procedures which must be followed; so as not to unnecessarily enlarge this Memoir, we will report only the results for all similar experiments. In a [straight] line, formed by twelve globes 2 inches in diameter [5.4 cm], we found that the quantity of electric fluid which the first globe contains, is to that which the second contains :: 1.50: 1.00; by comparing the first globe with the sixth or with that in the middle, we have found that the quantity of electric fluid which the first globe takes on, is to that which the sixth takes on :: 1.70: 1.00.

24.6 VI. Fourth Experiment. *Distribution of the Electric Fluid Between Twenty-four Globes 2 Inches in Diameter, Placed in Contact on the Same Line*

By always comparing by the same method the first globe with the second, I found that the quantity of electricity which the first globe contained, was to that which the second contained :: 1.56: 1.00; by comparing the first and the twelfth, or that of the middle, I found that the quantity of electricity which the first globe of the line contained, was to that of the middle globe :: 1.75: 1.00.

24.6.1 *Result of the Last Two Experiments*

It results from these two experiments that, whatever the number of globes placed in contact on a straight line, the average density varies considerably from the first to the second globe, but that it then varies very slowly from the second to that of the middle: in the fourth experiment, we had a line formed by 24 globes. The average density from the first to the second globe has diminished in the ratio of 1.56 to 1.00; but, from the second to the twelfth, it only varied in the ratio of 1.75 to 1.56.[857]

If we want to apply the above methods of approximation here, we will first take the line formed by twelve globes, by naming δ_1 the average density of the first globe, δ_2 that of the second, δ_3 that of the third, δ_4 that of the fourth, etc. we will have for the first equation which expresses the equilibrium at point a:

$$\delta_1 = \delta_2 + \frac{2}{3^2}\delta_3 + \frac{2}{5^2}\delta_4 + \frac{2}{7^2}\delta_5 + \frac{2}{9^2}\delta_6 + \frac{2\delta_6}{11^2} + \frac{2\delta_5}{13^2} + \frac{2\delta_4}{15^2} + \frac{2\delta_3}{17^2} + \frac{2\delta_2}{19^2} + \frac{2\delta_1}{21^2} .$$

But since the densities, from the second globe to that in the middle, vary slowly, since moreover the coefficients decrease according to a very convergent series, as we move away from δ_2; we can, without great error, suppose δ_3, $\delta_4 = \delta_2$;[858] then by summing the series, we will have very approximately $\delta_1 = 1.41\delta_2$; a quantity smaller by about one tenth than $\delta_1 = 1.55\delta_2$ which experiment has given as a difference produced, as we have explained, Section 24.3, by the fluid condensed at the last globe at the extremity of the axis; whereas in the others, it is at the equator that the *maximum* of condensation takes place.

To determine now the quantity of electricity which the sixth globe acquires, relatively to the first, I form, according to the method of Section 24.4, a Table of five equations which expresses the state of equilibrium at all points of contact.

Here are these five equations.

At point a:

$$-\delta_1 + \delta_2 + \frac{2}{3^2}\delta_3 + \frac{2}{5^2}\delta_4 + \frac{2}{7^2}\delta_5 + \frac{2}{9^2}\delta_6 + \frac{2}{11^2}\delta_6 + \frac{2}{13^2}\delta_5 + \frac{2}{15^2}\delta_4 + \frac{2}{17^2}\delta_3 + \frac{2}{19^2}\delta_2 + \frac{2}{21^2}\delta_1 = 0 .$$

At point a_1:

$$-\frac{2}{3^2}\delta_1 - \delta_2 + \delta_3 + \frac{2}{3^2}\delta_4 + \frac{2}{5^2}\delta_5 + \frac{2}{7^2}\delta_6 + \frac{2}{9^2}\delta_6 + \frac{2}{11^2}\delta_5 + \frac{2}{13^2}\delta_4 + \frac{2}{15^2}\delta_3 + \frac{2}{17^2}\delta_2 + \frac{2}{19^2}\delta_1 = 0 .$$

[857] The passage from here on to the end of this Section was not included in Potier's reprint of Coulomb's works, [Potier, 1884].

[858] That is $\delta_3 = \delta_2$ and $\delta_4 = \delta_2$. I believe that Coulomb will also assume $\delta_5 = \delta_2$ and $\delta_6 = \delta_2$ in order to sum the series. A few paragraphs later he writes he can take the "intermediate densities as equal".

At point a_2:

$$-\frac{2}{5^2}\delta_1 - \frac{2}{3^2}\delta_2 - \delta_3 + \delta_4 + \frac{2}{3^2}\delta_5 + \frac{2}{5^2}\delta_6 + \frac{2}{7^2}\delta_6 + \frac{2}{9^2}\delta_5 + \frac{2}{11^2}\delta_4 + \frac{2}{13^2}\delta_3 + \frac{2}{15^2}\delta_2 + \frac{2}{17^2}\delta_1 = 0 \ .$$

At point a_3:

$$-\frac{2}{7^2}\delta_1 + \frac{2}{5^2}\delta_2 - \frac{2}{3^2}\delta_3 - \delta_4 + \delta_5 + \frac{2}{3^2}\delta_6 + \frac{2}{5^2}\delta_6 + \frac{2}{7^2}\delta_5 + \frac{2}{9^2}\delta_4 + \frac{2}{11^2}\delta_3 + \frac{2}{13^2}\delta_2 + \frac{2}{15^2}\delta_1 = 0 \ .$$

At point a_4:

$$-\frac{2}{9^2}\delta_1 - \frac{2}{7^2}\delta_2 - \frac{2}{5^2}\delta_3 - \frac{2}{3^2}\delta_4 - \delta_5 + \delta_6 + \frac{2}{3^2}\delta_6 + \frac{2}{5^2}\delta_5 + \frac{2}{7^2}\delta_4 + \frac{2}{9^2}\delta_3 + \frac{2}{11^2}\delta_2 + \frac{2}{13^2}\delta_1 = 0 \ .$$

These five equations in terms of six unknowns being of the first degree, it is easy by ordinary methods to reduce them to a single one, which will represent the ratio of the average density of two globes taken at will in our line formed by 12 globes placed in contact; but, as we do not need much greater precision here than that provided by experiment, we will notice that the densities of the different globes do not vary considerably, except from the first to the second globe; thus we can take the intermediate densities as equal, when we introduce them into the calculation where they enter only as small fractions; thus, to have the ratio of the density of one of the globes of the line with the first, it is necessary to make sure to combine the equations, so that the coefficient of the density which we want to evaluate, is much greater than the others; but, if I add together the five preceding equations, I will have

$$0 = -1.38\delta_1 - 0.33\delta_2 - 0.07\delta_3 + 0.07\delta_4 + 0.33\delta_5 + 1.36\delta_6 + 0.38\delta_6 + 0.16\delta_5 + 0.10\delta_4 + 0.07\delta_3 + 0.05\delta_2 + 0.04\delta_1 \ .$$

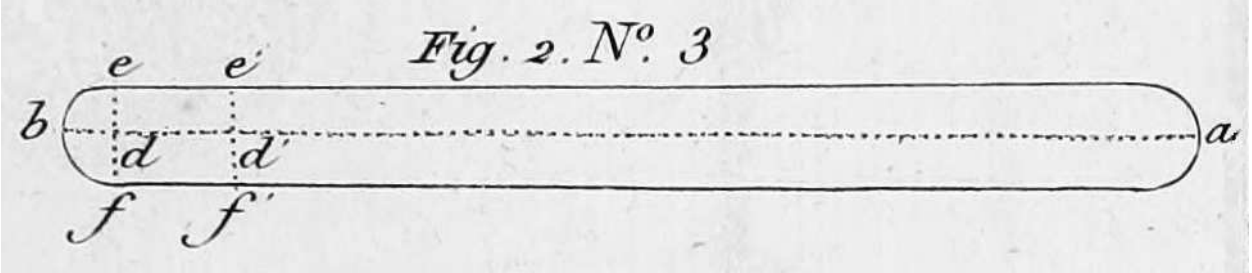
In this equation, we see that only the coefficients of δ_1 and δ_6 are considerable: we know moreover that the density varies little, except from the first to the second globe; thus we can suppose the average electric density, nearly the same in the terms whose coefficients are fractions from the second to the sixth globe; it will result from this supposition $1.34\delta_1 = 2.16\delta_6$, from which $\delta_1 = 1.54\delta_6$.

Experiment gave $\delta_1 = 1.70\delta_6$; so that the ratio of the quantity of the electric fluid of the first globe is, relatively to that of the middle globe, about a tenth larger by experiment than by calculation: a result consistent with everything we have found previously.

24.7 VII. Fifth Experiment. *Distribution of the Electric Fluid on the Surface of a Cylinder*

To suspend the needle of the electric balance in this experiment, we used a wire of gilded silver, whose force of torsion, under the same angle of torsion, was only a twentieth part of that of the thinnest copper wire, numbered 12 in commerce, used in the previous four experiments.

We placed, *Figure 2, No. 3*, a cylinder 2 inches in diameter [5.4 cm] and 30 inches in length [81.2 cm], terminated by two hemispheres, on an insulating support.



This electrified cylinder was made to touch a small disk of gilded paper, supported by a shellac thread which was then introduced into the [electric] balance according to the procedures already indicated in our Fifth Memoir, *volume of 1787, Plate 1, Figure 3*.[859] It resulted from this experiment, by touching alternately a point taken in the middle of the surface of the cylinder and a point taken at the end, that the density in the middle of the cylinder was to that at the end, as 1.00 : 2.30.

By comparing a point in the middle of the cylinder with a point 2 inches from the end, the [ratio of the] electric density in the middle of the cylinder to that at 2 inches from the end was found to be 1.00 : 1.25.

By comparing the middle point with a point on the great circle of the hemisphere which terminates the cylinder or at point e, at 1 inch from its end, the [ratio of] the densities was found to be 1:00 : 1.80.

24.7.1 *Result of This Experiment*

It results from this experiment, that within the two last inches at the end of the cylinder, the electric density is much more considerable than towards the middle of the cylinder; but that it varies little from the middle of the cylinder to [a point located at] two inches from its extremity.

24.8 VIII. *Theory of the Distribution of the Electric Fluid on the Surface of an Insulated Cylinder*

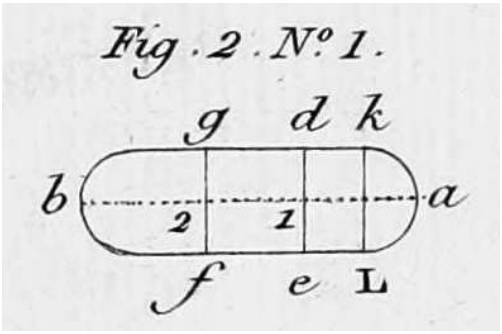
When a body is charged with electric fluid, and this fluid is in equilibrium, it is necessary that in dividing the body into two parts, and by calculating the action of these two parts on any point, this action being evaluated in the same direction, there is equilibrium [between the opposite actions]. Thus it suffices, to have the conditions of equilibrium of the electric fluid on the surface of a cylinder, to calculate the conditions of equilibrium relative to the axis of this cylinder.[860]

[859] This conducting disk of gilded paper suspended by an insulating shellac thread is a proof plane. See the representation of the proof plane in Figure 3 of the Fifth Memoir, page 335.

[860] The passage from here on to the end of Section 24.9 was not included in Potier's reprint of Coulomb's works, [Potier, 1884].

24.8.1 First Example. *Cylinder Two Inches in Diameter and Six Inches in Length*

The cylinder, *Figure 2, No. 1*, is 2 inches in diameter and 6 inches in length [5.414 and 16.242 cm, respectively].



It is divided at points 1 and 2 by planes perpendicular to the axis, into three equal parts;[861] it is terminated by a hemisphere at both ends. We assume that the average density on the surface of part *dae* is δ_1; that the one on the part *degf* is δ_2: that of the part *fgb* will be the same as that of *dae*.

But the action of the hemisphere *kaL* on point 1, in the direction *a*1, is $2\pi\delta_1(1-1/\sqrt{2})$, the radius of the cylinder being unity,[862] and 2π the ratio of the circumference to the radius; the portion of the cylinder *dkLe*, whose length is equal to the radius, has for action on point 1 in the direction *a*1, the quantity $2\pi\delta_1(1-1/\sqrt{2})$: the action of the portion *degf*, which has 2 inches of length on the same point 1 in the opposite direction, is equal to $2\pi\delta_2(1-1/\sqrt{5}) = 1.10\pi\delta_2$; the action of the portion *fbg*, on the same point 1, in the same direction, can without perceptible error be calculated as if it were united in the middle of 2*b* or at 3 inches from point 1; thus its action on point 1 is very close to $0.44\delta_1$, from which results, to express the equilibrium at point 1 of all the actions evaluated along the direction of the axis, the equation

$$0.59\delta_1 = 0.55\delta_2 + 0.22\delta_1 \ ,$$

from which results $\delta_1 = 1.49\delta_2$.

24.8.2 Second Example. *Cylinder Two Inches in Diameter and Twelve Inches in Length*

If the cylinder, *Figure 2, No. 2*, was 12 inches long [32.5 cm] and ended in a hemisphere at each end, to have an approximate value of the average density of its different parts, we would divide it into 6 equal parts of 2 inches each in length, and we would seek the equilibrium conditions in the direction of the axis for point 1 and for point 2.

[861] That is, it is divided into three parts of the same length, although their shapes are different.
[862] That is, radius = 1 inch = 2.707 cm.

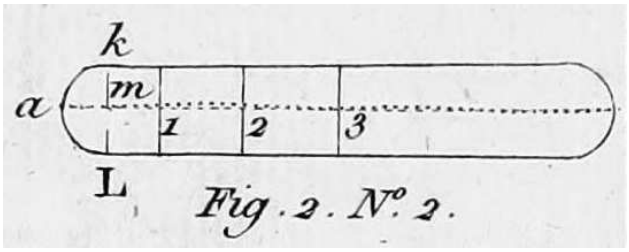


Fig. 2. N°. 2.

Let δ_1 be the average density on the part of the surface that responds to [portion] $a1$; δ_2 the average density over the surface that responds to [portion] 12; δ_3 on the surface that answers to [portion] 23; we will have, according to what is explained in the previous Section, the following two equations.

For the equilibrium at point 1, *first equation*:

$$0.59\delta_1 = 0.55\delta_2 + 0.22\delta_3 + 0.08\delta_3 + 0.04\delta_2 + 0.02\delta_1 \ .$$

For the equilibrium at point 2, *second equation*:

$$0.22\delta_1 = -0.55\delta_2 + 0.55\delta_3 + 0.22\delta_3 + 0.08\delta_2 + 0.04\delta_1 \ .$$

These two equations reduce to, *first equation*:

$$0.57\delta_1 = 0.59\delta_2 + 0.30\delta_3 \ ,$$

second equation:

$$0.18\delta_1 = -0.47\delta_2 + 0.77\delta_3 \ ,$$

from which results $\delta_1 = 1.60\delta_3$ and $\delta_1 = 1.55\delta_2$; slightly larger ratios, but still very close to those we found, *Sections 24.2 and 24.3*, for six equal globes placed in contact on a straight line. We feel indeed, according to the observations which we made, *Section 24.4*, in the theory of the distribution of the electric fluid on six globes in contact and in straight line, that the density being almost null at the points of contact of the globes and at the parts which adjoin, the average distribution of the electric fluid on each globe must be nearly the same as [the distribution] on a continuous cylinder which would be 12 inches in length, and terminated by two hemispheres.

24.9 IX. *Second Approximation Method to Determine by Theory, the Distribution of the Electric Fluid Along the Surface of a Cylinder Terminated by Two Hemispheres*

From the two examples which precede, and from the theory which we have given of the distribution of the electric fluid on a [straight] line formed by twelve globes of 2 inches in contact, it is easy, by dividing a cylinder into a number of parts each [with a length] equal to its diameter, to determine the average density of each of these parts; but when the cylinder has a great length, the following method is sufficient, and greatly simplifies the calculation.

I take as an example a cylinder 30 inches long [81.2 cm] and 2 inches in diameter, *Figure 2, No. 3*.[863] To have a first approximation, I will divide this cylinder into three unequal parts; the first formed from the hemisphere ebf, whose density is δ_1; the second, of the cylindrical portion $ee'ff'$, having 2 inches of length, whose density is δ_2; the third from d' to a, extremity of the axis of the cylinder whose density is δ_3. It is now necessary to calculate the action of these three parts on points d and d' in the direction of the axis. In this way we will derive the two [following] approximate equations.

First equation for the equilibrium at point d:

$$\frac{\delta_1}{2} = \delta_2 \left(1 - \frac{1}{\sqrt{5}}\right) + \delta_3 \left(\frac{1}{\sqrt{5}} - \frac{1}{29}\right) .$$

Second equation for the equilibrium at point d':

$$\frac{\delta_1}{\left(2\frac{1}{2}\right)^2} = -\delta_2 \left(1 - \frac{1}{\sqrt{5}}\right) + \delta_3 \left(1 - \frac{1}{27}\right) .$$

The derivation of these two equations is based on the fact that the action of the hemispherical surface ebf, on point d, which is at its center, is equal to $\pi\delta_1$; and on the fact that the action of a portion of the cylindrical surface, whose density D is uniform, acting in the direction of the axis, is equal to

$$2\pi Dr \left(\frac{1}{(r^2 + a^2)^{1/2}} - \frac{1}{(r^2 + x^2)^{1/2}}\right) ,$$

in which a is the distance from the point where the cylinder begins, to the point on which it acts; x is the distance from the point where the cylinder ends, to the point on which it acts; r is the radius of the cylinder; 2π is the ratio of the circumference to the radius.[864]

Applying this formula to our example, we derive the two preceding equations, from which results $\delta_1 = 2.09\delta_3$ and $\delta_2 = 1.14\delta_3$.

Thus, by this approximate calculation, we find that the average density of the electric fluid on the surface of the hemisphere which terminates the cylinder, is twice the average density on the surface of the cylinder. If we compare this result with the last experiment, *Section 24.7*, we found by this experiment, that the density at the extremity of the axis of the cylinder or at the pole of the hemisphere which terminates it, is to the density in the middle of the cylinder :: 2.30 : 1.00; we also found, *in the same Section*, that the density on the great circle of this hemisphere is to that in the middle of the cylinder :: 1.80 : 1.00. Thus, taking an average between these two values, one of which represents the greatest density of the hemisphere, and the other the smallest, this density decreasing from the pole of the hemisphere to the equator, we will have the ratio of the average density of the hemisphere to the average density on the cylinder, as [given by:]

$$\frac{2.30 + 1.80}{2} : 1.00 :: 2.05 : 1.00 ,$$

[863] See page 388.

[864] In the original, [Coulomb, 1791, p. 633], Coulomb mentions here that π represents "le rapport de la circonférence au rayon", that is, the ratio of the circumference to the radius, a magnitude equal to 6.28318.... As I stated in footnote 849 and in Section 25.2, I am replacing Coulomb's symbols Π or π with 2π, further emphasizing the modern definition of π as the ratio of the circumference to the diameter, that is, $\pi = 3.14159....$

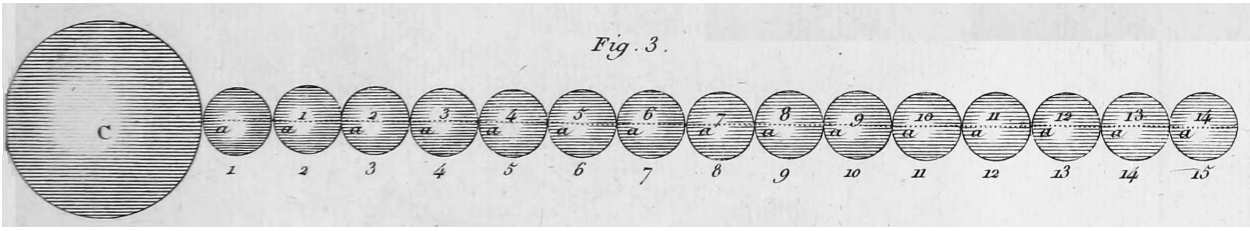
[that is,] almost exactly the same quantity given to us by theory; which must be the case, since the density varies very little from the middle of the cylinder to 2 or 3 inches from its extremity. We will give in the rest of this Memoir approximate methods, to determine in a sufficiently exact manner in practice, the laws of the variation of the electric density along the surface of a cylinder.

24.10 X. *Of the Manner in Which the Electric Fluid is Distributed Between a Certain Number of Equal Globes Placed in Contact on the Same Line Terminated by a Globe of a Larger Diameter*

The experiments in this Section run like the previous ones; we put on insulators the row of the small globes of 2 inches of diameter, as well as the globe of 8 inches [of diameter]. One of these little globes is placed at different locations on the line, and alternately in the [electric] balance.

24.11 XI. Sixth Experiment. *Distribution of the Electric Fluid Between Three globes in Contact, One Having 8 Inches in Diameter, and the Two Other 2 Inches in Diameter*

In following, *Figure 3*, in this experiment the procedures of the preceding Sections, I found that by placing only two globes, 1 and 2, of 2 inches in diameter, the centers of which were placed in [straight] line with that of globe C of 8 inches in diameter, the quantity of electricity of globe 2, the farthest from the large globe, was to that of globe 1 in contact with globe C :: 2.54: 1.00.



24.12 XII. *Theory of This Experiment*

If we analyze this experiment supposing,[865] as in the preceding Sections, that the mass of the electric fluid of each globe is spread uniformly over the surface of this globe; and if we represent by D the average density of the electric fluid on the surface of the large globe C, by R the radius of this globe; and if δ_1 represents the density of the electric fluid of the surface

[865] The text of this entire Section was not included in Potier's reprint of Coulomb's works, [Potier, 1884].

of the small globe r in contact with the large globe; and if δ_2 represents the electric density on the surface of globe 2, whose radius is r, we will have the following two equations:[866]

For the contact at point a, *first equation*:

$$D = \delta_1 + 0.22\delta_2 \ .$$

For the contact at point a_1, *second equation*:

$$\frac{2DR^2}{(R+2r)^2} = -\delta_1 + \delta_2 \ .$$

And since in our experiment globe C has a diameter of 8 inches,[867] and the two others only 2 inches, we will have, according to these two equations,

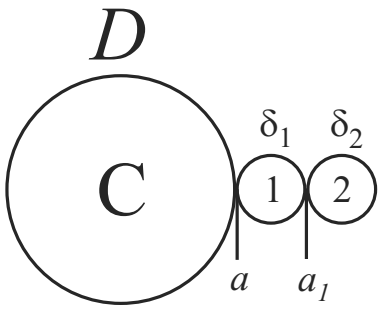
$$\delta_2 = 1.55D \ , \qquad \delta_1 = 0.67D \qquad \text{and} \qquad \delta_2 = 2.35\delta_1 \ .$$

We have just found by experiment $\delta_2 = 2.54\delta_1$, a quantity a little larger than that found by theory; which ought to effectively take place, as we said before, by virtue of the condensation of the electric fluid around point a', extremity of the axis of the small globe which terminates the row.

24.13 XIII. Seventh Experiment. *One 8-Inch Globe, and Four 2-Inch Globes in Contact*

We put, *Figure 3*, four globes 2 inches in diameter, [namely,] 1, 2 , 3 , 4, in contact with globe C, 8 inches in diameter, and we looked for the ratio of the quantities of electricity taken up by a 2-inch globe placed successively at 1 and 4.[868] By an average result between six alternate observations, we found that by placing 4 small 2-inch globes in a row in contact with globe C, the quantity of electric fluid taken up by a small 2-inch globe placed at the end of the line at 4, was to that of globe 1, immediately in contact with the 8-inch globe C, as 3.40 : 1.00.

[866]This situation is represented in the Figure of this footnote in which point a is the point of contact between globes C and 1, while point a_1 is the point of contact between globes 1 and 2:



[867]By an oversight in the original text it appeared here "radius" instead of "diameter". I have corrected that error.

[868]This situation is represented in the Figure of this footnote in which point a is the point of contact between globes C and 1, point a_1 is the point of contact between globes 1 and 2, point a_2 is the point of contact between globes 2 and 3, while a_3 is the point of contact between globes 3 and 4:

24.14 XIV. *Result and Theory of the Seventh Experiment*

To calculate by theory the seventh experiment,[869] where we put four 2-inch globes in contact with an 8-inch globe, we will construct, according to the methods which we have already explained, four equations which will express the state of equilibrium at points a, a_1, a_2 and a_3. As in the *previous Section*, let D be the average density of the electric fluid on the surface of the large globe C, whose radius is R; δ_1 the average density on the surface of globe 1; δ_2 that on globe 2; δ_3 that on globe 3; δ_4 that on globe 4, as in our experiment $R = 4r$: we will have the four [following] equations.

First equation. Equilibrium at point a gives:

$$D = \delta_1 + 0.22\delta_2 + 0.08\delta_3 + 0.04\delta_4 \ .$$

Second equation. Equilibrium at point a_1 [gives:]

$$0.89D = -\delta_1 + \delta_2 + 0.22\delta_3 + 0.08\delta_4 \ .$$

Third equation. Equilibrium at point a_2 [gives:]

$$0.50D = -0.22\delta_1 - \delta_2 + \delta_3 + 0.22\delta_4 \ .$$

Fourth equation. Equilibrium at point a_3 [gives:]

$$0.32D = -0.08\delta_1 - 0.22\delta_2 - \delta_3 + \delta_4 \ .$$

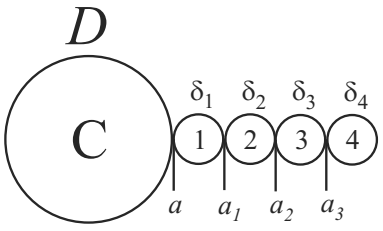
To solve these four equations, add the first to the fourth, and the second to the third, we will have;
first and fourth equations:

$$1.32D = 0.92\delta_1 - 0.92\delta_3 + 1.04\delta_4 \ ;$$

second and third equations:

$$1.39D = -1.22\delta_1 + 1.22\delta_3 + 0.30\delta_4 \ .$$

It is clear that in this operation, the coefficient of δ_2 disappears in the two results, and that in each equation, δ_1 and δ_3 have the same coefficient; thus, dividing the first result by 0.92, the second by 1.22 and adding one to the other, we will have[870]



[869] The text of this entire Section was not included in Potier's reprint of Coulomb's works, [Potier, 1884].
[870] In the original text this equation appeared as follows:

$$\left(\frac{1.32}{0.92} + \frac{1.39}{1.22}\right) D = \left(\frac{1.04}{0.92} + \frac{0.30}{1.22}\delta_4\right) \ .$$

$$\left(\frac{1.32}{0.92}+\frac{1.39}{1.22}\right)D=\left(\frac{1.04}{0.92}+\frac{0.30}{1.22}\right)\delta_4\ ;$$

from which we obtain $\delta_4 = 1.88D$.

If we substitute this value of δ_4 in the first three equations, and we continue the operation to determine the values of δ_3, δ_2, δ_1, we will find $\delta_1 = 0.60D$; $\delta_2 = 1.06D$; $\delta_3 = 1.28D$; $\delta_4 = 1.88D$; from which results

$$\frac{\delta_4}{\delta_1}=\frac{1.88}{0.60}=3.13\ ;$$

but we found by experiment $\delta_4 = 3.40\delta_1$; thus the ratio given by our theory is, as we see, less than about a tenth of that provided by experiment; which is in conformity with all that we have found previously, and with the reflections from which we saw that it would be necessary to correct our theory.

24.14.1 *Remark*

If we add together the densities of the four small globes, and take a quarter of this sum, we will have the average density of the four small globes, assuming that the sum of the densities is evenly distributed over the four globes. This average density would be equal to

$$\frac{0.60D+1.06D+1.28D+1.88D}{4}=1.205D\ .$$

24.15 XV. Eighth Experiment

To confirm the above theory, I tried to determine in a direct way by experiment, the ratio between the [electric] density of the large globe C of 8 inches in diameter, and that of the small globe 4 which terminates the line in the previous assumption of five globes in contact. Here is the procedure I followed in this comparison.

I first determined, as in the previous experiment, the density of globe 4 placed at the end of the line; I then separated globe C from the row of the four small globes, without destroying its electricity; and I made the large globe touch globe 4 which I then presented in the electric balance, to determine in a direct way the quantity of electricity that this globe 4 acquired by an immediate contact with the large globe. According to this process, I found that globe 4 placed at the end of the line of small globes, acquired a quantity of electricity which was to that which it acquired when we put it alone in immediate contact with the insulated globe C, :: 1.60 : 1.00. We find this ratio by theory :: 1.88 : 1.00; but the theory, as we have seen in the supposition of the uniform density on the surface of each globe, necessarily gives too small (a value), and according to the reflections and the experiments which precede, the corrected theory would have given very approximately this ratio as :: 2.00 : 1.00. To evaluate the result of the experiment, we must now remember, as we have seen in the previous Memoir, that a 2-inch globe placed in contact with an 8-inch globe takes on an average density, larger than that of the 8-inch globe, in the ratio of 1.30 to 1.00. Thus, to have the true ratio between the density of globe 4 placed last in the line, and that of globe C,[871] it is necessary to multiply $1.60D$, which represents the density that

[871] In the original text, C' appeared here instead of C.

globe 4 has taken on, in touching globe C, by 1.30, and we will find by experiment, between the average density of the small globe 4 placed the last in the line, and between the average density of the surface of the globe of 8 inches, the ratio as 2.08 : 1.00, almost exactly the same as just given by the corrected theory.

24.16 XVI. Ninth Experiment. *A Globe 8 Inches in Diameter, Placed in Contact with a Line of 24 Small Globes, Each 2 Inches in Diameter, Forming a Length of 48 Inches*

In this experiment, the different globes which form the line are compared to the twenty-fourth, that is to say, to the one which ends the line.

Twenty-fourth compared to the twenty-third.

By comparing the last to the penultimate, that is, the twenty-fourth globe of 2 inches to the twenty-third, it was found by an average between six tests, that the quantity of electricity, or the average density of the electric fluid on the surface of the twenty-fourth globe, was to that of the twenty-third, as 1.49 : 1.00.

Twenty-fourth compared to the twelfth.

By comparing the twenty-fourth globe with the twelfth or with that placed in the middle of the line, the average density of the twenty-fourth to that of the twelfth globe was found to be 1.70 : 1.00.

Twenty-fourth compared to the second.

By comparing the twenty-fourth with the second, it was found that the average quantity of electricity of the twenty-fourth globe was to that of the second, as 2.10 : 1.00.[872]

Twenty-fourth compared to the first.

By comparing the twenty-fourth globe with that immediately in contact with the 8-inch globe, the average amount of electricity of the twenty-fourth globe with that of the first was found to be 3.72 : 1.00.

The twenty-fourth globe compared to the 8-inch globe.

Finally, comparing by the corrected method, explained in the preceding Section, the average density of the electricity of the twenty-fourth 2-inch globe with that of the 8-inch globe, we found this ratio, to be 2.16 : 1.00, which differs, as we see, only very little from that which we had found in the preceding Section, for the fourth globe which terminated a line formed by 4 globes of 2 inches in contact with an 8-inch globe.

[872] In the original text, Coulomb wrote 2.10 : 10.

24.17 XVII. *Application of Analysis to the Preceding Experiments*

I have formed a Table of 24 equations which represent,[873] according to the method of the preceding Sections, the state of equilibrium at all the points of contact: this Table will be found at the end of this Memoir.[874] The reduction of the 24 equations to [an equation with] two unknowns, requires only patience, and presents no difficulty; but as the length of the calculation could tire most physicists, it is easy to construct different methods of approximation to shorten it; here are a few.

[873] The text of this entire Section was not included in Potier's reprint of Coulomb's works, [Potier, 1884].

[874] We included this Table at this point of the text. Moreover, to facilitate the visualization of this Table, its 24 equations are presented separately in Section 25.3, page 447.

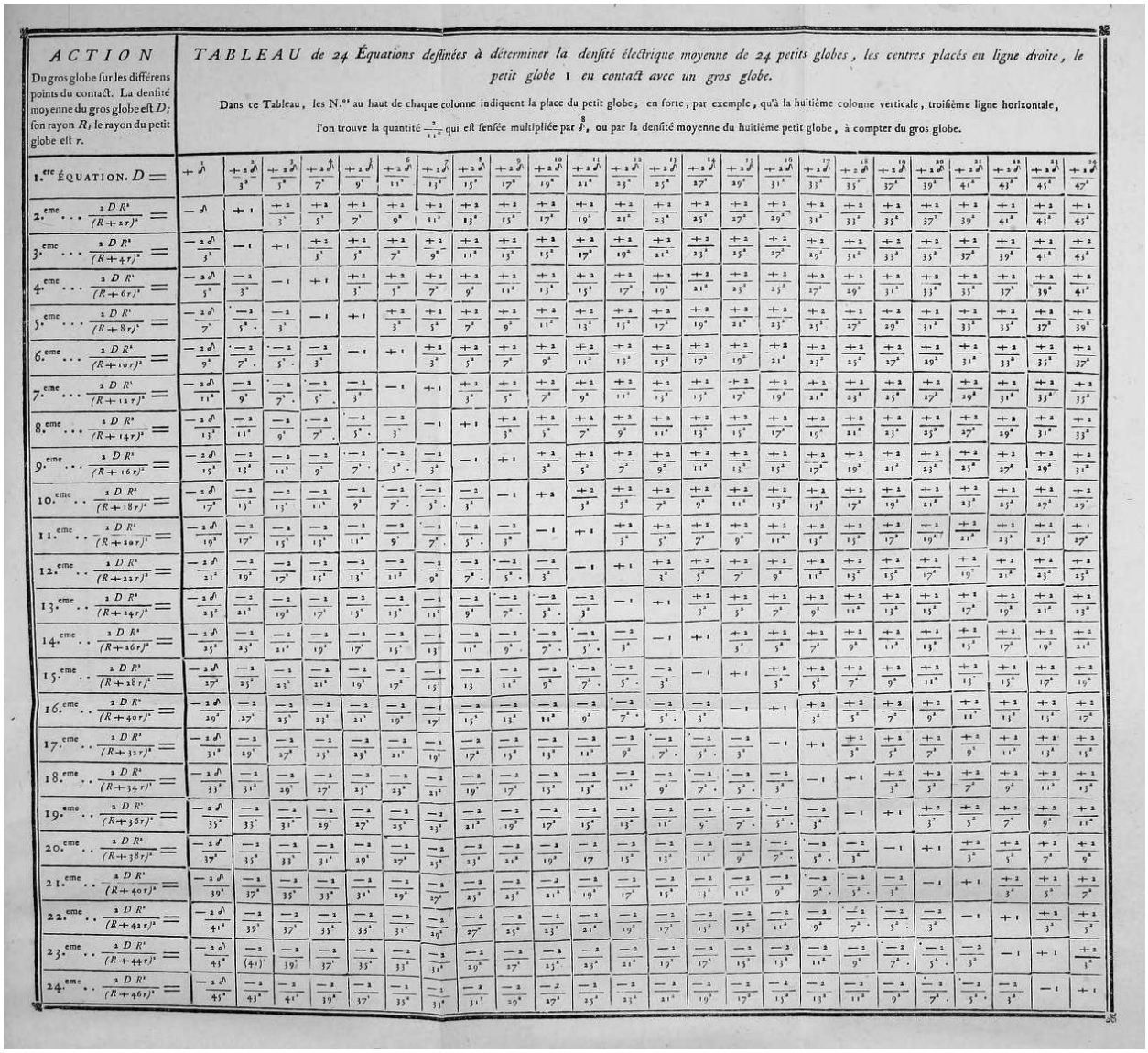
At the top left of the first line of this Table we have the following:

"Action of the large globe on the different points of contact. The average density of the large globe is D; its radius R, the radius of the small globe is r."

At the top right of the first line of this Table we have the following:

"*Table of 24 equations intended to determine the average electric density of 24 small globes, the centers placed in a straight line, the small globe 1 in contact with a large globe.* In this Table, the numbers at the top of each column indicate the position of the small globe; so that, for example, at the eighth vertical column, third horizontal line, we find the quantity $\frac{2}{11^2}$ which is supposed to be multiplied by δ_8, or by the average density of the eighth small globe, counting from the large globe."

In the second row of the first column we have "First equation. $D =$". In the third row of the first column we have "Second equation. $\frac{2DR^2}{(R+2r)^2} =$". Etc.



ACTION
Du gros globe sur les différens points du contact. La densité moyenne du gros globe est D; son rayon R; le rayon du petit globe est r.

TABLEAU de 24 Équations destinées à déterminer la densité électrique moyenne de 24 petits globes, les centres placés en ligne droite, le petit globe 1 *en contact avec un gros globe.*

Dans ce Tableau, les N.os au haut de chaque colonne indiquent la place du petit globe; en sorte, par exemple, qu'à la huitième colonne verticale, troisième ligne horizontale, l'on trouve la quantité $\frac{2}{11^2}$ qui est fenfée multipliée par $\frac{8}{5}$, ou par la densité moyenne du huitième petit globe, à compter du gros globe.

If we take the twenty-fourth equation from our Table, we will notice, from experiment and from the preceding theoretical observations, that the difference in the average electric density between the twenty-third and the twenty-fourth globe, is significant, but that the variation in density of the globes then decreases very slowly from the twenty-third to the twenty-second globe, and consecutively from the twenty-second to the twenty-first; it will be noticed moreover that in this twenty-fourth equation, the coefficients decrease very rapidly. Thus we can without great error suppose in this twenty-fourth equation, that the average density of all the small globes, from the twenty-third to the first, is equal, and from this will result[875]

$$\delta_{24} = \delta_{23}\left(1 + \frac{2}{3^2} + \frac{2}{5^2} + \frac{2}{7^2} + \text{etc.} + \frac{2}{45^2}\right) + 0.013D\ .$$

[875] Using also $R = 4r$ such that

$$\frac{2DR^2}{(R+46r)^2} = \frac{2D\cdot 4^2}{50^2} = 0.0128D \approx 0.013D\ .$$

As D is smaller than δ_{23}, we can neglect $0.013D$ and we will have:[876]

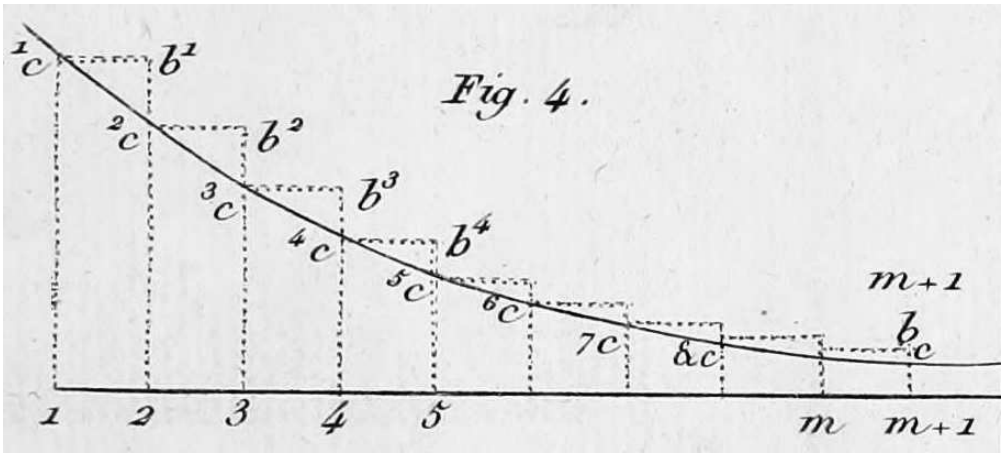
$$\delta_{24} = \delta_{23}\left(1 + \frac{2}{3^2} + \frac{2}{5^2} + \text{etc.} + \frac{2}{45^2}\right) = 1.44\delta_{23}\ ,$$

[while] experiment has given us in the preceding Section, $\delta_{24} = 1.49\delta_{23}$, which differs, as we see, very little from the result provided by the theory; but it must be remarked, that here the errors of the supposition on which we base our calculations, mutually compensate each other; we make the average density of the twenty-fourth globe smaller than it really is, as we have seen *Section 24.4*, but we also make the density of the twenty-third globe too small, since the same density is assumed from the twenty third to the first globe, instead of always decreasing.

If we want to obtain in an approximate way the values of the densities $\delta_1\delta_2\delta_3$ relatively to D, we will assume in the first four equations, that the densities are equal from δ_4 to δ_{24}, and in this case the first four equations of the Table will reduce to, *Figure 3*:

[876] [Note by Coulomb] To have in an approximate manner the value of this very convergent series, we can make use of a known method, very simple, sufficient in practice.

We see that the consecutive terms having the same numerator, the denominators follow the progression of the square of the odd numbers; thus, if we take one of the terms of the progression at the distance m from another term, such as, for example, that $1/7^2$, this sum integrated as forming an ordinary curved line, would give for its differential $2dm/(7+2m)^2$, and for the integral $k - \frac{1}{7+2m}$; where $k = 1/7$, because this quantity must vanish when $m = 0$. Thus, if the consecutive terms of the series differed little from each other, this integral would represent quite exactly the sum of the series. But it should be noted, to correct this value, that, if, *Figure 4*, c_1, c_2, etc., c_m, represent the curve that we have just integrated, whose base $1m$ is divided into parts equal to unity; and if we build on the divisions of this base, each term of the series, these terms will be represented by the parallelograms $12c_1b_1$, $23c_2b_2$, etc.; thus each term of the series will differ from the corresponding term in the differential of the surface of the curve of a small triangle $c_1b_1c_2$; and if each element c_1c_2, c_2c_3, etc. can be taken for a straight line, it is easy to see that the sum of the series differs from the integral of the curve, by an amount equal to the sum of all the small triangles $c_1b_1c_2$, $c_2b_2c_3$, etc. plus the last term of the series, represented in the Figure by the right-angled parallelogram $m(m+1)\cdot c_mb_m$.



But the sum of these small triangles, plus half of the last term or the small parallelogram $c_mb_m \cdot m(m+1)$, is equal to half of the first term or of the rectangular parallelogram c_1b_112; thus the sum of the terms which form the series, is equal to the integral of the curve, plus half of the first term, plus half of the last; so in our example:

$$1 + \frac{2}{3^2} + \frac{2}{5^2} + \frac{2}{7^2} + \text{etc.} + \frac{2}{45^2} = 1 + \frac{2}{3^2} + \frac{2}{5^2} + \left(\frac{1}{7} - \frac{1}{45}\right) + \frac{1}{7^2} + \frac{1}{45^2} = 1.44\ .$$

For the contact point a:

$$D = \delta_1 + 0.22\delta_2 + 0.08\delta_3 + 0.14\delta_4 \ .$$

For the contact point a_1:

$$0.89D = -\delta_1 + \delta_2 + 0.22\delta_3 + 0.22\delta_4 \ .$$

For the contact point a_2:

$$0.50D = -0.22\delta_1 - \delta_2 + \delta_3 + 0.44\delta_4 \ .$$

For the contact point a_3:

$$0.32D = -0.08\delta_1 - 0.22\delta_2 - \delta_3 + 1.44\delta_4 \ .$$

Let us do for these four equations the same operations as we did for *Section 24.15*, and we will have

$$\delta_1 = 0.54D; \qquad \delta_2 = 0.92D; \qquad \delta_3 = 1.04D; \qquad \delta_4 = 1.14D \ .$$

The values δ_2, δ_3 and δ_4 which differ little from each other, suggest that, without much error, we could assume equal densities from δ_4 up to δ_{24}, since the coefficients decrease according to a very convergent series; it is however clear that δ_4, found by this operation, is a little too large, since the densities increase[877] from δ_4 to δ_{24}, while according to our supposition they are equal.

The densities δ_1, δ_2, δ_3, δ_4, being determined relative to D, according to the preceding equations, if we substitute their values in the fifth, sixth, seventh and eighth equations of our Table, and suppose in these four equations all the other densities equal [to one another], from δ_8 to δ_{24}, we will be able, by following the same process, to determine, by means of these four new equations, the approximate densities of δ_5, δ_6, δ_7, δ_8; we shall then succeed, by the same method, in determining by approximation the values of δ_9, δ_{10}, δ_{11}, δ_{12}, etc. If in accord with these values thus determined, we wanted to obtain values of δ_1, δ_2, δ_3, δ_4, to a better approximation than that obtained by the first operation, we would substitute in the first four equations, the values which we would have found for δ_5, δ_6, δ_7, etc. and the first four equations combined together, would then give us in a very approximate manner the values of δ_1, δ_2, δ_3, etc., taking care to introduce into them the corrections indicated at the beginning of this Memoir.

24.18 XVIII. *Of the Manner in Which the Electric Fluid is Distributed Between a Globe and Cylinders of Different Lengths, but of the Same Diameter*

24.18.1 Tenth Experiment

We electrified a globe 8 inches in diameter [21.7 cm], and brought it into contact with a ball 9 lines in diameter [2.0 cm], insulated and supported by a shellac thread which we introduced as usual into the [electric] balance; the needle was driven out at 28 degrees, with a force of torsion, all included,[878] of 154 degrees.

[877] For a lapse in the original text it appeared here "decrease" instead of "increase". I fixed this error.

[878] That is, taking into account the torsion of the needle attached to the bottom of the suspension wire and also the torsion of the micrometer attached to the top of the suspension wire.

This 8-inch globe was immediately touched by a cylinder 2 inches in diameter [5.4 cm] and 30 inches long [81.2 cm], and on withdrawing the cylinder, the globe was touched by the small ball (measuring) 9 lines in diameter which has been introduced again into the balance; the needle was driven the same distance as the first time with a force, all included, of 68 degrees.

24.19 XIX. *Result of This Experiment*

The 8-inch globe before the contact of the cylinder, has a quantity of electricity which we find represented by 154 degrees; but it must be remarked that in the [time] interval of the observations, the quantity of electricity diminished by one fortieth by contact with the air; thus to compare the first observation with the second, it is necessary to reduce the quantity of electricity of the first observation to 150 degrees. But we find that by the contact of the cylinder these 150 degrees are reduced to 68 degrees; thus, by contact, the cylinder has taken 82 degrees of the electric mass of the globe, and has left it with only 68 degrees, so that the quantity of the electric fluid of the cylinder to that of the globe, is after this division, :: 82 : 68, [that is,] :: 1.21 : 1.00.

To obtain now the ratio of the average densities of the electric fluid spread on the surface of the cylinder to the density of the electric fluid on the surface of the globe, it will be noticed that the globe being 8 inches in diameter, and the cylinder 2 inches in diameter and 30 inches in length, the surface of the cylinder is to that of the globe, :: 60 : 64; thus the average densities of the electric fluid spread only on the surface of the bodies, being equal to the quantity of this fluid divided by the surface, the average density of this fluid on the surface of the cylinder, will be to that on the surface of the globe,

$$:: \frac{1.21}{60} : \frac{1.00}{64} ,$$

[that is,] :: 1.29 : 1.00.

Taking the mean of many other experiments, we obtain this ratio :: 1.30 : 1.00.

24.20 XX. Eleventh Experiment

We have determined by the same method the quantity of electricity taken by a cylinder which was only half or even a third of the length of the first; and it was found, by following the methods of the preceding experiment, that the mean density of a cylinder 15 inches [40.6 cm], and even ten inches in length [27.1 cm], was to the mean density of the same fluid on the 8-inch globe, in approximately the same ratio as we have just found for the 8-inch globe when it shares its electric fluid with a cylinder 30 inches in length.

It is only necessary to note that, when the globe is very large relative to the cylinder, and this cylinder is very short in length, then the average density of the small cylinder, relative to that of the globe, will be much less than when the cylinder will have a great length; thus, for example, when I placed a small cylinder 5 or 6 lines in length [1.13 to 1.36 cm] and 2 lines in diameter [0.45 cm] in contact with an 8 inch globe, the average density of the electric fluid on the surface of this cylinder was to that of the globe approximately in the ratio of 2 to 1; but if I put a cylinder 2 lines in diameter and more than 6 inches in length [16.2 cm] in contact with this same globe, the average density of the cylinder was to that of the 8-inch

globe, approximately :: 8 : 1 We will see in what follows that the theory agrees with this result.

24.21 XXI. *Remark*

It is easy to perceive that the theory ought to give approximately the results which have been furnished to us by the experiments which precede; for, if we suppose that we successively put our 8-inch globe in contact with a 30-inch cylinder, and then with a 15-inch cylinder, by electrifying this globe each time, so that after the contact with the two cylinders, it preserves in both cases the same quantity of electricity, it will be necessary, since there is equilibrium, that the quantity of electricity and its distribution on the cylinder of 15 inches, are such, that its action on the point of contact with the globe, be the same [action] as that of the 30-inch cylinder; but as the action is inversely proportional to the square of the distances in the 30-inch cylinder, all the parts placed beyond 15 inches are at a sufficiently considerable distance from the point of contact, so that their action is only a very small quantity, relative to the action of the first 15 inches near the contact. Thus, to preserve equilibrium in the two suppositions, the quantity of the electric fluid of the large globe being supposed the same, it is necessary that the fluid over the first 15 inches produce, in both cases, nearly the same action; thus, the electric fluid there must be in approximately the same quantity, and distributed nearly in the same way. Consequently, the ratio of the average density between the globe and the cylinders must be nearly the same in the two cases.

24.22 XXII. *Of the Manner in which the Electric Fluid is Divided Between an Electrified Globe and Cylinders of Different Diameters, but of the Same Length*

As the experiments intended for this Section are carried out by exactly the same methods as those which precede, I will report here only the results.

The globe, 8 inches in diameter, placed on insulating supports, being electrified, this globe was touched by three different cylinders thirty inches in length.

The first cylinder is 2 inches in diameter [5.414 cm]; the second cylinder is 1 inch in diameter [2.707 cm]; the third cylinder is only 2 lines in diameter [0.452 cm].

We first determine the quantity of electricity on the globe before it has been touched by a cylinder; we then determine the quantity of electricity after it has been touched by this cylinder; the difference of these two quantities of electricity, gives that which the cylinder takes up in contact, which, compared with that which remains in the globe, gives the ratio between the quantity of electricity of the globe and the average quantity of electricity of the cylinder after contact: but as the electric fluid is distributed, as we have proved, only on the surface of bodies, we will have the density of this fluid, by dividing the quantity [of electric fluid] by the surface of the body. By following this method of reduction, it has resulted from many experiments, that the average density on the surface of an 8-inch globe being represented by the number 1.00.

That of a cylinder two inches in diameter and 30 inches in length would be represented by 1.30.

That of a cylinder one inch in diameter by 2.00.

That of a cylinder 2 lines in diameter by 9.00.

In these results, the cylinder 2 lines in diameter having only the twelfth part of the diameter of the first, the average density of the electric fluid which covers the surface is 7 to 8 times greater than that of the cylinder two inches in diameter; whence it follows that this increase in density does not exactly follow the ratio of the diameters of the cylinders, but a smaller ratio. In practice, it seemed to me that one would have in a sufficiently exact manner the densities of different cylinders, put in contact with a globe whose electric density would be a constant quantity, supposing them to be in inverse ratio of the power 4/5 of the diameter of the cylinders; power which varies and seems to approach unity, when we compare cylinders whose diameter is very small relative to that of the globe, and which is smaller than unity, as the diameters of the cylinder increase relatively to that of the globe. We have just found that the electric density of a globe whose diameter is 4 times greater than that of a cylinder being represented by D, the average density of the cylinder is equal to $1.30D$; but we find by experiments analogous to those of which we have just reported the result, that when the diameter of the globe is only twice as large as that of the cylinder, the average density of the cylinder will be equal to $0.85D$: if finally the diameter of the globe is equal to that of the cylinder, we will find the average electric density of the cylinder equal to $0.60D$.

24.23 XXIII. *First Remark*

Reasoning, independently of any calculation, suggests the preceding result; that is, according to this reasoning, we perceive that the average density of two cylinders of different diameters, should not follow exactly the inverse [ratio] of the diameters, but a ratio a little smaller.

Let us take two cylinders equal in length, whose diameters are [to one another] as 2 : 1, and put them successively in contact with an electrified globe; suppose that the original quantity of electricity of this globe was such that after contact it retained in both cases the same quantity of electricity: if one divides the cylinders into a great number of parts equal in length, then in order that equilibrium holds at the points of contact of the globe and the cylinders, it is necessary that each corresponding part of the same length in the two cylinders experience the same electric force in order that equilibrium exists with the globe since the action of the globe is the same in the two cases. But it should be noted that the two cylinders being in contact with the globe at the extremity of their axes, the electric fluid spread over the surface of the two cylinders will act, in the parts which border the globe, more directly on the point of the axis in contact with the globe in a cylinder of a small diameter, than in a cylinder of a large diameter; thus it will not be necessary for equilibrium, [to have] precisely the same quantity of electric fluid on the surface of a cylinder of a small diameter, as on the surface of a cylinder of a larger diameter; thus the density on the surface of the globe being supposed the same after the contact of the two cylinders, the average density of the electric fluid on the surface of the small cylinder, will not be to that of a larger cylinder exactly in inverse ratio of the diameter of the cylinders, and the variation of this ratio will be all the greater, as the diameter of the cylinder will be greater relatively to that of the globe.

24.24 XXIV. *Second Remark*

A very interesting observation presents itself here, it is that of the action of points, or cylinders of a very small diameter applied at their extremity to an electrified body. Experiment

teaches that a body thus equipped with a point rapidly loses the greater part of its electricity. The above results explain this phenomenon.

We find, in fact, by experiment, that a cylinder two lines in diameter and 30 inches in length, placed in contact with an 8-inch globe, is surrounded by an electric fluid whose average density is 9 times greater than that of the globe. But we have seen before, *Section 24.7, fourth experiment*, that when a cylinder is electrified and terminated by a hemisphere of the same diameter as the cylinder, the density of the electric fluid at the end of the axis of the cylinder was to that on the middle of the cylinder, :: 2.30 : 1.00. This ratio should even be greater, as reasoning and experiment indicate, when this cylinder is very long and one of its extremities is in contact with a large globe; thus supposing the cylinder 2 lines in diameter, rounded at its extremity into a hemisphere, the electric density at the extremity of the axis of this cylinder, would be to that on the surface of the globe, 8 inches, as nine times 2.30 is to 1.00, [that is,] as 20.7 is to 1.0: but as air is a body of imperfect insulating capacity,[879] whose moving parts only resist the communication and penetration of the electric fluid as long as it is only brought to a very small degree of density,[880] it results that by touching the end of our cylinder of 2 lines in diameter, with the globe of 8 inches charged with electricity, the electric fluid must escape by the end of the cylinder more rapidly, as the electric density strengthens; and this electric density being still very great at the extremity of the cylinder, while it will be almost imperceptible on the surface of the globe, the globe must be stripped very quickly of almost all its electricity.[881] This in no way contradicts the law which we found in our Third Memoir, which gave us the successive decrease of the density of the small globes proportional to the density,[882] because, as we said at the time, this law takes place

[879] That is, as the air is an imperfect insulator.

[880] That is, to the extent that electricity or electric charge has a very small density.

[881] Coulomb is trying to explain the power of points on electrified bodies. The modern explanation of the electric discharge through the ends of electrified conductors is more complex. This high surface charge density that occurs at the ends of the conductors causes, in the vicinity of these ends, an ionization of the air. It is these charges produced by the ionization of the air that are set in motion. It should be emphasized here that the electric discharges in the air are not due to the stripping of electrons from the electrodes, as is sometimes erroneously claimed. For electrons to be emitted from metallic surfaces kept at low temperatures, forces per unit charge of the order of $10^8\ V/m$ are required. This force per unit charge is much greater than the value of $3 \times 10^6\ V/m$ needed to ionize air at atmospheric pressure, when electric discharge occurs. That is, the force per unit charge needed to ionize air is much smaller than the force per unit charge needed to remove electrons from cold electrodes. More details on the power or action of points can be found in Section B.9 (Discovery of the Power of Points) of the book *The Experimental and Historical Foundations of Electricity*, [Assis, 2010b], [Assis, 2010a], [Assis, 2011], [Assis, 2015b] and [Assis, 2017]; together with Chapter 9 (The Power of Points) and Section 11.7 (Gray, Franklin, the Power of Points and the Electric Nature of Lightning) of Volume 2 of the same book, [Assis, 2018b], [Assis, 2018a] and [Assis, 2019]. See also [Gray, 1732, p. 42] with Portuguese translation in [Boss et al., 2012, Chapter 7], [Savelyev, 1989, p. 249], [Ferreira and Maury, 1991, pp. 60-62], [Ferreira, s da, pp. 39-40], [Gaspar, 2003, pp. 239-243], [Blondel and Wolff, 2008b], [Laburú et al., 2008], [Silveira, 2010], [Silveira, 2011], [Blondel and Wolff, 2011d], [Silveira, 2016] and [Silveira, 2018].

[882] Let a sphere of radius r be electrified in time t with a charge q. In the Third Memoir, translated in Chapter 17, Coulomb studied the charge leakage of this sphere as a function of time when placed in the air. He found the following equation (see also Chapter 19):

$$\frac{d\delta}{\delta} = -m dt \ , \tag{24.1}$$

where δ can represent the value of the charge q or else the electric surface density of the sphere, $q/(4\pi r^2)$, with m being a positive constant. That is, the variation of δ as a function of time is proportional to δ, as mentioned here by Coulomb. The integration of this equation leads to an exponential decrease of δ as a function of time t.

only when the electric density is insignificant.

24.25 XXV. *Of the Manner in Which the Electric Fluid is Divided Between Globes of Different Diameters, and the Same Cylinder*

By following the same methods as in the preceding Sections in these experiments and in their reduction, it will be found that, when the globes are of a diameter much larger than that of the cylinder, as for example eight times and beyond, the electric densities of the different globes in contact with the cylinder being supposed equal to the same quantity D, the densities of the electric fluid which will envelop the cylinder will be like the diameter of the globes; so, for example, that if we take our 8-inch globe in contact with a one-inch cylinder, we saw in *Section 24.12*, that the density of the globe being D, that of the cylinder was about $2D$: but if instead of an 8-inch globe we put in contact with the same cylinder a globe whose diameter would be 24 inches, and whose density of the electric fluid spread over the surface of this globe, would be, as in the first case, equal to D, the average electric density of the electric fluid which would surround the cylinder, would be approximately equal to $6D$.

24.26 XXVI. *Result of the Previous Experiments*

If, according to the preceding experiments, we want to obtain the ratio between the electric density of the fluid spread over the surface of a globe and that of a cylinder of any diameter in contact at its extremity with this globe, it will suffice to observe that since for the same globe and different cylinders, according to *Section 24.22*, the electric densities of the different cylinders will be in inverse ratio of the power 4/5 of the diameters of the cylinder; power which comes very close to unity, when the globe has a diameter much larger than that of the cylinder, for different globes and the same cylinder, if the diameter of the globes is much larger than that of the cylinder, the density of the cylinder will follow the ratio of the diameter of the globes: assuming D the density of the globe, R its radius, δ the average density of the cylinder, r its radius, we will generally have

$$\delta = \frac{mDR}{r^{4/5}} ,$$

or $\delta = mDR/r$ when R is much larger than r. In this equation, m is a constant coefficient, which can easily be determined by experiment.

If indeed we observe that when we put; *Section 24.22*,[883] a globe 4 inches in radius in contact with a cylinder 30 inches in length and 2 lines in diameter, we had for the average density of the electric fluid which surrounds the cylinder $\delta = 9D$; we will see that in this example our equation $\delta = mDR/r$, by substituting the number 48 instead of R/r, will give $\delta = 48mD = 9D$; from which results $m = 9/48$.

[883] For a lapse in the original text we have here Section 33 instead of 22.

24.27 XXVII. *Application of This Result to the Electric Kite*

When in stormy weather we raise a kite, the string[884] of which is conductive or braided with a metal wire, we know that at the moment of the passage of a cloud charged with electric fluid in the region where the kite is, if the lower end of the string is insulated, or attached to an insulating body, the string of the kite throws electric sparks in all directions, and these sparks carry with the greatest violence and the greatest danger to all conducting bodies which are close to this string: it is easy to see that this phenomenon results necessarily from the experiments which precede and from the formula which we drew from it.[885]

Suppose, to serve as an example, that the cloud charged with electric fluid has the form of a globe of a thousand feet in radius [345 m]; that the kite string has a radius of one line;[886] that δ is the average density on the surface of the string: [in this case,] the equation

$$\delta = \frac{mDR}{r}$$

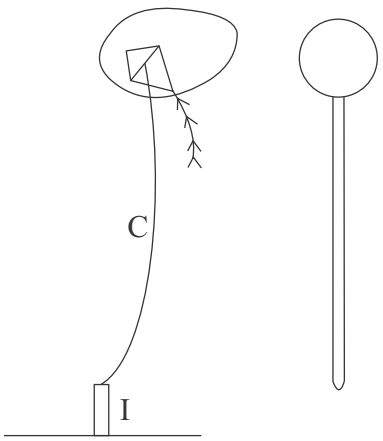
will give here[887]

$$\delta = \frac{9}{48} 1000 \cdot 12^2 D = 27000 D \ .$$

But we have seen, *Section 24.7, fourth experiment*; that the electric density, at the extremity of an electrified cylinder, terminated in a hemisphere, was to the average density of the cylinder, :: 2.30 : 1.00. Thus the electric density at the end of the string, would be equal to $62000D$, or sixty-two thousand times greater than the electric density of the fluid which is

[884] In the original: *corde*. This word can be translated as string, cord, rope, line or thread.

[885] This experiment is due to Benjamin Franklin, see footnote 80 on page 36. Coulomb here imagines a kite attached to a conducting wire C with the lower end of the wire attached to an insulator I, as illustrated in this footnote:



He will model the situation in which the kite is immersed in an electrified cloud as being analogous to the situation in which a long, thin conducting cylinder is in contact at its upper end with a large electrified conducting globe.

For a discussion of the electric kite and lightning rods, see Chapter 11 (Electric Discharges in Air) of Volume 2 of the book *The Experimental and Historical Foundations of Electricity*, [Assis, 2018b], [Assis, 2018a] and [Assis, 2019], along with the references cited there.

[886] 1 line = 0.226 cm. Coulomb is assuming the cloud to be a large conducting sphere in contact with the string holding the kite, assuming this string to be a long, thin conducting cylinder insulated at its lower end.

[887] We have 1 foot = 12 inches = 144 lines = 12^2 lines. Therefore D/r = (1000 feet)/(1 line) = $1000 \cdot 12^2$.

supposed to envelop the cloud. It must therefore necessarily happen, as it actually happens, that the electric fluid condensed at this degree of density along the string of the kite, sparks on all sides, especially towards the end of this string or towards its lower attachment, and carries itself with violence at distances often of several feet to all conducting bodies which are near it.

24.28 XXVIII. *Theoretical Determination of the Density of the Different Points and of the Average Density of a Cylinder Brought into Contact at Its Extremity with a Globe of a Larger Diameter than This Cylinder*

24.28.1 *Cylinders of Different Diameters and of the Same Length*

The simplest approximate way to determine the ratio between the electric density of the surface of the globe and that of the cylinder,[888] when the cylinder is very long, is to divide the whole length of the cylinder into parts equal to its diameter, and to look at each part as a small globe of the same diameter, to seek, as we did before, *Section 24.17*, the conditions of equilibrium at all the points of contact. Here are two other approximation methods.

24.29 XXIX. *First Method.* **First Example.** *Cylinder 30 Inches Long, 2 Inches in Diameter, in Contact at Its End with a Globe 8 Inches in Diameter*

If I want to determine the mean density or the density of the electric fluid at the middle of a cylinder 2 inches in diameter and 30 inches in length, in contact at its end with a globe 8 inches in diameter, I will suppose this cylinder terminated by two hemispheres, and divided into fifteen parts equal to the diameter of the cylinder; I will calculate the action at each division, as if the line were formed of fifteen small globes, each 2 inches in diameter. According to this supposition, the first fifteen equations of our Table[889] would give us the density of each small globe in the line: but if we want to obtain the approximate average electric density, we must determine this density towards the middle of the cylinder or towards the eighth globe. To simplify the calculation, we must observe that, according to our Table, if we add together the first eight equations, the coefficient of δ_8 will be greater than the coefficient of the terms which are far from it; that, moreover, the variation of the density from one globe to another towards the middle of the line is not significant. If, according to these reflections, we combine the first eight equations together, we will have for all the terms that follow δ_7, the following series:[890]

[888] Sections 28 to 38 were not included in Potier's reprint of Coulomb's works, [Potier, 1884].

[889] See footnote 874 on page 397 for the case of 24 small globes in contact with one large globe. In the current case of 15 small globes in contact with a large globe we would have an analogous Table with 15 equations, the terms of each equation going up to δ_{15}.

[890] According to footnote 876 on page 399, Coulomb replaced the equation

$$\frac{2}{7^2} + \frac{2}{9^2} + \ldots + \frac{2}{45^2}$$

$$\delta_8\left(1+\frac{2}{3^2}+\frac{2}{5^2}+\frac{2}{7^2}+\text{etc.}+\frac{2}{15^2}\right)=\delta_8\left(1.30+\frac{1}{7}-\frac{1}{15}+\frac{1}{7^2}+\frac{1}{15^2}\right)\ ,$$

$$\delta_9\left(\frac{2}{3^2}+\frac{2}{5^2}+\frac{2}{7^2}+\text{etc.}+\frac{2}{17^2}\right)=\delta_9\left(0.30+\frac{1}{7}-\frac{1}{17}+\frac{1}{7^2}+\frac{1}{17^2}\right)\ ,$$

$$\delta_{10}\left(\frac{2}{5^2}+\frac{2}{7^2}+\text{etc.}+\frac{2}{19^2}\right)=\delta_{10}\left(0.08+\frac{1}{7}-\frac{1}{19}+\frac{1}{7^2}+\frac{1}{19^2}\right)\ ,$$

$$\delta_{11}\left(\frac{2}{7^2}+\text{etc.}+\frac{2}{21^2}\right)=\delta_{11}\left(\frac{1}{7}-\frac{1}{21}+\frac{1}{7^2}+\frac{1}{21^2}\right)\ ,$$

$$\delta_{12}\left(\frac{2}{9^2}+\text{etc.}+\frac{2}{23^2}\right)=\delta_{12}\left(\frac{1}{9}-\frac{1}{23}+\frac{1}{9^2}+\frac{1}{23^2}\right)\ ,$$

$$\delta_{13}\left(\frac{2}{11^2}+\text{etc.}+\frac{2}{25^2}\right)=\delta_{13}\left(\frac{1}{11}-\frac{1}{25}+\frac{1}{11^2}+\frac{1}{25^2}\right)\ ,$$

$$\delta_{14}\left(\frac{2}{13^2}+\text{etc.}+\frac{2}{27^2}\right)=\delta_{14}\left(\frac{1}{13}-\frac{1}{27}+\frac{1}{13^2}+\frac{1}{27^2}\right)\ ,$$

$$\delta_{15}\left(\frac{2}{15^2}+\text{etc.}+\frac{2}{29^2}\right)=\delta_{15}\left(\frac{1}{15}-\frac{1}{29}+\frac{1}{15^2}+\frac{1}{29^2}\right)\ .$$

All the first terms of this equation represent the sum of the coefficients of the first eight equations of the Table; the second term on the right represents each series which forms these coefficients, summed according to the method which we explained in the Note of the Section 24.15.[891]

As the variation of the density of the small globes which are close to globe δ_8 is not significant and the coefficients decrease rapidly as we move away from δ_8, we can, by approximation, consider all densities from δ_8 to δ_{15} equal to one another, which will give the final equation for the sum of our series,

---

by

$$\frac{1}{7}-\frac{1}{45}+\frac{1}{7^2}+\frac{1}{45^2}\ .$$

Here he will use the same approximation, that is, he will replace the following equation

$$\frac{2}{m^2}+\frac{2}{(m+2)^2}+...+\frac{2}{n^2}$$

by

$$\frac{1}{m}-\frac{1}{n}+\frac{1}{m^2}+\frac{1}{n^2}\ .$$

[891] See footnote 876 on page 399.

$$\delta_8 \left(1.68 + \frac{3}{7} + \int \left(\frac{1}{7} + \frac{1}{9} + \text{etc.} + \frac{1}{15} \right) \right.$$

$$\left. - \int \left(\frac{1}{15} + \frac{1}{17} + \text{etc.} + \frac{1}{29} \right) + \frac{3}{7^2} + \int \left(\frac{1}{7^2} + \frac{1}{9^2} + \text{etc.} + \frac{1}{29^2} \right) + \frac{1}{15^2} \right) .$$

We will sum the series by approximation, either by taking average values, which we will multiply by the number of terms, or by following, as we are going to do, the method that we have given, in the *Note of Section 24.15*, which will give

$$\int \left(\frac{1}{7} + \frac{1}{9} + \text{etc.} + \frac{1}{15} \right) = \frac{1}{2\mu} \log \left(\frac{15}{7} \right) + \frac{1}{14} + \frac{1}{30} ,$$

where μ is the modulus of the logarithmic system:[892] by the same method

$$\left(\frac{1}{15} + \frac{1}{17} + \text{etc.} + \frac{1}{29} \right) = \frac{1}{2\mu} \log \frac{29}{15} + \frac{1}{30} + \frac{1}{58} .$$

In addition, we will have

$$\int \left(\frac{1}{7^2} + \frac{1}{9^2} + \text{etc.} + \frac{1}{29^2} \right) = \frac{1}{7} - \frac{1}{29} + \frac{1}{7^2} + \frac{1}{29^2} .$$

Thus, by combining all these values, and making μ equal to 0.434, module of the logarithmic system of the ordinary tables, the sum of all the coefficients of the first eight equations of our Table, from δ_8 to δ_{15}, will give $2.40\delta_8$.

To complete the equation, you must add together all the terms which, in the first eight equations, precede δ_8, from δ_7 to δ_1. We will notice, by examining the Table, that if all the densities were equal from δ_7 up to δ_1, all these terms would mutually cancel each other, the positive coefficient of δ_7 being equal to the negative coefficient of δ_1, and so on alternately: we will notice moreover, that except for the coefficients of δ_7 and of δ_1, the others are very small; that moreover the variation of the density from one globe to another only increases rapidly from the first globe whose density is δ_1, to the second globe whose density is δ_2. Thus we can, without great error, suppose that all the terms between δ_7 and δ_1 mutually cancel each other, and the very approximate value of the sum of all the terms which precede δ_8 in the first eight equations, will be represented by

$$(\delta_7 - \delta_1) \left(\frac{2}{3^2} + \frac{2}{5^2} + \text{etc.} + \frac{2}{13^2} \right) = 0.40 \, (\delta_7 - \delta_1) \ ;$$

but, from the above observations, δ_7 can be taken for δ_8. We found, *Section 24.14*, by approximation, $\delta_1 = 0.52D$.

Thus, by taking the sum of the first eight equations of our Table, we will have in an approximate way the equation

$$D \left(1 + \frac{2R^2}{(R+2r)^2} + \frac{2R^2}{(R+4r)^2} + \text{etc.} + \frac{2R^2}{(R+14r)^2} \right) + 0.21D = 2.80\delta_8 .$$

[892] See footnote 659 on page 288.

As in our experiments and in the assumption of this example, $R = 4r$, it will not take very long to calculate exactly the sum of the first term; but if we determine it by the approximation method of *Section 24.14*, which is sufficient, we will have in our example,

$$D\left(1 + \frac{16}{6} - \frac{16}{18} + \frac{16}{6^2} + \frac{16}{18^2}\right) + 0.21D = 3.48D = 2.80\delta_8 \ ;$$

whence finally results $\delta_8 = 1.24D$, a quantity which we have found, by experiments, equal to $1.30D$; thus theory and experiment differ from each other only by quantities too small for the approximate operations which precede to be able to evaluate them.

24.30 XXX. Second Example. *Cylinder Thirty Inches Long, Two Lines in Diameter, in Contact at Its End with an Eight Inch Globe*

If I now want to compare a cylinder 30 inches long and 2 lines in diameter, with a globe 8 inches [in diameter], instead of the cylinder I can suppose a line formed of 180 small globes 2 lines in diameter, in contact at its end with this globe. Thus, by following, according to this supposition, the method of the example which precedes, I will form a Table of 180 equations analogous to that of this example; and by taking the sum of the first 91 equations, I will have for all the terms which follow δ_{91}, the quantity

$$\delta_{91}\left(1.68 + \frac{3}{7} + \frac{3}{7^2} + \frac{1}{181} + \int\left(\frac{1}{7} + \frac{1}{9} + \text{etc.} + \frac{1}{181}\right)\right.$$

$$\left. - \int\left(\frac{1}{101} + \frac{1}{183} + \text{etc.} + \frac{1}{359}\right) + \int\left(\frac{1}{7^2} + \frac{1}{9^2} + \text{etc.} + \frac{1}{359^2}\right)\right) \ .$$

By taking the sums of the series, according to the method explained in the Note of the *Section 24.17*, this quantity will be reduced to $3.70\delta_{91}$.

If we now calculate the sum of the terms preceding δ_{91}, in the same member of the equation, we will find it equal to $0.46(\delta_{90} - \delta_1)$, conforming to the observations of the preceding Section: but δ_{90} can be taken equal to δ_{91}, and δ_3 can be without significant error for our operation, calculated according to the first two equations which will form the Table: assuming the densities equal from δ_2 to δ_{180}, these first two equations would be:

First equation:

$$D = \delta_1 + \left(\frac{2}{3^2} + \frac{2}{5^2} + \text{etc.} + \frac{2}{359^2}\right)\delta_2 = \delta_1 + 0.46\delta_2 \ .$$

Second equation:

$$1.84D = -\delta_1 + \left(1 + \frac{2}{3^2} + \frac{2}{5^2} + \text{etc.} + \frac{2}{357^2}\right)\delta_2 = -\delta_1 + 1.46\delta_2 \ ;$$

from which results $\delta_2 = 48D$ and $\delta_1 = 0.32D$. Thus $0.46\delta_1 = 0.15D$. Therefore, the second member of the sum of the first 91 equations, would be equal to $(3.70 + 0.46)\delta_{91} = 0.15D$.

It remains to calculate the sum of the first member formed from the addition of the first 91 terms which express the action of the large globe; this sum, from what we have said in the preceding Section, will be represented by the formula,

$$D\left(1+(48)^2\left(\frac{2}{(48+2)^2}+\frac{2}{(48+4)^2}+\text{etc.}+\frac{2}{218^2}\right)\right)\ ,$$

a quantity equal to, according to the approximation method of the Note of Section 24.14:

$$D\left(1+(48)^2\left(\frac{1}{50}-\frac{1}{228}+\frac{1}{50^2}+\frac{1}{228^2}\right)\right)=38.18D\ .$$

Thus by comparing this term with the second member, we will have $(38.18{+}0.15)D = 4.16\delta_{91}$, from which finally results $\delta_{91} = 9.21D$.

We have found, *Section 24.24*, that by placing a globe 8 inches in diameter in contact with a cylinder 2 lines in diameter and 30 inches in length, if the density of the globe, after contact, was D, the average density of the cylinder was, according to the experiment, equal to $9.00D$. Thus, the experiment in this example agrees with the theory as well as can be expected in investigations of this kind: we have found the same agreement in the preceding example between theory and experiment, although the cylinder is twelve times larger than in the present experiment; thus the truth of the two results is confirmed one by the other.

24.31 XXXI. *Cylinders of Different Lengths in Contact with the Same Globe*

We found in *Section 24.24*, that by theory we obtained the average density of a 30 inches long line of fifteen small globes, each globe 2 inches in diameter, in contact with a globe of 8 inches, equal to $1.24D$, where D expresses the density of the surface of the globe after contact; but we found, in the *Remark* of *Section 24.14*,[893] that by putting only four globes of 2 inches in diameter in contact with the same globe of 8 inches in diameter, the density of the surface of the globe being D, the average density for the line of the four globes, was by theory, equal to $1.21D$; thus the theory teaches us, that whatever the length of a cylinder or of a line formed by small globes of the same diameter as this cylinder, the average density is approximately equal; a result consistent with that given by the tenth and eleventh experiments.

24.32 XXXII. *Remark*

We only need to note that when the cylinders are very short in length, and are of a very small diameter relatively to that of the large globe, then the small cylinder takes on an average density much smaller than that which a cylinder of the same diameter but of greater length would take on. To be convinced of this truth, let us calculate the density of two small globes of only 2 lines in diameter each, forming a length of 4 lines in contact at its extremity with a globe 8 inches in diameter,[894] this supposition would yield the two equations:

First equation:

$$D = \delta_1 + 0.22\delta_2\ .$$

[893] See Subsection 24.14.1.

[894] This situation is represented in the Figure of footnote 866 on page 393.

Second equation:

$$1.84D = -\delta_1 + \delta_2 \ .$$

It results from these two equations, $\delta_1 = 0.48D$ and $\delta_2 = 2.36D$; thus the average density of the system of 4 lines in length, formed by the two small globes of 2 lines each in diameter, would be equal to $(0.48 + 2.36)D/2 = 1.42D$, a quantity that we found at *Section 24.30*, equal to $9.21D$ for a line of the same diameter, but 30 inches in length.

24.33 XXXIII. *Remark*

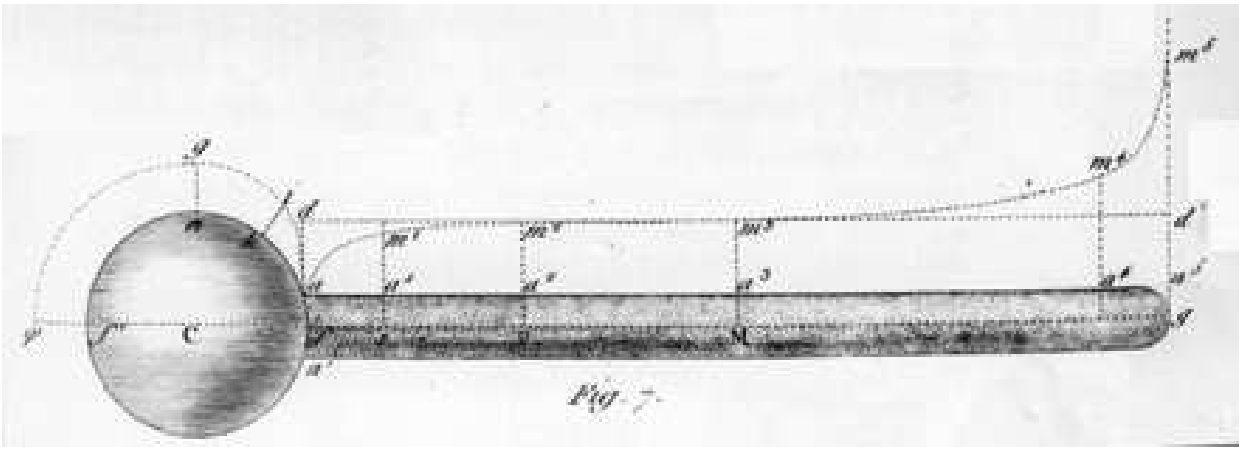
This theory, confirmed by experiment,[895] according to the methods which we have explained previously, accounts for an electrical result known for a long time. We know that when an electrified globe is armed with a needle or a tip, it rapidly loses its electricity, but much less rapidly when this needle is very short. Here is the explanation of this phenomenon; the coercive force that the air opposes to the flow of the electric fluid being limited, the greater the density of this fluid, the faster the fluid will flow. Thus, in our example, when the needle is 30 inches long and 2 lines in diameter, its average density is equal to $9.21D$: but it is only equal to $1.42D$, when the cylinder has 4 lines of length; thus the electric fluid must escape with much more rapidity through the first needle than through the second.

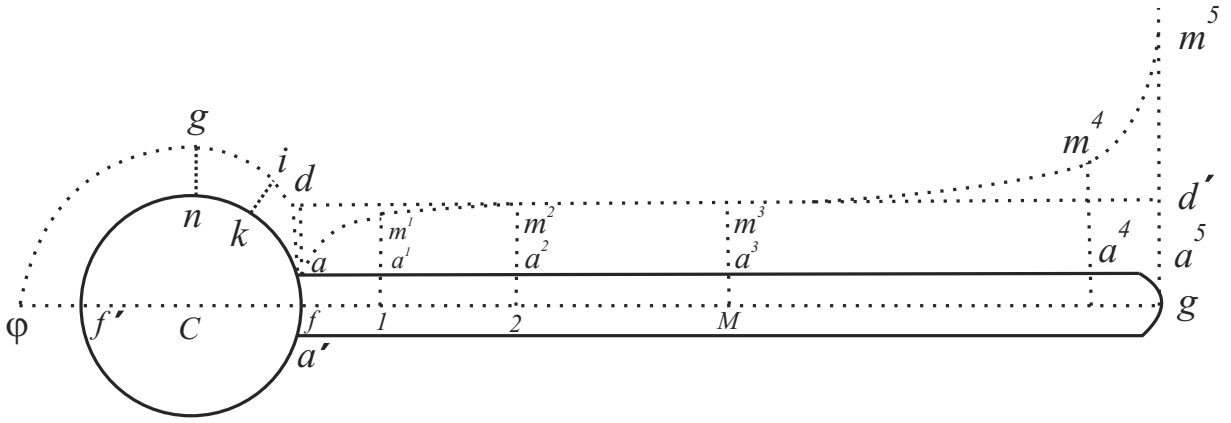
24.34 XXXIV. *Second Approximation Method for Determining the Variation of the Electric Density Along the Surface of a Cylinder in Contact at Its End with a Globe*

It is easy, from the preceding observations, to find various ways of approaching, by calculation, the variation of the electric density along the surface of a cylinder. As an example, imagine our cylinder, *Figure 7*, two inches in diameter and 30 inches in length, and suppose it, as in this Figure, to be in contact at its end with a globe 8 inches [in diameter].[896]

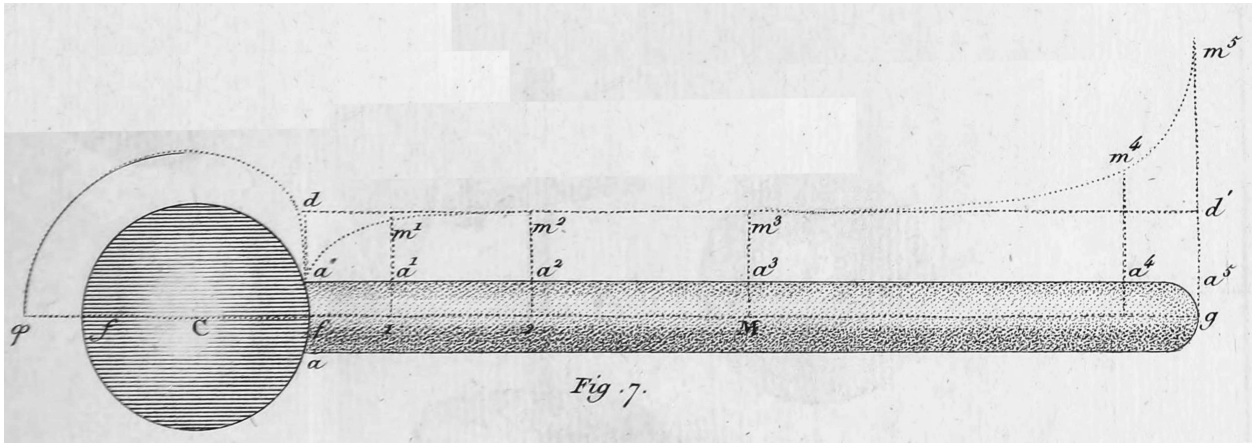
[895] By a lapse in the original text, this Section came out with the same number as the previous Section, namely, 32.

[896] In the image of this footnote I present another reproduction of Figure 7. It is clearer than the original Figure 7 that I placed in the middle of the text, but points n, g, k, i on the globe do not appear. Furthermore, points a and a' were inverted in the Figure of this footnote in relation to Coulomb's original Figure 7:





According to all that we have said in this Memoir and in the one that precedes, it is easy to see that the density of the electric fluid is zero in the circle of contact $a'a$ of the cylinder and the globe;[897] that for the globe, the electric density increases from the point a to the opposite pole f' where its *maximum* is located, that the increase of this density, which is rapid for the first 25 to 30 degrees, starting from point a, slows down considerably afterward, so that the increase in the electric density is almost imperceptible from the point n of the equator[898] to the pole f': on the cylindrical surface, the density is zero or at least insensible



[897] I placed below Figure 7 appearing in the middle of the text, another representation of the image in which I indicated more clearly the letters mentioned by Coulomb.

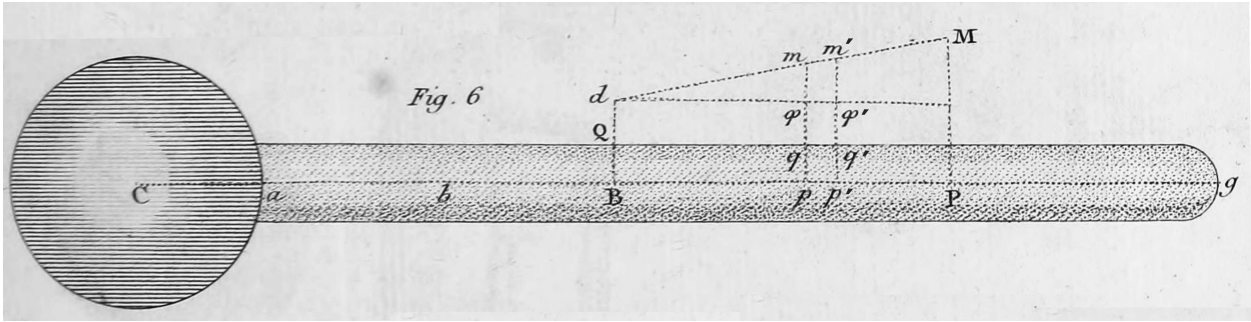
[898] In the original: *depuis le point μ de l'équateur.* I replaced in this sentence the letter μ with n, according

at point a; it then increases rapidly over the first two or three inches, then less rapidly as one approaches the middle of the cylinder, where the *maximum* of this increase is found; the variation of the increase in density then grows rather slowly up to two or three inches from the end, but very rapidly over these last two inches. The distance, *Figure 7*, from the dotted line to the globe and the cylinder, roughly indicates the locus of the electric density: taking the circle fkf', and the line aa_5 for axis, the ordinates ki, ng, $f'\varphi$; a_1m_1, a_2m_2, a_3m_3, etc. will represent the densities.

If we now divide the cylinder, *Figure 7*, into any number of equal or unequal parts, such as $f1$; 12, $2M$, etc., taking care to make the parts very short where the variation of the density must be considerable, it will be easy to form in the place of the curve $am_1m_2m_3m_4m_5$ a polygon, by joining points a, m_1, m_2, m_3 etc. by straight lines. In this polygon the densities will increase from one point to another, following a straight line, and the polygon will have as many sides as the cylinder has divisions; thus seeking at each division the state of equilibrium on the axis between the action of all the portions of the cylinder, and the action of the globe which terminates it, we shall have as many equations as the polygon has sides. It will consequently be possible, and even easy, as we shall see, to determine, relative to the axis, the inclination of each side of the polygon, and consequently to have the approximate variation of the density.

24.35 XXXV

Here are the principles of the calculation for each part of the cylinder: a portion (*Figure 6*) BP of a cylindrical surface acting on a point b of its axis, following the inverse of the square of the distances, the density at point Q being $Qd = \delta$, and then increasing along the line dM, we ask for the action of this surface on point b.



Let $bB = a$; $bp = x$; $Bp = x - a$, $\frac{\varphi m}{Bp} = n$; let R be the radius of the globe, r the radius of the cylinder and the ratio of the circumference to the radius equal to 2π;[899] we will have for the action of the zone of [the] elementary surface which corresponds to pp', this action evaluated in the direction of the axis, the quantity[900]

to Coulomb's original Figure 7, where the symbol μ does not appear at the equator of the sphere, there appearing point n.

[899] In the original, [Coulomb, 1791, p. 665]: "le rapport de la circonférence au rayon égal π". Again I am replacing Coulomb's symbols Π or π with 2π, further emphasizing the modern definition of π as the ratio of circumference to diameter, that is, $\pi = 3.14159....$ See also footnotes 849 and 864 on pages 383 and 391, respectively. And this also applies to the factor 2π in what follows on the next page.

[900] That is, the next equation represents the force exerted by the section pp' of the cylinder, with a length dx, when acting on a electrified particle located at point b.

$$[\delta + n(x-a)]\,\frac{2\pi r x dx}{(r^2+x^2)^{1/2}} = \frac{(\delta - na)2\pi r x dx}{(r^2+x^2)^{1/2}} + \frac{2\pi r x^2 dx}{(r^2+x^2)^{1/2}}$$

$$= (\delta - na)\frac{2\pi r x dx}{(r^2+x^2)^{1/2}} + \frac{2\pi r x^2 dx}{(r^2+x^2)^{1/2}} - \frac{2\pi r dx}{(r^2+x^2)^{1/2}} + \frac{2\pi r dx}{(r^2+x^2)^{1/2}}\ ,$$

whose integral taken so that it vanishes when $x = a$, will give, μ being the logarithmic modulus;[901]

$$2\pi r\delta\left[\frac{1}{(r^2+a^2)^{1/2}} - \frac{1}{(r^2+x^2)^{1/2}}\right] + 2\pi r n\left[\frac{a-x}{(r^2+x^2)^{1/2}}\right] + \frac{n2\pi r}{\mu}\log\frac{x+(r^2+x^2)^{1/2}}{a+(r^2+a^2)^{1/2}}\ .$$

24.36 XXXVI

Supposing now that I first want to determine the variation in density about the middle of my cylinder, which in our example is 2 inches in diameter and 30 inches in length; if, *Figure 7*, I pass a line dm_3d' parallel to the axis through point m_3, which I assume is in the middle of my cylinder, it will be easy to see from all the preceding observations, that the variation of the density towards the middle a_3 follows roughly a straight line up to a considerable distance from this point; and that as the action of the distant points diminish like the inverse ratio of the square of the distances, only a small error results from the supposition that the line which expresses the density, extends in a straight line to the extremities of the cylinder.

From this reflection, it will be easy to apply the formula to our example, noticing that here $a = 0$, and that half the length of the cylinder represented by x is much greater than r; thus we will have for the value of the action of half the cylinder Mg on point M, the quantity

$$2\pi r\delta\left(\frac{1}{r} - \frac{1}{x}\right) - 2\pi r n + \frac{n2\pi r}{\mu}\log\frac{2x}{r}\ .$$

To have the opposite action of the other half of the cylinder Mf, the quantity n must be negative, since the density decreases from a_3 to a, which will give[902]

$$2\pi r\delta\left(\frac{1}{r} - \frac{1}{x}\right) + 2\pi r n - \frac{n2\pi r}{\mu}\log\frac{2x}{r}\ ;$$

thus the action of the whole cylinder in the direction Mf will give

$$\frac{4n\pi r}{\mu}\log\frac{2x}{r} - 4n\pi r\ ;$$

a quantity which must be equal to the action of globe C, which, since x is half the length of the cylinder, will be $4\pi DR^2/(R+x)^2$; from which finally results the equation

[901] See footnote 659 on page 288.

[902] In the original text the next equation appeared as follows:

$$2\pi r\delta\left(\frac{1}{r} - \frac{1}{x}\right) + 2\pi r n - \frac{n2\pi r}{\mu}\log 2x\ .$$

$$\frac{4\pi DR^2}{(R+x)^2} = 4n\pi r \log \frac{2x}{r} - 4n\pi r \ ,$$

which will express the variation nr of the density in the middle of the cylinder, over a length equal to the radius. In our example, the diameter of the cylinder is 2 inches, its length 30 inches; the radius of the globe is 4 inches: thus $R = 4r = 1$, $x = 15$, from which results $n = 0.018D$; that is to say, over an inch of length, the electric density increases about the middle of the cylinder, by one fiftieth part of that of the globe.

If we take as a second example the cylinder with a radius of one line [0.226 cm], and 30 inches in length [81.21 cm], the average density of which we have determined by our experiments, then $R = 48r$, $x = 180r$, which gives $n = 0.09D$; that is, in a cylinder whose diameter would be twelve times smaller than the preceding one, the density over a length of one line would increase, in the middle of the cylinder, by 1/1000, and over 12 lines of length; it would therefore increase six times more for the cylinder 2 lines in diameter than for the cylinder 2 inches in diameter.

Finally, if we take for the radius of the cylinder a hundredth of a line,[903] we will find that over an inch of length, the density increases fifty times more than on a cylinder of one line of radius, and consequently three hundred times more than on a cylinder 2 inches in diameter.

If the cylinders had a much greater length than in the example which precedes, then the variations of the density in the middle of the two cylinders, for the same length of an inch, would be in a ratio closer to the inverse of the diameters of the two cylinders; thus, for example, if the length of the cylinders were 300 inches [812.1 cm], our formula would give us in the middle of the cylinders, the variation of the density of the cylinder with a radius of one line to that of the cylinder with a radius of one inch, approximately :: 8 : 1; a ratio which was :: 6 : 1, when the length of the cylinders was only 30 inches.

24.37 XXXVII

If we want, according to the method of the two preceding Sections, to determine in an approximate manner the variations of the density for the different points of our cylinder aa_5, *Figure 7*, we must divide this cylinder into different parts, and suppose that in each part the variation of the density follows a straight line. In order not to deviate in this supposition much from the truth, it is necessary that the first part aa_1 be very short. Thus the cylinder being 2 inches in diameter, I would make this first part aa_1, 2 inches in length; I would make the second [part] a_1a_2, 4 inches in length, the third [part] a_2a_3, 9 inches; which brings me to the middle of the cylinder. I take for the variation of the density, from the middle a_3 of the cylinder to its extremity a_5, the one that I just described; I then seek the conditions of equilibrium of these four parts of the cylinder relatively to the point of contact f, and to points 1 and 2 of the axis of the cylinder, which furnishes three equations which, combined together, determine the variation of the densities at each division of the cylinder, the density curve considered as a polygon; and as the density of the electric fluid is zero at the meeting point of the globe and the cylinder, it will be easy to conclude from this in an approximate manner the electric density at all the points of the surface of the cylinder. A fairly detailed application of this method will be found at the end of this Memoir.

[903] That is, $r = (1 \text{ line})/100 = 0.00226$ cm.

24.38 XXXVIII. *Theoretical Determination of the Ratio of the Average Electric Densities of Two Cylinders of a Very Small Diameter, of a Very Great Length, in Contact at Their Extremities with a Large Globe*

If we looked for the variation in the middle of two cylinders, the length of which would be $2a$, which we suppose of a different diameter, but very small, relatively to that of the globe, we would have according to the preceding formulas, *Section 24.36*:[904]

$$\frac{2DR^2}{(R+a)^2} = 2nr\left(\log\frac{2a}{r} - 1\right) \ ;$$

but it should be noted that, for any cylinder of the same length, but of another diameter, there would be in this formula only the quantities n and r which would vary, the diameter of the globe R, as well as half the length a of the cylinders, being assumed to be the same; from which it follows that if n_1 represents the variation of the density in the middle of the cylinder whose radius is r_1, and if n_2 represents the variation of the density in the middle of the cylinder whose radius is r_2, the lengths of the two cylinders being the same, we will have

$$n_1 r_1 = n_2 r_2 \ , \qquad \text{or} \qquad \frac{r_1}{r_2} = \frac{n_2}{n_1} \ ;$$

that is, the variations of the densities in the middle of the two cylinders will be between them like the inverse of the radii.

It is now easy to perceive that this same proportion must take place, if we divide the two cylinders equal in length, into a number of reciprocally equal parts, and compare in the two cylinders the parts corresponding each to each. Indeed, we conceive according to the principles on which all the preceding calculations are based, that if in one of the cylinders we have for the expression of the variation at any distance a' of the globe, the formula

$$\frac{2DR^2}{(R+a')^2} = n_1 r_1 A' \ ,$$

A' being a function of a', we will have, by taking a point at equal distance from the globe in the second cylinder, whose length is divided into a number of parts equal to those of the first, the formula

$$\frac{2DR^2}{(R+a')^2} = n_2 r_2 A' \ ,$$

the quantity A' being the same, since the distance a and the number of divisions are the same; thus for each corresponding point, [the ratio] $\frac{n_1 r_1}{n_2 r_2}$ will be the same quantity; thus the variations of the density for points at equal distance from the globe, will be in the two

[904] In the original text the next equation appeared as:

$$\frac{2DR^2}{(R+a)^2} = 2nr\left(\frac{\log 2a}{\mu} - 1\right) \ .$$

cylinders in inverse ratio of the radius of the cylinders,[905] and consequently the sum of these variations or the average densities of two cylinders of a very small diameter in contact with a large globe, are inversely proportional to the radii, as experiment has taught us.

24.39 XXXIX. *Of Two Conducting Bodies Placed at a Sufficiently Great Distance from Each Other, so that the Electricity Cannot Be Communicated Through the Layer of Air which Separates Them*

In the preceding Sections, we have determined the manner in which the electric fluid is distributed between two conducting bodies in contact; we are now going to seek the electric state of the different parts of a [conducting] non-electrified body presented to an electrified body at a sufficiently great distance, so that the electricity of the electrified body cannot be communicated to the non-electrified body through the layer of air that separates them. It has been known for a long time that in this arrangement the non-electrified body, if insulated [electrically from the ground], will give, by the sole influence of the electrified body, signs of electricity contrary to that of the electrified body, in the neighboring parts of this body, and signs of the same nature as the electrified body in the parts which are furthest removed from it. We also know that if the non-electrified body presented to an electrified body is not insulated [from the Earth], it will give on all points of its surface signs of electricity contrary to that of the electrified body.[906]

The evaluation of the electric state of the different parts of a non-electrified body, insulated or not, but presented at some distance from an electrified body, is the subject of this last Part of my Memoir.

24.40 XL. *Of the Two Types of Electricity*

Whatever may be the cause of electricity, all its phenomena will be explained, and the calculation will be found to conform to the results of experiments, assuming two electric fluids, the parts of the same fluid[907] repelling each other in inverse proportion to the square of the distances, and attracting the parts of the other fluid in the same inverse ratio of the square of the distances. This law was found by experiment for electric attraction and repulsion, in the First and Second Memoirs on Electricity, *volume of the Academy of 1785*;[908] according to this supposition, the two fluids in the conducting bodies always tend to unite until there is equilibrium, that is to say, until by their meeting, the attractive and repulsive forces compensate each other. It is the state in which all bodies are found in their natural state; but if by any operation whatsoever, a superabundant quantity of one of the electric fluids is passed into an insulated conducting body, it will be electrified, that is to say, it will

[905] In the original text, by a lapse, we have here "seront dans les deux cylindres en rayon inverse du rayon des cylindres". I replaced "in inverse radius" by "in inverse ratio".

[906] See Chapter 7 (Differences betweeen Conductors and Insulators) of the book *The Experimental and Historical Foundations of Electricity*, [Assis, 2010b], [Assis, 2010a], [Assis, 2011], [Assis, 2015b] and [Assis, 2017].

[907] In the original: *les parties du même fluide*.

[908] These Memoirs are translated in Chapters 11 and 14, respectively.

repel the electric parts of the same nature,[909] and will attract the electric parts of another nature than the superabundant fluid with which it is charged. If the electrified conducting body is brought into contact with another insulated conducting body, it will share with it the superabundant electric fluid in the proportions indicated in this Memoir and those which precede; but if it is made to communicate with a non-insulated body,[910] it will lose in an instant all its electricity, since it will share it with the globe of the Earth, whose dimensions relatively to it are infinite.

Mr. Aepinus has supposed in the theory of electricity,[911] that there was only one electric fluid, whose parts mutually repelled each other and were attracted by the parts of bodies with the same force as they repelled each other. But to explain the state of bodies in their natural situation, as well as the repulsion in the two kinds of electricity, it is necessary to suppose that the molecules of bodies repel each other with the same force as they attract electric molecules,[912] and that these electric molecules repel each other.[913] It is easy to perceive that the supposition of Mr. Aepinus gives, as regards the calculation, the same results as that of the two fluids. I prefer that of the two fluids which has already been proposed by several physicists, because it seems to me contradictory to admit at the same time in the parts of the bodies, an attractive force in inverse ratio to the square of the distances demonstrated by universal gravity,[914] and a repulsive force in the same inverse ratio of the square of the distances; a force which would necessarily be infinitely large, relatively to the attractive action from which gravity results.

The supposition of the two fluids is, moreover, in conformity with all the modern discoveries of chemists and physicists, who have made us acquainted with different gases whose mixture in certain proportions suddenly and completely destroys their elasticity; an effect which cannot take place without something equivalent to a repulsion between the parts of the same gas which constitutes their elastic state, and to an attraction between the parts of the different gases which makes them suddenly lose their elasticity.

As these two explanations have only a greater or lesser degree of probability, I warn, in order to protect the theory that will follow from any systematic dispute, that in the supposition of the two electric fluids, I have no other intention than to present with the fewest possible elements the results of the calculation and of the experiment, and not to indicate the true causes of electricity.[915] I will refer to the end of my work on electricity, the examination of the principal systems to which electric phenomena have given rise.

[909] In the original: *il repulssera les parties électriques de la même nature*. According to Gillmor, the word "parties" can be translated as "parts", "elements", "elementary parts" or "particles", [Gillmor, 1971a, pp. 159, 201 and 217]. See also footnotes 181, 395, 606 and 907 on pages 72, 176, 254 and 418, respectively.

[910] That is, if it is grounded when it comes into contact directly with the Earth, or with a conducting body connected to the Earth.

[911] See footnote 182 on page 73. See also [Aepinus, 1759] with English translation in [Aepinus, 1979].

[912] In the original: *les molécules électriques*.

[913] Two neutral bodies neither attract nor repel each other. Two positively charged bodies repel each other, just as two negatively charged bodies repel each other. To explain the non-existence of force in the first case, as well as the repulsion in the last two cases, Aepinus had to make these assumptions pointed out by Coulomb. In particular, Aepinus had to assume the existence of a repulsion between the material parts of two bodies, that is, between the molecules of two bodies.

[914] In the original: *par la pesanteur universelle*. That is, by the law of universal gravitation proposed by Isaac Newton in his book *Principia*, see footnotes 15 and 528 on pages 21 and 230, respectively.

[915] [Note by Bucciarelli] To claim that "two explanations have only a greater or lesser degree of probability" (of being true?) is a very friendly way of acknowledging a rival's theory. His claim that he does not intend to provide "the true causes of electricity" is reminiscent of Newton's claim — he makes no hypothesis about the true cause of universal gravitation.

24.41 XLI

In the Memoirs which precede,[916] as well as in the research which will follow, I have often caused different points of an electrified body to be touched by a small circular disk of insulated gilded paper,[917] which I then place in the [electric] balance to determine its action on the needle: in the results, I assumed that the electric [surface] density of the points touched was proportional to that which the small plane took in contact with the body. To know if this supposition can be admitted, it is necessary to determine according to what ratio the electric fluid is divided between a body and a small disk which touches it.

24.42 XLII. Experiment. *Distribution of a Single Electric Fluid Between a Globe and a Disk of Very Small Thickness, Which Touches the Globe Tangentially at the Center of the Disk*

I placed a globe 8 inches in diameter [21.656 cm] on an insulator described in the preceding Memoirs; I electrified it positively, as well as the needle of the balance.[918] By means of a small globe one inch in diameter [2.707 cm], with which I touched the large globe, and which I introduced into the balance, I determined the electric density of the 8-inch globe, which I found to be 144 degrees. I made the globe touch an insulated disk 16 inches in diameter [43.312 cm] and a quarter of a line thick [0.0565 cm], I immediately withdrew the disk; and by means of my little globe one inch in diameter, I again determined the electric density which remained at the 8-inch globe, I found it equal to 47 degrees.

24.43 XLIII. *Explanation and Result of This Experiment*

The original density of the electric fluid, or, what comes to the same thing, the quantity of the electric fluid diffused over the surface of the globe, was, before the contact of the disk, represented by 144 degrees. By contact with the disk, it was reduced to 47 degrees; thus in the division between the globe and the disk, the globe retains 47 parts, and the disk takes 97 parts; thus the quantity of fluid is divided between the disk and the globe, so that the quantity of the disk is double that of the globe. If we now calculate the surface of the globe 8 inches in diameter, we will find it equal to one of the two surfaces of the disk 16 inches in diameter;[919] thus, as this disk has two surfaces, it appears by this experiment that the electric fluid is distributed between the disk and the globe in proportion to the surfaces.

I have found by a very large number of experiments made with disks smaller than the preceding, that this result always took place; that is to say, whatever the diameter of the

[916] There is a numbering fault in the original article, with the Sections from the current Section onwards being numbered with an additional unit. That is, in the original text the present Section is numbered XLII instead of XLI and so on. To alleviate this problem, I decided to call the Experiment that follows it as Section XLII, since this Experiment does not appear as a numbered Section in Coulomb's article. Thus Sections XLIII onwards are in accordance with the original numbering.

[917] That is, by a proof plane, see footnotes 53 and 726 on pages 30 and 319, respectively.

[918] That is, the needle was also positively electrified.

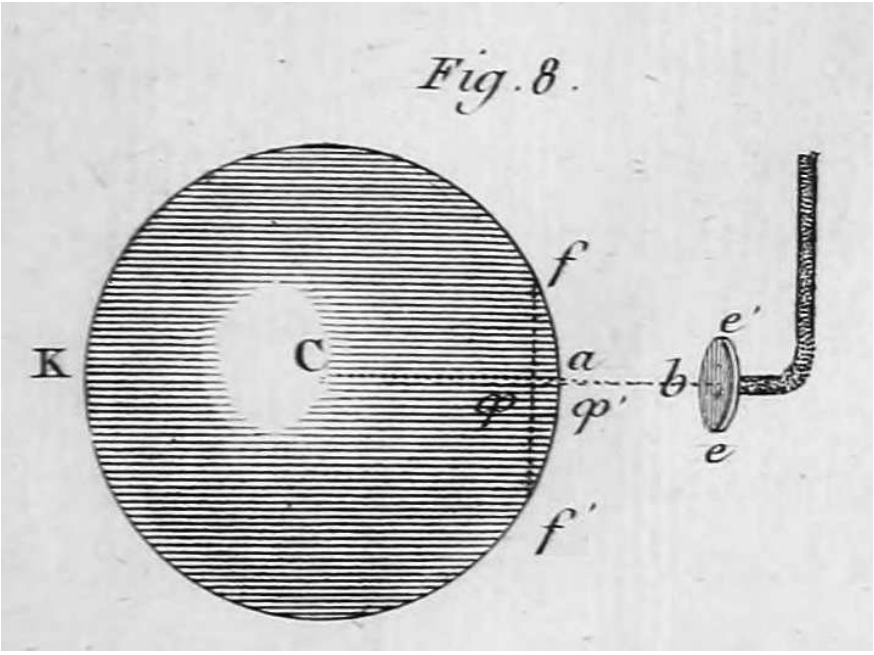
[919] That is, the area of the 4-inch-radius globe, $A_G = 4\pi 4^2 = 64\pi$ square inches, is equal to the area of one of the faces of the 8-inch-radius disk, $A_D = \pi 8^2 = 64\pi$ square inches.

globe and that of the disk, whenever the disk was brought into contact tangentially with the globe, it shared the electricity of the globe in the ratio of the sum of the extent of the two surfaces of the disk to that of the globe. Experiment has always given this result in a very exact manner, when the disk brought into contact with the globe was of a very small diameter, relatively to that of the globe; so that, when one touches, for example, the globe 8 inches in diameter [21.656 cm], with a small insulated disk 6 lines in diameter [1.356 cm], it takes on each of its surfaces an electric density equal to that of the surface of the globe, that is to say, this small disk of 6 lines in diameter is charged with a quantity of electricity double that of the portion of the surface of the globe which it has touched.[920]

24.44 XLIV. *Theory of This Experiment*

The result of this experiment is easy to explain theoretically, at least, when the touching disk is of a small diameter relative to that of the touched globe; this is the only case I consider here, because it is the only one that I will need in the experiments that will follow.

Let us place, *Figure 8*, a small disk b at a distance ab from the electrified globe C, small enough so that the interposed layer of air cannot prevent the electric fluid from passing from globe C to the small disk b.



This disk being very small, the action of the globe on point b in the direction ab, will be equal to $2DR^2 : (R + ab)^2$, D being supposed to represent the electric density of the surface of the globe, and R its radius.[921] As [the distance] ab is supposed to be very small relatively to the radius R of the globe, the action of the globe on point b is very nearly equal to $2D$;

[920] Later on Potier will point out that this assumption is wrong, see footnotes 927 and 969 on pages 423 and 441. A small proof plane acquires, on the surface it has touched, essentially the same amount of electricity as the amount of electricity that was contained in an area equal in size to the disk of the proof plane. See also footnote 53 on page 30, and [Gillmor, 1971a, pp. 200-210], [Heilbron, 1999, pp. 495-496] and [Blondel and Wolff, 2011c].

[921] See Section 23.4 on page 376 and footnote 796 on page 351. Coulomb is leaving out the 2π factor here.

but the action of a disk, whose radius is R' on a point at a distance a from the center of this disk, is equal to[922]

$$\delta\left(1-\frac{a}{(R'^2+a^2)^{1/2}}\right)\ ;$$

and if a is an infinitely small quantity, this action will reduce to δ, δ being the electric density of all the points of the disk. Thus, as there must be equilibrium at point b in the direction ba, between the action of the disk and that of the globe, we will have the equation $2D=\delta$; that is to say, that the density of the disk, or that the quantity of electricity which will pass to the disk at the moment that it is separated from the globe, will be double the quantity of electricity contained in a portion of the surface of the globe equal to this disk, which is found to be very much in accordance with the experiment.[923],[924]

24.45 XLV. *General Remark on the Theory of the Preceding Section, and on the Experiment from Which It Results*

The result which we have just found by experiment and by theory, for a small disk placed in contact with a globe, is general for all bodies terminated by a curved, convex surface of any shape whatsoever. Whatever may be the shape of the body, experiment teaches that a small disk placed in contact with these surfaces always takes, at the moment it is withdrawn from contact, a quantity of electricity double that of the portion of surface touched. Experiment gives again this same double ratio, by making a very small disk touch a large electrified plane.

This general result of the experiments for a small disk placed in contact with a conducting body, terminated by a surface of any figure whatever, could, as we shall see, have been foreseen by simple reasoning; but in this Memoir, as well as in the preceding ones, all the phenomena have been revealed by experiment before attempting to apply calculation to them. This is indeed what the theory indicates.

Instead of globe C, *Figure 8*, suppose for a body of any shape, that the small surface represented by faf', has been touched by the disk ebe';[925] we seek, after the small disk ebe' has been separated from faf', its electric density, or the quantity of electric fluid which it contains relative to that which the equal portion of surface faf' contains. Let us take two points φ and φ' at an infinitely small distance from point a and from the surface faf', one inside the other outside the body C, let δ be the electric density of the plane ff';[926]

[922] In this expression Coulomb is assuming a uniformly electrified disk. This can only happen if it is an insulator. On the other hand, the disk of Coulomb's proof plane must be made of a conducting material so that it can collect the charges from the point where it touches. A charged conducting disk is not uniformly electrified on its surface. Therefore, the theoretical formula used by Coulomb could not be applied to draw conclusions about the experiments he carried out with his proof plane.

[923] [Note by Potier] Coulomb has again suppressed the 2π factor in the previous formulas.

[924] See Section 23.4 on page 376. See also footnotes 796 and 802 on pages 351 and 352.

[925] Due to an error in the original text, *ebe* appeared here. Potier corrected it to ebe'.

[926] In the original: *plan* ff'. This expression can be translated by *disk* ff' or by *plane* ff'. Probably Coulomb is referring here to the small spherical cap faf'. He will then consider the force exerted by this small, uniformly electrified spherical cap acting on an electrified particle located at point φ which is to the left of point a in Figure 8, and acting on an electrified particle located at point φ' which is on the right side of point a, the distances φa and $a\varphi'$ being equal. Moreover, he will assume that these distances φa and $a\varphi'$ are infinitely small compared to the diameter ff' of the small cap.

the action of this small circular plane ff', decomposed along the direction $a\varphi$, and acting on point φ as well as on point φ', will be by calculation equal to δ, [with the distance] φa being assumed to be infinitely small relatively to ff': but the action of ff' on point φ must balance the action of the entire surface fkf'; thus the action of all this surface on point φ will also be equal to δ. This action of the entire surface fkf', will be the same on one point φ' placed outside the body, since [the distance] $\varphi\varphi'$ is assumed to be infinitely small; so point φ' experiencing at the same time the action of the body fkf' and that of the plane ff', it will experience a repulsion equal to 2δ. Thus, if we suppose that the small disk ee' is sufficiently close to point a so that the electricity can pass from the body to this small disk, through the layer of air which separates them; and if we take a point between a and b at an infinitely small distance of b, the action of the small circular surface be' on this point in the direction ba, will be, by naming D the electric [surface] density of the disk ebe', equal to D; thus by naming δ the density of the small plane ff', we will have for the action of the entire surface of the body $fafk$ on point b, the quantity 2δ, which must balance the action D of the disk ee' on the same point; thus we will generally have $2\delta = D$, that is to say, that the quantity of electricity of the small disk ee', whatever the shape of the surface of the body $fkf'a$, will be equal to a quantity of electricity twice that of the surface portion faf', with which the small disk ee' will have been brought into contact. Thus the theory is found to have a perfect agreement with the experiment.[927,928]

24.46 XLVI

As in the experiments which precede and in those which will follow, we principally determined the density of each point of the bodies by making them touch a small disk; it is clear, according to the experiments and the theory which we have just explained, that by comparing for the same distance the actions of our small disk on the electrified needle of our balance, after this small disk has been successively put in contact with different points of the surface of the body, we determine very exactly the ratio of the electric densities of two points successively touched.

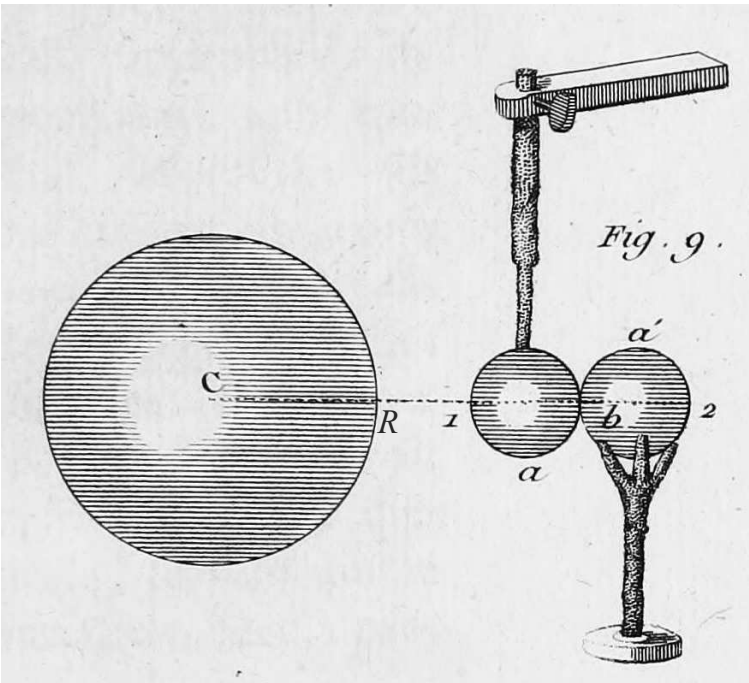
We will now move on to the search for the conditions of equilibrium of bodies which act on each other; these bodies being separated by an interval large enough to prevent the transfer of electric fluid from one to the other, through the layer of air which separates them.

24.47 XLVII. Experiment. *Two Small Globes, Figure 9, Insulated and Not Electrified, Are Placed at Any Distance from the Large Electrified Globe C*

We insulate, *Figure 9*, an electrified globe C 8 inches in diameter [21.656 cm].

[927] [Note by Potier] This whole theory is absolutely inadmissible. Coulomb does not take into account that when the disk and the globe touch, there is no electricity at the point of contact, a fact which was however well known to him, as we have seen previously. It is regrettable that he did not give the details of the experiments to which he alludes at the end of Section 24.43, which misled him. The thinner the disk, or proof plane, the better it presses against the body, and the closer the electric density, during contact, to that of the surface for which it is substituted.

[928] See also footnotes 920 and 969 on pages 421 and 441.



Two small [conducting] globes, each 2 inches in diameter [5.414 cm], are also insulated: one [small globe] a' is mounted on an insulating support formed of a glass cylinder, coated and surmounted by four arms of shellac; the other small globe a is carried by a vertical support, such that it can be introduced into the electric balance. We have described this support in the previous Memoirs. Having positively electrified the needle of the balance, as well as globe C,[929] the small globe a presented in the balance at the same distance from the needle, attracted the needle, after having been placed at a, exactly with the same force as it repelled it, when it was placed at a'.[930]

24.47.1 *Result of This Experiment*

It is easy to see that this result agrees perfectly with the principle explained in *Section 24.41*; for in our ninth Figure, globe C being positively electrified, part of the positive fluid of globe a passes into globe a'; and *vice versa*, some of the negative fluid from globe a' passes into globe a. But, as each of the globes acquires a portion of fluid equal to that of which the other is stripped and as the quantity of the two fluids necessary for saturation, that is to say, for there to be no electric action, subsists in both bodies, and as these fluids are only displaced, it follows that the attractive action of globe a, relatively to the needle of the balance, must be exactly equal to the repulsive action of the body a'.

[929] That is, globe C is also positively electrified.

[930] I prepared the image of this footnote to illustrate the situation in which the small globe carried by a vertical support that can be inserted into the electric balance is placed at the right end of the row of 3 globes. Assuming that globe C is positively electrified, if we introduce the small globe from the far right into the electric balance, it will repel the positively electrified needle of the balance:

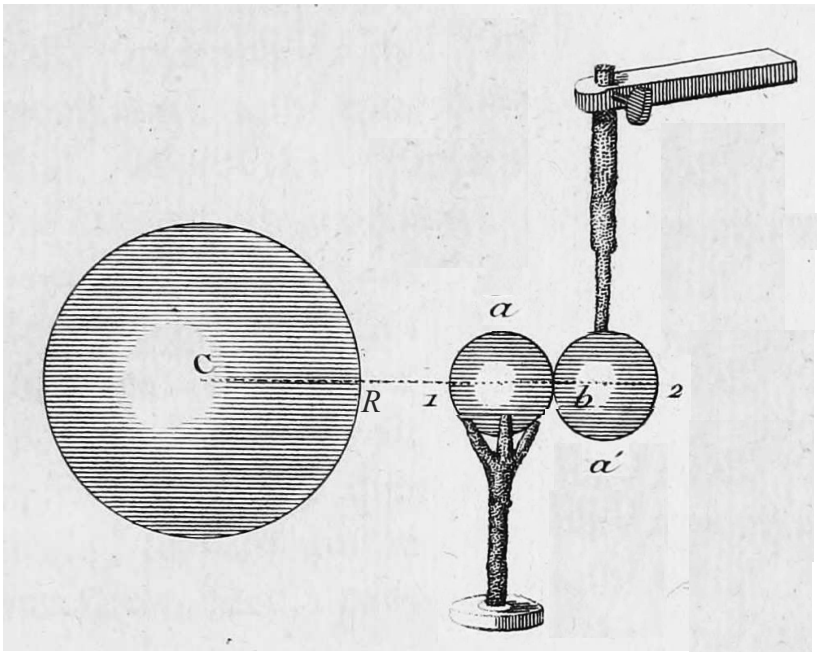
24.48 XLVIII. Experiment. *Comparison, Figure 9, of the Average Electric Density of the Globe Placed at a', and That of the Surface of Globe C*

This experiment is intended to determine, in accord with the ninth Figure,[931] the quantity of excessive positive electric fluid in the little globe a', etc., for a given distance $R1$.

To perform this experiment, globe C being 8 inches in diameter,[932] globes a and a' two inches [in diameter], the first globe a being placed two inches from globe C, I have presented the last globe a' in the balance, and I determined its repulsive action, which I found to be 21 degrees for a given distance.[933] I then caused globe C to touch globe a', and introducing it again into the balance, I determined the action of globe a' which, because the distance was the same as in the first operation,[934] was proportional to the quantity of electricity with which the small globe a' was charged during the contact with globe C.[935] I found that the needle was repelled, in this second experiment, with a force of 66 degrees.

24.49 XLIX. *Result and Theory of This Experiment*

The quantity of electric fluid, being proportional to its action when we compare the actions at equal distances — we let (*Figure 9*) δ be the mean density of the electric fluid spread over the surface of the first globe a, — an electricity which will be negative in our experiments



[931] Point R does not appear in the original Figure 9. I introduced this point in this Figure according to Figure 10.

[932] In the original text, C' appeared here instead of C. Potier fixed this error.

[933] That is, there was a total twist of 21° of the suspension wire for a certain angular distance between globe a' and the needle of the torsion balance electrified with a charge of the same sign as globe a'.

[934] That is, in both cases there was the same angular distance between globe a' and the needle of the balance.

[935] In the original text, C' appeared here instead of C. Potier fixed this error.

when the globe C is supposed electrified positively — and we let δ' be the positive electric density of globe a' which, in our Figure and in our experiment, is found to be electrified positively with the same quantity with which the globe a is electrified negatively.

If we look for the action of the three globes C, a, a', on the point of contact b of the two small globes, the point where there must be equilibrium, we will find that if the electric fluids were spread uniformly over the surface of the three globes, we would have for the equilibrium of action at point b, the [following] equation[936]

$$\frac{2D \cdot (CR)^2}{(Cb)^2} = -\delta + \delta' \ ;$$

but, as the quantity of natural positive electric fluid, of which globe a is stripped, is equal to the quantity of the superabundant fluid of globe a',[937] it follows that the sum of the quantities of fluid of the two globes is equal to 0: so we have $\delta' + \delta = 0$; thus, substituting in the first equation the value of δ, we will have

$$\frac{2D \cdot (CR)^2}{(Cb)^2} = 2\delta' \ .$$

It must now be remarked that, in the first equation, we have supposed that the fluid was uniformly spread over the surface of each globe, whereas these fluids, as we have seen at the beginning of this Memoir, have no action or are united to saturation at the point of contact b, and are separated and brought to their greatest degree of density at points 1 and 2. We have found in the same Section, that the corrected action of globe a, on point b, was measured by $0.60\delta'$, and not by δ'; it is the same for body a',[938] so our corrected equation[939] will give us:

$$\frac{2D(CR)^2}{(Cb)^2} = 1.20\delta' \ .$$

In our experiment, $CR = 4$ inches, $R1 = 2$ inches, the radius of globe $a = 1$ inch;[940] thus we will have $0.50D = 1.20\delta'$, hence $D = 2.40\delta'$.

We have found in our experiment, that the average density of the small globe a' being measured by 21 degrees, that of the same small globe, when it touched C,[941] was measured by 66 degrees: but we have seen in our Fifth Memoir, *volume of 1787, page 437*,[942] that when

[936] See Section 23.4 on page 376. See also footnote 796 on page 351. In this equation CR indicates the radius of globe C and Cb indicates the distance between the center C of the large globe and point b of Figure 9.

[937] For a lapse in the original text, a appeared here instead of a'.

[938] In the original text, a appeared here instead of a'.

[939] [Note by Potier] Here is how Coulomb arrives at this approximate result: If a mass M of electricity is spread uniformly on [the surface of] a sphere, the force exerted on a point of this surface is $\frac{M}{2R^2}$; if it is concentrated on the great circle of which this point is the pole, the action is $\frac{M}{2R^2\sqrt{2}}$, and finally, if it is concentrated at the opposite pole, [the action is given by] $\frac{M}{4R^2}$. These quantities are [to one another] like 1, $1/\sqrt{2}$ and $1/2$, and Coulomb admits that the real distribution produces an average force between $1/\sqrt{2}$ and $1/2$ of the force resulting from a uniform distribution; he takes 0.60 as a rough average between 0.50 and 0.707.

[940] We then have $Cb = CR + R1 + 1b = 4 + 2 + 2 = 8$ inches.

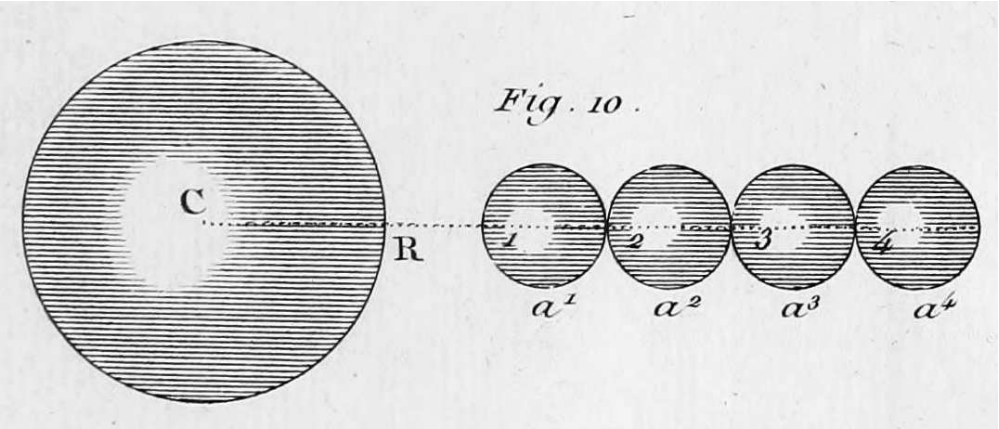
[941] In the original text, a appeared here instead of C.

[942] See the Table in Section 22.11 on page 344.

a globe one inch in radius touched a globe 4 inches in radius, the average density on the surface of the one inch globe, was to that of the 4-inch globe, approximately as 1.30 : 1.00; thus the average density of the small globe, after contact, being represented by 66 degrees, that of the large globe would be represented by 51 degrees: but it should be noted that by the division of electricity between the large globe and the small globe, at the moment of contact, the large globe loses roughly 1/12 of its electric fluid, that it loses moreover in the [time] interval of the observations about 1/20; thus the density of globe C, before contact, was roughly measured by 57 degrees. Now we had found by experiment the average density of globe a' placed as in the Figure, measured by 21 degrees; thus the average density of the positive electric fluid of the surface of globe C,[943] is to that on the surface of globe a' placed as in the ninth Figure, :: 57 : 21 :: 2.70 : 1.00; thus, from experiment, we have $\delta' = 2.70D$, a quantity we have just found equal to $2.40D$ by theory; therefore theory and experiment differ little from each other, and the errors can only be attributed to the imperfection of the operations.

24.50 L. Fourth Experiment. *Comparison, Figure 10, of the Electric Densities of Four Small Non-electrified Globes, 2 Inches in Diameter, Placed on an Insulator, 2 Inches Away from an Electrified Globe C′, 5 Inches in Diameter*

Figure 10 shows the position of the globes. We compared, according to the processes indicated in the previous experiment, the average density of the negative electricity of globe a_1 with the positive density of globe a_4, and that of globe a_4 with that of globe C; globe C being positively electrified.



We found, in naming δ_1 the average density of globe a_1, δ_4 the average density of globe a_4, D that of the large globe C, that

$$D = -1.50\delta_1 = 2.20\delta_4 \ .$$

[943] In the original text, C' appeared here instead of C. Potier corrected this error.

24.50.1 *Theory of This Experiment*

From[944] all that we have said in the preceding Sections, it is easy to see that in naming δ_1, the [average] density of globe a_1, δ_2 the average density of globe a_2, δ_3 the average density of globe a_3, δ_4 the average density of globe a_4, D that of globe C, R and r the radii of globes C and a', again making the distance $R1 = a$, we will have the three equations corrected according to the *Section 24.4* of this Memoir:

For the contact point 2:[945]

$$\frac{2DR^2}{(R+a+2r)^2} = -0.60\delta_1 + 0.70\delta_2 + 0.22\delta_3 + 0.08\delta_4 \ .$$

For the contact point 3:

$$\frac{2DR^2}{(R+a+4r)^2} = -0.22\delta_1 - 0.70\delta_2 + 0.70\delta_3 + 0.22\delta_4 \ .$$

For the contact point 4:

$$\frac{2DR^2}{(R+a+6r)^2} = -0.08\delta_1 - 0.22\delta_2 - 0.70\delta_3 + 0.60\delta_4 \ .$$

To obtain a fourth equation, it should be noted that the four small globes being originally in their natural state, their electric fluid is only disturbed by the influence of the action of the large globe, and that the sum of the electric fluids of the four small globes, is neither increased nor decreased; thus we will have the sum of the densities equal to zero for the four small globes; so we will have

Fourth equation:

$$\delta_1 + \delta_2 + \delta_3 + \delta_4 = 0 \ .$$

By means of these four equations, we can make three of the four unknowns δ_1, δ_2, δ_3, δ_4, disappear at will, and we will compare the fourth [density] with the density D of globe C: this calculation applied to our experiment will give $D = -1.53\delta_1$. Experiment gave us $D = -1.550\delta_1$; calculation will give $D = 2.12\delta_4$; experiment gave us $D = 2.2040\delta_4$. Theory and experiment again agree here as well as can be hoped for in operations of this kind.

We will not extend the experiments and calculations which precede, to a greater number of small globes placed in contact, and whose centers would make the same [straight] line with the center of a large globe; it is easy to apply the different methods of approximation that we have already presented to this case; but we believe it necessary to develop in considerable detail the manner in which the electric fluid appears to manifest itself with different degrees of density in the different parts of an uninsulated, or what amounts to the same, an insulated cylinder but of an infinite length, presenting one of its ends at a given distance from a large electrified globe, the extension of the axis of the [conducting] cylinder passing through the center of the globe. We feel that this research should have an immediate relationship with the theory of lightning rods.

[944] This Subsection was not included in Potier's reprint of Coulomb's works, [Potier, 1884].

[945] See Figure 10.

24.51 LI. *Result of Experiments Intended to Determine the Electric State of the Different Parts of the Surface of an Uninsulated Cylinder, of Very Great Length, Presented at One of Its Ends to a Large Electrified, Insulated Globe*

In the results which are about to be presented, I will not enter into the details of the experiments, except insofar as these details are sufficiently different from those which precede, so as to require a particular explanation.

We know, and it follows from the preceding researches, that when we present an uninsulated cylinder at a great enough distance from an electrified globe so that the electric fluid of the globe is unable to pass into the [conducting] cylinder, the surface of the cylinder gives signs of electricity contrary to that of the globe, and that the electric density of each point of the cylinder, or what comes to the same thing, the action of each of these points is all the greater, as the point of the cylinder is closer to the electrified globe: the object of this Part of my Memoir is to determine:

1. For the same cylinder placed at different distances from the same electrified globe, the electric density at the end of the cylinder closest to the globe and the law which this density follows; to compare this density, supposing it to be proportional to its degree of action, with that of the electrified globe, which is likewise supposed to be proportional to its degree of action.

2. By placing uninsulated cylinders of different diameters, at the same distance from an electrified globe, to determine according to what ratio the density at the extremity of the cylinder increases or decreases, relatively to the diameters of these cylinders.

3. According to which law the density of the different points of the same cylinder placed at a given distance from an electrified globe, decreases relatively to the distance of these points from the center of the electrified globe.

4. Finally according to what law the density of the surface of the cylinders increases relatively to the diameter of different globes, the electric density of the globes being the same.

24.52 LII. First Result. *An Uninsulated Cylinder Placed at Different Distances from an Electrified Globe*

If we place the same cylinder, not insulated, or, which comes to the same thing, insulated, but of infinite length, so that the axis of the cylinder is in the direction of the center of the electrified globe, we will find by experiment, by varying the distance from the center of the globe to the end of the cylinder nearest to this globe, that the electric density of this end will be in a ratio a little below the power 3/2 of the inverse ratio of the distance from this extremity to the center of the globe. The experiment which gave this result was made in two ways; either by touching with a small insulated disk the extremity of the cylinder, and then placing this small disk in the electric balance as usual; or by making the end of the cylinder touch a small globe of the same diameter as the cylinder, which is then introduced into the balance.

24.53 LIII. Second Result. *Electric Densities at the Extremity of Two Insulated Cylinders of Different Diameters, Placed Alternately at the Same Distance from the Center of an Electrified Globe*

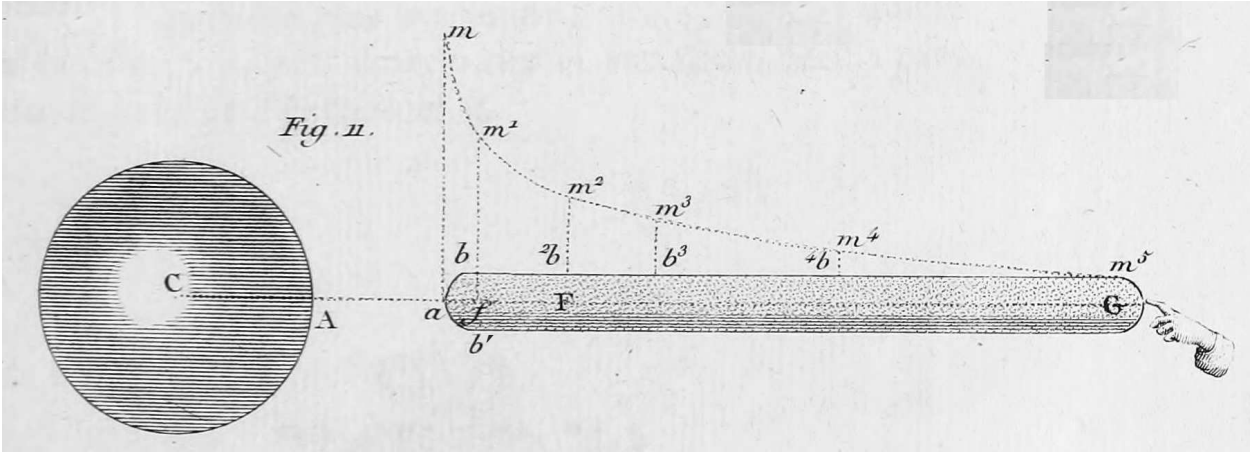
By successively placing two cylinders of different diameters at the same distance from the same electrified globe, it was found that the electric densities at the ends of the two cylinders were approximately in inverse proportion to the diameters of the two cylinders, provided, however, that the diameters of the cylinders were much smaller than the diameter of the globe.

24.54 LIV. Third Result. *Ratio of the Electric Densities at Different Points of the Surface of the Same Cylinder of a Great Length, and Not Insulated, According to Whether These Points Are More or Less Distant from the End of the Cylinder, Or from the Center of the Electrified Globe*

By placing a cylinder at a given distance from an electrified globe, we find that the electric density at different points on the surface of this cylinder is in inverse proportion to the square of the distance of these points from the center of the electrified globe.

This law is not followed towards the end of the cylinder which is close to the globe over a length equal to four or five diameters of the cylinder; we find by experiment that in this part the electric density increases as it approaches the end of the cylinder in a ratio much greater than the inverse square of the distances; and if, as in all the experiments we have made, the cylinder is terminated as in the *eleventh Figure*, by a hemisphere, we will find that the density at the extremity of the axis a, the nearest point of globe C, is approximately double that of point f, which is only distant from point a, at the end of the cylinder, by a quantity af[946] equal to the diameter of the cylinder, whatever be the distance Aa.

[946] In the original text, ae appeared here. I replaced it with af since there is no point e in Figure 11. Moreover, Coulomb will later mention that point f is away from point a by a distance equal to the diameter of the cylinder.



24.55 LV. Fourth Result. *The Same Non-Insulated Cylinder, Placed at the Same Distance from the Center of Two Electrified Globes of Different Diameters*

Supposing the electric density of two globes the same, it will be found by experiment that the density of the points of the cylinder placed at the same distance from the center of the two globes, will be like the square of the radii of these globes. The theory would give *a priori* the result that the experiment has just given; for the action of a spherical surface on any point placed outside the sphere is the same as if this surface were united at the center of the sphere;[947] thus its action on all the points placed outside the surface, will be in direct proportion to the extent of the surface multiplied by the electric density, and in inverse proportion to the square of the distance to the point on which the action is exerted. Now, as the cylinder is the same, and each point of the cylinder on which the action is evaluated, is supposed to be at the same distance from the center of the two globes; it follows that the electric densities of the same point of the cylinder placed at the same distance from the center of the two electrified globes, must always be in direct ratio composed of the electric density of the surface of the globes, and of the square of the radii of the globes.

24.56 LVI. *Formula Derived from the Previous Results*

In order to be able, according to the preceding results, to give a formula which immediately describes the electric state of the different points of an uninsulated cylinder, or touched at point G by an uninsulated body, as at *Figure 11*, at a very great distance from point a, we see that this ratio must be determined by experiment for a particular case, in order to obtain a constant coefficient. Among the different experiments which served to fix the four results which precede, I am going to choose one which will give me this coefficient.

24.56.1 Experiment

I insulated, *Figure 11*, globe C of 8 inches in diameter [21.656 cm]; I electrified it; I placed a cylinder aG, one inch in diameter [2.707 cm], on an insulator at $2\frac{1}{2}$ inches away [6.7675

[947] See Section 16.2 on page 268.

cm]: this cylinder was terminated by a hemisphere *bab*. As usual, I touched alternatively point a of the hemisphere and any point of globe C with a small insulated disk. Having electrified the needle of the balance with the same electricity as the globe,[948] the needle was attracted by the small disk when it touched [point] a, the end of the cylinder, and repelled when it touched the globe. By measuring the forces for the same distance,[949] I found that the attractive force of the small disk, when it had touched point a, was to the repulsive force of the same disk, when it had touched the globe, as 4.00 : 1.00.

When by the same method I compared point a with point f, placed at a distance of one inch from the end of the cylinder, I found that the electric density of point a was to that of point f, as 2.5 : 1.0; whence it is easy to conclude that the negative electric density one inch from the end of the cylinder will be to the positive electric density of the surface of the globe, approximately as 16 : 10.

24.57 LVII

Now bringing together the four preceding results, we find by experiment that the electric densities of the hemisphere which terminates different cylinders presented to an electrified globe, are of a nature contrary to that of the globe, are directly proportional to the density on the surface of the globe and to the square of the diameter of this globe, and inversely proportional to the radius of the cylinder and to the power 3/2 of the distance *ca*, *Figure 11*,[950] from the center of the globe to the end of the cylinder.

Thus, if D is the density of the positive electric fluid, spread over the surface of a globe, of which R is the radius; and if r is the radius of the cylinder; and if a is the distance between the center of the globe and the end of the cylinder,[951,952] we will have the [following] formula to express δ, the negative electric density at the end of the cylinder:

$$\delta = -\frac{mDR^2}{r(R+a)^{3/2}} \ ,$$

we will determine the value of the constant m, according to the experiment of the preceding Section. In this experiment $R = 4$ inches; $r = \frac{1}{2}$ inch; $a = 2.5$ inches, δ was found equal to $4D$: substituting these quantities in the formula, we will have

$$m = 2.07\sqrt{1 \text{ inch}} \ ,$$

and the general formula will be

$$\delta = 2.07DR^2 : r(R+a)^{3/2} \ ,$$

in which the values of a; of r, and of R must be given in inches.

[948] That is, Coulomb electrified globe C and the needle of his electric balance with electricities of the same sign.

[949] That is, when there was the same distance between the disk of the proof plane and the needle of the electric balance.

[950] Due to an error in the original text, Figure 4 appeared here instead of Figure 11.

[951] [Note by Potier] It is obviously the distance between the sphere and the cylinder that Coulomb designates by a.

[952] That is, a is the distance between point A on the surface of globe C and point a at the end of the cylinder that is closest to the globe, as indicated in Figure 11, such that the distance Ca is given by $R + a$.

24.58 LVIII. *Application of the Previous Formula to an Example Analogous to Lightning Rods*

Suppose that a cloud charged of electric fluid has the shape of a globe a thousand feet in radius [325 m], and passes 500 feet above the end of a cylinder one inch in diameter;[953] in this example $R = 1000$ feet; $a = 500$ feet; $r = \frac{1}{2}$ inch. These values substituted in the formula give

$$\delta = \frac{2.07 \cdot 12^2 \cdot (1000)^2}{\frac{1}{2}(1500)^{3/2}12^{3/2}} = 278D \ ;$$

that is to say, that the electric density at the end of the cylinder, of a nature contrary to that of the cloud, will be 278 times greater than that of the surface of the cloud.

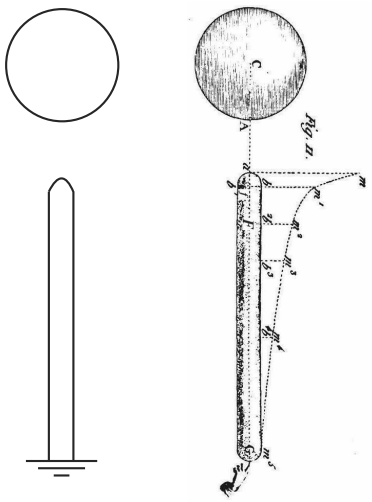
But as experiment has shown us, that in our formula $(R+a)$ was raised to a power smaller than 3/2, the density δ must be greater than $278D$. To be convinced of this, it suffices to suppose that in the formula $(R + a)$ is raised to the power of 1; for this case we will have $m = 1.23$, and the formula will give

$$\delta = 1.23DR^2 : r(R + a) \ ,$$

which, applied to our example, would give $\delta = 19680D$; so that a small variation in the power of $(R + a)$ would give a very large variation in the value of δ.

It follows from this observation that δ is greater than $278D$; but we do not know by how much, for as the experiments from which we have drawn the four results which precede, had as their limit globes of a foot in diameter and below, and cylinders from 4 lines up to 2 inches in diameter; and for greater ease in the calculations, we have supposed that the results were represented by a formula of a single term, which seemed to me to give values sufficiently approximate within the limits of the experiments. It may be, however, that this formula might not be extended to limits very distant from those within which the experiments are

[953] Coulomb imagines here a lightning rod consisting of a grounded vertical conductor close to an electrified cloud. He will model the lightning rod by the configuration of Figure 11 rotated by 90° in which a grounded cylindrical conductor has its upper end close to a large electrified globe, the cloud being modeled by this globe, as illustrated in this footnote:



contained; this will be easy to verify by the methods of theoretical approximations which will end this Memoir, and which will indicate to us in accordance with experiment, that the density at the end of the cylinder is greater for large globes which act on the end of an uninsulated cylinder of a small diameter, than that given by the formula, and that therefore in our example the density at the end of the cylinder one inch in diameter, shown at 500 feet of an electrified globe of 1000 feet radius, is greater than 278 times the electric density of the surface of that globe.

24.59 LIX. *Application of the Previous Result to the Effect of Lightning Rods*

From this it follows that the cloud, as well as the layer of very imperfectly insulating air interposed,[954] interposed between the cloud and the end of the cylinder, being composed of moving parts, those parts which are close to the end of the cylinder, must rush into it with very great rapidity, lose their electricity there, charge themselves with a strong electricity of a nature contrary to that of the cloud, then rush towards the cloud while fleeing the end of the cylinder, and so destroy the electricity of the parts of the cloud which they encounter. But as the diameter of the cylinder is very small, its action, although very great relative to the points which border the surface of the hemisphere which terminates it, is very small relative to the points which are 30 or 40 feet from the extremity of this cylinder. Thus, it must happen that the extremity of the cylinder will strip the parts of the cloud which adjoin it, without electric explosion,[955] and that all the bodies which will be below the extremity of this cylinder, at a not considerable distance from the cylinder, will be preserved from the explosion of the cloud.[956]

24.60 LX. *Theoretical Calculation Intended to Determine by Approximation the Electric State of an Uninsulated Cylinder, the Axis of Which Passes Through the Center of an Electrified and Insulated Globe, Placed at a Distance from This Cylinder, Large Enough so that the Electricity of the Globe Cannot Discharge Through the Layer of Air that Separates Them*

The different methods of approximation which we have used in the different Sections which precede can be employed here: we will give some examples of them.[957]

[954] In the original: *le nuage, ainsi que la couche d'air, très imparfaitement idio-électrique.* That is, the cloud and the layer of air are imperfect insulators.

[955] That is, without a large electric discharge in the form of lightning.

[956] That is, they will not suffer the consequences of lightning or electric discharge.

[957] Sections 60 and 61 were not included in Potier's reprint of Coulomb's works, [Potier, 1884].

24.60.1 First Example

Let us start by applying these methods to the experiment detailed in *Section 24.56*. In this experiment, *Figure 11*, an electrified globe C, 4 inches in radius, is placed at $2\frac{1}{2}$ inches from the hemispherical extremity a of a cylinder one inch in diameter and 30 inches in length. This cylinder is touched at its extremity G furthest from the globe, by a [conducting] body which communicates with the Earth.

I suppose that in *Figure 11*, $mm_1m_2m_3m_4m_5$ represents the curve of the electric densities of the different corresponding points of the surface of the cylinder. I seek according to the method explained, *Section 24.36* of this Memoir, the variations of the density of these various points, by supposing that this variation follows a straight line. I notice first that the action of the globe being in inverse proportion to the square of the distance to the points on which it acts, and the distance CG being significant, we can suppose the density null or at least very small, at point G, where the cylinder is touched by an undefined [conducting] body. According to this supposition, if we look for the variation nr in the middle of the cylinder, which is 30 inches in length or sixty times its radius r, we will have, *Section 24.37 and following*:[958]

$$\frac{2D \cdot 4^2}{(21.5)^2} = 2nr\left(\frac{1}{\mu}\log 60 - 1\right) = 2nr(3.09) \ ,$$

from which results $nr = 0.0112D$. In this equation, nr represents the variation or increase in density over a length equal to the radius of the cylinder. But as we assume the variation at point G to be zero, the average variation over a length of size, from point m_4, middle of the cylinder, to point G, will be 0.0056; thus the density at point m_4 would be equal to $0.168D$.

Now to determine the variation at point b_2 of the surface of the cylinder, a point which we assume to be 5 inches away from the extremity a, we mark off a length b_2b_3 of 5 inches on the other side of b_2; this leaves a length b_3b_4 of 5 inches from b_3 to the middle of the cylinder. As the variation at point m_4 was found for the length of radius r equal to $0.0112D$, the increase in density, from m_4 to m_3, would be by a first approximation for the length, of ten radii, equal to $0.112D$, which, added to the density found for m_4, will give for the density in m_3 the quantity $0.28D$.

If the density that we have just found for point b_3 were uniform up to the extremity G, we would have the action of the portion of the cylinder b_3m_5 on point F, represented by

$$0.28D\left(\frac{1}{10} - \frac{1}{50}\right) = 0.023D \ ,$$

a quantity a little too large, because the density decreases from b_3 to m_5. But as we have just assumed a constant instead of increasing variation, from the middle b_4 to point b_3, the two errors in opposite directions tend to compensate each other. Now if we seek the action of the portion of the cylinder ab_3 on point F placed 5 inches away from the end of the cylinder, assuming the variation in a straight line over all this length ab_3, and equal to $n'r$ for the

[958] To arrive at this numerical result, Coulomb used that $\log 60 = 1.778$ and that $\mu = 0.4343$, see footnote 659 on page 288. Therefore, $(1/\mu)\log 60 - 1 = 3.09$. By a lapse in the original text the next equation appeared as follows:

$$\frac{2D \cdot 4^2}{(21.5)^2} = 2nr\left(\log\frac{60}{\mu} - 1\right) = 2nr(3.09) \ .$$

length of one radius of the cylinder; we will have, because the portion b_3m_5 of the cylinder acts in the same direction as globe C, the electricity of the globe and of the cylinder being of a contrary nature, the [following] equation

$$\frac{2D4^2}{(11.5)^2} + 0.023D = 2n'r\left(\frac{\log 20}{\mu} - 1\right) ,$$

from which results $n'r = 0.082D$.

Thus, taking this average variation for that which extends from point b or from one inch, counting from the extremity a to 10 inches, that is to say, over a length of 18 radii, we will have for the increase, from point m_3 to one inch from the end, the quantity $1.476D$, which, being added to $0.28D$, density at 10 inches from point a, will give for the density, at one inch from the end of the cylinder, the quantity $1.72D$. Experiment has given us, *Section 24.56*, the density of this cylinder measured one inch from its extremity a, equal to approximately $1.6D$, which differs little from that given by the theory.

The variations of the different points of the cylinder being given by the preceding approximation, we can determine them again by a second approximation which will bring us very near to their true value. I suppose that we want to obtain in *Figure 4*,[959] the variation of the densities at point i, located mid-way between b_2 and b_3, I calculate according to the given formulas, *Section 24.36 and following*, the action that the middle portion of the cylinder b_2a, whose density, as well as its variation, are approximately known by the preceding approximation, exerts on the point of the axis which responds to i. I do the same operation for the cylinder portions b_3b_4 and b_4m_5. According to the actions obtained from this calculation, as well as according to the action of globe C; I form an equation analogous to those which precede; this equation gives me the variation nr of the average density between points b_2 and b_3. I perform the same operations for any other point of the cylinder, and I thus determine by a second approximation the variations at all the points of the surface, from which I obtain the electric density of the cylinder for each point of its surface. We could, by a third approximation, following the same method, come even closer to the truth, if we believed it necessary; but this precision does not appear to ever be useful to the practice in electric research.

24.61 LXI. Second Example

I suppose, *Figure 12*, that C is a globe of a thousand feet in radius [325 m],[960] that the density of the electric fluid spread over the surface of this globe is D; that the extremity a of the cylinder am is placed at 500 feet from this globe; that this cylinder of one inch radius and 60 feet of length [19.5 m] from a to m'', is inserted at point m'' perpendicularly to an indefinite plane surface Ho of a conducting body.

[959] This Figure 4 appears on footnote 876 on page 399.
[960] In the original text, C' appeared here instead of C.

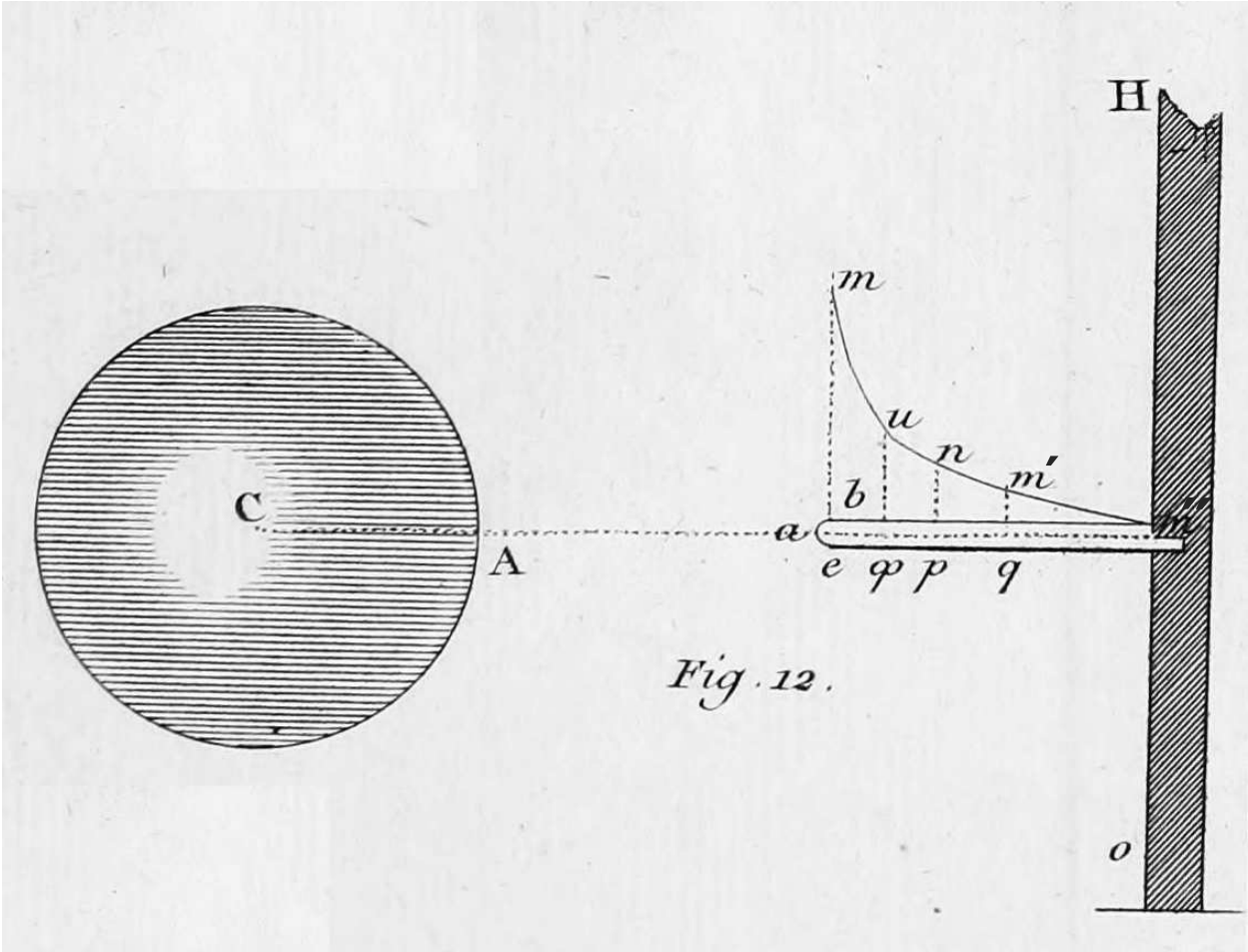


Fig. 12.

According to all that we have said, the uninsulated cylinder am'' will give throughout its length signs of electricity contrary to that of globe C.[961] Its electricity will be zero at point m'', where it joins the surface Ho, and this surface will itself be electrified with a kind of electricity contrary to that of the globe in the parts which border the globe; so that to have the variation of the density on any point q of the cylinder, it is necessary to determine this variation, according to the action of each part aq and qm'' of the cylinder on point q, and according to the action of globe C of the surface Ho on the same point q. If we did not take into account the action of the surface Ho, and if we calculated the variation in the middle q of the cylinder am'' at 30 feet from its extremity, according to the method of the previous Section, we would find for this variation over the length of one radius, $nr = 0.0766D$. Thus, if the variation were constant over the whole length of the cylinder, which is 60 feet or $720r$ in length, we would have the density at two or three inches from the end of the cylinder, equal to $55D$; but as the variation increases as one approaches the end a of the cylinder, the density at 2 or 3 inches from this end, will be much greater than that which we have just found. Calculating the variation for the different points of the cylinder, according to the methods given in the last four Sections of the previous Memoir,[962] I have found that the density five or six inches from the extremity a, would be by a first approximation, equal to more than $120D$, and that it would be equal to more than $300D$ on the hemisphere which ends the cylinder.

[961] In the original text, C' appeared here instead of C.
[962] This Fifth Memoir is translated in Chapter 22.

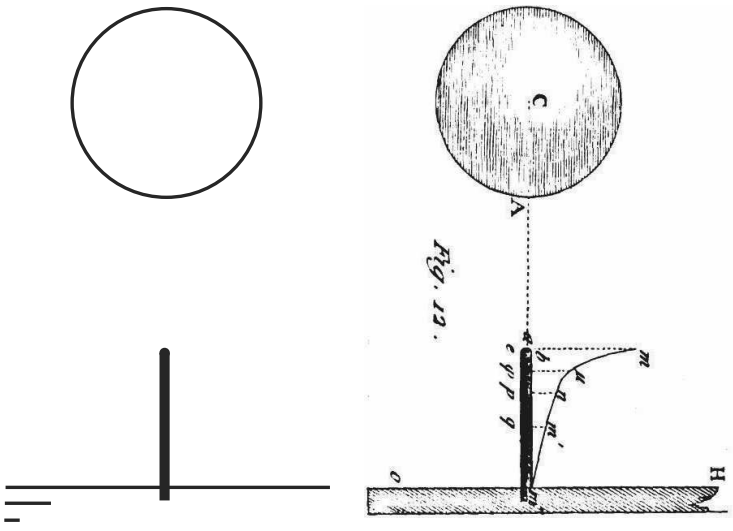
But it should be noted that we have not yet considered the action of the surface OH, whose electricity is of a nature contrary to that of globe C, and which unites with that of the globe, for pushing point q or any other point of the cylinder from q towards C; it must consequently increase the variation of the density for all the points of the cylinder; thus the density of each point of the cylinder must be even greater than that determined by the preceding calculation.[963]

Let us therefore seek the electric density of the surface HO, at a certain distance from an electrified globe C. Once we obtain the evaluation of this density, according to experiment and theory, it will be easy to determine the action of the surface HO on any point of the cylinder; and by uniting this action with that of the cylinder and the globe on the same point, to calculate in a fairly precise manner the electric density of the different parts of the surface of the cylinder.

24.62 LXII. *Electric State of a Non-Insulated Plane, Placed, Figure 13, at a Distance AB from an Electrified Globe, Great Enough for the Electricity Not to Be Communicated from the Globe to the Plane, Through the Layer of Air Which Separates Them*

Figure 13 shows the arrangement of globe C and of the [conducting] plane Bt presented to it: globe C is insulated and electrified; the plane B is sustained vertically by an insulating support efg.

[963] Coulomb is now introducing an additional element compared to Figure 11, namely, a conducting plane. His objective again is to study a vertical lightning rod having its lower end grounded and its upper end close to an electrified cloud. He will now discuss not only the influence of the electrified globe on the conducting cylinder next to it, but also the influence of the ground electrification on the cylinder, this ground electrification also being caused by the electrified globe. He will consider the situation in Figure 12 rotated 90° to be similar to the case he wants to study, as illustrated in this footnote:



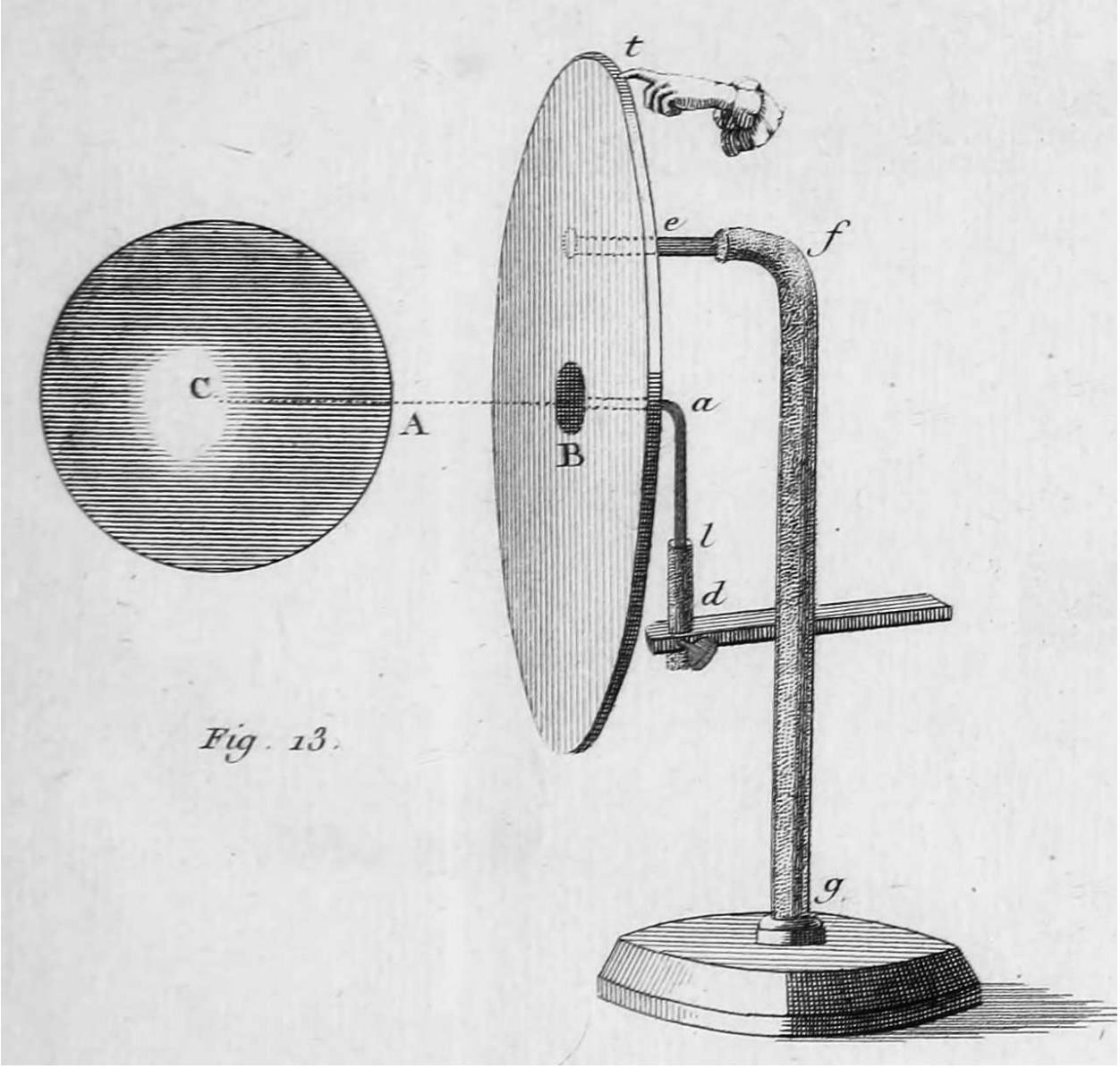


Fig. 13.

This disk [Bt] is pierced at B towards its center [with a hole] two inches in diameter, where a small disk of the same diameter as this hole is introduced. This small disk can then be placed in the electric balance.[964] I will give here only the detail of one experiment, and the general result of the others.

24.62.1 Experiment

Disk tB, *Figure 13*, was 16 inches in diameter [43.312 cm]; globe C was 8 inches in diameter; the center B of this disk was placed 4 inches from the surface of globe C; one touched with the finger at t the disk Bt, and by withdrawing the small circle B, one introduced it into the electric balance, whose needle was electrified with the same nature of electricity as globe

[964] The small conducting disk B in the center of the large conducting disk Bt is supported by an insulating handle *Bald* attached to a support. This set constitutes a proof plane, see footnotes 53, 726 and 757 on pages 30, 319 and 335, respectively. This proof plane can be taken inside the electric balance to analyze the sign and value of the electric charge collected by the small disk B.

C. This needle was attracted with a force which was measured by means of our torsion micrometer; we immediately touched globe C, with the same little disk B, which, presented in the balance, repelled the needle; this action was determined and compared for the same distance[965] with the first [action]. The result of this experiment was that the repulsive action of the small disk after having touched the globe, was four times greater than the attractive action of the same small disk, after having been placed at B in the center of the large disk Bt not insulated.

By a series of experiments analogous to the preceding one, by varying the distance CB, and by comparing the electric densities of point B, relative to this distance BC, I have found that the electric densities of point B, of a nature contrary to those of globe C, were exactly between them, in inverse proportion to the square of the distances from point B to the center of globe C.

24.63 LXIII. *Result of This Experiment*

It is easy to submit the results of the experiment to the calculation. Let D be the electric density of the surface of the globe, δ that of the plane in the parts which border point B: the action of a surface whose uniform density is δ acting in inverse ratio of the square of the distances, will be for a point at an infinitely small distance from this plane, equal to δ, that of a spherical surface acting at a distance a from the center of the sphere, if D is the density of the surface and R its radius, will be equal to $2DR^2/a^2$;[966] thus, if we put in equation the action of globe C and that of disk B,[967] the electric densities of the globe and of the plane being of a contrary nature, we will have

$$\left(\frac{2DR^2}{a^2}+\delta\right)=0 \qquad \text{or} \qquad -\delta=\frac{2DR^2}{a^2}\ ;$$

in the experiment of the preceding Section $R = 4$ inches, $a = 8$ inches; thus $-\delta = D/2$, a quantity which we seemed to find equal to $D/4$ by our experiment. But it must be remembered, as we proved above, *Figure 8*, that when the small disk B touches the globe, it takes up a quantity of electricity double that of the surface touched; whence it follows that the quantity of electricity of the small disk B, after having touched the globe, is double that of the surface: thus as in introducing the small disk B into the hole of the large disk Bt,[968] we only take a density equal to that of the disk, it follows that experiment, as well as theory, give $\delta = -D/2$.

The same formula teaches us that the density δ of the center B of disk Bt, must follow the direct compound ratio of the electric density of the large globe and its surface, and the inverse ratio of the square of the distance BC between the middle of the disk and the center

[965] That is, for the same distance between the proof plane disk and the needle of the electric balance.
[966] See Section 23.4 on page 376.
[967] In the original text, D appeared here instead of B.
[968] In the original text, BI appeared here instead of Bt.

of the globe; which is very exactly consistent with experiment.[969,970]

24.64 LXIV. *Remark*

In the experiment whose theory we have just given in the previous Section, there is a curious observation, it is that when the plane Bt (*Figure 13*) is touched at t,[971] globe C being electrified, only the surface of the disk which is on the side of the globe gives signs of electricity: the opposite surface remains in its natural state;[972] it is easy to prove this by experiment, by touching these two surfaces alternately with a small insulated disk[973] which is then presented to a very sensitive electrometer. When this small disk touches the large disk on a surface on the side of the globe, it gives signs of strong electricity; when the small disk touches it on the opposite side, he gives no sign of electricity.

This phenomenon is easy to explain by the considerations which we have made use of in the various Memoirs which precede, to prove that the electric fluid is distributed only on the surface of bodies. We will come back to this in the Memoir which will follow this one, and which will complete the work we have undertaken on electricity.[974,975] Its object will be to determine the manner in which the electric fluid is distributed and penetrates the surface of insulating bodies, as well as on conducting bodies which touch them or are near them.

---

[969] [Note by Potier] We know the rigorous solution of this question when the plane is indefinite; if a is the radius of the sphere and d the distance from the center [of the sphere] to the plane, by setting $k = \sqrt{d^2 - a^2}$ and $r = \frac{d-k}{a}$, we find that, to charge the sphere at potential 1, we must give it a charge

$$2k \sum_{n=1}^{n=\infty} \frac{r^n}{1 - r^{2n}} \; ;$$

the total negative charge of the plane is equal to it in absolute value, and the maximum density on this plane is

$$\frac{-k}{2\pi a^2} \sum_{n=1}^{n=\infty} \frac{r^n}{1 - r^{2n}} \left(\frac{1}{\frac{d}{a} - r\frac{1-r^{2n-2}}{1-r^{2n}}} \right)^2 ,$$

r tends to zero as d/a increases: if we could limit ourselves to the first term of the series, the ratio

$$\frac{\sum_{n=1}^{n=\infty} \frac{r^n}{1-r^{2n}}}{\sum_{n=1}^{n=\infty} \frac{r^n}{1-r^{2n}} \left(\frac{1}{\frac{d}{a} - r\frac{1-r^{2n-2}}{1-r^{2n}}} \right)^2}$$

of the mean density of the sphere to the maximum density of the plane would reduce to a^2/d^2, as Coulomb approximately verified. In the particular case where $d = 2a$, this ratio is really equal to 3.71, and not to 4; the deviation is of the same order as the deviations between Coulomb's observations and the results of Poisson's calculations; this experiment even shows that the charge of the proof plane is indeed that of the surface on which it is applied, and not the double.

[970] See also footnotes 727, 758, 782, 793, 920 and 927 on pages 320, 336, 343, 349, 421 and 423, respectively.

[971] That is, when disk Bt is touched at t by a grounded conductor, as indicated in Figure 13 by the finger of the person.

[972] That is, the opposite surface becomes neutral or electrically discharged.

[973] That is, with a proof plane.

[974] [Note by Potier] This Memoir was never published and Biot makes no reference to it.

[975] See Potier's Introduction to the reprint of Coulomb's Memoirs presented in Chapter 3.

24.65 LXV

Let us now return[976] to our second example, *Section 24.61*, in which C, *Figure 12*, is a 1000-foot globe,[977] placed 500 feet away from the end of a cylinder, whose length is 60 feet and radius one inch. In this example, since the plane Ho is 1560 feet from the center of globe C, its density D' will be represented by

$$D' = \frac{2D \cdot (1000)^2}{(1560)^2} = 0.822D \ ,$$

D being the density of the surface of the globe. If we want to determine the variation of [the electric surface density of] the cylinder by the preceding methods of approximation, we will find that the action of globe C on point q placed in the middle of the cylinder, will be equal to[978]

$$\frac{2D \cdot (1000)^2}{(1530)^2} = 0.854D \ ;$$

the action of the plane Ho on the same point q, assuming the radius of this disk to be very large relative to the distance $m''q$,[979] will be, as we have just found, equal to $D' = 0.822D$, action which pushes point q toward globe C while globe C' attracts the same point; thus the combined action of the globe and the plane Ho solicits point q in the direction qC' with a force equal to $(0.822 + 0.854)D$.

But we have seen, in the preceding Sections, that if the variation of the densities of the cylinder am'', *Figure 11*,[980] followed a straight line, the action of the whole cylinder on a point placed in its middle would be equal to

$$2nr \left(\frac{\log \frac{l}{r}}{\mu} - 1 \right) \ ,$$

where nr represents the variation of the density over a length equal to the radius r of the cylinder, l is the length of the cylinder, μ the modulus of the logarithmic system;[981] thus, in our example, as r is equal to one inch, and as the length of the cylinder is sixty feet or $720r$, we will have for the action of the cylinder on the point placed in the middle at q, the quantity $2 \cdot 5.58 \cdot nr$, action that operates on point q from q to m''; thus the equilibrium of this point will give

$$(0.822 + 0.854)D = 2nr \cdot 5.58 \qquad \text{or} \qquad nr = \frac{0.838}{5.58} D = 0.15D \ ,$$

[976] This last Section 65 was not included in Potier's reprint of Coulomb's works, [Potier, 1884].

[977] That is, with a radius of 1000 feet $= 3.25 \times 10^2$ m.

[978] In the original text this equation appeared as follows:

$$\frac{2D \cdot (1000)^2}{(1530)^2} D = 0.854D \ .$$

[979] That is, assuming the surface Ho to be a conducting disk that has a radius much greater than the distance $m''q$ between the center m'' of the disk and the center q of the cylinder aqm''.

[980] In the original text "Figure 5" appeared here. However, there is no Figure 5 in the original images relating to this article.

[981] See footnote 659 on page 288.

about twice the amount we found when we neglected the action of the surface Ho.

If I now want to have the variation at point φ, which I suppose is 5 feet from the extremity a of the cylinder am'', I would find that the action of the plane Ho on point φ is approximately equal to D' or $0.822D$, the distance $m''\varphi$ being very small relative to the electrified extent of the plane Ho: the action of the globe on point φ will be equal to $2D(1000)^2/(1505)^2$. To obtain now the action of the cylinder on point φ, I will suppose the cylinder divided into two parts; the first ap, being 10 feet long; the second pm, being 55 feet in length. If the variation of the density found in the middle of the cylinder, equal to $0.15D$, were extended to point p, 10 feet from the extremity a, we would have at this point the density δ equal to $600 \times 0.15D = 90D$.

Thus, if we wanted to calculate the action of the portion pm'' of the cylinder on point φ, the electric density being at p equal to δ, and decreasing by the quantity nr, over a length of the radius, we would have, as an expression for this action, by naming $\varphi p = a$ and $\varphi m'' = x$, according to the Sections which precede, the formula

$$\delta\left(\frac{r}{a} - \frac{r}{x}\right) + nr\left(\frac{a-x}{x}\right) + \frac{nr}{\mu}\log\left(\frac{x}{a}\right) \ .$$

It is necessary in this formula to substitute in place of δ, its value $90D$, in place of a, its value $60r$, in place of x, the quantity 660, and instead of nr, its value $0.15D$; from which will result for the action of this part mp of the cylinder, the quantity $1.14D$. If we now assume that the average variation over the last ten feet is $n'r$, we will have at point p, the middle of these last ten feet, for the action in the direction ap, of the last ten feet of the cylinder, the quantity

$$2n'r\left(\frac{\log 120}{\mu} - 1\right) = 7.58n'r \ .$$

Thus, by combining the action of the globe with that of the surface Ho and that of the portion of the cylinder pm'', which act in the same direction, we will have for the equilibrium at point φ, at 5 feet from end a, the equation

$$\frac{2D(1000)^2}{(1505)^2} + 0.82D + 1.14D = 7.58n'r \ ;$$

from which

$$n'r = \frac{1.42}{3.79}D = 0.37D \ .$$

Thus the variation at 5 feet from the extremity a would be, by this approximation, equal to 0.37; from which one would conclude that the average variation between a point taken in the middle of the cylinder where this variation is equal to $0.15D$, and a point 5 feet from the end where it is equal to $0.37D$, will be 0.26. Thus the increase in density, from the middle of the cylinder to five feet from the end, will be over twenty-five feet or $300r$ in length, equal to $0.26 \times 300D = 78D$. The variation from point q, at the middle of the cylinder, to point m'', is not quite equal to $0.15D$ which we found for the variation at the middle of the cylinder; but taking this value as a first approximation, we will have for the increase in density, from point m'' to the middle q of the cylinder or over $360r$ of length, the quantity $54D$ which will represent the density in the middle of the cylinder; thus by joining to it $68D$, an increase of

density which we have just found from the middle of the cylinder to point φ, placed 5 feet from the extremity a of the cylinder, we will have the density at this point φ equal to $132D$.

If, following the above method, I looked for the variation at 2 feet or $24r$ distance from the end a of the cylinder, I would find it for the length of one radius, equal to $1.07D$; I would then find the variation, over a length of one radius taken 6 inches from point a by the same method of approximation, equal to $5D$, or five times the density of the globe; and by concluding, according to these variations, the density at 2 or 3 inches from the extremity a of the cylinder, we would find it of more than $200D$.

If we now wanted to determine the electric density at the extremity a which terminates the cylinder, it would be easy to see, as experiment proves, that this density is at least twice as great as that of point b, which would only be apart from it by approximately a length equal to the half-diameter of the cylinder or to the radius of the hemisphere which terminates it. To prove it, suppose that from point b, which meets the equator of the hemisphere, to point m'', the density of the cylinder is uniform and equal to δ; that the density of the surface of the hemisphere ab is equal to δ'; we will find by a very easy calculation, that the action of the surface of the hemisphere ab on its center e in the direction am'', is equal to $\delta'/2$; it will be found that the action of the surface of the cylinder bm'' on point e in the same direction, is equal to δ. Thus, by uniting with this action, that of the sphere C[982] and that of the surface Ho, they must balance the action of the hemisphere ab, from which results the equation

$$\frac{\delta'}{2} = \delta + \frac{2D(1000)^2}{(1500)} + 0.82D \ ;$$

and as δ is much larger than D, it follows that δ' or that the electric density on the surface of the hemisphere which terminates at a the cylinder, is more than double that from the surface of this same cylinder at one inch from its extremity. This result is consistent with what experiment has given us, *Section 24.55*.

The calculations of this Section, though very imperfect, are sufficient in practice to analyze nearly all practical subjects relative to electricity, where it might be necessary to employ analysis. But the values given by this first approximation can be corrected and brought closer to the truth as much as desired by the preceding approximation, as we said at the end of *Section 24.60*, by viewing the curve of densities as a polygon, whose inclination of the sides would be defined relative to the *axis of the cylinder*: the calculation of the action of the different parts of the surface of the cylinder on a point of its axis, would be done by the methods explained in *Section 24.35 and following*, and would give the variation at the middle of a part of the cylinder, part that could be considered more or less large, depending on whether the point whose [density] variation we wanted to determine, would be more or less close to the extremity a.

[982] In the original text, C' appeared here instead of C.

Chapter 25

Remarks on Coulomb's Sixth Memoir

A. K. T. Assis

25.1 Coulomb's Experimental Procedure

In the experiment of Section 24.2 Coulomb uses his electric balance to estimate the amount of charge contained in an electrified globe C. In Figure 25.1 (a) we have the initial configuration of the experiment. This Figure represents the top-down view of the electric balance. The suspension wire has its projection at point a, which is the center of the horizontal needle attached to the lower end of the wire. There is a conducting disk b at the end of the insulating needle ab, and this disk will be electrified with a charge of the same sign as the charge on globe C. When the wire is not twisted, the needle is directed toward point o attached to the bottom of the balance. The arrow indicates the micrometer attached to the top of the suspension wire. Initially it is directed toward point S fixed at the top of the balance. I am assuming that when the wire is not twisted, points a, o, and S are in the same vertical plane.

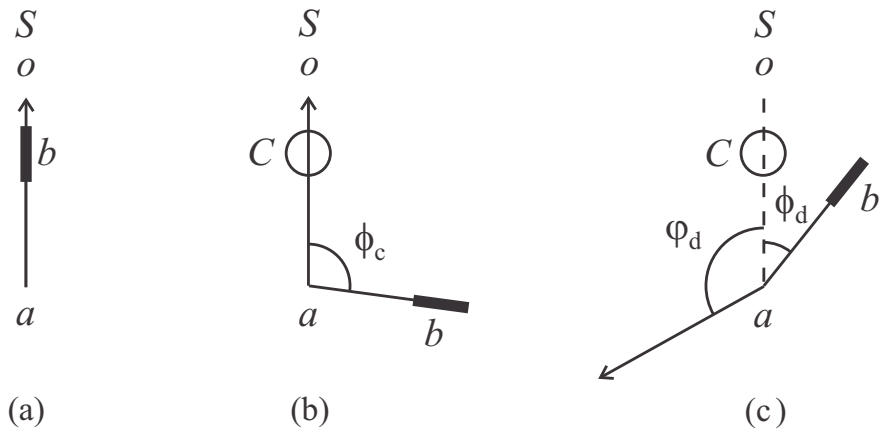


Figure 25.1: (a) Initial orientation of the needle ab and the micrometer indicator attached to the lower and upper parts of the suspension wire, respectively. (b) The electrified disk b of the needle is repelled clockwise by globe C electrified with a charge of the same sign. (c) By twisting the micrometer counterclockwise, disk b is brought closer to globe C.

Coulomb then introduces globe C into the balance, with the globe and the disk b of the needle being electrified with charges of the same sign. He puts the center of globe C in the

position where the center of the disk b was initially located. I am going to assume here that needle b was repelled clockwise. At equilibrium it is away from C by an angle ϕ_c, Figure 25.1 (b). In this situation the electric repulsion between C and b is balanced by the twisting force of the lower part of the suspension wire.

Coulomb turns the micrometer attached to the top of the suspension wire counterclockwise by an angle φ_d until C and b are separated by a pre-defined angle ϕ_d, Figure 25.1 (c). In this situation the total twist of the suspension wire is measured by $\phi_d + \varphi_d$.

Coulomb will estimate the amount of charges from different globes by always bringing them to the same angular distance ϕ_d from the disk of the needle, for example, with $\phi_d = 30°$. The force of repulsion between globe C and disk d of the needle is proportional to the product of their charges and inversely proportional to the square of the distance between their centers. This force is balanced by the total twist of the suspension wire. As the distance between C and b is always the same in all experiments, the amount of charge on globe C can be estimated by the total twist of the wire measured by $\phi_d + \varphi_d$.

25.2 The Force Acting on a Charge Placed Exactly on the Surface of a Uniformly Electrified Spherical Shell

In Section 24.4 of this Sixth Memoir Coulomb mentioned the force exerted by a spherical shell of radius R uniformly electrified with a surface charge density D[983] when acting on a point with charge μ located exactly on the surface. Coulomb then stated the following:[984]

> Dans cette supposition, l'action d'une surface sphérique, dont tous les points ont la même densité D, agissant sur un point de la surface dont la masse électrique seroit μ, seroit représentée par $\Pi D\mu$; Π étant le rapport de la circonférence au rayon.

Historically the letter π (or Π) was introduced to indicate the perimeter or circumference of a circle. During the 18th century, some authors began to use the letter π to represent the ratio of the circumference to the radius (that is, with $\pi = 6.28318...$). Among these authors was Coulomb as can be seen from this sentence.[985] Other authors started to use the letter π to represent the ratio of the circumference to the diameter (that is, with $\pi = 3.14159...$). This confusion regarding the meaning of the symbol π persisted during Coulomb's time. From the 19th century onwards, the letter π was adopted by most authors to represent only the ratio of the circumference of a circle to its diameter, that is, $\pi = 3.14159....$ In this English translation, the letter π has its modern meaning as the ratio of the circumference of a circle to its diameter, that is, with its value given by $\pi = 3.14159....$

Therefore, to make Coulomb's text understandable for a modern audience, the expression $\Pi D\mu$ in the translation of the above sentence was replaced with $2\pi D\mu$, as had been done by Potier.[986] The translation adopted for this sentence was as follows, see the page 383:

> In this supposition, the action of a spherical surface, all the points of which have the same density D, acting on a point of the surface whose electric mass would be

[983] D is the surface density of electricity, that is, the amount of charge per unit area.
[984] [Coulomb, 1791, p. 621], see also footnote 849 on page 383.
[985] See also footnotes 864 and 899 on pages 391 and 414, respectively.
[986] [Potier, 1884, p. 233].

μ, would be represented by $2\pi D\mu$, being 2π the ratio of the circumference to the radius.

What Coulomb is calling "action" here can be understood as "force" or "force per unit charge" in the usual Newtonian sense.

In this sentence Coulomb is including the factor 2π in the force exerted by a uniformly electrified spherical shell acting on an electrified particle located on that surface, see Section 23.4 on page 376. See also footnote 796 on page 351. In the Fifth Memoir he had left out this factor 2π.

Some authors have presented a discussion of Coulomb's work connected to the force acting on an electrified particle placed exactly on the surface of a uniformly electrified spherical shell.[987] There is also a recent discussion of the exact calculation of this force by means of analytical integrations.[988]

25.3 Set of 24 Equations

In Section 24.16 Coulomb performed the ninth experiment in which an 8-inch-diameter globe was placed in contact with a line of 24 small globes, 2-inch in diameter each, with the centers of all of them located along a straight line. These 24 small globes formed a length of 48 inches, as illustrated in Figure 25.2. The vertical lines in this Figure indicate the points of contact between two globes. Point a indicates the contact between the large globe and the first small globe. Point a_8, for example, indicates the point of contact between the eighth and the ninth small globe.

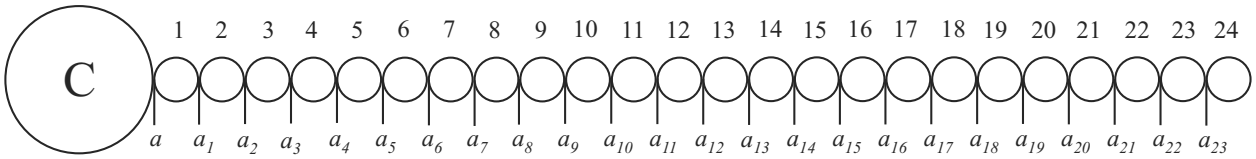


Figure 25.2: Large globe C, 8 inches in diameter, in contact with a line of 24 small globes, each 2 inches in diameter.

On page 398 of Section 24.17, the original Table was presented containing a set of 24 equations that describe the electrostatic equilibrium of this system of electrified globes according to Coulomb.

At the top left of the first line of this Table we have the following:

"Action of the large globe on the different points of contact. The average density of the large globe is D; its radius R, the radius of the small globe is r."

At the top right of the first line of this Table we have the following:

"*Table of 24 equations intended to determine the average electric density of 24 small globes, the centers placed in a straight line, the small globe 1 in contact with a large globe.* In this Table, the numbers at the top of each column indicate the position of the small globe; so, for example, that at the eighth vertical column, third horizontal line, we find the quantity $\frac{2}{11^2}$ which is supposed to be multiplied by δ_8, or by the average density of the eighth small globe, counting from the large globe."

[987] [Potier, 1884, p . ix] and footnote 59 on page 30 of this English translation. See also [Heilbron, 1999, pp. 494-496].

[988] [Lima, 2018] and [Lima, 2020].

In the second row of the first column we have "First equation. $D =$". In the third row of the first column we have "Second equation. $\frac{2DR^2}{(R+2r)^2} =$". Etc.

I present this Table here in the form of 24 equations to make clear what Coulomb had in mind. They can be obtained by following the procedure of Sections 23.4 and 24.4.

First equation, equilibrium at point a:

$$D = +\delta_1 + \frac{2\delta_2}{3^2} + \frac{2\delta_3}{5^2} + \frac{2\delta_4}{7^2} + \frac{2\delta_5}{9^2} + \frac{2\delta_6}{11^2} + \frac{2\delta_7}{13^2} + \frac{2\delta_8}{15^2} + \frac{2\delta_9}{17^2} + \frac{2\delta_{10}}{19^2} + \frac{2\delta_{11}}{21^2} + \frac{2\delta_{12}}{23^2}$$

$$+ \frac{2\delta_{13}}{25^2} + \frac{2\delta_{14}}{27^2} + \frac{2\delta_{15}}{29^2} + \frac{2\delta_{16}}{31^2} + \frac{2\delta_{17}}{33^2} + \frac{2\delta_{18}}{35^2} + \frac{2\delta_{19}}{37^2} + \frac{2\delta_{20}}{39^2} + \frac{2\delta_{21}}{41^2} + \frac{2\delta_{22}}{43^2} + \frac{2\delta_{23}}{45^2} + \frac{2\delta_{24}}{47^2} \ . \quad (25.1)$$

Second equation, equilibrium at point a_1:

$$\frac{2DR^2}{(R+2r)^2} = -\delta_1 + \delta_2 + \frac{2\delta_3}{3^2} + \frac{2\delta_4}{5^2} + \frac{2\delta_5}{7^2} + \frac{2\delta_6}{9^2} + \frac{2\delta_7}{11^2} + \frac{2\delta_8}{13^2} + \frac{2\delta_9}{15^2} + \frac{2\delta_{10}}{17^2} + \frac{2\delta_{11}}{19^2} + \frac{2\delta_{12}}{21^2}$$

$$+ \frac{2\delta_{13}}{23^2} + \frac{2\delta_{14}}{25^2} + \frac{2\delta_{15}}{27^2} + \frac{2\delta_{16}}{29^2} + \frac{2\delta_{17}}{31^2} + \frac{2\delta_{18}}{33^2} + \frac{2\delta_{19}}{35^2} + \frac{2\delta_{20}}{37^2} + \frac{2\delta_{21}}{39^2} + \frac{2\delta_{22}}{41^2} + \frac{2\delta_{23}}{43^2} + \frac{2\delta_{24}}{45^2} \ . \quad (25.2)$$

Third equation, equilibrium at point a_2:

$$\frac{2DR^2}{(R+4r)^2} = -\frac{2\delta_1}{3^2} - \delta_2 + \delta_3 + \frac{2\delta_4}{3^2} + \frac{2\delta_5}{5^2} + \frac{2\delta_6}{7^2} + \frac{2\delta_7}{9^2} + \frac{2\delta_8}{11^2} + \frac{2\delta_9}{13^2} + \frac{2\delta_{10}}{15^2} + \frac{2\delta_{11}}{17^2} + \frac{2\delta_{12}}{19^2}$$

$$+ \frac{2\delta_{13}}{21^2} + \frac{2\delta_{14}}{23^2} + \frac{2\delta_{15}}{25^2} + \frac{2\delta_{16}}{27^2} + \frac{2\delta_{17}}{29^2} + \frac{2\delta_{18}}{31^2} + \frac{2\delta_{19}}{33^2} + \frac{2\delta_{20}}{35^2} + \frac{2\delta_{21}}{37^2} + \frac{2\delta_{22}}{39^2} + \frac{2\delta_{23}}{41^2} + \frac{2\delta_{24}}{43^2} \ . \quad (25.3)$$

Fourth equation, equilibrium at point a_3:

$$\frac{2DR^2}{(R+6r)^2} = -\frac{2\delta_1}{5^2} - \frac{2\delta_2}{3^2} - \delta_3 + \delta_4 + \frac{2\delta_5}{3^2} + \frac{2\delta_6}{5^2} + \frac{2\delta_7}{7^2} + \frac{2\delta_8}{9^2} + \frac{2\delta_9}{11^2} + \frac{2\delta_{10}}{13^2} + \frac{2\delta_{11}}{15^2} + \frac{2\delta_{12}}{17^2}$$

$$+ \frac{2\delta_{13}}{19^2} + \frac{2\delta_{14}}{21^2} + \frac{2\delta_{15}}{23^2} + \frac{2\delta_{16}}{25^2} + \frac{2\delta_{17}}{27^2} + \frac{2\delta_{18}}{29^2} + \frac{2\delta_{19}}{31^2} + \frac{2\delta_{20}}{33^2} + \frac{2\delta_{21}}{35^2} + \frac{2\delta_{22}}{37^2} + \frac{2\delta_{23}}{39^2} + \frac{2\delta_{24}}{41^2} \ . \quad (25.4)$$

Fifth equation, equilibrium at point a_4:

$$\frac{2DR^2}{(R+8r)^2} = -\frac{2\delta_1}{7^2} - \frac{2\delta_2}{5^2} - \frac{2\delta_3}{3^2} - \delta_4 + \delta_5 + \frac{2\delta_6}{3^2} + \frac{2\delta_7}{5^2} + \frac{2\delta_8}{7^2} + \frac{2\delta_9}{9^2} + \frac{2\delta_{10}}{11^2} + \frac{2\delta_{11}}{13^2} + \frac{2\delta_{12}}{15^2}$$

$$+ \frac{2\delta_{13}}{17^2} + \frac{2\delta_{14}}{19^2} + \frac{2\delta_{15}}{21^2} + \frac{2\delta_{16}}{23^2} + \frac{2\delta_{17}}{25^2} + \frac{2\delta_{18}}{27^2} + \frac{2\delta_{19}}{29^2} + \frac{2\delta_{20}}{31^2} + \frac{2\delta_{21}}{33^2} + \frac{2\delta_{22}}{35^2} + \frac{2\delta_{23}}{37^2} + \frac{2\delta_{24}}{39^2} \ . \quad (25.5)$$

Sixth equation, equilibrium at point a_5:

$$\frac{2DR^2}{(R+10r)^2} = -\frac{2\delta_1}{9^2} - \frac{2\delta_2}{7^2} - \frac{2\delta_3}{5^2} - \frac{2\delta_4}{3^2} - \delta_5 + \delta_6 + \frac{2\delta_7}{3^2} + \frac{2\delta_8}{5^2} + \frac{2\delta_9}{7^2} + \frac{2\delta_{10}}{9^2} + \frac{2\delta_{11}}{11^2} + \frac{2\delta_{12}}{13^2}$$

$$+\frac{2\delta_{13}}{15^2}+\frac{2\delta_{14}}{17^2}+\frac{2\delta_{15}}{19^2}+\frac{2\delta_{16}}{21^2}+\frac{2\delta_{17}}{23^2}+\frac{2\delta_{18}}{25^2}+\frac{2\delta_{19}}{27^2}+\frac{2\delta_{20}}{29^2}+\frac{2\delta_{21}}{31^2}+\frac{2\delta_{22}}{33^2}+\frac{2\delta_{23}}{35^2}+\frac{2\delta_{24}}{37^2}\ . \quad (25.6)$$

Seventh equation, equilibrium at point a_6:

$$\frac{2DR^2}{(R+12r)^2}=-\frac{2\delta_1}{11^2}-\frac{2\delta_2}{9^2}-\frac{2\delta_3}{7^2}-\frac{2\delta_4}{5^2}-\frac{2\delta_5}{3^2}-\delta_6+\delta_7+\frac{2\delta_8}{3^2}+\frac{2\delta_9}{5^2}+\frac{2\delta_{10}}{7^2}+\frac{2\delta_{11}}{9^2}+\frac{2\delta_{12}}{11^2}$$

$$+\frac{2\delta_{13}}{13^2}+\frac{2\delta_{14}}{15^2}+\frac{2\delta_{15}}{17^2}+\frac{2\delta_{16}}{19^2}+\frac{2\delta_{17}}{21^2}+\frac{2\delta_{18}}{23^2}+\frac{2\delta_{19}}{25^2}+\frac{2\delta_{20}}{27^2}+\frac{2\delta_{21}}{29^2}+\frac{2\delta_{22}}{31^2}+\frac{2\delta_{23}}{33^2}+\frac{2\delta_{24}}{35^2}\ . \quad (25.7)$$

Eighth equation, equilibrium at point a_7:

$$\frac{2DR^2}{(R+14r)^2}=-\frac{2\delta_1}{13^2}-\frac{2\delta_2}{11^2}-\frac{2\delta_3}{9^2}-\frac{2\delta_4}{7^2}-\frac{2\delta_5}{5^2}-\frac{2\delta_6}{3^2}-\delta_7+\delta_8+\frac{2\delta_9}{3^2}+\frac{2\delta_{10}}{5^2}+\frac{2\delta_{11}}{7^2}+\frac{2\delta_{12}}{9^2}$$

$$+\frac{2\delta_{13}}{11^2}+\frac{2\delta_{14}}{13^2}+\frac{2\delta_{15}}{15^2}+\frac{2\delta_{16}}{17^2}+\frac{2\delta_{17}}{19^2}+\frac{2\delta_{18}}{21^2}+\frac{2\delta_{19}}{23^2}+\frac{2\delta_{20}}{25^2}+\frac{2\delta_{21}}{27^2}+\frac{2\delta_{22}}{29^2}+\frac{2\delta_{23}}{31^2}+\frac{2\delta_{24}}{33^2}\ . \quad (25.8)$$

Ninth equation, equilibrium at point a_8:

$$\frac{2DR^2}{(R+16r)^2}=-\frac{2\delta_1}{15^2}-\frac{2\delta_2}{13^2}-\frac{2\delta_3}{11^2}-\frac{2\delta_4}{9^2}-\frac{2\delta_5}{7^2}-\frac{2\delta_6}{5^2}-\frac{2\delta_7}{3^2}-\delta_8+\delta_9+\frac{2\delta_{10}}{3^2}+\frac{2\delta_{11}}{5^2}+\frac{2\delta_{12}}{7^2}$$

$$+\frac{2\delta_{13}}{9^2}+\frac{2\delta_{14}}{11^2}+\frac{2\delta_{15}}{13^2}+\frac{2\delta_{16}}{15^2}+\frac{2\delta_{17}}{17^2}+\frac{2\delta_{18}}{19^2}+\frac{2\delta_{19}}{21^2}+\frac{2\delta_{20}}{23^2}+\frac{2\delta_{21}}{25^2}+\frac{2\delta_{22}}{27^2}+\frac{2\delta_{23}}{29^2}+\frac{2\delta_{24}}{31^2}\ . \quad (25.9)$$

Tenth equation, equilibrium at point a_9:

$$\frac{2DR^2}{(R+18r)^2}=-\frac{2\delta_1}{17^2}-\frac{2\delta_2}{15^2}-\frac{2\delta_3}{13^2}-\frac{2\delta_4}{11^2}-\frac{2\delta_5}{9^2}-\frac{2\delta_6}{7^2}-\frac{2\delta_7}{5^2}-\frac{2\delta_8}{3^2}-\delta_9+\delta_{10}+\frac{2\delta_{11}}{3^2}+\frac{2\delta_{12}}{5^2}$$

$$+\frac{2\delta_{13}}{7^2}+\frac{2\delta_{14}}{9^2}+\frac{2\delta_{15}}{11^2}+\frac{2\delta_{16}}{13^2}+\frac{2\delta_{17}}{15^2}+\frac{2\delta_{18}}{17^2}+\frac{2\delta_{19}}{19^2}+\frac{2\delta_{20}}{21^2}+\frac{2\delta_{21}}{23^2}+\frac{2\delta_{22}}{25^2}+\frac{2\delta_{23}}{27^2}+\frac{2\delta_{24}}{29^2}\ . \quad (25.10)$$

Eleventh equation, equilibrium at point a_{10}:

$$\frac{2DR^2}{(R+20r)^2}=-\frac{2\delta_1}{19^2}-\frac{2\delta_2}{17^2}-\frac{2\delta_3}{15^2}-\frac{2\delta_4}{13^2}-\frac{2\delta_5}{11^2}-\frac{2\delta_6}{9^2}-\frac{2\delta_7}{7^2}-\frac{2\delta_8}{5^2}-\frac{2\delta_9}{3^2}-\delta_{10}+\delta_{11}+\frac{2\delta_{12}}{3^2}$$

$$+\frac{2\delta_{13}}{5^2}+\frac{2\delta_{14}}{7^2}+\frac{2\delta_{15}}{9^2}+\frac{2\delta_{16}}{11^2}+\frac{2\delta_{17}}{13^2}+\frac{2\delta_{18}}{15^2}+\frac{2\delta_{19}}{17^2}+\frac{2\delta_{20}}{19^2}+\frac{2\delta_{21}}{21^2}+\frac{2\delta_{22}}{23^2}+\frac{2\delta_{23}}{25^2}+\frac{2\delta_{24}}{27^2}\ . \quad (25.11)$$

Twelfth equation, equilibrium at point a_{11}:

$$\frac{2DR^2}{(R+22r)^2}=-\frac{2\delta_1}{21^2}-\frac{2\delta_2}{19^2}-\frac{2\delta_3}{17^2}-\frac{2\delta_4}{15^2}-\frac{2\delta_5}{13^2}-\frac{2\delta_6}{11^2}-\frac{2\delta_7}{9^2}-\frac{2\delta_8}{7^2}-\frac{2\delta_9}{5^2}-\frac{2\delta_{10}}{3^2}-\delta_{11}+\delta_{12}$$

$$+\frac{2\delta_{13}}{3^2}+\frac{2\delta_{14}}{5^2}+\frac{2\delta_{15}}{7^2}+\frac{2\delta_{16}}{9^2}+\frac{2\delta_{17}}{11^2}+\frac{2\delta_{18}}{13^2}+\frac{2\delta_{19}}{15^2}+\frac{2\delta_{20}}{17^2}+\frac{2\delta_{21}}{19^2}+\frac{2\delta_{22}}{21^2}+\frac{2\delta_{23}}{23^2}+\frac{2\delta_{24}}{25^2}\ . \quad (25.12)$$

Thirteenth equation, equilibrium at point a_{12}:

$$\frac{2DR^2}{(R+24r)^2}=-\frac{2\delta_1}{23^2}-\frac{2\delta_2}{21^2}-\frac{2\delta_3}{19^2}-\frac{2\delta_4}{17^2}-\frac{2\delta_5}{15^2}-\frac{2\delta_6}{13^2}-\frac{2\delta_7}{11^2}-\frac{2\delta_8}{9^2}-\frac{2\delta_9}{7^2}-\frac{2\delta_{10}}{5^2}-\frac{2\delta_{11}}{3^2}-\delta_{12}$$

$$+\delta_{13}+\frac{2\delta_{14}}{3^2}+\frac{2\delta_{15}}{5^2}+\frac{2\delta_{16}}{7^2}+\frac{2\delta_{17}}{9^2}+\frac{2\delta_{18}}{11^2}+\frac{2\delta_{19}}{13^2}+\frac{2\delta_{20}}{15^2}+\frac{2\delta_{21}}{17^2}+\frac{2\delta_{22}}{19^2}+\frac{2\delta_{23}}{21^2}+\frac{2\delta_{24}}{23^2}\ . \quad (25.13)$$

Fourteenth equation, equilibrium at point a_{13}:

$$\frac{2DR^2}{(D+26r)^2}=-\frac{2\delta_1}{25^2}-\frac{2\delta_2}{23^2}-\frac{2\delta_3}{21^2}-\frac{2\delta_4}{19^2}-\frac{2\delta_5}{17^2}-\frac{2\delta_6}{15^2}-\frac{2\delta_7}{13^2}-\frac{2\delta_8}{11^2}-\frac{2\delta_9}{9^2}-\frac{2\delta_{10}}{7^2}-\frac{2\delta_{11}}{5^2}-\frac{2\delta_{12}}{3^2}$$

$$-\delta_{13}+\delta_{14}+\frac{2\delta_{15}}{3^2}+\frac{2\delta_{16}}{5^2}+\frac{2\delta_{17}}{7^2}+\frac{2\delta_{18}}{9^2}+\frac{2\delta_{19}}{11^2}+\frac{2\delta_{20}}{13^2}+\frac{2\delta_{21}}{15^2}+\frac{2\delta_{22}}{17^2}+\frac{2\delta_{23}}{19^2}+\frac{2\delta_{24}}{21^2}\ . \quad (25.14)$$

Fifteenth equation, equilibrium at point a_{14}:

$$\frac{2DR^2}{(R+28r)^2}=-\frac{2\delta_1}{27^2}-\frac{2\delta_2}{25^2}-\frac{2\delta_3}{23^2}-\frac{2\delta_4}{21^2}-\frac{2\delta_5}{19^2}-\frac{2\delta_6}{17^2}-\frac{2\delta_7}{15^2}-\frac{2\delta_8}{13^2}-\frac{2\delta_9}{11^2}-\frac{2\delta_{10}}{9^2}-\frac{2\delta_{11}}{7^2}-\frac{2\delta_{12}}{5^2}$$

$$-\frac{2\delta_{13}}{3^2}-\delta_{14}+\delta_{15}+\frac{2\delta_{16}}{3^2}+\frac{2\delta_{17}}{5^2}+\frac{2\delta_{18}}{7^2}+\frac{2\delta_{19}}{9^2}+\frac{2\delta_{20}}{11^2}+\frac{2\delta_{21}}{13^2}+\frac{2\delta_{22}}{15^2}+\frac{2\delta_{23}}{17^2}+\frac{2\delta_{24}}{19^2}\ . \quad (25.15)$$

Sixteenth equation, equilibrium at point a_{15}:

$$\frac{2DR^2}{(R+40r)^2}=-\frac{2\delta_1}{29^2}-\frac{2\delta_2}{27^2}-\frac{2\delta_3}{25^2}-\frac{2\delta_4}{23^2}-\frac{2\delta_5}{21^2}-\frac{2\delta_6}{19^2}-\frac{2\delta_7}{17^2}-\frac{2\delta_8}{15^2}-\frac{2\delta_9}{13^2}-\frac{2\delta_{10}}{11^2}-\frac{2\delta_{11}}{9^2}-\frac{2\delta_{12}}{7^2}$$

$$-\frac{2\delta_{13}}{5^2}-\frac{2\delta_{14}}{3^2}-\delta_{15}+\delta_{16}+\frac{2\delta_{17}}{3^2}+\frac{2\delta_{18}}{5^2}+\frac{2\delta_{19}}{7^2}+\frac{2\delta_{20}}{9^2}+\frac{2\delta_{21}}{11^2}+\frac{2\delta_{22}}{13^2}+\frac{2\delta_{23}}{15^2}+\frac{2\delta_{24}}{17^2}\ . \quad (25.16)$$

Seventeenth equation, equilibrium at point a_{16}:

$$\frac{2DR^2}{(R+32r)^2}=-\frac{2\delta_1}{31^2}-\frac{2\delta_2}{29^2}-\frac{2\delta_3}{27^2}-\frac{2\delta_4}{25^2}-\frac{2\delta_5}{23^2}-\frac{2\delta_6}{21^2}-\frac{2\delta_7}{19^2}-\frac{2\delta_8}{17^2}-\frac{2\delta_9}{15^2}-\frac{2\delta_{10}}{13^2}-\frac{2\delta_{11}}{11^2}-\frac{2\delta_{12}}{9^2}$$

$$-\frac{2\delta_{13}}{7^2}-\frac{2\delta_{14}}{5^2}-\frac{2\delta_{15}}{3^2}-\delta_{16}+\delta_{17}+\frac{2\delta_{18}}{3^2}+\frac{2\delta_{19}}{5^2}+\frac{2\delta_{20}}{7^2}+\frac{2\delta_{21}}{9^2}+\frac{2\delta_{22}}{11^2}+\frac{2\delta_{23}}{13^2}+\frac{2\delta_{24}}{15^2}\ . \quad (25.17)$$

Eighteenth equation, equilibrium at point a_{17}:

$$\frac{2DR^2}{(R+34r)^2}=-\frac{2\delta_1}{33^2}-\frac{2\delta_2}{31^2}-\frac{2\delta_3}{29^2}-\frac{2\delta_4}{27^2}-\frac{2\delta_5}{25^2}-\frac{2\delta_6}{23^2}-\frac{2\delta_7}{21^2}-\frac{2\delta_8}{19^2}-\frac{2\delta_9}{17^2}-\frac{2\delta_{10}}{15^2}-\frac{2\delta_{11}}{13^2}-\frac{2\delta_{12}}{11^2}$$

$$-\frac{2\delta_{13}}{9^2}-\frac{2\delta_{14}}{7^2}-\frac{2\delta_{15}}{5^2}-\frac{2\delta_{16}}{3^2}-\delta_{17}+\delta_{18}+\frac{2\delta_{19}}{3^2}+\frac{2\delta_{20}}{5^2}+\frac{2\delta_{21}}{7^2}+\frac{2\delta_{22}}{9^2}+\frac{2\delta_{23}}{11^2}+\frac{2\delta_{24}}{13^2}\ . \quad (25.18)$$

Nineteenth equation, equilibrium at point a_{18}:

$$\frac{2DR^2}{(R+36r)^2}=-\frac{2\delta_1}{35^2}-\frac{2\delta_2}{33^2}-\frac{2\delta_3}{31^2}-\frac{2\delta_4}{29^2}-\frac{2\delta_5}{27^2}-\frac{2\delta_6}{25^2}-\frac{2\delta_7}{23^2}-\frac{2\delta_8}{21^2}-\frac{2\delta_9}{19^2}-\frac{2\delta_{10}}{17^2}-\frac{2\delta_{11}}{15^2}-\frac{2\delta_{12}}{13^2}$$

$$-\frac{2\delta_{13}}{11^2}-\frac{2\delta_{14}}{9^2}-\frac{2\delta_{15}}{7^2}-\frac{2\delta_{16}}{5^2}-\frac{2\delta_{17}}{3^2}-\delta_{18}+\delta_{19}+\frac{2\delta_{20}}{3^2}+\frac{2\delta_{21}}{5^2}+\frac{2\delta_{22}}{7^2}+\frac{2\delta_{23}}{9^2}+\frac{2\delta_{24}}{11^2}\ . \quad (25.19)$$

Twentieth equation, equilibrium at point a_{19}:

$$\frac{2DR^2}{(R+38r)^2}=-\frac{2\delta_1}{37^2}-\frac{2\delta_2}{35^2}-\frac{2\delta_3}{33^2}-\frac{2\delta_4}{31^2}-\frac{2\delta_5}{29^2}-\frac{2\delta_6}{27^2}-\frac{2\delta_7}{25^2}-\frac{2\delta_8}{23^2}-\frac{2\delta_9}{21^2}-\frac{2\delta_{10}}{19^2}-\frac{2\delta_{11}}{17^2}-\frac{2\delta_{12}}{15^2}$$

$$-\frac{2\delta_{13}}{13^2}-\frac{2\delta_{14}}{11^2}-\frac{2\delta_{15}}{9^2}-\frac{2\delta_{16}}{7^2}-\frac{2\delta_{17}}{5^2}-\frac{2\delta_{18}}{3^2}-\delta_{19}+\delta_{20}+\frac{2\delta_{21}}{3^2}+\frac{2\delta_{22}}{5^2}+\frac{2\delta_{23}}{7^2}+\frac{2\delta_{24}}{9^2}\ . \quad (25.20)$$

Twenty-first equation, equilibrium at point a_{20}:

$$\frac{2DR^2}{(R+40r)^2}=-\frac{2\delta_1}{39^2}-\frac{2\delta_2}{37^2}-\frac{2\delta_3}{35^2}-\frac{2\delta_4}{33^2}-\frac{2\delta_5}{31^2}-\frac{2\delta_6}{29^2}-\frac{2\delta_7}{27^2}-\frac{2\delta_8}{25^2}-\frac{2\delta_9}{23^2}-\frac{2\delta_{10}}{21^2}-\frac{2\delta_{11}}{19^2}-\frac{2\delta_{12}}{17^2}$$

$$-\frac{2\delta_{13}}{15^2}-\frac{2\delta_{14}}{13^2}-\frac{2\delta_{15}}{11^2}-\frac{2\delta_{16}}{9^2}-\frac{2\delta_{17}}{7^2}-\frac{2\delta_{18}}{5^2}-\frac{2\delta_{19}}{3^2}-\delta_{20}+\delta_{21}+\frac{2\delta_{22}}{3^2}+\frac{2\delta_{23}}{5^2}+\frac{2\delta_{24}}{7^2}\ . \quad (25.21)$$

Twenty-second equation, equilibrium at point a_{21}:

$$\frac{2DR^2}{(R+42r)^2}=-\frac{2\delta_1}{41^2}-\frac{2\delta_2}{39^2}-\frac{2\delta_3}{37^2}-\frac{2\delta_4}{35^2}-\frac{2\delta_5}{33^2}-\frac{2\delta_6}{31^2}-\frac{2\delta_7}{29^2}-\frac{2\delta_8}{27^2}-\frac{2\delta_9}{25^2}-\frac{2\delta_{10}}{23^2}-\frac{2\delta_{11}}{21^2}-\frac{2\delta_{12}}{19^2}$$

$$-\frac{2\delta_{13}}{17^2}-\frac{2\delta_{14}}{15^2}-\frac{2\delta_{15}}{13^2}-\frac{2\delta_{16}}{11^2}-\frac{2\delta_{17}}{9^2}-\frac{2\delta_{18}}{7^2}-\frac{2\delta_{19}}{5^2}-\frac{2\delta_{20}}{3^2}-\delta_{21}+\delta_{22}+\frac{2\delta_{23}}{3^2}+\frac{2\delta_{24}}{5^2}\ . \quad (25.22)$$

Twenty-third equation, equilibrium at point a_{22}:

$$\frac{2DR^2}{(R+44r)^2}=-\frac{2\delta_1}{43^2}-\frac{2\delta_2}{41^2}-\frac{2\delta_3}{39^2}-\frac{2\delta_4}{37^2}-\frac{2\delta_5}{35^2}-\frac{2\delta_6}{33^2}-\frac{2\delta_7}{31^2}-\frac{2\delta_8}{29^2}-\frac{2\delta_9}{27^2}-\frac{2\delta_{10}}{25^2}-\frac{2\delta_{11}}{23^2}-\frac{2\delta_{12}}{21^2}$$

$$-\frac{2\delta_{13}}{19^2}-\frac{2\delta_{14}}{17^2}-\frac{2\delta_{15}}{15^2}-\frac{2\delta_{16}}{13^2}-\frac{2\delta_{17}}{11^2}-\frac{2\delta_{18}}{9^2}-\frac{2\delta_{19}}{7^2}-\frac{2\delta_{20}}{5^2}-\frac{2\delta_{21}}{3^2}-\delta_{22}+\delta_{23}+\frac{2\delta_{24}}{3^2}\ . \quad (25.23)$$

Twenty-fourth equation, equilibrium at point a_{23}:

$$\frac{2DR^2}{(R+46r)^2}=-\frac{2\delta_1}{45^2}-\frac{2\delta_2}{43^2}-\frac{2\delta_3}{41^2}-\frac{2\delta_4}{39^2}-\frac{2\delta_5}{37^2}-\frac{2\delta_6}{35^2}-\frac{2\delta_7}{33^2}-\frac{2\delta_8}{31^2}-\frac{2\delta_9}{29^2}-\frac{2\delta_{10}}{27^2}-\frac{2\delta_{11}}{25^2}-\frac{2\delta_{12}}{23^2}$$

$$-\frac{2\delta_{13}}{21^2}-\frac{2\delta_{14}}{19^2}-\frac{2\delta_{15}}{17^2}-\frac{2\delta_{16}}{15^2}-\frac{2\delta_{17}}{13^2}-\frac{2\delta_{18}}{11^2}-\frac{2\delta_{19}}{9^2}-\frac{2\delta_{20}}{7^2}-\frac{2\delta_{21}}{5^2}-\frac{2\delta_{22}}{3^2}-\delta_{23}+\delta_{24}\ . \quad (25.24)$$

Chapter 26

Seventh Memoir on Electricity and Magnetism: On Magnetism

Coulomb[989]

26.1 I

In the six Memoirs which precede, printed successively since 1784 in the volumes of the Academy, I had mainly in view to submit to calculation the different phenomena of electricity. The thesis that I am presenting today aims to determine, by experiment and by theoretical analysis, the laws of magnetism.

It is necessary, for the operations that follow, to recall some results that I have already given, either in a Memoir on magnetic needles, printed in the ninth volume of the *Savants étrangers*,[990] or in a Memoir printed in our volume of 1785.[991]

In the first of these Memoirs, I proved, page 168,[992]

> that if a magnetic needle is suspended by its center of gravity, around which it can move freely in all directions, and if it is moved away from the magnetic meridian, it is always brought back there, by a constant force, whatever the angle of direction that the needle forms with the magnetic meridian.[993]

In this Memoir, I reported some experiments of different authors, from which I had deduced the above result; but *in 1785, volume of the Academy, page 603 and following*,[994] I confirmed it by means of my torsion balance, by an experiment which appears decisive, here is the experiment: we place in the magnetic balance, as it is described in this Memoir, a magnetic needle suspended horizontally by a copper wire, so that when the needle is pointed in the direction of the magnetic meridian, the angle of twist of the suspension wire is zero: the suspension wire is then twisted, by means of the micrometer described in the various

[989] [Coulomb, 1793] with Portuguese translation in [Assis, 2022]. This work was presented in 1789 to the French Academy of Sciences and published in 1793.

[990] [Coulomb, 1780]. This Memoir is translated in Chapter 5.

[991] [Coulomb, 1788d] with German translation in [Coulomb, 1890e] and with a partial English translation in [Coulomb, 1935b]. This Second Memoir is fully translated in Chapter 14.

[992] See page 56 in Subsection 5.0.2.

[993] A discussion and illustration of this first fundamental principle is found in Section 6.1.

[994] See page 251 on Subsection 14.5.1.

Memoirs which precede, and one observes for various angles of torsion, how much the needle moves away from its meridian, and we find that the force of torsion necessary to retain a needle at any [angular] distance from its meridian, is exactly proportional to the sine of the angle which the direction of the needle forms with this meridian, whence it obviously results that the resultant of the forces which bring the needle back to its meridian, is a constant quantity parallel to the meridian, which always passes through the same point of the needle.

I also proved, *ninth volume of the Savans Étrangers, page 170*,[995] that the magnetic forces of the globe of the Earth which solicit the different points of a magnetic needle, act in two opposite directions; that the part of the needle which, in our climates, points almost towards the [geographic] North, is drawn towards the North, while the austral part[996] of the needle is drawn towards the South; but however the needle was magnetized, or even for a half or any portion cut from the needle after being magnetized, the sum of the forces which urges the needle or portion we detached toward the North, is exactly equal to the sum of the forces which urge the needle or the cut portion towards the South of the magnetic meridian. I have deduced this result from several experiments, the simplest of which is that a needle weighed before and after having been magnetized has in both cases very exactly the same weight. Mr. Bouguer, in his *Voyage au Pérou, page 85 and following*,[997] had proved before me, by decisive experiments, this equality of opposite actions.

It is still a fact of experiment, as we have already said in the Memoirs cited, that magnetic needles are only susceptible to a certain degree of magnetism that they cannot exceed, however strong the magnets which are used successively to magnetize them.

Finally we have proved, *Memoirs of 1786*, that the attractive and repulsive actions of magnetic molecules were in direct proportion to the magnetic intensity and to the inverse of the square of their distances.[998]

All these facts being known, here are the principal objects which I have sought to determine in the Memoir which I am presenting.

1. The ratio of the directive forces[999] which bring back needles of different dimensions but of the same nature to the magnetic meridian when they are magnetized to saturation.[1000]

2. The magnetic intensity of each point of a needle.

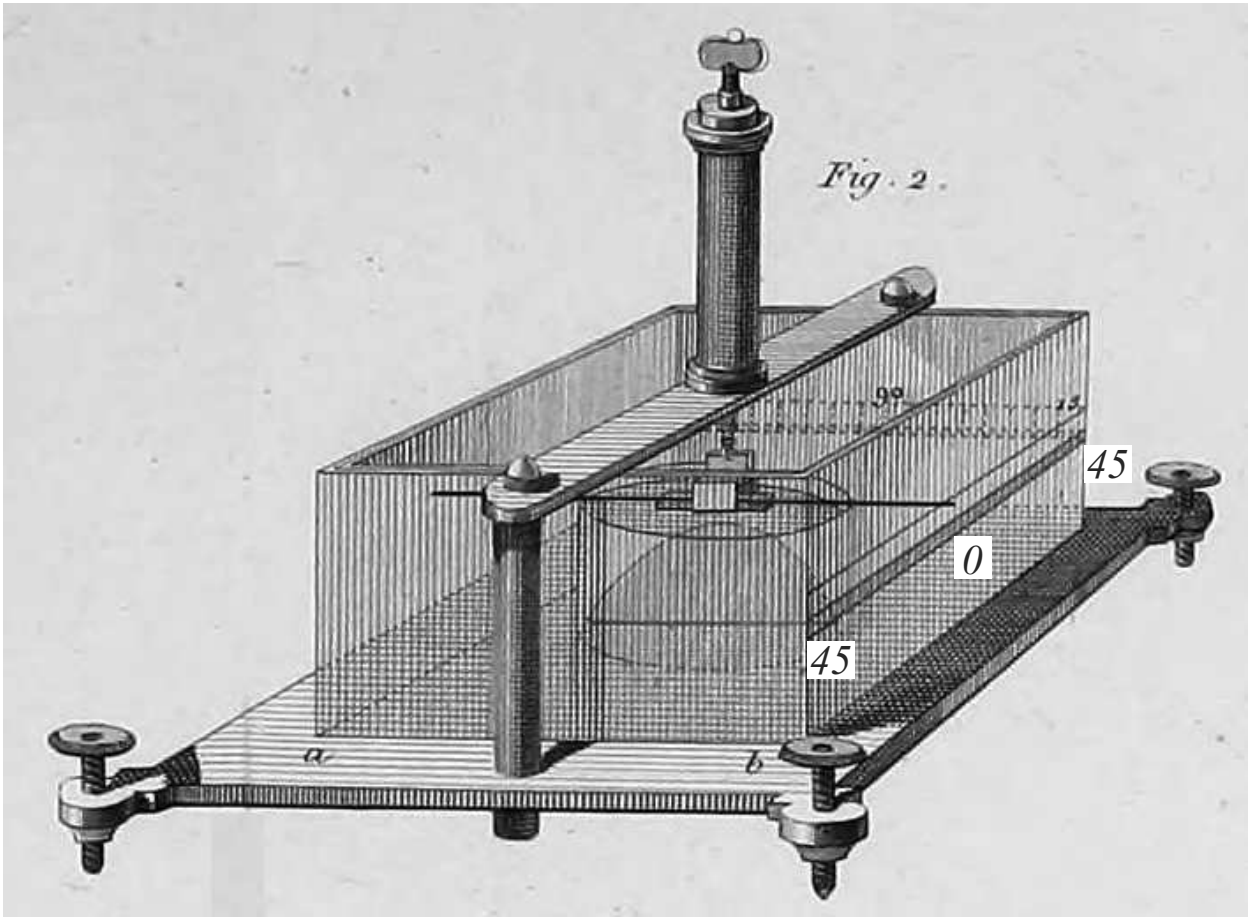
3. What limits must be placed on the hypotheses of attraction and repulsion of magnetic fluids[1001] so that the hypotheses fits with experiment.

4. The most advantageous practical means for magnetizing needles to saturation and for making artificial magnets of great strength as suggested by experiment and theory.

[995] [Coulomb, 1780, p. 170]. See page 57 of Subsection 5.0.3.
[996] In the original: *la partie australe*. See footnote 127 on page 56.
[997] Pierre Bouguer (1698-1758) was a French mathematician, physicist and astronomer. See [Bouguer, 1749].
[998] In the original: *les actions attractives et répulsives des molécules magnétiques étaient en raison directe de l'intensité magnétique et de l'inverse du carré de leurs distances*. That is, the attractive or repulsive force between magnetic particles is directly proportional to the product of the magnetic fluid intensities of these particles and inversely proportional to the square of the distance between them. See also Section 14.3.
[999] In the original: *forces directrices*. See footnote 286 on page 127.
[1000] That is, Coulomb wants to determine the magnetic torque exerted by the Earth on a cylindrical needle made of a certain material, magnetized to saturation, as a function of the length and thickness of the needle.
[1001] In the original: *fluides aimantaires*.

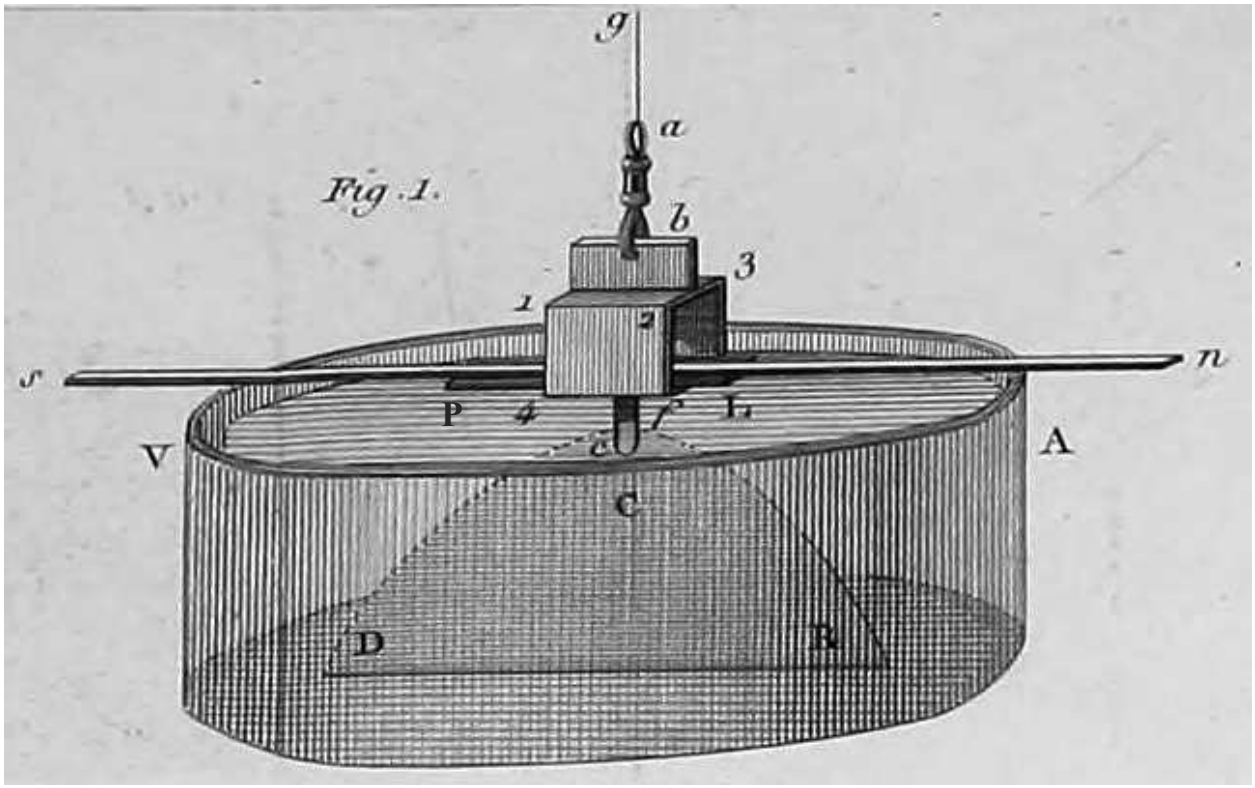
26.2 II

I used, *Figure 2*,[1002] in most of the experiments, a torsion balance exactly similar to the electric balance described in the various Memoirs that I have already published, *Volume of 1787*.



Only the shape of the support of the needle, *Figure 1*, is special as required by the new type of experiments for which it is intended.

[1002] This is one of Coulomb's magnetic balances. I added the numbers 0 and 45 that do not appear in his Figure. They represent the angles marked on the lateral strip.



In the drawing of this support, *Figure 1*, *ab*, represents the clamp which grasps by its upper part the [lower end of the] suspension wire *ag*; this wire, as we have said in the Memoirs on electricity, is grasped at its upper end by another clamp which forms part of the micrometer (*see the Volume of the Academy, 1785, page 569; 1787, page 421*);[1003] the clamp *ab* seizes at its lower extremity *b*, a stirrup, 1234, formed with a very light copper blade. In this stirrup we place a small plane of cardboard PL, covered, in its upper surface, with a coating of Spanish wax, on which we make an impression of the steel wire that we want to submit to the experiments,[1004] which makes it easy, in the successive tests, to always place the wire in the same place: under the middle of this stirrup, we weld at its upper extremity f, a copper wire ef, whose lower extremity e, is also soldered to a copper plane DCR, very wide and very light. This vertical plane DCR, is submerged in a vase VA, filled with water, so that the surface of the water is at least five or six lines [1.13 or 1.36 cm] above the vertex e of the plane. The resistance of the water against the plane is intended to promptly stop the oscillations of the needle *sn*; but it is necessary, as we have just said, that the plane be entirely immersed in the water, otherwise in the oscillations of the needle, the surface of the water, rising unevenly and adhering to the surface of the plane, could vary the magnetic direction of the needle.[1005,1006]

Figure 2 represents the device that we have just described, placed in the magnetic bal-

[1003] See pages 203 and 327.

[1004] I added in Figure 1 the letter P that does not appear in the original image.

[1005] [Note by Coulomb] In the volume of the Academy of 1785, I gave the description of a compass intended to observe the diurnal variations [of terrestrial magnetism]; in this Memoir I proposed to weld a small plane with the needle. The reasons exposed here indicate that it is necessary that this small plane is welded to a copper wire, which is in the same vertical as the wire of suspension, that it is necessary moreover that the plane is entirely submerged.

[1006] Coulomb is referring here to the Memoir of 1777 published in 1780 and which is translated in Chapter 5.

ance. This balance is positioned so that its side *ab* is directed along the magnetic meridian: the small band 45, *o*, 45, drawn on the side of the balance perpendicular to the magnetic meridian, is the tangent of a circle which would have its center at the wire of suspension, so that a vertical plane, passing through this wire of suspension and the point *o*, middle of the tangent, represents the magnetic meridian, the tangent, *o*, 45, is divided according to the degrees of the circle:[1007] to operate [the balance], one first places horizontally, in the stirrup *E*,[1008] a copper wire, and the micrometer being on the point *o*, one ensures that the torsion [of the suspension wire] being null, the copper wire is directed along the magnetic meridian. We have given in our Memoirs for 1787, methods which render this operation very easy; when the balance is thus arranged, the copper needle is replaced by a magnetic needle; then, by means of the torsion micrometer, we move this needle 20 to 30 degrees away from the meridian, and one observes the force of torsion necessary to retain the needle at a similar distance: when we then want to compare the directive force of this needle with that of another needle, we substitute this second for the preceding one, and we take care to move the second away from the magnetic meridian, precisely by as many degrees as we have moved away the first; it follows that the two needles, forming in the two experiments, the same angle with the magnetic meridian, the force of torsion will necessarily measure the *moment* of their restoring forces.[1009] When the angles of direction with the magnetic meridian are not the same in the two experiments, it is easy to evaluate them by calculation, according to the principles of the First Section.[1010]

The reader should be aware that to obtain precise results, it is necessary to proportion the force of torsion of the wires of suspension to the magnetic force[1011] of the needles, so that by moving the needles away at 30 degrees from their meridian, the force of torsion of the wires of suspension which retain the needle at this distance, is always at least 25 to 30 degrees: it is because of this that I have sometimes used silver wires and sometimes harpsichord[1012] copper wires of different sizes as they are found in commerce, and sometimes silver wires: in the needles of a very weak magnetism, where the silver wire would have given me only 2 or 3 degrees of torsion, I suspended the needles by a very fine silk thread, and counting the number of oscillations they made in a given time, I calculated their directing force, by means of the formulas of the oscillatory movement which I detailed in the *Memoir quoted in the ninth volume of the Savans étrangers.*[1013]

[1007] Points 45 and *o* are not marked in Coulomb's original Figure. I introduced these points in the Figure by representing them by the numbers 45 and 0, which indicate the angles 45° and 0°. See also footnote 745 on page 331.

[1008] The letter *E* does not appear in the original Figure 2. Coulomb is referring here to the stirrup 1234 of Figure 1.

[1009] In the original: "la force de torsion mesurera nécessairement les *momentum* de leurs forces directrices". The word "momentum" here can be understood as torque or moment. In the title of the next Section Coulomb will explicitly use the expression "momentum magnétique", see Section 26.4. One of Coulomb's objectives in this work is to determine the magnetic torque exerted by the Earth on a magnetized cylindrical needle, as a function of the length and thickness of the needle. Coulomb's experimental procedure was illustrated in Section 16.3.

[1010] That is, even when two magnetized needles are at different angles to the magnetic meridian, it is easy to compare their magnetic moments using the principles presented in the First Section.

[1011] In the original: *force aimantaire*.

[1012] In the original: *clavecin*. See footnote 341 on page 155. Coulomb is referring here to the suspension wire.

[1013] [Coulomb, 1780]. This work is translated in Chapter 5.

26.3 III

The ratio of the unequal forces of torsion of two different wires of suspension is easy to determine, either by the formulas and the experiments which we have given, Volume of the Academy of 1784,[1014] or more simply by means of the torsion micrometer by suspending successively the same magnetized needle in a horizontal position on the two wires; because if, for the two suspensions, we displace the magnetized needle by the same [angular] distance from its meridian, [the ratio of] the angles of torsion necessary to twist the two wires, necessarily measures the ratio of their forces of torsion, since they both retain the same magnetized needle by this degree of torsion at the same angular distance from its meridian.

In the experiments that follow, I mainly used, for the suspensions, a copper wire numbered 12, the thinnest that we find on the market, and a silver wire much finer and whose twisting force, at the same length, is only a thirtieth part of the copper wire; but all the experiments, of whatever kind of suspension we have used, have been compared by analysis to those which would have taken place with the same copper wire numbered 12 in commerce, 14 inches in length (37.89 cm): this wire weighs 0.83 grains with a length of one foot.[1015,1016]

26.4 IV. *Comparison of the Magnetic Moments of Different Steel Needles, of the Same Diameter and of Different Lengths*

26.4.1 First Experiment

*Steel wire weighing 38 grains per foot (*6.21 *g per meter).*

The steel wire which was used in this experiment, as well as in all those which will follow, is the steel wire from England, passed through the die,[1017] of a diameter, consequently, equal throughout its length.

The needle magnetized to saturation is placed, in the suspension stirrup, along the imprint directed along the magnetic meridian. One then twists, in all the tests, the wire of suspension, until the direction of the needle makes an angle of 30 degrees with the magnetic meridian, and one observes the angle of torsion: then cutting the steel needle successively at different lengths, and magnetizing the needle each time to saturation, one observes for each needle, the angle of torsion which retains them at 30 degrees from their meridian.[1018]

We used in this experiment, for the suspension, a very fine silver wire, the twisting force of which was only a thirtieth of the copper wire numbered 12; but by dividing by 30 the angle of torsion found by experiment, the results have been reduced to the numbers of degrees which would have been observed if the copper wire numbered 12 had been used. It is good to warn the reader again, that this scaling reduction took place in all the experiments which

[1014] [Coulomb, 1787]. This work is translated in Chapter 7.
[1015] [Note by Potier] One degree of torsion corresponds to a couple of 0.504 (C.G.S.).
[1016] The linear density of this copper wire is 0.83 grains/foot $= (0.83 \times 0.05311\ g)/(0.3248\ m) = 0.136\ g/m$. According to Potier, a torque of 0.504 $dyn \cdot cm$ due to this pair of forces is required to rotate by an angle of 1° the copper wire 37.89 cm long. See also footnotes 365 and 366.
[1017] See footnote 389 on page 174.
[1018] Coulomb's experimental procedure was illustrated in Section 16.3.

will follow, and we had:

First trial. The length of the magnetized steel wire, being 12 inches (32.48 cm), it required, to retain it at 30 degrees from its meridian, a twisting force of ..... 11.50°.
Second trial. With a 9 inches long steel wire ..... 8.50°.
Third trial. With a 6 inches long steel wire ..... 5.30°.
Fourth trial. With a 3 inches long steel wire ..... 2.30°.
Fifth trial. With a 2 inches long steel wire ..... 1.30°.
Sixth trial. With a 1 inch long steel wire ..... 0.35°.
Seventh trial. With a 1/2 inch long steel wire ..... 0.07°.
Eighth trial. With a 1/4 inch long steel wire ..... 0.02°.

26.5 V. Second Experiment

*Steel wire, weighing 865 grains per foot of length (*141.52 *g per meter), or about 2 lines in diameter (0.45 cm).*

First trial. The length of the steel wire, magnetized to saturation, being 18 inches (48.72 cm), it took, to retain it at 30 degrees from its meridian, a twisting force of ..... 288.00°.
Second trial. For a length of 12 inches ..... 172.00°.
Third trial. For a length of 9 inches ..... 115.00°.
Fourth trial. For a length of 6 inches ..... 59.00°.
Fifth trial. For a length of $4\frac{1}{2}$ inches ..... 34.00°.
Sixth trial. For a length of 3 inches ..... 13.00°.
Seventh trial. For a length of $1\frac{1}{2}$ inch ..... 3.00°.
Eighth trial. For a length of 1 inch ..... 1.46°.
Ninth trial. For a length of 1/2 inch ..... 0.32°.

26.6 VI. *Result of These Two Experiments*

In the first experiment, it was found that by displacing the steel needle, whose 12 inches in length weighs 38 grains, 30 degrees from the magnetic meridian, the force of torsion, which brought it back towards the meridian, was measured to be 11.50 degrees; for a length of 9 inches, this twisting force was 8.50 degrees; thus, in these two tests, the decrease in the directive force was 3 degrees or one degree per inch. Continuing this operation, we find that from 9 inches to 6 inches the diminution of the directing force was 3.2 degrees, still very close to one degree per inch; from 6 inches to 3 inches, the decrease was again 3 degrees; and from 3 inches to 1 inch it was two degrees, that is to say, still one degree per inch of decrease, whence it is easy to conclude that until the needle weighed 38 grains, or reduced to one inch in length, we find a constant ratio between the quantities by which the needles are diminished and those by which the directing forces diminish; but by comparing the lengths of the same needle below one inch, it would appear that for needles of length 1 inch to a quarter of an inch the *moment* [of the directing force] varies approximately as the square of the lengths of the needles.

In the second experiment, we find a result similar to that of the first. Because, comparing in this experiment the first trial with the second, we find that a diminution of 6 inches in

the needle 18 inches in length, produces in the *moment* of the directing force, a diminution of 116 degrees, or 19 one-third of a degree per inch.

By then reducing this same needle from 12 inches to 6 inches, we will still find in the *moment* a decrease of 19 degrees per inch: but from 6 inches in length to 4 and a half inches, the *moment* of the directing force decreases only 16.6 degrees per inch. Below 4 and a half inches to half an inch, it would appear that the *moment* varies approximately as the square of the lengths of the needles; so that we can, without great error, suppose in this second experiment, that the *moment* of steel needles 2 lines in diameter (0.45 cm), from 0 inch to 5 inches in length (13.53 cm), varies approximately as the square of their lengths; and that for a greater length of the needle, the increases in the *moment* are roughly proportional to the increases in the lengths. I say approximately, because when the needles are magnetized to saturation, we find that the increments of the moments are almost always a little greater than the increments of the lengths, but this variation is generally too slight to be detected by experiments of the kind of the two which precede.

26.7 VII. *Of the Moment of the Directive Force of the Needles, Relative to Their Diameter*

We have just seen the path followed by the *moment* of the directing forces of two needles of different lengths, but of the same diameter: we are now going to try to determine the ratios of the *moment* of the directing force of two needles magnetized to saturation, of different diameters: but I must begin by warning the reader that in the course of the experiments, I soon recognized that it was almost impossible to obtain two steel needles of different diameters, which had exactly the same degree of spring,[1019] and which were of a homogeneous nature: thus, to obtain the laws of magnetism in the needles of different diameters, I was obliged to form bundles of very fine needles drawn from the same wire. What greatly facilitated this operation was that by twisting around its axis an iron wire of approximately a half-line diameter [0.113 cm], such as we find in commerce, I saw that by this torsion it work hardened and [increased its] spring,[1020] and that it was susceptible of almost the same degree of magnetism, as a steel wire of the same diameter: in accord with this observation, I chose a very pure iron wire, such as it leaves the die[1021] before being annealed; it was about 120 feet long [39 m]; I cut it into different parts, which I twisted about their axes while holding them in tension in order to straighten them; I formed bundles of them of different diameters and of different lengths, which I magnetized to saturation. Then placing these bundles in the magnetic balance, we find from a great number of experiments, some of which we will describe, that for two needles of the same material and whose dimensions are homologous,[1022] the *moment* of the directing forces are between them like the cube of the homologous dimensions. If, for example, I take a [cylindrical] needle one line in diameter and 6 inches in length, and another needle 2 lines in diameter and 12 inches in length, whose corresponding dimensions are, therefore, as 1 : 2, the magnetic moments of these two needles magnetized, one and the other to saturation, will be between them as 1 is to 8, [that is, in the] ratio of the cubes of their homologous dimensions.

[1019] In the original: *degré de ressort*. This expression can also be translated as degree of elasticity.
[1020] In the original: *il prenoit de l'écrouissement et du ressort*. See footnote 375 on page 165.
[1021] In the original: *filière*. See footnote 389 on page 174.
[1022] In the original: *homologues*.

26.8 VIII. Third Experiment

We twisted around their axes, 36 wires of one foot length (32.48 cm), weighing 48 grains each one (2.55 g); we formed a bundle of these 36 needles joined together and bound with thread; this bundle was magnetized to saturation. Then suspending it horizontally in the stirrup of the magnetic balance, it was found that a torsion angle of 342 degrees was required to retain this bundle at 30 degrees from the magnetic meridian.

26.9 IX. Fourth Experiment

A second bundle was formed with 9 needles, each 6 inches in length (16.24 cm), but of the same nature and the same diameter as those which were used in the preceding experiment, it was found that to retain this bundle at 30 degrees of the magnetic meridian, a twisting force of 42 degrees was needed.

26.10 X. *Result of the Two Previous Experiments*

In the two previous experiments, an iron wire was used, such as it leaves the die, the purest that we could get; all the needles were cut from the same piece, so we are sure that they are of the same nature and of the same diameter, but the two bundles having their homologous sides proportional, in the ratio of 2 to 1, the diameters were as the square root of the number of needles: thus the cubes of the diameters are between them like 8 : 1; but we have just found that the *moment* of the directing forces of the two bundles, are [to one another] like 342 : 42 :: 8.14 : 1.00, a ratio which differs very little from that of 8 to 1, or from the [ratio of the] mass of the two bodies: the two preceding experiments were repeated, with bundles whose homologous dimensions were like 3 to 1, and like 4 to 1; and we have always found the same result, that is to say, the directing forces proportional to the cubes of the diameters of the two bundles.[1023]

26.11 XI. *Remark*

The previous result — from which we have learned that the moments of the directing force of the two needles whose dimensions are homologous were as the cube of their dimensions — when joined to the first result for needles of the same diameter but different lengths — which has showed us, if the needles had lengths 40 to 50 times their diameters, that the moments of the directing force increased in proportion to their increase in lengths — can immediately give the magnetic *moment* of all steel wires, of the same nature and at the same degree of hardness, of any diameter and of any length, provided that we know the *moment* of the directing force of a single of these needles, as well as the increase of its *moment*, relatively to the increases of its length.

Suppose, for example, that we wish to determine the *moment* of the directing force of a needle 48 inches in length and 6 lines in diameter, but of the same steel and of the same degree of temper as that of the second experiment, which had 2 lines in diameter; the question

[1023] That is, the magnetic torque exerted by the Earth on a needle magnetized to saturation is proportional to the volume of the needle.

consists in finding in the second experiment, the length of a needle 2 lines in diameter, which would have dimensions homologous with that of 48 inches in length and 6 lines in diameter; we would find that the needle of 2 lines in diameter would be 16 inches in length; but we find in the second experiment, that the magnetic *moment* of a needle 2 lines in diameter and 16 inches in length, would measure 250 degrees, and since the homologous dimensions of the two needles that we want compare, are as 3 is to 1, their cubes are :: 27 to 1, so that the *moment* of the directing force of the needle 6 lines in diameter and 48 inches in length, would be represented by $250 \times 27 = 6750$ degrees.

26.12 XII. *Of the Action of the Different Points of a Magnetized Needle, Depending on Whether These Points Are More or Less Distant from the End of the Needle*

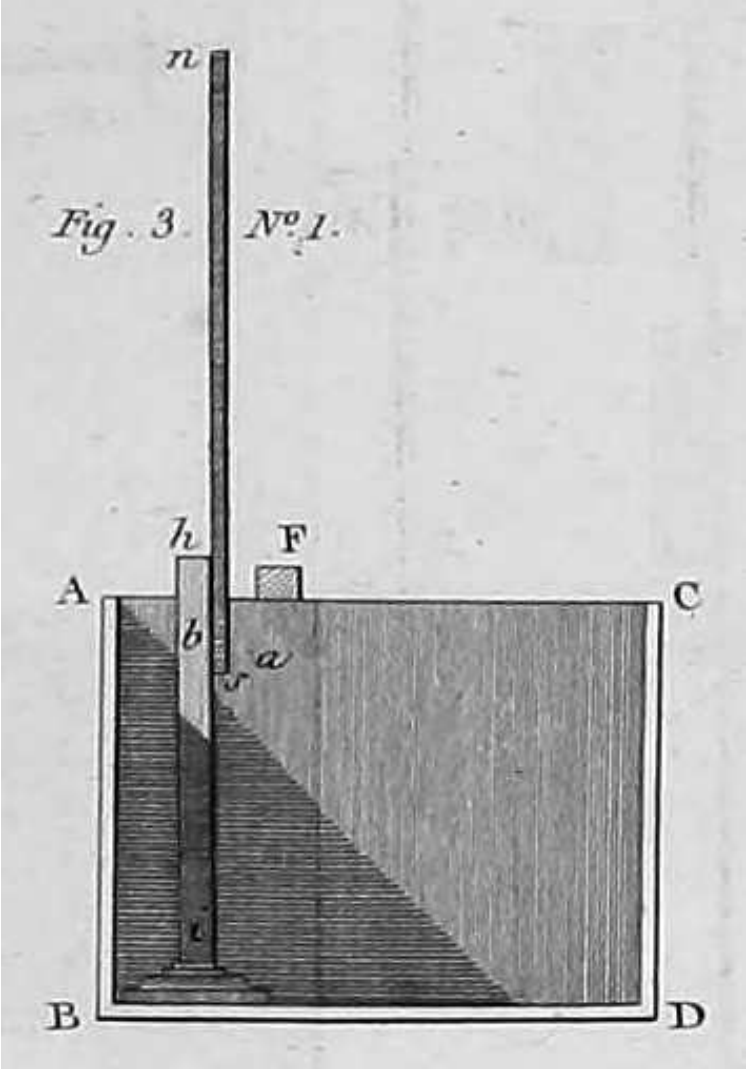
The preceding experiments, and those which we presented in 1785, in the Memoirs of the Academy,[1024] suffice to prove that in steel wires, the diameter of which is not significant in relation to the length, evidence of action of the magnetic fluid is concentrated towards the extremities: the first and second experiment even prove, as we will see presently, that whatever the length of the steel wires, provided that they have at least 40 to 50 times the length of their diameter, the curve which represents the magnetic action of each point of a needle, is the same, whatever the length of the steel wire, and that it extends approximately from the end of the needles, up to a distance from these ends, equal to 25 diameters; beyond that, to the middle of the needle, the action is very small, that is, the ordinates of the curve which would express this action almost merge with the axis of the needle.

I tried to confirm this result by direct experiments, by determining the law which the magnetic action of the different points of a wire follows when the wire is magnetized to saturation,[1025] from its end to the middle of the wire:[1026] we can perceive that for the success of such an experiment, it was necessary to arrange the tests in such a way that by presenting a steel wire to a very short needle, there was only a very small part of the wire whose action on the needle was considerable, in order to be able to conclude from it the magnetic density of the point of the wire presented to the needle.

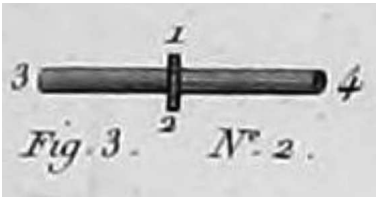
26.13 XIII

In a box, the section of which is represented in *ABCD*, *Figure 3, No. 1*, I suspended from the crosspiece F a small needle of steel, 2 lines in length (0.45 cm) and a quarter of line of diameter [0.0565 cm].[1027]

[1024] [Coulomb, 1787]. This Memoir is translated in Chapter 7.
[1025] For a lapse in the original text, *d'une aiguille aimantee à saturation* appears here. In the next Sections Coulomb will study the magnetic density along a *wire* magnetized to saturation.
[1026] In the original text it appears here *jusqu'au milieu de l'aiguille*.
[1027] A better view of the small needle a can be seen in Figure 3, Number 3, on page 465.



Below this needle, I attached at right angle, with a little wax, a small cylinder of red copper, 2 lines in diameter [0.45 cm] and one inch in length (2.71 cm), the whole was suspended horizontally by a silk thread one inch in length, as it emerges from the cocoon; I have proved elsewhere that the torsion force of such a thread was almost nil. The copper needle and cylinder are shown in plan in *Figure 3, No. 2*; [where] 1, 2 represents the steel needle,[1028] and 3, 4 the copper cylinder.



One then places fixedly in the box, *Figure 3, No. 1*, at 3 or 4 lines from the needle a, a vertical ruler hi; along this ruler, a steel wire, magnetized to saturation, of one or two lines in diameter [0.226 or 0.452 cm] is run vertically along the magnetic meridian of the needle

[1028]For a lapse in the original text it appears here that 1, 2 represents the steel wire.

a, so that the point b of the axis of this wire is only two or three lines away from it[1029]

When we want to determine the magnetic action of the point b, we first make the needle a oscillate, before presenting the steel wire ns to it;[1030] we count the number of oscillations that this needle makes, by virtue of the action of the globe of the Earth alone; then we place the end s of the magnetized steel wire at b, at the height of the needle a; we count in this position the number of oscillations which the needle has made in 60";[1031] we successively lower the end s of the steel wire, from six lines to six lines, and each time we count the number of oscillations that the needle has made in 60".

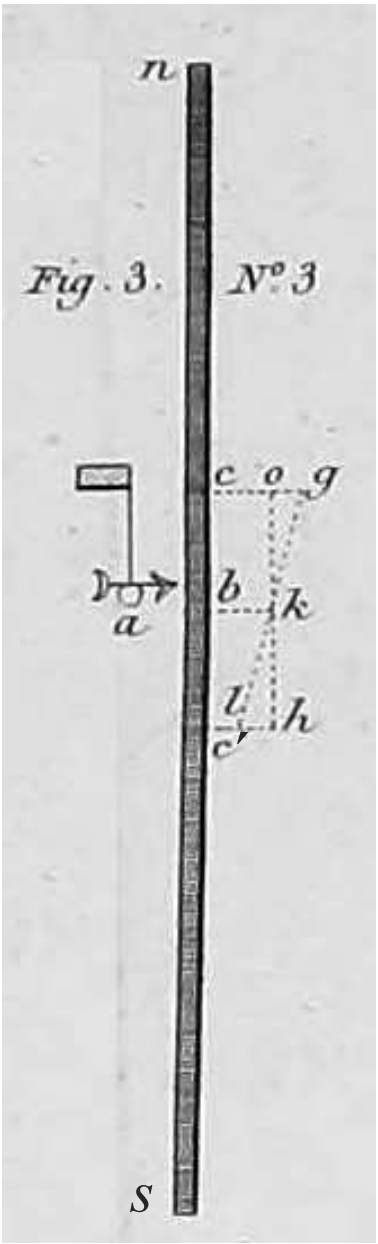
26.14 XIV

From this operation, it follows that if the needle a always remained in the same state of magnetism,[1032] the point b of the steel wire being only three lines away from this needle, there would be in the wire only the points which are close to b, whose action would be considerable on the needle a, since the action of the other points decomposed along a horizontal direction, decreases at equal density, as the [inverse] square ratio of the distances and the obliqueness of their action: thus by successively making the different points b of the wire[1033] slide along the ruler hi, it would result that the action of the different points b of the wire,[1034] would be approximately proportional to the square of the number of oscillations made by the needle a, in a constant time.[1035]

26.15 XV

Figure 3, No. 3 can be used to demonstrate the previous assertion.

[1029]That is, the point b on the wire axis is located at this distance from the center of the magnetized needle, see Figure 3, Number 3, on page 465.
[1030]I added the letter s in Figure 3, Number 3, see page 465. This letter does not appear in the original image.
[1031]That is, in 60 seconds.
[1032]That is, if it maintains the same magnetization throughout the entire experiment.
[1033]For a lapse in the original text "aiguille" appears here instead of "fil". Certainly Coulomb was referring to the points b of the steel wire ns of Figure 3, Number 1.
[1034]Once more we have here "aiguille" instead of "fil" in the original text.
[1035]See Section 16.1 on page 267.



[In *Figure 3, No. 3,*] ns represents the steel wire whose axis at b is placed 3 or 4 lines from the middle of the small needle a; if we take above and below the point b, two portions of wire bc and bc', very small, relatively to the total length of the wire, the magnetic density of this portion cc' can be, without noticeable error, represented by a straight line gkl; so that gc will be the density of point c; kb, that of point b; and lc', that of the point c': if we now draw through the point k, a line okh, parallel to the axis of the steel wire ns, the triangle gko, being equal to the triangle khl, it follows that the action of the portion cc' of the steel wire ns on the needle a, being decomposed in a horizontal direction, is the same as if the magnetic density had been uniform from c to c', and equal to bk, which represents the density of the center b. We shall see, however, by the experiments which follow, that the results found by the process which we have just indicated, require a correction, because the magnetic state of a needle a, whose dimensions are very small, and such as those of our

experiment, change as the points b presented to it are more or less magnetized.[1036]

26.16 XVI. Fifth Experiment

Steel wire 2 lines in diameter (0.45 cm) and 27 inches in length (73.08 cm).

We took a wire of excellent steel, 2 lines in diameter and 27 inches in length, of the same size and nature as that of our second experiment; it was magnetized to saturation by the method which we will prescribe at the end of this Memoir. Having placed it, as indicated in the two preceding Sections and by *Figure 3*, 3 lines away [0.678 cm] from the small needle a, which is 2 lines long and a quarter of a line in diameter, we made it slide vertically from 6 lines to 6 lines, observing each time the number of oscillations of the needle a.

First trial. The needle a, before the steel wire is presented to it, makes approximately one oscillation in 60".[1037]

Second trial. By placing the end s of the steel wire, at the level of the needle a, this needle makes 64 oscillations in 60".

Third trial. The end s lowered by 6 lines, the needle a makes 58 oscillations in 60".

Fourth trial. The end s lowered by 1 inch, the needle a makes 44 oscillations in 60".

Fifth trial. The end s lowered by 2 inches, the needle a makes 18 oscillations in 60".

Sixth trial. The end s lowered by 3 inches, the needle a makes 12 oscillations in 60".

Seventh trial. The end s lowered by 4 inches and a half, the needle a makes in 60" one or two oscillations. It is the same until the end s of the steel wire has been lowered, to a little more than 22 inches, that is to say, to 4 inches and a half (12.18 cm) from the other end n; in this case the needle a turns its poles by changing position end for end, and it gives towards this second end and in the corresponding points, approximately the same number of oscillations as at the other end.

26.17 XVII. Sixth Experiment

Steel wire 2 lines in diameter [0.452 cm] and 10 inches in length (27.07 cm).

By presenting to the needle a, at the same distance as in the preceding experiment, a steel wire of the same nature and of the same diameter, but only 10 inches in length, we find that the first three inches from each end of the 10-inch wire, give almost exactly the same action as the last three inches from the ends of the 27-inch wire, detailed in the previous experiment.

26.18 XVIII. Seventh Experiment

Wire 5 inches in length (13.53 cm) and 2 lines in diameter [0.452 cm].

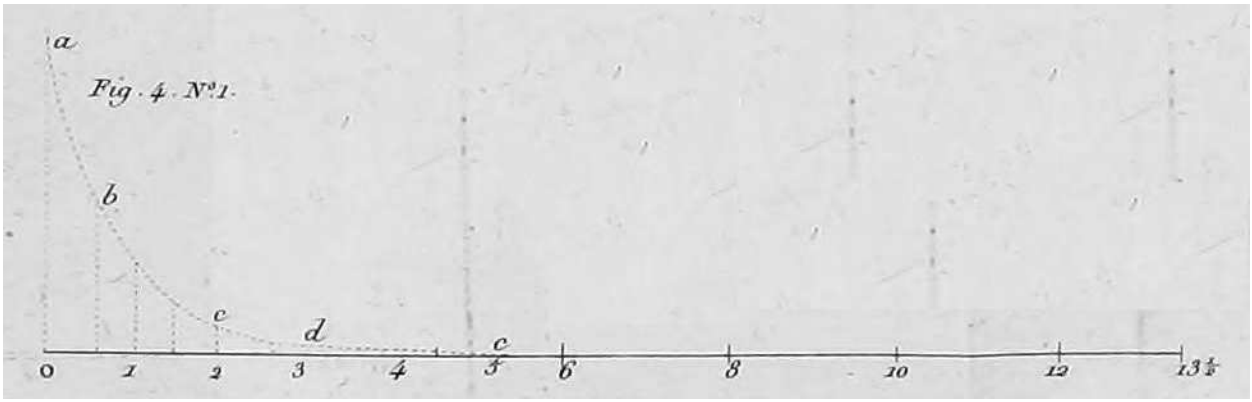
[1036] That is, Coulomb will show that the magnetization of the needle can be affected by the presence of the magnetized wire ns placed in the vicinity of the needle.

[1037] That is, in 60 seconds.

Finally, using a steel wire 5 inches in length, but of the same diameter as the preceding one, we still find at the ends of these wires, and even up to five or six lines [1.13 or 1.356 cm] from these ends, very nearly the same degrees of action as at the extremity of the wires[1038] of the two preceding experiments.

26.19 XIX. *First Remark*

The action which causes the needle to oscillate is measured as we know by the square of the number of oscillations made at the same time;[1039] according to this consideration, I have constructed, *Figure 4, [Number 1]*, taking the square of the number of oscillations, the curve *abcde*, which represents the geometric locus of the densities or magnetic actions of all the points of the half of a needle[1040] 27 inches long [73.1 cm] and 2 lines in diameter [0.45 cm].



In this *Figure*, [the segment between] 0 and $13\frac{1}{2}$ represents half the length of the needle, and the ordinates represent the magnetic densities: these ordinates decrease, as we see, rapidly, and are almost zero towards the fifth inch; from this point the curve of the densities merges with the axis up to the twenty-second inch, and over the five inches from the other extremity, they follow nearly the same law, but in a contrary direction; so that if the first extremity has a positive density, or whose action, on a pole of the same nature, is repulsive, that of the other extremity on the same pole will be attractive: in *Figure 4, [Number 1]*, we have doubled, at the end of the needle at *o*, the number which represents the square of the oscillations; it is easy to see, from the method of Section 26.15, that the true value of this density must be still greater, since at this point, by the position of the needle, the point *b* being *Figure 3, No. 1*, the end of the wire,[1041] there is action only on one side of *b*, and not on both sides, as [it happens] in all the other trials; moreover, the density decreases from the point *b*, when *b* is the end of the wire; whereas, in order to be able to compare the result of the square of the oscillations in this case with the other tests, it would be necessary, according to the observations of Section 26.14, that the density should be uniform, because there is no compensation of one side by the other.

[1038] For a lapse in the original text, "of the needles" appears here instead of "of the wires".
[1039] See Section 16.1 on page 267.
[1040] From now on, when Coulomb is using the word "needle", it should be understood as being the magnetized wire of which Coulomb studied how its magnetic density varied along its length.
[1041] For a lapse in the original text "aiguille" appears here instead of "fil". Coulomb is discussing here the situation where the point *b*, which is at the same height as the small needle *a*, corresponds to the lower end of the vertical magnetized wire *ns* of Figure 3, Number 1.

26.20 XX. *Second Remark*

From the sixth experiment, we will draw this interesting consequence, it is that the curve, *Figure 4, [Number 1]*, which describes the density of the magnetic action at each point of the wire near its ends, is exactly the same, whatever the length of the wires, provided they are more than 8 or 9 inches in length [21.66 or 24.36 cm]: from this we cannot yet conclude that when we measure, relatively to the magnetic meridian, the *moment* of the directing force of different steel needles,[1042] of different lengths, but of the same nature and of the same size, that these *moments* must differ between them by a quantity proportional to the decreases in the lengths of the needles; for, since the *moment* of the directing force of each needle, will be equal to the area which represents the sum of the magnetic densities, multiplied by the distance from the center of gravity of this area to the middle of the wire, which is the point of suspension, and moreover the area of the densities, as well as its dimensions are the same, whatever the lengths of the needles, it is clear that the *moment* of the directing force of the globe of the Earth, for each needle, will be represented by this area, multiplied by the distance from its center of gravity to the middle of the needle; but since the distance from this center of gravity to the end of the needle is constant, whatever the length of the needles, it follows that the *moment* of the needles will be measured by a constant quantity, which expresses the area of the densities multiplied by the length of the needle, minus the constant quantity which represents the distance from the center of gravity of the area of densities, to the tip of the needle. This result is exactly in line with what we found in the first and second experiments, when seeking the magnetic *moment* of several needles of the same diameter and of different lengths; because, we have seen, according to these two experiments, that the moments of the directing force increase in proportion to the increase in the lengths of the needles; which must necessarily take place, since by cutting a needle, and magnetizing it to saturation, the curve which represents the area of the magnetic densities, being the same for needles of different lengths, the center of gravity of this area approaches the middle of the needle by half the part of the length that was cut off, and therefore the decrease of the *moment* is proportional to that part cut off.

26.21 XXI. [*Third Remark*]

According to the preceding remark, it is easy, by means of the first and second experiments, which served us to know the law of the *moment* of the directing force of different needles of the same nature and of the same size, but of different lengths, to determine the place of the center of action, or, what comes to the same thing, the center of gravity of the curve of the magnetic densities of these needles.[1043]

Let us first take as an example the needle tested in the first experiment. This needle weighs 38 grains per foot of length; we found, Section 26.4, that when this needle was 12 inches in length, it required a twisting force of 11.50 degrees to hold it at 30 degrees from its magnetic meridian and when it was only 5 inches in length, it took a force of 2.30 [degrees] to hold it at the same [angular] distance. But, according to the preceding remarks, the area of the densities, *Figure 4, [Number 1]*, is the same for all the lengths of needle of the same thickness, thus the center of gravity of this area is in the two experiments at the same

[1042] Again these "different steel needles" should be understood to be "different steel wires".
[1043] In the original: "le centre de gravité de la courbe des densités magnétiques de ces aiguilles".

distance from the ends of the needle.

Let A be the area of this surface, let x be the distance from the center of gravity of this area to the end of the needle, naming l half the length of the needle, we will have for its magnetic *moment* the quantity[1044]

$$2A(l-x)\sin 30^\circ$$

and taking the two quantities found by the first experiment, for the *moment* of the directing forces of the two needles 12 inches and 3 inches in length; we will have the following two equations:

$$2A(6-x)\sin 30^\circ = 11.50$$

and

$$2A(1.5-x)\sin 30^\circ = 2.30\ .$$

Division of one by the other gives:

$$x = 0.36\ \textit{inches}\ (0.974\ \textit{cm})\ .$$

By doing the same operation for the steel needle of the second experiment, which weighs 865 grains per foot of length, we will derive its distance from the center of gravity of the area of the densities to the end of the needle, [namely,] $x' = 1.51$ inches (4.088 cm). In these two experiments, the diameters of the two steel wires are between them like the [square] roots of the weights, so they are in the ratio

$$:: \sqrt{865} : \sqrt{38} :: 4.8 : 1.0\ ;$$

but we find [the ratio of] the distance from the center of gravity to the ends of the needles to be :: 1.510 : 0.36 :: 4.2 : 1.0.[1045] Thus it would appear, according to these results, that the distances of the center of magnetic action[1046] of two needles, to the end of these needles, are approximately in the ratio, like the diameters of these needles.

26.22 XXII. *Fourth Remark*

A difficulty presents itself here which seems to deserve some attention; we have just seen that the curve, *Figure 4, No. 1*, which represents the magnetic density, near the end of the steel wire, 2 lines in diameter, has its center of gravity, at about 1.5 inches from its end. We saw, fifth experiment, that the magnetic density of this same needle, extends, in a very

[1044] In the original text, this equation appeared as follows, [Coulomb, 1793, p. 476]:

$$2A\sin 30^d(l-x)\ .$$

[1045] That is, the ratio between the distance from the center of gravity to the ends of the needles $= \frac{1.510}{0.36} = \frac{4.2}{1.0}$. For a lapse in the original text, ":: 1.510 : 36 :: 4.2 : 1.0" appeared here instead of ":: 1.510 : 0.36 :: 4.2 : 1.0".

[1046] In the original: *centre d'action magnétique*.

sensible way, only up to five inches, approximately, from the end of this steel wire: now, as 1.5 inches is the third of 4.5 inches, it would result from this comparison that the curve of the magnetic densities, would have its center of gravity placed almost at the same distance from its end, as if the shape of this curve was nearly a straight line: now, we find, from the fifth experiment, *Figure 4, No. 1*, that this curve is convex. Although these results are not contradictory, it must be observed that the fifth experiment shows us only the point where the magnetic density of the steel wire is insignificant; because, it is equal to 0 only in the middle of the steel wire. This experiment also indicates to us the points of two wires of magnetized steel, of the same size, where the magnetic density is the same; but we cannot draw the exact law of the magnetic densities of all the points of the steel wire from this fifth experiment, because it gives, for the high [magnetic] densities of the point b, *Figure 3*, too large quantities, relatively to the small densities of the other points of the needle, here is the reason:

When the needle a, *Figure 3*, is only one or two lines in length, and less than half a line in diameter, as in the fifth experiment; this needle shows only very weak signs of magnetism when oscillating freely after having been magnetized without any action foreign to the globe of the Earth; but if the steel wire ns is presented to it at a distance of 3 lines, as we did in the fifth experiment, its magnetic state increases as the point b of the steel wire is more charged with magnetism:[1047] so that, from one test to another, the needle a is not in a constant state of magnetism; but this state changes as the action of the point b is greater or smaller: whence it follows that in the successive tests of this fifth experiment, the action of the point b on the needle a, is not proportional to the magnetic density of the point b, but proportional to the this density and to the magnetic state of the needle a; so that, if the magnetic state of this needle increased in proportion to the magnetic density of the point b, then the action or the ordinates found by our curve, *Figure 4, No. 1*, would be like the square of the densities of the point b: that is to say, if this supposition could be admitted, it would be necessary that the ordinates which would represent the [magnetic] densities, were only proportional to the number of oscillations found by the tests of this fifth experiment.

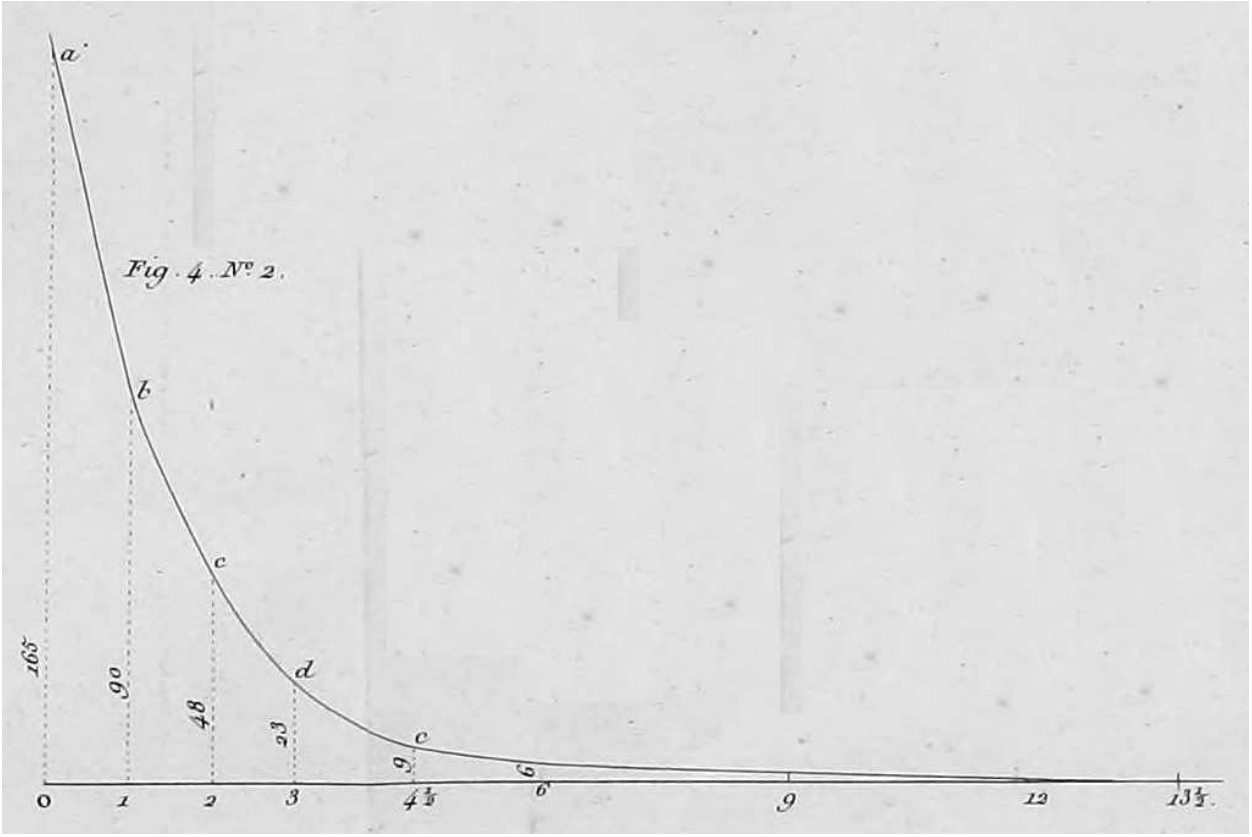
An experiment which proves in a convincing way the variation of the magnetic state of the small needle a, during the different tests, consists of presenting for a single moment the southern extremity, for example, of the needle a at a distance of one or two lines from the southern extremity of the steel wire ns; in this case, by the action of the wire ns, the South pole of the needle a becomes in an instant the North pole; moreover, by this operation, this small needle is magnetized to saturation, which will be easy to prove by the number of oscillations which it will make freely,[1048] that is to say, after having been presented at two lines of distance from the pole of the steel wire ns, or after having touched the pole of this steel wire or even a stronger magnet, in both cases, we will find that it makes, in the same time [interval], the same number of oscillations.

[1047] That is, the more intense the magnetization of the point b of the steel wire, the greater the magnetization it will produce in the small needle a placed nearby.
[1048] Due only to the magnetism of the terrestrial globe.

26.23 XXIII. Eighth Experiment, *Intended to Give More Approximate Results Than the Fifth Experiment*

Informed by the observations of the preceding remark, I sought to determine, by a new experiment, the densities of the wire *ns*, in a more approximate way than by the fifth experiment, of which we have just given the details and its shortcomings. It can be seen that I had to seek to substitute for the small needle *a*, whose magnetic state varied from one test to another, another needle whose magnetic resistance was greater, and at the same time whose magnetic action on the points *b* of the steel wire, *Figure 3*, was not large enough to alter, in a sensible way, the state of this wire; because the action being reciprocal between the needle *a* and the wire *ns*, the magnetic alteration is equally to be feared on both sides.

Here is how I arrived at an approximate result, after several tests, to determine the most suitable dimensions. In place of the little needle *a*, *Figure 3*, which, in our fifth experiment, was only two lines in length, and less than half a line in diameter, I hung a steel needle 3 lines in diameter (0.67 cm) and 6 lines in length (1.35 cm); I placed the point *b* of the steel wire *ns*, 8 lines away (2.03 cm) from the end of the needle *a*, and I followed all the procedures of the fifth experiment: by then calculating the action of the different points *b* of the steel wire *ns* on the needle *a*, according to the square of the oscillations, I found the densities of these different points as they are observed at *Figure 4, No. 2*.



In this Figure, the base [between] 0 and $13\frac{1}{2}$ inches represents half of the needle axis;[1049] the ordinates represent the magnetic densities of the corresponding points. The last ordinate

[1049] Again when Coulomb refers here to the "needle", the steel wire *ns* of Figure 3, Number 1, is to be understood.

oa, was determined by making *ba*, relatively to *bc*, the same angle as *bc* makes with *cd*; this last ordinate should probably be a bit larger, but the others are closer to the truth.

It results from this experiment, that the curve of the densities, *Figure 4, No. 2*, starting from the end of the needle, quickly approaches the axis, since in our experiment, the ordinate which represents the density of the point placed at four and a half inches from the end of the wire, is at least eighteen times smaller than that of this end: we see again that, from this point, the curve continues to approach the axis, which it intersects in the middle of the needle, to form, in an opposite direction at the other end of the needle, a curve absolutely similar to the first; by calculating the distance from the center of gravity of the curve of the densities, according to the ordinates of *Figure 4, No. 2*, we find it placed at 1.3 inches (3.52 cm) from the extremity *o*: we found it by the calculation of the second experiment, Section 26.21, at a distance of 1.5 inches from this extremity, a relation as exact as can be hoped, in experiments of this kind, which would only seem to indicate that the [magnetic] density of points placed near the middle of the needle, is a little larger than that indicated by our figure; which must result, as we proved in Section 26.22, from the magnetic influence of the strongly magnetized points of the steel wire *ns*, on the magnetic state of the needle *a*; because, although this state is not subject to variations as strong as those of the small needle of the fifth experiment; there will be, however, in the state of the needle *a*, an increase of magnetism all the more sensible, as the action of the point *b* of the steel wire *ns*, *Figure 3*, will be stronger.[1050]

26.24 XXIV. *Recapitulation*

Let us sum up in a few words the principal results furnished by the preceding experiments.

1°. The curve of the magnetic intensities can, in practice, be calculated as a triangle which extends only from the end of the needles up to a distance from this end, equal to 25 times the diameter of the needle: thus, in the needles which have a length greater than 50 times their diameter, the *moment* increases as the lengths of the needles increase.

2°. When the needles are less than 50 times their diameter in length, the moments of the directive forces can, in practice, be evaluated in a ratio of the square of the lengths of the

[1050] [Note by Potier] Biot proposed the formula

$$y = A\left(\mu^x - \mu^{L-x}\right)$$

to represent the result of Coulomb's observations; x being the distance *in inches* from one end of the magnet of length L, the value μ deduced from the curve *abcde*, shown above, would be 0.518 (or 0.784, if we take the centimeter as a unit).

From this formula we deduce for the moment of the terrestrial couple, if H is the horizontal component [of terrestrial magnetism, the following value:]

$$\frac{2AH}{l\cdot\mu}\left[\frac{L}{2}(1+\mu L)+\frac{1-\mu L}{l\cdot\mu}\right] .$$

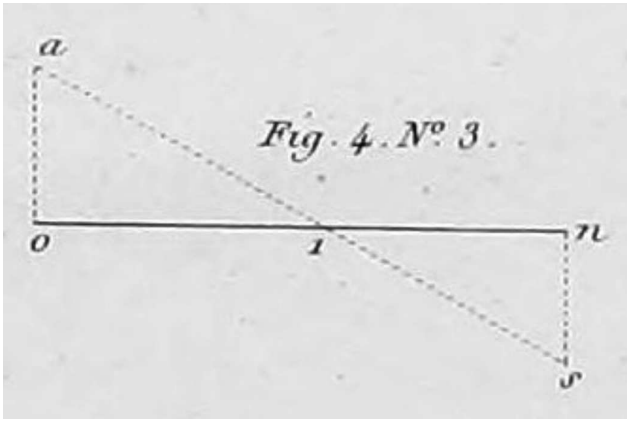
If we seek to determine the constants $\frac{2AH}{l\cdot\mu}$ and μ, so as to represent the experiments of Section 26.5, we find (inches and degrees of twist taken as units) $\frac{2AH}{l\cdot\mu} = 38.22$ and $\mu = 0.525$, numbers very close to the previous ones.

Applied throughout the first experiment (Section 26.4), the same formula would give $\frac{2A'H}{l\cdot\mu'} = 2$ and $\mu' = 0.0433$. The values of μ and μ' satisfy the relation $rl\cdot\mu = r'l\cdot\mu'$, being r and r' the radii of the two wires, and the ratio $\frac{A'}{l\cdot\mu'} : \frac{A}{l\cdot\mu}$ is substantially that of the [cross] sections.

needles. This result found in the *first and second experiment*, is confirmed by the *fifth, sixth and seventh*, where it is found that, whatever the length of the needles, the magnetic intensity of their end is substantially the same; thus, if the shape of the curve of the intensities is represented by a triangle whose tip is at the middle of the needle, and if one names, *Figure 4, No. 3*,[1051,1052] A the magnetic intensity ns at the extremities of the needles, and x half of the [length of the] needle, we will have, for the *moment* of the directing force of this needle,

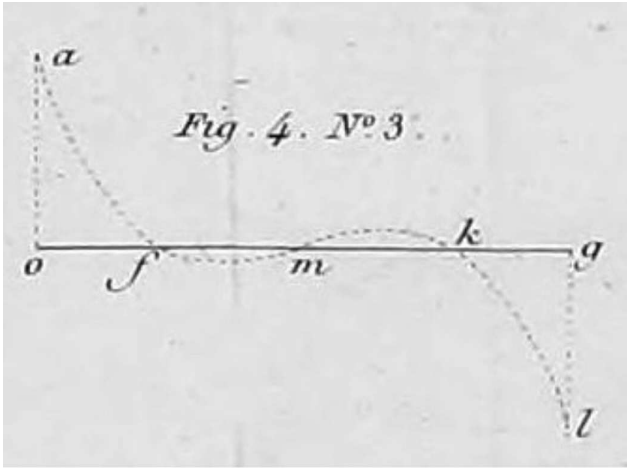
$$\frac{2Ax^2}{3}.$$[1053]

That is to say, that the *moments* of the directing force, are like the squares of the lengths of the needles, when [the lengths of] these needles are less than 50 times their diameter, and when the locus of the magnetic densities is roughly a straight line.



[1051]In this Figure 4, Number 3, the segment $0n$ represents the magnetized needle, the point 1 is its midpoint with the distance 01 being represented by x. The needle is supported by its midpoint. Moreover, $0a$ and ns represent the opposing magnetic intensities at the ends of this needle. Coulomb will indicate the absolute value of this intensity by the letter "A". The dashed line indicates the magnetic intensity along the length of the needle, being positive in one half (representing, for example, the boreal or northern fluid) and negative in the other half (representing, for example, the austral or southern fluid).

[1052]Among the images of Coulomb's Seventh Memoir, there is also another image numbered as Figure 4, Number 3. I don't know where this Figure should appear. Anyway, I put it in this footnote:



[1053]Section 27.1 presents a detailed calculation to arrive at this result.

3°. When we compare two needles of the same nature, whose dimensions are homologous, the [magnetic] *moments* of their directing force are like the cube of the homologous dimensions.

26.25 XXV. *Essay on the Theory of Magnetism, with Some New Experiments Tending to Clarify This Theory*

Physicists have long attributed the effects of magnetism to a vortex of fluid matter which made its revolution around magnets, whether artificial or natural, entering at one pole and leaving at the other.[1054] This fluid acted, it was said, on iron and steel because of the configuration of their parts, but it exerted no action on other bodies. As, in this system, some phenomena appeared which were inexplicable by a single vortex, several were imagined, or several magnets were combined together; they were given, according to the need, particular movements. It is on such hypotheses that three Memoirs on the cause of magnetism, crowned by the Academy in 1746, were based.[1055]

I believe I have proved, *ninth volume of the Savans étrangers, page 137 and 157*,[1056] how difficult it was to explain, by means of vortices, the different magnetic phenomena; it must therefore be seen whether, by simple suppositions of attractive and repulsive forces, these phenomena will be more easily explained. To avoid any discussion, I warn, as I have already done in the various Memoirs which precede, that any hypothesis of attraction and repulsion, according to any law whatsoever,[1057] should only be regarded as a formula which expresses a result of experiment; if this formula is deduced from the action of the elementary molecules of a body endowed with certain properties; if we can draw from this first elementary action all the other phenomena; if, finally, the results of the theoretical calculation are found to agree exactly with the measurements furnished by the experiments, we can perhaps hope to go further only when we have found a more general law which envelops in the same calculation bodies endowed with different properties, which, up to now, do not appear to us to have any connection between them.

Mr. OEpinus appears to be one of the first who sought to explain,[1058] by means of calculation, by attraction and repulsion, the magnetic phenomena. He thinks that the cause of magnetism can be traced to a single fluid which acts on its own parts by a repulsive force, and [acts] on the parts of the steel or the magnet by an attractive force. This fluid once engaged in the pores of the magnet, only moves with difficulty. This system has led Mr. OEpinus to this conclusion, that in order to explain different magnetic phenomena, it is necessary to suppose between the solid parts of the magnet a repulsive force. Since Mr. OEpinus, several physicists have admitted [the existence of] two magnetic fluids; they supposed that when a steel lamina was in its natural state, these two fluids were combined to saturation; that by the operation of magnetism, they separated and were carried to the two extremities of the lamina. According to these authors, the two fluids exert on each other an

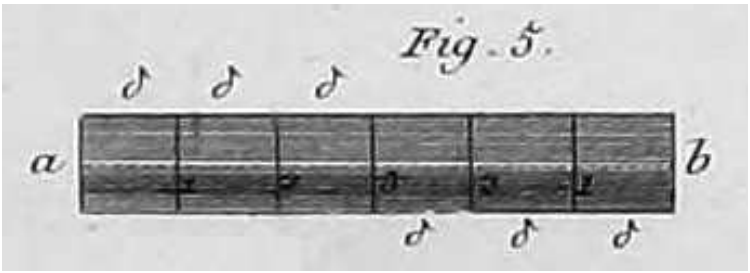
[1054] See Section 4.4.
[1055] See footnote 301 on page 134.
[1056] This page numbering seems strange to me. I believe Coulomb is referring to pages 173 and 257 of the original article: [Coulomb, 1780, pp. 173 and 257], translated in Chapter 5. See, in particular, pages 60 and 134 of this English translation.
[1057] That is, whatever the variation of the force as a function of the distance between the elementary molecules of a body that are interacting with one another.
[1058] See Section 4.1 and footnote 182 on pages 35 and 73, respectively.

attractive action; but they exert on their own parts a repulsive action; it is easy to perceive that these two systems[1059] must give, by theory, the same results.

It is now a question of seeing if the calculations founded on the hypotheses which precede, will agree exactly with the experiments; researches which it was not possible to attempt before knowing the law of attraction and repulsion of the magnetic molecules of magnetized bodies; law that we found, *Memoir of the Academy, for 1785, page 606 and following*,[1060] directly proportional to the density or to the magnetic intensity and inversely proportional to the square of the distances. It was equally impossible to verify any hypothesis, before having employed means which gave exact measurements in the experiments; as we have tried to do in those which precede.

26.26 XXVI. *Example for Determining, by Calculation, the Distribution of the Magnetic Fluid in a Cylindrical Steel Needle, According to the Systems Which Have Just Been Stated*

To simplify the results and put the calculations within reach of a greater number of readers, we are going to apply a method of approximation to a very simple example, but which will suffice to indicate to us at the same time the main results, given by the experiments which precede, and the course which we will be able to follow in more complicated examples. Suppose, *Figure 5*, that the cylindrical steel needle *ab*, has a length six times its diameter, and is divided into six equal parts.[1061]



Suppose this needle magnetized to saturation, and seek what must be the magnetic density[1062] of each part so that there is equilibrium at the point of the axis of each division; suppose moreover the magnetic density uniform in each part and different only from one part to another: according to this supposition, the point 3 being placed in the middle of the needle, the magnetic densities of the points on the two sides, [located] at equal distances from point 3, will be equal; but some will be positive and others negative. Let the limit of the coercive force which prevents the magnetic fluid from flowing from one part of the needle

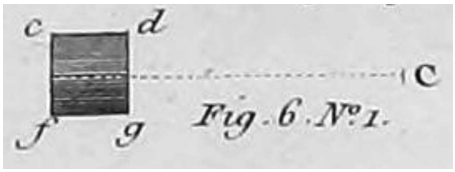
[1059] That is, the system with a single magnetic fluid and the system that admits the existence of two magnetic fluids.

[1060] See Section 14.2, page 237 and the following.

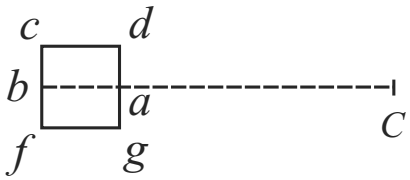
[1061] Section 27.2 presents a discussion of this Figure 5.

[1062] What Coulomb here calls the magnetic density (*densité magnétique*) must be understood as the volumetric density of the magnetic fluid.

to the other, a force which can be compared to friction in machines, or to coherence,[1063] be represented by the constant quantity A; to have the action of each part on a point of the axis, it is necessary to determine, by calculation, in *Figure 6, [Number 1]*,[1064] the action of the small cylinder $cdfg$, whose density is uniform, on the point C of the axis, assuming the action of all the points in inverse proportion to the square of the distances.



(a)



(b)

Let the radius of the cylinder $ag = r$, the distance $cb = a$, the distance $ca = b$, the length of the cylinder $ba = a - b$, [and] 2π the ratio of the circumference to the radius;[1065] the action of the cylinder $cdfg$, whose density is δ,[1066] acting on the point C of the axis, in the direction of the axis ac, will be expressed by the formula[1067]

$$2\pi\delta\left(a - b + \sqrt{b^2 + r^2} - \sqrt{a^2 + r^2}\right) .$$

Here is the type of calculation that gives this formula. The action of a circular zone, which would have, *Figure 6, No. 2*, $mn = dr$ in width, and $pm = r$ for radius, far from the point C on which it acts at the distance $pC = x$,[1068] would be represented by the quantity

$$\frac{2\pi\delta x r dr}{(r^2 + x^2)^{3/2}} .$$

[1063] In the original: *cohérence*. See footnote 322 on page 145.

[1064] Coulomb's original image appears in letter (a) of the image of this Figure 6, Number 1. In letter (b) I added the letters a and b which will be mentioned next by Coulomb. These letters a and b are located along the axis of the cylinder.

[1065] In the original, [Coulomb, 1793, p. 484], Coulomb represents by the lowercase c "*le rapport de la circonférence au rayon*", that is, the ratio of the circumference to the radius. I am here following Potier, [Potier, 1884, p. 299], and representing that ratio by 2π. The same substitution was made in the other formulas of this work.

[1066] That is, δ is the volumetric density of magnetic fluid, such that the amount of magnetic fluid contained in an infinitesimal volume dV is given by δdV.

[1067] In the original text, [Coulomb, 1789, p. 484], Coulomb defines the lowercase letter c to be the ratio of the circumference of a circle to the radius. Today this ratio is represented by 2π. Potier, [Potier, 1884, p. 299], replaced the letter c with 2π. Moreover, he replaced the expression ab in the first parenthesis of the next formula with $a - b$. I followed Potier's procedure in this translation. The next equation appeared in the original text as follows:

$$c\delta\left((ab) + (bb + rr)^{1/2} - (aa + rr)^{1/2}\right) .$$

[1068] By an oversight the expression $pm = x$ appears here in the original text. The "action" mentioned by Coulomb should be understood as the force component along the direction pC.

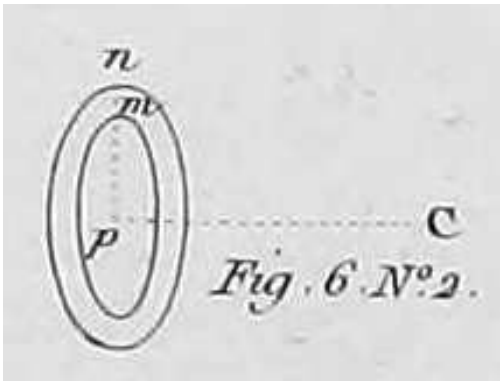


Fig. 6 N°2.

This quantity integrated in such a way that it vanishes when $r = 0$ will give for the action of the circle of which r is the radius, [the following quantity:]

$$2\pi\delta\left(1 - \frac{x}{\sqrt{r^2 + x^2}}\right) .$$

Multiplying by dx and integrating so that the value is complete when $x = a$, and vanishes when $x = b$, we obtain, *Figure 6, No. 2*, as an expression for the action of the small cylinder $cfgd$,[1069] on the point C, evaluated in the direction of the axis, the formula

$$2\pi\delta\left(a - b + \sqrt{b^2 + r^2} - \sqrt{a^2 + r^2}\right) .$$

Now applying this formula to our example, where each part of the cylinder [has a length] equal to $2r$, and where, *Figure 5*, there must be equilibrium at the points 1, 2 and 3 of the axis, between the magnetic forces and the resistance experienced by this fluid in passing from one point of the steel wire to another, we will derive the following three equations.

At point 1:

$$0.77\delta_1 = 0.74\delta_2 + 0.06\delta_3 + \frac{A}{2\pi r} .$$

At point 2:

$$0.13\delta_1 = -0.81\delta_2 + 0.65\delta_3 + \frac{A}{2\pi r} .$$

At point 3:

$$0.10\delta_1 = -0.22\delta_2 - 1.52\delta_3 + \frac{A}{2\pi r} .$$

By reducing these three equations, we find, for the magnetic densities, the following values,

$$\delta_1 = 2.41\frac{A}{2\pi r} ; \qquad \delta_2 = 0.72\frac{A}{2\pi r} ; \qquad \delta_3 = 0.19\frac{A}{2\pi r} .$$

[1069] Due to a lapse in the original text, cylinder $efgd$ appears here. Coulomb is certainly referring to the action of the small magnetized cylinder $cfgd$ of Figure 6, Number 1, acting on a particle placed at point C.

26.27 XXVII

If we suppose another needle whose coercive force, which depends on the nature and degree of tempering of the needle, is represented by A', whose radius is r', and whose length is equal to six times its diameter, we would have a needle all the dimensions of which would be homogeneous, or proportional to the dimensions of the one which has just served as a tool for our calculation, and naming d_1, d_2 and d_3 the densities corresponding to the three divisions of the half of this [new] needle, we will have the three values[1070]

$$d_1 = 2.41\frac{A'}{2\pi r'} \; ; \qquad d_2 = 0.72\frac{A'}{2\pi r'} \; ; \qquad d_3 = 0.19\frac{A'}{2\pi r'} \; .$$

Thus in the two needles, by comparing the corresponding densities, we will have:[1071]

$$\delta_1 : d_1 :: \delta_2 : d_2 :: \delta_3 : d_3 :: \frac{A}{r} : \frac{A'}{r'} \; ,$$

that is to say, that the densities of the corresponding portions of the two needles are between them

$$:: \frac{A}{r} : \frac{A'}{r'} \; ,$$

[that is,] in direct proportion to the coercive forces and in inverse [proportion] to the radii.

If the two needles which we wish to compare had, relative to their diameters, a greater length than the preceding one; but were of homologous dimensions, it is easy to see that we would obtain, by the method which precedes, as many equations as there would be divisions in the half of the needle, and as in each corresponding equation the coefficients of the parts similarly placed are the same, it follows that the densities of the parts similarly placed, will be in all cases between them

$$:: \frac{A}{r} : \frac{A'}{r'} \; .$$

26.28 XXVIII

It is now easy to calculate according to the theory, the ratio of the magnetic moments of the actions of the globe of the Earth, which restore two needles magnetized to saturation of homologous dimensions to the magnetic meridian; consider in these two needles two homologous parts whose radii are r and r', the masses of the homologous parts will be [among themselves as:]

$$:: r^3 : r'^3 \; ,$$

the masses of the magnetic fluid of these same parts[1072] will be like the [volumetric] densities multiplied by the cube of the radii: but the middle of each needle being, in our experiments,

[1070] For a lapse in the original text, A appears instead of A' in the next three equations. I fixed this.

[1071] That is:

$$\frac{\delta_1}{d_1} = \frac{\delta_2}{d_2} = \frac{\delta_3}{d_3} = \frac{A/r}{A'/r'} \; .$$

[1072] That is, the total amounts of magnetic fluid in each of these parts.

the center of rotation, around which each part solicited by the magnetic force of the Earth is brought back to its magnetic meridian, it follows that each part has, for the *moment* around this point, the product of its density, of the cube of the radius and the distance from this point to the center of rotation. But as the densities in two corresponding parts of two homologous needles are between them as

$$:: \frac{A}{r} : \frac{A'}{r'} ;$$

and as, moreover, for the parts similarly placed in the two homologous needles, the distances to the middle of the needles are like the radii; it follows that the magnetic *moments* which brings back two homologous needles to the magnetic meridian, are to one another in a direct ratio composed of the coercive force, and of the cube of the radius:[1073] but we have seen, Section 26.10, that it resulted from experiment, that in two needles of the same nature, and of homologous dimensions, the moments of the directing force were as the cubes of the radii, which is perfectly in accordance with the theory.

We have also found, Section 26.21, from experiment, that in two steel needles of the same nature, but of different diameters, the center of gravity of the curve which represented the densities of the magnetic fluid, was placed relative to the ends of these needles, at distances proportional to their diameter; the previous formulas give the same result.

26.29 XXIX

The agreement between the fundamental experiments and the calculation which we find here appears to give a great weight, either to the opinion of Mr. OEpinus, or to the system of the two fluids, such as we presented it; however, it must be admitted that there are some phenomena which seem to refute these hypotheses entirely; here is one of the principal ones.

We saw in Section 26.1, that when a magnetic needle was suspended freely, the sum of the boreal forces which solicited this needle along the magnetic meridian, was exactly equal to the sum of the austral forces;[1074] this result, founded on experiments that cannot be denied, is valid not only for a needle that has just been magnetized, but [also] if, after having magnetized it, we cut this needle in different parts; if we cut, for example, the extremity of the northern part, this suspended part will be solicited by exactly equal boreal and austral forces; but in the preceding hypotheses, this part would be solely charged with boreal fluid, and the action of the two magnetic poles of the globe of the Earth would combine to transport it towards the boreal pole; thus theory here finds itself in contradiction with experiment.[1075]

26.30 XXX

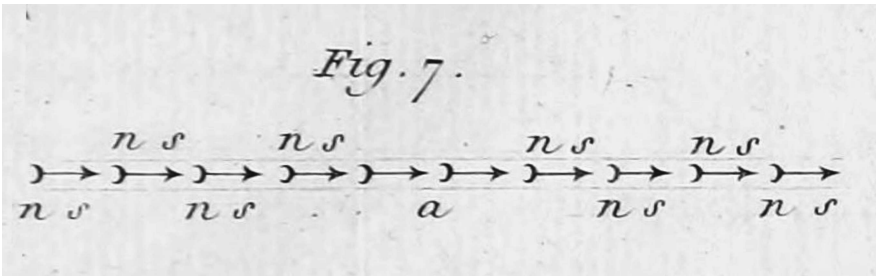
I believe that we could reconcile the result of the experiments with the calculation, by making some changes to the hypotheses; here is one which appears to be able to explain all the magnetic phenomena of which the preceding tests have given precise measurements.

[1073] That is, the torque exerted by the Earth on a magnetized needle is proportional to the coercive force acting on the magnetic parts of the needle and is also proportional to the volume of the needle.

[1074] These two forces act in opposite directions, along parallel lines, composing a pair of forces that generate a torque on the needle.

[1075] See the discussion of this argument in Section 4.4.

It consists in supposing, in the system of Mr. OEpinus, that the magnetic fluid is enclosed in each molecule or integral part of the magnet or of the steel;[1076] that the fluid can be transported from one end of this molecule to the other, which gives each molecule two poles; but that this fluid cannot pass from one molecule to another. Thus, for example, if a magnetized needle were of a very small diameter, or if, *Figure 7*, each molecule could be regarded as a small needle whose North extremity would be united to the South extremity of the needle which precedes it, only the two extremities n and s of this needle would give signs of magnetism; because it would only be at the two extremities where one of the poles of the molecules would not be in contact with the opposite pole of another molecule.



If such a needle after having been magnetized were cut in two parts in a, for example, the end a of the part na, would have the same force as the end s of the entire needle, and the end a of the part sa, would also have the same force as the end n of the whole needle had before being cut.[1077]

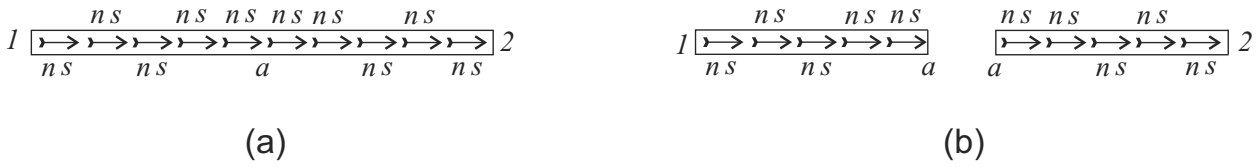
This fact is very exactly confirmed by experiment; for, if we cut a very long and very fine needle into two parts after having magnetized it, each part tested on the [magnetic] balance is found to be magnetized to saturation, and although we magnetize it again, it will not acquire a greater directing force.

Every part of our needle, in this new system, however magnetized or cut, will be directed into the magnetic meridian by perfectly equal austral and boreal forces; which appears to be one of the principal phenomena which the hypotheses must satisfy.

The hypothesis which we have just made appears very analogous to this well-known

[1076] In the original: *le fluide magnétique est renfermé dans chaque molécule ou partie intégrante de l'aimant ou de l'acier*. See footnote 395 on page 176. Another possible translation would be individual particle, integral particle or constituent particle, [Gillmor, 1971a, pp. 159, 201 and 217].

[1077] I will assume here needle 12 in Figure (a) of this footnote to have a North pole at end 1 and a South pole at end 2. At its center a there is no magnetic pole intensity, since the poles sn at this point cancel out as they are practically in contact:



By cutting the needle at its midpoint a and separating the two parts, we are left with two needles, $1a$ and $a2$, as shown in Figure (b). The part $1a$ has a North pole at end 1 and a South pole at end a. The part $a2$ has a North pole at the end a and a South pole at the end 2, as stated by Coulomb. The intensities of the magnetic poles at the ends of needle $1a$ are the same intensities as those of the original needle 12. The same is true for needle $a2$ compared to needle 12.

electrical experiment. When we electrify a glass plate covered with two metallic planes,[1078] however thin the planes may be, if they are taken away from the plate, they give signs of significant electricity: the surfaces of the glass, after the electricity of the fittings has been discharged, remain impregnated with the two contrary electricities, and form a very good electrophorus;[1079] this phenomenon takes place whatever thickness is given to the glass plate: thus, the electric fluid, although of a different nature on the two sides of the glass, penetrates only at an infinitely small distance from its surface; and this [glass] plate looks exactly like a magnetized molecule of our needle. And if, now, we placed one on the other a series of plates thus electrified so that, in the union of the plates, the positive side which forms the surface of the first plate is at a distance of several inches from the negative surface of the last plate; each surface of the extremities, as experiment proves, will produce, at fairly considerable distances, effects as perceptible as our magnetized needles; though the [electric] fluid of each surface of the plates of the extremities penetrates these plates only to an infinitely small depth, and that the electric fluids of all the surfaces in contact balance each other mutually, since one of the surfaces being positive, the other is negative.[1080]

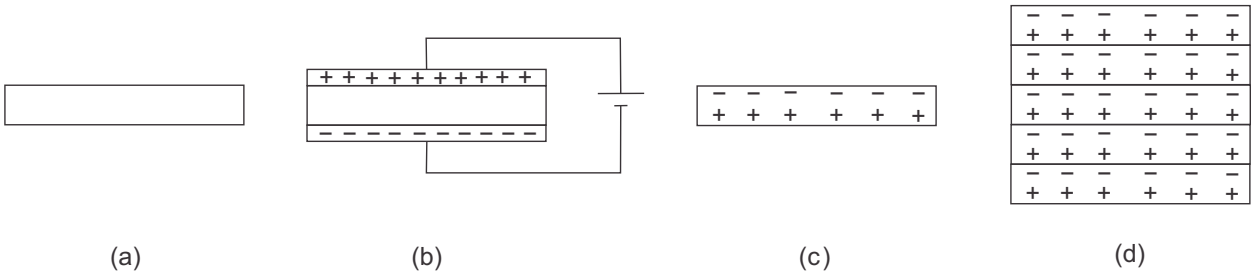
Finally, in any system of attraction and repulsion, we cannot suppose that one of the two magnetic fluids can pass from one steel bar to another, since the magnetized needles are always acted upon by exactly equal boreal and austral forces; however, if we fill a small pipe or a straw with steel filings, and magnetize it, we will find in this pipe a directing force, very perceptible, and which we will easily measure in our magnetic balance.[1081] The filings within the pipe are covered by our hypothesis, since the magnetic fluid cannot pass from one steel molecule to another.

Here is one more experiment in support of our opinion; along a wooden ruler, *Figure 8*, I place, in a row, five or six parallelepipeds of very soft iron in contact at their ends, forming together a length of eighteen to twenty inches [48.726 to 54.14 cm].

[1078] In the original: *Lorsque l'on charge un carreau de verre garni de deux plans métalliques.* The word "carreau" can be translated as plate, pane, square or tile. These parallel-plate condensers or capacitors became known as Franklin squares, [Heilbron, 1999, pp. 317, 333, 334, 368, 407, 408, 418 and 435].

[1079] For a detailed discussion of the electrophorus see Chapter 6 (The Electrophorus) of [Assis, 2018b], [Assis, 2018a] and [Assis, 2019].

[1080] An illustration of this experiment appears in the Figure of this footnote:



In letter (a) of the Figure of this footnote we have a neutral glass plate. The glass in this experiment has to behave as an insulator, see footnote 503 on page 223. In (b) this glass plate is covered with two conducting planes that are connected to the ends of a battery, remaining there for a certain time. When the electrified planes are removed, it is observed that the glass plate was electrically polarized, as shown in (c). When stacking several of these polarized plates, it is observed that the set is also polarized, with charges of opposite signs on its external faces, (d).

[1081] For a lapse in the original text, "electric balance" appears here.

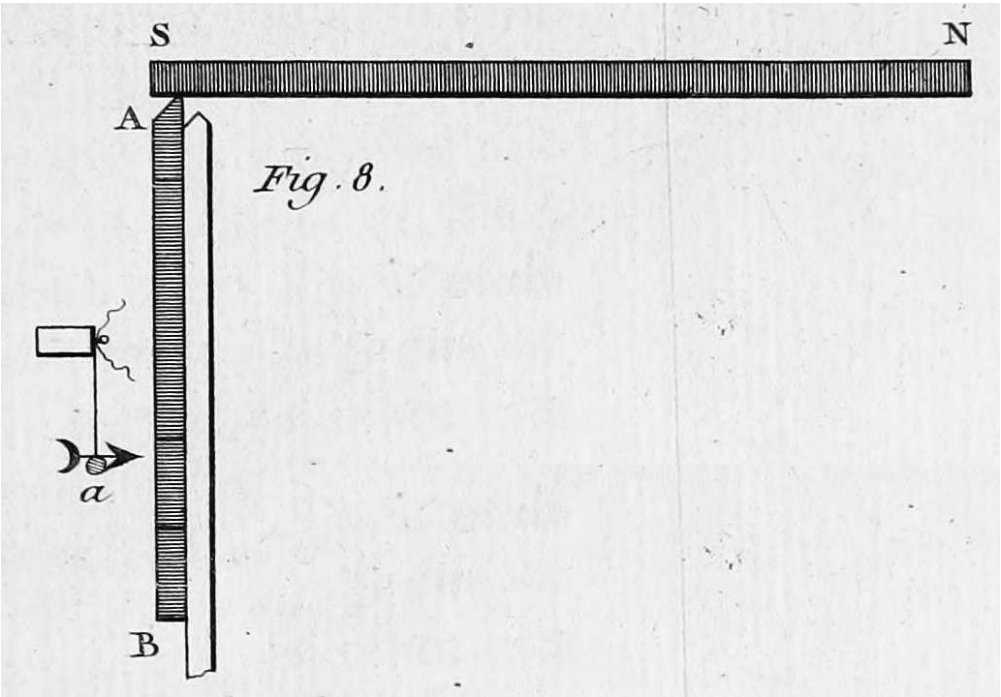


Fig. 8.

I apply the pole s of a bar magnet to the extremity A, and I slide, *Figure 3, [Number 3]*, the row AB of my parallelepipeds four or five lines away [0.904 or 1.13 cm] from a little magnetized needle a. Since the magnetic fluid cannot pass from one parallelepiped to another, each parallelepiped should have two poles. Experience teaches on the contrary, that the whole line AB gives the same nature of magnetism, as the pole s of the magnet sn in contact by this pole with the extremity A. This experiment is easily explained by our hypothesis.

26.31 XXXI

It is easy, from what we have just said, to determine the magnetic state of a magnetized plate; let *abcd*, *Figure 9, No. 1*, represent this lamina, which we suppose formed of an infinity of longitudinal elements.

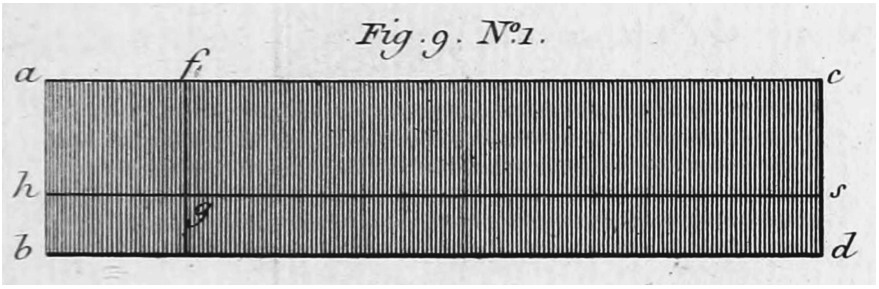
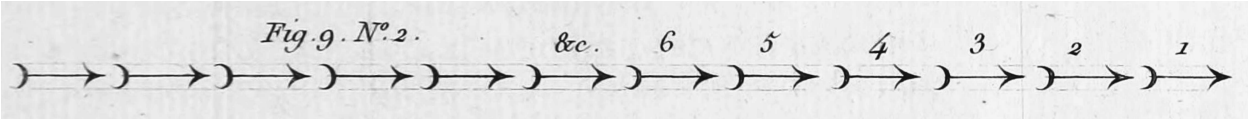


Fig. 9. N°1.

In *Figure 9, No. 2*, *hgs* is an elementary fiber seen larger, in which 1, 2, 3 represent small needles or elementary molecules.



In each molecule the magnetic fluid can be transported from one end to the other, but cannot leave the molecule: thus, in the first needle, if the magnetic fluid is condensed at the boreal end by the quantity a, in this same needle it will be dilated at the southern end beyond the state of neutralization of the quantity a; in needle 2 it may be condensed at the northern end by a quantity $a + b$; thus it will be dilated at the other end of the needle by the same amount $a + b$; in needle 3 it will be condensed at the northern end of the quantity $a + b + c$; thus at the other extremity of the same needle, it will be dilated by the same amount; it will be the same for all the other elements of this fiber.

Hence it follows that at the extremity of our fiber, the boreal force will be a; that at the boreal end of the second element, the boreal force will be reduced to b, the force a being destroyed by the negative force a of the southern end of element 1; at the northern end of element 3, the northern force will be reduced to c, the part $(a + b)$ being destroyed by the negative force of the southern pole of element 2.

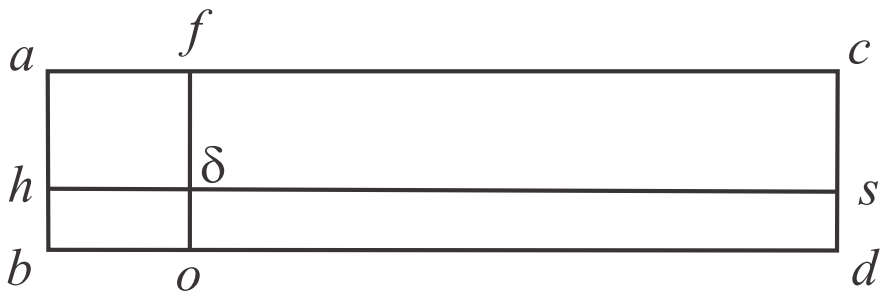
It is now easy, by replacing our fiber in *Figure 9, No. 1*, to see that taking in this fiber, on the northern side, for example, any point δ, of which the boreal force, reduced, according to the preceding observation, is represented by δ. If we draw through this point δ, a line of perpendicular to the length of the lamina; [then,] in the state of stability, the action of the whole part $abfo$ on the point δ, being decomposed in the direction $h\delta$, must balance the action of the whole remaining part $focd$, plus the coercive force that prevents the fluid from flowing into each element.[1082]

Thus, in our hypothesis, the calculation of the magnetic actions or the intensity of the magnetic forces of each point, must give us precisely the same result as that [calculation based on the assumption] of the transport of the magnetic fluid, from one end of a lamina to the other. Calculation which gives, as we have seen, the greatest conformity between experiments and theory, when the needles are magnetized to saturation.

26.32 XXXII

We have hitherto tried to determine by experiment and by theory the principal laws of the distribution of the magnetic fluid in needles of different lengths, and of different sizes; we

[1082] I present in the image of this footnote the letters mentioned by Coulomb as they seem to me to be located in the original Figure 9, Number 1:



have seen that by means of a few corrections, it was easy to bring the theory into line with the magnetic phenomena. We are now going to give some experiments intended to determine,

1. the most advantageous form of magnetized needles, intended to indicate the magnetic meridian;

2. the degree of quenching and annealing that is best suited for steel blades, to take on magnetism;

3. the degree of magnetism assumed by a bundle of magnetized plates, as well as [the degree of magnetism assumed by] each plate of this bundle, when it is detached from this bundle, and when, without magnetizing it again, its magnetic force is determined;

4. the means which have succeeded best for magnetizing steel needles to saturation, and for forming artificial magnets.

26.33 XXXIII. *Shape and Degree of Quench Hardening of Magnetized Needles*

Most authors have believed that the most advantageous form of magnetized needles was a steel blade having the shape of a rectangular parallelogram.

Experiment has shown me that at the same length, same weight and same thickness, an arrow-cut blade, *Figure 9, No. 3*, has a magnetic *moment*, greater than a rectangular parallelogram.

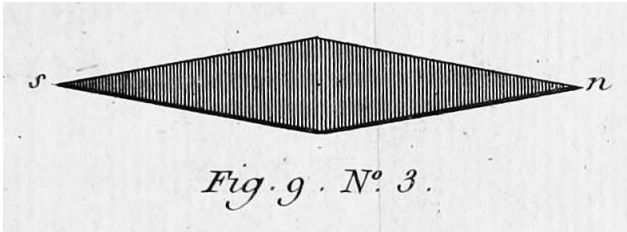


Fig. 9. N°. 3.

26.33.1 Eighth Experiment

In a steel blade, which we find in commerce under the name of *steel sheet from England*,[1083] we cut three needles of the length of six inches (16.24 cm).

The first a rectangular parallelogram of $9\frac{1}{2}$ lines wide (2.14 cm), weighing 382 grains (20.10 g).

The second, also a rectangular parallelogram, was $4\frac{3}{4}$ lines wide [1.07 cm], and weighing 191 grains [10.14 g].

The third one, cut like an arrow, was $9\frac{1}{2}$ lines wide [2.15 cm] in the middle and weighed 191 grains like the second one.

We successively suspended these three needles in the magnetic balance, after having magnetized them, and we obtained the following results:

[1083] In the original: *tole d'acier d'Angleterre*.

First trial. *The three needles tempered, red white.*

The parallelogrammatic needle, weighing 382 grains, was held at 30 degrees from its magnetic meridian, by a twisting force, measured as .....85 degrees.
The parallelogrammatic needle, weighing 191 grains, ...49 degrees.
The arrow needle, weighing 191 grains, ...53 degrees.

Second trial. *Needles annealed to the consistency of a purple spring.*

The parallelogrammatic needle, weighing 382 grains, was retained at 30 degrees from the magnetic meridian, by a twisting force of .....118 degrees.
The parallelogrammatic needle, weighing 191 grains, of .....65 degrees.
The arrow-shaped needle, weighing 191 grains, of .....68 degrees.

Third trial. *The annealed needles, water color.*

The parallelogrammatic needle, weighing 382 grains, was held at 30 degrees from the magnetic meridian, by a twisting force of .....126 degrees.
The parallelogrammatic needle, weighing 191 grains, of .....68 degrees.
The arrow needle weighing 191 grains, of .....3 degrees.

Fourth trial. *The needles annealed to a degree of heat, dark red.*

The parallelogrammatic needle, weighing 382 grains, was retained at 30 degrees from the magnetic meridian, by a force of torsion measured as .....134 degrees.
The parallelogrammatic needle, weighing 191 grains, as .....70 degrees.
The arrow-shaped needle, weighing 191 grains, as .....79 degrees.

Fifth trial. *Needles reddened to white and not tempered.*

By reddening the needles to white,[1084] and allowing them to cool slowly without quenching them, it was found that the degree of magnetism they could take on was about the same as when the needles were quenched red-white, as in the first trial.

26.34 XXXIV. *Remark on This Experiment*

We learn from this experiment that, in the first place, lamina quenched to great rigidity[1085] take on the least magnetism and that in this state, the magnetism is approximately the same as when the needle is annealed red-white: that starting from the state of the strongest hardening, the magnetism of the blades always increases through all the degrees of annealing, until the annealing is of a very dark red, and then the magnetism decreases as the blade is annealed at a greater degree of heat, that once it has reached the red-white state and cooled slowly, the blade being then magnetized, will take about the same degree of magnetism as after the hardest tempering without annealing .

[1084] In the original: *en faisant rougir les aiguilles à blanc.*
[1085] In the original: *dans les lames, l'état de trempe très-roide.* See footnote 405 on page 178.

This experiment also shows that in blades of the same thickness and of the same weight, the magnetic *moment* of that cut as an arrow, is a little greater than in parallelogrammatic needles.

Finally, it is easy to see from this experiment that in a parallelogram of the same thickness and length, but twice the width of another, the magnetic *moment* is not twice as large. This result was indicated by theory.

26.35 XXXV. *Magnetic State of a Bundle Composed of Several Blades*

26.35.1 Ninth Experiment

From[1086] the same sheet of steel which was used in the preceding experiments, we cut 16 rectangular parallelogrammatic needles, 6 inches long [16.24 cm], and 9 lines and a half wide [2.15 cm], each weighing 382 grains [20.29 g]. They have all been annealed to white without tempering them to be sure they are in the same state; because, as we have just seen, the magnetism varies according to the degree of quenching and annealing, and it would have been difficult to ensure that the state of spring would have been the same in all the blades if we had used a lower degree of annealing; each needle was magnetized to saturation in particular, and we then united them by joining together the poles of the same name,[1087] we formed, by this means, bundles of a certain number of needles, which were tied together with a very fine silk thread, strong enough to press them together. In each test, we placed the bundle in the magnetic balance, moving it away from its magnetic meridian by 50 degrees, we observed the force of torsion necessary to keep it at this [angular] distance.

First trial. To hold a single needle at 30 degrees from its magnetic meridian, required a twisting force of ..... 82°.

Second trial. Two needles joined together ..... 125°.

Third trial. Four needles joined together ..... 150°.

Fourth trial. Six needles joined together ..... 172°.

Fifth trial. Eight needles joined together ..... 182°.

Sixth trial. Twelve needles joined together ..... 205°.

Seventh trial. Sixteen needles joined together ..... 229°.

26.36 XXXVI. Tenth Experiment. *Decomposition of the Previous Needle*

I separated[1088] the 16 needles from the seventh trial of the previous experiment; I placed them successively in the magnetic balance, displacing them 30 degrees from the magnetic

[1086] For a lapse in the original text, this Section was numbered with the same number as the previous Section, namely, 34. Due to this fact, I changed the numbering of this Section and the following Sections so that they follow the ascending order, without repetitions and without gaps.

[1087] That is, they were placed side by side with all the North poles at one end and with all the South poles at the other end.

[1088] By a lapse in the original text, this experiment appeared with the same number as the previous one, that is, as the ninth. Because of this, I changed the numbering of this experiment and the next ones so that they followed the ascending order, without repetitions and without gaps.

meridian, and naming the "first needle" that one taken from one of the surfaces of the bundle, and in succession to the "sixteenth" taken from the other surface, I found:

First trial. The first needle is held at 30 degrees from its meridian, by a twisting force of 46°.

Second trial. Second needle ..... 39°.
Third trial. Third needle ..... $14\frac{1}{2}°$.
Fourth trial. Fourth needle ..... $44\frac{1}{2}°$.
Fifth trial. Fifth needle ..... 31°.
Sixth trial. Sixth needle ..... $32\frac{1}{2}°$.
Seventh trial. Seventh needle ..... $22\frac{1}{2}°$.
Eighth trial. Eighth needle ..... $30\frac{1}{2}°$.
Ninth trial. Ninth needle ..... 30°.
Tenth trial. Tenth needle ..... 26°.
Eleventh trial. Eleventh needle ..... $29\frac{1}{2}°$.
Twelfth trial. Twelfth needle ..... 34°.
Thirteenth trial. Thirteenth needle ..... 26°.
Fourteenth trial. Fourteenth needle ..... 32°.
Fifteenth trial. Fifteenth needle ..... 30°.
Sixteenth trial. Sixteenth needle ..... 48°.

All the needles were again bundled, without changing anything in their magnetic state, nor in the order in which they were in the seventh trial of the eighth experiment;[1089] placing the bundle in the magnetic balance, and displacing it 30 degrees from its meridian, it required, to retain it at this distance, a force of torsion of 229 degrees, exactly the same as before the disunion of the needles.

26.37 XXXVII. *Result of the Last Two Experiments*

The eighth experiment[1090] proves that the magnetic force of each bundle increases in a much smaller ratio than the number of blades, or than the thickness of the bundle. A single blade has, for the *moment* of its directing force, 82 degrees of torsion, while for 16 needles together, the mean magnetic *moment* of each has the measure $\frac{229}{16}$ degrees or 14.3 degrees, that is to say, approximately the sixth part of 82 degrees, [which is the] directing force of a single insulated blade magnetized to saturation. I have already drawn from this result a very important conclusion, in the *ninth volume of Savans étrangers*,[1091] relative to compass needles intended to indicate the meridian, and carried on caps and pivots:[1092] it is that the *moment* of the friction at the pivots increases, as I have then proved, in a greater ratio than the pressures, while the magnetic *moment* grows in a much less ratio than the masses or the pressures of the pivots, [therefore,] needles thin and very light are preferable to all the others at the same length. We see indeed by our experiment, that by supposing even the *moment* of the frictions proportional to the pressures, if the friction could produce on a single blade

[1089] Probably Coulomb was referring here to the ninth experiment.
[1090] Probably Coulomb was referring here to the ninth experiment.
[1091] [Coulomb, 1780]. See, in particular, Section 5.5 on page 101.
[1092] In the original: *portée sur des chapes et des pivots.* See footnote 141 on page 58.

magnetized to saturation, an error of 4' in its position relatively to the magnetic meridian, [then,] according to our experiment, it would produce an [error] six times greater, or about 24', if a bundle of sixteen blades had been used.

It is useless to examine here the laws which follow the magnetic *moment* of the bundles of blades which we subjected to the experiments; it would be necessary, to have this law, to extend the work that we have done, eighth experiment, for a particular case, to blades of different lengths and different widths: but it seems easy to us to predict these results in a sufficiently exact manner in practice, by following the research methods which we presented at the beginning of this Memoir, in similar cases, for steel cylinders of different sizes and lengths.

By now examining the Table given by the ninth experiment, we see that the two blades at the surfaces of the decomposed bundle have a greater magnetic force than the others. The first being measured by 46 degrees, and the sixteenth by 48 degrees, we also see that the average *moment* of all the other blades is approximately equal and measured by 30 degrees. For although the magnetic *moment* of the third blade in this experiment was found to be only 14 and a half degrees, this decrease is compensated by the *moment* of the neighboring needles; the second having as measure of its directing force 39 degrees, and the fourth 44 degrees;[1093] so that the mean *moment* of these three needles is[1094]

$$\frac{39 + 14\frac{1}{2} + 44}{3} = 31\frac{1}{6} .$$

By repeating this experiment, and replacing the third blade by another, I no longer found any irregularity, and this third blade had a directing force measured by 32 degrees like the others.

But a very curious observation presented by this ninth experiment is that the sum of the particular *moments* of all the blades gives us a quantity more than double that of the compound bundle. If indeed we add together the *moments* of all the blades of the ninth experiment, we find this sum equal to 516 degrees; while by uniting all the needles, the bundle thus composed gives us only 229 degrees.

This last result could be explained, in our theory, by the constrained state of the magnetic fluid, repelled from the extremities of each element in the compound bundle, by the action of all the blades united, and above all by that of the surfaces; action which takes place in a sensible manner only at the extremities of the bundle. When the bundle is decomposed, the action of the parts remote from the extremities, which remains nearly the same as in the compound blades, pushes the magnetic fluid towards the extremities; whence results the increase in *moment* which we have just found by experiment.

26.38 XXXVIII. *Eleventh Experiment*

Decomposition of a bundle of four blades.

[1093] In the tenth experiment it appears that the twisting force of the fourth needle was 44.5°.

[1094] The correct result of this calculation is

$$\frac{39 + 14\frac{1}{2} + 44}{3} = 32.5 .$$

I have the impression that Coulomb arrived at the result $31\frac{1}{6}$ by using, by an oversight, the value 40 instead of 44 in the third term of the numerator.

I collected only four of the previous needles, after having magnetized them to saturation; the bundle, displaced 30 degrees from its meridian, was brought back [to the meridian] by a force measured as ..... 150°.

These separated needles were brought back to the meridian: The first, by a force of *moment*, measured by ..... 70°.

The second, by ..... 44°.

The third, by ..... 44°.

The fourth, by ..... 60°.

26.39 XXXIX. *Twelfth Experiment*

Decomposition of a bundle of eight blades.

Eight united needles were brought back to the magnetic meridian, from which they were displaced 30 degrees, by a force of ..... 183°.

The needles were separated.

The first was brought back by a force measured by ..... 48°.

The second, by ..... 36°.

The third, by ..... 35°.

The fourth, by ..... 33°.

The fifth, by ..... 34°.

The sixth, by ..... 38°.

The seventh, by ..... 35°.

The eighth, by ..... 51°.

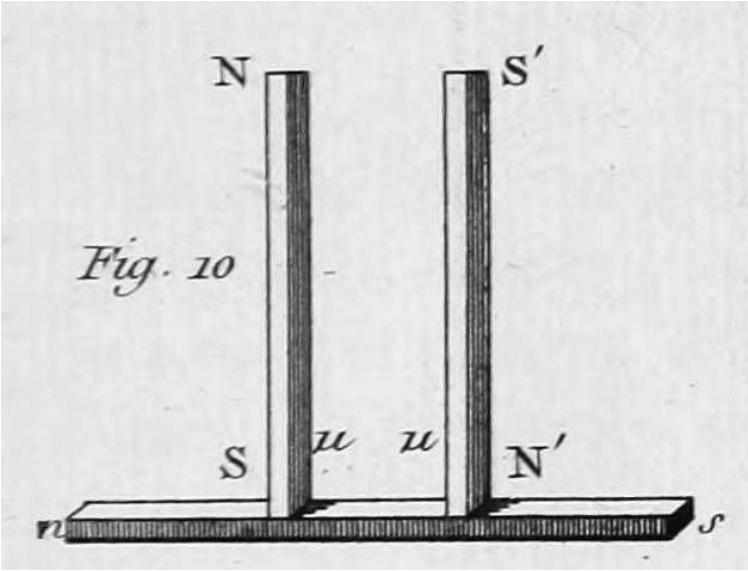
It is useless to dwell on these two experiments. They give results analogous to those we have developed in the preceding Sections: we are going to pass to the methods for magnetizing the blades to saturation, and for forming artificial magnets.

26.40 XL. *How to Magnetize*

I am going to present the means that have been most successful for me, to construct, with little expense, artificial magnets of very great strength; it will be easy to see that I have been guided by the preceding experiments and observations.

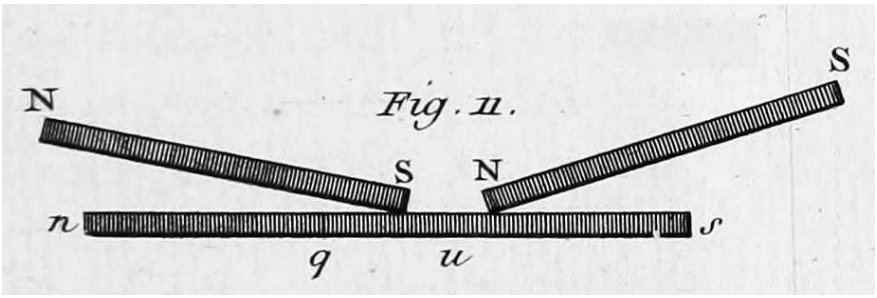
When we want to magnetize a steel wire or blade, we note that the advantage, when we use two bars to magnetize, to make the action of the two poles of these bars cooperate. This is what led to the idea of the double touch method.[1095] *Figure 10* shows how it was first practiced.

[1095] See Section 4.6.



On the needle ns that we wanted to magnetize, we placed two bars SN and $S'N'$ vertically at approximately 7 or 8 lines apart [1.58 or 1.81 cm] depending on the strength of the magnets: the points S and S' represent the South poles, and N and N' the North poles. The two bars in this configuration are moved from one end of the needle ns to the other.[1096]

Mr. OEpinus[1097] has noticed that in this method the center of action[1098] of the two magnets NS and $N'S'$, being necessarily located at some distance from their extremities, at the point μ, for example, the action on the points of the needle, located between the two [magnetized] bars, are made very obliquely, and consequently do not give to this needle the degree of magnetism which it might receive. Thus, instead of placing the two bars vertically in this operation, Mr. OEpinus advises to incline them on the needle, as in *Figure 11*, and to slide them in this configuration from one end of the needle to another.



I actually found, by means of the magnetic balance, which I described at the beginning of this Memoir, that the method of Mr. OEpinus was preferable to the first; but at the same time, I have found that it does not quite produced magnetic saturation in the needles; more often, when the needle is long, several poles form in the intermediary parts, whose action was, in truth insignificant, but still perceptible. I attribute the cause to the particular action of each magnet, which tends to produce on the points passed by the two magnets, an effect

[1096] The two magnets NS and $S'N'$ are moved together maintaining a fixed separation between them. When the procedure is finished, the left end of the needle acquires a North pole, n, while the right end acquires a South pole, s.

[1097] See footnote 182 on page 73.

[1098] See footnote 568 on page 243.

contrary to that sought. In our *Figure 11*, the pole S, for example, placed on the needle, tends to give at the same time to the point q which is placed under the pole S,[1099] the same nature of magnetism, as to the point u;[1100] that is to say, in the hypothesis of the two magnetic fluids, which can be transported towards the two extremities of the needles, if the point u[1101] is drawn towards the point n, the neighboring point q will be drawn towards the point s, after this point q will have been passed by the two magnets: in our hypothesis, where the magnetic fluid can only move in the integral parts,[1102] the molecules u[1103] and q, which are adjacent, tend to magnetize each other in the opposite manner; which must produce a diminution of magnetism towards the extremities of the needles, where the magnetic fluid must be the most condensed, and which may, in very long needles, as experiment proves, give rise to several poles. This observation, which could only be the fruit of the exact measurements given by our experiments, obliged me to change the method of magnetizing of Mr. OEpinus; and here, after several attempts, is the method which, according to the magnetic balance, has appeared to be the most advantageous.

I make use, for my operation, of four very strong magnets, made according to a method which I will presently detail. I place, *Figure 12*, my two strongest magnets NS and NS in a straight line, on a horizontal plane so that they are separated from each other by a few lines less than the length of the needle ns, which I want to magnetize.

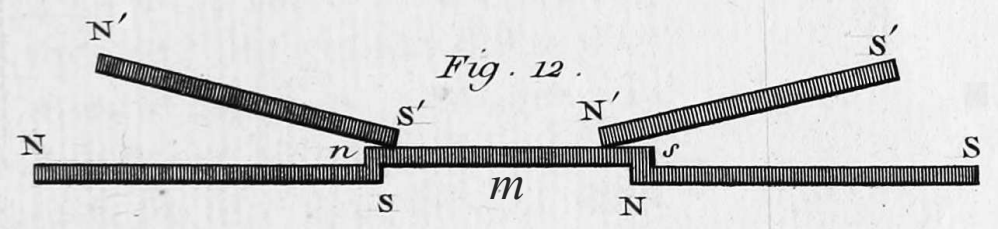


Fig. 12.

I then take the two magnets $N'S'$, and tilting them as in the method of Mr. OEpinus, I place them down first, almost joining their poles on the middle m of the needle;[1104] I then pull each magnet, without changing its inclination, to the end of the needle, and I repeat this operation five or six times on the different faces of the needle. It is clear that in this operation, the poles of the needle ns remain fixed and invariable at the extremities of the needle, by means of the two strong magnets NS, on which this needle is placed: the effect produced, by these two magnets, can only be increased by the action of the two upper magnets which combine to magnetize all the molecules of the needle in the same direction.

As with the preceding operation, the needle ns, placed between the two large magnets, acquires, through the action of the four magnets, a stronger polar force than that which it can retain when separated from these magnets, it follows that at the moment of this separation, the needle loses a part of the magnetism which it owes to these forces, and that its magnetism decreases until the magnetic action of the whole needle, at each of its points, is in equilibrium with the coercive force. Thus, by separating the needle from the magnets,

[1099] I replaced "under the pole s" with "under the pole S".
[1100] In the original text, μ appears here instead of u.
[1101] In the original text, μ appears here instead of u.
[1102] That is, in which the magnetic fluid can only move within each molecule of the bar, without passing from one molecule to another. See further [Gillmor, 1971a, pp. 159, 201 and 217] and footnote 395 on page 176.
[1103] In the original text, μ appears here instead of u.
[1104] I added in Figure 12 the letter m that does not appear in the original image.

it is magnetized to saturation.

I also found that by magnetizing by our method, we were more certain to give to the surfaces of the blades intended to form needles, to indicate the magnetic meridian, an equal degree of magnetism; a point which should receive great attention if the needle is suspended sideway in the construction of compasses.

26.41 XLI. *Construction of Artificial Magnets*

I took, *Figure 13*, about thirty steel blades quenched and tempered to spring consistency, 5 or 6 lines wide (1.1 to 1.3 cm), by 2 or 5 lines thick (0.45 to 0.68 cm), and 36 inches in length (97.45 cm).

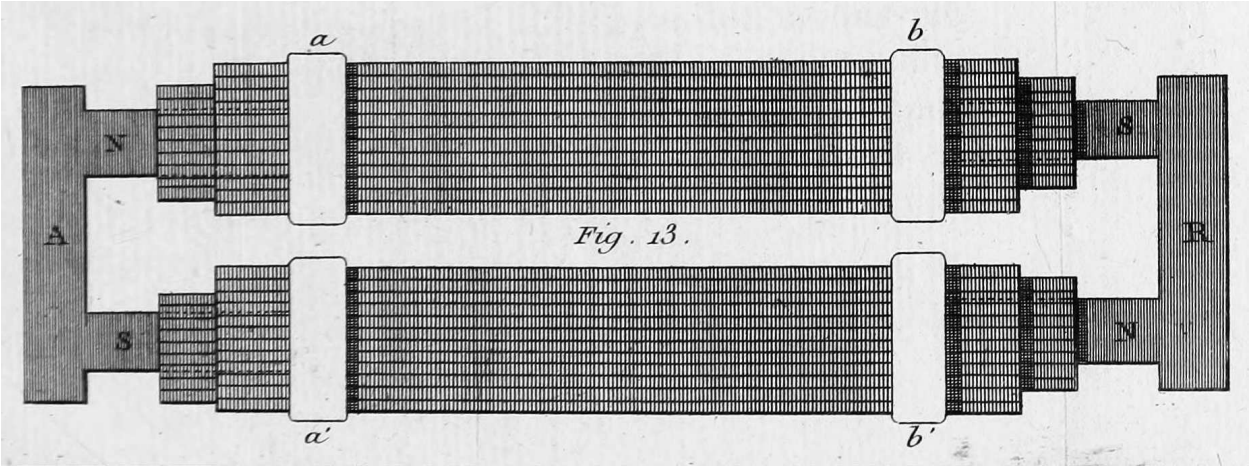


Fig. 13.

Foil blades, such as are found in the commerce, make fairly good magnets. English steel sheet, cut in one inch [2.7 cm] wide strips, quenched and annealed to spring consistency, to the grades given in *Section 26.33*, is preferred. When I use only 15 or 20 pounds (7 to 10 kg) of steel for each magnet, it suffices to make the blades 50 to 56 inches long (81 to 97 cm).

I magnetize each blade singularly, according to the method prescribed in the preceding Section: I then take two rectangular parallelepipeds of very soft and very well polished iron, six inches long (16.24 cm), 20 to 24 lines wide (4.5 to 5.4 cm), and from 10 to 12 lines thick (2.2 to 2.7 cm); I form, with these two parallelepipeds, represented, *Figure 13*, in N and S, the armor of my magnet, by wrapping one end of each parallelepiped with a layer of my magnetized steel blades, so that the end of the parallelepipeds exceeds the end of the blades, by 20 to 24 lines, and such that the other end of the parallelepipeds is surrounded by the end of the blades. On this first layer of steel blades, 3 to 4 lines thick [0.68 to 0.90 cm], I place a second [layer] which is 3 inches [8.12 cm] less in length than the first, so that the first exceeds this second by 18 lines [4.07 cm], of each side; the whole is fixed at the extremities, by means of two copper rings which press the blades against each other and which prevents the armor from escaping.

Figure 13 represents two artificial magnets, composed according to the method which we have just prescribed; N and S, are the two extremities of the two iron parallelepipeds; the two other extremities, engaged between the steel blades, are shown as dotted lines in this same Figure. Each magnet thus composed, is firmly fixed by copper rings which are marked

on the two magnets at a, b, a', b', the contacts placed at A and B,[1105] unite the poles of the armors.

Experiment has taught me that with a device of this shape, each magnet weighing 15 or 20 pounds [7 to 10 kg], requires a force of 80 to 100 pounds to separate the contacts: that by placing ordinary compass needles on the two ends of our two bars, composed as in *Figure 12*, they magnetized each other to saturation, without it being necessary to rub them with superimposed magnets; needless to say, when we wish to obtain magnets of a greater force, it will be necessary, as we multiply the number of steel blades, to increase their length, and the dimensions of the iron parallelepipeds, which serve as armor. It would be easy to estimate the different dimensions that magnets must have in a sufficiently exact manner in practice, according to the laws of magnetism and the position of the center of action of steel wires of different lengths and thicknesses, all of which we have presented in the course of this Memoir.

[1105] In Figure 13, the letter R appears instead of the letter B.

Chapter 27

Remarks on Coulomb's Seventh Memoir

A. K. T. Assis

27.1 Calculation of the Magnetic Moment of a Magnetized Needle When the Density of the Magnetic Fluid Varies Linearly Along Its Length

In Section 26.24 Coulomb presented the result of the magnetic moment of a magnetized needle when he assumed the density of the magnetic fluid to vary linearly along the length of the needle, see footnote 1053 on page 473. I will detail here how Coulomb arrived at this result.

Initially I suppose a magnetic dipole composed of two particles separated by a distance $2x$ along the ℓ axis. One of these particles has a North fluid of intensity μ and is located at $\ell = x$, while the other particle has a South fluid of intensity $-\mu$ and is located at $\ell = -x$, Figure 27.1.

Figure 27.1: Magnetic dipole.

This configuration is called a magnetic dipole. The magnetic dipole moment m is defined by

$$m = \sum_{i=1}^{2} \ell_i \cdot \mu_i = (-x)(-\mu) + x\mu = 2x\mu \ . \tag{27.1}$$

Figure 27.2 presents a magnetized needle sn of length $2x$. Coulomb assumed in Section 26.24 a density of magnetic fluid varying linearly along the length of the needle and

having maximum intensity A at the tip of the needle. Representing this linear density of magnetic fluid by the letter λ, its variation along the ℓ axis is given by Figure 27.2.

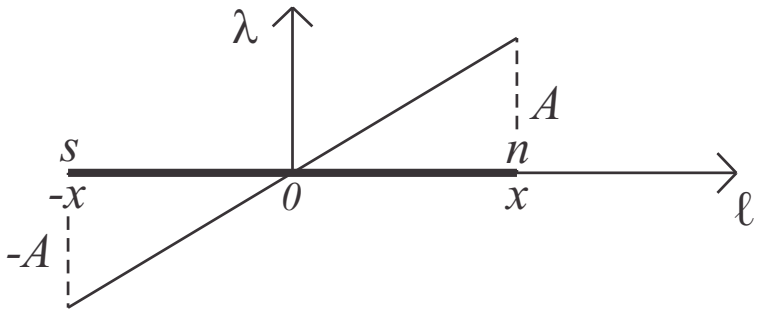


Figure 27.2: Magnetized needle with a density of the magnetic fluid varying linearly along its length.

Considering this needle along the ℓ axis, with its center at the origin 0 of this axis, we can assume that λ varies linearly as a function of ℓ as follows:

$$\lambda = \frac{A\ell}{x} \ . \tag{27.2}$$

According to Equation (27.2) we have $\lambda = -A$ when $\ell = -x$, $\lambda = 0$ when $\ell = 0$ and $\lambda = A$ when $\ell = x$. The unit of λ and A is the unit of magnetic fluid per unit of length. The magnetic dipole moment of this needle is obtained by Equation (27.1) by replacing the sum with an integral along the needle:

$$m = \int_{\ell=-x}^{x} \ell \cdot d\mu = \int_{\ell=-x}^{x} \ell(\lambda d\ell) \ , \tag{27.3}$$

where $\lambda = d\mu/d\ell$ is the linear density of magnetic fluid. Using Equation (27.2) and integrating we get:

$$m = \int_{\ell=-x}^{x} \ell \frac{A\ell}{x} d\ell = \frac{A}{x} \int_{\ell=-x}^{x} \ell^2 d\ell = \frac{A}{x} \left[\frac{\ell^3}{3} \right]_{\ell=-x}^{x} = \frac{2Ax^2}{3} \ . \tag{27.4}$$

This was the equation presented by Coulomb on page 473, see footnote 1053.

27.2 Figure 5 Reworked

Figure 5 that appears on page 475 of Section 26.26 represents a magnetized cylindrical needle. I have reworked this image in Figure 27.3 to indicate the letters more clearly.

Also, I have included the letters α, β, γ, σ, ϕ, and θ to distinguish the needle parts. Points 1, 2 and 3 are along the axis of the cylinder. Point 1 is at the junction between the parts α and β, as well as at the junction between the parts ϕ and θ. Point 2 is at the junction between the parts β and γ, as well as at the junction between the parts σ and ϕ. Point 3 located in the center of the cylinder is at the junction between the parts γ and σ.

This cylindrical needle ab is magnetized to saturation along its axis. Coulomb assumes that the density of magnetic fluid in each part is uniform. By symmetry, he further assumes

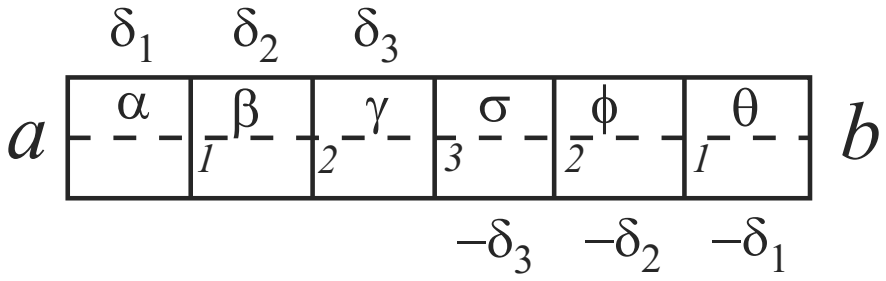


Figure 27.3: A new Figure 5.

that the magnetic density in the first part, α, is equal and opposite to the magnetic density in the last and sixth part θ. That is, if the magnetic density of the first part is δ_1, then the magnetic density of the sixth part will be $-\delta_1$. Likewise the density δ_2 in the second part, β, is equal and opposite to the density of the fifth part ϕ. The density δ_3 of the third part, γ, is equal and opposite to the density of the fourth part σ. For this reason, I replaced in this new Figure the densities of the parts σ, ϕ and θ with $-\delta_3$, $-\delta_2$ and $-\delta_1$, respectively, instead of utilizing the densities δ_3, δ_2 and δ_1 that appeared in Coulomb's original Figure.

Chapter 28

Theoretical and Experimental Determination of the Forces that Bring Different Magnetic Needles Magnetized to Saturation Back to Their Magnetic Meridian

Coulomb[1106]

1. In the various Memoirs that I presented to the former Academy of Sciences, I found, by means of my torsion balance, by experiments which appear to be decisive, the principal laws of action of the elements of the magnetic fluid.

2. It follows from these experiments that, whatever the cause of magnetic phenomena, all these phenomena could be explained and subjected to calculation, assuming in the steel laminae or in their molecules, two magnetic fluids,[1107] the parts of each fluid repelling each other in direct proportion to their density, and in inverse proportion to the square of their distance, and attracting the molecules of the other fluid in the same ratio; so that each lamina of iron or steel contains in each molecule, before being magnetized, a sufficient quantity of both fluids to saturate or balance each other, that the two fluids thus combined no longer exert any action on each other.[1108]

3. It follows from this supposition that the whole art of magnetizing a lamina consists in separating the two fluids, and I have proved in the Memoirs I have just cited that whether they are only separated in each molecule of the steel, or whether they are transported from one end of the lamina to the other, the results are the same as regards the calculation.

4. But as these two supposedly separate fluids in the magnetized laminae, act to reunite each other; they would indeed come together, if there were not in the magnetized laminae

[1106] [Coulomb, 1801a] with Portuguese translation in [Assis, 2022]. This work was read in 1799 to the French Institute of Sciences and published in 1801.

[1107] In the original: *fluides aimantaires*. See also [Gillmor, 1971a, p. 216].

[1108] [Note by Bucciarelli] I was struck by Coulomb's reference to molecules, both of steel and of the magnetic fluid(s) and how the latter adhere to the former unless separated by the action of the magnetic fluid itself. I conjecture that Coulomb was influenced by the work of Laplace who set out to explain "all terrestrial phenomena" by means of "sensible forces at insensible distances" among molecules (in all kinds of stuff). See [Bucciarelli and Dworsky, 1980, Chapters 1 and 6] for a discussion on the subject.

some force which prevented this reunion. The simplest supposition to satisfy this condition, is a force of adhesion[1109] of this fluid to the molecules of the steel, which prevents it from moving. But if this force of adhesion exists, it has a limit: so whenever the action of the magnetic fluid on a molecule of this fluid, will be more considerable than its adhesion to the steel, this molecule will move, and this displacement will continue until there is equality between the forces that act on each magnetic molecule[1110] to move it, and the force of adhesion opposed to this displacement.

5. It follows from the preceding Article that the distribution of magnetic fluid in a magnetized lamina, offers to the calculation an indeterminate problem; because this fluid can be distributed in any way possible, provided there is no point in the lamina where the action which tends to move it is greater than the adhesion of the fluid to the steel molecules. Among all assumptions that can be made for the distribution of this fluid, and which make this problem determined, there is one where we can say that the needle is magnetized to saturation: it is the one where each point of the fluid experiences from all the fluid of the lamina an action which tends to move it, precisely equal to the one that the cohesion opposes to this displacement. This condition determines, as we see, the disposition of the fluid, and for this case the question can be submitted to calculation.

6. We manage to magnetize to saturation, or at least to approach this state in steel laminae, either by the method of the double touch,[1111] or by the one which I have used.[1112,1113] By this last method, the magnetic fluid is transported from one end of the lamina to the other, and is consequently separated by the combined forces of the opposite poles of four strong magnets. When we then separate the magnetized lamina from the magnets, the fluid is found to have, at the ends of the lamina, more density than in the state of saturation, that is, all the fluid spread in the lamina acts on each of its molecules with a force greater than the resistance of the adhesion: thus the magnetic fluid moves from each point of the needle, until there is equality between the action that tends to move it and the adhesion which opposes this movement.

It sometimes happens in laminae that are very long relatively to their other dimensions, and especially in those which are strongly tempered, that several magnetic centers are formed;[1114] or that the magnetic center is not located in the middle of the needle. We will report on this effect in another Memoir; we will only say that it is due to this difficulty of placing the magnetic center in the center of gravity of the laminae, which we must attribute a fact absolutely necessary to know in the construction of compass needles. Here is what it consists of. When a long thin steel lamina, which would, for example, be 330 millimeters in length, 10 millimeters wide and 1 millimeter thick, is white tempered, we find that the directive force which brings it back into its meridian is much less than when the needle has returned to spring-like consistency. The opposite takes place in the small needles: it is necessary, for the moment of the directive force to be a *maximum*, that it be tempered red-white.

[1109] In the original: *force d'adhérence*.
[1110] In the original: *molécule aimantaire*. This magnetic molecule would be a particle containing only one of the magnetic fluids. It would be equivalent to the modern concept of a magnetic monopole. See also Section 4.5.
[1111] See Section 4.6 on page 48.
[1112] [Note by Coulomb] Volume of the Academy of Sciences for 1789, p. 504.
[1113] [Coulomb, 1793, p. 504]. See Section 26.40 on page 489.
[1114] See footnote 146 on page 59.

I had already discovered some of these facts;[1115,1116] but I had, at the same time, went too far in generalizing the results. I will have to come back to it in one or two Memoirs that will follow immediately this one, and which will complete the work which I have undertaken on the laws of magnetism and their uses in the construction of magnetic needles.

7. I return to the subject of the Memoir that I am submitting today to the judgment of the Institute. In one of the experiments described in the Memoir that I have just cited, I bundled together several needles of iron wire into a bundle, and by magnetizing them to saturation thus united, I found that by forming similar bundles, or, which comes to the same thing, one whose corresponding dimensions were proportional, these bundles were brought back to the magnetic meridian by forces whose *moment*[1117] was like the cube of the similar dimensions.[1118] I then tried to prove, by a method of trial and error, that, relatively to the axis of two cylinders magnetized to saturation, the theory gives the same result.

I have today for my object to prove that, whatever the shape of two magnetized needles, as long as the figures are similar, that is to say the corresponding parts are proportional to each other, it results from the experiment that the moment of their directive force towards the magnetic meridian is like the cube of their corresponding dimensions.

I will prove then, by a rigorous method, that, according to the theory that I have just explained, this result must take place. Together, these two proofs leave no doubt — not about the causes of magnetism, which still provides a wide open field [of questions] to all systems — but on the laws according to which we must rigorously calculate and determine all magnetic phenomena.

28.1 First Experiment

8. I obtained from the same sheet of laminated steel two parallelogram needles; they were 250 millimeters long, 30 millimeters wide, and about one millimeter thick.

These two needles have been united by their plane, strongly binding the two ends in such a way as to keep them in contact; we then magnetized them to saturation, we placed them [together] in the torsion balance of which we will speak presently, and we have found that to keep them 27 degrees away from their meridian, a force of torsion of 332 degrees was necessary.

28.2 Second Experiment

9. I cut from the same steel plate a third lamina, which had precisely half the length and width of the first. As it had been obtained from the same plate, it had necessarily half the thickness of the two laminae combined. This lamina being magnetized to saturation, a force of torsion of 42 degrees was needed to keep it, like the first, at 27 degrees from its magnetic meridian.

[1115] [Note by Coulomb] Volume of the Academy for 1789, p. 494.

[1116] [Coulomb, 1793, p. 494]. See Section 26.33, page 485.

[1117] In the original: *momentum*. This word can be translated as "moment", "moment of a force" or "torque". See also Section 4.5 and footnote 150 on page 60.

[1118] That is, the magnetic torque exerted by the Earth was proportional to the volume of the magnetized needle.

28.3 Explanation and Result of This Experiment

10.[1119] I explained, in the Memoirs of the Academy for 1789,[1120] all the details of the construction of a torsion balance based on the laws of the force of torsion of metal wires. Here is the precise details of this construction. When a cylinder is suspended from a very fine metal wire, so that the axis of this cylinder is in the extension of the thread and the point of attachment, there will be a position where this cylinder will stop, and this orientation is that where the torsion of the thread is null; but if, without disturbing the axis of the vertical situation where the cylinder is located, we make this cylinder turn around this axis, the wire will be twisted, and the force of torsion, when the cylinder is released, will force it to rotate and oscillate around this axis. However, if we observe with a seconds watch the times of the oscillations, we will find that no matter if the torsion angle is only of a few degrees or of several circles,[1121] the oscillations will be isochronous; whence it follows, by a theory known to all geometers, that the forces of torsion of the same wire are proportional to the angle of torsion. The absolute value of this force of torsion is then determined in weight in an exact way, according to the time of the oscillations of the cylinder, of which we know the weight and the radius. I have proved[1122,1123] that by determining the force of torsion of a metal wire, from the oscillations of a suspended cylinder to this wire, and rotating around its axis by means of this force of torsion, I found that the moment of this force was equal to $\left(\frac{Pa^2}{2\lambda}\right)$ multiplied by the angle of torsion, where P is the weight of the cylinder, a its radius, and λ the length of the pendulum that beats oscillations isochronous with the oscillations of the cylinder. We find in the volume of the Memoirs of the Academy for 1784, all the experimental details and calculation necessary to determine the force of torsion of the threads of suspension relative to their length, size and nature.

11. Now, to use the force of torsion of a metal wire to determine the ratio of the force that brings two needles back to their magnetic meridian, we only need to know that the force of torsion for the same wire is proportional to the angle of torsion. Accordingly, we suspend in a box, horizontally and successively by means of a metal wire, the two magnetized needles, making sure that, when the needles are in their magnetic meridian, the torsion is null. We then twist the wire at its top by means of a clasp which holds it tight and which carries a pointer which measures the angle of twist, making sure, in our two experiments, that the twist be such that the magnetized lamina [at the bottom], in each case, forms the same angle with respect to the magnetic meridian, then the moment of the force [tending to] bring back the two needles to the meridian is proportional to the angle of twist. To obtain the true angle of torsion, it is necessary to subtract from the angle of twist measured by the pointer [at the top], the angular displacement [at the bottom] of the magnetized lamina from the meridian caused by the torsion.[1124] We will find in the volume of the *Mémoires de l'Académie* for 1789, all the details according to which we can determine the magnetic laws by means of the torsion balance: it is only necessary to warn [the reader] that, in the use

[1119] Articles 10 and 11 were not included in the reprint of Coulomb's works edited by Potier, [Potier, 1884].
[1120] [Coulomb, 1793]. This Memoir is translated in Chapter 26.
[1121] That is, the angle of torsion can be that of a few degrees, or a large one of several complete turns around the axis of the cylinder.
[1122] [Note by Coulomb] *Mémoires de l'Académie* for 1784.
[1123] [Coulomb, 1787]. This Memoir is translated in Chapter 7.
[1124] That is, the true angle of torsion is given by the angle indicated by the pointer attached to the top of the wire, minus the angle of the needle in relation to the magnetic meridian. I present in Section 16.3 an illustration of this procedure.

of this instrument, we must observe the needles to the right and to the left of the meridian, and take an average to correct the error that may result, either from the uncertainty of the meridian line drawn on the middle of the needle, or of the initial angle of torsion relative to this meridian.

12. Here is the result of the experiment which precedes. The needle, made up of two large laminae, in the first experiment, had all the dimensions double the size of the small lamina of the second experiment: thus the cubes of these dimensions were between them :: 8 : 1. We find, for the forces of torsion, the numbers 322 and 41; which are very close to each other :: 81 : 10. Thus the moments of the forces which bring back the two needles to their magnetic meridian, are between them as the cube of their homologous dimensions.

28.4 Third Experiment

13. I put together three laminae similar to the two of the first experiment, and to move this needle thus composed, away of 21 degrees from its meridian, I found that a torsion of 340 degrees was necessary.

28.5 Fourth Experiment

14. A lamina taken from the same plate, but which had only a third of the width and length of the three previous ones, was held at 21 degrees from its meridian by a force of about 13.5 degrees.

15. In the last two experiments, the cubes of homologous dimensions are between them :: 27 : 1. The forces of torsion are in a ratio slightly greater than 25 to 1, quantities which can be considered as very approximate in experiments of this kind.

16. Finally, to have no doubt about the continuity of this law, I wanted to compare needles, either parallelogram or cylindrical, whose ratio of cubes was represented by a very large number like, for example 150 to 1. Moreover, in the preceding experiments, my first needles were of several pieces, and I wanted to compare needles of a single piece, to know if the needles or magnets, composed of one or more pieces, had the same strength as the others; but I realized, according to the results of the preceding experiments, that by placing very small needles in the yoke of the magnetic balance[1125] which is intended to carry these needles, I would have, by moving these small needles of 20 to 30 degrees from their meridian, only very small angles of torsion, and that the errors of observation would then introduce uncertainty into the results. I decided, in this case, to use the method of oscillations which is suitable for this kind of experiment, and the calculation of which is very easy when we only want to compare simple shapes which have the same number of equal fibers throughout their length.

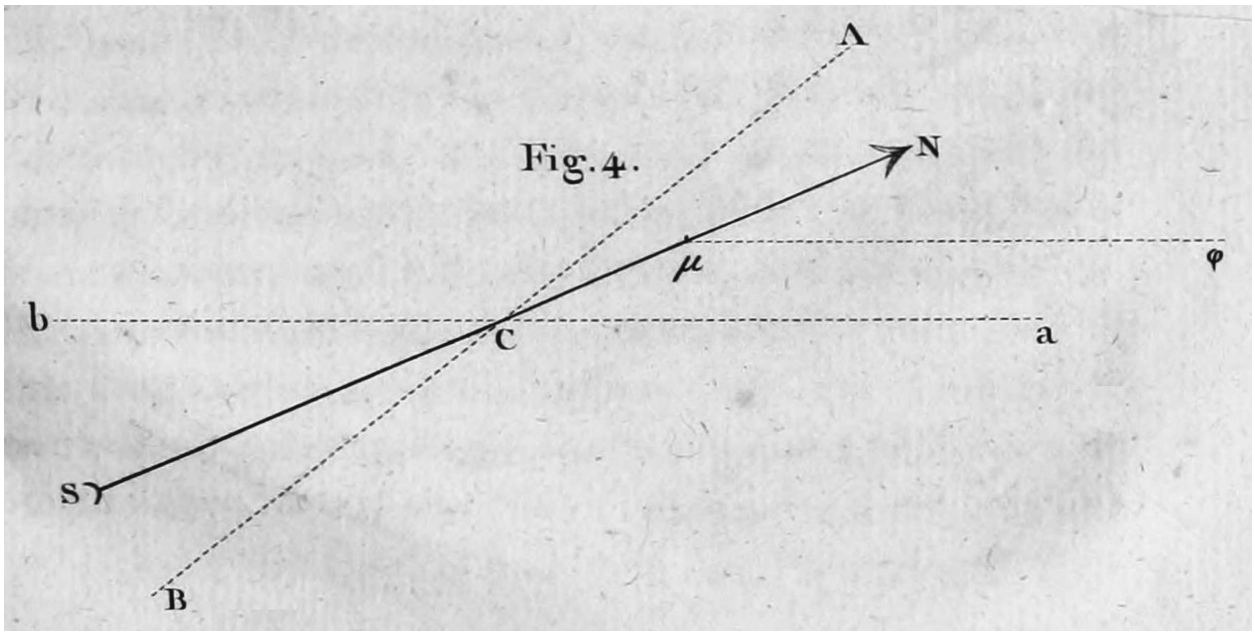
17. Here is what this method consists of. Euler[1126] had found before me, and I developed this theory in the ninth volume of the *Mémoires des Savans étrangers*,[1127] that when a magnetic needle, either parallelogram or cylindrical in shape, oscillates forming small angles

[1125] In the original: *en plaçant de très petites aiguilles dans la chape de la balance magnétique.* The word "chape" can also be translated here as cover, cap or frame.
[1126] See footnote 164 on page 66.
[1127] [Coulomb, 1780]. This work is translated in Chapter 5.

with the magnetic meridian, the moment of the forces which bring it back to this meridian was quite exactly represented by the formula[1128]

[1128][Note by Coulomb] Here is the proof of this result. In Figure 4, ab represents the magnetic meridian; $ACB = A$, the angle that the needle forms with its meridian, when it begins to oscillate around its center C, angle that we suppose very small; φ the magnetic force of the Earth which acts on point μ parallel to the magnetic meridian; NC the position of the needle at the end time t:

$$ACN = s \; ; \qquad NCa = (A - s) \; ; \qquad C\mu = r \; .$$



Being μ a magnetic molecule placed at μ, we have for the moment of the action of the Earth which brings the needle back to its meridian CA, [the following expression:]

$$(A - s)dt \int \varphi \mu r \; ;$$

and u being the angular velocity, we will have rdu for the acceleration of point μ, and $du \cdot \int \mu r^2$ for the moment of acceleration of the whole needle; from which results

$$(A - s) \int \varphi \mu r \cdot dt = du \int \mu r^2 \; ,$$

or

$$(A - s)dt \frac{\int \varphi \mu r}{\int \mu r^2} = du \; .$$

But if an ordinary pendulum swings, we have [by assuming L to be the length of this simple pendulum:]

$$(a' - s')dt \frac{g}{L} = du \; .$$

Thus, if we suppose, which is very permitted, that the two equations are identical, the needle and the pendulum will make their oscillation at the same time, and we will have in this case

$$\frac{Pl^2}{3\lambda}\,,$$ [1129]

multiplied by the angle from which it is distant from this meridian, where P is the weight of the needle, l half of its length, and λ the length of a pendulum swinging with oscillations isochronous to that of the needle.

So if, in the experiments where we want to compare two similar needles, we make P the weight of the first, l its length, and λ the [length of the] pendulum which beats oscillations isochronous to the vibrations of this needle; P', l' and λ' the corresponding quantities of the second needle; if we call φ the magnetic moment of the first, and φ' that of the second, we will have

$$\frac{\varphi}{\varphi'} = \frac{Pl^2 \cdot \lambda'}{P'l'^2 \cdot \lambda}\,.$$

But since the length of two pendulums is in the ratio of the square of the time of the oscillations, if T is the time when the first needle makes a certain number of oscillations, and T' the time where the second does the same number of oscillations, we will have $\frac{\lambda}{\lambda'} = \frac{T^2}{T'^2}$. Therefore

$$\frac{\varphi}{\varphi'} = \frac{Pl^2 \cdot T'^2}{P'l'^2 \cdot T^2}\,.$$

But since we want to compare needles here, either parallelogram or cylindrical, of similar dimensions, it follows that $\frac{P}{P'} = \frac{l^3}{l'^3}$. Thus

$$\frac{\varphi}{\varphi'} = \frac{l^5 \cdot T'^2}{l'^5 \cdot T^2}\,.$$

And if φ/φ' were, as we found it, by the previous experiments, proportional to l^3/l'^3, we would have, according to this formula,

---

$$\frac{\int \varphi\mu r}{\int \mu r^2} = \frac{g}{L}\,.$$

But if h is the surface which represents the section of the needle, section of which the dimensions are assumed to be very small relatively to the length of the needle, we will have $\int \mu r^2 = \int hr^2 \cdot dr$, the integral of which is $\left(\frac{hr^3}{3}\right)$, and R being half of the length of the needle, we will have for the whole needle $\frac{2hRR^2}{3}$. Therefore

$$\int \varphi\mu r = g2hR\frac{R^2}{3L}\,;$$

and as $g2hR$ represents the weight of the needle, we have

$$\int \varphi\mu r = \frac{PR^2}{3L}\,,$$

quantity that represents the moment of the magnetic action that the Earth's magnetic force exerts to bring the needle back to its magnetic meridian.

[1129] Footnote 1128 was not included by Potier in the reprint of Coulomb's works, [Potier, 1884, p. 326]. Due to a slip in that footnote, Coulomb wrote $ACB = A$ instead of $ACa = A$. I fixed this bug in Coulomb's Note.

$$\frac{l^3}{l'^3} = \frac{l^5 \cdot T'^2}{l'^5 \cdot T^2}\ , \qquad \text{or} \qquad \frac{l'}{l} = \frac{T'}{T}\ ;$$

that is, assuming that the moments of the magnetic forces of two needles of similar dimensions be, as previous experiments have shown us, proportional to the cube of these dimensions, we must find the times of the oscillations proportional to the lengths of the laminae.

It will therefore be easy to verify, by this very simple relation, whether the law which has been indicated to us in the previous experiments, is still valid when the number that represents the ratio of the cubes of these dimensions is very big.

28.6 Fifth Experiment

18. I took two rectangular parallelogrammatic laminae of cast steel: the first weighed 100.31 grams; the second, 0.61 grams. The cubic roots of these weights are between them :: 5.5 : 1.0; it is also the ratio that we gave to their similar dimensions. The first one had 321 millimeters of length, the second had 58 millimeters;[1130] the other dimensions were in the same ratio. These laminae were both magnetized to saturation, the first one made 30 oscillations in 300", the second one made 30 oscillations in 55".[1131]

28.7 Result of This Experiment

19. If we take the cubic root of the weight of the two needles, we find these roots very approximately :: 55 : 10; the lengths, widths and thicknesses being in the same proportions, we will find the [ratio of the] time of the same number of oscillations :: 300 : 55, [that is,] very approximately :: 55 : 10. Thus the times of the same number of oscillations being like the length of the needles, it results from the calculation of the preceding Article that the moments of the directive forces are between them like the cubes of the dimensions.

The [ratio of the] cubes of the dimensions, and consequently the ratio of the forces, is found here :: 164 : 1; which leaves no doubt about the truth of the result that we established according to the experiment.

28.8 Sixth Experiment

20. I took two cylindrical needles of excellent cast steel, such as are commonly found in commerce.

The first one weighed 46.388 grams; its length was of 322 millimeters. The small one weighed 2.159 grams; it was 115 millimeters long.

The big needle made 10 oscillations in 90"; the small needle made 10 oscillations in 32".

[1130] The ratio of these lengths was $321/58 = 5.5/1.0$.

[1131] That is, the first lamina performed 30 oscillations in 300 seconds, the second in 55 seconds.

28.9 Result of This Experiment

21. The ratio of the cubic roots of the weights of the two needles is approximately :: 28 : 10; that of the lengths of the needles :: 28 : 10; that of the same number of oscillations :: 90 : 32 :: 28 : 10.

These three rigorously calculated ratios, using a larger number of digits, are so close that, in experiments of this kind, they can be regarded as equal.

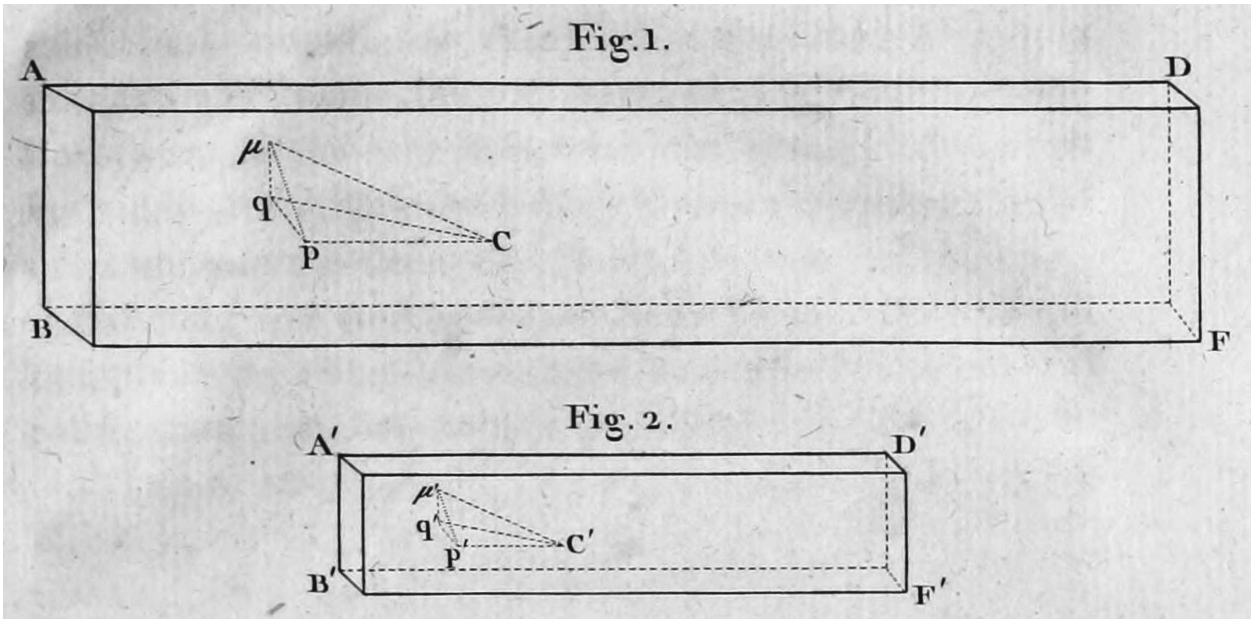
22. I will not unnecessarily lengthen this Memoir by including a number of experiments which all gave me the same result; I only warn [the reader] that, to make them succeed, it is absolutely necessary that the needles be in the same state, that is to say, either annealed red-white, or tempered red-white. The first state is preferable;

- in the first place, because in needles so annealed, unless they have a very great length relative to their other dimensions, it is very rare that their magnetic center do not coincide with the midpoint, or that they have several centers.[1132] This is what must always be verified before making the comparison of the experiments.

- In the second place, if it is very difficult to give, by tempering two needles, precisely the same degree of temper, it is even more difficult, by annealing them to the state of a spring, to give them the same degree of annealing: and therefore, the state of the steel not being the same in the two needles, the adhesion of the magnetic molecules to that of the steel, is not the same.[1133]

22*.[1134] It remains for me, to fulfill the object of this Memoir, to show the agreement of the theoretical calculation with the experiments which precede.[1135]

Figures 1 and 2 represent two parallelepipeds whose sides are homologous.



[1132] See footnote 146 on page 59.

[1133] See footnotes 375 and 405 on pages 165 and 178, respectively.

[1134] By a lapse in the original text, this Article received the same number 22 as the previous Article. I changed its number to 22*.

[1135] The remainder of this Article until the end of Article 24 was not included by Potier in the reprint of Coulomb's works, [Potier, 1884].

I choose these figures because of their simplicity. We will soon see that, whatever bodies are compared to each other, provided that the two figures are similar, the demonstration that follows can be applied to them.

I relate any point c[1136] to its three coordinates perpendicular to each other, and parallel to the faces of the parallelepiped. I make $cp = x$, $pq = y$ and $q\mu = z$.

I then take in the parallelepiped $A'B'D'F'$ a point c', placed in a position homologous to the first one.

I divide each parallelepiped into an infinite number of parallelepipeds similar to the parallelepipeds $ABDF$ and $A'B'D'F'$; so that each parallelepiped contains an equal number of them.[1137]

According to these assumptions, the action of an elementary molecule placed at μ on point c, will be represented by the mass of this molecule multiplied by its density,[1138] and divided by the square of its distance [up to point c].

And if we decompose this force parallel to the axis cP, we will have the force decomposed along the direction of this axis, equal to

$$\frac{\delta dxdydzx}{(x^2+y^2+z^2)^{3/2}} ,$$

where δ is the density of the magnetic fluid in μ. We will have for the small parallelepiped, by naming the same letters with an accent, the corresponding quantities:

$$\frac{\delta' dx'dy'dz'x'}{(x'^2+y'^2+z'^2)^{3/2}} .$$

But since the molecules are assumed, in the two parallelepipeds, in equal number and similar to the parallelepipeds which they compose, it results from this assumption that

$$\frac{x}{x'} = \frac{y}{y'} = \frac{z}{z'} = \frac{l}{l'} = \frac{dx}{dx'} = \frac{dy}{dy'} , \text{ etc.},$$

where l and l' are the lengths of the two parallelepipeds. Thus the force which acts in the second parallelepiped becomes

$$\frac{\delta' l'}{l}\frac{(xdxdydz)}{(x^2+y^2+z^2)^{3/2}} .$$

Whence it results that the action of a magnetic molecule, in the first parallelepiped on point c, is to the corresponding action, in the second parallelepiped on a point c', similarly placed $:: \delta : \frac{\delta' l'}{l}$.

But we will observe that the two parallelepipeds each contain the same number of similar parallelepipeds and placed similarly, relative to points c and c', and that the adhesion being the same in the two parallelepipeds, the sum of the actions of all the magnetic molecules which act along pc in the large parallelepiped, are equal to the magnetic action which acts

[1136] By an oversight in the original text, here appears point μ instead of point c.

[1137] That is, the parallelepipeds $ABDF$ and $A'B'D'F'$ will be composed of the same number of similar small parallelepipeds.

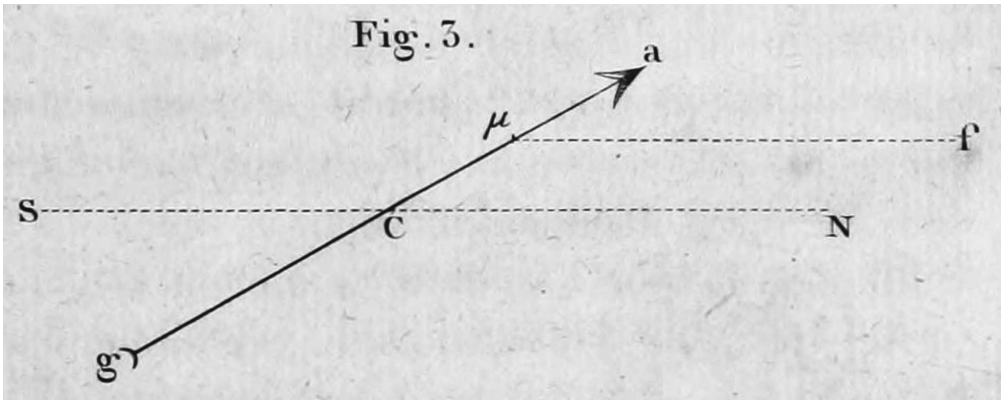
[1138] Coulomb is referring here to the magnetic force exerted by a magnetic particle μ acting on another magnetic particle located at point c. The amount of magnetic fluid in the particle μ is given by its volumetric density of magnetic fluid, δ, multiplied by the infinitesimal volume $dxdydz$ occupied by the particle μ.

similarly on point c' in the small parallelepiped: which will take place if we suppose that the corresponding molecules in the two parallelepipeds, exert on points c and c' an equal action; from which results $\delta' l' = \delta l$.

Thus the magnetic densities of the corresponding points in two similar parallelepipeds, are between them inversely proportional to the lengths of these two parallelepipeds.

23. It is now necessary to prove that, according to this ratio of densities, the moments of the magnetic forces which bring back two similar needles to their meridian, are between them like the cubes of homologous dimensions.

In *Figure* 3, NS represents the magnetic meridian, ag a longitudinal fiber taken in the length of the needle, μ a molecule of this fiber, on which the magnetic force of the Earth acts along μf, parallel to the magnetic meridian.



However, as the center of action from the Earth is at a distance that we can consider as infinite, relative to the length ga of the needle, it follows that it[1139] will be everywhere proportional to the fluid density of the molecule μ, multiplied by its volume. The moment of this force, if the needle forms the angle AcN with its magnetic meridian, will be equal to $\delta\mu c\mu \cdot \sin aCN$.[1140]

If we compare this first result with what would have happened for a corresponding fiber, and similarly placed in the small parallelepiped, we would have for this corresponding fiber [the torque given by:][1141] $\delta'\mu' c'\mu' \cdot \sin aCN$.

Thus the moments of the two corresponding molecules in the two parallelepipeds,[1142] are between them for the same angle aCS :: $\delta \cdot c\mu \cdot \mu : \delta' \cdot c'\mu' \cdot \mu'$. However, the molecules being similar to parallelepipeds,

$$\frac{\mu}{\mu'} = \frac{l^3}{l'^3} \qquad \text{and} \qquad \frac{c\mu}{c'\mu'} = \frac{l}{l'} \ .$$

We have found just now that $\delta l = \delta' l'$. Therefore we will have $\delta c\mu \cdot \mu : \delta' c'\mu' \cdot \mu' :: \delta l^4 : \delta' l'^4 :: l^3 : l'^3$,[1143] as originally we have learned from experiment.

[1139] That is, the magnetic force exerted by the Earth.

[1140] That is, the torque will be given by the volumetric density of magnetic fluid, δ, multiplied by the volume of the particle, μ, multiplied by the distance $c\mu$ and the sine of the angle aCN.

[1141] Due to a printing error, in the original text we have here: $\delta'\mu' c\mu' \cdot \sin aCN$.

[1142] That is, the magnetic torques exerted by the Earth on the two corresponding molecules located on the two parallelepipeds.

[1143] That is,

Thus it also results from experiment and theory, that the moments of the driving forces of two steel parallelepipeds, of the same nature and with the same degree of annealing and tempering, are in the same proportion as the cubes of their homologous dimensions.[1144]

24. We thought it necessary to present the theory which precedes in a particular example where the elementary calculations are very simple; but it is easy to see, and this remark is not for those who are accustomed to deal with these kinds of questions, that the same result takes place in all the bodies of similar figures, since we can always take points similarly placed in the two similar bodies, and suppose each body divided into molecules whose mass is proportional to the total mass of the body; what will give at the same time an equal number of molecules in every body, and all the preceding results. This is again what experiment proves: for by comparing between them magnetic needles of similar figures, such as those used in compasses, which are usually, or parallelepipeds of long and flattened rectangles, or cylindrical needles, or arrow needles, flat or conical, I have always found that the moments of their directive forces were like the cube of homologous dimensions.[1145]

25. When comparing two similar needles, but which are not of the same nature, in this case the adhesion of the fluid in the molecules of the two steel needles, is not the same, and, in the results of Article 23, instead of making $\delta l = \delta' l'$, it is necessary, so that the balance remains, to make $\delta l : \delta' l' :: A : A'$, or $\delta l A' = \delta' l' A$, assuming that A is the force adhesion in the first needle, and A' that of the second;[1146] and in this case, to have the ratio of the moments of the directive force, it will be necessary, instead of $\delta' l' = \delta l$, to put $\delta' l' = \frac{A' \delta l}{A}$; which gives the ratio of the moments of the magnetic forces of the two similar needles, but of a different nature

$$:: \delta l^4 : \frac{A'}{A} \delta l l'^3 :: A l^3 : A' l'^3 \ .$$

Thus, in two similar needles, but of different nature, the moments of the directive force are between them in a composed ratio of the adhesion of the magnetic fluid to the molecules of the steel and of the cube of one of the dimensions.

26. The analytical method that I have just submitted to the eyes of the Institute is, in all respects, elementary; it leads to this observation. Most topics of physics exhibit phenomena of attraction, of repulsion and of cohesion, concerning which it is nearly always more intriguing than useful to determine the causes, and we rarely succeed in this; but it is not the same with respect to the laws of attraction and of repulsion according to which bodies act one upon the other. These laws once known, whatever the position of the molecules, if this position is given, the question is reduced to a problem of analysis most often very difficult to solve, especially when many elements act on each other according to different laws; but there are almost always in every question points of view which simplify them, and which are sufficient to verify the laws which serve as a basis for the calculations, and in which an often elementary analysis can be made.

$$\frac{\delta c \mu \cdot \mu}{\delta' c' \mu' \cdot \mu'} = \frac{\delta l^4}{\delta' l'^4} = \frac{l^3}{l'^3} \ .$$

[1144] That is, the magnetic torques exerted by the Earth on these two parallelograms are proportional to the cubes of their similar dimensions, that is, they are proportional to their volumes.

[1145] That is, the torques exerted by the Earth were proportional to the cubes of similar dimensions.

[1146] The remainder of this work was not included by Potier in the reprint of Coulomb's works, [Potier, 1884, p. 329].

Chapter 29

Result of the Different Methods Employed to Give Steel Blades and Bars the Greatest Degree of Magnetism

Coulomb[1147]

1. We have seen[1148] in the various Memoirs that I have successively presented, either to the Academy of Sciences or to the Institute, that the bars and blades of steel, magnetized by certain methods, soon acquire a magnetic force that they could not exceed. This state is that of magnetic saturation; it takes place when the resultant of the magnetic action of all magnetized points which tend to neutralize the magnetism of each magnetized point[1149] is in equilibrium with the coercive force, or with the adhesion of the magnetic fluid to each molecule of steel of the magnetized blade.

2. I proved in volume III of the *Memoirs of the Institute*, p. 176 and following,[1150] that when bars or blades of homologous dimensions are magnetized to saturation, it follows from experiment, as well as from theory, that the moments of the magnetic forces which direct them toward the magnetic meridian are in the ratio of the cube of one of their dimensions, or, if you like, as [the ratio of] their volume or as [the ratio of] their weight.

Experiment as early as 1789 (*Mém. de l'Académie des sciences*, p. 466),[1151] had given me this result — one easy to verify by the relevant analysis which shows that the times required for the same number of oscillations are in the ratio of one of the dimensions; so that if the corresponding dimensions of the two bars that we want to compare are to each other as 2 is to 1, the times of the duration of the same number of oscillations will also be to each other as 2 is to 1.

3. This result, as well as all those of the different experiments that I communicated, either to the former Academy of Sciences, or to the Institute, proves, as I have already said several times, that all magnetic phenomena can be calculated, supposing *two magnetic*

[1147] [Coulomb, 1806] with Portuguese translation in [Assis, 2022]. This work was read in 1802 to the French Institute of Sciences and published in 1806.
[1148] Potier did not include Articles 1 to 13 of this paper in his reprint of Coulomb's works, [Potier, 1884].
[1149] Magnetic fluids of opposite types attract each other. Therefore they tend to neutralize each other.
[1150] [Coulomb, 1801a]. This work is translated in Chapter 28.
[1151] [Coulomb, 1789, p. 466], see Section 22.34 on page 368.

fluids whose molecules attract each other reciprocally in inverse proportion to the square of the distances [between them], and repel their own molecules in the same ratio;[1152] and by further assuming that *the two fluids, separated by any means, can be retained in this state of separation by their adhesion to steel: the limit of this adhesion determines the maximum [level] of magnetism, that is, the state of saturation.* I do not pretend, as I have already said several times, to explain the cause of magnetism, but a law certainly exists, whatever the cause which produces it.

4. To be sure that a needle is magnetized to saturation, it is necessary to have recourse to a means of determining exactly the magnetic intensity of this needle, by magnetizing it according to the various methods in use until now. This is the subject of the Memoir that I am presenting today to the Institute.

5. For a long time, we have been content to measure magnetic intensity by supporting, at the tip of a magnetized bar, a piece of iron that one loads successively with different weights; but this method, which only determines the adhesion of the iron at the point of the bar with which it is in contact, and which varies according to whether the iron and the bar are polished — more or less — cannot be admitted in precise investigations which need to be verified by calculation. The most exact method for determining the magnetic intensity by experiment consists in suspending the magnetic bar horizontally, by means of an untwisted silk thread; one counts the time taken by a certain number of oscillations, and one deduces the force which directs the bar along its magnetic meridian, from the duration of each oscillation.

Here is the basis of this method: the globe of the Earth being a natural magnet, whose center of action on the oscillating bar is at an infinite distance relatively to the length of the bar, the frequency of the oscillations[1153] will necessarily increase as the magnetic intensity of each point of the bar increases. We can see, volume III of the *Mémoires de mathématiques et de physique de l'Institut*, p. 186,[1154] that when a bar or blade of the same size throughout its entire length oscillates about its magnetic meridian, the moment of the force which directs it in this meridian is expressed by the formula $\frac{Pl^2}{3\lambda}$, where P is the weight of the bar, l half of its length, and λ the length of a pendulum which would make oscillations of the same duration as those which the magnetic force causes the bar to make.

Thus, if we magnetize the same bar by two different methods, and if we observe the times of the duration of the same number of oscillations of the bar magnetized by these two methods; the lengths of two pendulums being in the same ratio as the square of the times of the same number of oscillations,[1155] the moments of the forces which will direct the bar in the magnetic meridian will be like the inverse ratio of the length of the pendulums, and consequently the moments of the forces will be like the inverse ratio of the square of the times of the same number of oscillations; which, as we see, makes the evaluation of the directing force depend on a very simple calculation.

6. Today, because not all physicists have a torsion balance at their disposal — the method which I used in my first Memoirs, and whose use is absolutely necessary in various researches of the same kind — I prefer to determine the magnetic force by [the method which counts] the oscillations of the bar, and by observing a very large number of oscillations we have a degree of precision which it would be difficult to hope for by any other means. Besides, to

[1152] That is, repelling molecules that have the same type of magnetic fluid.
[1153] In the original: *la vitesse des oscillations.* Coulomb is referring here to the frequency of the oscillations.
[1154] [Coulomb, 1801a, p. 186], see footnote 1128 on page 504.
[1155] See Section 16.1 on page 267.

determine the directing force by means of oscillations, only a band of cloth or paper, into which the bar is introduced and supported by an untwisted silk thread is required to have it oscillate — a device which can always be procured without care or a workman. I pass on to the various methods in use for magnetizing the blades and the bars.

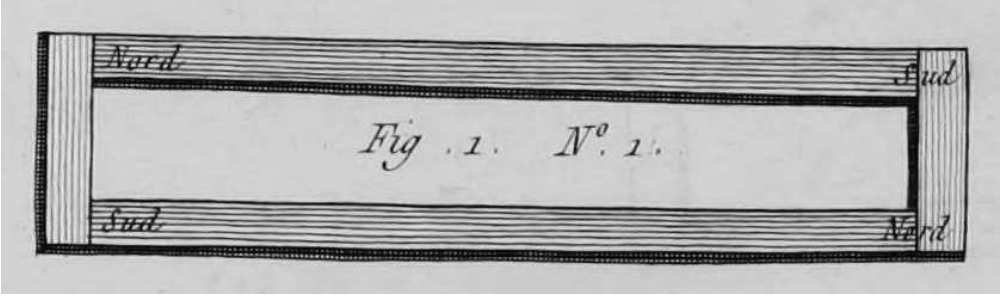
7. In the first method, which was the only one in use for a long time, the plate or the bar which we wanted to magnetize was made to slide at right angles to one of the poles of a natural or artificial magnet;[1156] we shall presently see that this method can magnetize to saturation only needles of very small thickness.

8. After various attempts, the method which seems to have approached the theory is due to Mr. Knigth. We find the details of it in the *Philosophical Transactions*, in 1745.[1157] This physicist placed two strongly magnetized bars in a straight line, joining the North pole of one of the bars to the South pole of the other; he then placed a small bar tempered [to the color] light cherry[1158] along the large bars, the middle of the small bar meeting the junction of the other two; then by sliding the large bars each on their side to the ends of the small bar, the small bar was charged with a greater magnetic force than was previously possible.

By this method, if the large bars which have been used are very strongly magnetized, the small bars, when they are very short and have little thickness, will attain, approximately, the level of magnetic saturation; but it will be impossible by this means to give a somewhat longer bar the state of saturation, whatever its thickness.

9. The little bars of Mr. Knigth, which populate the offices of physicists, induced several physicists at this time to seek other means of providing bars with the same degree of magnetism. Mr. Duhamel, member of the Academy of Sciences, having joined with M. Antheaume in this research, succeeded in doing so by the following means.[1159]

He formed (*Figure 1, [Number 1]*) a rectangular parallelogram with two steel bars and two parallelepipeds of soft iron which were much shorter than the bars.[1160]



Taking afterwards two bundles of bars already magnetized, he places the poles of different names onto the middle of one of the bars of the parallelogram; then tilting these bundles as seen in Figure 1, No. 2, he slides them slowly, each on their side, to the end of the bars.[1161]

[1156] See Figure 4.11 on page 51.
[1157] See Section 4.6 on page 48. See also [Knight, 1744a], [Knight, 1744b] and [Knight, 1746].
[1158] In the original: *un petit barreau trempé cerise clair.*
[1159] See Section 4.6 on page 48. See also [Du Hamel, 1745], [Du Hamel, 1750] and [Antheaulme, 1760].
[1160] On the bars it is written from top to bottom and from left to right: North, South, South and North. However, at the beginning of the process these long steel bars are not yet magnetized. This is how the bars will become magnetized at the end of the process described in Figure 1, Number 2.
[1161] That is, the left-angled bundle moves from the center to the left end of the lying bar, while the right-angled bundle moves from the center to the right end of the lying bar. At the end of the process, the left end of this lying bar will be magnetized with a North pole and its right end will be magnetized with a South

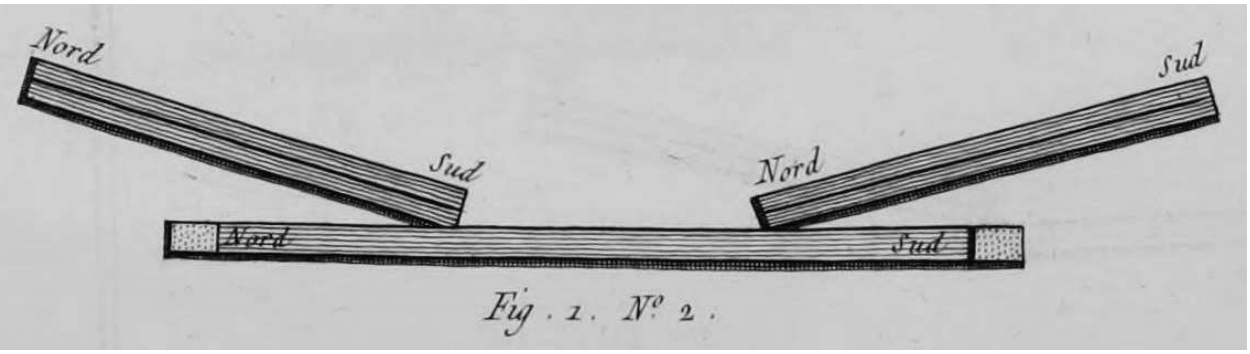


Fig. 1. N° 2.

The first Figure indicates the position of the poles and the detail of the operation much better than a longer description.

We will see presently that this slightly modified method is the best that can be used to magnetize the needles of compasses and the blades which are not more than 2 or 3 millimeters thick, provided that we employ, for the bundles which magnetize, bars strongly magnetized.

10. About the same time as Mr. Duhamel occupied himself in Paris with this research, Messrs. Michel and Canton took the same approach in England.[1162]

Mr. Michel made use of two bundles of strongly magnetized bars, connected parallel to each other, the poles of different names united at each extremity, in such a way, however, that there remained between them an interval of 7 to 8 millimeters; he then placed several equal bars in a straight line, and he caused one of the extremities of the bundle to slide at right angles along the line formed by the bars he wanted to magnetize. He found by this method, that the intermediate bars in the line took on a great magnetic force, which is true, although this degree of magnetism never gives the maximum [value] or the point of saturation.

11. Mr. Canton placed the bar which he wanted to magnetize in a parallelogram similar to that which we have described in relating the process of Mr. Duhamel; to magnetize his bars, he first used Mr. Michel's method; then separating the two bundles, he inclined them on the bar, like Mr. Duhamel, and made them slide like him to its extremities. This last method, which increased, he said, the magnetic force of the bar, was; as we see, precisely that of Mr. Duhamel; and the process which preceded it, which never gives the degree of saturation, was useless.

12. I do not think it necessary to speak of several other means which all relate to the preceding ones; but we must not confuse with these methods that of Mr. OEpinus, which is, according to the author, a correction of that of the double touch of Mr. Canton.[1163] This method is founded on a theory on electricity analogous to that of Mr. Franklin.[1164] It has a very great advantage over all the other methods, when it is required to magnetize very large bars with bundles which do not have a very great magnetic force: here is what it consists of.

After having formed the parallelogram of Mr. Duhamel with the magnetic bars and two small iron parallelepipeds, one inclines two bundles of magnetic bars, so that they form each on their side an angle of 15 or 20 degrees on the bar that we want to magnetize; their South and North poles are placed at a very short distance from each other; and in this state,

pole.

[1162]See Section 4.6 on page 48. See also [Michell, 1750], [Canton, 1752a] with German translation in [Canton, 1752b], [Michell and Canton, 1752], [Fara, 1995] and [Reich and Roussanova, 2022].

[1163]See Section 4.6 and [Aepinus, 1979].

[1164]See footnote 80 on page 36.

without changing the distance from these poles, the two bundles are made to slide together alternately from the middle of the bar to its extremity, always starting, at each operation, from the middle of the bar.[1165]

We see that by this method not only is the action of the two poles of the bundles combined to transport or separate the magnetic fluids from one end of the bar to the other, as by the methods of Messrs. Michael and Canton; but even that by inclining the bundles a great deal, this action is brought about in the most advantageous manner. This method has the advantage over all the others, when we want to magnetize very large bars with weak magnetic bundles, although it never produces in any case precisely the point of saturation: because, if we place blades or bars magnetized by this method, under a sheet of paper, on which we spread [iron] filings, we will see that the magnetic center[1166] will never be placed in the middle of the blade, but always moved a few millimeters closer to the end of the blade which was last magnetized.[1167]

However, as the bars magnetized by this method have, according to experiment, a directing force along their meridian which differs barely by a thirtieth part from that which gives the state of saturation, it is, as we will soon see, preferable to all the others, if we magnetize very large bars, and if we do not have at our disposal bundles of a great magnetic intensity.

13. After familiarizing myself with the preceding methods, and aligning myself as much as possible with the dictates of theory, here is the apparatus which I use to magnetize with facility the bars and the blades of all the dimensions. I have already described this device in the *Memoirs of the Academy of Sciences of Paris*, year 1789.[1168]

By means (*Figure 2*), of ten steel bars tempered light cherry, each 5 to 6 decimeters long, 15 millimeters wide and 5 millimeters thick, I form two layers of five bars each, separated at the two ends by a parallelepiped of very soft iron, which is 75 millimeters wide, that is to say a width equal to that of the five bars united flat, 80 millimeters long and 7 to 8 millimeters thick. This parallelepiped [of soft iron] is sandwiched between the two layers of bars at about half its length;[1169] so that the bars having been each magnetized separately before being united, these parallelepipeds of iron form a kind of shield of these bars.

Figure 2, No. 1, represents this magnet with its armors, top view.[1170]

[1165] The word "alternately" here means that the two bundles are rubbed together against the blade, sliding together from the center to one end of the blade, then sliding together from the center to the second end of the blade, sliding together again from the center to the first end of the blade, and so on.

[1166] See footnote 146 on page 59.

[1167] In Aepinus' method, the previously magnetized bundles are rubbed together against the blade to be magnetized, sliding together from the center of the blade to one of its ends. Then the bundles are placed back in the center of the blade, sliding together to the other end. This procedure is alternated until the blade reaches the desired magnetization. The problem with this method is that the magnetic center of the blade, that is, the neutral point where it is not magnetized between its North and South poles, is not located exactly in the center of the blade, being a little closer to the end of the blade whose portion was last rubbed.

[1168] [Coulomb, 1793]. This Memoir is translated in Chapter 26. See, in particular, Section 26.40.

[1169] In the original: *sur la moitié de sa longueur*. I think here it should be "about half of its width," instead of "about half of its length".

[1170] On the left side, we see 5 bars side by side, already magnetized, with their poles S at the left end and their poles N at the right end. Below them, another 5 bars are placed side by side, also with their poles S at the left end and their poles N at the right end. This set of 10 bars is going to be one of the magnets Coulomb is going to mention next.

On the right side, we see another 5 bars side by side, already magnetized, with their poles S at the left end and their poles N at the right end. Below them, another 5 bars are placed side by side, also with their poles S at the left end and their poles N at the right end. This other set of 10 bars is going to be one of the other magnets Coulomb is going to mention next.

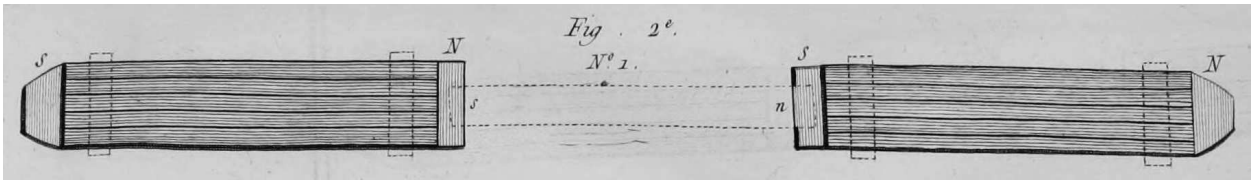
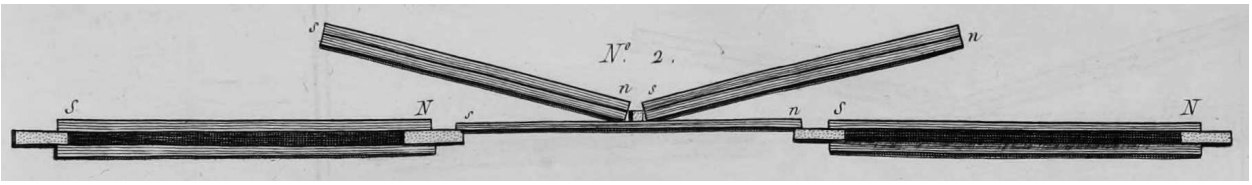


Figure 2, No. 2, is the cross-section along its length.[1171]



The two magnets,[1172] as well as the two bundles, are placed as they should be when starting to magnetize a bar or a blade. The letters N, S and n, s, represent the nature and position of the different poles. Thus, during the operation, the large magnets remain motionless, placed in a straight line, separated by a distance approximately equal to the length of the blade that is being magnetized,[1173] and this blade should extend only 4 or 5 millimeters over the ends of the shields.

If we employ the method of Mr. OEpinus, the lower poles of the bundles intended to slide on the plate which we wish to magnetize, must not be separated from each other more than 5 to six millimeters. They are maintained at this constant distance, during the whole operation, by means of a small lead blade which separates them; but again, I note that when the blade which we magnetize is at most only 2 or 3 millimeters thick, like almost all compass needles, we can, with more certainty, obtain the degree of magnetic saturation of these blades by placing, as in the Figure, the two North and South poles of the two bundles in the middle of the blade, and by sliding, each on their side, the two bundles to its ends,

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The armors are represented by the white parallelograms, two of them in each set of 10 bars, separating the top 5 bars from the bottom 5 bars.

Between these two sets, there is a long horizontal dotted rectangle that represents the blade to be magnetized. The poles s and n indicated on this blade represent the magnetic poles that it will acquire after magnetization.

[1171]The horizontal central blade is the steel blade that will be magnetized. The letters s and n above it indicate how it will become magnetized at the end of the process.

We have two inclined bundles in this Figure, already magnetized, with their inverted poles placed on the central horizontal blade that will be magnetized. The left bundle has an upper pole s and a lower pole n. The right bundle has an upper pole n and a lower pole s. There is a small square between these bundles that represents a lead blade that will hold them together, with a constant separation between them, as they are slid together from the center of the horizontal blade to its left end, or else from the center of the horizontal blade to its right end, in the cases where Coulomb will use Aepinus' magnetization method.

On the left side of this Figure 2, Number 2, the letters S and N represent the first set of 10 magnet bars seen from above in Figure 2, Number 1. On the right side of this Figure, the letters S and N represent the second set of 10 bars viewed from above in Figure 2, Number 1.

The four armors separating the upper bars from the lower bars of each magnet are represented by the four small horizontal rectangles with internal scattered dots.

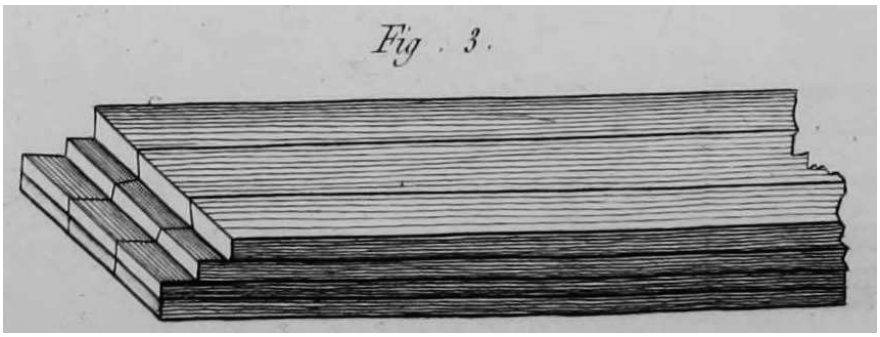
[1172]Each magnet is made up of a set of the previously mentioned 10 horizontal magnetized bars.

[1173]The distance Coulomb refers to here is the distance between the right end of the left magnet and the left end of the right magnet, that is, between the central points N and S in Figure 2, Number 2. In Figure 2, Number 2, it is possible to see that this distance is a little greater than the length of the central blade that will be magnetized.

under an inclination of 20 to 30 degrees.[1174]

Each of the sliding bundles which I use are composed, in the ordinary operations, of four bars 400 millimeters long, 5 [mm] thick and 15 [mm] wide. After having strongly magnetized them, I join two of them together on the width[1175] and two on the thickness;[1176] which makes each bundle 30 millimeters wide and 10 millimeters thick.

Before joining them [to form the two bundles], they are tempered light cherry, and magnetized to saturation. When I want to magnetize large bars, I have to form my bundles with a greater number of bars placed on top of each other in steps, set back 10 or 12 millimeters in the direction of the thickness. Figure 3 shows the end of such a bundle.



The steel of these bundles is from a steel graded as seven stars; its quality is mediocre; but I have observed, as has already been noted, that tempered steels, unless they were of a very poor quality, all took on nearly the same degree of magnetism.

Having sufficiently described the different magnetizing methods used up to the present, I will submit them to experiment and compare them.

29.1 First Experiment

14. A steel wire 300 millimeters in length, one millimeter in diameter, sliding at right angles over the pole of a single bar magnet, 400 millimeters in length, 15 millimeters in width and 5 millimeters in thickness, being placed in oscillation in a horizontal plane, and suspended on a very fine silken thread, made ten oscillations in 74".[1177]

Sliding at right angles on the pole of four and ten bars joined together, it also makes ten oscillations in 74".

By magnetizing this wire by the method[1178] of Mr. Duhamel or that of OEpinus, it also makes ten oscillations in 74".

[1174] In this last case, in which the bundles separate from one another, going from the center to the ends of the blade to be magnetized, Coulomb is using the improved Duhamel's method. This procedure does not use the lead blade which, in the case of Aepinus' method, keeps the lower ends of the bundles at a constant distance from each other during the process.

[1175] That is, placed side by side.

[1176] That is, with two bars placed side by side joined on top of two bars placed side by side.

[1177] That is, in 74 seconds. It should be remembered that each oscillation for Coulomb lasts half the time of the modern definition of the period of oscillation, see Section 6.3 on page 141.

[1178] [Note by Coulomb] I will always call, in the continuation of this Memoir, method of Mr. Duhamel, that where, by placing a blade on my apparatus described in Article 13, the two bundles are made to slide in opposite directions as far as the shield; I will call the method of Mr. OEpinus that where the poles of the bundles which slide on the plate which we magnetize, always remain at a [fixed] distance of 5 or 6 millimeters.

Thus all the methods for steel wires of such a small diameter, give the same degree of magnetism, which is that of saturation.

29.2 Second Experiment

15. An annealed steel blade, 300 millimeters in length, 8 [mm] in width, $\frac{6}{10}$ millimeter thick, sliding at right angles on the pole of a single bar, made ten oscillations in 77".

[Sliding] on the poles of two joined bars, [made] ten oscillations in 75".

[Sliding] on the poles of ten joined bars, [made] ten oscillations in 75".

[Sliding] with a single [magnetized] bar on each side, by the methods of Messrs. Duhamel and OEpinus, [made] ten oscillations in 75".[1179]

29.3 Third Experiment

16. A steel blade 164 millimeters in length, 9 [mm] in width, $\frac{6}{10}$ [mm] thick, tempered light cherry, after sliding at right angles on the poles of two bars joined together, made ten oscillations in 51".

[After sliding] on the poles of four bars joined together, [made] ten oscillations in 49".

[After sliding] on the poles of eight and of ten bars joined together, [made] ten oscillations in 47.5".

But, using only two [magnetized] bars joined together, and sliding them under an angle of inclination of 15 to 20 degrees on the blade, it also made ten oscillations in 47.5".

By the methods of Messrs. Duhamel and OEpinus, the magnetized blade with a single bar on each side, also made ten oscillations in 47.5".

I need only remark that, by the method of Mr. OEpinus, we find a duration of half a second and sometimes a second longer than in that of Mr. Duhamel.

29.4 Remark on the Three Preceding Experiments

17. In the first two experiments, the steel wire, as well as the blade, were annealed light cherry; in this state, two bars united by the same poles, and even a single bar, sliding at right angles to the steel wire or the blade, were enough to magnetize them to saturation; but in the third experiment, where the blade was tempered light cherry, it was only with a bundle of eight or ten bars that we were able to magnetize this blade to saturation, by sliding the blade at a right angle to the end of the bundle; but by giving the direction of the action of the bundle a more advantageous position, that is to say, by inclining it from 15 to 20 degrees to the blade, two bars united by the same pole sufficed to give the saturation degree.

18. In the last two experiments the blades had only $\frac{6}{10}$ millimeter of thickness: they were easily penetrated by the magnetic action of a single bundle throughout their thickness. We should not therefore be surprised if all the methods are equally good, provided that we

[1179] When Coulomb says that he has a single bar on each side, I believe he is referring to the horizontal magnetized bars that are on each side of the steel blade that he wishes to magnetize, as shown in Figure 2, Numbers 1 and 2.

employ bundles of strong magnetic intensity. In the following experiments, the blades and the bars have a greater thickness, and are tempered light cherry.

29.5 Fourth Experiment

19. A blade 202 millimeters long, 14 millimeters wide, 1 millimeter thick, after sliding several times at right angles on the pole of a single bar, made ten oscillations in 73".

[After sliding] on the pole of four bars joined together, [made] ten oscillations in 62".

[After sliding] on the pole of ten bars joined together, [made] ten oscillations in 59".

But with a single bundle of two [magnetized] bars, sliding under an inclination of 15 degrees with the blade, it made ten oscillations in 53".

[A bundle with the] same inclination with four [magnetized] bars joined together, [the blade made] ten oscillations in 49".

[A bundle with the] same inclination with eight and ten [magnetized] bars [joined together, the blade made] ten oscillations in 49".

[After being magnetized] by the methods of Messrs. Duhamel and OEpinus, with a single [magnetized] bar on each side, or [with] a greater number [of magnetized bars on each side, the blade made] ten oscillations in 49".

29.6 Remark on This Experiment

20. As it is here the same blade magnetized by different methods, the force which directs it along its meridian is measured by the inverse of the square of the times [required to make] the same number of oscillations.[1180] Thus we see that, even by bringing together ten [magnetized] bars and making them slide at right angles, it is far from being magnetized to saturation; but this [state of saturation] is easily achieved with a single bundle [composed] of four bars, by giving its magnetic action on the blade a more advantageous direction, that is to say, an inclination of 15 to 20 degrees. Two bars suffice, employing the methods of Messrs. Duhamel and OEpinus, to give this blade the state of saturation: but a very important observation is that, as there is almost always disadvantage, as I have often noticed in different Memoirs which preceded this one, to use blades more than a millimeter thick to form compass needles; provided that four or six strongly magnetized bars are united, they will always suffice to give these needles [up to 1 mm thick] the degree of magnetic saturation.

21. Wanting to magnetize several blades similar to the preceding one, by joining them together before magnetizing them, I believe that one ought not use any other process than that of Messrs. Duhamel and OEpinus to give them a degree of saturation in accord with the results which I have just found. In the following experiments, the blades are each 302 millimeters long, 28 millimeters wide and 1.07 millimeters thick: they are tempered light cherry.

[1180] See Section 16.1 on page 267.

29.7 Fifth Experiment

22. A single blade, magnetized with [two] bundles of two [previously magnetized] bars each, made, by the two methods,[1181] ten oscillations in 72".

Same result with bundles of a greater number of bars. There have been some small variations when using the method of Mr. OEpinus; but there have never been any when using that of Mr. Duhamel.

29.8 Sixth Experiment

23. Two blades united and forming a thickness of 2.14 millimeters, magnetized by the method of Mr. Duhamel, with two bundles of two bars each, made ten oscillations in 80".

[Magnetizing these two blades] with two bundles of four bars each, [the two united blades made] ten oscillations in 78".

[Magnetizing them] with two bundles of ten bars each, [the two united blades made] ten oscillations in 78".

[Magnetizing them] by the method of Mr. OEpinus, with bundles of two, four or ten bars each, [the two united blades made] also ten oscillations in 78".

29.9 Seventh Experiment

24. Four blades similar to the preceding ones, were joined together and formed a bundle 300 millimeters in length, 28 millimeters in width and 4.28 millimeters in thickness.

I only succeeded in magnetizing such a bundle of blades by the method of Mr. Duhamel, by employing eight bars in each bundle. By suspending the four blades thus joined, they made ten oscillations in 91".

By the method of Mr. OEpinus, two bundles of two bars each suffice to magnetize these blades to saturation. Thus, when we have to magnetize blades or bars more than 4 to 5 millimeters thick, unless we make use of two bundles of very great magnetic intensity to magnetize them, the method of Mr. OEpinus is still preferable to all the others despite the small defect of this method, which we have pointed out, Article 13.[1182,1183]

29.10 Eighth Experiment

25. I wanted, in this experiment, to magnetize one of the bars which make up the bundles which I use to magnetize [the blades]: they are, as I have already said, 400 millimeters in length, 14 [mm] in width and 5 [mm] thick; they are tempered light cherry.

I only managed to magnetize this bar by Mr. Duhamel's method with two bundles of four bars each.

[1181] That is, both by Duhamel's method and by Aepinus' method.

[1182] Coulomb is referring to the defect mentioned in Article 12 and not in Article 13, see footnote 1167 on page 515.

[1183] [Note by Potier] The neutral line always approaches the pole that was rubbed last.

But by that of Mr. OEpinus, a single bar on each side produces the state of saturation in the bar which one magnetizes, because it makes in this case ten oscillations in 110".[1184]

And by bringing together, to magnetize this bar, a greater number of bars,[1185] it also makes ten oscillations in 110".

29.11 Ninth Experiment

26. After having magnetized blades and bars 5 millimeters thick, I tried to magnetize thicker ones. That of this experiment was 400 millimeters in length, 25 [mm] in width, and 9 [mm] in thickness. This bar was tempered light cherry. It is about the size of the largest bars which are usually used for magnetizing [other blades]. It was impossible for me to magnetize this bar by the method of Mr. Duhamel, even by employing two bundles of ten bars each. Using this method, the degree of magnetism of the bar was such that it made ten oscillations in 162".

It is not possible to magnetize it, by the method of Mr. OEpinus, with bundles of two bars each; but, magnetized with two bundles of four or ten bars each, it also makes ten oscillations in 153".

Thus, to magnetize such bars, only Mr. OEpinus' method should be used. But we will see presently that when we want to obtain artificial magnets of great strength, there is no case where we must use tempered bars of such great thickness, and that there is always a very great advantage in forming coarse magnets by the union of a large number of bars of a lesser thickness.

29.12 Tenth Experiment

27. In this experiment I wanted to learn what would be the difference in the results by magnetizing several bars separately, and then bringing them together; or by magnetizing them after having united them. As I had in this experiment to magnetize bars of a greater thickness than in most of the preceding experiments, I contented myself with employing the method of Mr. OEpinus. We can add, if we want, a thirtieth part to the directing force, to obtain, according to the note of Article 13, the state of saturation.[1186]

A single bar 400 millimeters long, 14 [mm] wide and 5 [mm] thick, magnetized with two bundles of ten bars each, made ten oscillations in 108".

Two such bars joined together, forming a bundle 28 millimeters wide by 5 [mm] thick, each magnetized separately before being joined together, made, after their joining, ten oscillations, in 115".

Thus united, I magnetized them in the opposite direction, changing the poles end for end, and, after this operation, the bundle composed of the two bars also made ten oscillations in 115".

[1184] That is, Coulomb places a single horizontal magnetized bar on each side of the horizontal bar to be magnetized, as illustrated in Figure 2. After the central bar has been magnetized, it performs ten oscillations in 110 seconds.

[1185] On each side of the bar to be magnetized.

[1186] This last sentence was not included by Potier in his reprint of this paper, [Potier, 1884, p. 367]. Again Coulomb is referring here to Article 12 and not to Article 13. The note mentioned by Coulomb appears in the last two paragraphs of Article 12, see, in particular, page 515 of this English translation.

Thus, since I have the same result by magnetizing the two bars separately before joining them, or by magnetizing them in the opposite direction after having joined them, the two processes are here perfectly equal.

29.13 Eleventh Experiment

28. Four bars similar to the preceding ones, forming a bundle of the same length, but 28 millimeters wide by 10 [mm] thick, each magnetized separately before being united; the bundle, after the joining together of the four bars, made ten oscillations in 130".

Having wanted, in this state of reunion, to change the poles end for end, I had [after this inversion] ten oscillations in 133".

I have never been able, by changing the poles of four bars thus united, to succeed in giving them precisely the same degree of directing force as by reuniting them after having magnetized each one separately. The result was nearly the same, although the four bars joined together had not been magnetized before their union.

29.14 Twelfth Experiment

29. I joined four other bars to those which had been used in the preceding experiments: magnetized each one separately, the eight bars joined together formed a bundle 28 millimeters wide, 20 [mm] thick. This bundle suspended horizontally, like the preceding ones, by untwisted silk threads, and glued together with a little gum, made ten oscillations in 166".

29.15 Remarks on These Experiments

30. If we compare here the different results given by the preceding experiments, and if we want to deduce from them the directing force which returns the same bar to its magnetic meridian, when it is alone or when it is united in a bundle of several bars, we will find that in the state of saturation:

An isolated bar makes ten oscillations in .....108".
Two bars together, [make] ten oscillations in .....115".
Four bars together, [make] ten oscillations in .....130".
Eight bars together, [make] ten oscillations in .....166".

Thus, considering a single bar, either alone or joined to several others, since the force which holds it aligned with its magnetic meridian follows the inverse of the square of the times of the same number of oscillations, we will have, letting 1000 represent the directing force of the isolated bar, the following Table:

	Directing force
For the isolated bar	1000
For the same bar joined to another one	882
The same bar joined with three others	692
The same bar joined with seven others	433

I have given in another Memoir, the law in accord with theory and experiment that the directing force follows for each bar of a bundle of a given thickness and width. All that

we ought to conclude from the preceding result, relative to the object of this Memoir, is that there are very few advantages to be hoped for in increasing the thickness of artificial magnets, when this thickness exceeds 10 or 12 millimeters.

29.16 Second Remark

Finally there is a very interesting result to be drawn from the preceding experiments: it is the ratio of the directing force of a large bar magnetized to saturation, and of a bundle of the same dimensions.

We have just seen, eleventh experiment, that four bars joined together, forming a bundle 400 millimeters long, 28 millimeters wide and 10 millimeters thick, magnetized to saturation, makes 10 oscillations in 130"; but we saw, in the ninth experiment, that a single bar of the same length, but 25 millimeters wide and 9 [mm] thick, made 10 oscillations in 153". Thus, although the width and thickness of the bundle are greater than those of the bar, the lengths being approximately equal, we find, for each part of the large bar reduced to the same dimensions as a single bar of the bundle, a smaller directing force than in the bundle. The ratio of the directing forces is as the inverse of the square of the times for the same number of oscillations, we find this ratio is $\overline{153}^2$ to $\overline{130}^2$, [that is,] approximately 14 to 10, in favor of the bundle, although of larger dimensions than those of the large bar.

As this Memoir is only intended to direct physicists and artisans who wish to manufacture artificial magnets of very great force, or to magnetize compass needles to saturation, for all theoretical analyses, I refer [the reader] to the various Memoirs that I have already published, either in the collection of *Memoirs of the Academy of Sciences of Paris*, or in those of the Institute.

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This book contains complete and commented translations of the main works of Charles-Augustin de Coulomb (1736-1806) on torsion, electricity, and magnetism. They include the 1777 prize winning work on the best method of making magnetic needles, the 1784 paper with the law of torsion of metal wires and his 7 major Memoirs on electricity and magnetism. In these works he arrived experimentally at the law of force between electrified bodies varying with the inverse of the square of their distance (known in textbooks as Coulomb's law), at the law of force between magnetic poles also varying with the inverse square of their distance, at the exponential law of charge leakage, at the distribution of electricity over the surface of charged conducting bodies in various configurations of electrostatic equilibrium, at advanced methods of magnetization and the production of artificial magnets.

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