On the Mechanism of Railguns

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Two mechanisms utilized to explain the operation of railguns are explained: one based on Ampère's force and the other on the transfer of momentum through electromagnetic waves. It is shown how the former is compatible with the data while the latter has problems with the quantitative figures and the case of the immobilized projectile.

Introduction

Railguns have been known for quite a long time. A schematic diagram is given in Fig. 1. The railgun is fed by a current source S (the usual lead-acid battery, a capacitor bank, etc). The current flows through the metallic rails A and C, which are rigidly fixed in the laboratory, as is the source S. The projectile B forms a conducting bridge between the rails. When the current flows through this closed circuit a resultant ponderomotive force acts on the projectile along the positive Y direction. If the projectile is mobile and free to move along this direction (assuming the spring K to be absent) it can achieve higher velocities than those obtainable with conventional guns using chemical explosives.

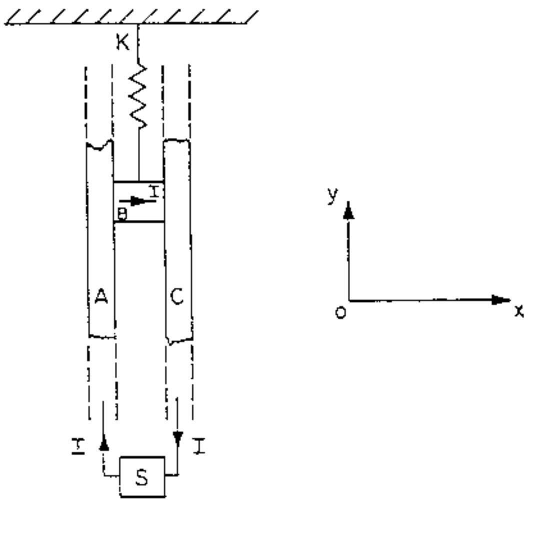

Fig. 1. Schematic diagram of the railgun considered in this paper.

The main question in the operation of a railgun is to find the source of the ponderomotive force responsible for the motion of the projectile. Recently a discussion on this topic has appeared in the literature [1–4]. A possible answer is that this ponderomotive force on the projectile is nothing more than Ampère's force exerted by the rails A and C on the projectile B. According to Ampère the ponderomotive force exerted by a current element $I_2 d\mathbf{l}_2$ on another current element $I_1 d\mathbf{l}_1$ is given by [5]

$$d\mathbf{F} = -\frac{\mu_0}{4\pi} I_1 I_2 \frac{\hat{\mathbf{r}}}{r^2} [2d\mathbf{l}_1 \cdot d\mathbf{l}_2 - 3(\hat{\mathbf{r}} \cdot d\mathbf{l}_1)(\hat{\mathbf{r}} \cdot d\mathbf{l}_2)] \qquad (1)$$

where $r \equiv |\mathbf{r}_1 - \mathbf{r}_2|$ and $\hat{\mathbf{r}} \equiv (\mathbf{r}_1 - \mathbf{r}_2)/r$. A second explanation is based on the reflection of electromagnetic waves by the projectile [3, 4]. This paper agrees with the first explanation, but not with the second, and an attempt will be made to justify this position.

The first reason why Ampère's force seems to be the correct mechanism is that it satisfies Newton's Third Law (action and reaction) in the strongest form. This means that a closed circuit cannot exert a net force on itself. This is in agreement with the fact that not a single experiment has ever found conclusive evidence for such a bootstrap effect in metallic currents in closed conductors. Moreover, it predicts that the reaction force should act in the rails, and not in the magnetic field itself. This is in agreement with the experiments resulting in the buckling of thin rails after a railgun operation, and also with the correlated mechanical inefficiency of the railgun [1].

We must now refute the claim that if Ampère's force were the correct mechanism, this would prove the "inadequacy of classical electrodynamics in explaining the operation of railguns" [6]. This paper by Allen was written in answer to Graneau's paper [7]. But in his paper Graneau was discussing the "inadequacy of the field-energy momentum concept," not that of "classical electrodynamics." If we understand classical electrodynamics as the set of Maxwell's equations then Ampère's force is compatible with that. As a matter of fact Maxwell himself wrote in connection with (1) [emphasis added]: "The experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science. The whole, theory and experiment, seems as if it had leaped, full grown and full armed, from the brain of the 'Newton of electricity.' It is perfect in form, and unassailable in accuracy, and *it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electrodynamics*" [8].

Let us now discuss why we cannot accept the explanation of the railgun based on electromagnetic momentum (see [3],[4]). We will utilize the data of the experiment of Deis, Scherbarth and Ferrentino [9]. In this experiment the energy released by the current source was 16.3 MJ, stored originally in the rotor of a homopolar generator. The maximum current produced was 2.1 MA. A projectile of mass 0.317 kg was accelerated to a velocity of 4200 m/s, so that its acquired momentum was 1331.4 kgm/s. Its acquired kinetic energy was $2.8 \cdot 10^6$ J, which represents only 17% of the stored energy in the generator. This shows that energy was lost elsewhere (Joule heating, mechanical deformation of the rails, etc.) We will examine this problem based on the reflection of electromagnetic waves using an argument first developed by Pappas [10]. The sum of linear momentum of the projectile and the backward radiation (which propagates at light velocity) must be conserved. If we represent the

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equivalent electromagnetic mass of this radiation field by m_e, the conservation of momentum would imply $m_ec = 1331.4$ kgm/s, so that $m_e = 4.44 \cdot 10^{-6}$ kg. But this would imply a stored energy in the field given by $E = m_ec^2 = 3.99 \cdot 10^{11}$ J, which is $2.4 \cdot 10^4$ times larger than the whole energy initially available in the system. This explanation is therefore invalid because there is no source of the extra energy required by this model.

We can also reverse the argument. In an ideal situation (the most favorable from the viewpoint of the field-energy momentum explanation), the whole energy of the source would be transformed into electromagnetic waves. This would mean $m_ec^2 = 16.3$ MJ, so that $m_e = 1.8 \cdot 10^{-10}$ kg. This means that the maximum momentum associated with the electromagnetic field would be given by ($p = m_ec$) $5.4 \cdot 10^{-2}$ kgm/s. Once more this is much smaller than the actual momentum acquired by the projectile. It does not help to say that the much larger momentum of the projectile is gained by many reflections of photons or quanta of radiation [4]. Allen [3,4,6] argues that the electromagnetic waves are reflected at the projectile end and at the breech end of the railgun. But even if this were the case, it could not transmit more momentum than it has, no matter how many times they are reflected. The point is that this momentum of $5.4 \cdot 10^{-2}$ kgm/s is the maximum momentum possible for the electromagnetic radiation here, because it is due to the whole energy stored in the system. There is no more energy to generate more waves in this ideal situation. As the projectile acquires more momentum than this maximum amount, this only means that the mechanism for this moment transfer cannot be accomplished by reflection of electromagnetic waves or photons.

Other aspects are worthy of consideration. In the first place we are not sure that there is an electromagnetic wave when there is a DC current. A disturbance in the electric current and in the corresponding electromagnetic fields is certainly generated when we close the switch, but this is a transient situation. After the steady state has been attained, there is no reason for having an electromagnetic wave propagating in space. But even if there is an electromagnetic wave, it should propagate in all directions, and not only backwards. And this makes Pappas' argument even stronger. The reason is that in the ideal situation of the previous paragraph, we considered the whole energy of the source being converted in electromagnetic waves propagating in a single direction, and even though the maximum momentum associated with the field was found to be much smaller than the real one acquired by the projectile. If we had taken into account the other sources of energy depletion (energy spent to buckle the rails, or the work of deformation; the energy spent in Joule heating; the energy stored in the electromagnetic waves which move in other directions and cannot contribute to the acceleration of the projectile; etc.), we would observe that the available momentum of the photons or electromagnetic waves which could accelerate the projectile would become even smaller.

The explanation given by Allen and Jones [4] for a short-circuited line or immobilized projectile is also difficult to agree with. This is represented in Fig. 1 where now the projectile is kept at rest. Let us suppose, in accordance with their theory, that the railgun is fed by a *constant* current source. The ponderomotive force on the projectile is counterbalanced by an elastic force generated by the spring K (made of non-conductive material) fixed in the laboratory. This arrangement can be utilized to measure the resultant ponderomotive force on the projectile (as a matter of fact a slight modification of this scheme has been utilized experimentally elsewhere [11, 12]). The main aspect to note here is that this fixed closed metallic current generates no electric field. In fact $\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t$, but $\phi = 0$ (neutral current) and $\partial\mathbf{A}/\partial t = 0$ (constant and stationary current), so that the electric field is identically zero at any external point. The general proof of this fact in steady conduction currents of arbitrary form has been given elsewhere using Lorentz force and Liénard-Wiechert potentials [13]. Due to this fact the circuit of Fig. 1 with spring and fixed projectile can only generate a constant magnetic field at any external point. Allen and Jones [4] pointed out correctly that the Poynting vector and its associated momentum are both zero in this case. But beyond that, the absence of an electric field in space implies that there is no wave at all, so that the DC field *cannot* be resolved into two waves, one incident and one reflected, having parameters E_i and $-E_i$ (E_i is the electric field strength [6]), as they claimed [4]. This can only be true in this situation if $E_i = 0$, but then the power density of the wave and its corresponding momentum flow would identically equal zero according to their Eq. (1). Moreover, their relation $B = \mu_0 E/Z$ (Z being the impedance of free space, $Z = (\mu_0/\epsilon_0)^{1/2}$) cannot be true, for although this wire generates no electric field, $E = 0$, it does generate a magnetic field, $B \neq 0$. This shows that their reasoning to arrive at the relation $P = B^2/2\mu_0$ (P being the force per unit area acting on the projectile) cannot be correct.

A better way of showing a contradiction would be to consider the forward momentum of the projectile, the backward momentum of the recoil mechanism (or in terms of energy, the work of buckling the rails), and the momentum of the electromagnetic field. The sum of all these three moments must be zero. For the time being we will not develop this analysis due to the difficulties involved in calculating the momentum of the electromagnetic field. This calculation has to be carried out by integrating the density of electromagnetic radiation, given by $\mathbf{E} \times \mathbf{H}/c^2$, over all space. Although only the component in the direction of the motion of the projectile of this momentum would have to be calculated, a proper calculation is far too involved in this situation.

In conclusion one can say that although it is not certain that Ampère's force is *the* correct mechanism responsible for the operation of railguns, there is no refutation of this explanation. On the other hand, there seem to be real problems with an explanation based on transfer of momentum through electromagnetic waves.

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