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Schrödinger's Potential Energy and Weber's Electrodynamics

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We discuss Schrödinger's approach to gravitational interactions and Weber's electrodynamics. Then we make use of this model to calculate the energy of a charge moving inside and outside an ideal capacitor. This results in an ultimate speed, c, and in the variation of the mass of the particle with the electrostatic potential and its velocity.

1. INTRODUCTION

In this paper we discuss the ultimate speed of a particle moving inside an ideal capacitor. We utilize the potential energy for gravitation proposed by E. Schrödinger to implement Mach's principle [1], and a velocity dependent potential energy for electric charges postulated by W. Weber [2].

2. GRAVITATIONAL INTERACTION

In 1925 Erwin Schrödinger proposed a gravitational energy between two point masses m and m' which is a function of their distance r_{ij} and radial velocity $\dot{r}_{ij} = dr_{ij}/dt$ [1]. This energy W_{ij} is given by

$$W_{ij} = -\frac{Gmm'}{r_{ij}}\left[3 - \frac{2}{(1-\dot{r}_{ij}^2/c^2)^{3/2}}\right]$$

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$$= -\frac{Gmm'}{r_{ij}}\left[3 - 2\left(1 - \frac{1}{c^2}\frac{(\vec{r}_{ij}\cdot\vec{v}_{ij})^2}{r_{ij}^2}\right)^{-3/2}\right]. \quad (1)$$

In this expression $c = 2.99 \times 10^8$m/s and G is the gravitational constant ($G = 6.67 \times 10^{-11}$m^3/s^{2}kg). With $\dot{r}_{ij} = 0$ we recover the Newtonian potential energy.

Expanding eq. (1) up to second order in $\dot{r}_{ij}/c$ yields the usually presumed unexplained portion of the advance of perihelion of the planets and the implementation of Mach's principle [1,3].

Schrödinger has shown that integrating eq. (1) for a test mass m interacting with an homogeneous and isotropic universe with uniform matter density ρ_0 yields the result (except for an unimportant constant)

$$W = A\frac{mc^2}{\sqrt{1 - v^2/c^2}}. \quad (2)$$

In this equation $v = |\vec{v}|$ is the velocity of m relative to the frame of reference in which the universe as a whole is stationary. The constant A is given by $4\pi\rho_0 G R_0^2/c^2$, where R_0 is the characteristic length of the universe. If R_0 is estimated as c/H_0, where H_0 is Hubble's constant, then A may be taken as roughly unity, $A \simeq 1$. From now on we will take $A = 1$.

3. ELECTROMAGNETIC INTERACTION

We analyse here a charged particle moving orthogonally to the plates of an ideal capacitor with surface charge densities $\pm\sigma$ on the plates situated at $\pm z_0$ (Figure 1).

We utilize an electromagnetic energy given by

$$U_{ij} = \frac{q_i q_j}{4\pi\varepsilon_0}\frac{1}{r_{ij}}\left(1 - \frac{1}{2}\frac{\dot{r}_{ij}^2}{c^2}\right). \quad (3)$$

The equation above was proposed by W. Weber [2]. With $\dot{r}_{ij} = 0$ we recover Coulomb's potential energy.

The interaction energy of a charge q moving along the z axis is obtained integrating eq. (3) on both plates. The result of this integration [4] is given by

$$U(\pm z - z_0 > 0) = \pm q\frac{\sigma}{\varepsilon_0} z_0\left(1 + \frac{v^2}{2c^2}\right), \quad (4)$$

$$U(-z_0 \leq z \leq z_0) = q\frac{\sigma}{\varepsilon_0} z\left(1 + \frac{v^2}{2c^2}\right). \quad (5)$$

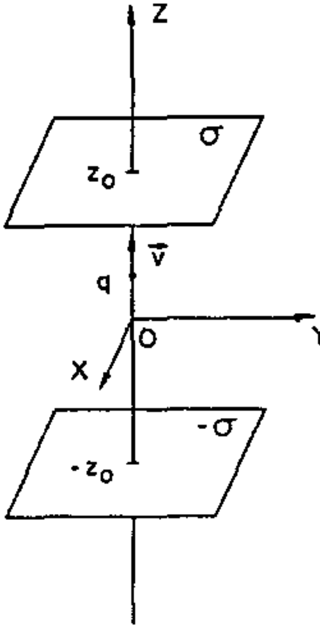

Figure 1. Geometry of the problem. A test charge q moving orthogonally to the plates of an ideal capacitor with surface charge densities $\pm\sigma$ at the plates situated at $\pm z_0$.

4. TOTAL ENERGY

Adding eq. (2) to eqs. (4) and (5) yields the total conserved energy E of charge q, namely

$$E(z \leq -z_0) = -\frac{q\Delta\varphi}{2} - \frac{q\Delta\varphi}{4}\frac{v^2}{c^2} + \frac{mc^2}{\sqrt{1-v^2/c^2}}\,, \tag{6}$$

$$E(-z_0 \leq z \leq z_0) = \frac{q\Delta\varphi}{2}\frac{z}{z_0} + \frac{q\Delta\varphi}{4}\frac{v^2}{c^2}\frac{z}{z_0} + \frac{mc^2}{\sqrt{1-v^2/c^2}}\,, \tag{7}$$

$$E(z_0 \leq z) = \frac{q\Delta\varphi}{2} + \frac{q\Delta\varphi}{4}\frac{v^2}{c^2} + \frac{mc^2}{\sqrt{1-v^2/c^2}}\,. \tag{8}$$

In these expressions $\Delta\varphi$ is the voltage between the two plates of the capacitor, $\Delta\varphi = 2\sigma z_0/\varepsilon_0$.

We now study the problem of an electron ($q = -e$) entering the capacitor at $z = -z_0$ with a negligible velocity ($v/c \simeq 0$), being accelerated between the plates and leaving at $z = z_0$ with a final velocity v. As Weber's electrodynamics is compatible with the conservation of energy [2], we can equate (6) and (8). This yields

$$\frac{e\Delta\varphi}{2} + mc^2 = -\frac{e\Delta\varphi}{2} - \frac{e\Delta\varphi}{4}\frac{v^2}{c^2} + \frac{mc^2}{\sqrt{1 - v^2/c^2}}, \tag{9}$$

or

$$\frac{e\Delta\varphi}{mc^2} = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)\frac{1}{1 + v^2/4c^2}. \tag{10}$$

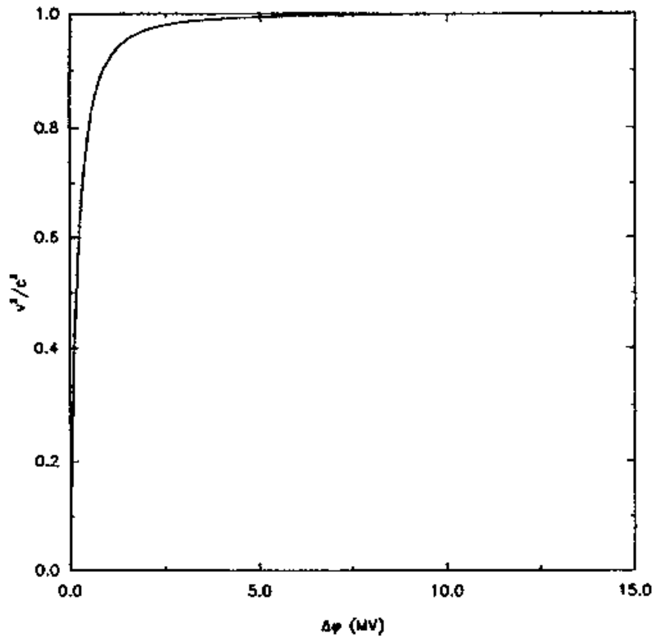

Figure 2. Behaviour of eq. (10) where we are supposing an electron being accelerated from rest inside the ideal capacitor. We can see that the final velocity goes to c when $\Delta\varphi \to \infty$.

A graphical analysis of this equation is presented in Figure 2. The main result is that the velocity of the accelerated charge is always smaller than c, and this is the asymptotic speed as $\Delta\varphi \to \infty$.

5. DISCUSSION AND CONCLUSION

Choosing the zero of the potential at $z = 0$, the middle of the plates, yields $\phi(z \leq -z_0) = -\Delta\varphi/2 = -\sigma z_0/\varepsilon_0$, $\phi(-z_0 \leq z \leq z_0) = \Delta\varphi z/2z_0 = \sigma z/\varepsilon_0$, $\phi(z \geq z_0) = \Delta\varphi/2 = \sigma z_0/\varepsilon_0$, where ϕ is the electrostatic potential at any point. With this choice we can write eqs. (6) to (8) as

$$E = q\phi + \frac{m_{ei}v^2}{2}, \tag{11}$$

where m_{ei} for $v \neq 0$ is defined by

$$m_{ei} \equiv \frac{q\phi}{c^2} + \frac{2mc^2}{v^2}\frac{1}{\sqrt{1 - v^2/c^2}}. \tag{12}$$

Equation (11) is equivalent to the classical result (Coulomb's potential plus $T = mv^2/2$) with the mass m replaced by an effective inertial mass m_{ei} defined by (12), which is a function of the electrostatic potential where the test charge is located and of the velocity of the charge. If we had chosen the zero of the potential at $z = -z_0$ eq. (11) would be written as $E = q\phi + m_{ei}v^2/2 - q\sigma z_0/\varepsilon_0$, where in this case

$$m_{ei} = \frac{q\phi}{c^2} + \frac{2mc^2}{v^2\sqrt{1 - v^2/c^2}} - \frac{q\sigma z_0}{\varepsilon_0 c^2}. \tag{13}$$

Experiments to test the variation of the effective inertial mass of a charge with the electrostatic potential where it is located have been proposed in [5,6].

In Figures 3a–3c we compare four models. In all of them we utilize conservation of energy in the form $E = T + U$. For the kinetic energy we can have the classical one, $T_c = mv^2/2$, or the value $mc^2/\sqrt{1 - v^2/c^2}$, apart from a constant. This last expression is analogous to the relativistic one and has also been obtained by Schrödinger from a different approach [1]. It will be called T_r or T_s accordingly. For the electric potential energy we have Coulomb's expression, eq. (3) with $\dot{r}_{ij} = 0$, represented here by U_c, and Weber's potential energy (3), represented here by U_w. When we integrate these expessions for a charge interacting with an ideal capacitor we get (4) and (5) for Weber (or for Coulomb, without the terms in v^2).

The problem analysed by all these models is this one of an electron being accelerated from rest since $z = -z_0$ until $z = z_0$, orthogonally to the plates of an ideal capacitor (Figure 1). In the first model, $T_c + U_c$, we

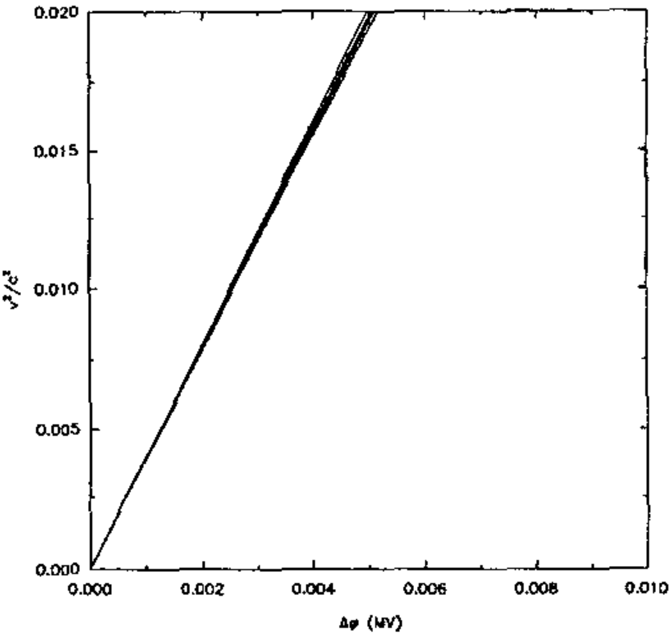
0.020
0.015
0.010
0.005
0.000
v^2/c^2
0.000
0.002
0.004
0.006
0.008
0.010
Δφ (MV)

Fig. 3a.

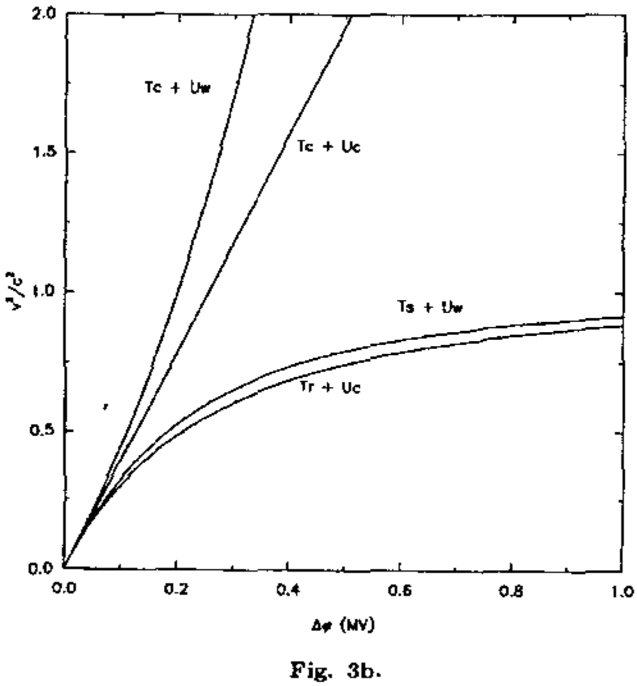
2.0
1.5
1.0
0.5
0.0
v^2/c^2
Tc + Uw
Tc + Uc
Ts + Uw
Tr + Uc
0.0
0.2
0.4
0.6
0.8
1.0
Δφ (MV)

Fig. 3b.

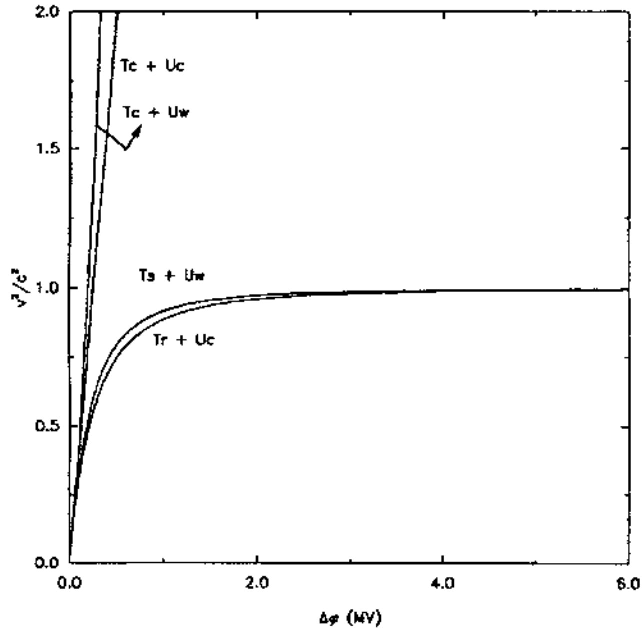

Fig. 3c.

Figure 3. Final velocity of an electron being accelerated from rest inside an ideal capacitor as a function of the voltage $\Delta\varphi$ between the plates, according to four models. We utilize the classical, relativistic and Schrödinger's kinetic energies (T_c, T_r and T_s, respectively). For the electric potential energy we utilize the Coulombian and Weberian ones (U_c and U_w, respectively).

have classical kinetic energy and Coulomb's potential. Then v^2 is a linear function of $\Delta\varphi$ and grows indefinitely without limits. In the second model, $T_r + U_c$, we have the relativistic kinetic energy and Coulomb's potential. Then the velocity tends asymptotically to c. In the third model, $T_c + U_w$, we have classical kinetic energy and Weber's potential. In this case there is no upper limit in v^2 and it diverges as $\Delta\varphi \to 1$ MV [4]. In the last model, $T_s + U_w$, we have Schrödinger's kinetic energy plus Weber's electromagnetic potential energy. Now the result is very close to the second model and the most striking feature is that even with Weber's potential energy for electromagnetism we obtained a limiting velocity for the electron, $v \to c$, as the potential difference between the plates goes to infinity. Only the second and fourth models are physically reasonable and compatible with the experimental findings.

In all these models we neglected border effects, energy losses due to electromagnetic radiation and induction of currents in the plates of the capacitor as the electron is accelerated between them.

There is an important experiment carried out by Bertozzi, where he measured the resulting time-of-flight velocity of an electron being accelerated in a van der Graaff electrostatic generator and a linear accelerator [7]. There were five runs in his experiment in which the kinetic energies of the electrons (in MeV) were as follows: 0.5, 1.0, 1.5, 4.5 and 15. From the measured time-of-flight of the electrons in each run and the traversed distance of 8.4 meters he obtained the following values of v^2/c^2, respectively: 0.752, 0.828, 0.922, 0.974 and 1.000. In Figure 4 we compare the theoretical models discussed in this paper along with Bertozzi's experimental results. We plot v^2/c^2 against the potential difference accelerating the electrons. As we can see from this figure and the previous one, only the relativistic and Schrödinger–Weber models are compatible with the data. As there are only five experimental points, and border effects etc. were not taken into account in the four models discussed in this paper, we cannot decide between these two last theoretical curves for this experiment.

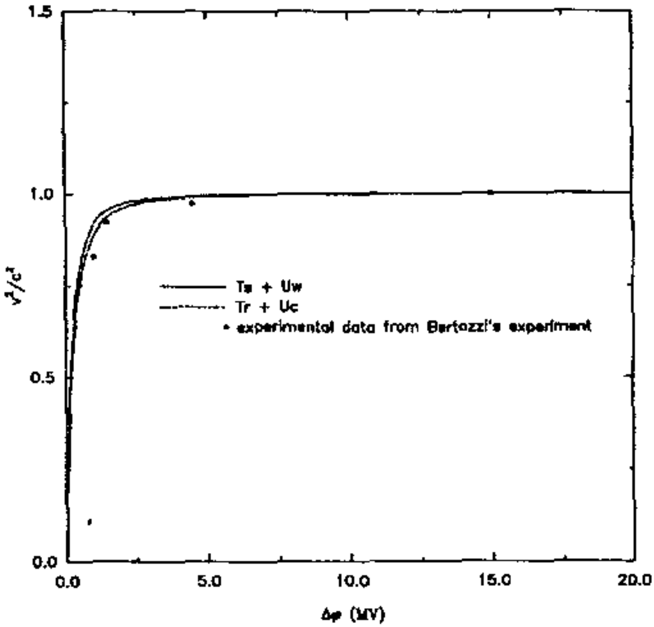

Figure 4. A comparison between the theoretical predictions of the Schrödinger-Weber model ($T_s + U_w$) and relativistic model ($T_r + U_c$) against the experimental data taken from Bertozzi's experiment [7].

Recently Wesley has entertained some similar possibilities (Ref. 8, p.259-272, Table 6.3, Ref. 9). He worked with Weber's electrodynamics and with another potential energy for gravitation. His approach has also been shown to be compatible with Bertozzi's experiment.

Futher research in all these approaches is essential for a bettter understanding of this situation.

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