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THE REALITY OF NEWTONIAN FORCES OF INERTIA

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Abstract

Abstract - We present a historical analysis of the inertial forces and of the theories developed to explain the origin of inertia. Then we introduce the principle of dynamic equilibrium and its relevance to mechanics. We analyse the Mach-Weber model which implements Mach's principle with Newtonian simultaneous interactions and we argue for the reality of the inertial forces, that is: the force of free fall $(-ma)$, the centrifugal force and the Coriolis force. Lastly we present an experimental consequence of the Mach-Weber model which differs from Newton's law of gravitation.

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1. Historical Introduction

Since the time of Newton a profound change has taken place in the teaching of mechanics. What was once the important force of inertia has, by coordinate transformations, been reduced to a pseudo force, or a fictitious force, or no force at all. Feynman (Feynman, Leighton and Sands 1963, Vol. 1, p. 12.11) states: "Another example of pseudo force is what is often called centrifugal force." A few textbooks still express doubt on this score. French (1971, p. 509), for example, confesses confusion when, with reference to the centrifugal force, he writes: "Once again the inertial force is 'there' by every criterion we can apply (except our inability to find another physical system as its source)."

Newton's Definition III of the force of inertia reads (Newton, 1962, p. 2):

"The *vis insita*, or inate force of matter, is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be of rest, or moving uniformly forward in a right line."

He goes on to say: "This force is always proportional to the [mass of the] body whose force it is." ... "But a body only exerts this force when another force, impressed upon it, endeavors to change its condition."

In Newton's mind this change of condition was acceleration of the body relative to absolute space. It is puzzling that Newton never mentioned the fact that the force of inertia, which opposes acceleration, violates his Third Law of Motion, for absolute space cannot sustain the required reaction force.

Two hundred years later Mach (1960) went to great length to strip mechanics of absolute space. This made relativity the powerful principle it has become since. The magnitude of inertia forces was not affected by relativity theory and there was no need for engineers to take note of the philosophical shift. Mach pointed out that the 'fixed stars' could be substituted for the term 'absolute space.' Mach had unhesitatingly accepted Newtonian attractions and repulsions, which are the basis of Newton's third law, and, therefore, held that the force of inertia is a mutual simultaneous far-action between a particle in the laboratory and huge amounts of matter in the distant universe. This has become known as Mach's Principle.

Einstein was impressed by Mach's opposition to absolute space. This helped to inspire the development of relativity theories. Their author hoped to incorporate Mach's principle into general relativity via his equivalence of gravitational and inertial forces. Later he admitted (Einstein, 1961) that this attempt had failed because the particle of general relativity would experience forces of inertia even when it was assumed that no matter existed in the remote universe with which it could interact.

Notwithstanding the so-called fictitious nature of the force of inertia, a number of physicists were intrigued by the cosmological consequences of Mach's principle and tried to trace the origin of inertia in greater detail. Amongst them were Sciama (1953) and

Dicke (1964). Both accepted the reality of the force of inertia as a force which opposed acceleration of matter relative to other matter in the distant universe. Unlike Mach, however, they abandoned Newtonian attractions, and his Third law, in favor of field contact action and the energy-impact mechanism which this implied.

With the velocity of light delays in the transmission of energy across space they were unable to pin down the locality of the inertial reaction forces. In this respect the inertia force fared no better than the Lorentz force of relativistic electromagnetism. The latter should have a reaction force in the local field and this force should slow down energy momentum. But this reaction force has been shown to be far too small to account for the measured electrodynamic forces on metallic conductors of electricity (Graneau, 1987).

The first to attempt an explanation of inertia forces by retarded action at a distance of cosmic matter on objects in the laboratory were Moon and Spencer (1959). In 1959 they proposed the addition of an inertia-gravity force to Newton's law of gravitation. The new term depended on the product of the two interacting masses and the acceleration of the laboratory body with respect to the cosmic substratum. This force between two particles, or bodies, opposed acceleration of the laboratory object relative to what was essentially Newton's absolute space. All the acceleration dependent forces on the laboratory mass summed to the force of inertia as given by Newton's second law of motion. The sum came to zero when the acceleration, or deceleration, ceased.

In 1982 Burniston Brown (Brown 1982) improved the Moon and Spencer theory, also proposing retarded action-at-a-distance. He made the inertia-gravity force an attraction or repulsion between the particle A on earth and the particle B in the distant universe. It was an attraction proportional to the relative acceleration with which the distance between A and B increased, and a repulsion for relative deceleration which decreased the $A - B$ distance. Surprisingly, all the inertia-gravity forces on A summed vectorially to a single force of inertia of the correct magnitude which opposed acceleration of A relative to the reference frame provided by the distant galaxies, or approximately, relative to the fixed stars. To achieve this result the galaxies had, however, to be distributed isotropically around the earth which then found itself in the center of the universe. Strangely, Burniston Brown paid only lip service to the retardation of the action of the universe on the earth laboratory. All his calculations ignored this retardation by millions of years of the gravity-inertia forces. Therefore the calculations equally well reflected the situation that would have arisen in a Newtonian world of simultaneous far-actions.

In 1971 French speculated on the origin of inertia in his textbook of *Newtonian Mechanics* (French, 1971, Chapter 12). He later attributed (private letter to P. G., March 1991) all facts of this discussion to Morrison (1965). French's speculation appears to be based on Newtonian attractions and repulsions, but no specific statement to this effect was made.

The Morrison-French treatment starts with an analogy to Coulomb's law in its Newtonian - and not quantum electrodynamic - connotation. It goes on to consider the case when the first charge is accelerated with respect to the second, which causes an additional interaction force on the second charge proportional to the acceleration and the charge product, and inversely proportional to the distance between the two charges. It should be emphasized that in classical electromagnetism this component of the electromagnetic force does not comply with Newton's third law. In this example, in particular, although the second charge experiences a force from the first charge proportional to its acceleration, the first charge does not experience an opposite force due to the second charge if the second charge is at rest or in uniform motion. If Newton's law of gravitation takes the place of Coulomb's law and an acceleration-dependent force arises again, then this latter force, when summed for one particle in the laboratory over all matter in the universe, could take the place of the inertia force. On this basis it was argued that the distant mass causing inertia must be much larger than the mass of the earth, or the sun, or the Milky Way galaxy. It would have to be predominantly the mass of matter residing outside our own galaxy.

In the Morrison-French analysis the particle pair interactions were not summed vectorially, as they should have been. This casts some doubt on the validity of their calculations. The other important aspect is the lack of symmetrical action and reaction in their force law.

In 1989 Assis (Assis 1989) published an implementation of Mach's principle in accordance with Newtonian simultaneous interactions. It involved a gravity-like attraction of two mass particles which was proportional to their relative acceleration and to the product of their masses, and inversely proportional to the distance of separation. A new step with respect to all previous attempts of explaining the origin of inertia was the contention that, when viewed from a laboratory on earth, all matter in the universe can be divided into (1) an isotropic and (2) an unisotropic distribution. The unisotropic distribution of matter, consisting primarily of the earth, moon and sun, is responsible for the normal gravitational effects of Newton's law of gravitation. The isotropically distributed matter which, as far as we know, seems to reside largely outside the Milky Way galaxy, together with the acceleration-dependent attraction, generates Newton's *vis insita* which opposes acceleration of a laboratory body relative to the whole of the isotropic matter distribution in the distant universe.

2. The Principle of Dynamical Equilibrium

It is unfortunate that Newton failed to state explicitly that his second law is the law of the acceleration resisting inertia force $\vec{F}_i$ acting on a body, or particle, of mass m which is subject to an impressed or applied force $\vec{F}_a$. On the basis of Mach's principle, if the body can move relative to the distant matter in the universe, then whatever it is in the distant universe that controls the force of inertia will adjust the acceleration $\vec{a}$ so that the inertia force of Newton's second law balances the applied force. In mathematical terms this can be expressed as

$$\vec{F}_i = -m\vec{a} = -\vec{F}_a \ . \tag{1}$$

We fully accept Mach's principle and augment it with the principle of dynamic equilibrium which reads:

The acceleration of a body, or particle, relative to the distant universe adjust itself to a magnitude and direction which ensures that the force of inertia on the body, or particle, is in equilibrium with the force impressed on it.

If more than one force is impressed on the test body, then the inertia force balances, of course, the resultant of all the applied forces.

When another body prevents the test body from moving independently relative to the distant universe, then the inertia force is replaced by a mechanical reaction force F_r. This complies with Newton's third law. For example, a body resting on the laboratory bench is subject to the downward applied gravitational force F_a and is pushed up by the bench with the reaction force F_r, so that

$$F_a + F_r = 0 \ . \tag{2}$$

The force of inertia should be treated as a dynamical reaction force. There are instances when both the force of inertia F_i and a mechanical reaction force F_r oppose an impressed force F_a. An example is the fall of a body in a viscous liquid. In this more general case the principle of dynamic equilibrium takes the form

$$F_a + F_r + F_i = 0 \ , \tag{3}$$

all in the direction of F_a. Equation (3) holds for all kinds of forces that may be applied to a material object. It must be remembered, however, that F_i and F_r are never applied forces but reactions which restore force equilibrium. Furthermore, it is claimed that objects would fall much faster than they do if the motion was not impeded by simultaneous interactions with matter in the remote universe.

As has been repeatedly pointed out in the past three hundred years, Newton's first law is really redundant, or at best a corollary of the second law, for without an impressed force the inertia force is zero and the test body simply coasts along in a straight line at constant velocity relative to the distant universe.

While the majority of mechanics textbooks continue to refer to "fictitious" inertia forces, all serious researchers of the subject have accepted the reality of these forces as spelled out by Newton in his Definition III and embedded in Mach's principle. Dynamic equilibrium is a natural consequence of the reality of inertia forces.

3. The Mach-Weber Model

This model to implement Mach's principle can be stated as follows (see Assis 1989 and 1992):

The sum of all forces of any kind (gravitational, electric, magnetic, contact, nuclear, inertial) on any body is always zero in all frames of reference, even when the test body is in motion and accelerated.

This is a generalization of the principle of static equilibrium because it involves situations of motion. It is also in agreement with Mach's point of view of treating the inertial forces as real forces due to some kind of interaction of the test body with the remainder of the universe.

According to the Mach-Weber model, all inertial forces ($-m_i\vec{a}$, the centrifugal force, the Coriolis force) are due to a *gravitational* interaction of any body with the remainder of the universe. This at once explains the proportionality between the inertial mass m_i (i. e., the mass which appears in the expressions of the linear and angular momentum, in the kinetic energy, and in the inertial forces), and the gravitational mass m_g (i. e., the mass which appears in the gravitational potential energy, in the weight of a body and in Newton's law of universal gravitation).

In classical mechanics there is a set of frames of reference in which Newton's second law of motion is valid and has the simplest form. These are called inertial frames of reference. It is an observational fact that the best inertial frame we have happens to be the frame of the distant galaxies (Schiff, 1964), that is, the frame relative to which the set of distant galaxies does not rotate and has no linear acceleration as a whole. Let us analyse the Mach-Weber model in this frame of the distant galaxies, taking into account the Seeliger-Neumann exponential term (Assis 1992):

The generalized potential energy U between two point particles m_1 and m_2 is given by

$$U = -H_g \frac{m_1 m_2}{r}\left(1 - 3\frac{\dot{r}^2}{c^2}\right)e^{-\alpha r} \ , \tag{4}$$

where H_g is an arbitrary constant and its value depends on the system of units. The simplest choice is that for which $H_g = G$ (where G is the constant of universal gravity, namely $G = 6.67 \times 10^{-11}\ m^3/kgs^2$). From now on we will follow this choice. Moreover, m_1 and m_2 are the gravitational masses of the particles 1 and 2, and c is a constant with the same value as the velocity of light in vacuum. The distance between the two particles is r, and their relative velocity along r is denoted by $\dot{r} = dr/dt$. The exponential decay $e^{-\alpha r}$, with a constant α, was first introduced in the Newtonian potential energy (equation (4) without the term in $\dot{r}$) by H. Seeliger and C. Neumann in order to avoid the gravitational paradox (North 1965, Jaakkola 1987 and 1991). We introduced this term in equation (4) for similar reasons (Assis 1992). We assume that $\alpha = 0$ if there is a complete vacuum between the interacting bodies.

The force between these two particles ($F = -dU/dr$) is found to be

$$F = -G\frac{m_1 m_2}{r^2}\left[1 - 3\frac{\dot{r}^2}{c^2} + 6\frac{r\ddot{r}}{c^2} + \alpha r\left(1 - 3\frac{\dot{r}^2}{c^2}\right)\right]e^{-\alpha r} , \tag{5}$$

where $\ddot{r} = d^2r/dt^2$ is the radial acceleration between them. The negative sign of this expression stands for a force of attraction. This force is always along the straight line connecting the two particles and follows Newton's third law.

We now calculate with this expression in the frame of distant galaxies, described above, the net force on a test body of gravitational mass m due to unifomly distributed matter in the whole boundless universe. This is found to be (Assis 1992)

$$\vec{F} = -Am\vec{a} , \tag{6}$$

where $\vec{a}$ is the acceleration of the body m relative to the frame of distant galaxies. In this expression A is a dimensionless constant given by

$$A = 8\pi\frac{G}{c^2}\frac{\rho}{\alpha^2} , \tag{7}$$

where ρ is the mean matter density in the universe.

According to the principle of dynamical equilibrium, if the sum of all non-inertial forces on m is represented by $\sum_n \vec{F}_n$ we then have (see Figure 1):

$$\sum_n \vec{F}_n - Am\vec{a} = 0 . \tag{8}$$

We can only recover Newton's second law of motion if $A = 1$, which means that

$$G = \frac{c^2\alpha^2}{8\pi\rho} = \frac{H_o^2}{8\pi\rho} , \tag{9}$$

where (Assis 1992): $\alpha = H_o/c$, with H_o being Hubble's constant. This relation between G, H_o and ρ was first observed by P. A. M. Dirac in the 1930's, and has been confirmed since then. It is an empirical fact which had no explanation in classical physics, and was utilized by Dirac as the basis of his large number theory. The same fact is a consequence of the Mach-Weber model.

We will now apply this formulation to some specific problems of dynamical equilibrium, namely: The force of free fall, problems involving the centrifugal force, and problems involving the gyroscope or the spinning top. In all these cases the gravitational interaction between the earth and a body on its surface (an apple or a top), or between the sun and a planet, will be represented by

$$F = -G\frac{m_1 m_2}{r^2} . \tag{10}$$

This is a simplification of Eq. (5) valid for low relative velocities, low relative accelerations and negligible matter density, or vaccum, between the interacting bodies.

4. The Forces of Free Fall

In Newtonian mechanics this would be considered a two-body problem, but in the Mach-Weber model the same situation is to be treated as a many-body interaction: The test body, the earth, and the distant galaxies. Suppose the test body falls in vacuum to the surface of the earth. We can write eqs. (8), (9) and (10) as (see Figure 2):

$$G\frac{mM_E}{R_E^2} - ma = 0 \; , \tag{11}$$

where m is the gravitational mass of the test body, M_E is the gravitational mass of the earth, and R_E is the radius of the earth.

The first fact to be observed from this expression is that the mass m of the test body will disappear from the expression, so that the acceleration of fall in vacuum will be the same for all objects. This extremely important fact was first pointed out by Galileo, and was confirmed in great detail by Newton with his famous pendulum experiments. This is the basis of the principle of equivalence, which was utilized by Einstein as the foundation of his general theory of relativity. Historically, no explanation of this fact was given, but it is a natural consequence of the Mach-Weber model, according to which the inertial force $-m\vec{a}$ is a real force of *gravitational* origin due to the relative acceleration between the test body and the distant galaxies.

Another relevant fact depicted in figure 2 is that even the inertial force $-m\vec{a}$ is paired with a force of equal magnitude but opposite direction, as any other fundamental force in classical physics. The paired force is in the matter which composes the distant universe and obeys Newton's third law. In classical mechanics the inertial forces were not associated with a counter-force on another body.

5. The Centrifugal Force

From the textbooks of classical mechanics we learn that the centrifugal forces do not exist as such, and are only "fictitious" forces which appear in non-inertial frames of reference. On the other hand, according to Mach, they are considered to be real forces which arise due to the relative rotation between the test body and the distant universe. Some quotations from Mach (Mach 1960) emphasizing this point are: "Try to fix Newton's bucket and rotate the heaven of fixed stars and then prove the absence of centrifugal forces" (p. 279); "The principles of mechanics can, indeed, be so conceived, that even for relative rotations centrifugal forces arise" (p. 284); "Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces but that such forces are produced by its relative rotation with respect to the mass of the earth and the other celestial bodies" (p. 284). We agree with Mach on these points and not with the textbooks. Let us analyse this problem with the Mach-Weber model.

We will study the orbit of a planet around the sun. For simplicity we will assume a circular orbit and that the mass of the planet is much smaller than the mass of the sun so that the sun can be considered at rest in the center of mass of the system, as shown in figure 3. If the tangential velocity of the planet is v and its distance to the center of the sun is r, than its centripetal acceleration is v^2/r pointing towards the sun. According to the principle of dynamical equilibrium and equations (8), (9) and (10) this situation of dynamical equilibrium will be as that represented in figure 3 and given by

$$G\frac{mM_S}{r^2} - m\frac{v^2}{r} = 0 \ , \tag{12}$$

where the forces are along the line connecting the sun to the planet, m is the mass of the planet and M_S is the mass of the sun.

Once more the mass of the planet will disappear from the final expression. And the inertial force will have its reaction in the distant universe.

An analogous situation occurs when a body is attached to a string of length l fixed in one of its extremeties to a frictionless table, rotates at a constant angular velocity ω. In this situation (see figure 4) the tension T in the string is balanced by the centrifugal force $m\omega^2 l$. The reaction to the tension T acts on the table (and eventually on the earth) through its point of support, while the reaction to the centrifugal force is in the distant universe.

6. The Coriolis Force

We will now analyse a spinnig top or a flywheel from this new point of view. To simplify the analysis we will represent the flywheel by only four particles of the wheel A, B, C, and D (see figure 5). The flywheel rotates with an angular velocity ω around the axis of symmetry $x - y$.

Let us analyse the situation when the flywheel, spinning about its axis, is held with its axis initially at rest at an angle $\theta_o = 90^o$ with the vertical and then released. The material point D closest to the observer in figure 5 will change its linear velocity from v to v'. For a very small angle α, the increment in velocity, $v_r = v' - v$, for this mass with respect to the laboratory will be horizontal and to the left (see figure 5). This acceleration is caused by the weight and the structure of the flywheel-shaft combination. The force of inertia which opposes this acceleration is horizontal to the right.

Now consider point mass B which is the farthest from the observer. The velocity increment for this mass during the same period is to the right, as is its acceleration. Consequently the force of inertia which opposes this acceleration is to the left, according to eq. (1).

These two forces of inertia create an inertial torque given by $2rF_i$, where $F_i = -ma$ is the inertial force acting on each of these masses and r is the radius of the flywheel.

Calling T_i the total inertial torque acting on all atoms composing the flywheel, this inertial torque will be in the direction Ω of its precession (see figure 5). This torque (the precession torque) exists only while the wheel is falling. It is responsible for the acceleration of the wheel in the Ω-direction. This torque ceases to exist when the fall of the wheel is arrested by another inertial torque (the lift torque) which prevents the wheel from falling to the ground.

As the Mach-Weber model complies with Newton's third law, there must exist a simultaneous torque $-T_i$ in the distant universe.

We now consider another aspect of the spinning top or flywheel, that is, when it is spinning with constant angular velocity ω and precessing with constant angular velocity Ω. Usually there will be a nutation of the flywheel, but we will analyse here only the special case in which there is no nutation. As in the previous example, we will suppose that the axis of the flywheel makes an angle $\theta_o = 90^o$ with the vertical, as shown in figure 6.

For a very small angle α along Ω the uppermost mass C (see figure 5) is associated with an increment of velocity $v' - v = v_r$ in the direction of the axis, pointing from x to y. This requires an acceleration in this direction which is caused by the internal structure of the flywheel. This acceleration of the mass C will be opposed by an inertial force which is parallel to the axis but pointing from y to x.

On the other hand the bottom mass A, during the same small time interval, is associated with an increment of velocity in the opposite direction, namely, parallel to the axis but pointing from y to x. This implies an acceleration in this direction which will be opposed by an inertial force parallel to the axis but pointing away from x to y.

Let the inertial torque due to precession be $T_i = 2rF_i$, then for dynamic equilibrium we must have $T_i + Wb = 0$, where b is the distance of the flywheel from support x and W its weight. If the precession is forcibly blocked, the inertial lift-torque T_i will cease to exist and the wheel will fall to the ground. In the Weber-Mach model there arises, of course, an equal and opposite reaction torque in the distant universe.

7. An Experimental Consequence of the Mach-Weber Model

We would now like to present an experimental consequence of the Mach-Weber model.

It arises from the concept that the centrifugal force derives from a *relative* rotation between the test body and the surrounding matter (see the Mach's quotations listed before). Accordingly, if we rotate a spherical matter shell about the z-axis, a centrifugal force must be exerted on any *stationary* internal body which is not on the axis of rotation (figure 7). Let us calculate the magnitude of this force. For a spherical shell of radius R, thickness dR, made of a material of mass density ρ, spinning with an angular velocity ω, the centrifugal force on a stationary mass m located at a distance $r < R$ from the axis of rotation is found to be (Assis 1989 and 1992):

$$F = \xi \frac{4\pi}{3} \frac{G}{c^2} m \rho R dR w^2 r \ , \tag{13}$$

where $\xi = 6$. Suppose the spherical shell has a radius of 1 m, a thickness of 0.1 m, is spinning at $\omega = 2\pi(100\ Hz)$, and made of iron ($\rho = 8 \times 10^3\ kg/m^3$). Let the test particle have a mass of 1 kg and be located at a distance $r = 0.5\ m$ from the axis of rotation. These conditions yield the centrifugal force $F = 3 \times 10^{-18}\ N$. It is a very small force, but at least it is a definite and quantitative prediction of the model. This force does not exist according to Newton's law of gravitation, but it must be present if Mach is correct. The advantage of the Mach-Weber model is that we can calculate its magnitude.

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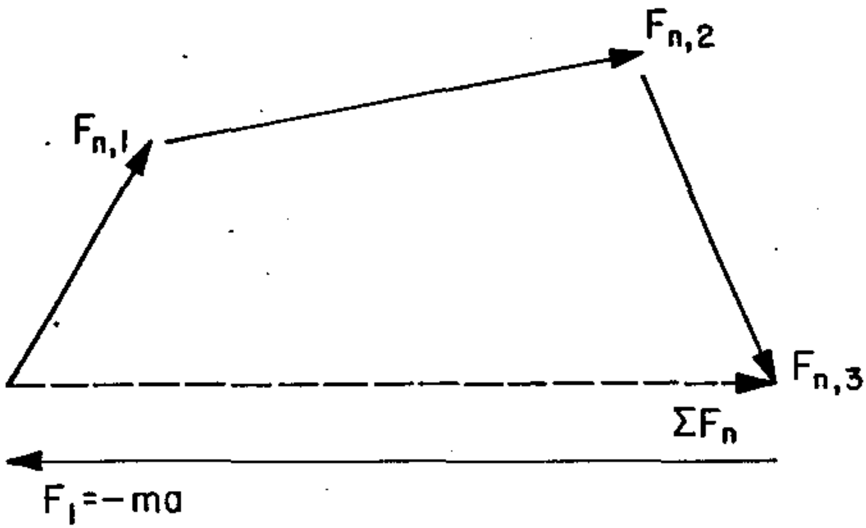

Fig. 1: The inertial reaction force F_i

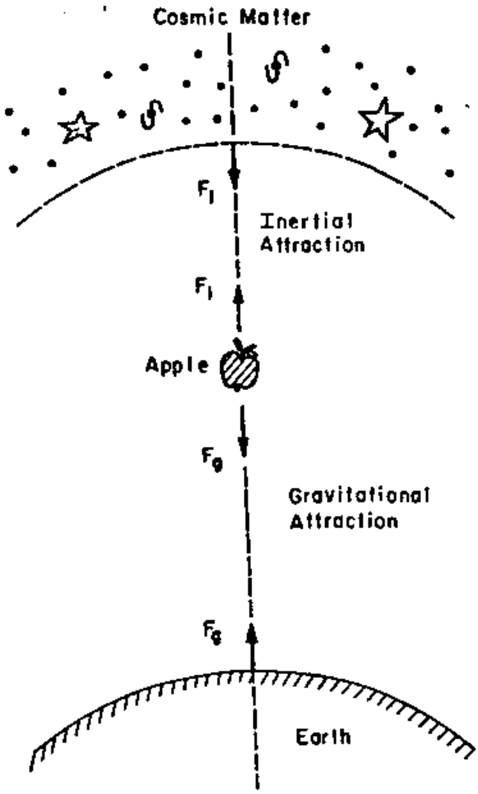

Fig. 2: The forces of free fall

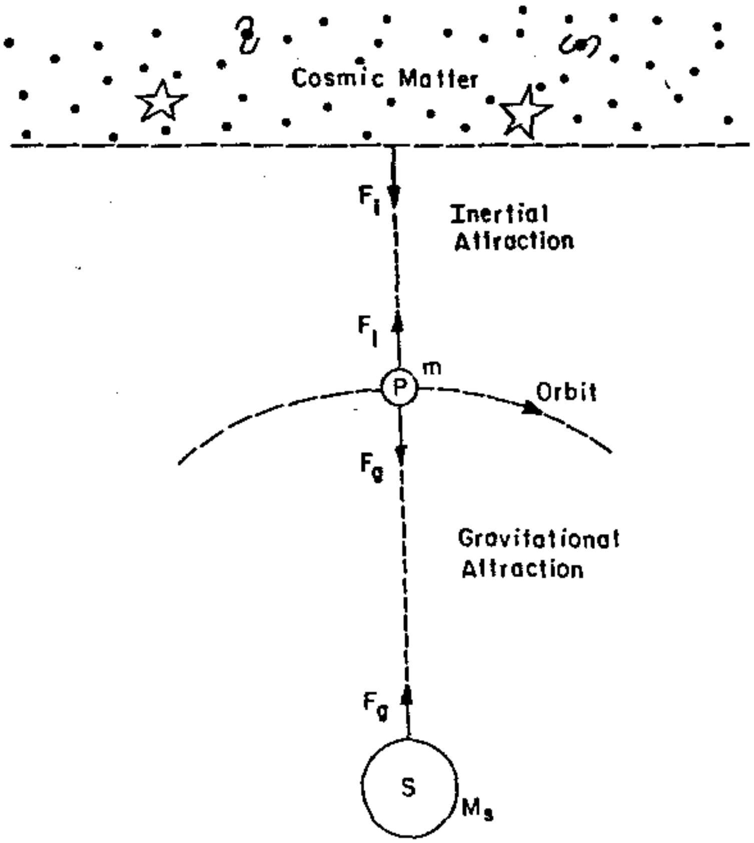

Fig. 3: Centrifugal force F_i due to inertial attraction

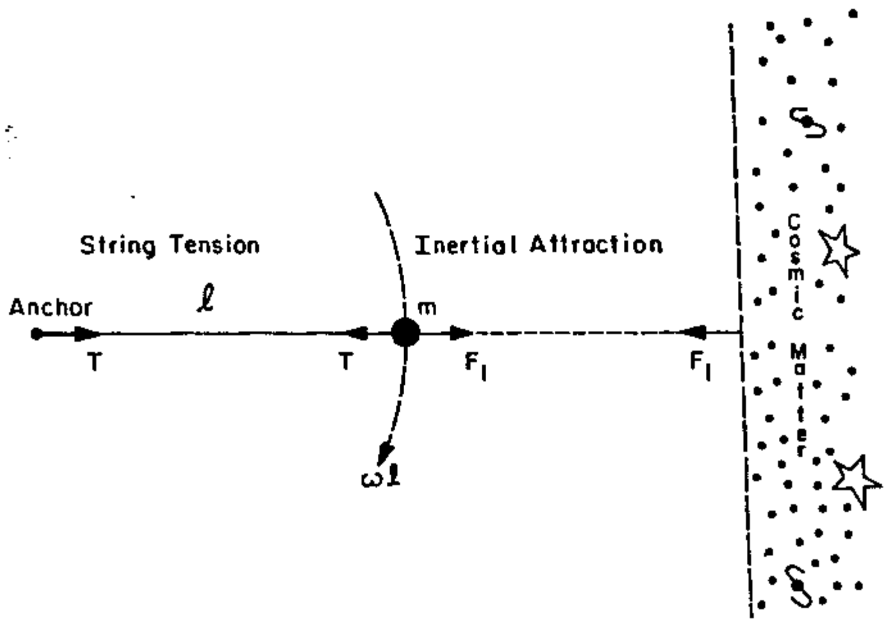

Fig. 4: String tension T balanced by centrifugal attraction

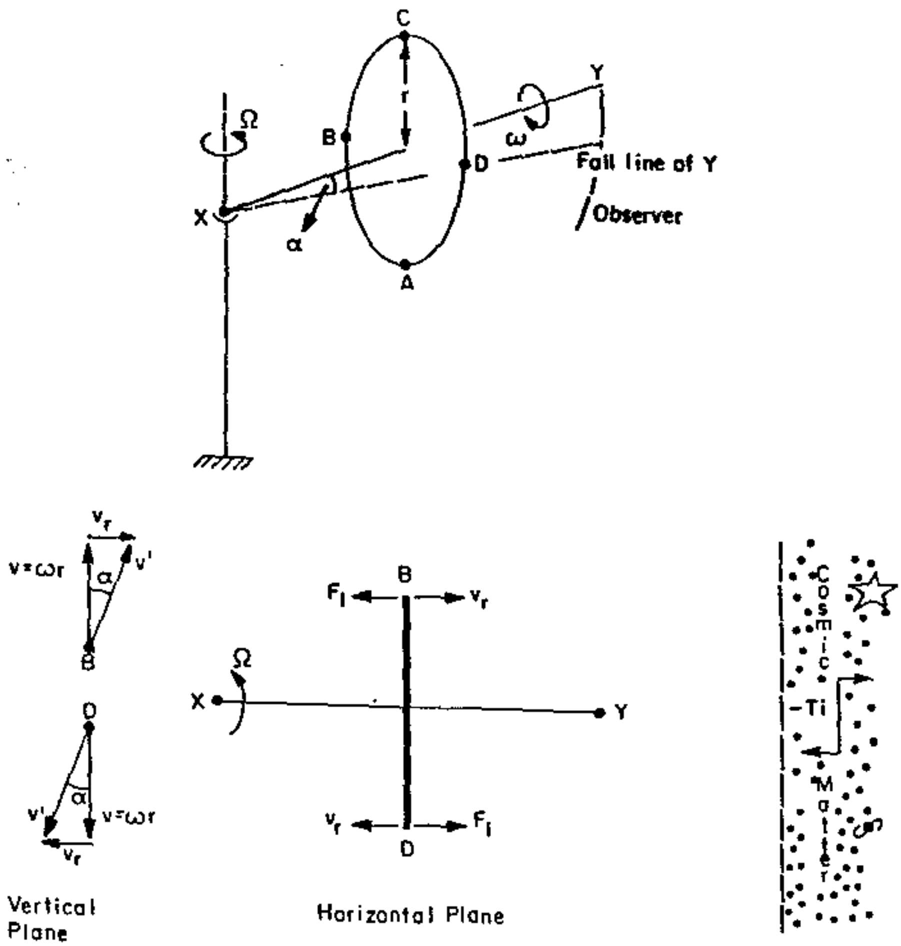

Fig. 5: Precession torque $T_i = 2rF_i$ of flywheel

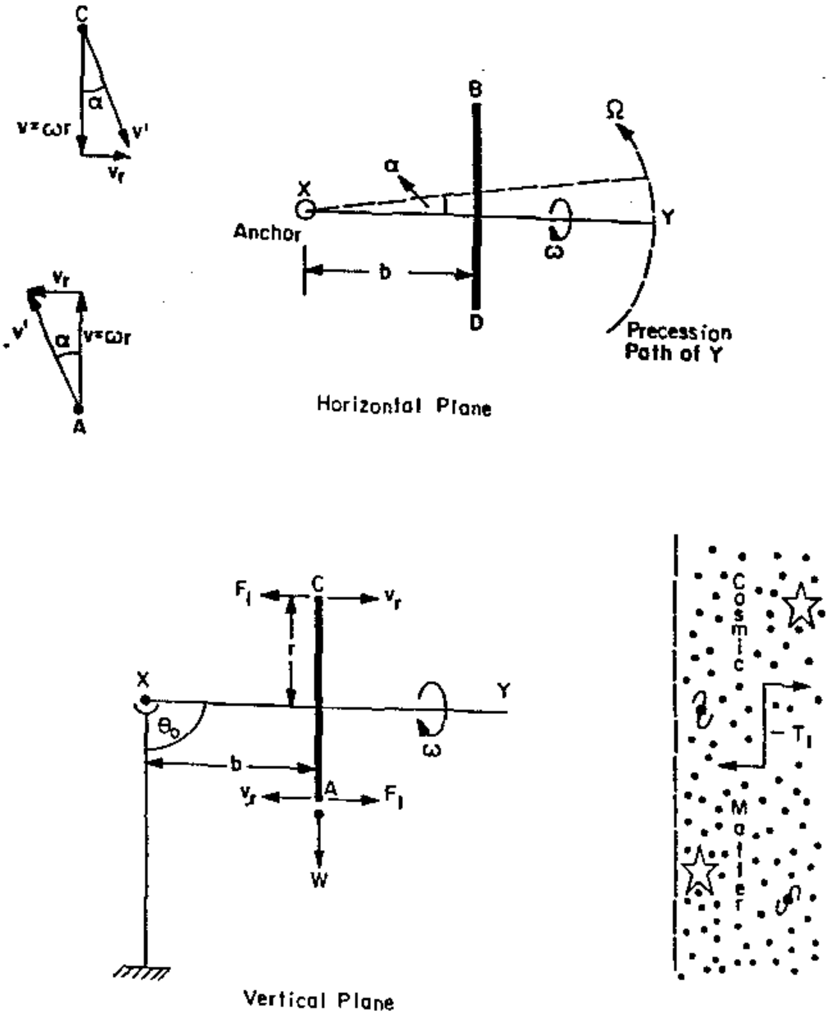

Fig. 6: Lift-torque $T_l = 2rF_l = -Wb$

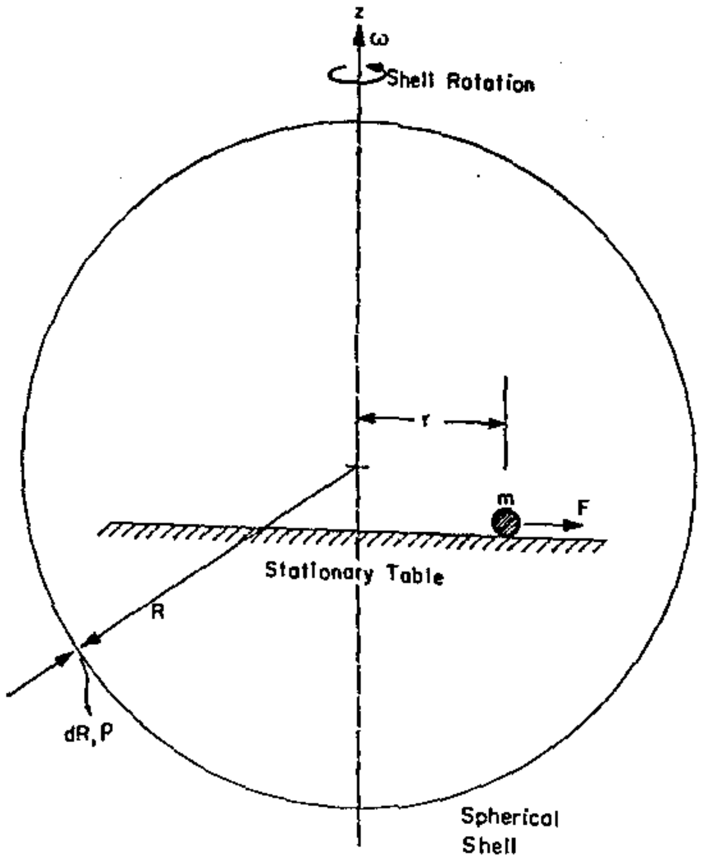

Fig. 7: Test of Mach-Weber model

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