Transactions Briefs

Surface Charges and Fields in a Resistive Coaxial Cable Carrying a Constant Current

A. K. T. Assis and J. I. Cisneros

Abstract—**We calculate the surface charges, potentials, and fields in a long cylindrical coaxial cable with inner and outer conductors of finite conductivities and finite areas carrying a constant current. It is shown that there is an electric field outside the return conductor.**

Index Terms—**Classical electrodynamics, coaxial cable, Surface charges.**

I. Introduction

In the study of *dc* and low frequency *ac* circuits, the following subjects are seldom analyzed in electromagnetic books: electric fields outside the conductors, surface charges on the wires, and energy flow from the sources to the conductors where energy is dissipated. There are two main reasons for this: 1) The scalar electric potential is the solution of Laplace's equation with frequently complicated boundary conditions; and 2) the solution of elementary circuits, based on Ohm's law, is obtained by the application of Kirchhoff's rules. As these rules utilize only the values of current and potential inside the conductors, the discussion of the subjects listed above is unnecessary. However, some authors have treated these topics in the past few years (see [1]–[3] and references therein). The case of a long coaxial cable has been treated by Sherwood, [3], Marcus, [4], Sommerfeld, [5] (German original from 1948), Griffiths, [6], and a few others. All of these works considered an equipotential return conductor either with an infinite area or with an infinite conductivity. Our goal in this work is to generalize these assumptions considering a return conductor with finite area, finite conductivity, and a variable electric potential along its length. We first discussed our work at the International Conference on Relativistic Physics and Some of Its Applications in 1998[7]. We calculate at all points in space the scalar and vector potentials, the electric and magnetic fields and analyze the energy flow by means of the Poynting vector. We also calculate the surface electric charges. To our knowledge the only one who has considered these generalizations before has been Jefimenko, [2]. However, he restricted his analysis to the fields inside the cable. He did not calculate or mention the electric field outside the cable and this is our main contribution here. We show that this electric field has not only a longitudinal component parallel to the cable but also a radial component due to surface charges distributed along the outer surface of the resistive return conductor, even for constant currents.

The geometry of the problem is that of Fig. 1. A constant current I flows uniformly in the z direction along the inner conductor (radius a and conductivity g_1), returning uniformly along the outer conductor (internal and external radii b and c, respectively, and conductivity g_3). The conductors have uniform circular cross sections and a length $\ell \gg c > b > a$ centered on $z = 0$. The medium outside the conductors is considered to be air or vacuum with $\varepsilon = \varepsilon_o = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}. The potentials at the top extremities ($z = \ell/2$) of the inner and outer conductors are maintained at the constant values ϕ_A and ϕ_B, respectively. The potentials at the bottom extremities ($z = -\ell/2$) of the outer and inner conductors are maintained at the constant values ϕ_C and ϕ_D, respectively.

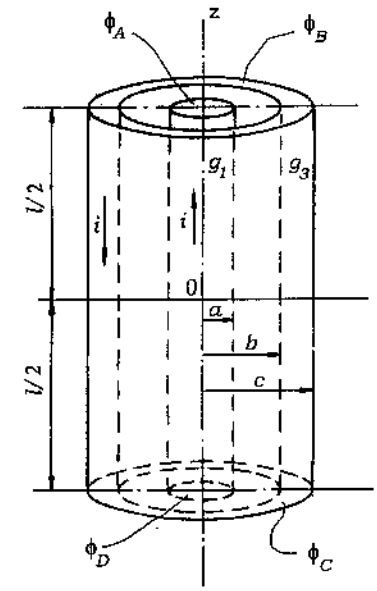

Fig. 1. Geometry of the problem.

In the previous works quoted above (with the exception of that by Jefimenko), the authors considered only a particular case: an equipotential outer conductor ($\phi_C = \phi_B = 0$) with an infinite area (Sommerfeld, $c \to \infty$) or with an infinite conductivity (Griffiths, $g_3 \to \infty$). We are interested in calculating the potentials and fields in a point $\vec{r} = (\rho, \varphi, z)$ such that $\ell \gg \rho$ and $\ell \gg |z|$, so that we can neglect border effects (ρ, φ and z are the cylindrical coordinates). All solutions presented here were obtained in this approximation. With this approximation and geometry we then have the potential as a linear function of z, [8]. In order to have uniform currents flowing in the z direction along the inner and outer conductors, with a potential satisfying the given values at the extremities, we have

$$\phi(\rho \leq a, \varphi, z) = \frac{\phi_A - \phi_D}{\ell} z + \frac{\phi_A + \phi_D}{2} \tag{1}$$

$$\phi(b \leq \rho \leq c, \varphi, z) = \frac{\phi_B - \phi_C}{\ell} z + \frac{\phi_C + \phi_B}{2} \tag{2}$$

where, by Ohm's law (R_1 and R_3 being the resistances of the inner and outer conductors, respectively)

$$\phi_D - \phi_A = R_1 I = \frac{\ell I}{\pi g_1 a^2} \tag{3}$$

$$\phi_B - \phi_C = R_3 I = \frac{\ell I}{\pi g_3 (c^2 - b^2)}. \tag{4}$$

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In the four regions ($\rho < a, a < \rho < b, b < \rho < c$, and $c < \rho$) the potential ϕ satisfies Laplace's equation $\nabla^2\phi = 0$. By (1) and (2) we have the value of ϕ in the first and third regions, which also supply the boundary conditions at $\rho = a$ and at $\rho = b$ in order to find ϕ in the second region. To find ϕ in the fourth region we need another boundary condition, in addition to the value of ϕ at $\rho = c$, which is given by (2). We then impose the following boundary condition:

$$\phi(\rho = \ell, \varphi, z) = 0. \tag{5}$$

This is the main nontrivial boundary condition for this problem. It says that the potential goes to zero at a radial distance $\rho = \ell$ so that the length ℓ of the cable appears in the solution. The usual condition $\phi(\rho \to \infty, \varphi, z) = 0$ does not work in the situation considered here. We first tried this last condition, but could not obtain a correct solution for the potential. We only discovered (5) working backward. That is, from the work of Russell we knew that, in general, the density of the surface charges on a system of long parallel homogeneous conductors in steady-state (as is the case of the coaxial cable being considered here) varies linearly with distance along the direction of their common axis [8]. That is, if d represents a, b or c, the surface charge densities at these surfaces must be given by $\sigma_d(z) = A_d z + B_d$, with the constants A_d and B_d characterizing each surface. We then obtained the potential at all points in space by

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_o} \sum_{j=1}^{3} \iint_{S_j} \frac{\sigma(\vec{r}_j)\, da_j}{|\vec{r} - \vec{r}_j|}. \tag{6}$$

Here the sum goes over the three surfaces $\rho = a$, b, and c, extending from $z = -\ell/2$ to $z = \ell/2$. After solving these integrals we discovered that ϕ went to zero, not at infinity, but at $\rho = \ell$. Although this difference is important mathematically in order to arrive at a working solution, physically we can say that the potential going to zero at $\rho = \ell$ is equivalent to it going to zero at infinity. As we are supposing $\ell \gg c > b > a$, we are essentially imposing that the potential goes to zero at a large distance from the cable, which is reasonable.

Here we are reversing the arguments, as this is more straightforward. That is, we are beginning with the boundary conditions for ϕ, obtaining the solutions of Laplace's equation, the electric field $\vec{E} = -\nabla\phi$ and then σ by Gauss's law.

The boundary conditions are then the values of ϕ at $\rho = a$, $\rho = b$, $\rho = c$, and $\rho = \ell$. They are given by (1), (2) and (5). The solutions of Laplace's equation $\nabla^2\phi = 0$ for $a \le \rho \le b$ and for $c \le \rho$ in cylindrical coordinates satisfying these boundary conditions yield

$$\phi(a \le \rho \le b, \varphi, z) = \left[\frac{\phi_A - \phi_D + \phi_C - \phi_B}{\ell} z + \frac{\phi_A + \phi_D - \phi_C - \phi_B}{2}\right] \frac{\ln(b/\rho)}{\ln(b/a)} + \left[\frac{\phi_B - \phi_C}{\ell} z + \frac{\phi_C + \phi_B}{2}\right] \tag{7}$$

$$\phi(c \le \rho, \varphi, z) = \left[\frac{\phi_B - \phi_C}{\ell} z + \frac{\phi_C + \phi_B}{2}\right] \cdot \frac{\ln(\ell/\rho)}{\ln(\ell/c)}. \tag{8}$$

The electric field $\vec{E} = -\nabla\phi$ is given by

$$\vec{E}(\rho < a, \varphi, z) = \frac{\phi_D - \phi_A}{\ell}\hat{z} \tag{9}$$

$$\vec{E}(a < \rho < b, \varphi, z) = \left[\frac{\phi_A - \phi_D + \phi_C - \phi_B}{\ell} z + \frac{\phi_A + \phi_D - \phi_C - \phi_B}{2}\right] \frac{1}{\ln(b/a)} \frac{\hat{\rho}}{\rho} + \left[\frac{\phi_C - \phi_B}{\ell} + \frac{\phi_D - \phi_A + \phi_B - \phi_C}{\ell} \cdot \frac{\ln(b/\rho)}{\ln(b/a)}\right]\hat{z} \tag{10}$$

$$\vec{E}(b < \rho < c, \varphi, z) = \frac{\phi_C - \phi_B}{\ell}\hat{z} \tag{11}$$

$$\vec{E}(c < \rho, \varphi, z) = \left[\frac{\phi_B - \phi_C}{\ell} z + \frac{\phi_C + \phi_B}{2}\right] \frac{1}{\ln(\ell/c)} \frac{\hat{\rho}}{\rho} + \frac{\phi_C - \phi_B}{\ell} \frac{\ln(\ell/\rho)}{\ln(\ell/c)}\hat{z}. \tag{12}$$

Equations (7) and (10) had been obtained by Jefimenko, [2], who also discussed the flow of energy in this system. He considered $\phi_C = 0$, $\phi_D = V$, $\phi_A - \phi_B = RI$, $\phi_B - \phi_C = R_b I$, $\phi_D - \phi_A = R_a I$, $I = V/(R_a + R_b + R)$ and his z is equal to our z plus $\ell/2$. Here R_a is the resistance of the inner conductor, R_b is the resistance of the outer one, V is the constant voltage maintained by a battery, and R is an external resistance between A and B. With these replacements in our (7) and (10) we recover his solutions. The main aspect to be emphasized here are our solutions (8) and (12). They were not obtained by Jefimenko (who did not study the fields outside the cable). They show the existence of an electric field outside the resistive cable, even when it is carrying a constant current.

Assis discussed elsewhere another kind of electric field outside conductors carrying constant currents [9], [10]. It is usually called motional electric field and is proportional to second order in v_d/c, where v_d is the drifting velocity of the electrons and $c = 3 \times 10^8$ m/s. However, its order of magnitude is much smaller than this one considered here (proportional to the potential difference along the cable, or to its current or to the drifting velocity of the conduction electrons). For this reason we do not need to take it into account here.

The surface charges densities σ along the inner conductor ($\rho = a$, $\sigma_a(z)$) and along the inner and outer surfaces of the return conductor ($\rho = b$, $\sigma_b(z)$ and $\rho = c$, $\sigma_c(z)$) can be obtained easily utilizing Gauss's law

$$\oint\!\!\!\oint_S \vec{E} \cdot d\vec{a} = \frac{Q}{\varepsilon_o} \tag{13}$$

where $d\vec{a}$ is the surface element pointing normally outward the closed surface S and Q is the net charge inside S. This yields $\sigma_a(z) = \varepsilon_o E_{2\rho}(\rho \to a, z)$, $\sigma_b(z) = -\varepsilon_o E_{2\rho}(\rho \to b, z)$, and $\sigma_c(z) = \varepsilon_o E_{4\rho}(\rho \to c, z)$ where the subscripts 2ρ and 4ρ mean the radial component of $\vec{E}$ in the second and fourth regions, $a < \rho < b$ and $c < \rho$, respectively. This means that

$$\sigma_a(z) = \frac{\varepsilon_o}{a} \frac{1}{\ln(b/a)} \left[\frac{\phi_A - \phi_D + \phi_C - \phi_B}{\ell} z + \frac{\phi_A + \phi_D - \phi_C - \phi_B}{2}\right] \tag{14}$$

$$\sigma_b(z) = -\frac{a}{b}\,\sigma_a(z) \tag{15}$$

$$\sigma_c(z) = \frac{\varepsilon_o}{c} \frac{1}{\ln(\ell/c)} \left[\frac{\phi_B - \phi_C}{\ell} z + \frac{\phi_C + \phi_B}{2}\right]. \tag{16}$$

Once more, Jefimenko could obtain only σ_a and σ_b but not σ_c, which has been calculated here.

An alternative way of obtaining ϕ and $\vec{E}$ is to begin with the surface charges as given by (14)–(16). We then calculate the electric potential ϕ (and $\vec{E} = -\nabla\phi$) through (6).

We checked our results with this procedure. To this end we needed essentially the following integral (with M and N constants and where d can represent the radii a, b, or c)

$$K \equiv \frac{1}{4\pi\varepsilon_o} \int_{\varphi_2=0}^{2\pi} \int_{z_2=-\ell/2}^{\ell/2} \cdot \frac{(Mz_2 + N)d\, d\varphi_2\, dz_2}{\sqrt{\rho^2 + d^2 - 2\rho\, d\cos(\varphi_2 - \varphi) + (z_2 - z)^2}}. \tag{17}$$

With $\ell \gg \rho$, $\ell \gg d$ and $\ell \gg |z|$ this yields, for $\rho \leq d$ and $\rho \geq d$, respectively

$$K(\rho \leq d) = \frac{d(Mz + N)}{\varepsilon_o} \ln\frac{\ell}{d} \tag{18}$$

$$K(\rho \geq d) = \frac{d(Mz + N)}{\varepsilon_o} \ln\frac{\ell}{\rho}. \tag{19}$$

We can calculate the vector potential utilizing this integral in the usual expression

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \iiint \frac{\vec{J}(\vec{r}\,')\, dV'}{|\vec{r} - \vec{r}\,'|} \tag{20}$$

where $\mu_o = 4\pi \times 10^{-7}$ kg m C^{-2} and dV' is a volume element. With the approximation above and uniform current densities in the inner and outer conductors we obtain

$$\vec{A}(\rho \leq a, \varphi, z) = -\frac{\mu_o I}{2\pi} \left[\frac{\rho^2}{2a^2} - \frac{c^2\ln(c/a) - b^2\ln(b/a)}{c^2 - b^2}\right] \hat{z} \tag{21}$$

$$\vec{A}(a \leq \rho \leq b, \varphi, z) = \frac{\mu_o I}{2\pi} \left[\frac{c^2\ln(c/\rho) - b^2\ln(b/\rho)}{(c^2 - b^2)} - \frac{1}{2}\right] \hat{z} \tag{22}$$

$$\vec{A}(b \leq \rho \leq c, \varphi, z) = \frac{\mu_o I}{2\pi} \left[\frac{c^2\ln(c/\rho)}{c^2 - b^2} - \frac{c^2 - \rho^2}{2(c^2 - b^2)}\right] \hat{z} \tag{23}$$

$$\vec{A}(c \leq \rho, \varphi, z) = 0. \tag{24}$$

The magnetic field can be obtained either through the magnetic circuital law $\oint \vec{B} \cdot d\vec{\ell} = \mu_o I$, or through $\vec{B} = \nabla \times \vec{A}$. Both approaches yield the same result, namely

$$\vec{B}(\rho \leq a, \varphi, z) = \frac{\mu_o I}{2\pi} \frac{\rho}{a^2} \hat{\varphi} \tag{25}$$

$$\vec{B}(a \leq \rho \leq b, \varphi, z) = \frac{\mu_o I}{2\pi} \frac{\hat{\varphi}}{\rho} \tag{26}$$

$$\vec{B}(b \leq \rho \leq c, \varphi, z) = \frac{\mu_o I}{2\pi} \frac{c^2 - \rho^2}{c^2 - b^2} \frac{\hat{\varphi}}{\rho} \tag{27}$$

$$\vec{B}(c \leq \rho, \varphi, z) = 0. \tag{28}$$

This completes the solution of this problem. It should be stressed once more that all these equations are valid at a point $\vec{r} = (\rho, \varphi, z)$ such that $\ell \gg \rho$ and $\ell \gg |z|$.

II. The Symmetrical Case

We now consider two equal batteries symmetrically located on both ends, such that $\phi_D = -\phi_A = \phi_1$ and $\phi_B = -\phi_C = \phi_3$. In this case, the potential is simply proportional to z without any additive constant. We can then write it in terms of the currents and conductivities as given by

$$\phi(\rho \leq a) = -\frac{Iz}{\pi g_1 a^2} \tag{29}$$

$$\phi(a \leq \rho \leq b) = -\frac{I}{\pi} \frac{z}{\ln(b/a)} \cdot \left[\frac{\ln(b/\rho)}{g_1 a^2} - \frac{\ln(\rho/a)}{g_3(c^2 - b^2)}\right] \tag{30}$$

$$\phi(b \leq \rho \leq c) = \frac{Iz}{\pi g_3(c^2 - b^2)} \tag{31}$$

$$\phi(c \leq \rho) = \frac{I}{\pi} \frac{\ln(\ell/\rho)}{\ln(\ell/c)} \frac{z}{g_3(c^2 - b^2)}. \tag{32}$$

A plot of $\phi(\rho)$ versus ρ is given in Fig. 2. In order to obtain this plot we utilized the following data: $a = 0.0010$ m, $b = 0.0040$ m, $c = 0.0047$ m, $I = 50$ A, $g_1 = 5.7 \times 10^6$ m^{-1} Ω^{-1}, $g_3 = 2 \times 10^6$ m^{-1} Ω^{-1} and $\ell = 1$ m. There are two curves, one for $z = 0.003$ m and another for $z = 0.006$ m. We see that the potential is constant for $0 \leq \rho \leq a$, increases between a and b, is constant between b, and c, decreasing for $\rho > c$. As we saw before, it goes to zero at $\rho = \ell$. As $\vec{E} = -\nabla\phi$, the z component of $\vec{E}$ is given by $E_z = -\phi(\rho)/z$. The point where $\phi(\rho) = 0$ is $\rho = \xi$, where

$$\xi = \exp\frac{g_1 a^2 \ln(a) + g_3(c^2 - b^2)\ln(b)}{g_1 a^2 + g_3(c^2 - b^2)}. \tag{33}$$

Sommerfeld or Griffiths's solutions are recovered taking $g_3(c^2 - b^2) \to \infty$, such that $\xi \to b$, $\sigma_c(z) \to 0$, $\vec{E}(\rho > b) \to 0$, and $\phi(\rho \geq b) \to 0$ for any z. The opposite solution when the current flows in an inner conductor of infinite conductivity, returning in an outer conductor of finite area and finite conductivity is also easily obtained from above yielding $\xi \to a$, $\vec{E}(\rho < a) \to 0$, and $\phi(\rho \leq a) \to 0$ for any z. In Fig. 3 we plotted the equipotentials with the same data as above, in SI units. The values of the surface charges at $z = 0.001$ m obtained from (14)–(16) are: $\sigma_a = -6.541\,74 \times 10^{-12}$ Cm^{-2}, $\sigma_b = 2.616\,70 \times 10^{-11}$ C m^{-2}, and $\sigma_c = 4.490\,27 \times 10^{-13}$ C m^{-2}. As the surface charges vary linearly with z, it is easy to find their values at any other distances from the center of the cable.

From Fig. 3 we can see that the equipotentials for $\rho > c$ are inclined relative to the z-ρ axis, indicating once more the existence of longitudinal and radial components of the external electric field.

In order to illustrate an asymmetric case, we plotted in Fig. 4 the equipotentials for $\phi_A = -1.0$ V, $\phi_B = 1.5$ V, $\phi_C = 0.0$ V and $\phi_D = 1.0$ V. They were obtained from (1)—(8). The values of a, b, c, and ℓ are the same as above, $g_1 = 5.70 \times 10^6$ m^{-1} Ω^{-1} and $g_3 = 1.25 \times 10^6$ m^{-1} Ω^{-1}. As we obtained algebraic solutions for the fields, potentials, and surface charges, it is easy to apply them for commercial cables. In this way we can know the orders of magnitude of these quantities for several standard cables.

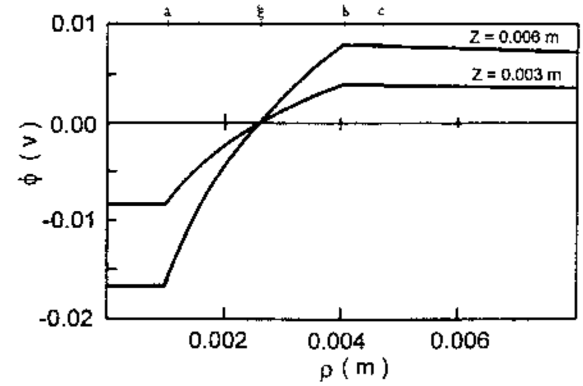

Fig. 2. Electric potential as a function of ρ.

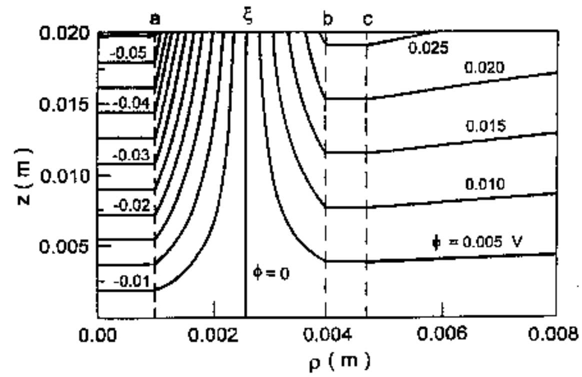

Fig. 3. Equipotentials for a symmetric case.

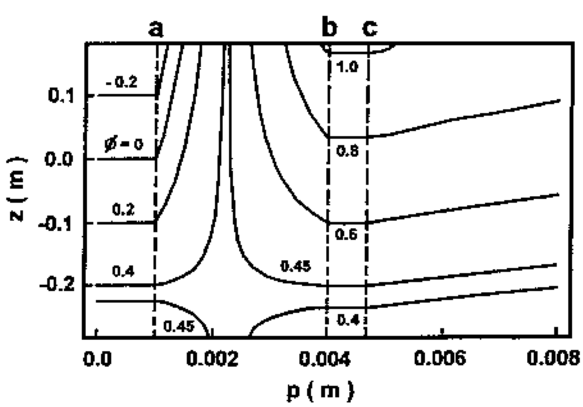

Fig. 4. Equipotentials for an asymmetric case.

III. Discussion

The distribution of charges given by (14)–(16) are equivalent to equal and opposite charges in the facing surfaces. That is, the charge at the position $\rho = a$, z in a length dz, $dq_a(z) = 2\pi a dz \sigma_a(z)$ is equal and opposite to the charge at the position $\rho = b$, z, in the same length dz: $dq_b(z) = 2\pi b\, dz \sigma_b(z) = -dq_a(z)$. The electric field outside the coaxial cable depends then only on the surface charges at the external wall of the return conductor, $\sigma_c(z)$

$$\phi(c \leq \rho, \varphi, z) = \frac{c}{\varepsilon_o}\, \sigma_c(z)\, \ln \frac{\ell}{\rho}. \tag{34}$$

The flux of energy from the Poynting vector $\vec{S} = \vec{E} \times \vec{B}/\mu_o$ is also represented in Fig. 3. That is, the lines of the Poynting flux lie in the equipotential surfaces, as had been pointed out by [1] and [3]. The classical view is that the energy comes from the batteries (not represented in Fig. 1). In Fig. 3 it would come from the top of the graph moving downwards toward decreasing values of z, along the equipotential lines. It would then enter the conductors and move radially in them. In the inner conductor it would dissipate as heat while moving radially from $\rho = a$ to $\rho = 0$, while in the outer conductor it also moves radially from $\rho = b$ to $\rho = c$, being completely dissipated as heat along this journey. The only region where the lines of Poynting flux do not follow the equipotential surfaces is for $\rho > c$. In this region there is no magnetic field. Although we have obtained an electric field and equipotential lines here, the Poynting vector goes to zero.

Our analysis was restricted to constant currents and voltages. Despite this fact it can also be applied to low-frequency ac circuits. A detailed study of the range of validity of this type of analysis for alternating currents has been given by Jackson [3] so we will not repeat it here. His main conclusion is that it should be applicable for frequencies ω such that $\omega\tau_1 \ll 1$ and $\omega\tau_2 \ll 1$, where τ_1 and τ_2 are the inductive and capacitive relaxation times given by $\tau_1 = L/R$ and $\tau_2 = RC$, L and R being the inductance and resistance of the circuit. When these conditions are satisfied, the skin effect is negligible. For constant currents and voltages, the surface densities of charge, the potentials and the fields will also be constant quantities in time. In the low-frequency regime, the equations obtained in this work for these quantities will remain valid replacing a constant I by $I_o \sin \omega t$. The displacement current will also be negligible in this case.

Beyond the generalizations of the previous works, the main nontrivial conclusion of this analysis are (12), (24) and (28). They show that although there is no vector potential nor magnetic field outside a coaxial cable, the electric field will not be zero when there is a finite resistivity in the outer conductor. As the previous works quoted above considered only the case of a return conductor with zero resistivity, this aspect did not appear. To our knowledge the first to mention this external electric field outside a resistive coaxial cable was Russell in his important paper of 1983 [11]. Our work presents a clear analytical calculation of this field, which Russell could only estimate. Our paper might be considered the quantitative implementation of his insights.

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References

[1] M. A. Heald, "Electric fields and charges in elementary circuits," *Amer. J. Phys.*, vol. 52, pp. 522–526, 1984.

[2] O. D. Jefimenko, *Electricity and Magnetism*, 2nd ed. Star City: Electret Scientific, 1989.

[3] J. D. Jackson, "Surface charges on circuit wires and resistors play three roles," *Amer. J. Phys.*, vol. 64, pp. 855–870, 1996.

[4] A. Marcus, "The electric field associated with a steady current in long cylindrical conductor," *Amer. J. Phys.*, vol. 9, pp. 225–226, 1941.

[5] A. Sommerfeld, *Electrodynamics*. New York, NY: Academic, 1964.

[6] D. J. Griffths, *Introduction to Electrodynamics, 2nd Ed.*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[7] A. K. T. Assis and J. I. Cisneros, "The problem of surrface charges and fields in coaxial cables and its importance for relativistic physics," in *Open Questions in Relativistic Physics*, F. Selleri, Ed. Montreal, Canada: Apeiron, 1998, pp. 177–185.

[8] B. R. Russell, "Surface charges on conductors carrying steady currents," *Amer. J. Phys.*, vol. 36, pp. 527–529, 1968.

[9] A. K. T. Assis, "Can a steady current generate an electric field?," *Phys. Essays*, vol. 4, pp. 109–114, 1991.

[10] ——, *Weber's Electrodynamics*. Dordrecht, The Netherlands: Kluwer, 1994.

[11] B. R. Russell, "Surface charges and leaky shields," *Amer. J. Phys.*, vol. 51, pp. 269–270, 1983.