

Influence of Rotation on the Weight of Gyroscopes as an Explanation for Flyby Anomalies

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Abstract: We consider two models which lead to the prediction of a weight change of gyroscopes depending on the rate of rotation: mass-energy equivalence and Weber's force for gravitation. We calculate the order of magnitude of this effect in both models and show that Weber's model predicts a weight change depending on the spin axis orientation resembling close similarities to observed Earth flyby anomalies. However, our predicted effect is much smaller than the observed effect, which could explain why flyby anomalies were not detected anymore in recent spacecraft trajectories.

Key Words: Gyroscope. Weber's law. Equivalence between mass and energy. Flyby anomaly.

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1. Introduction

The influence of rotation on gravity is generally known as frame-dragging, shifting satellites in polar orbit or causing precession on spinning gyroscopes as demonstrated by the LAGEOS and Gravity-Probe B space missions.^{1,2} We will show that rotation can also influence weight providing a physical basis for the observed Earth flyby anomalies, which are unexplained velocity jumps of 3.9, -4.6, 13.5, -2, 1.8 and 0.02 *mm/s* observed near closest approach during the Earth flybys of six spacecrafts. According to Anderson *et al*,³ the change in velocity depends on the incoming δ_i and outgoing δ_o geocentric latitudes and can be expressed as

$$\frac{\Delta v_\infty}{v_\infty} = \frac{2\omega_E R_E}{c} \cos(\delta_i - \delta_o) \quad (1)$$

where R_E and ω_E are the Earth's radius and angular velocity, respectively. Lämmerzahl *et al*⁴ analyzed possible errors from e.g. the solar wind or tidal forces on Earth without finding an explanation for the anomaly. McCulloch found a relationship with similar angular dependence but different prefactor using a modification of inertia due to a Hubble-scale Casimir effect,⁵ and Nyambuya suggested a new gravity model called Azimuthally Symmetric Theory of Gravitation⁶ that also shows a similar relationship but relies on measured values for its numerical prefactor. So up to now, equation (1) is an empirical guess without a physical basis obtained by data fitting.

2. Weight Change According to Mass-Energy Equivalence

According to the mass-energy equivalence, the energy E of a body is related to its mass m by the well-known formula $E = mc^2$, where $c = 2.998 \times 10^8 \text{ ms}^{-1}$ is the value of light velocity in vacuum. Therefore, if the energy of a body increases, its mass should also increase.

Consider a homogeneous sphere of radius r and mass m when at rest. Its kinetic energy K when it spins uniformly relative to an inertial frame of reference S around an axis passing through its center with an angular velocity ω is given by:

$$K = \frac{I\omega^2}{2} = \frac{mr^2\omega^2}{5} \quad (2)$$

where $I = 2mr^2/5$ is the moment of inertia of a solid sphere about its central axis.⁷

With the mass-energy equivalence we can predict an increase in the mass of the spinning sphere, Δm , by a factor $\Delta m = K/c^2$. Therefore a rotating sphere should have an effective mass $m_{\text{eff}} = m + K/c^2 = m(1 + r^2\omega^2/5c^2)$.

Utilizing the proportionality between gravitational and inertial masses, not only its inertial mass should increase, but also its weight. Suppose the sphere is interacting gravitationally with the Earth of mass M when the center of the sphere is located at a distance R to the center of the Earth. The weight of the sphere when not spinning will be represented by F_{Mm} and is given by $F_{Mm} = GMm/R^2$, where $G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ is the constant of universal gravitation.

When the sphere is spinning uniformly around its axis, its effective weight should increase due to the principle of equivalence. That is, the weight of the spinning sphere should be given by:

$$F_{Mm_{\text{eff}}} = \frac{GMm_{\text{eff}}}{R^2} = \frac{GMm}{R^2} \left(1 + \frac{r^2 \omega^2}{5c^2} \right) = F_{Mm} + \Delta F \quad (3)$$

The fractional change of weight is given by:

$$\frac{\Delta F}{F} = \frac{r^2 \omega^2}{5c^2} \quad (4)$$

In principle the weight change should not depend on the orientation of the rotation of the sphere relative to the ground.

3. Weight Change According to Weber's Law for Gravitation

We now consider the same problem utilizing Weber's law for gravitation. Consider particles 1 and 2 of gravitational masses m_{g1} and m_{g2} located at the position vectors \vec{r}_1 and \vec{r}_2 relative to the origin O of an inertial frame of reference S , moving relative to S with velocities $\vec{v}_1 = d\vec{r}_1/dt$ and $\vec{v}_2 = d\vec{r}_2/dt$, and with accelerations $\vec{a}_1 = d\vec{v}_1/dt$ and $\vec{a}_2 = d\vec{v}_2/dt$, respectively. Weber's gravitational force \vec{F}_{21} exerted by 2 on 1 is given by:⁸⁻¹⁰

$$\vec{F}_{21} = -Gm_{g1}m_{g2} \frac{\hat{r}_{12}}{r_{12}^2} \left[1 - \frac{6}{c^2} \left(\frac{\dot{r}_{12}^2}{2} - r_{12}\ddot{r}_{12} \right) \right] = -\vec{F}_{12} \quad (5)$$

where $r_{12} = |\vec{r}_1 - \vec{r}_2|$ is the distance between 1 and 2, $\hat{r}_{12} = (\vec{r}_1 - \vec{r}_2)/r_{12}$ is the unit vector pointing from 2 to 1, $\dot{r}_{12} = dr_{12}/dt$ is the relative radial velocity between them, $\ddot{r}_{12} = d\dot{r}_{12}/dt$ is the relative radial acceleration between them and \vec{F}_{12} is the force exerted by 1 on 2. A similar equation was also proposed by Erwin Schrödinger who could correctly derive with his weberian expression the advanced of Mercury's perihelion.^{11,12} According to Schrödinger the presence of the Sun has, in addition to the gravitational attraction, also the effect that the planet has a somewhat greater inertial mass radially than tangentially.

We want to calculate the gravitational force exerted by the Earth of mass M and radius R on a sphere of mass m and radius r according to Weber's law. We consider the Earth at rest relative to the inertial frame S with its center coinciding with the origin O of S . The center of the sphere will always be considered close to the surface of the Earth and located at $\vec{r}_o = R\hat{z}$ along the z axis. When the sphere is also at rest relative to S , Weber's force reduces to Newton's law of gravitation. Therefore the force exerted by the Earth on the sphere is given simply by $\vec{F}_{Mm} = -GMm\hat{z}/R^2$ pointing towards the center of the Earth.

We now consider the change in the weight of the sphere due to its rotation relative to the inertial frame S . In a first situation the sphere we suppose the sphere rotating uniformly around the z axis with an angular velocity $\vec{\omega} = \omega\hat{z}$, figure 1 (a).

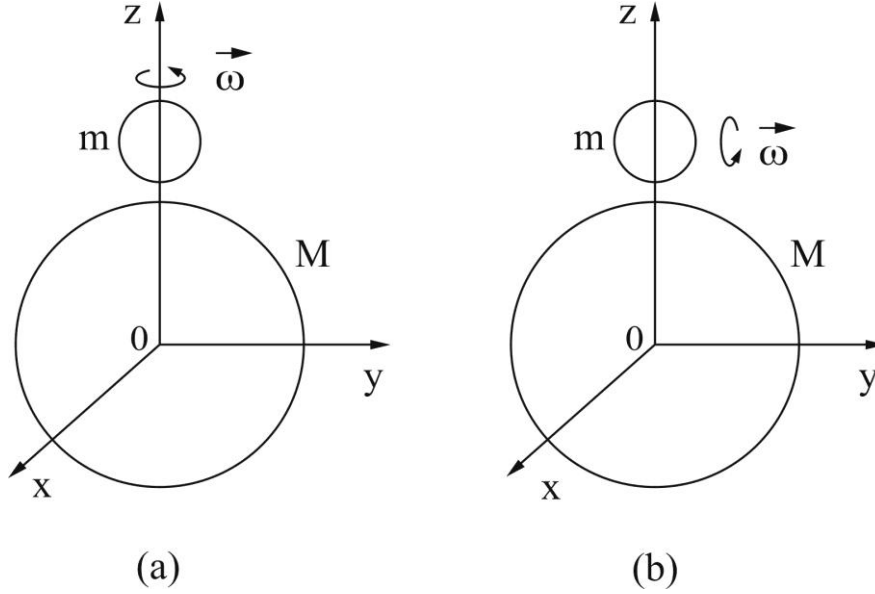


Figure 1: (a) Sphere rotating uniformly around the z axis with an angular velocity $\vec{\omega} = \omega \hat{z}$. (b) Sphere rotating uniformly around the y axis, $\vec{\omega} = \omega \hat{y}$.

We consider the whole mass of the Earth as concentrated on its center in order to simplify the calculations and to get an order of magnitude for the effect. In the situation of figure 1 (a) the distance r_{12} from each point of the spinning sphere to the center O of the Earth does not change as a function of time, $\dot{r}_{12} = 0$ and $\ddot{r}_{12} = 0$. Therefore the force exerted by the Earth on the spinning sphere of mass m is simply its normal weight \vec{F}_{Mm} no matter the value of the angular velocity ω , namely:

$$\vec{F}_{Mm} = -G \frac{Mm}{R^2} \hat{z} \quad (6)$$

We conclude that, according to Weber's law, there will be no change of weight when comparing a stationary sphere and a sphere spinning uniformly around an axis connecting the center of the sphere with the center of the Earth, namely:

$$\frac{\Delta F}{F} = 0 \quad (7)$$

Another situation is when the sphere rotates uniformly around the y axis, as in figure 1 (b). In this case it is necessary to take into account the velocity and acceleration of each portion of the sphere relative to the center O of the Earth. Weber's force complies with action and reaction. Therefore, the force exerted by the Earth on the spinning sphere is equal and opposite the force exerted by the spinning sphere on the Earth. The force $\vec{F}_{m,M}$ exerted by a spinning spherical shell of mass m_s and radius r_s centered at $\vec{r}_o = R \hat{z}$ and acting on an external point particle of mass M representing the Earth, located at the origin of coordinates, which is at rest relative to the center of the shell has already been calculated.⁸⁻¹⁰ In the situation of figure 1 (b) the force reduces to:

$$\vec{F}_{m_s M} = G \frac{M m_s}{R^2} \hat{z} \left(1 + \frac{r_s^2 \omega^2}{c^2} \right) = -\vec{F}_{M m_s} \quad (8)$$

where $\vec{F}_{M m_s}$ represents the force exerted by the point particle of mass M acting on the spinning shell.

Replacing the mass m_s of the shell by $4\pi r_s^2 \rho dr_s$, where $\rho = 3m/4\pi r^3$ is the volume density of mass of the spinning shell and integrating over the volume $V = 4\pi r^3/3$ of the spinning shell yields the force \vec{F}_{mM} exerted by the spinning shell and acting on M as given by:

$$\vec{F}_{mM} = G \frac{Mm}{R^2} \hat{z} \left(1 + \frac{3r^2 \omega^2}{5c^2} \right) = -\vec{F}_{Mm} \quad (9)$$

where \vec{F}_{Mm} represents the force exerted by the Earth of mass M acting on the spinning sphere of mass m .

According to Weber's force law and equations (6) and (9), the fractional change of weight when comparing a stationary sphere and a sphere spinning uniformly around an axis which is orthogonal to the direction connecting the center of the sphere to the center of the Earth as in figure 1 (b), is given by:

$$\frac{\Delta F}{F} = \frac{3r^2 \omega^2}{5c^2} \quad (10)$$

The same fractional change of weight happens when comparing the weight of a sphere spinning uniformly around an axis connecting the center of the sphere to the center of the Earth as in figure 1 (a) with the weight of the sphere of figure 1 (b).

The increase of weight given equation (4) has the same order of magnitude of the increase of weight given by equation (10) although the numerical value is not the same. Moreover, while the prediction based on the mass-energy equivalence does not depend on the direction of rotation, the same is not valid from the prediction based on Weber's force, as can be seen comparing equations (7) and (10).

4. Experimental Test of This Effect

In order to estimate the order of magnitude of this tiny effect we consider the tensile stress induced at a section of an horizontal bar of length $2r$ and uniform cross section A , rotating in the xy horizontal plane about a vertical axis z passing through its center with a constant angular velocity ω relative to the inertial frame of reference S . The volume density of this homogeneous bar of mass m_b and volume $V_b = 2rA$ is given by $\rho = m_b/(2rA)$. The centrifugal force dF produced at the bar by an element of length ds and mass $dm = \rho A ds$ located at a distance s from the center of the bar is given by $dF = dm s \omega^2$. By integrating this expression for the mass between s and r we obtain the centrifugal force acting on the cross-section at a

distance s from the center as given by $F = \rho A(r^2 - s^2)\omega^2/2$. The tensile stress σ induced at this section is the force per unit area:

$$\sigma = \frac{F}{A} = \frac{\rho(r^2 - s^2)\omega^2}{2} \quad (11)$$

The maximum value of σ happens at the origin, being given by $\sigma_{\max} = \rho r^2\omega^2/2$.

Replacing $r^2\omega^2$ in equations (4), (7) and (10) by $2\sigma_{\max}/\rho$ we obtain that the predicted fractional change of weight by comparing a rotating sphere with a sphere without rotation has the following order of magnitude:

$$\frac{\Delta F}{F} = \alpha \frac{r^2\omega^2}{c^2} = \frac{2\alpha\sigma_{\max}}{\rho c^2} \quad (12)$$

In this equation α is a dimensionless constant. Its value is $\alpha = 1/5$ according to the mass-energy relation, equation (4), no matter the direction of rotation of the sphere. According to Weber's law for gravitation, on the other hand, we have $\alpha = 0$ with a rotation of the sphere around the z axis, equation (7), or $\alpha = 3/5$ with a rotation of the sphere around the y axis, equation (10).

Consider a spinning sphere made of stainless steel. Its specific tensile stress, or specific strength, is given by $\sigma_{\max}/\rho \approx 2.5 \times 10^5 m^2 s^{-2}$. According to equation (12) we are then led to:

$$\frac{\Delta F^{steel}}{F} = \frac{2\alpha\sigma_{\max}^{steel}}{\rho c^2} \approx 5 \times 10^{-12} \alpha \quad (13)$$

This effect is very small and below detectability on Earth. The weight of horizontal spinning gyroscopes has been measured previously and no effect was observed up to $\Delta F/F = \pm 3.5 \times 10^{-5}$ according to Nitschke and Wilmarth.¹³ The best mass comparators available today (Mettler-Toledo M_One, Sartorius CCL1007) allow the measurement of a fractional change of weight of $\Delta F/F = \pm 9.5 \times 10^{-10}$, which is about 2 orders of magnitude above our steel example. Only carbon nanotubes have material properties that give $\Delta F/F = 1 \times 10^{-9} \alpha$, which is close to the resolution of mass comparators. However, we require a gyroscope mass of 1 kg to use the full scale of mass comparators and a single carbon nanotube with a length of 1 mm weights only $8.6 \times 10^{-21} \text{ kg}$.

But what about space measurements? A spinning satellite with its rotation axis tilted towards the Earth should feel a different gravitational attraction when compared with another satellite with its rotation axis orthogonal to the direction connecting the satellite to the center of the Earth. Likewise, the spinning Earth should attract bodies slightly different around the equator compared to its attraction of bodies around the poles. Although the Earth's angular velocity is quite small with $\omega_E = 7.3 \times 10^{-5} \text{ rad/s}$, its large mass and radius still make it by far the dominant contributor with respect to a spinning satellite. For example, the Galileo spacecraft (which showed the Earth flyby anomaly) had a launch mass of 2380 kg and a maximum spin rate of 10.5 rpm . Under optimal conditions (i.e. horizontal spin axis with respect to Earth), we

can expect according to Weber's law $\Delta F / F_{Galileo, \max} = 8.1 \times 10^{-18}$ assuming an average diameter of 1 m , in order to get the order of magnitude. On the other hand, the Earth would produce an effect of $\Delta F / F_{Earth, \max} = 1.41 \times 10^{-12}$. Therefore only this last contribution counts when analyzing satellite trajectories.

If a satellite is approaching Earth at a geocentric latitude less than 90 deg , its velocity should increase due to an increase in gravitational attraction. If this force is different due to the spinning Earth, we would indeed expect a different velocity after the flyby manouver v_∞ . A different gravitational pull will lead to a different kinetic energy and therefore spacecraft velocity. By equalling the kinetic and gravitational potential energy, we will estimate the order of magnitude by comparing both the Weber gravitational potential with the standard Newtonian one, which leads to

$$\frac{v_\infty^2}{2} = \frac{GM}{R} \quad \text{and} \quad \frac{v_{\infty, Weber}^2}{2} = \frac{GM}{R} \left(1 + \frac{3r^2 \omega^2}{5c^2} \right) \cos(\delta) \quad (14)$$

Here we assumed that the Weber modification of the gravitational potential will vary with the cosine of the geocentric latitude to account in order to account for the directionality of the effect. By using

$$\frac{v_{\infty, Weber}}{v_\infty} = \frac{v_\infty + \Delta v_\infty}{v_\infty} = \sqrt{1 + \frac{3r^2 \omega^2}{5c^2} \cos(\delta)} \approx 1 + \frac{3r^2 \omega^2}{10c^2} \cos(\delta) \quad (15)$$

we can express the increase in speed taking a different incoming and outgoing geocentric angle into account as

$$\frac{\Delta v_\infty}{v_\infty} \approx \frac{3r^2 \omega^2}{10c^2} \cos(\delta_i - \delta_o) \quad (16)$$

Remarkably, the prefactor is up to 10 % exactly the square of the empirical prefactor found by Anderson in equation (1).³ If this equation is correct, the effect should be also well below present satellite trajectory analysis capabilities as it predicts a maximum velocity shift due to Earth's spin of $(\Delta v_\infty) / v_\infty = 1.4 \times 10^{-12}$, which is 6 orders of magnitude below the reported Earth flyby anomaly and 4 orders of magnitude below the resolution limit.

Although Earth flyby anomalies were reported for a number of spacecraft including Rosetta in the past, two more recent Rosetta flybys did not show up an anomaly which is puzzling the community.⁶ According to Weber's law, the flyby velocity shifts should indeed be presently undetectable due to its extremely small order of magnitude, although it must exist at a lower order of magnitude with the same characteristic. The effect may reliably be detected by the following effects:

- Studying flybys at Jupiter which should increase the effect at least by 3 orders of magnitude (which may still be below present detection capabilities).
- Increasing the tracking accuracies e.g. by optical atomic clocks in space.

- Building better balances on Earth and testing weight changes of spinning gyroscopes in horizontal orientation.
- Performing a dedicated space experiment e.g. with gyroscopes in vertical and horizontal orientation with respect to the Earth's surface.

If, as suggested by Anderson,³ the square-root of the prefactor is correct, it should be possible to find the effect more easily. The best limit of weight changes with horizontal spinning gyroscopes today is an order of magnitude above the predicted effect. But today's mass comparators would allow to increase the sensitivity by three orders of magnitude if precession and vibration is properly taken into account.¹⁴ This would allow to test the Anderson effect in an Earth-based experiment.

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