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The Influence of Temperature on Gravitation.

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Summary. — We present some general reasonings using the mass-energy equivalence and Weber's force law for gravitation to show that the weight of a body should increase with its temperature. The two approaches predict a fractional change of weight or, equivalently, of the gravitational constant G, of one part in 10^{14} per degree.

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According to Einstein's relation, $E = Mc^2$, if we increase the energy content of a body, automatically its mass is increased. Using the proportionality between inertial and gravitational masses, we conclude that not only its inertial mass but also its weight should increase if its temperature is raised.

We now calculate this value. To this end, we suppose two spherical solid bodies 1 and 2, with isotropic distributions of mass $\rho_1(r_1)$ and $\rho_2(r_2)$, separated by a distance R. We will represent their «cold masses», that is, their masses at zero degrees kelvin, by M_{10} and M_{20}. From Newton's universal law of gravitation we have at this temperature $F = -GM_{10}M_{20}/R^2$. To arrive at temperatures T_1 and T_2 they need to receive a thermal energy. We utilize the law of Dulong-Petit [1] to estimate roughly this amount of energy increase. If body 1 has N_1 and body 2 has N_2 molecules, we obtain $\delta E_1 = 3N_1 k_B T_1$ and $\delta E_2 = 3N_2 K_B T_2$, where k_B is Boltzmann's constant. Using Einstein's relation we conclude that at these temperatures Newton's law will read

$$F = -\frac{G}{R^2} M_{10}\left(1 + 3\,\frac{N_1 k_B T_1}{M_{10} c^2}\right) M_{20}\left(1 + 3\,\frac{N_2 k_B T_2}{M_{20} c^2}\right). \tag{1}$$

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This means that, in general, if we increase the temperatures of bodies 1 and 2 by δT_1 and δT_2 there will be a fractional change of the gravitational force given by

$$\frac{\delta F}{F} = 3\left(\frac{k_B \delta T_1}{m_{10} c^2} + \frac{k_B \delta T_2}{m_{20} c^2}\right). \tag{2}$$

In this expression we utilized that $M_{i0} = N_i m_{i0}$, where m_{i0} is the average mass of the molecules constituting body i.

We now analyse the same problem using Weber's law [2-7]. This is essentially Coulomb's law (or Newton's law) plus terms which depend on the velocity and acceleration between the interacting bodies. There has been a renewed interest in this force law because from it we can derive not only Faraday's law of induction but also Ampère's force between current elements. In the last few years a large number of experiments have been performed trying to distinguish Ampère's force from Grassmann's force [8-11] but this is still an open question [12-13]. When applied to gravitation, Weber's law reads [14]

$$\boldsymbol{F} = -G m_{10} m_{20} \frac{\hat{r}_{12}}{r^2}\left[1 - \frac{\xi}{c^2}\left(\frac{\dot{r}^2}{2} - r\ddot{r}\right)\right] = \tag{3}$$

$$= -G m_{10} m_{20} \frac{\hat{r}_{12}}{r^2}\left[1 + \frac{\xi}{c^2}\left(\boldsymbol{v}_{12}\cdot\boldsymbol{v}_{12} - \frac{3}{2}(\hat{r}_{12}\cdot\boldsymbol{v}_{12})^2 + \boldsymbol{r}_{12}\cdot\boldsymbol{a}_{12}\right)\right].$$

In this expression $\boldsymbol{F}$ is the force exerted by m_{20} on m_{10}, $\boldsymbol{r}_{12} \equiv \boldsymbol{r}_1 - \boldsymbol{r}_2$, $r \equiv |\boldsymbol{r}_{12}|$, $\hat{r}_{12} \equiv \boldsymbol{r}_{12}/r$, $\boldsymbol{v}_{12} \equiv \mathrm{d}\boldsymbol{r}_{12}/\mathrm{d}t$, $\boldsymbol{a}_{12} \equiv \mathrm{d}\boldsymbol{v}_{12}/\mathrm{d}t$, c is the light velocity in vacuum and $\xi = 6$.

In order to calculate the gravitational interaction between the two previous spheres at temperatures T_1 and T_2, we make the following: In the first place we suppose each molecule vibrating harmonically around an equilibrium position. For instance, if the equilibrium position of molecule 2 is the origin and that of molecule 1 is $y_{10}\hat{y}$, a possible motion is $\boldsymbol{r}_2 = A_2 \sin(\omega_2 t + \theta_2)\hat{x}$ and $\boldsymbol{r}_1 = [y_{10} + A_1 \sin(\omega_1 t + \theta_1)]\hat{y}$. For simplicity we suppose $\omega_1 = n\omega_2$, where n is an integer, and $y_{10}^2 \gg A_1^2 + A_2^2$, which means that the distance R between the two spheres is much greater than the amplitudes of oscillation. We then utilize these values of $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ in eq. (3) and perform two averages. The first one is on time, integrating from $t = 0$ to $t = T_2 = 2\pi/\omega_2$, and dividing by T_2, and the second one is on the phases, allowing any value between 0 and 2π for θ_1 and for θ_2. We suppose also that $y_{10}^2 \gg c^2/\omega_1^2 + c^2/\omega_2^2$. The average value is found to be

$$\langle\langle \boldsymbol{F} \rangle\rangle = -G m_{10} m_{20} \frac{y_{10}\hat{y}}{|y_{10}|^3}\left(1 + \frac{\xi}{4}\frac{A_1^2\omega_1^2}{c^2}\right). \tag{4}$$

We perform this average for the nine combinations of oscillation along the main directions (m_1 oscillating along X, Y and Z, and the same for m_2). The final average value is found to be

$$\langle\langle\langle \boldsymbol{F} \rangle\rangle\rangle = -G m_{10} m_{20} \frac{y_{10}\hat{y}}{|y_{10}|^3}\left(1 + \frac{\xi}{12}\frac{A_1^2\omega_1^2 + A_2^2\omega_2^2}{c^2}\right). \tag{5}$$

A molecule in a solid has six degrees of freedom so that $\langle E\rangle = 6(k_B T/2)$. As $\langle E\rangle = 6\langle mv_x^2/2\rangle$, we get $\langle mv_x^2\rangle = k_B T$ or $\langle m\boldsymbol{v}\cdot\boldsymbol{v}\rangle = 3k_B T$. In the case under consideration $\langle v_x^2\rangle = A^2\omega^2/2$ so that, applying these results in (5) yields, after integrating for the solid bodies,

$$\langle\langle\langle F\rangle\rangle\rangle = -G\,\frac{M_{10}M_{20}}{R^2}\left[1+\frac{\xi}{6}\left(\frac{N_1 k_B T_1}{M_{10}c^2}+\frac{N_2 k_B T_2}{M_{20}c^2}\right)\right]. \tag{6}$$

The fractional change of force according to Weber's law is then given by

$$\frac{\delta\langle F\rangle}{\langle F\rangle} \equiv \frac{\xi}{6}\left(\frac{k_B\,\delta T_1}{m_{10}c^2}+\frac{k_B\,\delta T_2}{m_{20}c^2}\right). \tag{7}$$

This result is analogous to (2) and has the same order of magnitude.

To have an idea of this fractional change of gravitational force, which can be considered as a change of weight, of mass or of G, we take into consideration two solid spheres of iron. Then $\delta\langle F\rangle/\langle F\rangle \simeq 10^{-14}/°C$. If we change the temperature of the bodies by 100 °C, then $\delta\langle F\rangle/\langle F\rangle \simeq 10^{-12}$, which is near to the limit of precision of the modern analytic balances. We propose this experiment in order to know if these effects occur simultaneously or if only the mass-energy equivalence is compatible with the experiments.

In conclusion, we could say that two different ideas and concepts lead to the same conclusion that the gravitational force between two bodies should be a function of their temperatures. Moreover, the expected fractional change of force is of the order $k_B\,\delta T/mc^2$.

Recently, Massa studied some theoretical consequences of a gravitational constant which depends on temperature [15]. In this work we restrict ourselves to pointing out two different models indicating the possible existence of this effect. Other ramifications of this idea are outside the scope of the present work.

It is worthwhile pointing out that Poynting and Phillips examined experimentally a possible dependence of gravitational force on temperature before the coming of Einstein's relation $E = Mc^2$, see reference [16]. They showed that if the weight of a body changes due to a modification of its temperature, this change corresponds to less than one part in 10^{10} per degree. There is no contradiction between this result and the effect calculated in this work because we predict a change of only one part in 10^{14} per degree.

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