On Forces that Depend on the Acceleration of the Test Body

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Abstract

We discuss Waldron's arguments against force laws that depend on the acceleration of the test body and show that they are incorrect. In contrast to Waldron's point of view, we show that these force laws are compatible with Newton's second law of motion.

Key words: Newton's second law, forces that depend on the acceleration of the test body, Weber force, Weber's law

This work is an answer to Waldron's criticism against force laws that depend on the acceleration of the test body.(1) According to Waldron, force laws that depend on the acceleration of the test body are in conflict with Newton's second law of motion. We do not agree with this statement, but before we present our own arguments, let us present Waldron's point of view in the simplest way. In all examples utilized here we will analyze only unidimensional motions. (The generalization to three dimensions utilizing vectorial analysis is straightforward.)

If a force F acts on a body of mass m, Newton's second law of motion requires that

$$F = ma, \tag{1}$$

where a is the acceleration of mass m relative to an inertial frame. The simplest situation analyzed by Waldron is that in which the force F is of the type

$$F = A + Ba, \tag{2}$$

where A and B may depend on the position and velocity of m, but they do not depend on its acceleration a. Then he says: "If the force is multiplied by a factor $n = n(t)$ (e.g., by multiplying the voltage of an electrode by n), the acceleration will be multiplied by the same factor, and Newton's second law of motion will become

$$n(t)\mathbf{F} = m[n(t)\mathbf{a}]." \tag{3}$$

According to him, Eq. (2) would become (in this simplified situation)

$$nF = n(A + Bna), \tag{4}$$

which would imply

$$F = A + nBa. \tag{5}$$

Since Eq. (5) contradicts Eq. (2), unless $B = 0$, he concludes that "the acceleration [of the test body] cannot apply in a force law in a universe in which Newton's second law of motion holds."

From our point of view there are two flaws in this argument. The first and most obvious one is the passage from Eq. (3) to Eqs. (4) and (5), which is equivalent to the passage from Eq. (3) to Eqs. (4) and (5) of his paper. Equation (3) here is nothing more than Newton's second law, because dividing both sides of (3) by n yields (1). This means that instead of (4), he should have written [remembering that Eq. (3) can also be written as $nF = n(ma)$], utilizing (1) and (2),

$$nF = n(A + Ba), \tag{6}$$

which would then be equivalent to (2). So there is no contradiction in having a force law like Eq. (2) together with Newton's second law of motion.

The second flaw in his argument is the statement quoted above: multiplying the voltage of an electrode by n will result in an acceleration n times larger for the test charge. This is not true with force laws that depend on the acceleration of the test body (as is the case with the force laws of

Riemann and Weber, for instance). We can see this analyzing Weber's force law.[2]–[9] For a charge moving normally to the plates of an ideal plane capacitor (with surface charge densities $\pm\sigma$ on the plates situated at $\pm x_0$), Weber's law predicts that the resultant force on the internal test charge q is given by[10]

$$F = A + Ba, \tag{7}$$

where $A = -(q\sigma/\epsilon_0)(1 + v^2/2c^2)$, and $B = -q\sigma x/(\epsilon_0 c^2)$. In this equation, x, v, and a are, respectively, the position, velocity, and acceleration of the test charge q relative to an inertial frame (usually we can consider the laboratory where the capacitor is at rest as this inertial frame), and c is the ratio of electromagnetic to electrostatic units of charge (which was found experimentally by Weber and Kohlrausch to have the same value as light velocity in a vacuum). With (7) and Newton's second law, (1), we get

$$a = A/(m - B). \tag{8}$$

The voltage V of the capacitor is given by $V = 2\sigma x_0/\epsilon_0$, where $2x_0$ is the distance between the plates. In order to multiply the voltage by n, we need to multiply the charge density $\pm\sigma$ of the plates by n. In this new situation the acceleration of the test charge, a_n, will be given by [according to (8) and the previous definitions of A and B]

$$a_n = nA/(m - nB). \tag{9}$$

Obviously $a_n \neq na$ because now $B \neq 0$ (except at $x = 0$).

This shows that Waldron's statement that the acceleration is multiplied by n when the voltage becomes n times larger, which is valid for Coulomb's force, cannot be applied to force laws that depend on the acceleration of the test charge, as is the case with Weber's force. Despite this fact, Weber's force is still compatible with Newton's second law of motion, as we have shown in this paper.

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Résumé
On discute les arguments de Waldron contre les lois de force qui dependent de l'accélération du corp échantillon, et on montre qu'ils ne sont pas corrects. On montre aussi, contrairement à son point de vue, que ces forces sont compatibles avec la deuxième loi de mouvement de Newton.

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