WEBER'S LAW AND MASS VARIATION

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Due to the renewed interest of the last few years in Ampére and Weber's laws of force we decided to apply Weber's law to the situation of Bucherer's experiment. We calculate, according to Weber's expression, the force on a charge in regions of uniform electric and magnetic fields. Then we show that Weber's law agrees with the relativistic calculation only up to second order, inclusive, in v/c. So the special theory of relativity remains as the only model in perfect agreement with Bucherer's experiment.

1. Introduction

In the last few years there has been a renewed interest in Ampére's law of force between current elements [1–6] and in Weber's law of force between point charges [7]. As is well known, with Weber's law of force we can deduce Ampére's law as a special case valid when each current element has a zero net charge [8,9]. It was also shown that Faraday's law of induction for closed circuits can be derived from Weber's law [8]. Although Ampére's and Biot-Savart's laws of force between current elements are not equivalent, they do give the same force exerted by a closed circuit on a current element of another circuit [10]. As a result, we cannot distinguish between these two laws when dealing with different circuits. Weber also discovered that his law of force can be derived from a velocity-dependent potential and so it is consistent with the principle of conservation of energy [8].

The renewed interest in the basic laws of electromagnetism prompted us to calculate the forces on a charge for the arrangement of Bucherer's experiment [11], devised to show the variation of mass with velocity. Then we compared this calculation with the well tested and verified predictions of special relativity. We conclude that Weber's law is only an approximation, valid up to second order in v/c. Special relativity still remains as the only explanation for this experiment which is valid in all orders of v/c.

2. Forces in regions of uniform electromagnetic fields

Weber's law of force states that a charge q_j exerts a force on a charge q_i given by

$$\boldsymbol{F}_{ji}=\frac{q_i q_j}{4\pi\epsilon_0}\frac{\hat{\boldsymbol{r}}_{ij}}{r_{ij}^2}\left(1-\frac{\dot{r}_{ij}^2}{2c^2}+\frac{r_{ij}\ddot{r}_{ij}}{c^2}\right), \tag{1}$$

where

$$\boldsymbol{r}_{ij}\equiv\boldsymbol{r}_i-\boldsymbol{r}_j,\quad r_{ij}\equiv|\boldsymbol{r}_i-\boldsymbol{r}_j|,$$

$$\hat{\boldsymbol{r}}_{ij}\equiv\frac{\boldsymbol{r}_{ij}}{r_{ij}},\quad \dot{r}_{ij}\equiv\frac{\mathrm{d}}{\mathrm{d}t}r_{ij},\quad \ddot{r}_{ij}\equiv\frac{\mathrm{d}^2}{\mathrm{d}t^2}r_{ij},$$

$c\equiv$ light velocity.

A relevant characteristic of Weber's law is that it only depends on the relative distance, velocity and acceleration between the two charges. In this way it has the same value for an observer in a completely arbitrary state of motion, and is completely relational in its nature [12].

In order to make predictions for the situation of Bucherer's experiment we need to calculate the forces in regions of uniform electronic and magnetic fields. In classical electromagnetism we utilize Coulomb's force (which is eq. (1) without the terms in the velocity and acceleration) to calculate the force in a region of constant electric field. If we have an infinite capacitor with surface charge density $+\sigma_A$ and $-\sigma_A$ on the plates situated at $+x_0$ and $-x_0$, respec-

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tively, then according to Coulomb's law the force on a charge q_1 between the plates will be given by [13]

$$F=-q_1 \frac{\sigma_A}{\epsilon_0} \hat{x} , \tag{2}$$

where $\hat{x}$ is a unit vector pointing from the plate with $-\sigma_A$ to the plate with $+\sigma_A$.

In this paper we are interested in the predictions according to Weber's law. We therefore calculate, using eq. (1), the force exerted by the capacitor on q_1. As in the previous case we replace q_j by $\pm\sigma_A$ da, where da is an area element in the plate, and integrate over the surface of both plates. The result of the integration when q_1 is inside the plates is

$$F=-q_1 \frac{\sigma_A}{\epsilon_0} \{\hat{x}+c^{-2}[\tfrac{1}{2}v_1^2\hat{x}-x_1 a_1 + 2x_1 a_{1x}\hat{x}-v_{1x}(v_{1y}\hat{y}+v_{1z}\hat{z})]\} , \tag{3}$$

where r_1, v_1 and a_1 are, respectively, the position, velocity and acceleration of q_1 relative to an origin in the middle of the two plates, and $\hat{y}$ and $\hat{z}$ are unit vectors along the y and z axes, respectively.

It is important to note that according to Weber's law the force in a region of stationary electric field will depend on the velocity and acceleration of q_1. This never happens when we only utilize Coulomb's law.

We now calculate the force in a region of uniform magnetic field. For this we suppose an infinitely long cylindric shell of radius ρ, composed of a surface charge density σ_B at rest and of a surface charge density $-\sigma_B$ which circulates uniformly around the axis of the cylinder, which we call the z axis, with velocity $-\rho\omega\hat{\varphi}$, where $\hat{\varphi}$ is the unit azimuthal vector. According to classical electromagnetism this distribution of charges generates no electric field but only a uniform magnetic field inside the cylinder with a value given by

$$B=\mu_0\rho\omega\sigma_B\hat{z}\equiv B\hat{z} . \tag{4}$$

With the Lorentz force we obtain the force on the charge q_1 [13],

$$F=q_1 v_1 \times B=q_1 B(v_{1y}\hat{x}-v_{1x}\hat{y}) , \tag{5}$$

where v_1 is the velocity of q_1 relative to an inertial observer.

We calculated the force in this situation using eq. (1) and found that when q_1 is on the axis of the cylinder the force exerted on it is given by

$$F=q_1\mu_0\rho\omega\sigma_B(v_{1y}\hat{x}-v_{1x}\hat{y}) . \tag{6}$$

This is the same result as (4) and (5) but with a difference. As Weber's force is of a completely relational nature, the velocity which appears in eq. (6) is the velocity of q_1 relative to the axis of the cylinder and not the velocity relative to an observer. But for an observer at rest in the laboratory the two forces in eqs. (5) and (6) are indistinguishable for this situation.

3. Bucherer's experiment according to relativity and to Weber's law

In Bucherer's experiment [11] there is a source of β-rays (electrons) placed in the middle of a capacitor of linear dimensions L and separation $2x_0$, with $L \gg 2x_0$. This capacitor generates a uniform electric field given by (according to classical electromagnetism)

$$E=-\frac{\sigma_A}{\epsilon_0}\hat{x} , \tag{7}$$

where $\pm\sigma_A$ are the surface charge densities of the plates of the capacitor. Superimposed on this there is a uniform magnetic field B pointing along the z axis. The only electrons which can leave the capacitor are the ones on which a zero resultant force acts in the x direction and which also have zero initial velocity in this direction, otherwise they would collide with the plates. From the Lorentz force we obtain the force in the x direction:

$$F_x=-e[E_x+(v_1\times B\hat{z})_x]=-e(E_x+v_{1y}B) . \tag{8}$$

The electrons which leave the capacitor are then those whose velocity is given by

$$v_{1y}=\sigma_A/\epsilon_0 B . \tag{9}$$

After the electrons leaves the capacitor it is only under the influence of the magnetic field and then follows a circular path with radius given by

$$|-ev_1\times B|=m|a_1|=mv_1^2/r , \tag{10}$$

or

$$r = mv_1/eB\,, \tag{11}$$

where m is the mass of the electron. Here, v_1 is a constant equal to the velocity with which the electron leaves the capacitor. (The velocity remains constant because the magnetic field does no work.) This velocity is given by (9) and applying it to (11) we obtain

$$r = m\sigma_A/\epsilon_0 eB^2\,. \tag{12}$$

But from the special theory of relativity we know that the mass of a body is a function of its velocity:

$$m = \frac{m_0}{(1 - v_1^2/c^2)^{1/2}}\,, \tag{13}$$

where m_0 is the rest mass of the body. Applying this result in eq. (12), we obtain

$$r = \frac{\sigma_A}{\epsilon_0 eB^2}\frac{m_0}{(1 - v_1^2/c^2)^{1/2}}\,. \tag{14}$$

Utilizing a photographic plate after the capacitor to collect the electrons, Bucherer could determine the radius and obtained excellent agreement with expression (12). This was and continues to be a great experimental proof in favour of the special theory of relativity.

We now show that should be expected from this experiment according to Weber's law. As the source of electrons in Bucherer's experiment is in the middle of the plates of the capacitor, $\boldsymbol{r}_1 = 0$, we find from (3) and (6) that in order to leave the capacitor the velocity of an electron which leaves in the y direction must satisfy

$$e\frac{\sigma_A}{\epsilon_0}(1 + v_{1y}^2/2c^2) - eBv_{1y} = 0\,, \tag{15}$$

where we have used the fact that (5) and (4) are equivalent to (6), and replaced q_1 by the electron charge $-e$. Applying (15) in (11), we obtain

$$r = \frac{m\sigma_A}{\epsilon_0 eB^2}(1 + v_1^2/2c^2)\,. \tag{16}$$

Expanding (14) in powers of v_1/c, we obtain

$$r = \frac{m_0\sigma_A}{\epsilon_0 eB^2}(1 + v_1^2/2c^2 + \tfrac{3}{8}v_1^4/c^4 + \tfrac{5}{16}v_1^6/c^6 + \ldots)\,. \tag{17}$$

Eq. (16) tells us what the radius should be as a function of the velocity according to Weber's law and eq. (14) or eq. (17) what it should be according to relativity.

4. Discussion

From eqs. (16) and (17) we see that up to second order in v/c there are two equivalent interpretations of Bucherer's experiment. The first is that the electric field is calculated using Coulomb's law and the mass of a particle changes with velocity according to eq. (13). The second interpretation is that the mass is a constant for any velocity but the forces should be calculated according to Weber's law. In this case we have shown that the force in a region of uniform electric field will have a component proportional to the square of the velocity of the charge.

But of course this is only valid up to second order in v_1/c, as we can see from eqs. (16) and (17). For higher orders the two laws disagree and the experiments of Bucherer and others clearly show that it is the relativitistic result which agrees with the experiments. This is also confirmed by the modern experiments in linear and circular accelerators in which the charges attain velocities higher than $0.9c$.

The conclusion is that Weber's law is only an approximation valid up to second order in v/c. To this date there is no other explanation of Bucherer's experiment than that supplied by special relativity.

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