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Proof of the Identity Between Ampère and Grassmann's Forces

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Abstract

We consider a poloidal current flowing over the surface of a conducting cylinder of length l and radius a. We calculate exactly the force on a surface current element belonging to this cylinder due to the remainder of the circuit with Ampère and Grassmann's expressions. We conclude that the formulae agree with one another for any value of l/a. We then generalize this result for any current element belonging to a closed surface or volumetric currents of arbitrary forms.

1. Statement of the controversy

Whittaker showed that there is a family of expressions describing the electrodynamic interaction between current elements, one formula differing from another by a total derivative whose contribution to a line integral around a closed contour is always zero [1], Vol. 1, pp. 67–93. Then, if we have the interaction of a closed circuit with another one, or just with a current element belonging to this other circuit, all the differences between the expressions cited above vanish. And what would happen if the current element was part of the closed circuit? In this situation we cannot use linear current elements as a diverging problem will appear. The correct way of dealing with this kind of interaction is making use of surface or volumetric current densities [2, 3].

Although there is a family of equivalent expressions, we prefer to discuss just the ones that are inside the context of an electrodynamic theory. Therefore, in this work we shall just discuss two force expressions: those of Ampère and Grassmann.

These are the two main expressions for calculating forces between current carrying circuits. The first one was obtained by Ampère between 1820 and 1826, based on his experiments to explain Oersted's discovery of the deflection of a magnetized needle by a current carrying conductor, in 1820: [1], Vol. 1, pp. 84–88 and [4, 5]. The second expression was proposed theoretically by Grassmann in 1845: [6, 7].

In the International System of Units MKSA, Ampère and Grassmann's expressions for the force exerted by the current element $I_j\,\mathrm{d}\boldsymbol{r}_j$, localized at $\boldsymbol{r}_j$, acting on the current element $I_i\,\mathrm{d}\boldsymbol{r}_i$, localized at $\boldsymbol{r}_i$, can be written as, respectively:

$$\mathrm{d}^2\boldsymbol{F}_{ji}^{\mathrm{A}} = \frac{\mu_0 I_i I_j}{4\pi}\frac{\hat{\boldsymbol{r}}_{ji}}{r_{ij}^2}[3(\mathrm{d}\boldsymbol{r}_i\cdot\hat{\boldsymbol{r}}_{ij})(\mathrm{d}\boldsymbol{r}_j\cdot\hat{\boldsymbol{r}}_{ij}) - 2(\mathrm{d}\boldsymbol{r}_i\cdot\mathrm{d}\boldsymbol{r}_j)], \tag{1}$$

$$\begin{aligned}\mathrm{d}^2\boldsymbol{F}_{ji}^{\mathrm{G}} &= I_i\,\mathrm{d}\boldsymbol{r}_i \times \mathrm{d}\boldsymbol{B}_j(\boldsymbol{r}_i)\\ &= I_i\,\mathrm{d}\boldsymbol{r}_i \times \left(\frac{\mu_0}{4\pi}\frac{I_j\,\mathrm{d}\boldsymbol{r}_j\times\hat{\boldsymbol{r}}_{ij}}{r_{ij}^2}\right)\\ &= \frac{\mu_0 I_i I_j}{4\pi}\frac{1}{r_{ij}^2}[(\mathrm{d}\boldsymbol{r}_i\cdot\hat{\boldsymbol{r}}_{ij})\,\mathrm{d}\boldsymbol{r}_j - (\mathrm{d}\boldsymbol{r}_i\cdot\mathrm{d}\boldsymbol{r}_j)\hat{\boldsymbol{r}}_{ij}],\end{aligned} \tag{2}$$

where $\mu_0 \equiv 4\pi \times 10^{-7}\,\mathrm{kgmC^{-2}}$ is the vacuum permeability, $r_{ij} \equiv |\boldsymbol{r}_i - \boldsymbol{r}_j|$ is the distance between the current elements and $\hat{\boldsymbol{r}}_{ij} \equiv \boldsymbol{r}_{ij}/r_{ij}$ is the unit vector pointing from $I_j\,\mathrm{d}\boldsymbol{r}_j$ to $I_i\,\mathrm{d}\boldsymbol{r}_i$. In Grassmann's expression $\mathrm{d}\boldsymbol{B}_j(\boldsymbol{r}_i)$ is the magnetic field at $\boldsymbol{r}_i$ due to the current element $I_j\,\mathrm{d}\boldsymbol{r}_j$. This expression for the magnetic field was first proposed by Biot and Savart in 1820.

Ampère's expression is the only one compatible with Weber's force and Weber's electrodynamics [8], Chapter 4. Grassmann's expression, on the other hand, is the only one compatible with Lorentz's force and classical electrodynamics. Nowadays, it is difficult to find Ampère's force in any textbook dealing with electromagnetism. Despite this fact, it was accepted during the last century, as can be seen from Maxwell's statement (see [9], Vol. 2, Art. 528, p. 175):

> "The experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science. The whole, theory and experiment, seems as if it had leaped, full grown and full armed, from the brain of the 'Newton of Electricity'. It is perfect in form and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electrodynamics."

It should be remarked that Maxwell knew not only Ampère's force but also Grassmann's one (see [9], Vol. 2, Art. 526, p. 174). In his *Treatise*, Maxwell compared these

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two expressions and two others which he himself had proposed theoretically (see [9], Vol. 2, Art. 527, p. 174):

> "Of these four different assumptions that of Ampère is undoubtedly the best, since it is the only one which makes the forces on the two elements not only equal and opposite but in the straight line which joins them."

We would like to point out two main properties which distinguish these two expressions: (*I*) while the principle of action and reaction, $d^2\boldsymbol{F}_{ji} = -d^2\boldsymbol{F}_{ij}$, is always satisfied by Ampère's force, the same does not happen in general with Grassmann's one; (*II*) Grassmann's force is always perpendicular to the direction of the current in the element which suffers the force, no matter the direction of the magnetic field. This means that it does not allow longitudinal interactions between colinear and parallel current elements, while these longitudinal interactions are predicted by Ampère's force, at least between current elements.

Despite these differences, it has been known since last century that both expressions are equivalent to one another when we calculate the net force exerted by a closed circuit of arbitrary form on an external current element. The reason for this remarkable fact is that the difference between Ampère and Grassmann's expressions is an exact differential which integrates to zero around any closed circuit. A proof of this fact can be found in [8], Section 4.5; [10], pp. 55–58; and [11]. This means that we cannot distinguish between these two expressions when dealing with two or more closed circuits. Recently, people have been trying to distinguish Ampère's from Grassmann's force dealing with a single closed circuit composed of mobile or free parts, as in Figs 1(A) and (B).

The circuits abcda are closed ones where a current I flows, generated by a battery or by the discharge of a capacitor bank. The rigid metallic part abc is called the bridge [Fig. 1(A)] or the pi frame [Fig. 1(B)]. The rigid metallic part cda is called the support, usually fixed in the laboratory. At a and c there are gaps disconnecting mechanically the wire abc to the support cda. These gaps may be a simple

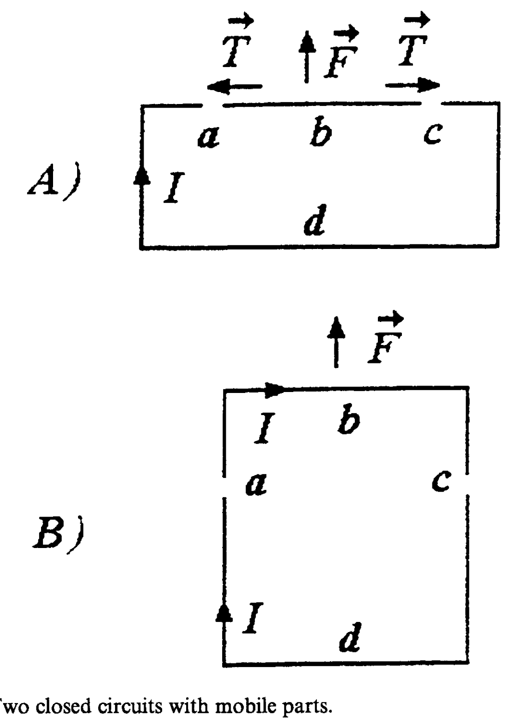

Fig. 1. Two closed circuits with mobile parts.

air separation of typically 1 cm length (where electric arcs or sparks will appear closing the electrical current). Alternatively they may be filled with liquid mercury which allow the passage of current. With both kinds of gaps the part abc is free to move or we can measure the net force acting on it (due to the remainder of the current) employing a torsion balance, for instance.

The main question is: in these situations, will the net force acting on abc calculated with Ampère's expression always agree with that calculated with Grassmann's one? There is no clear answer to this question up to now, despite the 150 years of the controversy. The answer to this question is one of the main topics of this work. The principal difficulty which arises when we consider this question theoretically are the divergences which appear when we try to utilize eqs (1) and (2) to calculate the forces. Only recently Wesley has proposed a method to handle these divergences [2, 12–14] and [15], Chapter 6. Utilizing his method and a few other techniques [3] it is possible to achieve a definitive conclusion regarding the controversy Ampère versus Grassmann's force.

To give an idea of the order of magnitude of these forces we quote here experiments by Peoglos utilizing these two topologies (rectangular or square circuits). With continuous currents of 1 A, copper wires of 1.2 mm diameter and a square circuit 10 cm wide, he measured with a torsion balance forces between 10^{-7}–10^{-6} N [16]. Moyssides, working with continuous currents in the range 35–140 A, copper wires of 1–3 mm and a rectangular circuit like that of Fig. 1(B) with sides of 48 and 124 cm measured forces between 10^{-3} and 10^{-2} N [17].

Beyond the net force on a piece of a closed circuit, it has also been recently discussed the tension T represented in Fig. 1(A). According to Grassmann's force, $d\boldsymbol{F}^G = I\, d\boldsymbol{r} \times \boldsymbol{B}$, there can never exist a force parallel to I $d\boldsymbol{r}$, no matter the value and direction of the magnetic field $\boldsymbol{B}$. Accordingly, a straight current carrying wire should not suffer any tension. On the other hand, if we calculate with Ampère's expression the force between ab and bc, see Fig. 1(A), we obtain that ab and bc repel one another, and this could generate a tension T in the wire. Modern experiments have confirmed the existence of such a tension.

These experiments are known as exploding wire phenomena. Although these explosions have been known for a long time [18], in the last decades the greatest interest has been in the quantitative explanation of this fact. A typical experiment is that of Graneau [19], which can be represented by Fig. 1(A). The straight wire abc is made of aluminium with 1 mm diameter and 1 m length. At a and c there are air gaps of 1 cm. The current source is a capacitor bank not represented in the figure. When it is discharged, electrical arcs appear at a and c closing the current, while keeping abc disconnected mechanically from cda. In Graneau's experiment the peak value of the current was 7000 A, and abc was broken in several pieces. The main question is to know the source of the force or tension responsible for this remarkable effect.

A qualitative explanation for this fact could be the heating and melting of the wire due to the Joule effect. This explanation has been ruled out after the microscopic analysis of the fragments, which revealed breaking in solid state due to a mechanical tension [19].

There are other features of these experiments: the force responsible for the breaking of the wire is proportional to I^2; the piece abc breaks approximately in the middle, then each half breaks again in the middle and so on, until the limit when the length of the fragment is of the same order of magnitude as its diameter. Graneau has been able to explain quantitatively all these features utilizing Ampère's force.

We can illustrate these explanations with Fig. 2. We have a straight wire of length l and diameter ω carrying a current I. We divided it in two arbitrary parts 1 and 2 of lengths x and $l - x$, respectively. Calculating the force between 1 and 2 with Grassmann's expression yields a null result, as expected. On the other hand, the force between 1 and 2 with Ampère's expression is found to be given by [3]:

$$F \approx \frac{\mu_0 I^2}{4\pi}\left[\ln\left(\frac{x}{\omega}\right) + \ln\left(\frac{l-x}{l}\right) + C\right]. \tag{3}$$

Here C is a dimensionless constant of the order of unity. Its specific value depends on the form of the cross section of the wire. This force is repulsive if $F > 0$ and attractive if $F < 0$.

If $l \gg \omega$, as usual, this expression indicates a repulsion between 1 and 2, generating a tension T in the wire. Moreover, this tension is found to be proportional to I^2. Fixing l and ω and varying x, the greatest tension happens at $x = l/2$. This indicates that the wire should break first around this point. The order of magnitude of this tension can be obtained utilizing Graneau's data: $I = 7000\,\mathrm{A}$, $l = 1\,\mathrm{m}$, $\omega = 1.2\,\mathrm{mm}$ and $x = l/2$. This yields $T \approx 30\,\mathrm{N/mm^2}$. This is of the right order of magnitude necessary to break an aluminium wire in this case [20], p. 48. Fixing $x = l/2$, shows that the tension tends to zero when $l \approx \omega$, so that the wire should stop breaking when becoming so small. And all these facts are observed experimentally [21].

Until today no other explanation has been able to account quantitatively for so many facts.

Another kind of experiment is known as railgun. In this case we observe compression instead of tension. A description of this experiment can be found in Fig. 3, adapted from [22].

We have a rectangular circuit made up by the metallic rails A and B fixed in the laboratory by the lateral wood supports D. The metallic projectile a closes the circuit and can move along the rails. The current source at C is a capacitor bank. The metallic conductors B are thin and do not support compression, contrary to what happens with the thick conductors A. The rails A and B were pinned together to the wood beams at p.

When closing the switch S, with the fixed projectile, a current flowed along the circuit, deforming the thin rails B. In this experiment capacitor banks of $8\,\mu\mathrm{F}$ charged to $80\,\mathrm{kV}$ yield current pulses of $100\,\mathrm{kA}$. The length of the projectile was 25 cm, of the rail A 200 cm and of the thin rail B 30 cm, all made of copper.

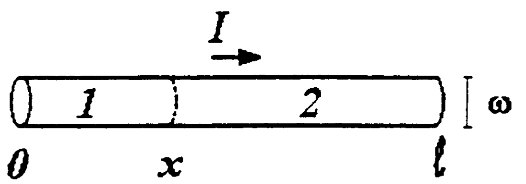

Fig. 2. A straight current carrying wire.

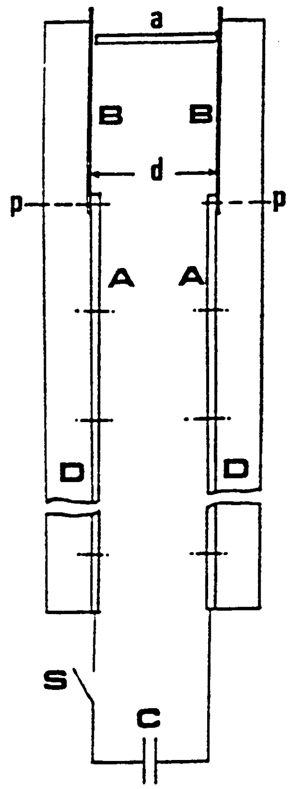

Fig. 3. A railgun experiment.

Calculating the net force on the projectile due to the whole circuit yields the same result with Ampère or Grassmann's expressions: the force is orthogonal to the projectile pointing away from the capacitor bank. The main question is: where is the reaction to this force and what object suffers the recoil force? Is it the rails A and B, or the source at C, or the magnetic field? To say that the reaction is stored in the electromagnetic field violates momentum or energy conservation, as shown in [23–25]. On the other hand, the experiment showed that the rails B suffered a force of compression, as evidenced by their longitudinal deformation. This compression cannot be due to Grassmann's force as it is always perpendicular to the current. Graneau explained quantitatively this compression utilizing Ampère's force [24]. According to him, this is due to the fact that the projectile a exerts a vertical downward force on the rails B utilizing Ampère's expression. As the rails B are pinned to A and cannot move, they suffer a compression due to this force.

These two experiments (exploding wire and railgun) show the existence of longitudinal forces in current carrying conductors. Grassmann's expression can never explain these effects. In this work we show that Ampère's force cannot account for them either, as there is a complete equivalence between these two expressions when considering closed circuits.

2. Preliminary results

Let us consider the surface cylinder in Fig. 4. The cylinder has a length l and its cross section is a circumference of radius a. There is an uniform poloidal current flowing over its surface, with a surface current density $\boldsymbol{K} = (I/l)\hat{\boldsymbol{\phi}}$. We shall calculate the force exerted by the whole cylinder on a surface current element of itself. Instead of using the linear

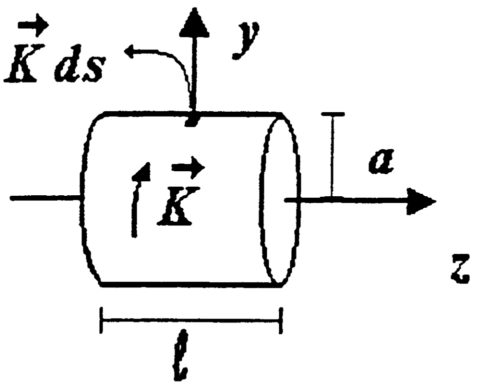

Fig. 4. A cylinder with a poloidal surface current density.

eqs (1) and (2), we replace on them the linear current elements $I\,\mathrm{d}\boldsymbol{r}$ by $\boldsymbol{K}\,\mathrm{d}s$, where $\mathrm{d}s$ is an element of area (see details in [3]). To simplify the calculation we consider the surface current element at $(a, \pi/2, 0)$ [in cylindrical coordinates (ρ, ϕ, z)]. By symmetry, the force exerted by the cylinder on this current element is in the direction $\boldsymbol{y}$. Ampère and Grassmann's force are, respectively:

$$\boldsymbol{F}^{\mathrm{A}} = \hat{\boldsymbol{y}}\,\frac{\mu_0 I^2 a^2\,\mathrm{d}s}{4\pi l^2}\int_{-l/2}^{l/2}\mathrm{d}z\int_0^{2\pi}\mathrm{d}\phi \times\left(\frac{3a^2\cos^2\phi(1-\sin\phi)}{[z^2+2a^2(1-\sin\phi)]^{5/2}} - \frac{2\sin\phi(1-\sin\phi)}{[z^2+2a^2(1-\sin\phi)]^{3/2}}\right), \tag{4}$$

$$\boldsymbol{F}^{\mathrm{G}} = \hat{\boldsymbol{y}}\,\frac{\mu_0 I^2 a^2\,\mathrm{d}s}{4\pi l^2}\int_{-l/2}^{l/2}\mathrm{d}z\int_0^{2\pi}\mathrm{d}\phi \times\frac{1-\sin\phi}{[z^2+2a^2(1-\sin\phi)]^{3/2}}. \tag{5}$$

On evaluating the integrals above the results are:

$$\boldsymbol{F}^{\mathrm{A}} = \boldsymbol{F}^{\mathrm{G}} = \hat{\boldsymbol{y}}\,\frac{\mu_0 I^2\,\mathrm{d}s}{2\pi l^2}\,\mathbf{K}\!\left(\frac{4\mathrm{i}a}{l}\right), \tag{6}$$

where $\mathrm{i} = \sqrt{-1}$ and $\mathbf{K}$ is the complete elliptic integral of the first kind [26], pp. 907–908.

It is important to note that the result in eq. (6) is exact for any value of l/a. This means that the force on the surface current element of the cylinder, exerted by the whole cylinder, is the same either for Ampère's force or Grassmann's one.

If we make $l \to 0$ in eq. (6) the result goes to infinity. This shows what we have cited before: the divergence that appear when using the expressions (1) and (2) for calculating forces between touching current elements.

Now, we shall generalize this equivalence for any geometry.

3. Complete equivalence

We shall prove here the following theorem.

Theorem: The net force on a current element due to the closed circuit of arbitrary form to which it belongs, has the same value according to Grassmann and Ampère's expressions. Moreover, this net force will always be orthogonal to the current element.

To this end we shall employ a geometric proof. The generalization of this reasoning for circuits with volumetric current densities can be easily carried out by analogy.

Consider the generic closed circuit Γ of Fig. 5(a) in which flows a current I over its surface. We want to compare Ampère and Grassmann's forces exerted on the surface current element δ of area $\mathrm{d}s$. The radius of curvature at δ is represented by a. As the force exerted by the surface current element δ on itself is zero with both expressions, we only need to know the force exerted by the remainder of the circuit on it. In Fig. 5(b) we have represented a cylinder Γ_1 of length l and radius a and a circuit Γ_2. This circuit Γ_2 is similar to Γ at all points, except those near the cylinder Γ_1. There is a uniform distance d between the cylinder Γ_1 and the equivalent cylinder of the circuit Γ_2. When $d \to 0$ we have $\Gamma_1 + \Gamma_2 \to \Gamma$. That is, Γ_1 plus Γ_2 reduce to Γ, Fig. 5(c).

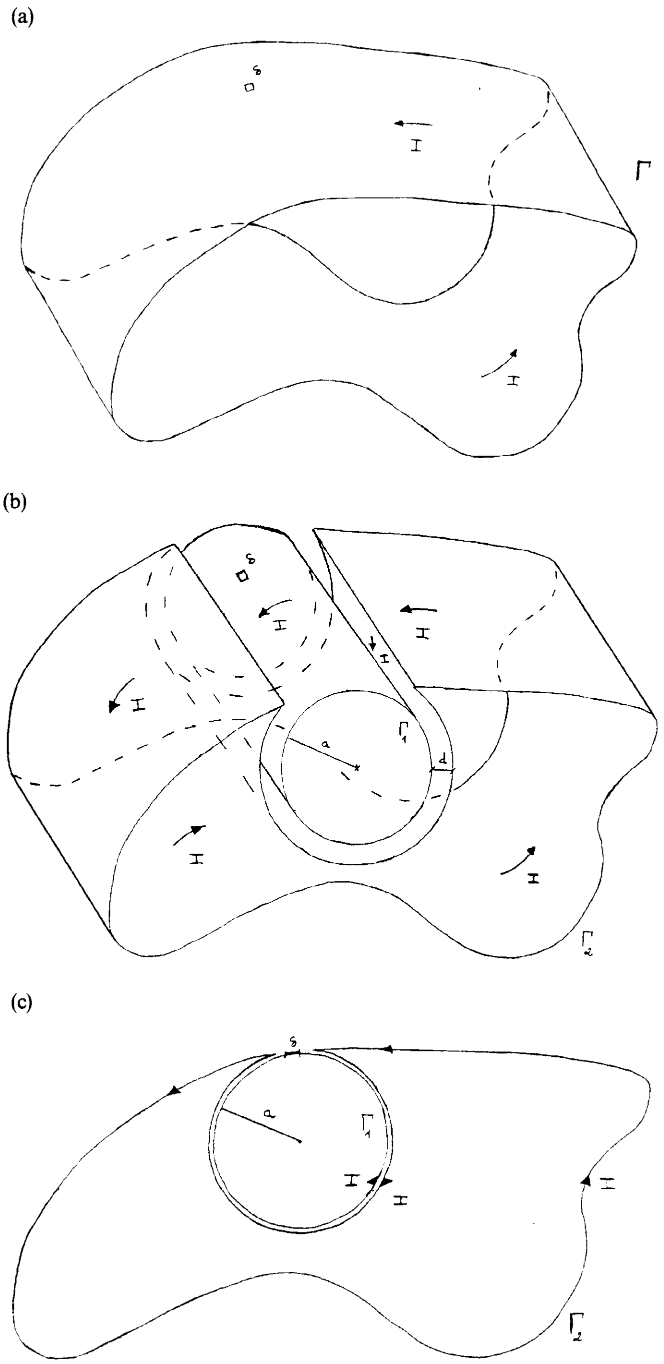

Fig. 5. Circuits utilized to prove the equivalence between Ampère and Grassmann's forces.

The direction of the current flow in each circuit is represented in Fig. 5(b).

Proof of the theorem: The net force on δ in Fig. 5(b) can be divided in two parts: the first one, due to the cylinder Γ_1; and the second one, due to the closed circuit Γ_2. The force exerted by Γ_2 on δ has the same value for Ampère and Grassmann's expressions as it is a closed circuit acting on a current element external to it: [8], Section 4.5; [10], pp. 55–58; and [11]. As regards the force exerted by the cylinder Γ_1 in δ, we have showed in Section 2 utilizing the circuit of Fig. 4 that it also has the same value for both expressions [eq. (6)]. This means that the net force on δ of Fig. 5(b) is given by the same value according to Grassmann or Ampère's expressions. Although this net force depends on the value of d, the equivalence between Ampère and Grassmann's expressions is independent of d. And when $d \to 0$, the force exerted by Γ_1 plus the force exerted by Γ_2 on δ tend to the force exerted by Γ [Fig. 5(a)] on δ, as can be seen in Fig. 5(c).

We have then proved that the force exerted by a closed circuit of arbitrary form on a current element belonging to it has the same value for Ampère and Grassmann's expressions. As Grassmann's force is always orthogonal to the element suffering the force, the same will be valid for Ampère's force in the case of a closed conductor. We have shown these facts in greater details with specific circuits in [3].

These are curious results. On the one hand, Grassmann's force has incorporated Ampère's force property of yielding no net force in the whole closed circuit. On the other hand, Ampère's force has incorporated Grassmann's force property of yielding no longitudinal forces.

4. Conclusions

We have shown that Ampère and Grassmann's expressions always yield the same results when considering the net force on any current element of a closed circuit of arbitrary form, against the opinion of some authors [2, 27, 28].

The equivalence between these expressions for a single circuit has been recently claimed by some authors. Jolly [29], Ternan [30] and Christodoulides [31] presented some demonstrations which are restricted to magnetostatic cases ($\nabla \cdot \boldsymbol{J} = 0$). As in most experiments of exploding wires there are discharge of capacitor banks with pulsed currents, their reasonings cannot be applied here. Moreover, people have pointed out mathematical problems (related with convergences and singularities) in Christodoulides's demonstration, [28, 32]. The demonstration we have presented here is not restricted to magnetostatic cases and does not have the problems pointed out by Pappas and Cornille against Christodoulides's proof. In this way, we expect to have settled this question definitively.

We are then left with the problem of explaining the source of tension responsible for the explosion of the wires. We have recently tried to explain it by applying directly Weber's force acting on the lattice of the conductor [33]. We concluded in this work that the tension caused by Weber's force is two orders of magnitude smaller than the required one to break the wire. Therefore Weber's force was not considered responsible for the explosions. The only possibility for Weber's force to explain these explosions, would be a great reduction in the tensile stress necessary to break a wire. This decrease in the tensile stress of the wire might be due to the increase of temperature in the metal during the discharge. It is reasonable to expect this behaviour in any metallic wire. We did not perform this analysis in [33] as we were unable to find tables giving the tensile stress of metals as a function of temperature.

As these experiments (exploding wires and railgun) cannot distinguish Grassmann and Ampère's expressions, a possibility to distinguish these different formulations of electromagnetism is to consider directly Weber and Lorentz's forces. As we have seen, from Weber's force we derive only Ampère's force, but not Grassmann's one, while from Lorentz's force (or from Liénard-Schwarzschild's one) we derive only Grassmann's force, but not Ampère's one, [8], Sections 4.2, 4.4 and 6.3. If one can show which one of these forces (Weber or Lorentz) is the wrong one, then we could distinguish between Weber's electrodynamics and classical electrodynamics, which is based on Lorentz's force. Some experiments involving charged particles moving inside dielectric charged spherical shells were proposed in order to distinguish Weber's force from Lorentz's one, as they yield different predictions in these cases [34, 35]. This might lead to important new results in the near future.

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