PRAMANA © Indian Academy of Sciences Vol. 55, No. 3
— journal of September 2000
physics pp. 393–404

Comparison between Weber's electrodynamics and classical electrodynamics

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MS received 20 January 2000; revised 25 April 2000

Abstract. We present the main aspects of Weber's electrodynamics and of Maxwell's equations. We discuss Maxwell's point of view related to Weber's electrodynamics. We compare Weber's force with Lorentz's force. We analyse the relation between Weber's law and Maxwell's equations. Finally, we discuss some experiments performed and proposed with which we can distinguish Weber's force from Lorentz's one.

Keywords. Weber's electrodynamics; Maxwell's equations; Lorentz's force; classical electromagnetism.

PACS Nos 41.20.Bt; 41.20.-q; 03.50.De

1. Weber's electrodynamics

Wilhelm Weber (1804–1891) developed his electrodynamics during the same period in which James Clerk Maxwell (1831–1879) was putting together what are known as Maxwell's equations. In this work we compare these two electrodynamics.

We begin presenting the main aspects of Weber's electrodynamics. For complete references, see [1] and [2]. Suppose we have a point charge q_2 located on $\vec{r}_2$ relative to the origin of an inertial frame of reference S, moving with velocity $\vec{v}_2$ and acceleration $\vec{a}_2$, and another point charge q_1 located on $\vec{r}_1$ and moving with velocity $\vec{v}_1$ and acceleration $\vec{a}_1$ relative to S. Weber's force (1846) exerted by 2 on 1 is given by

$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\ddot{r}}{c^2}\right)$$
$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left[1 + \frac{1}{c^2}\left(\vec{v}\cdot\vec{v} - \frac{3}{2}(\hat{r}\cdot\vec{v})^2 + \vec{r}\cdot\vec{a}\right)\right] . \quad (1)$$

In this equation $\varepsilon_0 \equiv 8.85 \times 10^{-12}$ C^2 $N^{-1}m^{-2}$ is called the permittivity of free space, $r \equiv |\vec{r}_1 - \vec{r}_2|$ is the distance between the charges, $\vec{r} \equiv \vec{r}_1 - \vec{r}_2$ is the vector pointing from

2 to 1, $\hat{r} \equiv \vec{r}/r$ is the unit vector pointing from 2 to 1, $\vec{v} \equiv \vec{v}_1 - \vec{v}_2$ is the relative vectorial velocity, $\vec{a} \equiv \vec{a}_1 - \vec{a}_2$ is the relative vectorial acceleration, $\dot{r} \equiv \mathrm{d}r/\mathrm{d}t = \hat{r} \cdot \vec{v}$ is the relative radial velocity, $\ddot{r} \equiv \mathrm{d}^2 r/\mathrm{d}t^2 = [\vec{v} \cdot \vec{v} - (\hat{r} \cdot \vec{v})^2 + \vec{r} \cdot \vec{a}]/r$ is the relative radial acceleration and $c = 3 \times 10^8\,\mathrm{ms}^{-1}$ is the ratio of electromagnetic and electrostatic units of charge. The experimental value of c was first determined by Weber and Kohlrausch in 1856.

In 1848, Weber presented a velocity dependent potential energy from which this force might be derived, namely:

$$U = \frac{q_1 q_1}{4\pi\varepsilon_0} \frac{1}{r} \left(1 - \frac{\dot{r}^2}{2c^2}\right) . \tag{2}$$

The relation between U and $\vec{F}_{21}$ is $\vec{F}_{21} = -\hat{r}\mathrm{d}U/\mathrm{d}r$.

These two expressions form the basis of Weber's electrodynamics.

2. Maxwell–Lorentz's electrodynamics

Based mainly on the works of Coulomb, Ampère, Gauss, Neumann and Faraday, Maxwell put together between 1855 and 1864 a set of four equations relating the electric and magnetic fields, $\vec{E}$ and $\vec{B}$, to their sources: the volumetric charge density ρ and the current density $\vec{J}$. In vectorial notation, in differential form, supposing the sources and fields in vacuum, these four equations can be written as

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} , \tag{3}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} , \tag{4}$$

$$\nabla \cdot \vec{B} = 0 , \tag{5}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} . \tag{6}$$

In these equations $\mu_0 \equiv 4\pi \times 10^{-7}\ \mathrm{kgmC}^{-2}$ is called the vacuum permeability.

These equations describe how the sources ρ and $\vec{J}$ create the electric and magnetic fields, but not how these fields act on another test charge q moving with velocity $\vec{v}$ relative to an inertial frame of reference. This last equation was given by H A Lorentz (1853–1928) in 1895:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} . \tag{7}$$

In this way we have the main aspects of Maxwell–Lorentz electrodynamics. In order to compare it with Weber's electrodynamics, we need expressions describing the force exerted by a point charge q_2 on another point charge q_1. This was obtained by the works of Liénard, Wiechert and Schwarzschild between 1898 and 1903. The final expression valid up to second order in v/c and taking into account retardation of time, radiation phenomena and relativistic effects is given by (see ([3], vol. 1, pp. 215–223), [4] and [5]):

$$\vec{F}_{21} = q_1\vec{E}_2(\vec{r}_1) + q_1\vec{v}_1 \times \vec{B}_2(\vec{r}_1) = q_1 \left\{ \frac{q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\left[\hat{r}\left(1 + \frac{\vec{v}_2\cdot\vec{v}_2}{2c^2}\right.\right.\right.$$

$$\left.\left.\left. - \frac{3}{2}\frac{(\hat{r}\cdot\vec{v}_2)^2}{c^2} - \frac{\vec{r}\cdot\vec{a}_2}{2c^2}\right) - \frac{r\vec{a}_2}{2c^2}\right]\right\} + q_1\vec{v}_1 \times \left\{\frac{q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\frac{\vec{v}_2\times\hat{r}}{c^2}\right\} . \quad (8)$$

The energy of interaction between 1 and 2 in Maxwell–Lorentz's electrodynamics is obtained from Darwin's lagrangian of 1920, namely ([6], pp. 150–151 and [7], Section 12.7, pp. 593–595):

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}\left[1 - \frac{\vec{v}_1\cdot\vec{v}_2 + (\vec{v}_1\cdot\hat{r})(\vec{v}_2\cdot\hat{r})}{2c^2}\right] . \quad (9)$$

In the next section we present Maxwell's points of view as regards Weber's electrodynamics and then begin the comparison between these two formulations.

3. Maxwell's points of view on Weber's electrodynamics

In his first work dealing with electromagnetism, published in 1855, Maxwell praised highly Weber's electrodynamics. After presenting Faraday's ideas which he was trying to follow, Maxwell said:

> There exists however a professedly physical theory of electro-dynamics, which is so elegant, so mathematical, and so entirely different from anything in this paper, that I must state its axioms, at the risk of repeating what ought to be well known. It is contained in M W Weber, *Electrodynamic Measurements*, and may be found in the Transactions of the Leibniz-Society, and of the Royal Society of Sciences in Saxony [8]. The assumptions are (...). From these axioms are deducible Ampère's laws of the attraction of conductors, and those of Neumann and others, for the induction of currents. Here then is a really physical theory, satisfying the required conditions better perhaps than any yet invented, and put forth by a philosopher whose experimental researches form an ample foundation for his mathematical investigations ([9], pp. 207–208).

In his famous article of 1864 in which he completed his electromagnetic theory of light, Maxwell presented similar points of view. After saying that the most natural theories of electromagnetism are based on forces acting directly between the charges he said:

> In these theories the force acting between the two bodies is treated with reference only to the condition of the bodies and their relative position, and without any express consideration of the surrounding medium. These theories assume, more or less explicitly, the existence of substances the particles of which have the property of acting on one another at a distance by attraction or repulsion.

> The most complete development of a theory of this kind is that of M W Weber, who has made the same theory include electrostatic and electromagnetic phenomena. In doing so, however, he has found it necessary to assume that the force between two electric particles depends on their relative velocity, as well as on their distance. This theory, as developed by M W Weber and C Neumann, is exceedingly ingenious, and wonderfully comprehensive in its application to the phenomena of statical electricity, electromagnetic attractions, induction of currents and diamagnetic phenomena; and it comes to us with more authority, as it served to guide the speculations of one who has made so great an advance in the practical part of electric science, both by introducing a consistent system of units in electrical measurement, and by actually determining electrical quantities with an accuracy hitherto unknown ([10], pp. 526–527).

But if Maxwell knew so well Weber's electrodynamics and praised it so much, why he did not work with it and develop its properties and applications? Weber presented his force law in 1846. It was the first example appearing in physics of a force depending not only on the distance between the interacting bodies but also on their velocity (Lorentz's force would appear only in 1895). Only one year after this Helmholtz published his famous and influential work dealing with the conservation of energy. The principle of the conservation of energy had been established by J R Meyer in 1842 and by J P Joule in 1843. In his work of 1847, Helmholtz put this principle in a firm theoretical basis by developing the consequences of central forces. Let us explain some terms which he utilized. At that period the usual name for mv^2 was Leibniz's living force, *vis viva*, but in this article Helmholtz says explicitly that he will call $mv^2/2$ (our kinetic energy) by *vis viva*, as this last quantity appeared more frequently in mechanics and was more useful. What we call nowadays by potential energy (like mgh, $kx^2/2$ etc), Helmholtz called by tension. The main results of his work were presented as:

> The preceding proposition may be collected together as follows:
>
> 1. Whenever natural bodies act upon each other by attractive or repulsive forces, which are independent of time and velocity, the sum of their *vires vivae* and tensions must be constant; the maximum quantity of work which can be obtained is therefore a limited quantity.
>
> 2. If, on the contrary, natural bodies are possessed of forces which depend upon time and velocity, or which act in other directions than the lines which unite each two separate material points, for example, rotatory forces, then combinations of such bodies would be possible in which forces might be either lost or gained *ad infinitum* ([11], p. 126).

Based on this statement, Maxwell and others thought that Weber's electrodynamics did not comply with the principle of conservation of energy (after all, although Weber's expression is a central force, it depends on the velocity of the charges). This can be seen in the sequence of Maxwell's statements presented above, as he presents this as the only problem of Weber's electrodynamics. For instance, in the sequence of his 1855 paper, Maxwell said: 'There are also objections to making any ultimate forces in nature depend on the velocity of the bodies between which they act. If the forces in nature are to be reduced to forces acting between particles, the principle of the conservation of force (energy) requires that these forces should be in the line joining the particles and functions of the distance only ([9], p. 208)'. In the sequence of his article of 1864, Maxwell wrote (our emphasis): '*The mechanical difficulties*, however, which are involved in the assumption of particles acting at a distance with forces which depend on their velocities are such as to *prevent me from considering this theory as an ultimate one*, though it may have been, and may yet be useful in leading to the coordination of phenomena ([10], p. 527)'.

Maxwell was wrong on this regard. Although Weber had presented his potential energy (2) in 1848, one year after Helmholtz's paper, he did not prove at this time the conservation of energy. It was only in 1869 and 1871 that he proved in detail that his force law did comply with the principle of conservation of energy. Maxwell only changed his points of view in 1871, after Weber's proof. In ([12], pp. 96–97) there is a reproduction of a post card from Maxwell to Tait, dated 1871, in which Maxwell informs Tait that Weber was right when stating that his (Weber's) electrodynamics did comply with the principle of conservation of energy. In this post card he says: 'Weber has reason, his force has a potential which involves the square of the relative velocity. Hence in any cyclic operation no motion is spent or gained.'

Helmholtz's proof cannot be applied to Weber's force because this expression depends not only on the distance and velocity of the charges but also on their accelerations. And this more general case had not been considered by Helmholtz.

When Maxwell wrote the *Treatise on Electricity and Magnetism* in 1873, he presented his new point of view that Weber's force was consistent with the principle of conservation of energy. In the last chapter of his book Maxwell discusses Weber's electrodynamics and other models which had been proposed to electromagnetism based on action at a distance. As regards conservation of energy, Maxwell says the following, after presenting the forces of Weber and Gauss (see [13], vol. 2, articles 852 and 853, p. 484):

> *Article 852*. The two expressions lead to precisely the same result when they are applied to the determination of the mechanical force between two electric currents, and this result is identical with that of Ampère. But when they are considered as expressions of the physical law of the action between two electrical particles, we are led to enquire whether they are consistent with other known facts of nature.
>
> Both of these expressions involve the relative velocity of the particles. Now, in establishing by mathematical reasoning, the well-known principle of the conservation of energy, it is generally assumed that the force acting between two particles is a function of the distance only, and it is commonly stated that if it is a function of anything else, such as the time, or the velocity of the particles, the proof would no longer hold.
>
> Hence a law of electrical action, involving the velocity of the particles, has sometimes been supposed to be inconsistent with the principle of the conservation of energy.
>
> *Article 853*. The formula of Gauss is inconsistent with this principle, and must therefore be abandoned, as it leads to the conclusion that energy might be indefinitely generated in a finite system by physical means. This objection does not apply to the formula of Weber, for he has shown [14] that if we assume as the potential energy of a system consisting of two electric particles

$$\Psi = \frac{ee'}{r}\left[1 - \frac{1}{2c^2}\left(\frac{\partial r}{\partial t}\right)^2\right] ,$$

> the repulsion between them, which is found by differentiating this quantity with respect to r, and changing the sign, is that given by formula
>
> $$\frac{ee'}{r^2}\left[1 + \frac{1}{c^2}\left(r\frac{\partial^2 r}{\partial t^2} - \frac{1}{2}\left(\frac{\partial r}{\partial t}\right)^2\right)\right] .$$
>
> Hence the work done on a moving particle by the repulsion of a fixed particle is $\Psi_0 - \Psi_1$, where Ψ_0 and Ψ_1 are the values of Ψ at the beginning and at the end of its path. Now Ψ depends only on the distance, r, and on the velocity resolved in the direction of r. If, therefore, the particle describes any closed path, so that its position, velocity, and direction of motion are the same at the end as at the beginning, Ψ_1 will be equal to Ψ_0, and no work will be done on the whole during the cycle of operations.
>
> Hence an indefinite amount of work cannot be generated by a particle moving in a periodic manner under the action of the force assumed by Weber.

It should be emphasized that what Maxwell wrote as $\partial r/\partial t$ would be written nowadays as $\mathrm{d}r/\mathrm{d}t$, as is evident from what he wrote in ([13], article 847).

After this statement in his book, Maxwell discussed another criticism of Helmholtz against Weber's electrodynamics. As we already discussed this in ([1] §7.3: Charged spherical shell) and in [15], we will not analyse these criticism here again. In these works we showed how to overcome Helmholtz's criticisms against Weber's law.

As we have seen, Maxwell was wrong (from 1855 to at least 1864) as regards the only negative aspect which he saw in Weber's electrodynamics. In the end he corrected himself, as we saw above.

There are other examples where he praised Weber's experimental and theoretical works. We quote a few here:

Weber introduced the electrodynamometer in 1846. Maxwell described it in ([13], article 725: Weber's electrodynamometer, pp. 367–371), from which we quote:

> The instrument originally constructed by Weber is described in his *Elektrodynamische Maasbestimmungen*. (...) The experiments which he made with it furnish the most complete experimental proof of the accuracy of Ampère's formula as applied to closed currents, and form an important part of the researches by which Weber has raised the numerical determination of electrical quantities to a very high rank as regards precision.
>
> Weber's form of the electrodynamometer, in which one coil is suspended within another, and is acted on by a couple tending to turn it about a vertical axis, is probably the best fitted for absolute measurements.

Weber's theory of diamagnetism was adopted by Maxwell and is still adopted in its main aspects today, see ([13], vol. 2, chap. VI: Weber's theory of induced magnetism, articles 442–448, pp. 79–94).

As regards the absolute system of units, which was created by Gauss jointly with Weber, Maxwell stated ([13], article 545, pp. 193, 194):

> The introduction, by Weber, of a system of absolute units for the measurement of electrical quantities is one of the most important steps in the progress of the science. Having already, in conjunction with Gauss, placed the measurement of magnetic quantities in the first rank of methods of precision, Weber proceeded in his *Electrodynamic Measurements* not only to lay down sound principles for fixing the units to be employed, but to make determinations of particular electrical quantities in terms of these units, with a degree of accuracy previously unattempted. Both the electromagnetic and the electrostatic systems of units owe their development and practical application to these researches.

We conclude from all these quotations that Maxwell always had a deep admiration as regards Weber's experimental and theoretical works.

4. Conceptual and philosophical differences

The main difference between these two formulations of electromagnetism lies in the mechanism of interaction between the charges. According to Weber's electrodynamics we have a direct action between each pair of charges, no matter their distance nor their motion. We do not need to speak in electric nor in magnetic fields. While Weber starts with force between the charges directly, Maxwell's approach is interaction via the field. Maxwell believed that each charge generated electric and magnetic fields, which would move in space tipically at light velocity. These fields would act on the other charges. According to him there would not be any direct action between two charges separated in space. The action between them would be performed by the fields. Maxwell believed in a material medium filling all space, the ether, which would be the responsible for carrying the action of one charge until the other and vice-versa. For instance, his last two sentences in the *Treatise* state ([13], vol. 2, article 866, p. 493, our emphasis): 'In fact, whenever energy is transmitted from one body to another in time, *there must be a medium or substance* in which the energy exists after it leaves one body and before it reaches the other, for energy, as Torricelli [16] remarked, 'is a quintessence of so subtle a nature that it cannot be contained in any vessel except the inmost *substance of material things*.' Hence all these theories lead to the conception of a medium in which the propagation takes place, and if we admit this medium as a hypothesis, I think it ought to occupy a prominent place in our investigations, and that we ought to endeavour to construct a mental representation of all the details of its action, and this has been my constant aim in this treatise."

Since 1905 with Einstein's special theory of relativity, the ether disappeared from Maxwell–Lorentz's electrodynamics. In its place we speak in electric and magnetic field flowing in empty space, in vacuum. At a later section we discuss a specific experiment which might distinguish between Weber's and Maxwell's formulations. It involves a situation in which a group of source charges generate no electric nor magnetic field, so that they cannot act on a test charge according to Maxwell–Lorentz's electrodynamics, but in which Weber's electrodynamics predicts a measurable effect.

As regards the conservation laws, Weber's force complies with action and reaction ($\vec{F}_{21} = -\vec{F}_{12}$), so that the conservation of linear momentum of the interacting charges is automatically satisfied. This does not happen with Lorentz's force (7) nor with

Schwarzschild's force (8). That is, usually $\vec{F}_{21} \neq -\vec{F}_{12}$, except in some very particular situations. This means that according to Lorentz's expression the linear momentum of a group of interacting charges may be gained or lost although they are not interacting with any external bodies. Usually it is argued in these cases that the linear momentum gained or lost by the charges is lost or gained by the electromagnetic field they generate. Weber's expression is also a central force directed along the straight line connecting the charges, which means conservation of angular momentum. This does not happen with Schwarzschild's force (8) as it has a component pointing towards the acceleration of the source charge. This means that according to this last theory the angular momentum of a system of interacting charges may be gained or lost without any exchange with external bodies. There is conservation of energy for a group of interacting charges in both theories, although with different values: eqs (2) and (9). All these three things are difficult to test in specific experiments.

When we compare the mathematical expressions (1) and (8) we see that Weber's expression depends on the velocity squared of both charges, while Schwarzschild's expression only depends on the square of the velocity of the source charge q_2, but not on v_1^2. As regards the accelerations, Weber's expression depends on a_1 and on a_2, while Schwarzschild's expression depends only on a_2 (the acceleration of the source charge). Later on we discuss a specific experiment based on this effect in which Weber's electrodynamics can be distinguished experimentally from Maxwell–Lorentz's one.

Usually it is claimed that Weber's electrodynamics cannot deal with radiation effects, antenna phenomena etc due to the fact that it is an action at a distance theory, while Maxwell's electrodynamics is based on electromagnetic field propagating at light velocity. However, several aspects need to be clarified here. The first one is that the electromagnetic quantity $c = 1/\sqrt{\mu_0 \varepsilon_0}$ was first introduced in physics by Weber in 1846, in his force law. The first measurement of this quantity was also performed by Weber, in collaboration with Kohlrausch, in 1856. Maxwell only measured this in 1868 [17]. This quantity was introduced by Maxwell in his displacement current in 1861–62, borrowed from Weber's electrodynamics. Moreover, the first to obtain that an electromagnetic signal would propagate in an electric circuit at light velocity were Weber and Kirchhoff in 1857, working independently of one another, but both of them based on Weber's action at a distance electrodynamics [18–22] ([23], vol. 1, pp. 144–146 and 296–297, [24] and [25]). And this was obtained prior to Maxwell's introduction of the displacement current and prior to his own derivation of the wave equation in 1864.

5. Maxwell's equations from Weber's electrodynamics

If there is no motion between the interacting charges, $\dot{r} = 0$ and $\ddot{r} = 0$, Weber's force reduces to Coulomb's one. And from Coulomb's force we derive Gauss's law (3) by a standard procedure ([7], §1.3). This means that the first of Maxwell's equations, eq. (3), can be derived from Weber's law.

From Weber's force we derive as well Ampère's force between current elements, namely (see [26], ([1], §4.2: Derivation of Ampère's force from Weber's force) and [27]):

$$\mathrm{d}^2\vec{F}_{21} = -\frac{\mu_0}{4\pi} I_1 I_2 \frac{\hat{r}}{r^2} \left[2(\mathrm{d}\vec{\ell}_1 \cdot \mathrm{d}\vec{\ell}_2) - 3(\hat{r} \cdot \mathrm{d}\vec{\ell}_1)(\hat{r} \cdot \mathrm{d}\vec{\ell}_2) \right] . \quad (10)$$

This is Ampère's force exerted by the current element $I_2 \mathrm{d}\vec{\ell}_2$ located at $\vec{r}_2$, on the current element $I_1 \mathrm{d}\vec{\ell}_1$ located at $\vec{r}_1$ relative to the origin of a coordinate system S. Ampère obtained it between 1820 and 1826. It was based on this force law and on Coulomb's force that Weber obtained his own force law (1) in 1846. We can also reverse the argument, namely, begin with Weber's force and derive Ampère's force.

From this expression it is possible to derive the magnetic circuital law with the displacement current, (4), and also the law of non-existence of magnetic monopoles, (5): see ([1], §4.7: Derivation of the magnetic circuital law and of the law of nonexistence of magnetic monopoles). So two other Maxwell's equations can be derived from Weber's electrodynamics.

Faraday's law of induction, eq. (6), can also be derived from Weber's electrodynamics, as was recognized by Maxwell himself ([13], article 856, p. 486):

> After deducing from Ampère's formula for the action between the elements of currents, his own formula for the action between moving electric particles, Weber proceeded to apply his formula to the explanation of the production of electric currents by magneto-electric induction. In this he was eminently successful, and we shall indicate the method by which the laws of induced currents may be deduced from Weber's formula.

A detailed proof of this fact can be found in ([1], §5.3: Derivation of Faraday's law from Weber's force).

These aspects show that Weber's electrodynamics is compatible with all Maxwell's equations. Further discussion of the relation between Weber's electrodynamics and Maxwell's equations can be found in [28] and [29].

Despite this fact in the next section we discuss a specific situation in which Weber's force can be distinguished experimentally from Lorentz's force.

6. Experimental distinction between Weber's force and Lorentz's force

In most situations, especially those related with closed circuits, Weber's force and Lorentz's one will yield the same result. We proved this, for instance, considering the similar predictions of Ampère's force between current elements (which can be derived from Weber's force but not from Lorentz's one) as compared with Grassmann–Biot–Savart's one (which can be derived from Lorentz's force but not from Weber's one): [30–34] and [27].

But here we analyse a possible experiment to distinguish these two theories and which highlights the main conceptual difference between these two electrodynamics.

A stationary uniformly charged spherical shell, made of a dielectric material, generates no electric nor magnetic field in its interior, as is well known (the equivalent proof of this fact for the gravitational force goes back to Newton in 1687). According to Lorentz's force (7), a test particle moving inside this shell will not experience any force from the charged shell. That is, it will not feel the shell, as if it did not exist. If the test charge has an inertial mass m and is interacting with other bodies which exert on the test charge a net force $\vec{F}$, its acceleration inside the shell will be given by (with Newton's second law of motion):

$$\vec{a} = \frac{\vec{F}}{m} \; . \tag{11}$$

Weber's force, on the other hand, predicts a force exerted by the charged shell on the charged particle moving in its interior whenever the test charge is accelerated by other bodies (a spring, a magnet, a current carrying wire, etc.) The expression for this force is given by (see [35] and [36]): $\vec{F}_{\rm shell} = \mu_0 q Q \vec{a}/12\pi R$, where q is the charge of the test particle moving with acceleration $\vec{a}$ relative to the shell of radius R charged with a total charge Q. If the force exerted by the other bodies on q is represented as above by $\vec{F}$, its acceleration according to Weber's electrodynamics and Newton's second law of motion $\vec{F} + \vec{F}_{\rm shell} = m\vec{a}$ will be given by

$$\vec{a} = \frac{\vec{F}}{m - \mu_0 q Q/12\pi R} \ . \tag{12}$$

That is, the test body will behave as having an effective inertial mass which depends on its charge, on the charge of the shell and on the radius of the shell.

This might be tested experimentally. If the test charge is an electron, Weber's electrodynamics predicts that its effective mass will double if the shell is charged to a potential of 1.5×10^6 V, considering the infinity at zero potential. Suppose the test charge q of mass m is moving with velocity v and describing a circular orbit with a Larmor radius $r = mv/qB$ due to an uniform magnetic field B when the spherical shell is not charged. Weber's electrodynamics predicts that the radius of curvature will double when the system is involved by the charged spherical shell at 1.5×10^6 V, whenever the electron is moving at the same speed v as before. No change in the radius of curvature should happen according to Lorentz's force.

Ideally the charged spherical shell should be made of a non-conducting material. Although the uniformly charged spherical shell exerts no force on internal test charges according to Lorentz's force, the internal test charge may affect the shell if it is metallic (electrostatic induction due to image charges, Foucault currents if the internal charge is in motion, etc.) This would disturb the distribution of surface charges in the shell so that the internal electric and magnetic fields would not be zero anymore. And this might mask the effect which is under investigation (a possible effective inertial mass of internal charges due to surrounding charges). To avoid this it would be advisable to work with charged dielectric shells. But usually it is difficult to charge uniformly dielectric shells to high potentials. For this reason we consider here the effect on the internal test charge due to image charges related with a metallic shell. By the method of images it is possible to estimate the force exerted on an internal test charge q at a distance $r < R$ from the center of the metallic shell of radius R due to electrostatic induction on the shell (although the total charge on the shell will be zero, the internal charge will cause a redistribution of charges on the surface of the shell, which will produce an electrostatic force on the internal charge). By standard electrostatic theory this force is given by

$$F = \frac{q^2}{4\pi\varepsilon_0} \frac{1}{R^2} \frac{r/R}{[1-(r/R)]^2} \ . \tag{13}$$

This force is relevant only when the test charge is close to the shell. If we have a shell of 1 meter radius and the test charge is an electron $(q = -1.6 \times 10^{-19}$ C), then if $r \leq 0.5$ m this force will be at most of the order of 10^{-28} N. If the electron is describing a circular orbit (Larmor radius) due to a magnetic field B of the order of 0.01 to 1 T, with typical velocities ranging from 10^4 to 10^7 m/s, the magnetic force will be of the order of 10^{-17}

to 10^{-12} N, many orders of magnitude larger than that due to electrostatic induction. This shows that the experiment can be performed with metallic spherical shells provided the whole setup is close to the center of the shell, away from its surface.

We now discuss a very recent experiment which looked for this effect (the first experiment of this kind known to us). Mikhailov published a very important paper showing that the inertia of electrons is changed when they are placed inside a charged Faraday cage [37]. He inserted a neon glow lamp RC-oscillator inside a cage which was charged from -3×10^3 V to $+3 \times 10^3$ V. The capacitor C was charged and discharged continually, for each value of the surrounding electrostatic potential of the shell. The oscillation period of the circuit was observed to vary linearly with the electrostatic potential of the surrounding shell. In principle this effect should not appear according to classical electromagnetism as there are no electric nor magnetic fields inside the stationary charged shell (the region inside the metallic shell is an equipotential). For this reason the circuit and its period of oscillation should not be affected by the surrounding charges.

The period of oscillation is a sum of the discharge time of the capacitor C and of its charge time. In his experiment $C = 0.0051$ μF is the capacitance and $R = 1.8$ MΩ is the resistance of the charging circuit and R_i the interior resistance of the neon lamp. The charging time is of the order of $RC = 9 \times 10^{-3}$ s (as a matter of fact the total period T_0 in his experiment was 7×10^{-3} s). The main aspect is that the period of oscillation is proportional to a characteristic resistance of the circuit which is inside the cage. By Drude's classical relation, which should work reasonably well under these conditions, the resistance is proportional to the electron's mass. This means that Mikhailov's result can be interpreted as showing that the effective inertial mass of the electrons depends on the surrounding charges.

An effect like this had been predicted back in 1992 ([35] and [36]). If $qQ > 0$ the inertia of the test particle should decrease, increasing if $qQ < 0$ (q is the charge of the internal test electrons and Q is the total charge in the spherical shell). All of this was observed by Mikhailov by varying the potential of the cage and by working with positive and negative potentials. Also the order of magnitude of the detected effect coincided with the prediction of Weber's electrodynamics.

In conclusion we can say that if Mikhailov's result is confirmed by independent researches, it 'may well be a landmark,' as described by Costa de Beauregard and Lochak [38].

Acknowledgements

The authors wish to thank Proyecto Internacionalizacion Universidad de Tarapaca for financial support.

References

[1] A K T Assis, *Weber's electrodynamics* (Kluwer Academic Publishers, Dordrecht, 1994) ISBN: 0-7923-3137-0

[2] A K T Assis, *Relational mechanics* (Apeiron, Montreal, 1999) ISBN: 0-9683689-2-1

[3] A O'Rahilly, *Electromagnetic theory — A critical examination of fundamentals* (Dover, New York, 1965)

[4] J M Pearson and A Kilambi, *Am. J. Phys.* **42**, 971–975 (1974)
[5] W F Edwards, C S Kenyon and D K Lemon, *Phys. Rev.* **D14**, 922–938 (1976)
[6] V V Batygin and I N Toptygin, *Problems in electrodynamics* (Academic Press, London, 1964)
[7] J D Jackson *Classical electrodynamics* (John Wiley, New York, second edition, 1975)
[8] When this was written, I [Maxwell] was not aware that part of M Weber's Memoir is translated in Taylor's *Scientific Memoirs*, vol. V Art. XIV. The value of his researches, both experimental and theoretical, renders the study of his theory necessary to every electrician
[9] J C Maxwell, *On Faraday's lines of force* edited by W D Niven, *The scientific papers of James Clerk Maxwell* (New York, Dover, 1965) vol. 1, p. 155–229. Article originally published in 1855
[10] J C Maxwell, *The scientific papers of James Clerk Maxwell* edited by W D Niven (New York, Dover, 1965) vol. 1, pp. 526–597. Article originally published in 1864
[11] H von Helmholtz, *Scientific Memoirs* edited by J Tyndall and W Francis (New York, 1966) vol. 7, p. 114–162, Johnson Reprint Corporation
[12] P M Harman *Energy, force, and matter – The conceptual development of nineteenth-century physics* (Cambridge University Press, Cambridge, 1982)
[13] J C Maxwell, *A treatise on electricity and magnetism* (Dover, New York, 1954)
[14] Pogg. Ann., lxxiii. p. 229 (1848)
[15] J J Caluzi and A K T Assis, *Foundations of physics* **27**, 1445–1452 (1997)
[16] Lezioni Accademiche (Firenze, 1715) p. 25
[17] J C Maxwell, *The scientific papers of James Clerk Maxwell* edited by W D Niven. Dover (New York, 1965) vol. 2, p. 125–143. Article originally published in 1868
[18] G Kirchhoff, *Philos. Mag.* **13**, 393–412 (1857)
[19] P Graneau and A K T Assis, *Apeiron* **19**, 19–25 (1994)
[20] L Rosenfeld, *Il. Nuovo Cimento*, Suppl. **4**, 1630–1669 (1957)
[21] F Kirchner, *Am. J. Phys.* **25**, 623–629 (1957)
[22] L Rosenfeld, Kirchhoff and Gustav Robert, *Dictionary of scientific biography* edited by C C Gillispie (Scribner, New York, 1973) vol. 7, p. 379–383
[23] C Jungnickel and R McCormmach, *Intellectual mastery of nature – theoretical physics from Ohm to Einstein* (University of Chicago Press, Chicago, 1986) vol. 1–2
[24] S D'Agostino, *Physis* **33**, 5–51 (1996)
[25] A K T Assis, *Proc. of the Int. Conf. Galileo Back in Italy II* edited by R Monti pp. 23–36, Bologna, 2000 Soc. Ed. Andromeda
[26] A K T Assis, *Hadronic J.* **13**, 441–451 (1990)
[27] Marcelo Bueno and A K T Assis, *Cálculo de Indutância e de Força em Circuitos Elétricos*. Editora da UFSC/Editora da UEM, Florianópolis/Maringá, 1998. ISBN: 85-328-0119-6
[28] E T Kinzer and J Fukai, *Found. Phys. Lett.* **9**, 457–461 (1996)
[29] J Fukai and E T Kinzer, *Galilean Electrodynamics* **8**, 53–55 (1997)
[30] Marcelo A Bueno and A K T Assis, *J. Phys.* **D28**, 1802–1806 (1995)
[31] A K T Assis and Marcelo A Bueno, *IEEE Trans. Magn.* **32**, 431–436 (1996)
[32] Marcelo Bueno and A K T Assis, *Can. J. Phys.* **75**, 357–362 (1997)
[33] Marcelo Bueno and A K T Assis, *Helv. Physica Acta* **70**, 813–821 (1997)
[34] Marcelo Bueno and A K T Assis, *Physica Scr.* **56**, 554–559 (1997)
[35] A K T Assis, *Commun. Theor. Phys.* **18**, 475–478 (1992)
[36] A K T Assis, *J. Phys. Soc. Jap.* **62**, 1418–1422 (1993)
[37] V F Mikhailov, *Annales de la Fondation Louis de Broglie* **24**, 161–169 (1999)
[38] O Costa de Beauregard and G Lochak, *Annales de la Fondation Louis de Broglie* **24**, 159–160 (1999)