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MODE CONVERSION OF GLOBAL MODES IN A UNIFORM CYLINDRICAL MAGNETIZED PLASMA

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ABSTRACT

The propagation of electromagnetic waves in a uniform magnetized plasma, bounded by a cylindrical conducting wall is studied using the two-fluid theory, with the pressure term included. Taking the cylindrical coordinates and perturbation of the form $f(r)\exp(ikz-in\theta-i\omega t)$ we obtain a sixth order equation for the electric field component $E_z(r)$. Its solution is a sum of three Bessel's or modified Bessel's functions. With appropriate boundary conditions, we obtain the dispersion relation which is solved numerically. The main result of this study is that the pressure term causes the mode conversion of a backward wave to another backward wave. The backward waves are modes which propagate in a frequency range between the plasma and upper-hybrid frequencies.

1. INTRODUCTION

In this paper we study the global modes of electromagnetic oscillation in a cylindrical waveguide. This is an important research topic not only for practical purposes (fusion devices) but also for basic research in electromagnetic theory [1]. We include the electron temperature in the Trivelpiece and Gould problem [2,3], and our model is not restricted to the slow wave cases ($\omega^2/k^2 \ll c^2$) so that a greater number of modes is analysed. One important result is that we generalize the dispersion relation of Ghosh and Pal [4].

II. THE MODEL AND DISPERSION RELATION

In our model the plasma is treated as an adiabatic fluid in which the ions are at rest (approximation valid in the high frequency limit, $\omega \gg \omega_{pi}$ and $\omega \gg \omega_{ci}$). We include a constant external magnetic field, $\vec{B}_o$, along the waveguide. We apply a linearization process in the form $f(\vec{r}) = f_o + f(r)\exp(ikz-im\theta-i\omega t)$, where k is the wave number, n is an integer, ω is the angular frequency of the electromagnetic field, and where we utilize cylindrical coordinates (the z axis of the coordinate system is the waveguide axis). In the absence of an equilibrium electrostatic fielc, $\vec{E}_o = 0$, and of an electron drift velocity, $\vec{u}_o = 0$, the first order equations to be solved are (equations of continuity, of momentum transfer and Maxwell's equations):

$$i\omega p_1 = n_o m U^2 \nabla \cdot \vec{u}_1,$$

$$i\omega n_o m \vec{u}_1 = n_o e(\vec{E}_1 + \vec{u}_1 \times \vec{B}_o) + \nabla p_1,$$

$$\nabla \times \vec{E}_1 = i\omega\mu_o \vec{H}_1,$$

$$\nabla \times \vec{H}_1 = -i\omega\epsilon_o \vec{E}_1 - n_o e \vec{u}_1,$$

where p_1, n_o, m, $U(=(\gamma k_B T_o/m)^{1/2})$, γ, k_B, T_o, $\vec{u}_1$, $-e$, $\vec{E}_1$, $\vec{H}_1$, μ_o and ε_o are, respectively, the perturbed pressure, fluid density, electron mass, electron thermal velocity, ratio of specific heats (usually γ = 5/3), Boltzmann's constant, electron temperature, perturbed fluid velocity, electron charge, perturbed electric and magnetic fields, vacuum magnetic permeability and vacuum dielectric constant. To obtain these equations we assumed also that the electron collision frequency is much smaller than the wave frequency ω.

Assuming that $\vec{B}_o = B_o\vec{z}$ we get from these equations [5-7]:

$$(\nabla_\perp^6 + b_1 \nabla_\perp^4 + b_2 \nabla_\perp^2 + b_3)E_z = (\nabla_\perp^2 + k_1^2)(\nabla_\perp^2 + k_2^2)(\nabla_\perp^2 + k_3^2)E_z = 0,$$

where

$$b_1 = 2k_e^2 + k_s^2 - \frac{\omega_c^2}{\omega^2}\frac{\omega^2 - k^2U^2}{U^2},$$

$$b_2 = k_e^4 + 2k_e^2k_s^2 - \frac{\omega_c^2}{\omega^2}\left[(k_e^2 + k_f^2)\frac{\omega^2 - k^2U^2}{U^2} + k^2\omega_p^2\frac{c^2 - U^2}{c^2U^2}\right],$$

$$b_3 = k_e^4k_s^2 - \frac{\omega_c^2}{\omega^2}k_f^2\left(k_e^2\frac{\omega^2 - k^2U^2}{U^2} + k^2\omega_p^2\frac{c^2 - U^2}{c^2U^2}\right).$$

and
$$\omega_p = \left(\frac{n_o e^2}{\epsilon_o m}\right)^{\frac{1}{2}}, \omega_c = \frac{eB_o}{m}.$$

$$k_f = \left(\frac{\omega^2}{c^2} - k^2\right)^{\frac{1}{2}}, \qquad k_s = \left(\frac{\omega^2 - \omega_p^2}{U^2} - k^2\right)^{\frac{1}{2}}.$$

$$k_e = \left(\frac{\omega^2 - \omega_p^2}{c^2} - k^2\right)^{\frac{1}{2}},$$

$$\nabla_\perp^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{n^2}{r^2}.$$

and k_1 , k_2 , k_3 are analytic functions of b_1, b_2 and b_3, obtained by Cardan's formula [8]. All other field components can be obtained in term of $E_z(r)$.

The solution of this equation is

$$E_z = A_n J_n(rk_1) + B_n J_n(rk_2) + C_n J_n(rk_3).$$

where $J_n(x)$ is the n-th-order Bessel function of first kind. Applying the boundary conditions $E_z(R) = E_\theta(R) = 0$, and $u_r(R) = 0$, where R is the radius of the waveguide, we obtain the general dispersion relation given by [5-7]:

$$\begin{aligned}
&n^2[F_1(L_2 - L_3) + F_2(L_3 - L_1) + F_3(L_1 - L_2)] + n[P_1(F_3 - F_2) - Q_1(L_3 - L_2)] \\
&\quad \times \frac{J_n'(Rk_1)}{J_n(Rk_1)} + n[P_2(F_1 - F_3) - Q_2(L_1 - L_3)]\frac{J_n'(Rk_2)}{J_n(Rk_2)} \\
&\quad + n[P_3(F_2 - F_1) - Q_3(L_2 - L_1)]\frac{J_n'(Rk_3)}{J_n(rk_3)} + (Q_1P_2 - P_1Q_2)\frac{J_n'(Rk_1)J_n'(Rk_2)}{J_n(Rk_1)J_n(Rk_2)} \\
&+ \ (Q_2P_3 - P_2Q_3)\frac{J_n'(Rk_2)J_n'(Rk_3)}{J_n(Rk_2)J_n(Rk_3)} + (Q_3P_1 - P_3Q_1)\frac{J_n'(Rk_3)J_n'(Rk_1)}{J_n(Rk_3)J_n(Rk_1)} = 0,
\end{aligned}$$

where

$$F_j = \frac{U^2[k_e^2(k_e^2 - k_j^2) + (k_+^2 - k_j^2)(k_-^2 - k_j^2)]}{Rk(c^2 - U^2)},$$

$$\begin{aligned}
P_j = &\frac{-k_j}{k\omega^2\omega_p^2 G(c^2 - U^2)}[\omega^2U^2(k_e^2 - k_j^2)(c^2G + k_c^2\omega_p^2) \\
&+ \ k^2\omega_p^2(c^2 - U^2)(\omega^2k_e^2 - \omega_c^2k_j^2) + U^2\omega^2\omega_p^2(k_+^2 - k_j^2)(k_-^2 - k_j^2)],
\end{aligned}$$

$$\begin{aligned}
L_j = &\frac{U^2}{Rk\omega\omega_c\omega_p^2 G(c^2 - U^2)} \\
&\times[\omega_c^2(k_e^2 - k_j^2)(c^2G + k_j^2\omega_p^2) + c^2(\omega^2k_e^2 - \omega_c^2k_j^2)(k_+^2 - k_j^2)(k_-^2 - k_j^2)],
\end{aligned}$$

$$\begin{aligned}
Q_j = &\frac{-k_j}{c^2(c^2 - U^2)\omega_p^2 k\omega\omega_c} \\
&\times[k^2\omega_c^2\omega_p^4(c^2 - U^2) + \omega_c^2k_j^2U^2\omega_p^2c^2(k_e^2 - k_j^2) + c^4U^2(\omega^2k_e^2 - \omega_c^2k_j^2)(k_+^2 - k_j^2)(k_-^2 - k_j^2)],
\end{aligned}$$

and where $j = 1, 2$ or 3.

III. NUMERICAL RESULTS

In figure 1 we show the dispersion relation for a magnetized plasma waveguide with radius R=0.085 m, plasma frequency $\omega_p = 1.2.10^{10} s^{-1}$, electron cyclotron frequency $\omega_c = 1.5.10^{10} s^{-1}$, electron temperature T_o= 40. eV and azimuthal wavenumber n=1. The figure shows the mode conversion pattern for values of the wave number k around 1/R. We also see the mode conversion for kR ~ 4. It is interesting to observe the occurence of the mode conversion at a low temperature.

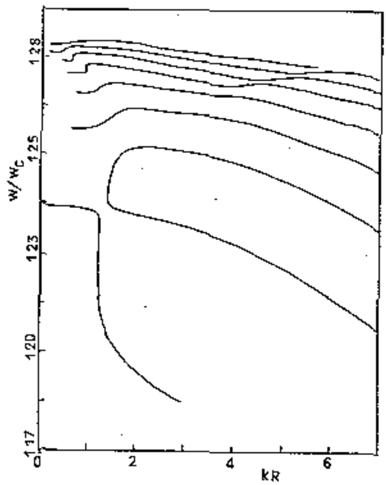

Fig. 1

Dispersion Relation

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