THE INFLUENCE OF THE ELECTRIC FIELD OUTSIDE A RESISTIVE SOLENOID ON THE AHARONOV-BOHM EFFECT[1]

Héctor Torres S.[2] Andre K.T. Assis[3]

ABSTRACT

It is a known fact that there is no magnetic field outside an infinite solenoid carrying a constant current, although there is magnetic vector potential non-null outside it. The existence of the Aharonov-Bohm effect (AB) is usually considered as proof of the relevance of the vector potential for quantum mechanics. In this paper we will see that there is a non null electric field outside an infinite resistive solenoid carrying a constant current and its possible relevance to the analysis of the Aharonov-Bohm effect. When calculating, we introduce the Galilean invariance of Maxwell's equations and then we obtain the contribution to the AB effect due to electric potential.

INTRODUCTION

In classical electromagnetism there are the concepts of the scalar electric potential and the magnetic vector potential. Despite this fact, the relevant quantities are the electric and magnetic fields as they are the ones which appear in Maxwell's equations and in Lorentz' force law. On the other hand, in quantum mechanics these potentials are relevant, as they appear in the Hamiltonians, which describe electromagnetic interactions between charged particles.

In 1959 Aharonov and Bohm proposed an experimental test of the relevance of the magnetic vector potential in a region free of the electromagnetic fields, [1]. As an example, suppose an infinite solenoid along the z-axis carrying a constant poloidal current in the ϕ direction.

It generates a constant magnetic field $\vec{B}$ at all points in its interior pointing along the z-axis and no magnetic field outside it. Despite this fact, there is a magnetic vector potential $\vec{A}$ outside the solenoid pointing in the ϕ direction.

The Aharonov-Bohm effect was first experimentally confirmed by Moellenstedt and Bayh, [2]. Since then this effect has been considered as a proof of the essential importance of the magnetic vector potential for quantum mechanics. This is a curious result from the classical point of view, as the magnetic force on any charged particle is zero.

Here we wish to call attention to a fact, which has always been neglected in this connection. If the solenoid mentioned above is made of a normal resistive wire like copper and the current is generated by a battery, there will be an external electric field, although the external magnetic field is zero, [3]. This electric field will exist outside the solenoid, even in the case of stationary solenoid carrying a constant current.

Jefimenko was able to show experimentally the existence of electric field outside conductors carrying constant currents utilizing grass seeds, which align themselves with the lines of electric field, in analogy with iron fillings, which map magnetic fields, [4.5]. He also measured potentials outside these current carrying wires utilizing an electronic electrometer connected to a radioactive alpha-source, [6]. Historical analysis of this whole subject has been presented in [7].

The case of the solenoid was considered theoretically by Heald, [3]. He was able to obtain analytic expressions for the electric potential inside and outside the solenoid in the case, in which there is a "line" battery driving current azimuthally in a uniform cylindrical resistive sheet. From this potential the electric field can be easily obtained. This electric field has radial and poloidal components. The important aspect is that the poloidal component follows the direction of the current just outside the solenoid, as the magnetic vector potential mentioned above. Considering the typical configuration of the Aharonov-Bohm effect, we will have an electron beam moving in the general direction of the electric field and the other electron beam moving opposite to

[1] This work was supported by proyecto Internacionalización Universidad de Tarapacá and Fondecyt 1010300, Chile
[2] Universidad de Tarapacá, Av. 18 de Septiembre 2222, Arica, Chile, e-mail: htorres@uta.cl
[3] Instituto de Fisica "Gleb Wataghin", Universidade Estadual de Campinas, 13083-970 Campinas, Sao Paulo, Brasil, e-mail: assis@ifi.unicamp.br.

the electric field. In the first case the electrons will be retarded by the electric field, while in the second case they will be accelerated by it. This might be relevant to the interpretation of the Aharonov-Bohm effect.

Our goal is to call attention that the region outside an infinite solenoid is not free of electromagnetic fields, as usually supposed and how it can be calculated. Although the external magnetic field is zero, the electric field is different from zero and it can play a role in the Aharonov-Bohm effect. To do that, first we derive the Galilean invariance of Maxwell's equations [8] [9], and then we obtain the force due to the magnetic vector potential and the electric potential. From the framework of classical electrodynamics we postulate this invariance, because it may give forces, which are of longitudinal type, i.e. is directed along motion of the charge, oppositely to Lorentz force, which is of transversal type [9].

Dimensional analysis of Maxwell's equations implies a wave propagation speed of c, defined as $(\mu_0\varepsilon_0)^{-\frac{1}{2}}$ But Maxwell's equations in and of themselves say nothing about the value of c in any particular observer's frame of reference. The generally accepted frame-invariance of c, and hence μ_0 and ε_0, constitutes an *assumption*. Lorentz transformations allow the preservation of the form of Maxwell's equations in any inertial frame of reference (IFR) under this assumption, raised to the status of a postulate by Einstein. It is likely only the experimental means by which we measure c, μ_0 and ε_0, that produces the observed invariance of light's velocity.

By the principle of equivalence, any experiment performed in a uniformly moving reference frame should produce the same results as if performed in a "stationary" frame. Unless one is willing to assume the existence of an ether or preferred reference frame, all experiments will result in a measured "velocity" of c in any uniformly moving frame of reference, regardless of the actual behavior of the light itself. Observers in different IFRs measuring values for μ_0 and ε_0 will each obtain the same result. Thus each of several observers in different IFRs will measure the velocity of light from a distant source to be intersecting their apparatus at a velocity of c.

In light of the above, the second postulate can be modified to state: "The *observed* velocity of light is constant from all inertial frames of reference, and is independent of the motion of the source".

Electromagnetic radiation propagates from its source at all velocities from zero to some undetermined upper value C. Only that component of this radiation, that passes a physical observer at a relative velocity of c in the observer's frame of reference, produces physical interaction and is detected. Any observer in motion relative to the first observer will, in general, detect a different component of the radiation, that component being the one that has a relative velocity of c in its frame of reference.

THE GALILEAN INVARIANCE OF MAXWELL'S EQUATIONS

One considers three quantities, length, time and the speed of electromagnetic (EM) propagation in transforming Maxwell's equations between reference frames. Einstein assumed the velocity of EM propagation to be strictly c, requiring the Lorentz transformations to keep the form of Maxwell's equations consistent. This was at the expense of standard concepts of length, time, and simultaneity; each becoming distorted to accommodate the constancy of c. RCM simply adds the word *observed* to the second postulate, and derives the Galilean invariance of Maxwell's equations.

In Fig. 1, observers in the stationary (K) and moving (K') frames are at the origins S and S' respectively. The origins are initially coincident at the time of a flash at A, a distance x from S. In RCM, where $0 \leq c \leq C$, the component velocity of light in the non-moving K frame is c. As measured in the K' frame (moving with a constant velocity v), $c' = c$, but in the K frame c' is $c + v$. Restricting motion of the K' frame to the x axis, the Galilean transformations become:

$$x' = x + vt\ ;\ y' = y\ ;\ z' = z \tag{1}$$

$$t' = t \tag{2}$$

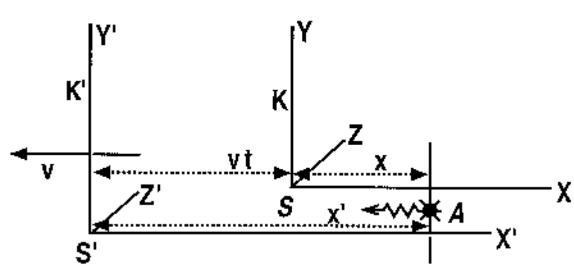

Fig.1.- Galilean systems: Observers in the stationary (K) and moving (K') frames are at the origins S and S' respectively

From a treatment of wave mechanics, for wave propagation in the x direction with $E_Z = 0$ and c the velocity of propagation, we write:

$$\frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_y}{\partial x^2} \qquad (3)$$

If a flash of light occurs some distance x from the origin of K, we can let x be represented by c times the time it takes light to reach an observer at K's origin. Thus, we derive the following relations:

$$x = ct; \quad x' = x + vt = ax \qquad (4)$$

$$a = (c+v)/c \qquad (5)$$

Combining equations (1) and (4), we obtain the following useful relations, where we can pull α out of the partial as a constant:

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t}; \qquad \frac{\partial}{\partial x'} = \frac{\partial}{\partial \alpha x} = \frac{\partial}{\alpha \partial x} \quad (6)$$

We can also derive two useful relations from (4), whereby we express ∂x in terms of ∂t and c, where the last expression holds since $t' = t$:

$$\begin{aligned} &\partial x = c\partial t; \\ &\partial x' = \partial \alpha x = \\ &\alpha \partial x = \alpha c \partial t = c' \partial t = c' \partial t' \end{aligned} \qquad (7)$$

Now we wish to examine the wave equation for the same wave in the K' system. We have:

$$\frac{1}{c'^2}\frac{\partial^2 E_{y'}}{\partial t'^2} = \frac{\partial^2 E_{y'}}{\partial x'^2} \qquad (8)$$

Substituting (6) into (8) and comparing with (2) yields:

$$\begin{aligned} &\frac{1}{c'^2}\frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_y}{\alpha^2 \partial x^2}, \quad or \\ &\frac{\alpha^2}{c'^2}\frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_y}{\partial x^2} \end{aligned} \qquad (9)$$

Equation (9) implies:

$$\frac{\alpha^2}{c'^2} = \frac{1}{c^2}, \quad or \quad c' = \alpha c = \frac{c+v}{c} c = c + v \; (10)$$

Equations (9) and (10) demonstrate the frame invariance in going from the K frame to K', provided that the velocity observed in K', as me assured in K, is $c + v$. This wave has a velocity as observed in K' of c, as required by experiment. The wave we are considering must have a velocity with respect to the source of c plus the velocity, v, of the K' system. Since v can assume any value, the light must leave the source in a continuum of velocities such that $0 \leq c \leq C$, where we place no constraints on the upper bound of C.

One interesting consequence of the SR Lorentz group is the invariance of the metric:

$$\begin{aligned} &c dt'^2 - dx'^2 - dy'^2 - dz'^2 \\ &= c^2 dt^2 - dx^2 - dy^2 - dz^2 \end{aligned} \qquad (11)$$

However, since c' is forced to transform into c, the left side of equation (10) could simply begin with c'^2 and such a statement then holds under a Galilean transformation, where we use the substitution dx = cdt.

$$\begin{aligned} &c' dt'^2 - dx'^2 - dy'^2 - dz'^2 \\ &= (c^2 dt^2 + 2vc dt^2 + v^2 dt^2) \\ &- (dx^2 + 2v dx dt + v^2 dt^2) - dy^2 - dz^2 \\ &= c^2 dt^2 - dx^2 - dy^2 - dz^2 \end{aligned} \qquad (12)$$

Now we consider the transformation of Maxwell's equations to ensure that the assumed wave equation of (8) is actually valid. Maxwell's equations may be expressed as:

$$div\, \mathbf{E} = 4\pi\rho, \qquad curl\, \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad (13)$$

$$div\, \mathbf{B} = 0, \quad curl\, \mathbf{H} = -\frac{1}{c}\frac{d\mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{J} \qquad (14)$$

J is a vector quantity of current density, equal to the net amount of positive charge crossing a unit area of surface per second. Using the Galilean transformation, the transformation of $\boldsymbol{J'}$ is as follows:

$$J' = \rho v', \quad J_{x'} = \rho v_{x'} = \rho \alpha v_x = \alpha J_x ,$$
$$J_{y'} = J_y , \quad J_{z'} = J_z \tag{15}$$

All that remains is to show that primed equations in K' remain form invariant under the Galilean transformations of (1). We will demonstrate the transform for one quantity. All other equations transform similarly:

$$-\frac{\partial H_{z'}}{\partial y'} = \frac{1}{c'}\frac{\partial D_{x'}}{\partial t'} - \frac{4\pi}{c'} J_{x'} \Rightarrow$$
$$-\frac{\partial H_z}{\partial y} = \frac{\partial D_{\alpha x}}{\alpha c \partial t} - \frac{4\pi}{\alpha c}\alpha J_x \tag{16}$$
$$= \frac{\partial D_{\alpha x}}{\partial \alpha x} - \frac{4\pi}{c} J_x = \frac{\partial D_x}{\partial x} - \frac{4\pi}{c} J_x$$
$$= \frac{1}{c}\frac{\partial D_x}{\partial t} - \frac{4\pi}{c} J_x$$

Thus, we see that Maxwell's equations are indeed form invariant under the Galilean transformation we have proposed. Next we will compare the wave as observed in the K' system with that observed in the K system.

THE FORCE ON A MOVING CHARGE ABOVE A CURRENT CARRYING WIRE

We desire to solve the problem of the force on a moving charged particle above a neutral, current carrying wire without resorting to Lorentz transformations. We begin with the formula for the force on a moving particle outside an infinitely long current carrying wire:

$$\mathbf{F} = q\mathbf{v}_0 \times \mathbf{B} + q\nabla V \tag{17}$$

The magnetic force on a wire is due only to the movement of the charges in it, and thereby depends only on the total current, and not the amount of charge carried by each particle or even its sign. Thus we must be careful, in considering different reference frames, to keep track of both the positive and negative currents in the wire.

We define v_o as the velocity of the charged particle with respect to the mass of the wire carrying the current producing charge density ($\nabla V = 0$). Thus v_o will not change as we, the observers, change our reference frame. By convention, v_o is positive in the same direction as the flow of a current defined by moving negative charges. We further define the velocity of the current due to moving negative charges in the frame of the wire as v. If we observe a wire moving opposite to the flow of a negative charge current with respect to our reference frame at a velocity of -2v, and a charge above that wire moving the same direction at a velocity of -1.5v with respect to our reference frame, then v_o will be equal to $-1.5v-(-2v) = v/2$ in the wire's reference frame. This is illustrated in Fig. 2.

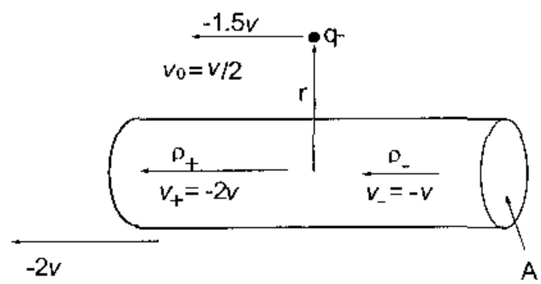

Fig. 2.- Flow of negative and positive charge densities in a wire with cross-section A

Generally, we attribute a current I in a wire stationary with respect to our reference frame to the motion of the negative conduction electrons, while the positive nuclear charges stay fixed with respect to the wire. Depending on our frame of reference, we have two currents, one due to the flow of the negative charge density, the other due to the flow of the positive charge density, each with respect to our IFR. In the following expression for the total current in the wire, v_- and v_+ are the velocities of the negative and positive charge densities, respectively, with respect to our reference frame, and A is the cross-sectional area of the wire:

$$I = (\rho_- v_- + \rho_+ v_+)A \tag{18}$$

In (17) we replace **B** with the equation for the field at a distance r due to a current I:

$$F = \frac{1}{4\pi\varepsilon_o c^2}\cdot\frac{2Iqv_o}{r} \tag{19}$$

Substituting (18) into (19) yields the expression for the force on a charged particle moving above a current carrying wire. An example shows that (20) is valid when viewed from any inertial frame of reference.

$$F = \frac{1}{4\pi\varepsilon_o c^2}\cdot\frac{2qA(\rho_+ v_+ + \rho_- v_-)v_o}{r} \tag{20}$$

In Fig. 2, we are moving at a velocity with respect to the wire of $2v$. Thus the total current is given by:

$$\begin{aligned} I &= \{\rho_+(-2v)+\rho_-(-v)\}A \\ &= \{\rho_-(2v)-\rho_-(v)\}A = \rho_- vA \end{aligned} \tag{21}$$

This current is the same current we observe when stationary with respect to the wire. The velocity of the charged particle with respect to the wire is $v/2$. Thus the force on the charged particle is given by:

$$\begin{aligned} F &= \frac{1}{4\pi\varepsilon_o c^2}\cdot\frac{2qA(\rho_+ v_+ + \rho_- v_-)v_o}{r} = \\ &\frac{1}{4\pi\varepsilon_o c^2}\frac{2qA(\rho_- v)v/2}{r} = \frac{qAv^2\rho_-}{4\pi\varepsilon_o c^2 r} \end{aligned} \tag{22}$$

The force on a moving charged particle due to a current carrying wire is the same regardless of the reference frame of the observer. More importantly, the force is a magnetic force in all frames of reference. In SRT the length contraction experienced by one charge density (the one arbitrarily chosen to be in motion by our choice of reference frame), but not the other causes an increased positive charge density and the wire becomes charged. This is unsettling at best. As we move from one reference frame to another, we see what was a magnetic field effect vanish while an electric field arises, and a neutral wire acquires excess positive charge. Equally confusing is that the velocity of the particle, v_o, is measured with respect to our arbitrary frame of reference, rather than the frame of the current carrying wire.

Now, if we apply these concepts to an infinite solenoid along the z-axis carrying a constant poloidal current in the ϕ direction, it generates a constant magnetic field $\vec{B}$ at all points in its interior pointing along the z-axis and no magnetic field outside it. Despite this fact there is a magnetic vector potential $\vec{A}$ outside the solenoid pointing in the ϕ direction given by.

$$A_\varphi = B\frac{a^2}{2r}, \quad A_r = 0, \quad A_z = 0 \tag{23}$$

where B is the field inside the solenoid.

The Schrodinger equation for an electron in a magnetic field is given by

$$\frac{1}{2m}(\overline{p} - q\vec{A})\Phi = E\Phi \tag{24}$$

so in a region where $\overline{B} = \nabla x\vec{A}$ vanishes, the solution is

$$\Phi = \Phi_0 \exp\left[\frac{iq}{\hbar}\int_1^2 \vec{A}\bullet d\vec{l}\right], \tag{25}$$

where Φ_0 is the solution when $\vec{A} = 0$

The phase difference between any two paths with the same initial and final positions gives the AB effect

$$\Delta\Phi_m = \frac{q}{\hbar}\oint \vec{A}\bullet d\vec{l} \tag{26}$$

which represents the total magnetic flux contained in the solenoid. Thus we have the typical existence of the AB effect due to the vector potential A, but now we extended this result to a resistive solenoid, where we considered the contribution of the electrical potential V. Within the approach presented here we can obtain the force due to ∇V . The magnitude of the electric field outside the solenoid is

$$F_e = \frac{V_0}{\pi}\frac{a}{r\rho} \tag{27}$$

Now the change in phase due to the electric field is

$$\Delta\Phi_e = \frac{1}{\hbar}\int_{t_1}^{t_2}\int_{C_{12}} \overline{F_e}\bullet d\vec{l}dt \tag{28}$$

where r and ρ are the polar radii measured from the center (axis) and from the battery respectively. Here the particle of charge q and mass m is emitted at time t_1 with position C_1 and detected at the point C_2 at a later time t_2.The battery is located at $\phi = \pm\pi$ and its terminals are at potentials $\pm V_0/2$. This last equation is the main result, which in neutral systems is not considered. The electric Aharonov-Bohm effect founded here can be compared with the magnetic AB effect, (eqs. 26). For typical copper wire, if we take for a resistive solenoid: $B = \mu_0 Ni / l$ where N=1000 turns, l=10mts, a=1cm, ρ=3 cm, r=1cm, $V_0 = 1$ volt, we can estimate from equations (23), (26) and (28), that $\frac{\Delta\Phi_e}{\Delta\Phi_m} \leq 0.1$.

This shows that the electric field may be important when the magnetic contribution vanishes.

CONCLUSION

In this paper we have studied the Aharonov-Bohm effect, when there is a non null electric field outside an infinite resistive solenoid carrying a constant current To the calculation we introduce the Galilean invariance of Maxwell's equations and then we obtain the contribution $\Delta\Phi_e$ to the AB effect due to the electric potential. This analysis shows that the AB effect has not a purely mathematical origin and it has physically observable consequences.

REFERENCES

[1] Aharonov Y. and Bohm D.; "Significance of electromagnetic potentials in quantum theory", Physical Review 115, 485, 1959.

[2] Moellenstedt G. and Bayh W.; "Messung der kontinuierlizhen Phasetrschiebung im kraftfreiem Raum durch das magnetische Vektor Potential", Naturwissenschaften , pp. 49, 81,1962.

[3] Heald M. A.; "Electric fields and charges in elementary circuits", American Journal of Physics 52, 522, 1984.

[4] Jefimenko O.; "Demonstration of the electric fields of current-carrying conductors", American Journal of Physics 30, 19, 1962.

[5] Jefimenko O. D., Electricity and magnetism (Electret Scientific Company, Star City, Second edition, 1989).

[6] Jefimenko O., Barnett T. L. and Kelly W. H.; "Confinement and shaping of electric fieds by current-carrying conductors", Proceedings of the West Virginia Academy of Science, pp. 34, 163, 1962.

[7] Assis A. K. T., Rodrigues W. A. Jr. and Mania A. J.; "The electric field outside a stationary resistive wire carrying a constant current", Foundations of Physics 29, 729, 1999.

[8] Renshaw C. E.; "The Effects of Motion and Gravity on Clocks,"IEEE: Aerospace and Electronics Systems, Vol. 10, Number 10, October 1995.

[9] Assis A. K. T. and Torres Silva H.; "Comparison between Weber's electrodynamics and classical electrodynamics", Pramana Journal of Physics, Vol. 55, 393, 2000.