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The Influence of Ernst Mach in the Teaching of Mechanics

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ABSTRACT. We present Newton's main ideas for the formulation of classical mechanics as given in the *Principia*. Then we discuss Ernst Mach's criticisms of Newtonian mechanics as contained in his book *The Science of Mechanics*. We analyze the influence of Mach's ideas in the teaching of classical mechanics considering five representative textbooks: those of Kittel, Knight and Ruderman; Marion and Thornton; Symon; Feynman, Leighton and Sands; and Goldstein. We conclude that the influence of Mach's ideas has been very great, being incorporated in the textbooks, although not always with the deserved acknowledgment.

1. INTRODUCTION

Classical mechanics, as presented in modern textbooks, bears little resemblance with the way in which it was originally formulated. During the three hundred years since Isaac Newton (1642–1727) published, in 1687, his *Mathematical Principles of Natural Philosophy*, usually known by its first Latin name, *Principia* (Newton 1934), its formalism has changed and important concepts were reformulated. No one will find, for instance, the expression $\vec{F} = m\vec{a}$ in Newton's writing and, for those used to vectors and calculus, the geometrical language used by him demands an almost insurmountable level of effort.

On the conceptual side, rarely there will appear, in modern textbooks, references to Newton's absolute space and to his definition of mass. As substitutes for them we have nowadays, respectively, the distant celestial bodies and the operational definition of mass. The main influence for these changes, although seldom acknowledged, is to be found in Ernst Mach's criticisms of the *Principia*, as we intend to show in this paper.

2. NEWTONIAN MECHANICS

Newton's presentation of mechanics in the *Principia* begins with 8 definitions, a Scholium, the three laws of motion, 6 corollaries and then 3 books (the motion of bodies, the motion of bodies in resisting mediums, and the system of the world in mathematical treatment).

His first definition is that of quantity of matter, what we would call today the inertial mass of the body, which Newton defines as the product of the density and volume of the body. His second definition is the quantity of motion, or what we call today the linear momentum of the body, defined as the product of the inertial mass of the body by its velocity. He then defines the inertia of a body, an impressed force, a centripetal force, the absolute quantity of a centripetal force, the accelerative quantity of a centripetal force and the motive quantity of a centripetal force.

In the Scholium after these definitions he distinguishes between the absolute and relative concepts of time, space and motion: "Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year. Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies; and which is commonly taken for immovable space; such is the dimension of a subterraneous, an aerial, or celestial space, determined by its position in respect of the earth." (. . .) "Place is a part of space which a body takes up, and is according to the space, either absolute or relative." (. . .) "Absolute motion is the translation of a body from one absolute place into another; and relative motion, the translation from one relative place into another."

Newton knew the difficulties posed by the conception of absolute space, which he conceived as infinite, homogeneous and isotropic. Since parts of such a space are indistinguishable from one another by our senses, we usually refer motion to visible bodies: "And so, instead of absolute places and motions, we use relative ones; and that without any inconvenience in common affairs; but in philosophical disquisitions, we ought to abstract from our senses, and consider things themselves, distinct from what are only sensible measures of them."

Newton believed that the credibility of such an elusive notion such as absolute space could be enhanced by considering the centrifugal effects shown in circular motion, a point that he illustrated with a discussion of his famous bucket experiment: The surface of the water in a bucket is flat when both are at rest relative to the earth. However, when both are rotating together relative to the earth with a constant angular velocity, the surface of the water is concave (if we perform the calculations it is easy to show that it will be a paraboloid of revolution). According to Newton this behavior can only be due to the rotation of the water relative to absolute space in the second situation, and not relative to its rotation relative to the bucket, to the earth or to the distant stars: "The effects which distinguish absolute from relative motion are, the forces of receding from the axis of circular motion. For there are no such forces in a circular motion purely relative, but in a true and absolute circular motion, they are greater or less, according to the quantity of motion." (...) "And therefore this endeavor does not depend upon any translation of the water in respect of the ambient bodies, nor can true circular motion be defined by such translation."

Newton's efforts in defending the existence of absolute space can be understood when one considers that it had a logical function in his theory of motion, by establishing a conceptual requisite for the validity of the First Law. It provided the ultimate, as we call it today, inertial frame of reference, the ideal condition in which his laws of motion could be applied in an absolutely rigorous form, the background against which movement could be described in a rational way.

After presenting his laws of motion, Newton introduces the concept of an inertial frame (although not calling it by any name) in his fifth corollary: "The motion of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion." That is, we can apply Newton's laws of motion not only in absolute space, but also in any frame of reference moving with a constant velocity relative to it.

3. MACH'S IDEAS

The idea of absolute space was not taken lightly by Newton's contemporaries George Berkeley and Leibniz (Zylbersztajn 1994), but the most influential criticism were to come through the work of Ernst Mach (1838–1916) *The Science of Mechanics – a Critical and Historical Account of its Development* (Mach 1960), first edited in German in 1883.

Mach was an important philosopher-scientist at the turn of the century, who contributed to different areas of physics. He had a strong interest in education and founded and coedited, in 1887, what is probably the first science education journal published (Matthews 1994, pp. 95–99). He believed that science should be taught historically and wrote *The Science of Mechanics* from that point of view.

He didn't accept Newton's concepts of absolute space and time, considering them metaphysical obscurities. In the Preface of the seventh German edition of his book, published in 1912, he wrote: "The character of the book has remained the same. With respect to the monstrous conceptions of absolute space and absolute time I can retract nothing. Here I have only shown more clearly than hitherto that Newton indeed spoke much about these things, but throughout made no serious application of them. His fifth corollary contains the only practically usable (probably approximate) inertial system." Instead of Newton's absolute space Mach proposed the rest of matter in the universe (Mach 1960, pp. 336–337): "I have remained to the present day the only one who insists upon referring the law of inertia to the earth, and in the case of motions of great spatial and temporal extent, to the fixed stars."

In his critique of absolute space, Mach's efforts were directed at defusing Newton's argument based on the rotating bucket (Mach 1960, p. 284): "Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produce no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the earth and the other celestial bodies." That is, we don't need to invoke rotation of the water relative to an absolute space disconnected from all other matter in order to explain this experiment. Mach went a step further claiming that the centrifugal forces are real forces which appear in any frame of reference relative to which the set of distant stars is spinning. On page 274 we read: "Try to fix Newton's bucket and rotate the heaven of fixed stars and then prove the absence of centrifugal forces." And on page 284: "The principles of mechanics can, indeed, be so conceived, that even for relative rotations centrifugal forces arise."

Mach's relational views and his assumption about the connection between the overall distribution of matter in the Universe with the motion and, consequently the mass of every single body, became later known as Mach's Principle, and were of heuristic value for Einstein's development of his general theory of relativity.

He was also against Newton's definition of inertial mass (Mach 1960, p. 300): "Definition I is, as has already been set forth, a pseudo-definition. The concept of mass is not made clearer by describing mass as the product of the volume into the density, as density itself denotes simply the mass of unit volume. The true definition of mass can be deduced only from the dynamical relations of bodies." He had presented this dynamical definition in page 266: "All those bodies are bodies of equal mass, which, mutually acting on each other, produce in each other equal and opposite accelerations. We have, in this, simply designated, or named, an actual relation of things. In the general case we proceed similarly. The bodies A and B receive respectively as the result of their mutual action the accelerations $-\varphi$ and $+\varphi'$, where the senses of the accelerations are indicated by the signs. We say then, B has φ/φ' times the mass of A. If we take A as our unit, we assign to that body the mass m which imparts to A m times the acceleration that A in the reaction imparts to it. The ratio of the masses is the negative inverse ratio of the counter-accelerations. That these accelerations always have opposite signs, that there are therefore, by our definition, only positive masses, is a point that experience teaches, and experience alone can teach. In our concept of mass no theory is involved; "quantity of matter" is wholly unnecessary in it; all it contains is the exact establishment, designation, and determination of a fact." In order to apply Mach's definition we need to remember that the correct reference frame to consider the accelerations is the frame of fixed stars, as he pointed out before.

Mach's criticisms were deeply rooted in his philosophy which assumed that only sensations can be known and are real. He also assumed that the purpose of science is to describe and to relate appearances in the simplest possible way (see Blackmore 1972; Cohen and Seeger 1970). In the twenties his ideas became a reference for logical-positivism, a philosophy of science grounded on radical empiricism that, after being influential for decades, is now considered to be superseded. The same is true for his account of the historical development of mechanics.

On the other hand, one cannot fail to praise his critical sense and the influence of his ideas on the development of science (even today Mach's Principle is a matter of debate in cosmology) and, as we shall show, in the teaching of mechanics.

4. INFLUENCE OF MACH'S IDEAS IN THE TEACHING OF MECHANICS

We are now in a position to analyze Mach's influence in the modern teaching of classical mechanics. We consider 5 representative books for this analysis: Kittel et al. (1965), Marion and Thornton (1995), Symon (1971), Feynman et al. (1963) and Goldstein (1980).

We begin with the first book. Kittel et al. (1965) discuss the concept of an inertial reference frame in Chapter 3 of their book, when presenting Newton's laws of motion. They say that Newton's second law, force equals to the time derivative of momentum, is valid for an observer stationed in an unaccelerated reference frame, which they call an inertial reference frame, or inertial reference system. Later on they say: "it is an established convention to speak of the fixed stars as a standard unaccelerated reference system." On pages 63–64 they discuss absolute and relative acceleration and Newton's bucket experiment. They call by Mach's principle the conjecture that only acceleration relative to the fixed stars has any significance, as opposed to the Newtonian point of view that acceleration relative to absolute space disconnected from any matter is what is significant. According to them there are not confirmations neither objections to Mach's point of view and that some physicists, including Einstein, judged it attractive *a priori*, whilst others did not. For them Mach's Principle is a matter of speculative cosmology.

Their discussion of the mass concept appear in pages 79–80 and in Chapter 14. In this Chapter, for instance, they say: "We may determine the mass of a body by measuring the acceleration produced on it by a known force: $M_i = F/a$. The mass determined in this way is known as the inertial mass and is denoted by M_i."

The influence of Mach is very clear in this last definition of inertial mass (although they do not cite Mach in this connection) and in the discussion of inertial frames of reference as those unaccelerated relative to the fixed stars. The only mention of Mach's name, however, is on page 64 when referring to Mach's principle, but one of the main consequences of the principle, that inertial forces are real is not explicitly made. Moreover, the term "fictitious forces" is used when discussing movement in non inertial frames of reference, indicating the option for a non-Machian interpretation.

Marion and Thornton's (1995) book discuss the concepts of mass and of frames of reference in Chapter 2 of their book. When presenting the concept of inertial mass they assume almost literally Mach's idea: "To demonstrate the significance of Newton's third law, let us paraphrase it in the following way, which incorporates the appropriate definition of mass: III'. If two bodies constitute an ideal, isolated system, then the accelerations of these bodies are always in opposite directions, and the ratio of the magnitudes of the accelerations is constant. This constant ratio is the inverse ratio of the masses of the bodies." On pages 53 and 54 they discuss the frames of reference and make reference to the fixed stars: "We may, however, consider the "fixed" stars to define a reference frame that approximates an "absolute" inertial frame to an extent quite sufficient for our present purposes." They do not discuss Newton's bucket experiment.

Once more the influence of Mach is very clear. Although they quote Mach's book on page 49, they do not discuss any of his ideas and do not seem to be aware that they are in fact adopting many of his concepts.

Symon's (1971) book presents essentially Mach's definition of inertial mass in Section 1.3 of his book, not mentioning Mach's name in this connection. For example, they say: "Thus the ratio of the masses of any two bodies is the negative inverse of the ratio of their mutual accelerations, independently of the unit of mass chosen." His definition of an inertial coordinate system appears only on Chapter 13, when discussing the special theory of relativity. No mention is made of the fixed stars. But on the next Chapter on relativistic dynamics, especially on Section 14.8, Mach's points of view are clearly expressed. He points out that Ernst Mach proposed that the effects observed in accelerated frames may be ascribed to the acceleration relative to the rest of the matter in the universe. He concludes: "According to Mach, an inertial coordinate system is one in which the matter in the universe is not accelerated on the average."

Mach's influence could be seen once more but, as before, Mach's definition of inertial mass, although adopted, do not receive proper credit.

Mach's name is not mentioned in Feynman et al.'s (1963) book. They present Newton's second law of motion in Section 9.1, without any deep discussion of the mass concept, nor of the inertial frame of reference. However, in Section 15.1 they discuss the inertial frames of reference, quoting Newton's fifth corollary. They do not discuss the experimental fact that the best inertial frame know to us is the frame of distant matter, nor any possible relevance for this fact. In Section 16.1 they discuss absolute and relative velocities and accelerations. In particular they emphasize that in a closed laboratory we cannot detect our absolute velocity nor our relative velocity relative to the stars and nebulae around, but that we can detect our acceleration relative to the stars and nebulae. They also stress that this comes only from experience.

There is no clear influence of Mach's criticisms of Newtonian mechanics in this book, maybe because they do not discuss critically the concept of inertial mass.

Our last book is that of Goldstein (1980). The only mention of Mach's book is on page 30, in the suggested references to Chapter 1. He also does not present any critical discussion of the mass concept. On page 2 he defines an inertial system with the words: "A reference frame in which Eq. ($\vec{F} = d\vec{p}/dt$) is valid is called an inertial or Galilean system. Even within classical mechanics the notion of an inertial system is something of an idealization. In practice, however, it is usually feasible to set up a coordinate system that comes as close to the desired properties as may be required. For many purposes a reference frame fixed in the Earth (the "laboratory system") is a sufficient approximation to an inertial system, while for some astronomical purposes it may be necessary to construct an inertial system by reference to the most distant galaxies." There is no further discussion of the possible connection between the distant galaxies and the local properties of bodies (their mass etc.). There is also no discussion of Newton's bucket experiment. The reason for this lack of a deeper analysis can be found in the author's goal, summarized in page 1: "Basic to any presentation of mechanics are a number of fundamental physical concepts, such as space, time, simultaneity, mass, and force. In discussing the special theory of relativity the notions of simultaneity and of time and length scales will be examined briefly. For the most part, however, these concepts will not be analyzed critically here; rather, they will be assumed as undefined terms whose meanings are familiar to the reader."

Despite this fact, Mach's influence can be felt in Goldstein's discussion of inertial systems.

5. CONCLUSION

Mach's influence can be felt in almost all of these books, especially when discussing the dynamical operational definition of inertial mass (Mach's definition has been incorporated in almost all of them). Here Mach's ideas are directly relevant, although not a single one of them cited him as the originator of this definition of inertial mass.

As far as the matter of inertial frames of reference is concerned, the analysis of Mach's influence is more complex. On one end we find Feynman et al.'s (1963) book, that, apparently see the issue as unproblematic and, therefore, not deserving any questioning. On the other extreme there is Symon's (1971) book, where Mach's points of view are clearly expressed. It is worth noticing, however, that his ideas are left for Chapter 14, that deals with relativistic dynamics, a topic that is not studied in most courses of mechanics, for which this book is directed. One can ask why Mach's Principle is not introduced in earlier chapters of the book in connection with classical mechanics, since it was formulated as a critique to the foundations of Newton's theory.

Kittel et al. (1965) present a better critical discussion of the fundamentals of classical mechanics, including Newton's bucket experiment. Even so, the reality of the so called "fictitious forces", one of the main implications of Mach's Principle, is not explicitly mentioned. That certainly weakens the discussion and reduces the importance of Mach's criticism.

Marion and Thornton's (1995) book, as well as Goldstein's (1980), mention the far away stars as an inertial frame of reference, without any reference to Mach and his critical discussions. In doing so they show his influence, but only to a very limited extent. We would argue that they assume the far away stars frame of reference from an implicit Newtonian perspective: absolute space does exist, and the stars frame of reference is either still or is moving with constant (or nearly constant) velocity in relation to it. Our interpretation is that those textbooks dissociate, maybe for the lack of historical knowledge, the far away stars frame of reference from the critical context in which it was assumed by Mach, therefore "Newtonizing" this frame of reference.

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