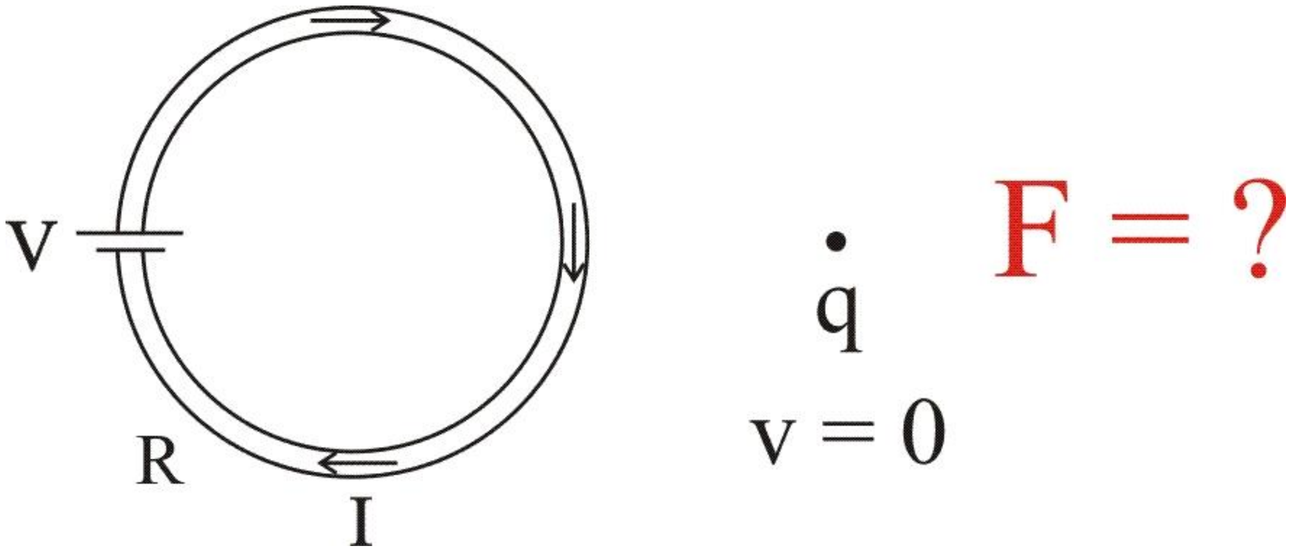
The Electric Force of a Current

Andre K. T. Assis
University of Campinas, Brazil

www.ifi.unicamp.br/~assis

A steady current I flows in a wire of resistance R connected to a battery of emf V. Consider an external charge q at rest relative to the circuit. Some simple questions:



- Does the circuit exert a force on the stationary external charge?
- Does the wire generate an external electric field?
- Is the wire neutral on all of its points?

J. C. Maxwell (Treatise of Electricity and Magnetism, 1873, article 848):

“Such an action has never been observed.”

R. Clausius (1877):

“We accept as criterion the experimental result that a constant current in a stationary conductor exerts no force on stationary charge.”

R. Feynman, The Feynman Lectures on Physics, Vol. 2, Section 13-6 (1964):

"In a normal conductor, like copper, the electric currents come from the motion of some of the negative electrons. (...) There is thus no electric field outside the wire."

J. D. Jackson, Classical Electrodynamics, 1975, 2nd edition, Exercise 14.13:

"For a real circuit the stationary positive ions in the conductors will produce an electric field which just cancels that due to the moving charges."

R. Skinner, Mechanics (1969):

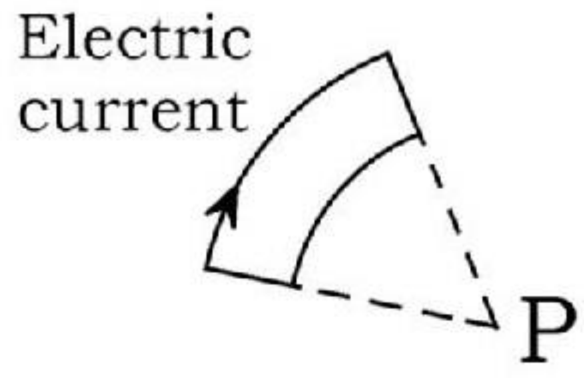
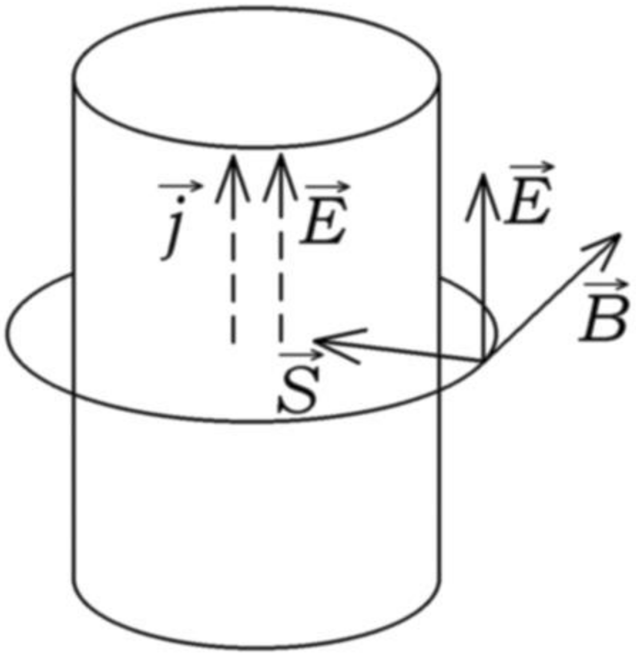


Figure caption:
"A crucial test of Weber's force law."

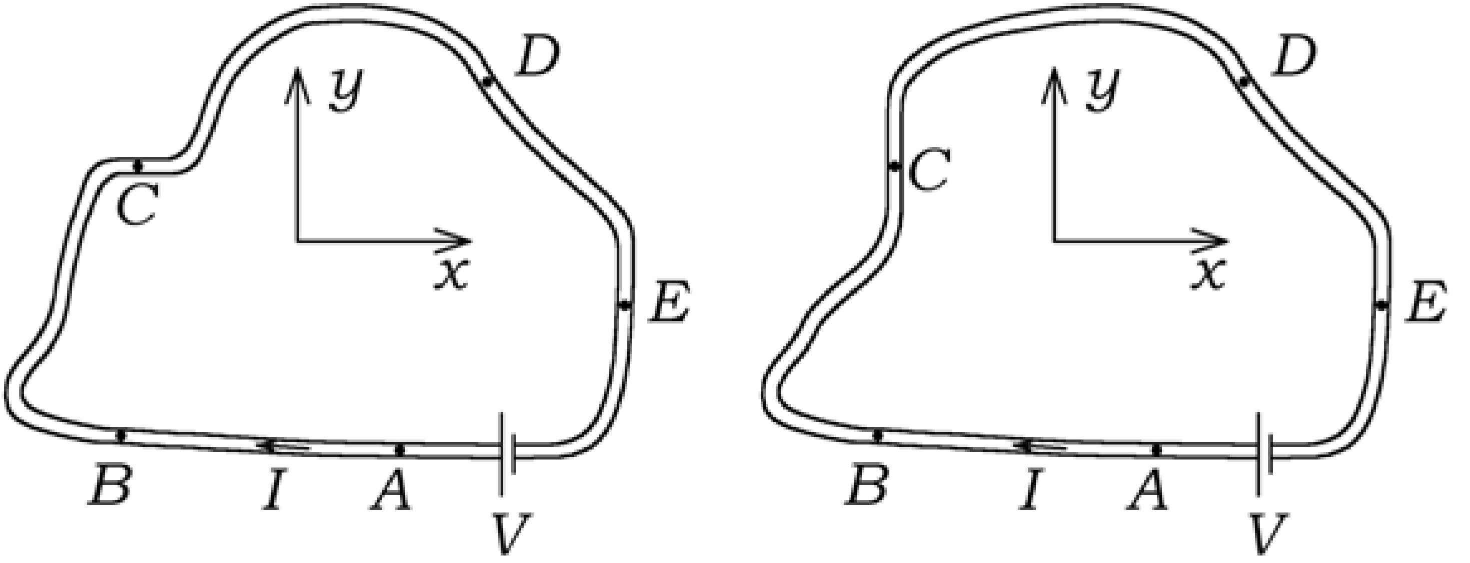
"According to Weber's force law, the current of this figure would exert a force on an electric charge at rest at point P. And yet a charge at P does not experience any force."

Contrary to the opinion of these authors, there are two main reasons for the existence of an electric field outside resistive wires carrying steady currents:

a) Continuity of the tangential component of the electric field E:

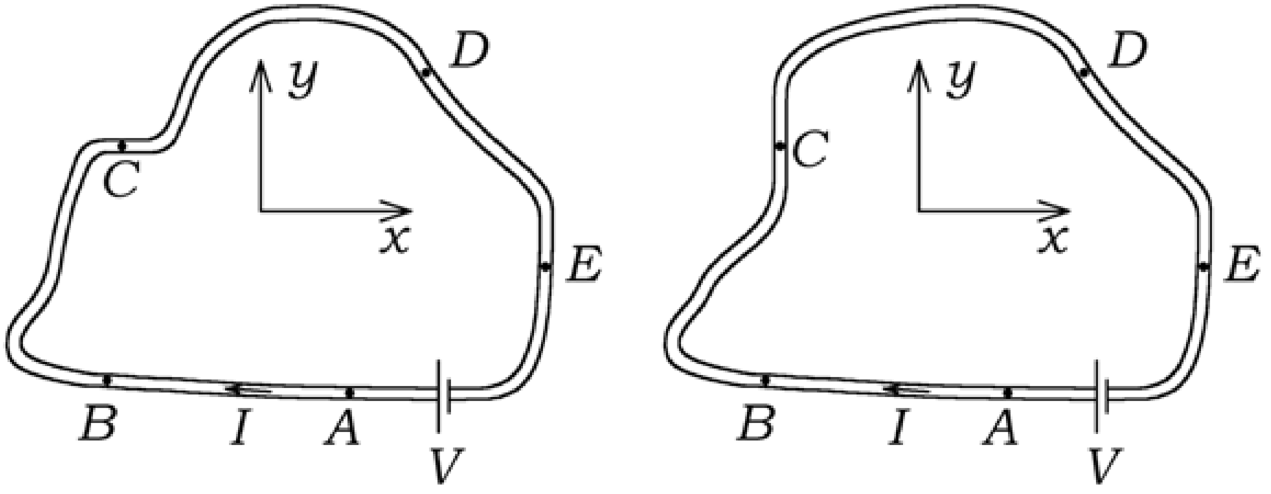


b) Consider the electric field at a point C located inside a current carrying wire in two configurations:



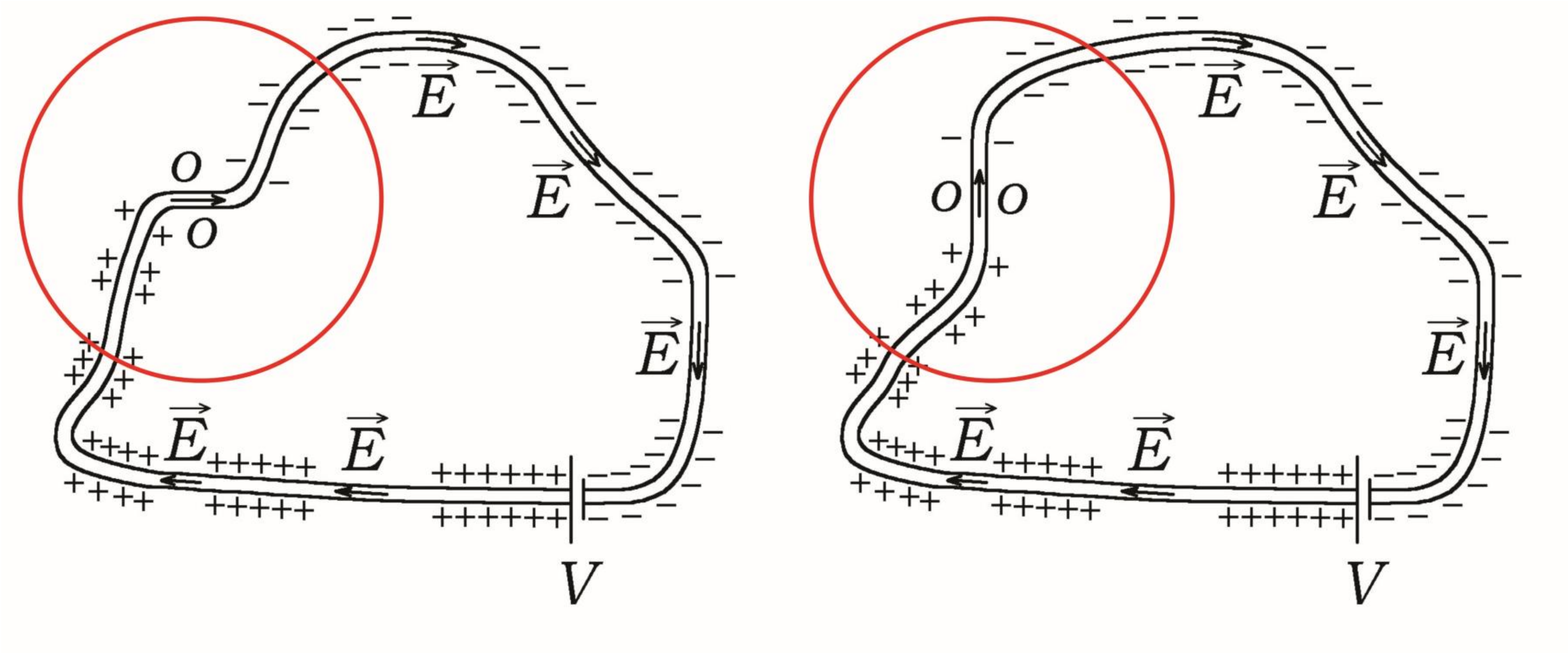
Ohm's law: $$\vec{E} = \rho \vec{J}$$

At the left side the electric field at point C is along the x direction, while at the right side it is along y.

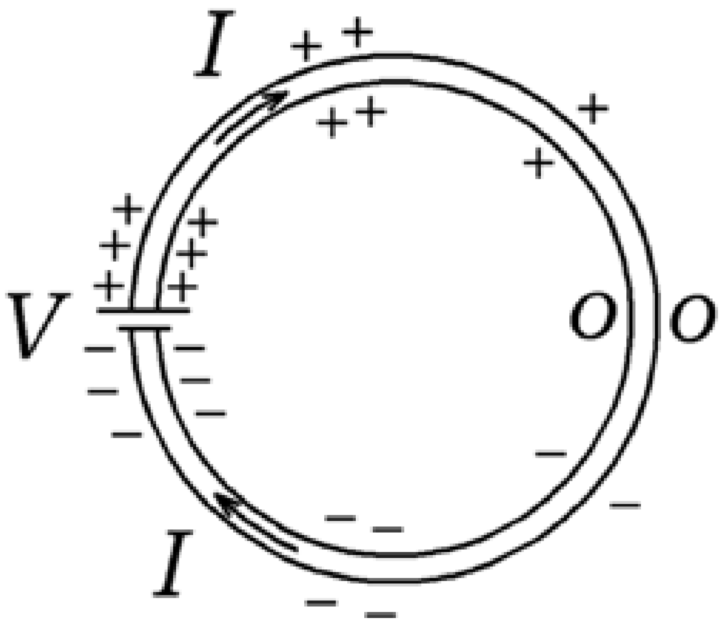


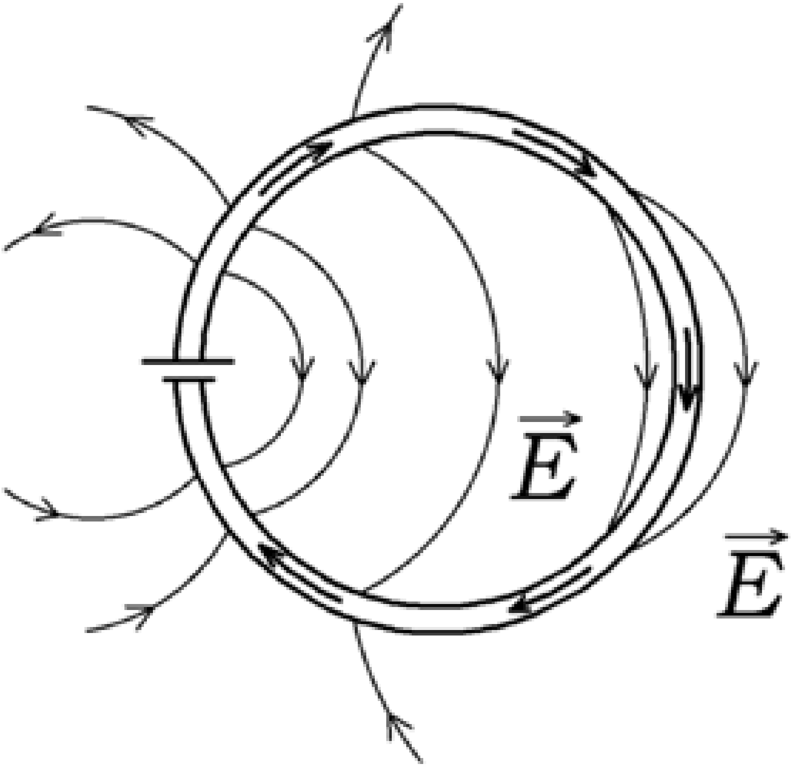
Therefore, the electric field at point C cannot be due only to the charges located at the battery. After all, the electric field at the points A, B, D and E closer to the battery points along the same directions in both situations (it does not change in these points, but only at point C).

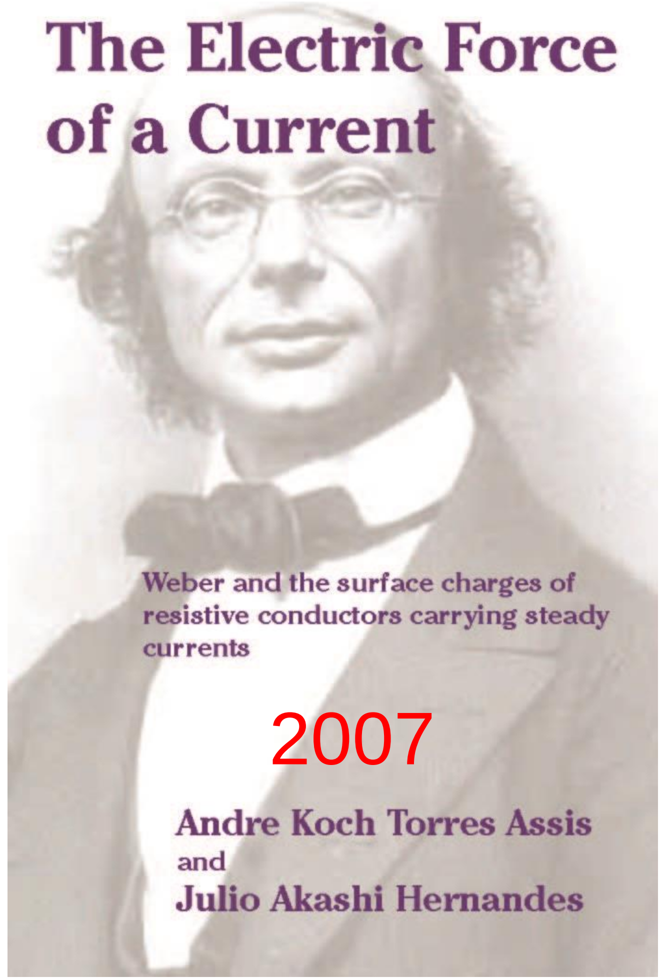
Weber and Kirchhoff proposed around 1850 that the electric field inside the wire is due to the presence of charges spread all over the surface of the wire. The charges creating the electric field in any point are essentially located close to this point:



Consequences of Weber's idea and essence of this work:







2007

Available in pdf format at

www.ifi.unicamp.br/~assis

There are 2 procedures to solve this problem

a) To give the charge density on the surface of the wire and then calculate the potential in space:

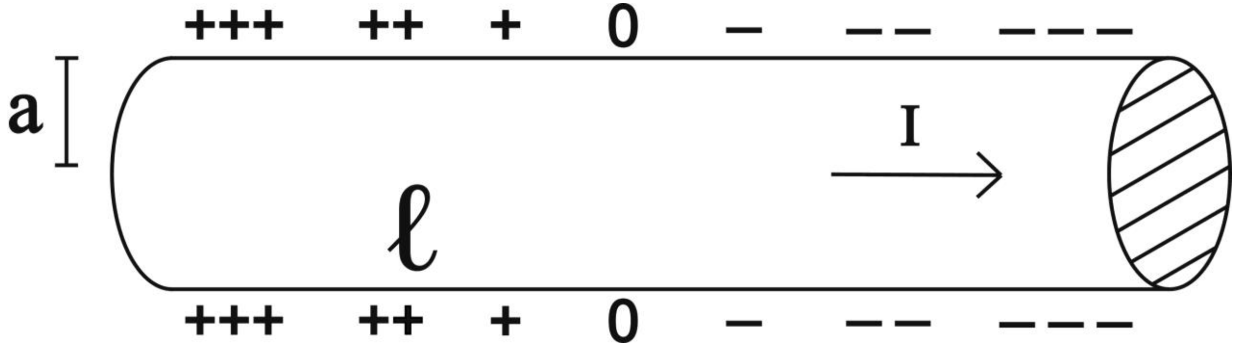
$$\phi(\vec{r}) = \frac{1}{4\pi\,\varepsilon_0}\frac{q'}{|\vec{r}-\vec{r}\,'|} = \frac{1}{4\pi\,\varepsilon_0}\iint\frac{\sigma(r')da'}{|\vec{r}-\vec{r}\,'|}$$

b) To give the potential on the surface of the wire and then calculate the potential in space through Laplace's equation:

$$\nabla^2\phi = 0$$

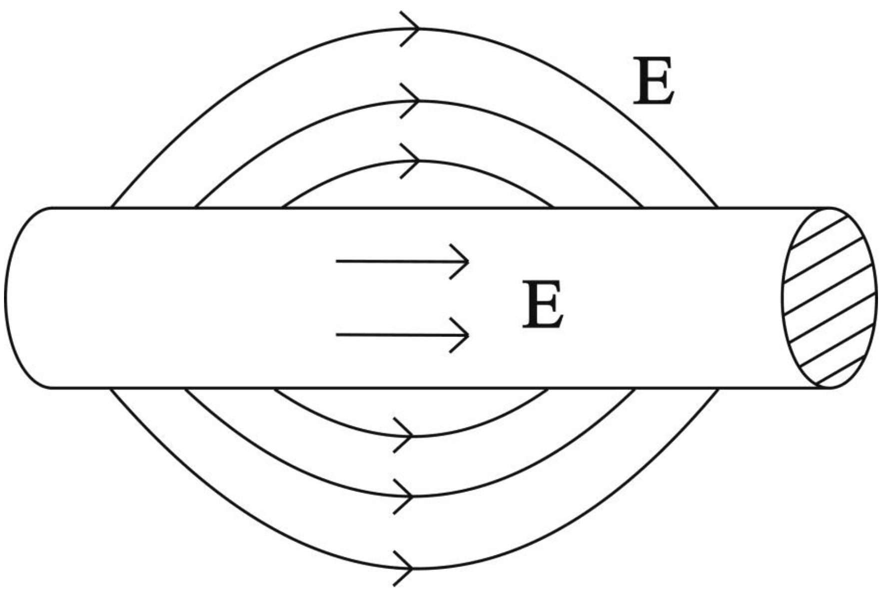
Wilhelm Weber in 1852 was the first to obtain a solution for the density of surface charges in the case of a straight wire carrying a steady current:

$$\sigma = -\sigma_0 \frac{z}{\ell}$$

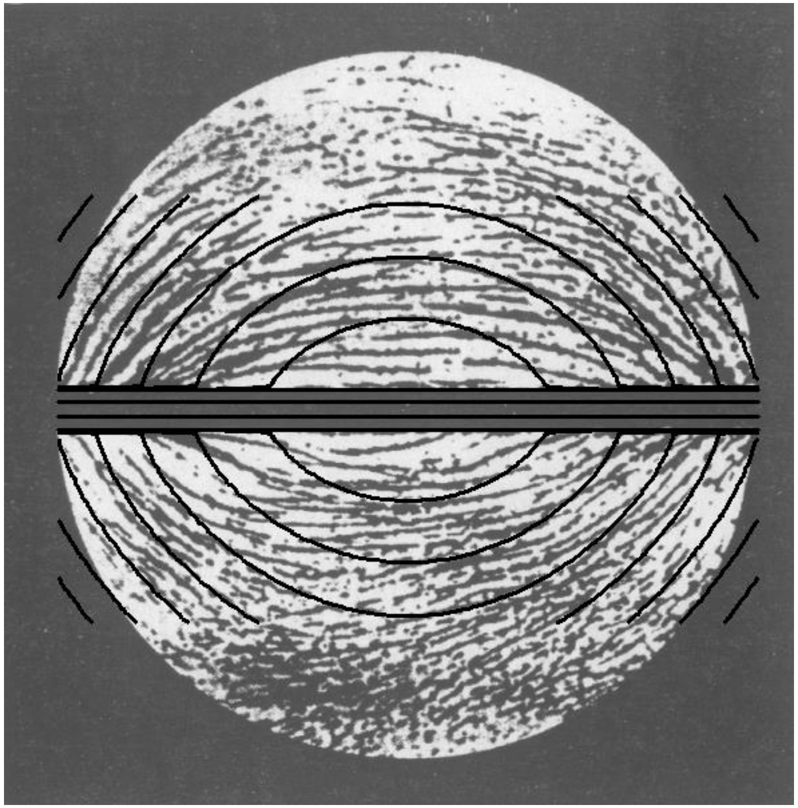


$$\vec{E}_{inside} = E_0 \hat{k} = \frac{\Delta V}{\ell} \hat{k}$$

$$\vec{E}_{outside} = \frac{\Delta V}{\ell \ln \frac{\ell}{a}} \left(\ln \frac{\ell}{\rho} \hat{k} - \frac{z}{\rho} \hat{\rho} \right)$$

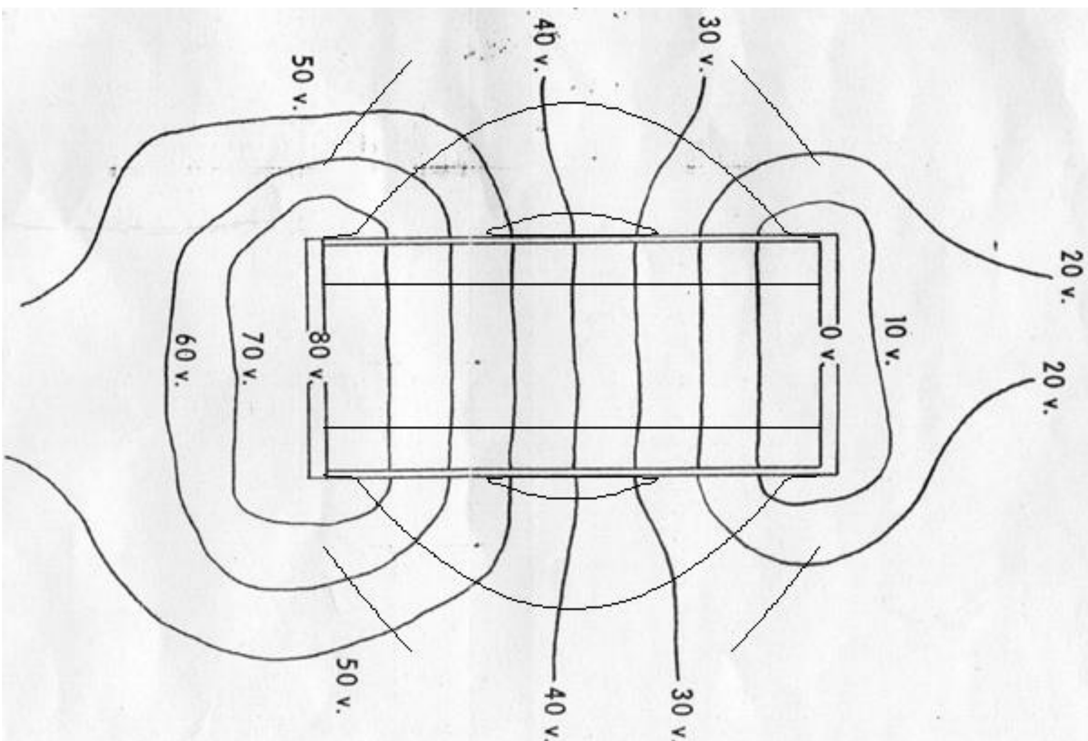


Experiment by Bergmann and Schäfer (Elektrizität und Magnetismus, 1950). Constant current flowing along a high resistance paper strip surrounded by oil with semolina. $\Delta V = 30$ kV.



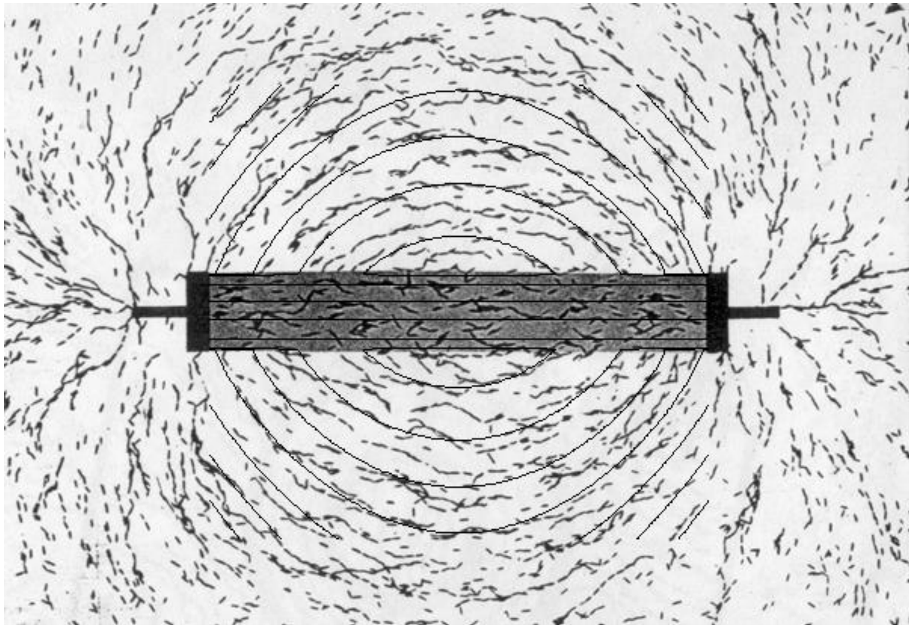
Experimental lines of electric field compared with theoretical calculations.

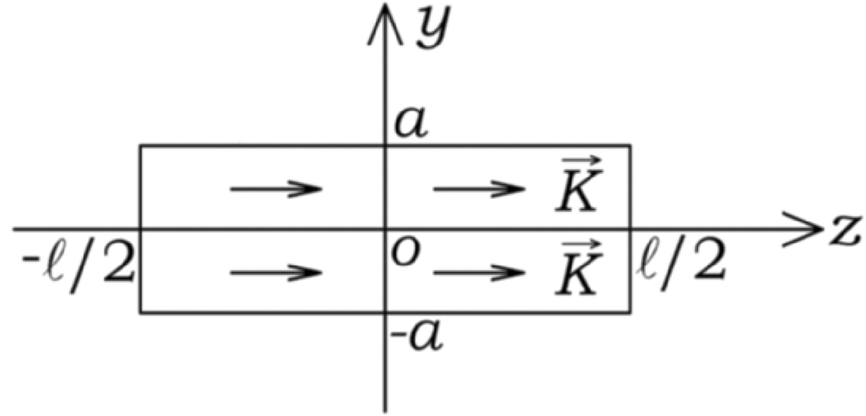
Equipotentials measured with an electrometer in a hollow rectangular chamber with graphite paper ($I = 5x10^{-2}$ A) or photographic film ($I = 4x10^{-6}$ A) on the walls. Jefimenko et al., Proc. West Virginia Acad. Sci. <u>34</u>, 163 (1962).



Experimental equipotential lines compared with calculated lines of electric field.

Resistive strip:



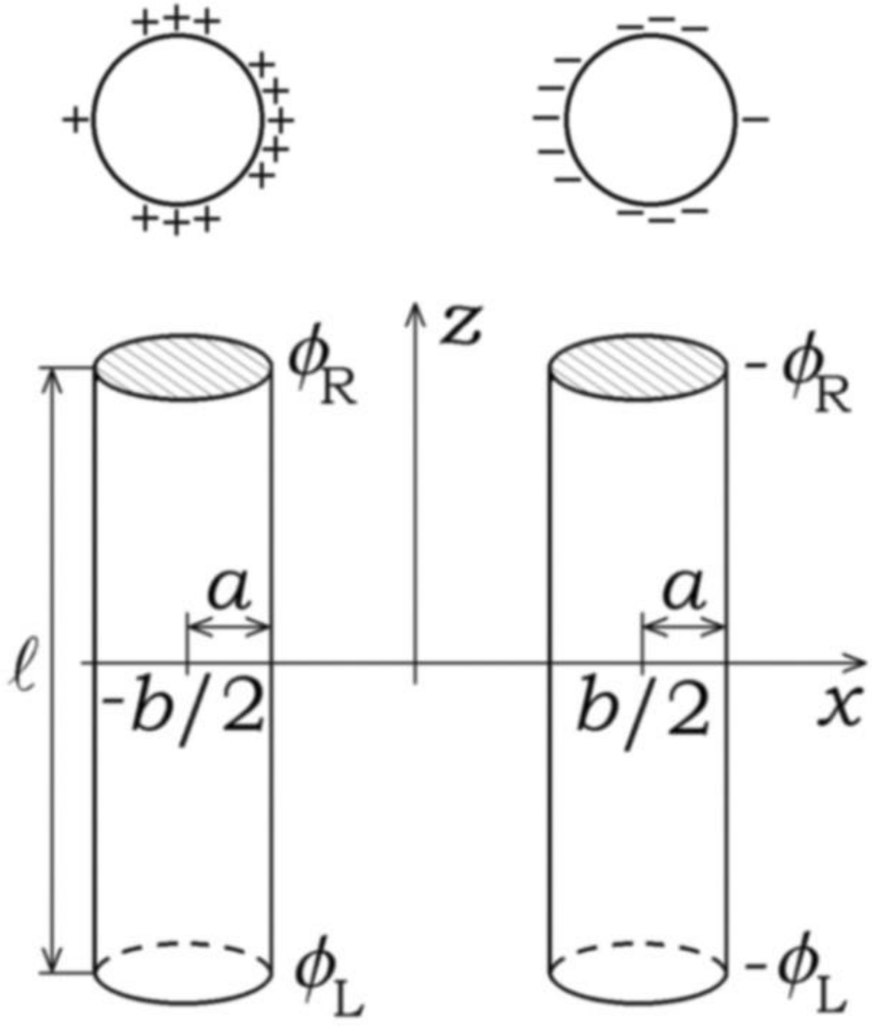


$$\sigma = \frac{k_1 + k_2 z}{\sqrt{a^2 - y^2}}$$

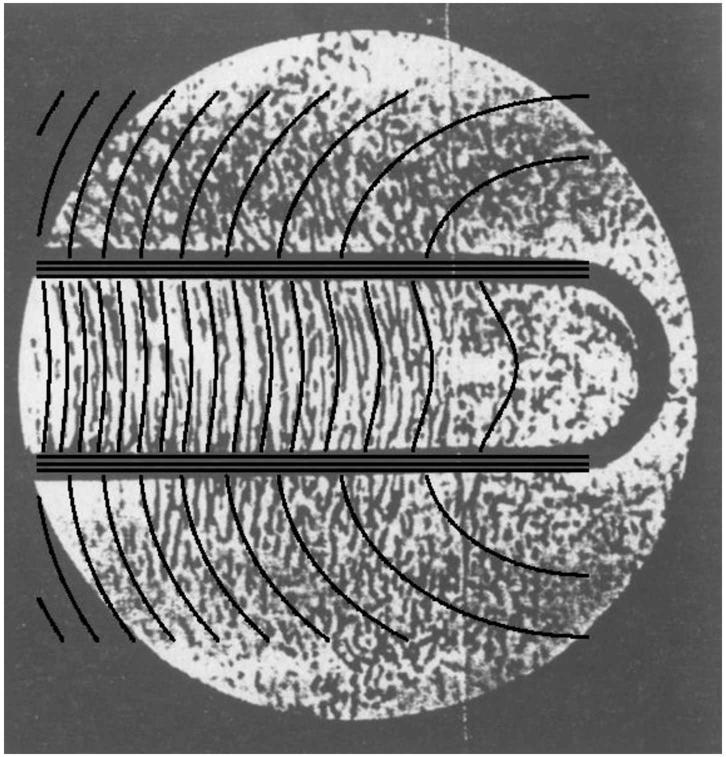
Jefimenko, AJP 30, 19 (1962). $\Delta V = 10$ kV.

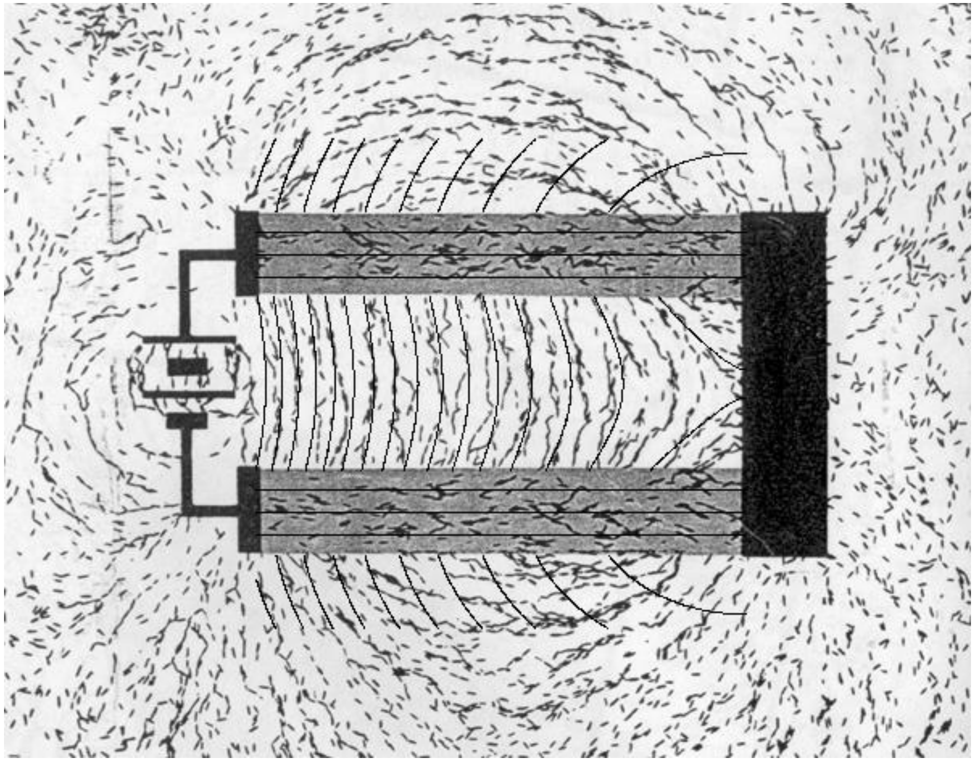
Experimental lines of electric field obtained with grass seeds spread on glass, compared with calculated lines of E.

Transmission line:

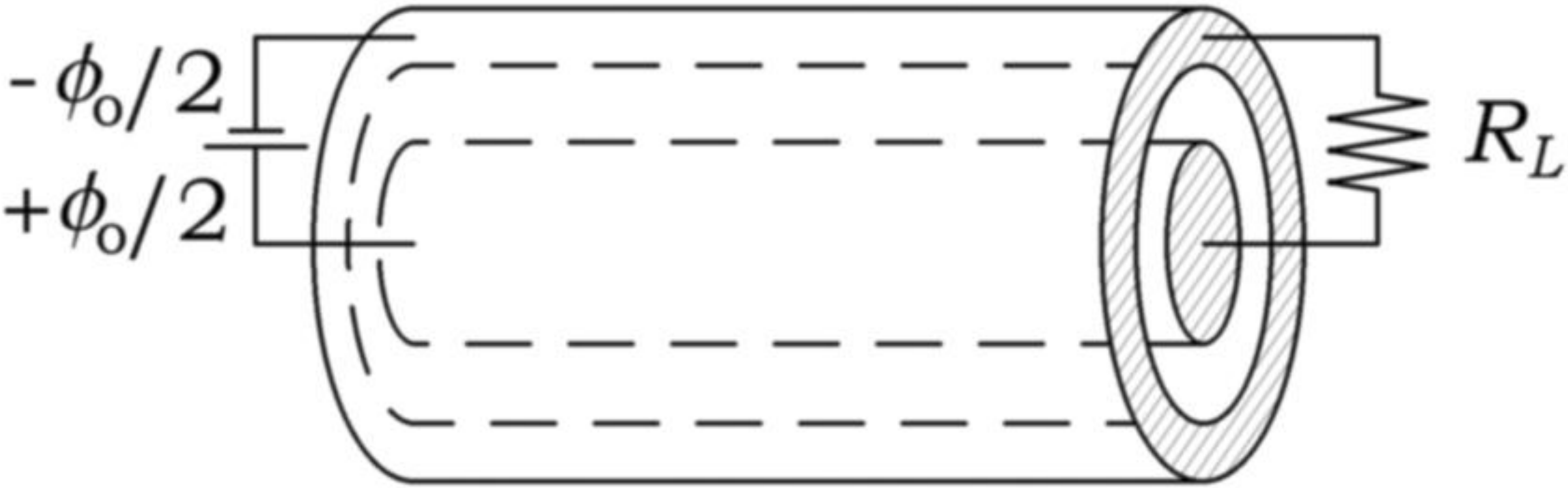


Experiments by Bergmann, Schäfer and Jefimenko for a transmission line showing the external electric field, compared with theoretical solution:





Coaxial cable: The situation is analogous to that of a straight wire.



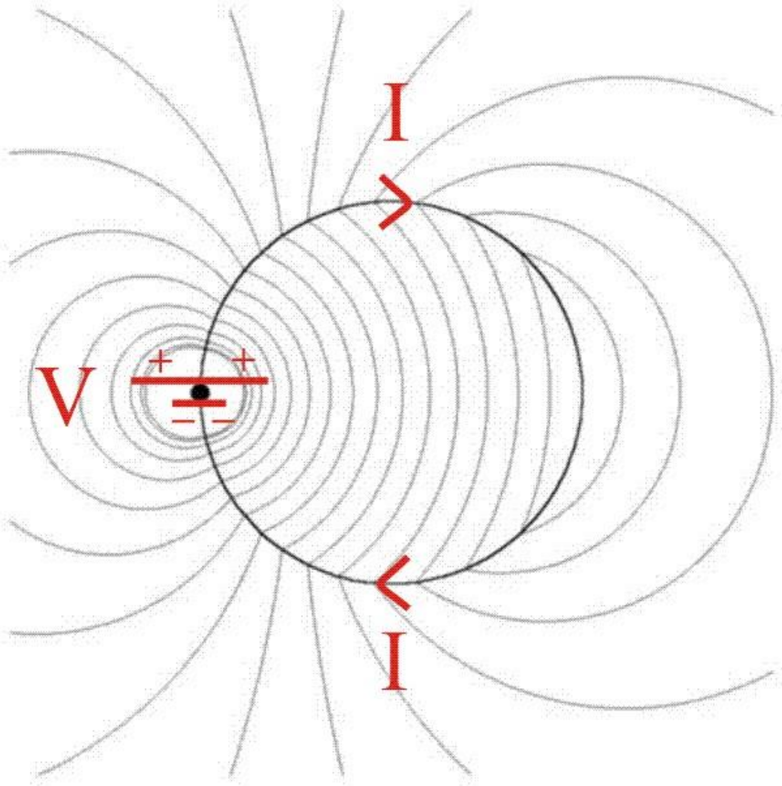
However: B outside = 0, but E outside ≠ 0.

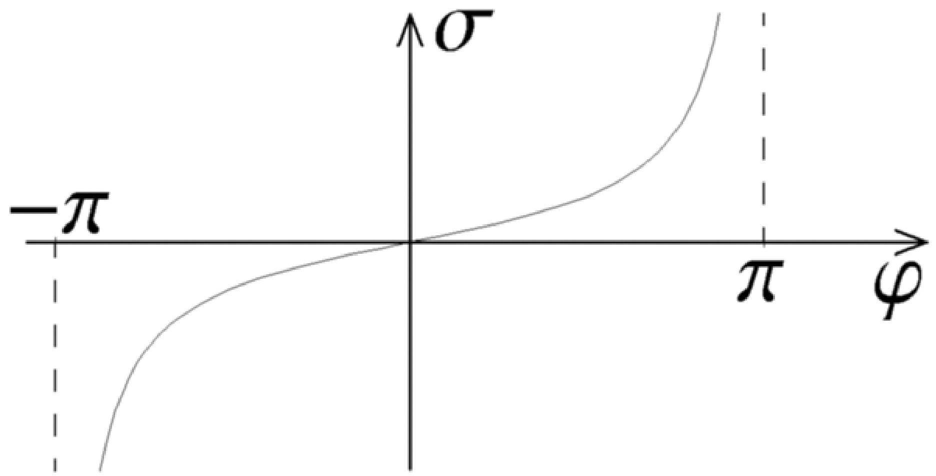
Therefore, we predict the electric interference between 2 coaxial cables.

Infinite solenoid, Heald, AJP 52, 522 (1984):

Given: $$\phi = \frac{V}{2}\frac{\varphi}{\pi}$$

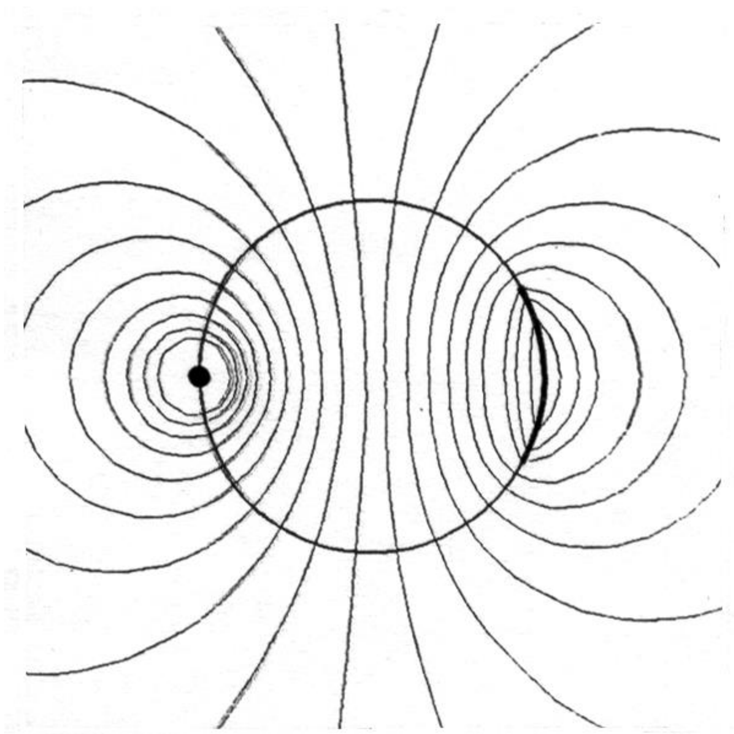
He obtained: $$\sigma = \sigma_0 \tan\frac{\varphi}{2}$$

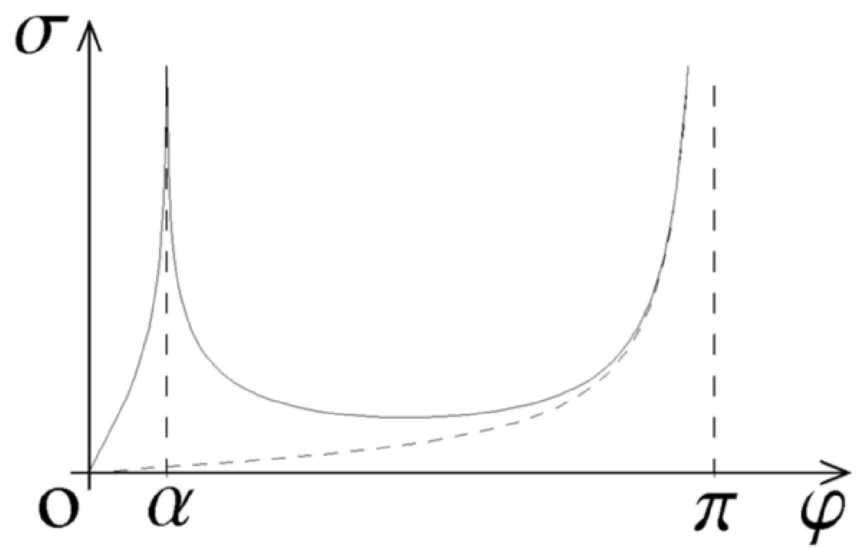




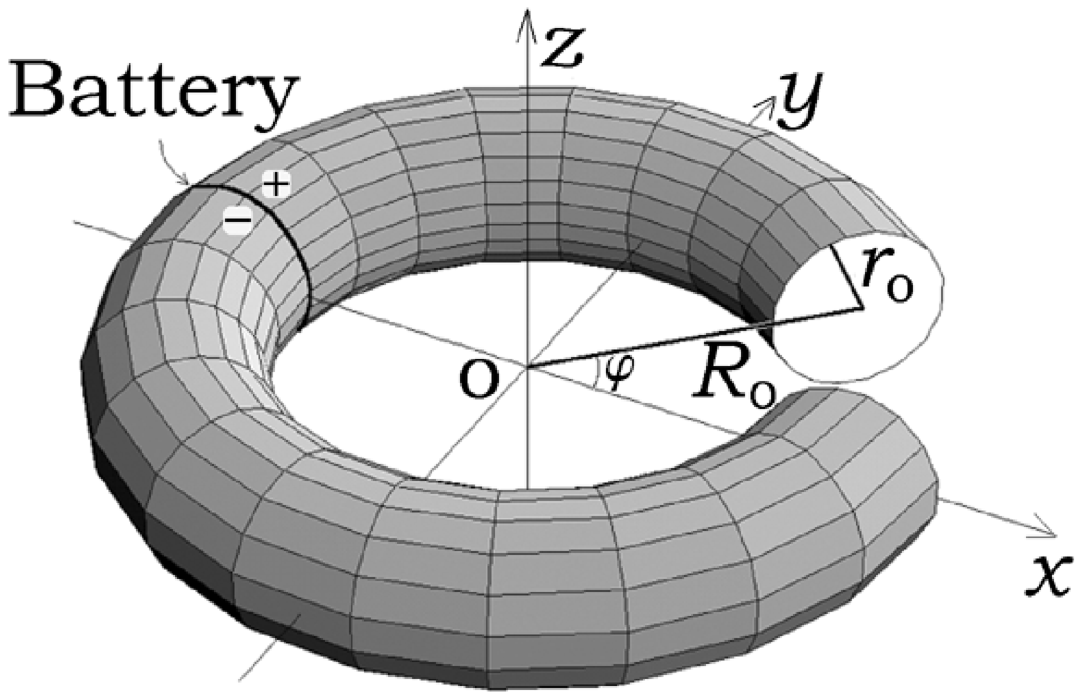
E outside ≠ 0. Moreover, the density of surface charge does not vary linearly with the azimuthal angle.

Lumped resistor:





Resistive ring with steady azimuthal current:



J. A. Hernandes and A. K. T. Assis,

Phys. Rev. E 68, 046611 (2003).

Toroidal coordinates: η, χ, φ

$$\tan\eta = \frac{2a\sqrt{x^2+y^2}}{x^2+y^2+z^2+a^2},\ \tan\chi = \frac{2za}{x^2+y^2+z^2-a^2},\ \tan\varphi = \frac{y}{x}$$

Potential ϕ outside the ring with the method of R-separation:

$$\nabla^2\phi = 0, \qquad \phi = \sqrt{\cosh\eta - \cos\chi}\, H(\eta)X(\chi)\Phi(\varphi)$$

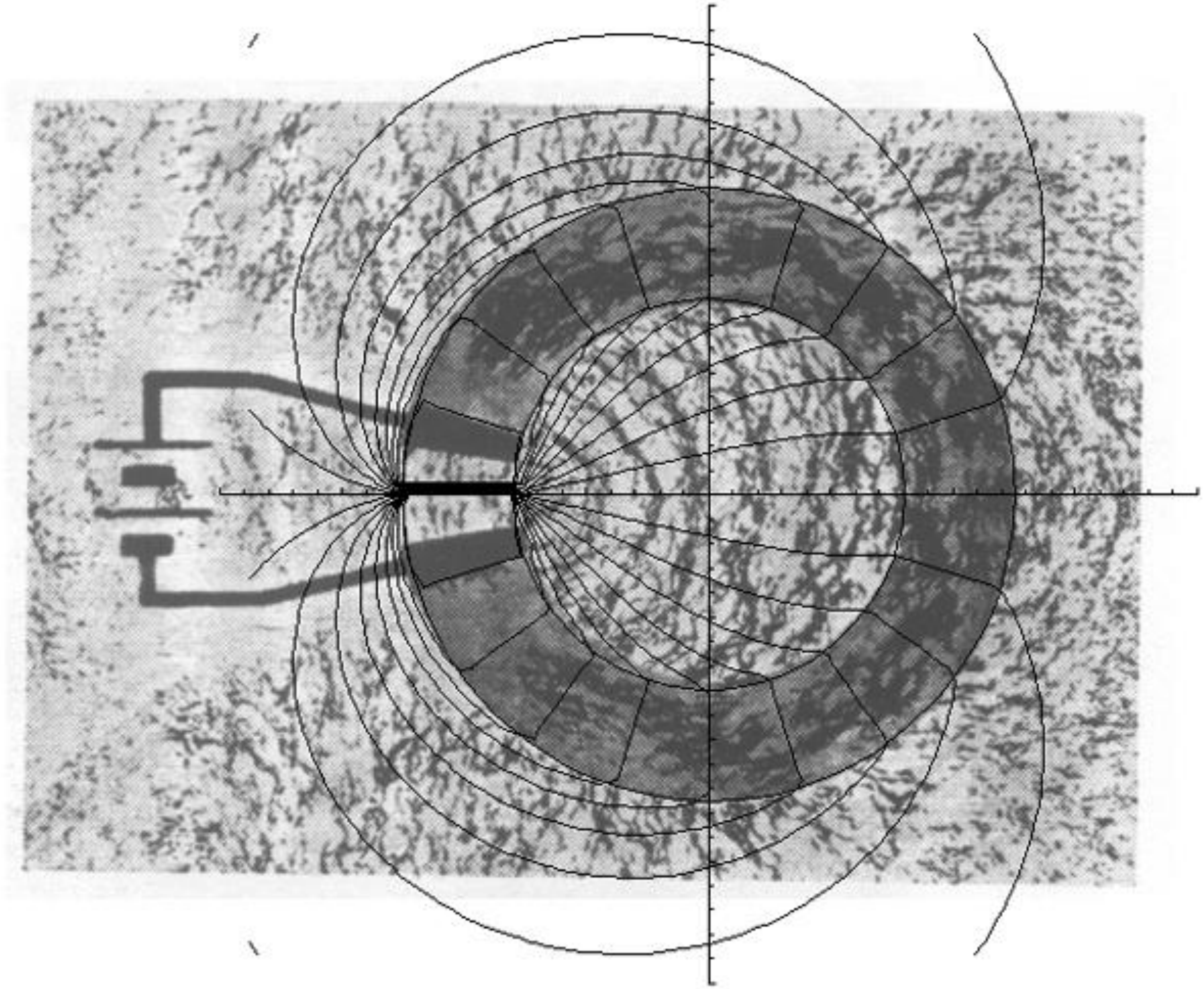
Solution for the external potential ϕ in toroidal coordinates:

Two infinite series of Legendre functions of hyperbolic cosines of a toroidal variable:

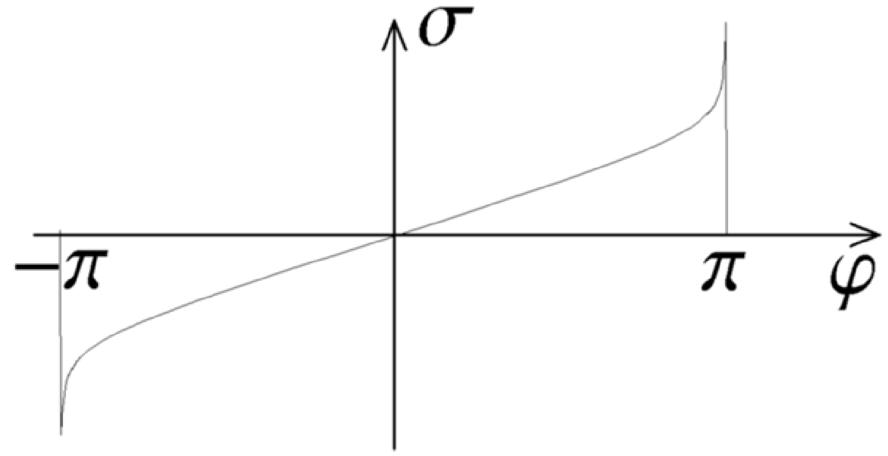
$$\phi = \sqrt{\cosh\eta - \cos\chi}\left\{\sum_{p=0}^{\infty} A_p \cos(p\chi) P_{p-1/2}(\cosh\eta)\right.$$

$$\left. + \sum_{q=1}^{\infty} \sin(q\varphi)\left[\sum_{p=0}^{\infty} B_{pq} \cos(p\chi) P_{p-1/2}^{q}(\cosh\eta)\right]\right\}$$

We compare our theoretical equipotential lines (plotted numerically) with the experimental lines of electric field obtained in 1962 by Jefimenko utilizing grass seeds spread on glass around a resistive ring carrying a steady current:



Density of surface charges in the resistive ring:

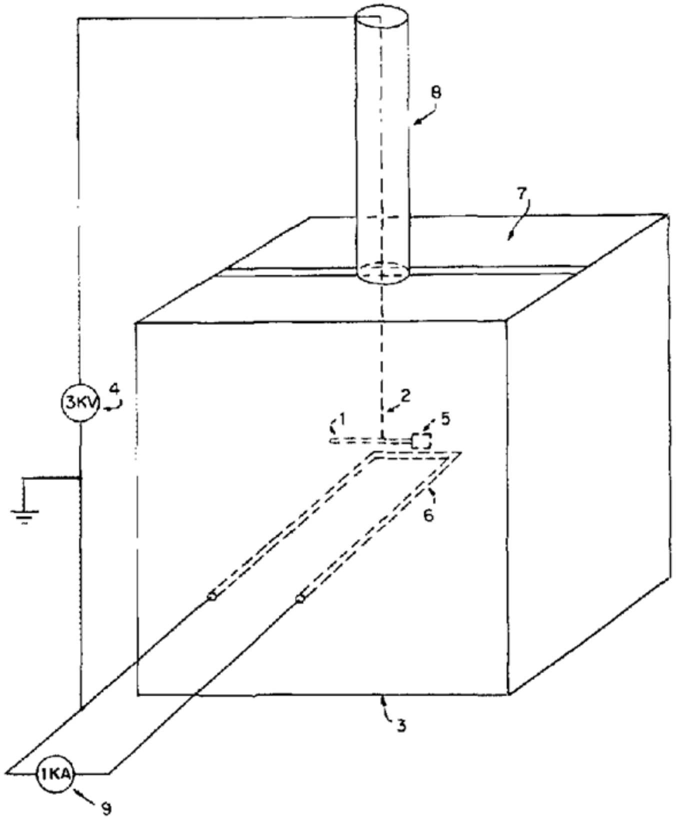


Weber was the first to calculate approximately the surface charges for this case. In 1852 he arrived at the following remarkable conclusion:

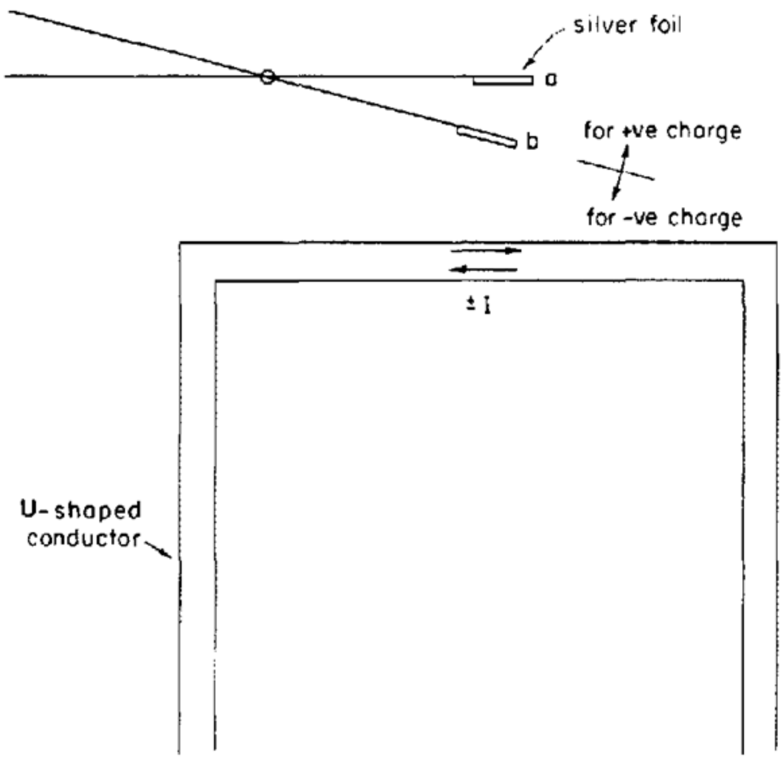
“The electric charge of the circuit increases from the neutral point to the contact point not uniformly, but accelerates gradually.”

Measurement of the force exerted by a wire carrying a steady current acting on a stationary external charge:

Sansbury, Rev. Scientific Instruments 56, 415 (1985):

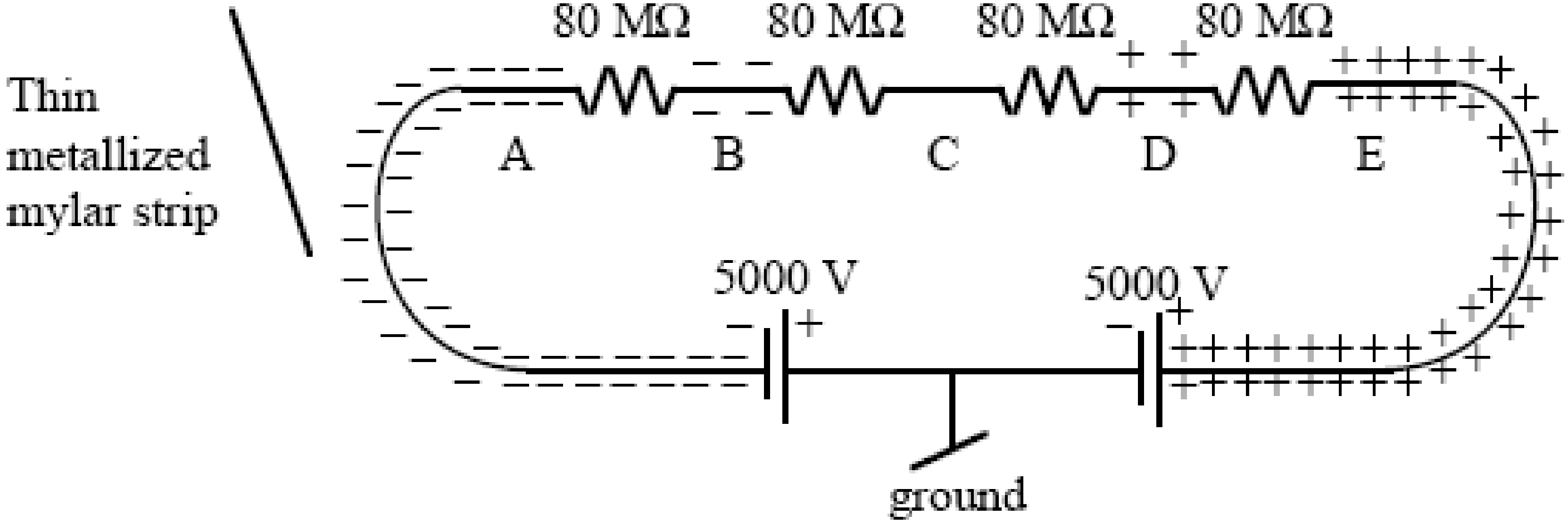


$q \approx 10^{-10} C$



$$q \approx 10^{-10} C, \quad I \approx kA, \quad F \approx 10^{-7} N$$

Experiment by Chabay and Sherwood collecting the surface charges as described in the book: Electric and Magnetic Interactions (Wiley, 2002).



Conclusion:

- There are charges spread on the surface of a resistive wire carrying a steady current. The surface charge density is constant in time, but varies along the length of the wire. It is positive close to the + terminal of the battery, goes to zero at an intermediate point, and is negative close to the – terminal of the battery.

- This surface charge density varies linearly with the distance along a straight wire. However, for a bent wire, it does not vary linearly with the angle.

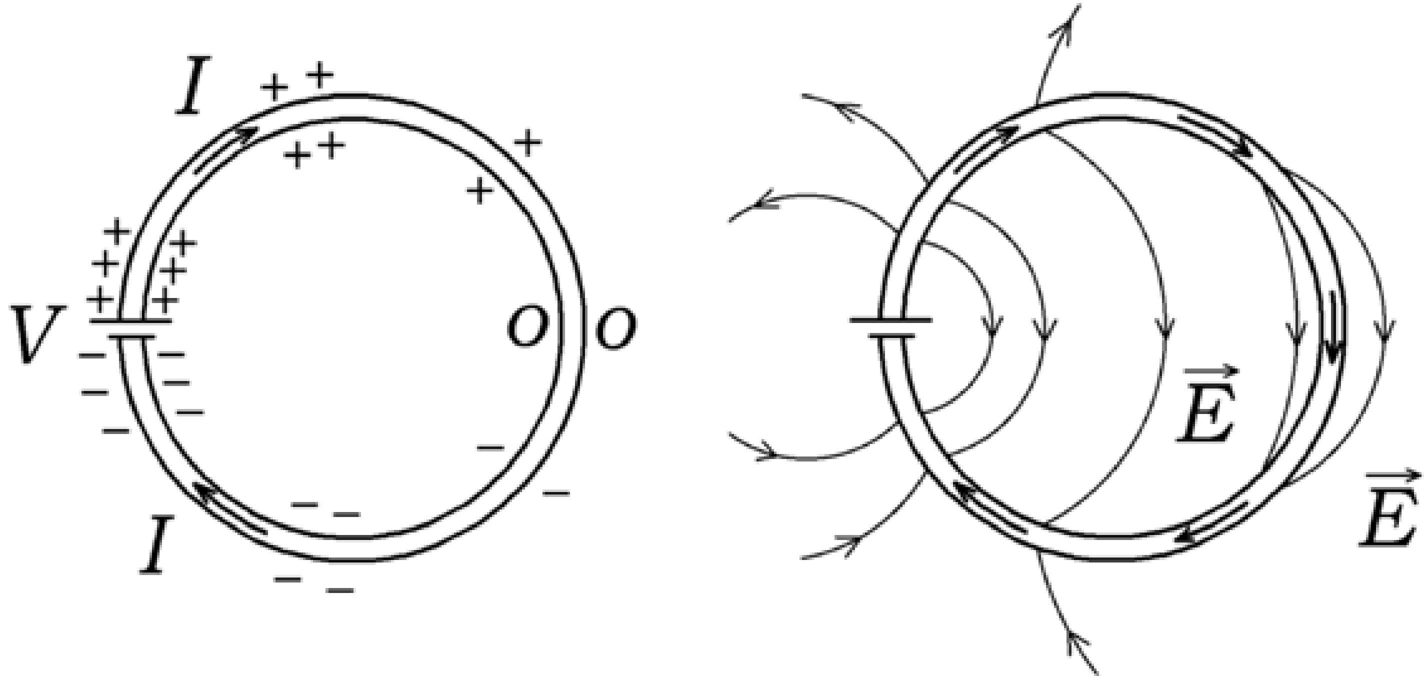
Conclusion:

- These surface charges generate not only the internal electric field responsible for Ohm's law, but also an external electric field at all points in space.

- Therefore, a resistive wire carrying a steady current exerts a force on an external charge at rest relative to the circuit.

- These facts were first calculated by Wilhelm Weber 150 years ago, although his pioneering work has been totally neglected!

Conclusion:

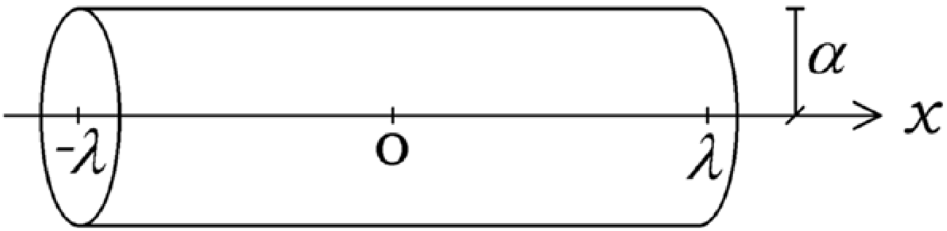


But Weber got there first!

www.ifi.unicamp.br/~assis

Extra material

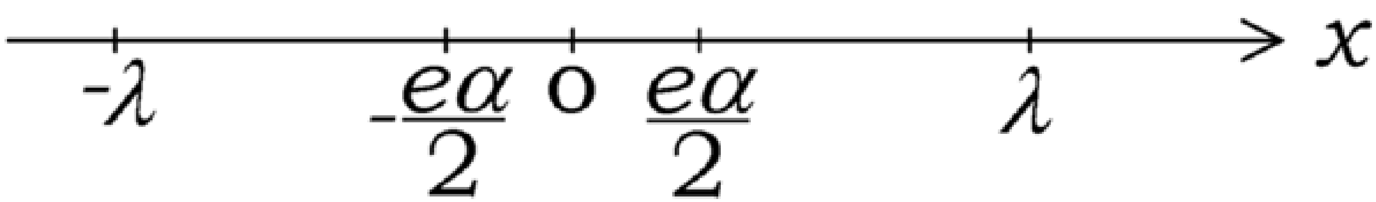
Weber in 1852:



$$\sigma = \sigma_A + \frac{\sigma_B}{\lambda} x$$

$$\vec{E}(x) = \frac{\alpha \sigma_B}{\varepsilon_o \lambda} \left(\ln \frac{\lambda}{e\alpha} \right) \hat{i}$$

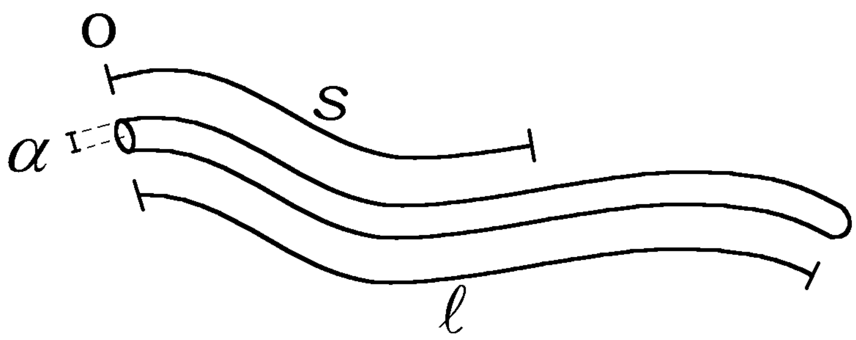
The same electric field is obtained with:



Linear charge density = $\sigma_L = 2\pi\alpha \left(\sigma_A + \frac{\sigma_B}{\lambda} x \right)$

The propagation of electromagnetic signals was first obtained by Weber and Kirchhoff in 1857, before Maxwell. They worked with Weber's electrodynamics. In particular, they obtained the telegraphy equation:

$$\vec{J} = g\vec{E} = -g\left(\nabla\phi + \frac{\partial\vec{A}}{\partial t}\right)$$



$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\frac{\partial^2\xi}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2\xi}{\partial t^2} = \frac{2\pi\varepsilon_0 R}{\ell\ln\frac{\ell}{\alpha}}\frac{\partial\xi}{\partial t}$$

with $\xi = I, \sigma, \phi, A$

The constant c had been introduced by Weber in 1846. Its value was first measured by Weber and Kohlrausch in 1855-56.

For a wire of negligible resistance Weber and Kirchhoff obtained in 1857 the wave equation for an electric signal:

$$\frac{\partial^2 \xi}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} = 0$$

Kirchhoff in the Annalen der Physik (1857):

"The velocity of propagation of an electric wave is here found to be independent of the cross section, of the conductivity of the wire, also, finally, of the density of the electricity: its value is 41950 German miles in a second [3.11 × 10^8 m/s], hence very nearly equal to the velocity of light in vacuo."

Maxwell introduced the displacement current in “Ampère’s” circuital law in 1864-1873:

$$\nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

However:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

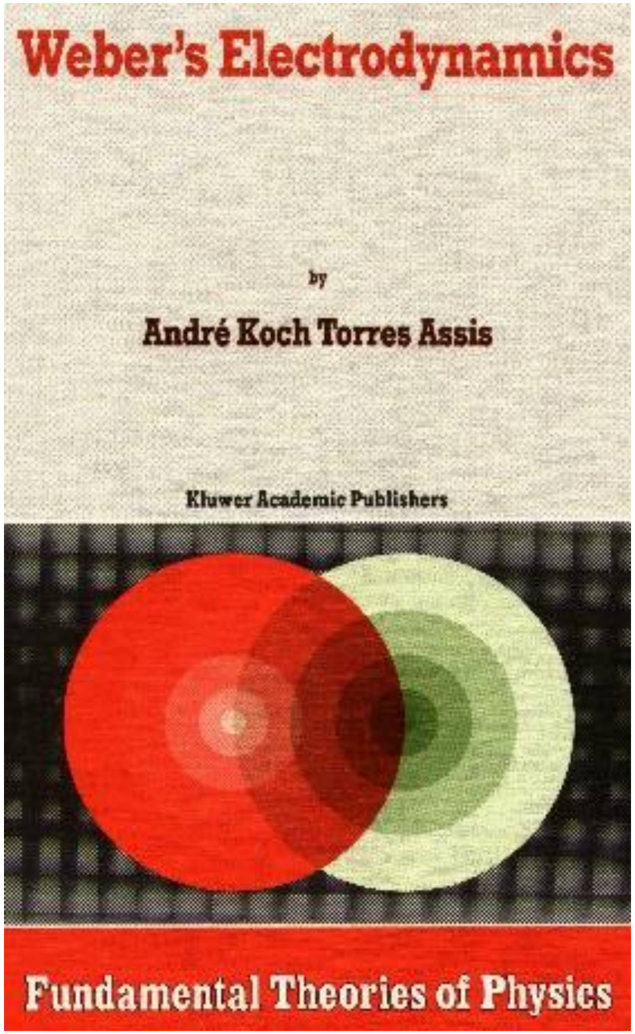
He utilized the constant ***c*** which had been introduced by Weber in 1846.

$$c = 3 \times 10^8 \frac{m}{s}$$

He knew the value of this constant as first measured by Weber in 1856.

$$\frac{\partial^2 \xi}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} = 0$$

He knew the wave equation which had been first obtained by Weber and Kirchhoff in 1857 from Weber’s electrodynamics.



www.ifi.unicamp.br/~assis