UNIPOLAR INDUCTION AND WEBER'S ELECTRODYNAMICS

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INTRODUCTION

Unipolar induction is the generation of current on a conductor for the case in which the conductor and the magnet are in relative rotatory motion. A typical case of unipolar induction is shown in figure 1.

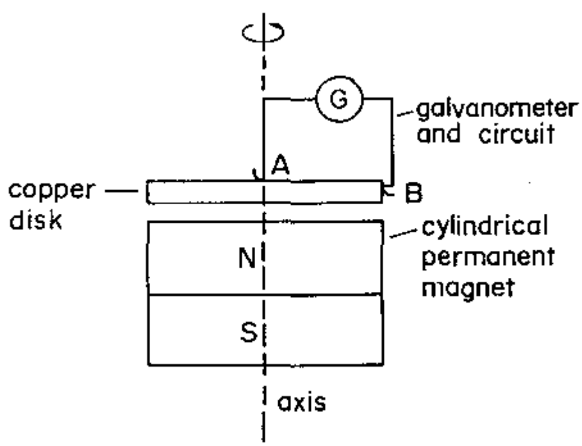

Figure 1. Apparatus used to investigate unipolar induction. The sliding contacts in A and B connect the galvanometer to the copper disk. Copper disk and magnet are free to rotate.

Frontiers of Fundamental Physics, Edited by M. Barone and F. Selleri, Plenum Press, New York, 1994

PP. 409 _ 414

Since Faraday's experiments[1] of 1832 on electromagnetic induction on rotating systems there are intense debates concerning the location of the seat of the electromotive force (*emf*)[2].

In this work whenever we speak of "rotation" it should be understood "rotation relative to the earth or laboratory."

Let us see what happens in the laboratory. When we rotate only the disk an *emf* is produced on the galvanometer-disk circuit (the magnet is fixed in the laboratory), as wee can see on the galvanometer. When we rotate only the magnet (the disk is fixed in the laboratory) no current flows by the galvanometer. When we rotate both the disk and the magnet there is a current in the galvanometer.

These results have lead some scientists, like Kennard[3,4], to think of a special frame of reference at which the systems shows unexpected effects when under acceleration relative to it. A classical example is the disk-magnet system when we observe a polarization when both rotate together. Kennard makes no consideration about inductions on the galvanometer. This means that he does not consider the galvanometer as part of the seat of induction.

The galvanometer consideration has been made[5] but the seat of the *emf* (disk or galvanometer and circuit) is a matter of controversy. It should be observed that in all the experiments which have been made, the galvanometer is fixed in the laboratory.

WEBER'S ELECTRODYNAMICS

In the last few years there has been a renewed interest in Ampère's force between current elements[6,7] and in Weber's force between point charges[8,9,10]. As we know, Ampère's force between current elements can be derived from Weber's force between point charges. It has also been shown that Faraday's law of induction for closed circuits can be derived from Weber's force[11].

The renewed interest in the basic laws of electromagnetism prompted us to study unipolar induction.

Weber's force states that a charge q_j exerts a force on a charge q_i given by:

$$\vec{F}_{ji} = \frac{q_i q_j}{4\pi\epsilon_0} \frac{\vec{r}_{ij}}{r_{ij}^3} \left[1 - \frac{\dot{r}_{ij}^2}{2c^2} + \frac{r_{ij}\ddot{r}_{ij}}{c^2} \right], \tag{1}$$

where $\vec{r}_{12} \equiv \vec{r}_1 - \vec{r}_2$, $r_{ij} \equiv |\vec{r}_i - \vec{r}_j|$, $\hat{r}_{ij} \equiv \frac{\vec{r}_{ij}}{r_{ij}}$, $\dot{r}_{ij} \equiv \frac{dr_{ij}}{dt}$, $\ddot{r}_{ij} \equiv \frac{d\dot{r}_{ij}}{dt}$, $\epsilon_0 = 8.85\times10^{-12}$ F/m and c is the ratio of electromagnetic and electrostatic units of charge which was found experimentally to have the same value of the light velocity in vacuum.

UNIPOLAR INDUCTION BY WEBER'S LAW

We will analyse unipolar induction in a region of uniform magnetic field. This can be obtained rotating a uniformly charged spherical shell at a constant angular velocity.

Two shells of radius R and $R + dR$ made up of non-conducting material and uniformly charged with charges Q and $-Q$, respectively, are rotating with constant angular velocities $\vec{\omega}_M$ and $\vec{\omega}_M + \vec{\omega}_N$ ($\vec{\omega}_M$ and $\vec{\omega}_N$ at the same direction).

According to Weber's electrodynamics the force exerted by the first shell (radius R, charge Q, $\vec{\omega}_M$) on an internal charge q located at $\vec{r}$ ($r < R$), moving relative to the laboratory with velocity $\vec{v}$ and acceleration $\vec{a}$ is given by ([10]):

$$\vec{F}(r < R) = \frac{qQ}{12\pi\epsilon_0 c^2 R}\left[\vec{a} + \vec{\omega}_M \times (\vec{\omega}_M \times \vec{r}) + 2\vec{v} \times \vec{\omega}_M + \vec{r} \times \frac{d}{dt}\vec{\omega}_M\right]. \tag{2}$$

Since we are interested in the force upon a free charge of a spinning conductor we make $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{\omega}$ is the angular velocity of the conductor.

The net force on the charge q is obtained by adding the contributions of the two shells. Considering that $dR << R$, that $d\vec{\omega}_M/dt = d\vec{\omega}_N/dt = 0$, and utilising (2) this yields ($\vec{\omega}_M = \omega_M\hat{z}$, $\vec{\omega}_N = \omega_N\hat{z}$, $\vec{\omega} = \omega\hat{z}$):

$$\vec{F}_1 = \frac{q_1 Q_M}{12\pi\epsilon_0 c^2 R}[\omega_N^2 + 2\omega_N(\omega_M - \omega)]\vec{\rho}. \tag{3}$$

In this expression $\vec{\rho}$ is the position vector to the axis of rotation, so that ρ is the distance between q and this axis.

Classically this situation of a double shell would give rise to a uniform magnetic field $\vec{B} = B\hat{z}$ inside the shells given by

$$B(r < R) = -\frac{\mu_0 Q\omega_N}{6\pi R}. \tag{4}$$

We may consider $\vec{\omega}_M$ as the rotation of the magnet itself as usually the positive charges are fixed in the lattice. So $\vec{\omega}_N$ may be considered as the drifting angular velocity of the electrons responsible for the current and for the magnetic field. In Faraday's experiments and in all other experiments on unipolar induction we had $\omega_N^2 << \omega_N(\omega_M - \omega)$, where ω represents the angular velocity of the copper disk. For this reason we can write (3) as (by (4)):

$$\vec{F}(r < R) = -qB[\omega_M - \omega]\vec{\rho}. \tag{5}$$

We can see that the force on the charge q is completely dependent of the relative motion between the magnet and q. Equation (5) is the basic expression for understanding unipolar induction with Weber's electrodynamics.

In equation (5) ω_M represents the rotation of the magnet relative to the laboratory, and B has been defined by equation (4). Moreover, ρ is the distance of the charge q (an electron) to the axis of rotation (z axis). When this electron belongs to a spinning disk we have $\omega = \omega_D$, where ω_D represents the rotation of the disk relative to the laboratory. In this case equation (5) reads

$$\vec{F}(r < R) = -qB[\omega_M - \omega_D]\vec{\rho}. \tag{6}$$

When this electron belongs to the circuit connected to the galvanometer (AGB in figure 1) we have $\omega = \omega_G$, where ω_G represents the rotation of this circuit relative to the laboratory. In this case we have

$$\vec{F}(r < R) = -qB[\omega_M - \omega_G]\vec{\rho}. \tag{7}$$

We can now analyse the situation of figure 1 by the device of figure 2, where the magnet has been replaced by the double shell of zero net charge.

There will be a polarization of the disk or of the open circuit connected to the galvanometer whenever $\omega_M - \omega_D \neq 0$ or $\omega_M - \omega_G \neq 0$, respectively. However when $\omega_D = \omega_G$ no current will flow in the closed circuit composed by the disk and the galvanometer. This is because even when $\omega_M - \omega_D \neq 0$ we will have in this case the same polarization of the disk and the open circuit, so that the net *emf* in the closed circuit is zero. A net *emf* only happens in the closed circuit when $\omega_D \neq \omega_G$.

From equations (6) and (7) we can construct the table 1 where "ω" and "0" represents the "presence" or "absence" of rotation relative to the laboratory, respectively. In table 1 in the column of the galvanometer the simbol (I) indicates that Weber's electrodynamics predicts a current through the galvanometer.

Let us calculate the *emf* in situation 2 of table 1, for the others the procedure is the same. From equation (6) we have, with $\omega_D \equiv \omega_0$

$$\vec{E} = \frac{\vec{F}}{q} = B\omega_0\vec{\rho}. \tag{8}$$

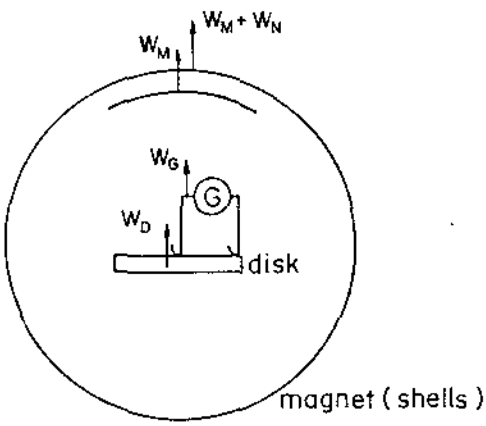

Figure 2. Two shells (Q and $-Q$) under rotation ($\vec{\omega}_M$ and $\vec{\omega}_M + \vec{\omega}_N$) generate a uniform magnetic field for $r < R$.

If the disk has the radius a the voltage between its center and the border will be

$$\triangle\phi = \int_0^a \vec{E}\cdot d\vec{r} = \frac{B\omega_0 a^2}{2}. \tag{9}$$

This will be the *emf* in the closed circuit. If there is a resistance R in the closed circuit composed of the galvanometer and disk, the current I flowing through the galvanometer will be given by

$$I = \frac{B\omega_0 a^2}{2R}, \tag{10}$$

When there is a current in table 1, this is its typical predicted value.

Table 1. Predictions for the current in the galvanometer.

	ω_G	ω_D	ω_M	Galvanometer
1	0	0	0	0
2	0	ω	0	I
3	0	0	$-\omega$	0
4	$-\omega$	0	0	I
5	ω	ω	0	0
6	$-\omega$	0	$-\omega$	I
7	0	ω	ω	I
8	ω	ω	ω	0

The experiments which have been performed up to now, to our knowledge, had always the galvanometer at rest relative to the laboratory. These are situations $1, 2, 3$ and 7 of table 1. The observed values of the currents agree with table 1 and equation (10).

With Weber's electrodynamics we can easily predict the situations 4, 5, 6 and 8 of table 1. If $\omega_G = \omega_0 \neq 0$ the predicted currents in these cases is given by (10), provided that $\omega_D = \omega_0$ or $\omega_M = \omega_0$ when they are also spinning relative to the laboratory.

We propose these experiments as a test of Weber's electrodynamics. A qualitative experiment of this kind might be easily performed if the galvanometer were replaced by a small lamp which is visible under a current of the order of equation (10).

ACKNOWLEDGMENTS

One of the authors (AKTA) wishes to thank FAPESP and CNPq for financial support in the past few years. He thanks also the Organizing Commitee of the International Conference Frontiers of Fundamental Physics for financial support.

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