The Velocity in Weber's Electrodynamics Versus the Velocities in Different Field Theories

A. K. T. Assis

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Chapter 1

The Velocity in Weber's Electrodynamics Versus the Velocities in Different Field Theories

A. K. T. Assis[1]

In this Chapter I compare the velocity which appears in Weber's electrodynamics with the velocities which appear in different field theories.

Before that I show that when one gives different interpretations to the magnitudes appearing in the same mathematical formula, then one has different theories.

1.1 Different Theories Described by the Same Mathematical Formula

Suppose a force proportional to the following expression:

$$\frac{mm'}{r^2} . \tag{1.1}$$

Is this force a mathematical representation of Newton's law of gravitation? The answer to this question depends on the meanings assigned to the magnitudes m, m' and r.

If m and m' refer to the masses of the interacting particles, and r is the distance between them as given by Figure 1.1 (a), then one can say that this formula is a mathematical representation of Newton's law of universal gravitation.

If, on the other hand m and m' indicate the surface areas of the two interacting bodies, or their volumes, then Equation (1.1) no longer represents Newton's law of gravitation. With this new interpretation of magnitudes m and m' one would have a theory which is different from Newton's theory of gravitation, although represented by the same algebraic formula.

In Figure 1.1 (b) I consider an observer O and the point P midway between m and m'. If the magnitude r to be utilized in Equation (1.1) is defined as the distance between O and P, as indicated in Figure 1.1 (b), then this Equation is no longer a mathematical representation of Newton's law of universal gravitation. In this case Equation (1.1) would

[1]Homepage: www.ifi.unicamp.br/~assis

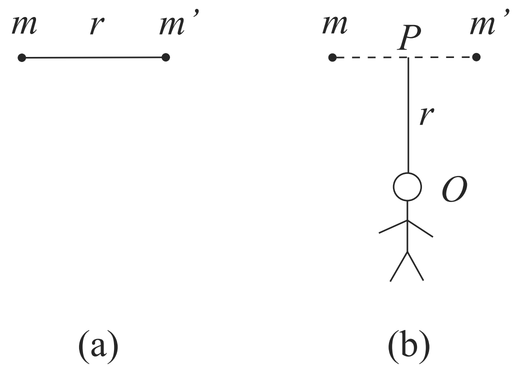

Figure 1.1: Different definitions of the magnitude r. (a) As the distance between the particles m and m'. (b) As the distance between P and the observer O, where P is the middle point between m and m'.

be a mathematical representation of a different theory and should not be called *Newton's* law of gravitation.

1.2 Force Acting on an Electrified Body based on Electromagnetic Fields

Newton presented the universal law of gravitation in terms of a force acting between material bodies. In gravitational field theory, on the other hand, the gravitational force is usually expressed in terms of a gravitational field $\vec{g}$ generated by a source gravitational mass M_g. When this field reaches another test gravitational mass m_g it generates a force $\vec{F}$ on this mass given by:

$$\vec{F} = m_g \vec{g} \ . \tag{1.2}$$

Analogously, in electromagnetic field theory, the force $\vec{F}$ acting on an electrified body which has a charge q in the presence of an electric field $\vec{E}$ and a magnetic field $\vec{B}$ is given by:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \ . \tag{1.3}$$

The history of this force and the different meanings of the velocity $\vec{v}$ which appears in this equation will be discussed in Section 1.3.

1.3 Origins and Meanings of the Velocity $\vec{v}$ which Appears in the Classical Electromagnetic Force Law $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Classical electromagnetism is composed of two main portions, namely, Maxwell's equations and the electromagnetic force acting on an electrified particle with charge q in the presence of an electric field $\vec{E}$ and a magnetic field $\vec{B}$. Maxwell's equations relate the fields $\vec{E}$ and $\vec{B}$

with the sources of these fields, namely, the volume charge density ρ and the volume current density $\vec{J}$.

The electromagnetic force $\vec{F}$, on the other hand, specifies how the fields $\vec{E}$ and $\vec{B}$ act on a particle with charge q. In the International System of Units (SI or MKSA) this so-called Lorentz force is normally expressed by the following equation:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \ . \tag{1.4}$$

Most textbooks when presenting for the first time this force express themselves as Feynman, Leighton and Sands:[2]

> We can write the force $\mathbf{F}$ on a charge q moving with a velocity $\mathbf{v}$ as
>
> $$\mathbf{F} = \mathbf{q}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \ . \qquad (\mathbf{1.1})$$
>
> We call $\mathbf{E}$ the *electric field* and $\mathbf{B}$ the *magnetic field* at the location of the charge.
>
> [...]
>
> The force on an electric charge depends not only on where it is, but also on how fast it is moving. [...] The total electromagnetic force on a charge can, then, be written as
>
> $$\mathbf{F} = \mathbf{q}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \ . \qquad (\mathbf{13.1})$$
>
> This is called the *Lorentz force*.

Statements such as this one by Feynman, Leighton and Sands are found in most textbooks on electromagnetism.[3] However, these statements have no meaning because they do not specify what is the velocity which appears in Equation (1.4). Velocity is not an intrinsic property of any body. It is a relation between the electrified particle and another body relative to which the charge is moving. For this reason one and the same electrified body can have several different velocities simultaneously. For instance, it can be simultaneously at rest relative to the ground, moving towards a second electrified particle, moving away from a certain magnet etc. Which of these velocities should one apply when utilizing Equation (1.4)? My goal is to discuss the meaning of this velocity $\vec{v}$ which appears in this equation.[4]

Consider for instance the situation of figure 1.2, where all velocities are relative to the ground. The test particle with charge q moves with velocity $\vec{v}_q$, the magnet NS moves with velocity $\vec{v}_m$, the circuit c carrying a current I moves with velocity $\vec{v}_c$, the observer O moves with velocity $\vec{v}_o$ and the magnetic field detector d moves with velocity $\vec{v}_d$. In particular, the velocity $\vec{v}$ which appears in Equation (1.4) should be understood relative to what object, body or entity? That is, $\vec{v}$ is the velocity of the particle q relative to what?

Some possible answers to this fundamental question:

- Relative to Newton's absolute space.

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[2] [FLS64, pp. 1.2 and 13.1].

[3] Several examples are quote in [AP92].

[4] [Ass92, Appendix (A): The Origins and Meanings of the Magnetic Force $\vec{F} = q\vec{v} \times \vec{B}$], [AP92], [Ass94, Appendix A: The Origins and Meanings of the Magnetic Force $\vec{F} = q\vec{v} \times \vec{B}$], [Ass13, Section 14.5], [Ass14, Section 15.5: Origins and Meanings of the Velocity $\vec{v}$ which Appears in the Magnetic Force $q\vec{v} \times \vec{B}$] and [Ass15a, Appendix B: Origins of the electromagnetic force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ and the different meanings given to the velocity $\vec{v}$].

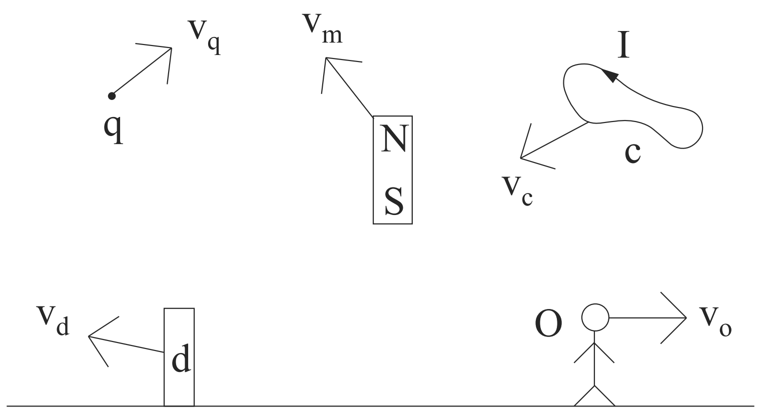

Figure 1.2: Velocities of several bodies relative to the ground.

- Relative to the laboratory, relative to the ground or relative to the Earth.
- Relative to the frame of fixed stars.
- Relative to the universal frame of distant galaxies.
- Relative to the macroscopic source of the magnetic field (that is, relative to the magnet or relative to the current carrying wire).
- Relative to the average velocity of the microscopic charges (normally electrons) which generate the magnetic field.
- Relative to the magnetic field itself.
- Relative to the detector of the magnetic field.
- Relative to the observer.
- Relative to any inertial frame of reference.
- Relative to an arbitrary frame of reference, which does not need to be inertial.
- Relative to the ether.
- Etc.

In this Section I discuss the history of this force and the different interpretations which have been given to the velocity appearing in this formula along the years by several authors.

1.3.1 Meaning of the Velocity According to Maxwell

The force given by Equation (1.4) is usually called Lorentz force in the textbooks. However, it seems that Maxwell was the first to obtain it.[5]

Maxwell presented this force in 1861-1862 in his article on physical lines of force.[6] Moreover, he discussing it in 1864-1865 in his paper with a dynamical theory of the electromagnetic

[5] [Mar90, p. 31], [Rib08], [Cur09, Section 4.6: On the paternity of Lorentz force, pp. 122-128], [Hur10, p. 22], [Tom12a], [Tom12b] and [Yag20].

[6] [Max62, p. 342, Equation (77)] and [Max65b, Equation (77)].

field,[7] and also in his main book of 1873, *A Treatise on Electricity and Magnetism*.[8] He was considering the force acting on an electrified body. Sometimes he referred to this test body as a conductor, as a dielectric or insulator, as a particle, as a current element of an electric circuit, or simply as electricity.

Maxwell interpreted this velocity as being the velocity of the test body relative to the magnetic field.

As regards the magnetic field, Maxwell himself mentioned that:[9]

> The theory I propose may therefore be called a theory of the *electromagnetic field*, because it has to do with the space in the neighbourhood of the electric and magnetic bodies, and it may be called a *dynamical* theory, because it assumes that in that space there is matter in motion, by which the observed electromagnetic phenomena are produced.

On page 166 of his paper of 1861-1862 he said:[10] "μ is a quantity bearing a constant ratio to the density." The "density" here refers to the supposed density of vortices in the medium. On page 283: "To determine the motion of a layer of particles separating two vortices. Let the circumferential velocity of a vortex, multiplied by the three direction-cosines of its axis respectively, be α, β, γ, as in Prop. II." On page 288: "Let P, Q, R be the forces acting on unity of the particles in the three coordinate directions, these quantities being functions of x, y, and z." On page 342: "[...] F, G, and H are the values of the electrotonic components for a fixed point of space, [...]" The force per unit charge, analogous to $\vec{F}/q$ of Equation (1.4), was originally written as follows in his paper of 1861-1862:[11]

$$\left.\begin{array}{l} P = \mu\gamma\frac{dy}{dt} - \mu\beta\frac{dz}{dt} + \frac{dF}{dt} - \frac{d\Psi}{dx} \ , \\ Q = \mu\alpha\frac{dz}{dt} - \mu\gamma\frac{dx}{dt} + \frac{dG}{dt} - \frac{d\Psi}{dy} \ , \\ R = \mu\beta\frac{dx}{dt} - \mu\alpha\frac{dy}{dt} + \frac{dH}{dt} - \frac{d\Psi}{dz} \ . \end{array}\right\} (77) \tag{1.5}$$

Soon after this equation he wrote, our emphasis:[12]

> The first and second terms of each equation indicate the effect of the *motion of any body in the magnetic field*, the third term refers to changes in the electrotonic state produced by the alterations of position or intensity of magnets or currents in the field, and Ψ is a function of x, y, z, and t, which is indeterminate as far as regards the solution of the original equations, but which may always be determined in any given case from the circumstances of the problem. The physical interpretation of Ψ is, that it is the *electric tension* at each point of space.

In the paper of 1864-1865 the force per unit charge, analogous to $\vec{F}/q$ of Equation (1.4), was presented as follows, our emphasis:[13]

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[7] [Max65, p. 484, Equation (D)] and [Max65a, Equation (D)].
[8] [Max54, Vol. 2, §§598-599, pp. 238-241, equations (B) and (10)].
[9] [Max65, p. 460] and [Max65a, p. 527].
[10] [Max62].
[11] [Max62, p. 342, Equation (77)].
[12] [Max62, p. 342, soon after Equation (77)].
[13] [Max65, p. 484].

The complete equations of *electromotive force on a moving conductor* may now be written as follows:

Equations of Electromotive Force.

$$\left.\begin{array}{l} P = \mu\left(\gamma\frac{dy}{dt} - \beta\frac{dz}{dt}\right) - \frac{dF}{dt} - \frac{d\Psi}{dx}\,, \\ Q = \mu\left(\alpha\frac{dz}{dt} - \gamma\frac{dx}{dt}\right) - \frac{dG}{dt} - \frac{d\Psi}{dy}\,, \\ R = \mu\left(\beta\frac{dx}{dt} - \alpha\frac{dy}{dt}\right) - \frac{dH}{dt} - \frac{d\Psi}{dz}\,. \end{array}\right\} \text{(D)} \tag{1.6}$$

The first term on the right-hand side of each equation represents the electromotive force arising from the motion of the conductor itself. This electromotive force is perpendicular to the direction of motion and to the lines of magnetic force; and if a parallelogram be drawn whose sides represent in direction and magnitude the velocity of the conductor and the magnetic induction at that point of the field, then the area of the parallelogram will represent the electromotive force due to the motion of the conductor, and the direction of the force is perpendicular to the plane of the parallelogram.

The second term in each equation indicates the effect of changes in the position or strength of magnets or currents in the field.

The third term shows the effect of the electric potential Ψ. It has no effect in causing a circulating current in a closed circuit. It indicates the existence of a force urging the electricity to or from certain definite points in the field.

In his *Treatise on Electricity and Magnetism* Maxwell defined the electric field $\vec{E}$, which he represented by the German letter $\mathfrak{E}$, on articles 44 and 68. He also called this electric field by the name "electromotive intensity:"[14]

The Electric Field.

44.] The Electric Field is the portion of space in the neighbourhood of electrified bodies, considered with reference to electric phenomena. [...]

Let e be the charge of the body, and F the force acting on the body in a certain direction, then when e is very small F is proportional to e, or

$F = Re$,

where R depends on the distribution of electricity on the other bodies in the field. If the charge e could be made equal to unity without disturbing the electrification of other bodies we should have $F = R$.

We shall call R the Resultant Electromotive Intensity at the given point of the field. When we wish to express the fact that this quantity is a vector we shall denote it by the German letter $\mathfrak{E}$.

Analogously, on article 68 he mentioned that:[15]

Resultant Intensity at a Point.

68.] In order to simplify the mathematical process, it is convenient to consider the action of an electrified body, not on another body of any form, but on an indefinitely

[14] [Max54, Vol. 1, §44, pp. 47-48].
[15] [Max54, Vol. 1, §68, p. 75].

> small body, charged with an indefinitely small amount of electricity, and placed at any point of the space to which the electrical action extends. By making the charge of this body indefinitely small we render insensible its disturbing action on the charge of the first body.
>
> Let e be the charge of the small body, and let the force acting on it when placed at the point $(x,\ y,\ z)$ be Re, and let the direction-cosines of the force be l, m, n, then we may call R the resultant electric intensity at the point $(x,\ y,\ z)$.
>
> If X, Y, Z denote the components of R, then
>
> $X = Rl, \quad Y = Rm, \quad Z = Rn.$
>
> In speaking of the resultant electric intensity at a point, we do not necessarily imply that any force is actually exerted there, but only that if an electrified body were placed there it would be acted on by a force Re, where e is the charge of the body.[16]
>
> This force not only tends to move a body charged with electricity, but to move the electricity within the body, so that the positive electricity tends to move in the direction of R and the negative electricity in the opposite direction. Hence the quantity R is also called the Electromotive Intensity at the point $(x,\ y,\ z)$.
>
> When we wish to express the fact that the resultant intensity is a vector, we shall denote it by the German letter $\mathfrak{E}$. [...]

In Maxwell's *Treatise* the vector magnetic induction was represented by $\mathfrak{B}$ and its components along the x, y and z direction by a, b and c, respectively.[17] In modern vector notation this vector would be written as $\vec{B}$ and its components as B_x, B_y and B_z. The vector-potential of magnetic induction was represented by $\mathfrak{A}$ and its components by F, G and H, respectively.[18] In modern notation this magnetic vector potential would be written as $\vec{A}$ and its components as A_x, A_y and A_z. The vectors $\vec{B}$ and $\vec{A}$ were related by:[19]

$$\vec{B} = \nabla \times A \ . \tag{1.7}$$

The electromotive force E due to induction acting on the secondary circuit was written as follows:[20]

$$E = \int \left(P\frac{dx}{ds} + Q\frac{dy}{ds} + Rdzds \right) ds \ . \quad (5) \tag{1.8}$$

Chapter VIII of Volume 2 of Maxwell's *Treatise on Electricity and Magnetism* was devoted to an exploration of the field by means of the secondary circuit. He mentioned on page 229 that:[21] "[...] the electromagnetic action between the primary and the secondary circuit depends on the quantity denoted by M, which is a function of the form and relative position of the two circuits." He wished to study the electrokinetic momentum of the secondary circuit depending on the primary current i_1, which he denoted by $p = Mi_1$. On page 230 he

[16] [Note by Maxwell:] The Electric and Magnetic Intensities correspond, in electricity and magnetism, to the intensity of gravity, commonly denoted by g, in the theory of heavy bodies.

[17] [Max54, Vol. 2, §400, p. 25].

[18] [Max54, Vol. 2, §405, p. 29].

[19] [Max54, Vol. 2, §405, p. 29, Equation (21) and §591, p. 233, Equation (A)].

[20] [Max54, Vol. 2, §598, p. 239].

[21] [Max54].

mentioned that "the part contributed by the element ds of the circuit is Jds, where J is a quantity depending on the position and direction of the element ds." On page 232 he said that the electrokinetic moment at the point $(x,\ y,\ z)$ was identical to the vector-potential of magnetic induction.

The force per unit charge representing the equations of electromotive intensity, analogous to $\vec{F}/q$ of Equation (1.4), was expressed in the *Treatise* as follows:[22]

$$\left.\begin{array}{l} P = c\frac{dy}{dt} - b\frac{dz}{dt} - \frac{dF}{dt} - \frac{d\Psi}{dx}\ , \\ Q = a\frac{dz}{dt} - c\frac{dx}{dt} - \frac{dG}{dt} - \frac{d\Psi}{dy}\ , \\ R = b\frac{dx}{dt} - a\frac{dy}{dt} - \frac{dH}{dt} - \frac{d\Psi}{dz}\ . \end{array}\right\} \text{(B)} \tag{1.9}$$

He summarized these equations, which he denoted by the letter (B), as follows:[23]

> The electromotive intensity, as defined by equations (B), may therefore be written in the quaternion form,
>
> $\mathfrak{E} = \mathrm{V}.\mathfrak{G}\mathfrak{B} - \dot{\mathfrak{A}} - \nabla\Psi\ .$ (10)

Maxwell's Equation (10) can be written in modern vector notation as follows:

$$\vec{E} = \vec{v} \times \vec{B} - \frac{\partial \vec{A}}{\partial t} - \nabla\Psi\ . \tag{1.10}$$

Maxwell's Equation (1.10) is then analogous to Equation (1.4), expressing the force per unit charge, namely, $\vec{F}/q$.

In his *Treatise*, soon after presenting his equations (B) for the electromotive force, that is, our Equation (1.9), he said the following, our emphasis:[24]

> The terms involving the new quantity Ψ are introduced for the sake of giving generality to the expressions for P, Q, R. They disappear from the integral when extended round the closed circuit. The quantity Ψ is therefore indeterminate as far as regards the problem now before us, in which the electromotive force round the circuit is to be determined. We shall find, however, that when we know all the circumstances of the problem, we can assign a definite value to Ψ, and that it represents, according to a certain definition, the *electric potential* at the point $(x,\ y,\ z)$.
>
> The quantity under the integral sign in equation[25] (5) represents the electromotive intensity acting on the element ds of the circuit.
>
> If we denote by $T.\mathfrak{E}$, the numerical value of the resultant of P, Q, and R, and by ϵ, the angle between the direction of this resultant and that of the element ds, we may write equation (5),
>
> $$E = \int T.\mathfrak{E} \cos\epsilon ds\ . \quad (6)$$
>
> The vector $\mathfrak{E}$ is the electromotive intensity at the moving element ds. Its direction and magnitude depend on the position and motion of ds, and on the variation of the magnetic field, but not on the direction of ds. Hence we may now disregard the circumstance that ds forms part of a circuit, and consider it simply as a portion of a

[22][Max54, Vol. 2, §598, p. 239, Equation (B)].
[23][Max54, Vol. 2, §599, p. 241, Equation (10)].
[24][Max54, Vol. 2, §598, pp. 239-241].
[25]That is, our Equation (1.8).

> *moving body*, acted on by the electromotive intensity $\mathfrak{E}$. The electromotive intensity has already been defined in Art. 68. It is also called the resultant electrical intensity, being the force which would be experienced by a unit of positive electricity placed at that point. We have now obtained the most general value of this quantity in the case of *a body moving in a magnetic field* due to a variable electric system.
>
> If the body is a conductor, the electromotive force will produce a current; if it is a dielectric, the electromotive force will produce only electric displacement.
>
> The electromotive intensity, or the force on a particle, must be carefully distinguished from the electromotive force along an arc of a curve, the latter quantity being the line-integral of the former. See Art. 69.

Maxwell continued his book as follows, our emphasis:[26]

> 599.] The electromotive intensity, the components of which are defined by equations (B), depends on three circumstances. The first of these is *motion of the particle through the magnetic field*. The part of the force depending on this motion is expressed by the first two terms on the right of each equation. It depends *on the velocity of the particle transverse to the lines of magnetic induction*. If $\mathfrak{G}$ is a vector representing the velocity, and $\mathfrak{B}$ another representing the magnetic induction, then if $\mathfrak{E}_1$ is the part of the electromotive intensity depending on the motion,
>
> $$\mathfrak{E}_1 = \text{V}.\mathfrak{G}\mathfrak{B} \ , \quad (7)$$
>
> or, the electromotive intensity is the vector part of the product of the magnetic induction multiplied by the velocity, that is to say, the magnitude of the electromotive force is represented by the area of the parallelogram, whose sides represent the velocity and the magnetic induction, and its direction is the normal to this parallelogram, drawn so that the velocity, the magnetic induction, and the electromotive intensity are in right-handed cyclical order.

Maxwell's Equation (7) would nowadays be written in vector notation as follows:

$$\vec{E} = \vec{v} \times \vec{B} \ . \tag{1.11}$$

It is important to emphasize some aspects here. Maxwell's equations (B) of the *Treatise*, our Equation (1.9), is analogous to the force given by Equation (1.4). Maxwell's seems to have been the first to write down this equation, publishing his results between 1861 and 1873.

The magnetic component of this force, namely, $\vec{F}_m = q\vec{v}\times\vec{B}$, seems to have been obtained by Maxwell after considering Ampère's electrodynamic force exerted by a closed circuit C_2 acting on a current element $i_1 d\vec{\ell}_1$, namely:

$$d\vec{F}_{21} = \frac{\mu_o}{4\pi} i_1 i_2 \oint_{C_2} \frac{\hat{r}}{r^2} \left[3(\hat{r}\cdot d\vec{\ell}_1)(\hat{r}\cdot d\vec{\ell}_2) - 2(d\vec{\ell}_1 \cdot d\vec{\ell}_2) \right] = i_1 d\vec{\ell}_1 \times \oint_{C_2} \frac{\mu_o}{4\pi} \frac{i_2 d\vec{\ell}_2 \times \hat{r}}{r^2} \ . \tag{1.12}$$

Maxwell, but not Ampère, then defined the magnetic field $\vec{B}$ at the location $\vec{r}_1 = (x_1, \ y_1, \ z_1)$ of the test element $i_1 d\vec{\ell}_1$ and being due to the closed current carrying circuit C_2 as follows:

[26] [Max54, Vol. 2, §599, pp. 240-241].

$$\vec{B}(\vec{r}_1) \equiv \oint_{C_2} \frac{\mu_o}{4\pi} \frac{i_2 d\vec{\ell}_2 \times \hat{r}}{r^2} \ . \tag{1.13}$$

With this definition, Ampère's Equation (1.12) exerted by the closed circuit C_2 on the current element $i_1 d\vec{\ell}_1$ could then be written as follows:

$$d\vec{F}_{21} = i_1 d\vec{\ell}_1 \times \oint_{C_2} \frac{\mu_o}{4\pi} \frac{i_2 d\vec{\ell}_2 \times \hat{r}}{r^2} = i_1 d\vec{\ell}_1 \times \vec{B} \ . \tag{1.14}$$

Maxwell then finally replaced this current element $i_1 d\vec{\ell}_1$ by $q\vec{v}$, where q is the charge of the electrified body and $\vec{v}$ its velocity. The magnetic force $\vec{F}_m$ acting on this charged body moving in a magnetic field would then be written as:

$$\vec{F}_m = q\vec{v} \times \vec{B} \ . \tag{1.15}$$

Moreover, Maxwell interpreted that this velocity $\vec{v}$ which appears in equations (1.4) or (1.15) as the velocity of the particle with charge q relative to the magnetic field $\vec{B}$. As shown above, in his papers of 1861 and in article 598 of his Treatise, he mentioned explicitly the force acting on "a body moving in a magnetic field."

1.3.2 Meaning of the Velocity According to Thomson and Heaviside

In 1881 J. J. Thomson (1856-1940) obtained theoretically the magnetic force as given by $q\vec{v} \times \vec{B}/2$.[27] This velocity $\vec{v}$ in his theory was interpreted as the velocity of the particle with charge q relative to the medium through which it was moving, a medium whose magnetic permeability was μ. For Thomson this velocity of the particle with charge q was not a velocity relative to the magnetic field, nor relative to the luminiferous ether, nor relative to the magnet which generated the magnetic field $\vec{B}$, nor the velocity of the particle with charge q relative to the observer. He called this velocity the "actual velocity" of the electrified particle. He said the following on page 248 of his original article:[28]

> It must be remarked that what we have for convenience called the actual velocity of the particle is, in fact, the velocity of the particle relative to the medium through which it is moving [...], medium whose magnetic permeability is μ.

In 1889, O. Heaviside (1850-1925) deduced theoretically the magnetic force as $q\vec{v} \times \vec{B}$. This is the same value obtained earlier by Maxwell and twice the value obtained by Thomson. He accepted Thomson's interpretation for the meaning of the velocity $\vec{v}$, as can be seen from the title of his paper:[29]

> On the electromagnetic effects due to the motion of electrification through a dielectric.

This title shows that for him this $\vec{v}$ was the velocity of the particle with charge q relative to the dielectric material through which it was moving.

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[27] [Tho81] and [Whi73, pp. 306-310].
[28] [Tho81, p. 248].
[29] [Hea89].

1.3.3 Meaning of the Velocity According to Lorentz

In 1895 H. A. Lorentz (1853-1928) presented the force acting on a particle with charge q as follows:[30]

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \ . \tag{1.16}$$

As his deduction contains only two pages, I will present it in full. I will quote from the English translation:[31]

The second part of the force acting on ponderable matter

§ 12. A current element as the one previously considered, may be located in a magnetic field generated by external causes. According to a known law it suffers an electrodynamic force

$$[\mathrm{ids}\cdot\mathfrak{H}],$$

for which we also can write now

$$[\Sigma\ \mathrm{e}\mathfrak{v}\cdot\ \mathfrak{H}],$$

or

$$\Sigma\{\mathrm{e}[\mathfrak{v}\cdot\ \mathfrak{H}]\}.$$

This action results (according to our view) from the forces, which will be exerted by the aether upon the ions of the current element. It is thus near at hand, to assume for the force acting on a single ion

$$\mathrm{e}[\mathfrak{v}\cdot\mathfrak{H}],$$

a hypothesis, which we still want to extend in a way, so that we *generally* assume a force acting on ponderable matter of the volume element $d\tau$

$$\rho d\tau[\mathfrak{v}\cdot\mathfrak{H}]$$

In unit charge this would be

$$\mathfrak{E}_2 = [\mathfrak{v}\cdot\mathfrak{H}]$$[32].

By putting this vector together with $\mathfrak{E}_1$ that was considered earlier (§ 9), we obtain for the total force exerted on the unit charge, *i.e.* for the *electric force*,

$$\mathfrak{E} = 4\pi V^2\mathfrak{d} + [\mathfrak{v}\cdot\mathfrak{H}]. \qquad \text{(V)}$$

We refuse to express the thus stated law by words. By elevating it to a general fundamental-law, we have completed the system of equations of motion (I)—(V), since the electric force, in connection with possible other forces, determines the motion of ions.

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[30] [Lor95, §12, pp. 21-22 and table after page 138], [Pai82, p. 125] and [Pai86, p. 76].

[31] Available at https://en.wikisource.org/wiki/Translation:Attempt_of_a_Theory_of_Electrical_and_Optical_Phenomena_in_Moving_Bodies, see also [Lor95, §12, pp. 21-22 and table after page 138].

[32] [Note by Lorentz:] If we don't want to consider an ordinary electric current as a convection current, then we must substantiate this formula by the assumption, that a body in which a convection takes place, experiences the same electrodynamic actions as a corresponding current conductor.

> Concerning the latter, we still want to introduce the assumption, that the ions never rotate.[33,34]

Lorentz presented a similar deduction in a course of lectures delivered in Columbia University, New York, in March and April, 1906. They were published in his famous book *The Theory of Electrons*, first published in 1909.[35] I quote from the second edition of 1915. Passages in square brackets are our words and the modern rendering of some of his formulas (for instance Lorentz' $[\mathbf{a}\cdot\mathbf{b}]$ is nowadays usually represented by $\vec{a}\times\vec{b}$). He utilized the cgs system of units. What he called "electron" represented a generic electrified particle (the specific particle called nowadays the "electron," with a charge of $q = -1.6\times 10^{-19}\ C$ and mass $m = 9.1\times 10^{-31}\ kg$, was only discovered in 1897). Here are his words with our emphasis:[36]

> However this may be, we must certainly speak of such a thing as the force acting on a charge, or on an electron, on charged matter, whichever appellation you prefer. Now, in accordance with the general principles of Maxwell's theory, *we shall consider this force as caused by the state of the ether, and even, since this medium pervades the electrons, as exerted by the ether on all internal points of these particles where there is a charge*. If we divide the whole electron into elements of volume, there will be a force acting on each element and determined by the state of the ether existing within it. We shall suppose that this force is proportional to the charge of the element, so that we only want to know the force acting per unit charge. This is what we can now properly call *the electric force*. We shall represent it by $\mathbf{f}$. The formula by which it is determined, and which is the one we still have to add to (17)-(20) [Maxwell's equation's], is as follows:
>
> $$\mathbf{f} = \mathbf{d} + \frac{1}{c}[\mathbf{v}\cdot\mathbf{h}]. \qquad \left[\vec{f} = \vec{d} + \frac{\vec{v}\times\vec{h}}{c}\right] . \qquad (23)$$
>
> Like our former equations, it is got by generalizing the results of electromagnetic experiments. The first term represents the force acting on an electron in an electrostatic field; indeed, in this case, the force per unit charge must be wholly determined by the dielectric displacement. On the other hand, the part of the force expressed by the second term may be derived from the law according to which an element of a wire carrying a current is acted on by a magnetic field with a force perpendicular to itself and the lines of force, an action, which in our units may be represented in vector notation by
>
> $$\mathbf{F} = \frac{s}{c}[\mathbf{i}\cdot\mathbf{h}], \qquad \left[\vec{F} = \frac{i\vec{s}\times\vec{h}}{c}\right] ,$$
>
> where $\mathbf{i}$ is the intensity of the current considered as a vector, and s the length of the element. According to the theory of electrons, $\mathbf{F}$ is made up of all the forces

[33] [Note by Lorentz:] In an earlier published derivation of the equations of motion (La théorie électromagnétique de Maxwell et son application aux corps mouvants), I have discussed the necessary conditions.

[34] [Lor92].

[35] [Lor09, pp. 14-15] and [O'R65, p. 561].

[36] [Lor15, pp. 14-15].

with which the field $\mathbf{h}$ acts on the separate electrons moving in the wire. Now, simplifying the question by the assumption of only one kind of moving electrons with equal charges e and a common velocity $\mathbf{v}$, we may write

$$s\mathbf{i} = Ne\mathbf{v} \ , \qquad [i\vec{s} = Ne\vec{v}] \ ,$$

if N is the whole number of these particles in the element s. Hence

$$\mathbf{F} = \frac{Ne}{c}[\mathbf{v}\cdot\mathbf{h}], \qquad \left[\vec{F} = \frac{Ne\vec{v}\times\vec{h}}{c}\right] \ ,$$

so that, dividing by Ne, we find for the force per unit charge

$$\frac{1}{c}[\mathbf{v}\cdot\mathbf{h}]. \qquad \left[\frac{\vec{v}\times\vec{h}}{c}\right] \ .$$

As an interesting and simple application of this result, I may mention the explanation it affords of the induction current that is produced in a wire moving across the magnetic lines of force. The two kinds of electrons having the velocity $\mathbf{v}$ of the wire, are in this case driven in opposite directions by forces which are determined by our formula.

9. After having been led in one particular case to the existence of the force $\mathbf{d}$, and in another to that of the force $\frac{1}{c}[\mathbf{v}\cdot\mathbf{h}]$, we now combine the two in the way shown in the equation (23), going beyond the direct result of experiments by the assumption that in general the two forces exist at the same time. If, for example, an electron were moving in a space traversed by Hertzian waves, we could calculate the action of the field on it by means of the values of $\mathbf{d}$ and $\mathbf{h}$, such as they are at the point of the field occupied by the particle.

O'Rahilly made two very important comments related to this deduction of Lorentz. These comments can also be applied to Maxwell's earlier and similar deduction presented in Subsection 1.3.1. It is difficult to disagree with O'Rahilly, when he noted that "the ordinary proof of the formula is extremely unsatisfactory", adding that:[37]

> There are two overwhelming objections to this alleged generalization. (1) The two 'particular cases' he 'combined' are quite incompatible. In the one case we have charges at rest, in the other the charges are moving; they cannot be both stationary and moving. (2) Experiments with a 'wire carrying a current' have to do with *neutral* currents, yet the derivation contradicts this neutrality.

O'Rahilly then quotes several current textbooks showing that the proofs they present of Lorentz force are equally unsatisfactory.

I can add a third very unsatisfactory aspect of Maxwell and Lorentz' deductions of their force laws, namely, they do not specify clearly the meanings associated with the velocity of the electrified particle. In order to grasp the meanings they gave to this velocity one needs to study carefully their papers, paying close attention to their words.

[37][O'R65, p. 561].

They begin with the force acting on a current element $Id\vec{\ell}$ and replace it by $q\vec{v}$. As pointed out by O'Rahilly, when Ampère obtained his expression for the force exerted by a closed circuit acting on a current element of another circuit, he was dealing with neutral current elements. Therefore it is wrong to replace $Id\vec{\ell}$ by $q\vec{v}$. Moreover, when there is a current flowing along a metal wire, the electrified particles are moving relative to the matter of the conductor, for instance, relative to the copper wire. Therefore, the first idea would be to consider this velocity $\vec{v}$ of the particle electrified with charge q as the drift velocity of the charge relative to the metal wire. But this is not what Maxwell and Lorentz did. As was shown in Subsection 1.3.1, Maxwell considered it as the velocity of the particle relative to the magnetic field.

Lorentz, on the other hand, considered it as the velocity of the particle relative to the ether. This can see from the above quotation ("[...] force as caused by the state of the ether, and even, since this medium pervades the electrons, as exerted by the ether [...]"). Therefore, for Lorentz the velocity $\vec{v}$ meant originally the velocity of the electrified particle relative to the ether and not, for instance, relative to the observer or frame of reference. He did not interpret this velocity as being the velocity of the test electrified particle relative to the magnetic field, nor relative to the observer or frame of reference. In Lorentz theory the ether was in a state of absolute rest relative to the frame of fixed stars.[38]

A conclusive proof of this interpretation of the velocity which appears in the magnetic force $q\vec{v} \times \vec{B}$ can be found in another work of Lorentz, *Lectures on Theoretical Physics*. This work is based on a course of Maxwell's theory presented in 1900-1902 and on another course on the principle of relativity for uniform translations presented in 1910-1912 which were first published in 1925 and 1922, respectively. Figure 1.3 shows our representation of the two situations he was considering. Lorentz considered three bodies, namely, the Earth E, the circuit carrying a constant current I, and the test particle with charge q. In situation (a) these three bodies were considered at rest relative to the ether and relative to the frame of fixed stars. In situation (b) these three bodies are moving together relative to the ether and relative to the frame of fixed stars with a common velocity $\vec{v}$.

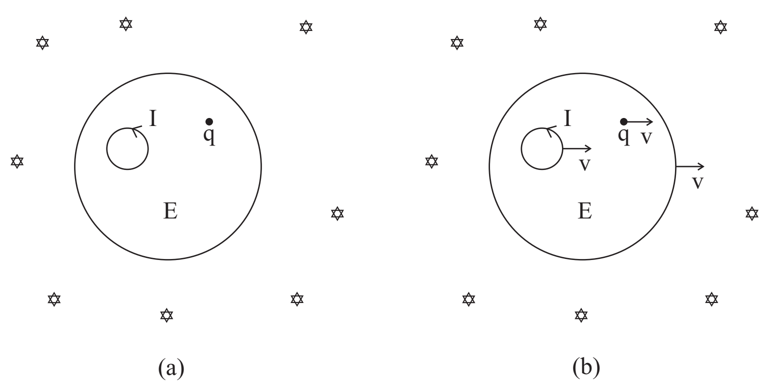

Figure 1.3: Earth E, circuit carrying a constant current I, and test particle with charge q. In configuration (a) these three bodies are at rest relative to the ether and relative to the fixed stars. (b) These three bodies move together with velocity $\vec{v}$ relative to the ether and fixed stars.

In this work Lorentz said:[39]

[38] [Pai82, p. 111].
[39] [Lor31, Volume 3, p. 306] and [O'R65, Volume 2, p. 566].

> 8·9. There is yet one problem worth of attention. Imagine an electric current flowing in a closed circuit without resistance. Will this current act upon a particle carrying a charge e which is placed in its neighbourhood? We purposely speak of a circuit without resistance. For, if it had a resistance, a certain electromotive force would be necessary to sustain the current, and this would unavoidably give rise to a potential gradient and to charges (no matter how small) spread over the conductor which would act upon the electrified particle. In fine, our question concerns the effect of the current *as such* upon the particle.
>
> The answer to this question was, of course, that the current did not act upon the particle. It would act upon a magnetic needle placed in the neighbourhood, since it is surrounded by a magnetic field, but there is no trace of an electric field. This is certainly correct so long as the current and the electrified particle are at rest. Suppose, however, that both share in some motion, *e.g.* the Earth's motion. What then? To begin with, the charged particle will move with a certain velocity through the magnetic field of the current and it will thus be acted upon by some force.

In this work Lorentz said that when a current carrying wire and an external electrified particle are at rest relative to one another, and also at rest relative to the ether, then no magnetic force would act on the particle. In his words: "the current did not act upon the particle."

On the other hand, if the current carrying wire and the electrified particle were at rest relative to one another, but if both were moving with the same velocity $\vec{v}$ relative to the ether, then there would be a magnetic force acting on the particle. In his words: "the charge particle will move with a certain velocity through the magnetic field of the current and it will thus be acted upon by some force." In this last situation there was no motion between the electrified particle and the current carrying circuit, nor between the particle and the Earth or laboratory, nor even between the particle and the observer or detector of magnetic field (who are supposedly at rest in the laboratory). But to Lorentz, even in this case there would be a magnetic force acting on the particle. He could only consider this possibility because he supposed $\vec{v}$ to be the velocity of the electrified particle relative to the ether or relative to the fixed stars. As the fixed stars did not cause any electromagnetic net force on the particle with charge q, all that remained was the force acting on the particle and being exerted by the ether.

In conclusion, to Lorentz the velocity appearing in his force given by Equation (1.16) was the velocity of the electrified particle relative to the ether. Moreover, he assumed this ether to be at rest relative to the frame of fixed stars.

1.3.4 Meaning of the Velocity According to Einstein

The velocity $\vec{v}$ of the test particle with charge q which appears in the force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ had several meanings according to different authors. According to Maxwell, it was the velocity relative to the magnetic field, according to Thomson and Heaviside it was the velocity of the electrified particle relative to the medium of magnetic permeability μ, while to Lorentz it was the velocity of the particle relative to a very specific medium, the ether. These three authors considered that this force did arise due to an interaction of the test electrified particle with three different entities, namely, (a) the magnetic field, (b) with a specific material medium of permeability μ, or (c) with the ether. As the electrified particle was supposed to be interacting with these material media, it was natural to interpret $\vec{v}$ as the velocity of the particle relative to these media.

Einstein changed all this with his paper of 1905 on the special theory of relativity. What Einstein proposed in this paper was that the velocity $\vec{v}$ which appears in Equation (1.4), should be interpreted as the velocity of the electrified particle relative to the observer.[40] He initially obtained Lorentz transformations for the spatial coordinates and for time. These transformations relate the magnitudes in one inertial frame to another inertial frame moving relative to the first frame with a constant linear velocity. Einstein then obtained these transformations also for the electric and magnetic fields. He applied these transformations for the electric and magnetic fields in the electromagnetic force given by Equation (1.4). In this way Einstein began to utilize the velocity $\vec{v}$ as being the velocity of the particle with charge q relative to the observer, or as a velocity relative to the inertial frame of reference. For instance, in this paper Einstein gave the difference between the old paradigm of electromagnetism and the new one based on his theory of relativity (passages in the footnotes are our words):[41]

> As to the interpretation of these equations[42] we make the following remarks: Let a point charge of electricity have the magnitude "one" when measured in the stationary system K,[43] i.e. let it when at rest in the stationary system exert a force of one dyne upon an equal quantity of electricity at a distance of one cm. By the principle of relativity this electric charge is also of the magnitude "one" when measured in the moving system. If this quantity of electricity is at rest relatively to the stationary system, then by definition the vector $(X,\ Y,\ Z)$[44] is equal to the force acting upon it. If the quantity of electricity is at rest relatively to the moving system (at least at the relevant instant), then the force acting upon it, measured in the moving system, is equal to the vector $(X',\ Y',\ Z')$. Consequently the first three equations above[45] allow themselves to be clothed in words in the two following ways:
>
> 1. If a unit electric point charge is in motion in an electromagnetic field, there acts upon it, in addition to the electric force,[46] an "electromotive force" which, if we neglect the terms multiplied by the second and higher powers of v/c, is equal to the vector-product of the velocity of the charge and the magnetic force, divided by the velocity of light.[47] (Old manner of expression.)[48]
>
> 2. If a unit electric point charge is in motion in an electromagnetic field, the force acting upon it is equal to the electric force which is present at the locality of the

[40] See footnote 4.

[41] [Ein52, p. 54] and [Ein78, p. 71].

[42] [Note by AKTA:] Equations for the Lorentz transformations of the electric and magnetic field components in two different inertial frames of reference which move relative to one another with a constant velocity.

[43] [Note by AKTA:] A system of coordinates in which the equations of newtonian mechanics hold good.

[44] [Note by AKTA:] This vector $(X,\ Y,\ Z)$ represents the electric force per unit charge. That is, it is the vector electric field, which nowadays would be expressed as: $\vec{E} = (E_x,\ E_y,\ E_z)$.

[45] [Note by AKTA:] That is, equations for the transformation of the electric and magnetic field components in two different inertial systems which move relative to one another with a constant velocity.

[46] [Note by AKTA:] That is, beyond the force $q\vec{E}$.

[47] [Note by AKTA:] This "electromotive force" would then be given by $q\vec{v} \times \vec{B}$ in the International System of Units.

[48] [Note by AKTA:] Therefore, in this old manner of expression, the net force on the test particle with charge q would be given by $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$.

charge, and which we ascertain by transformation of the field to a system of coordinates at rest relatively to the electrical charge. (New manner of expression.)

Following Einstein, let us call K the stationary inertial system and k the inertial system which is moving relative to K with a constant velocity v. I will utilize primed symbols for the magnitudes expressed in k. According to Einstein, the charge of the electrified particle has the same value in both coordinate systems, $q' = q$. Moreover, the particle moves with velocity $\vec{v}$ relative to K and is stationary relative to k, that is, $\vec{v}\,' = 0$. Therefore, according to Einstein, the net force acting on the electrified particle in K would be given by $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. On the other hand, the net force acting on the electrified particle in the frame k would be given by $\vec{F'} = q'\vec{E}' + q'\vec{v}\,' \times \vec{B}' = q\vec{E}'$, as $q' = q$ and $\vec{v}\,' = \vec{0}$.

This is the crucial passage in which Einstein introduced frame-dependent forces, that is, forces whose value depend on the motion between the test body and the observer. He presented here a completely new meaning for the velocity $\vec{v}$ which appears in Equation (1.4), namely, velocity of the electrified particle relative to the observer or relative to the frame of reference.

The introduction of physical forces which depend on the state of motion of the observer has created many problems and paradoxes for the explanation of several simple phenomena of nature. Unfortunately it has been part of theoretical physics ever since that time. No experiment has suggested or forced this new interpretation. This whole interpretation arose from Einstein's mind. The usual expression for the magnetic force might have been maintained, interpreting $\vec{v}$ as the velocity of the test electrified particle relative to the magnet or relative to the current carrying wire, without any contradictions with experimental data. Instead of adopting this reasonable point of view, Einstein decided to change the interpretation of this velocity. He created an enormous confusion with this new interpretation which has plagued theoretical physics ever since 1905.

1.4 Forces Depending on Velocity in Newtonian Mechanics

In newtonian mechanics there are also forces which depend on the velocity of the test body. However, these forces depend only on the *relative velocity between the interacting bodies*. That is, these forces do not depend on the velocity of the test body relative to the observer, nor on the velocity of the test body relative to the frame of reference. I present here two examples, one in mechanics and the other in electromagnetism.

Suppose a parachutist falling to the ground after leaving an airplane which was flying horizontally. Its initial vertical velocity relative to the ground is zero. Due to its weight, the person is initially accelerated downwards. An upward force due to air resistance begins to act on the body. This dragging force increases with the velocity of the body relative to air. The vertical velocity increases until it reaches a terminal constant velocity relative to the ground. In this last situation the weight is balanced by the dragging force exerted by the air. This dragging force depends only on the *relative velocity* $\vec{v}_r$ between the test body and the air around it. This relative velocity for a rigid body is given by $\vec{v}_r = \vec{v} - \vec{v}_f$, where $\vec{v}$ is the velocity of the body relative to the ground, while $\vec{v}_f$ is the velocity of the surrounding fluid relative to the ground. Suppose air is at rest relative to the ground, $\vec{v}_f = \vec{0}$. Let $\vec{v}_t$ represent the terminal velocity of the parachute relative to the ground. Let us analyze the problem from the terrestrial point of view when the body is falling with this terminal constant velocity relative to the ground. The dragging force acting on it will depend on the relative velocity $\vec{v}_r = \vec{v} - \vec{v}_f = \vec{v}_t - \vec{0} = \vec{v}_t$. By equating the weight of the body with the upward dragging force, one can relate the terminal velocity of the body with its weight, shape, air density etc. Let me now analyze the problem from the parachutist point of view when he is falling with the terminal velocity relative to the ground. Although the parachutist is not moving relative to himself, the dragging force acting on it will not be zero from his own point of view. Let S' be the frame of the parachutist. As he is at rest relative to himself, he has a zero velocity, $\vec{v}\,' = \vec{0}$. The air around him, on the other hand, is moving upwards and has a velocity $\vec{v}_f\,'$ different from zero. When the parachutist is falling at terminal velocity relative to the ground, the air around him is moving upwards relative to him with a constant velocity given by $\vec{v}_f\,' = -\vec{v}_t$. Therefore, the relative velocity between the parachutist and the fluid around him will be given by $\vec{v}_r\,' = \vec{v}\,' - \vec{v}_f\,' = \vec{0} - (-\vec{v}_t) = \vec{v}_t$. That is, this relative velocity has the same value it had in the terrestrial frame of reference. This means that one can solve the problem not only in the terrestrial frame of reference, but also in the frame of reference of the parachutist falling with its terminal velocity. No paradoxes appear here and it is not necessary any transformation of the gravitational field. It is also not necessary any transformation of the dragging force in two different inertial frames of reference.

The second example is that of Ohm's law. When a potential difference $\Delta\phi$ is applied between the terminals of a circuit with a resistance R, a constant current I will flow along the circuit as given by $\Delta\phi = RI$. Microscopically this Ohm's law can be written at a certain point inside the conductor as $\vec{J} = -\sigma\nabla\phi = \sigma\vec{E}$, where $\vec{J}$ is the volume current density, σ is the conductivity of the medium and $\vec{E}$ is the force per unit charge acting at this point. In the case of metals, only conduction electrons move relative to the wire. The volume current density can then be written as $\vec{J} = \rho_-\vec{v}_-$, where ρ_- is the volume density of negative charges and $\vec{v}_-$ is the *relative velocity* between the electron and the wire. This relative velocity can be written as $\vec{v}_- = \vec{v}_q - \vec{v}_w$, where $\vec{v}_q$ represents the velocity of the conduction electron relative to the ground, while $\vec{v}_w$ represents the velocity of the wire relative to the ground. As in the previous case of the parachutist, one can analyze Ohm's law not only in a frame of reference at rest relative to the wire, but also in a frame of reference which is at rest relative to a specific conduction electron. In both cases there will be the same force exerted by the wire on the electron, as the relative velocity between them is the same, no matter the inertial frame of reference.

1.5 These Different Meanings Given to the Velocity in the Classical Electromagnetic Force Law Imply *Different* Field Theories

The so-called Lorentz force can be written as follows:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \; . \tag{1.17}$$

I have been dealing with the meaning of the velocity in this force since my undergraduate years:[49]

[49][Ass21, pp. 16-17].

> During my undergraduate and graduate studies in physics I was introduced to the so-called Lorentz force. As usually mentioned in the textbooks, if an electrified particle with charge q is moving with velocity $\vec{v}$ in the presence of an electric field $\vec{E}$ and a magnetic field $\vec{B}$, then the Lorentz force $\vec{F}$ acting on this charge is given by $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. The textbooks usually do not specify the meaning of this velocity. Is it the velocity of the charge q relative to what? This force can only be applied or utilized when we know the meaning of this velocity. When I discovered that nowadays this velocity $\vec{v}$ is interpreted as the velocity of the charge q relative to the observer, I did not accept it. This interpretation was against my physical intuition, after all the charge q is not interacting with the observer. It is interacting with other electrified particles.

I then began to make a historical search related to the meaning of this velocity in the so-called Lorentz force.[50] I concluded my 1994 analysis of this topic with the following words:

> This change in the meaning of $\vec{v}$ in $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ is very strange, confusing and unusual in physics.

In the last few years I have been finally aware that this is not only strange, confusing and unusual in physics. As I show in Section 1.1, by changing the meaning of the magnitudes appearing in the same mathematical formula, one changes the underlying theory. In particular, Equation (1.1) is a mathematical representation of Newton's law of gravitation when one interprets the distance r in this equation as given by Figure 1.1 (a), but it represents a different theory when one interprets the distance r as given by Figure 1.1 (b). Likewise, it is now clear to me that all of these different meanings given to the velocity appearing in Equation (1.17) imply different theories.

Lorentz, in particular, as shown in Subsection 1.3.3, interpreted the velocity appearing in Equation (1.17) as the velocity of the electrified particle relative to the ether. As in modern electromagnetic textbooks the velocity appearing in this Equation is interpreted as the velocity of the charge relative to the observer, it is **wrong** *to call it Lorentz force, as is the case in every textbook on electromagnetism.*

After I was convinced that Maxwell himself had deduced a similar formula before Lorentz, I decided to call this equation the Maxwell-Lorentz's force.[51] *However, as shown in Subsection 1.3.1, Maxwell interpreted the velocity appearing in this equation as the velocity of the electrified particle relative to the magnetic field. As in modern electromagnetic textbooks the velocity appearing in this equation is interpreted as the velocity of the charge relative to the observer, it is* **wrong** *to call it the Maxwell-Lorentz force, as I did in 2013 and 2014.*

The four different interpretations of the velocity discussed in Subsections 1.3.1, 1.3.2, 1.3.3 and 1.3.4 imply four different theories. These different theories should be called by different names. Maxwell, Heaviside, Lorentz and Einstein expressed the force acting on an electrified particle by the same formula, Eq. (1.17). However, as they gave different interpretations for the meanings of the velocity appearing in this equation, each of these force laws should receive a different name. In particular, if one interprets this velocity as the velocity of the particle relative to the observer or frame of reference, as is the case in all textbooks on electromagnetism, the names of Maxwell, Heaviside and Lorentz should not be associated with this force law.

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[50] See footnote 4.

[51] [Ass13, p. 240] and [Ass14, p. 274].

1.6 Comparison Between Lorentz and Weber's Deductions of Their Force Laws

In the last 130 years Weber's force (1846) has been replaced by the so-called Lorentz force (1895) in electromagnetic textbooks. The so-called Lorentz force has been presented side by side with Maxwell's equations as the basis of classical electromagnetic theory. In this Section I will compare the deduction of these two force laws. A comparison with similar conclusions might be made between Maxwell and Weber's deductions of their different force laws.

Lorentz presented the deduction of his force in 2 pages of his 1895 paper, and similarly in 2 pages of his book *The Theory of Electrons*.[52] Weber, on the other hand, presented the deduction of his force in 25 pages, Sections 18-21, of his 1846 treatise.[53]

1.6.1 Experimental Proof of Ampère's Force

To my knowledge Lorentz never performed a single experiment to arrive at his force law of 1895 presented in Subsection 1.3.3.

Weber, on the other hand, utilizes the first 50 pages of his paper of 1846 presenting a detailed experimental proof of Ampère's force for the interaction between electric currents.[54] To this end he introduced his famous electrodynamometer. According to Maxwell:[55]

> The experiments which he made with it furnish the most complete experimental proof of the accuracy of Ampère's formula as applied to closed currents, and form an important part of the researches by which Weber has raised the numerical determination of electrical quantities to a very high rank as regards precision. Weber's form of the electrodynamometer, in which one coil is suspended within another, and is acted on by a couple tending to turn it about a vertical axis, is probably the best fitted for absolute measurements.

He then utilized his electrodynamometer to test Faraday's law of induction. He also used it to many other applications.

1.6.2 Lorentz Utilized the Force Exerted by a Closed Circuit on a Current Element, While Weber Utilized the Force Between Two Current Elements

In order to obtain the magnetic component of his force, Lorentz considered a current element $id\vec{s}$ in a magnetic field $\vec{H}$ generated by external causes and said that according to a known law it suffers an electrodynamic force given by

$$id\vec{s} \times \vec{H} \ . \tag{1.18}$$

The unspecified external causes can be a magnet or a closed circuit. André-Marie Ampère (1775-1826) obtained in 1822 the final expression of his fundamental force law between two

[52] [Lor95, §12, pp. 21-22 and table after page 138], [Lor09, pp. 14-15] and [Lor15, pp. 14-15].

[53] [Web46, pp. 132-157 of Weber's *Werke*] with English translation in [Web21, pp. 129-149].

[54] [Web46] with a partial French translation in [Web87] and complete English translations in [Web07] and [Web21].

[55] [Max54, Vol. 2, Article 725, p. 371].

current elements. He then proved for the first time, experimentally and theoretically, that a current carrying closed circuit of arbitrary shape exerts a force on an external current element which is always orthogonal to this element and also orthogonal to a certain straight line which he called directrix. The direction of this directrix coincides with the direction of the magnetic field of modern theories, although it should be emphasized that Ampère himself never worked with the magnetic field concept. Later on Ampère also proved for the first time the equivalence between a magnetic dipole layer and a current carrying closed circuit, so that any magnet can be replaced by a system of closed loops carrying steady currents.[56]

In essence, Equation (1.18) represents the result first obtained by Ampère that the force acting on a current element due to a magnet or to a closed circuit of arbitrary shape is always orthogonal to this element and to the straight line given by Ampère's directrix.

Weber, on the other hand, in order to arrive at his force law decided to begin directly with Ampère's force between two current elements. This is a more basic force law, as Equation (1.18) was deduced by Ampère from his force between current elements. According to Maxwell, Ampère's force between current elements should always remain the most important formula of electrodynamics:[57]

> The experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science. The whole, theory and experiment, seems as if it had leaped, full grown and full armed, from the brain of the 'Newton of electricity.' It is perfect in form, and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electro-dynamics.

Moreover, in the deduction of his force law Weber began not only with Ampère's force between current elements, but also with Faraday's law of induction.

1.6.3 Lorentz Replaced a Neutral Current Element by Particles with a Net Charge, While Weber Replaced a Neutral Current Element by Oppositely Charged Particles

Lorentz then replaced the current element $id\vec{s}$ in Equation (1.18) by a sum of particles, each with a net charges e and moving with velocity $\vec{v}$, namely, $\sum e\vec{v}$. He then assumed that this would also be valid for a single ion, namely:

$$e\vec{v} \times \vec{H} \ . \tag{1.19}$$

O'Rahilly correctly pointed out that this proof is extremely unsatisfactory because there is an overwhelming objection to this generalization, namely, experiments with a wire carrying a current have to do with neutral currents, yet Lorentz derivation contradicts this neutrality by replacing $id\vec{s}$ with $e\vec{v}$ which has a net charge e.[58]

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[56] Ampère's masterpiece was published in 1826, [Amp26] and [Amp23]. There is a complete Portuguese translation of this work, [Cha09] and [AC11]. Partial English translations can be found at [Amp65] and [Amp69]. Complete and commented English translations can be found in [Amp12] and [AC15]. A huge material on Ampère and his force law between current elements can be found in the homepage *Ampère et l'Histoire de l'Électricité*, `http://www.ampere.cnrs.fr` and [Blo05], at the homepage of the Friends of André-Marie Ampère, `https://saama.fr`, and at the homepage of the Ampère Museum, `https://amperemusee.fr/en`.

[57] [Max54, Vol. 2, article 528, p. 175].

[58] [O'R65, p. 561].

Weber did not make this obvious mistake in his deduction of his own force law. He began with Ampère's force between two current elements ids and $i'ds'$. He then replaced the first current element by two equal and opposite electrified particles with charges e and $-e$ moving with velocities u and $-u$, while the second current element was replaced by two equal and opposite electrified particles with charges e' and $-e'$ moving with velocities u' and $-u'$. That is, each current element remained electrically neutral with Weber's replacement, which was not the case with Lorentz replacement.

1.6.4 The Different Meanings of the Velocities of the Electrified Particles Composing a Current Element

Lorentz and Weber began with the force acting on a current element ids of length ds. This current element belonged to a wire carrying a current i.

Lorentz replaced the current element with an electrified particle having charge e and moving with velocity v. That is, he replaced ids with ev. However, he did not consider this velocity as the drift velocity of the particle with charge e relative to the wire. As shown in Subsection 1.3.3, he considered it as the velocity of the charge relative to the ether.

Weber, on the other hand, replaced ids with particles with charges e and $-e$ moving with velocities u and $-u$, respectively. He considered these velocities as the velocities of the electrified particles relative to the material body of the conductor, as it is clear from the context of his detailed discussion in his 1846 paper. This conductor might be, for instance, a copper wire. Therefore the velocities u and $-u$ in Weber's deduction meant the drift velocities of the electrified particles relative to this copper wire.

Three further aspects should be emphasized here.

- In 1846 Weber assumed a double current in each current carrying wire. In particular, he assumed Fechner's hypothesis of 1845, namely, he supposed in each current element equal and opposite electrified particles moving relative to the wire with equal and opposite drift velocities.[59] This was a common assumption at that time. However, it can be shown that one also deduces Ampère's force between current elements from Weber's force between electrified particles without imposing any restrictions on their velocities. That is, one can assume current element ids composed of charges e and $-e$ moving relative to the wire with arbitrary velocities u_1 and u_2, while current element $i'ds'$ is composed of charges e' and $-e'$ moving relative to the wire with arbitrary velocities u'_3 and u'_4. These four velocities are each one of them arbitrary and independent from one another. This means that, beginning with Weber's force, one arrives at Ampère's force between current elements even in metallic circuits in which the positive charges are fixed in the lattice ($u_1 = u'_3 = 0$), while only the moving electrons are responsible for the current. This will also happen when the positive and negative charges move in opposite directions with velocities of different magnitudes (as in the situations of electrolysis).[60]

- Later on Weber changed his mind and assumed a single current in each current carrying wire. In particular he began to suppose that the particles electrified with charges of one sign remained at rest relative to the lattice of the conductor, while only the particles

[59] [Fec45] with English translation in [Fec21].
[60] [Ass90], [Wes90], [Ass94, Section 4.2: Derivation of Ampère's force from Weber's force] and [Ass15b].

electrified with charges of the other sign were responsible for the current, moving with a drift velocity relative to the wire. He was not sure if the positive charges should be assumed at rest, while only the negative charges would be moving relative to the wire; or if the negative charges should be assumed at rest, while only the positive charges would be moving relative to the wire. A detailed discussion of this topic with many quotations can be found in Section 1.5 (The Evolution of Weber's Conception of an Electric Current: From a Double Current to a Simple Current) of our book Weber's Planetary Model of the Atom.[61]

- The third fact to emphasize is that the electron was only discovered in 1897, six years after Weber's death (1891). In any event with this assumption of a single current Weber was much ahead of his time.

1.6.5 The Final Velocities Appearing in Lorentz and Weber's Force Laws

Lorentz final expression of his force law depends on the velocity of electrified particle relative to the ether, as was shown in Subsection 1.3.3.

We now consider Weber's force law. Weber began with Ampère's force law between two current elements. He then replaced the first current element ids by the particles oppositely electrified with charges e and $-e$, moving relative to the wire with drift velocities u and u'. The second current element $i'ds'$ was similarly replaced by electrified particles e' and $-e'$ moving relative to the wire with velocities u' and $-u'$. However, in the development of his force law, he ended up with the *relative velocities*. That is, if r is the distance between any electrified particle of one current element and any electrified particle of the other current element, Weber's final force law depends only on the product of the values of these charges, on their distance r, on their *relative velocity* dr/dt and also on their *relative acceleration* d^2r/dt^2.

Newton and Coulomb's force laws depend on the distance r between the interacting particles. Weber's force is a generalization of Coulomb's law depending also on the *relative* velocity, dr/dt, and *relative* acceleration, d^2r/dt^2, between the interacting particles.[62] These are *relational magnitudes* which have the same values for all observers and for all frames of reference. These magnitudes r, dr/dt and d^2r/dt^2 have the same value even when comparing an inertial frame of reference with a non inertial frame of reference. They are intrinsic magnitudes related only to the interacting bodies.[63] Moreover, Weber's force points along the straight line connecting the two interacting particles and complies with Newton's action and reaction law. These were the main reasons which made me work with Weber's electrodynamics ever since I encountered it in my undergraduate studies.[64]

1.6.6 Analysis and Synthesis in Weber's 1846 Treatise

After breaking down Ampère's force between current elements and Faraday's law of induction in order to deduce his force law between electrified particles, Weber reverts the arguments.

[61] [AWW11] with Portuguese translation in [AWW14] and German translation in [AWW18].

[62] See footnote 54.

[63] [Ass13, Section 2.8] and [Ass14, Section 2.8 (Weber's force between electrified bodies) and Appendix A (Relational magnitudes)].

[64] [Ass21, pp. 16-17].

In the final 57 pages of his 1846 treatise, Sections 22-32, he postulates his force law and deduces a series of consequences.[65] He begins Section 22 with the following words:

> Having attained the *fundamental electrical law* expressed in the previous Section, we can place it at the head of the theory of electricity, and from it synthetically derive a system of consequences, which is the ultimate purpose of such a law.
>
> [...]
>
> For moving electricity, first the uniform motion of the electricity of galvanic currents in conductors at rest is to be considered, to which Ampère's law relates. Now, since the above fundamental electrical law was developed analytically from Ampère's law, Ampère's law must in turn follow synthetically from this fundamental law. This derivation is actually to be given here.

He then deduces several consequences from his force law:

- The laws of Coulomb and Poisson when there is no motion between the interacting particles.
- Ampère's force between two current elements.
- The law of the electrodynamic action of a closed circuit on a current element.
- The law of the electromagnetic action of a magnet on a current element.
- Ampère arrived at his force between two current elements by considering the interaction between wires carrying steady currents. Weber now shows that Ampère's force will also be valid with variable current intensities.
- Faraday's law of induction.
- Etc.

All of this reminds us of Newton's approach to inverse problems.[66]
In the last Query of his book *Opticks* Newton expressed himself as follows:[67]

> As in mathematicks, so in natural philosophy, the investigation of difficult things by the method of analysis, ought ever to precede the method of composition. This analysis consists in making experiments and observations, and in drawing general conclusions from them by induction, and admitting of no objections against the conclusions, but such as are taken from experiments, or other certain truths. For hypotheses are not to be regarded in experimental philosophy, and although the arguing from experiments and observations by induction be no demonstration of general conclusions; yet it is the best way of arguing which the nature of things admits of, and may be looked upon as so much the stronger, by how much the induction is more general. And if no exception occur from phaenomena, the conclusion may be pronounced generally. But if at any time afterwards any exception shall occur from

[65] [Web46, pp. 157-214 of Weber's *Werke*] with English translation in [Web21, pp. 149-203].
[66] [Ass98], and [Ass11] with Portuguese translation in [Ass17].
[67] [New79, pp. 404-405] with Portuguese translation in [New96, pp. 292-293].

> experiments, it may then begin to be pronounced with such exceptions as occur. By this way of analysis we may proceed from compounds to ingredients, and from motions to the forces producing them, and from particular causes to more general ones, till the argument end in the most general. This is the method of analysis: and the synthesis consists in assuming the causes discovered, and established as principles, and by them explaining the phaenomena proceeding from them, and proving the explanations.
>
> In the first two books of these Opticks, I proceeded by this analysis to discover and prove the original differences of the rays of light in respect of refrangibility, reflexibility, and colour, and their alternate fits of easy reflexion and easy transmission, and the properties of bodies, both opake and pellucid, on which their reflexions and colours depend. And these discoveries being proved, may be assumed in the method of composition for explaining the phaenomena arising from them: an instance of which method I gave in the end of the first book.

Newton formalized his general approach in science at the Preface of the first edition of his book *Mathematical Principles of Natural Philosophy*:[68]

> I offer this work as the mathematical principles of philosophy, for the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena; and to this end the general propositions in the first and second books are directed. In the third book I give an example of this in the explication of the system of the world; for by the propositions mathematically demonstrated in the former books, in the third I derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and the several planets. Then from these forces, by other propositions which are also mathematical, I deduce the motions of the planets, the comets, the moon, and the sea.

In the third book of the *Principia*, Newton presented six phenomena comprising Kepler's laws. From these phenomena he deduced that the force of gravitation is inversely proportional to the square of the distances. After arriving at this result, he begins the opposite process. That is, starting from a force of gravitation varying as $1/r^2$, he deduces Kepler's laws and other results. On example:[69]

> Proposition XIII, Theorem XIII. *The planets move in ellipses which have their common focus in the centre of the sun; and, by radii drawn to that centre, they describe areas proportional to the times of description.*
>
> We have discoursed above on these motion from the phenomena. Now that we know the principles on which they depend, from those principles we deduce the motions of the heavens *a priori*.

1.6.7 Comparison Between Lorentz and Weber's Force Laws

In the previous Subsections I compared Lorentz and Weber's deductions of their force laws.

[68] [New34, p. xvii] with Portuguese translation in [New90, pp. I-II], [New08] and [New10].

[69] [New34, p. 420] with Portuguese translation in [New08, p. 210].

In earlier works I made a direct comparison of their final expressions. There I show, for instance, situations in which the so-called Lorentz force does not comply with Newton's action and reaction law; the prediction of a missing torque utilizing Lorentz force which has never been observed experimentally, etc.[70]

Now anyone can compare Lorentz and Weber's deductions and final expressions of their force laws by reading their original German texts or their English translations.[71]

Everyone can then make their own decision about which expression is best, which should be developed theoretically and explored experimentally. Which of these theories will you dedicate your time and effort to?

[70] [Ass92], [Ass94, Chapter 6: Forces of Weber and of Lorentz], [Ass95] and [Ass15b].

[71] [Lor95] with English translation in https://en.wikisource.org/wiki/Translation:Attempt_of_a_Theory_of_Electrical_and_Optical_Phenomena_in_Moving_Bodies, [Lor09] and [Lor15]; and [Web46] with English translation in [Web21].

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