Remarks on Neumann's Theory of Induced Currents

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Abstract

English translation of Wilhelm Weber's 1849 paper "Bermerkungen zu Neumann's Theorie inducirter Ströme", [Web49].

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By Wilhelm Weber[1,2,3]

Meeting of 17 March

Mr. *Wilhelm Weber* presents a treatise on *Electrodynamic Measurements*[4] as a continuation of his studies published under the same title in the treatises published by the Princely Jablonowsky Society at the request of the Royal Saxon Society of Sciences. The former treatise[5] had the measurements of the electrodynamic forces; the latter,[6] on the other hand, deals with the measurements of electrodynamic resistances. The [latter] treatise is divided into three parts:

1. resistance measurements according to given basic units;

2. attributing absolute units to resistance measurements;

3. relationship of the resistance measurements with the other electrodynamic measurements.

Since the content of the treatise is not suitable for a short report, and the treatise itself will soon appear in print,[7] the present communication is to be limited to a remark attached at the end of Neumann's treatise: "On a general principle of the mathematical theory of induced electric currents", especially printed from the writings of the Berlin Academy of Science of 1847; Reimer 1848.[8] In this treatise Neumann has established the following theorem:

> "If a closed, unbranched, conducting curved system $A_|$ is transferred from one form and position to another $A_{||}$ by an arbitrary

[1] [Web49].

[2] Translated by Elisabeth Becker-Schmollmann, Germany, and edited by A. K. T. Assis, www.ifi.unicamp.br/~assis. We thank L. Hecht and R. W. Gray for relevant suggestions.

[3] The Notes by H. Weber, the editor of the third volume of Weber's *Werke* are represented by [Note by HW:], while the Notes by A. K. T. Assis are represented by [Note by AKTA:]. AKTA has also introduced the words between square brackets in the middle of some sentences.

[4] [Note by AKTA:] The general title of Weber's eight major memoirs was *Elektrodynamische Maassbestimmungen*.

[5] [Note by AKTA:] [Web46] with partial French translation in [Web87] and a complete English translation in [Web07].

[6] [Note by AKTA:] [Web52].

[7] [Note by AKTA:] It was published in 1852, [Web52].

[8] [Note by AKTA:] [Neu48] and [Neu49].

> displacement of its elements, but without cancellation of the conductive connection of the same, and if this change from $A_|$ to $A_{||}$ occurs under the influence of an electric current system $B_|$, which simultaneously experiences a change in position, form and [current] intensity from $B_|$ to $B_{||}$ by any displacement of its elements, then the sum of the electromotive forces induced in the conductive curved system by the changes, is equal to the differences (multiplied by the induction constant ε) between the potential values of the current $B_{||}$ with respect to $A_{||}$ and of the current $B_|$ with respect to $A_|$ when $A_{||}$ and $A_|$ are thought to be traversed by one unit of current."

After Neumann has developed this theorem and its conclusion in the first four paragraphs of this treatise, he continues with § 5:

> "W. Weber has paved the way in his treatise: "Electrodynamic Measurements" and so on, which will lead across the gap in our knowledge of the electrostatic and electrodynamic effect of electricity. He shows how Ampère's laws for the action of two current elements[9] can be derived from the action of the positive and negative electricity of one element on the two electricities of the other element. This analysis of Ampère's law led to the fundamental law for the action of two electric masses, according to which it depends not only on their relative distance, but also on their relative velocity and its change. As Weber has shown, this fundamental law also explains the induction phenomena and gives their laws. The object of this paragraph is to prove to what extent the results contained in the foregoing agree with the laws of induction derived from Weber's fundamental law of electric action."

Neumann now develops the general expression of induction from the fundamental law of [Weber's] "Electrodynamic Measurements" and then applies it to various types of induction:

- in the case where neither the current nor the conductor elements suffer a change of location and arrives at a law which is identical to his own;
- in the case where the induction is excited solely by the displacement of the conductor elements, which takes place under the influence of a stationary or constant current;

[9] [Note by AKTA:] André-Marie Ampère (1775-1836). See [Amp23] and [Amp26], with a complete English translation in [AC15].

- in the case where the induced conductor is at rest and the induction is excited by the displacement of an entire conductor carrying a constant current.

In all these cases, the laws are perfectly consistent with Neumann's laws.

> "The situation is different", continues Neumann, "with the equation which expresses the induced electromotive force [produced] by a simple circulating current[10] consisting of a moving conductor section and a resting one... The sum of the electromotive force that gets excited during the circulation of the elements of the inductor is the same according to both formulas, but the direction of the induced current is the opposite."

The observation decides in favor of Neumann's formula. [Neumann continues:]

> "It must therefore be investigated what was missing when deriving the formula from Weber's fundamental law. The fact that the contradiction in question occurs only for the inductor with sliding contacts leads the consideration immediately to this [inductor with sliding contacts]. Here new elements enter or leave the path in which the current strength changes within a very short time from 0 to i or from i to 0, and which by this change in their [current] intensity they produce an inducing effect, which is already contained in my formulas, but which must still be considered with the application of Weber's fundamental law."

Neumann really finds the error of the derivation in the neglect of an essential part of the induction; the difference of the results, however, is only half compensated by consideration of this error.

Notwithstanding now the experiments made by Mr. Neumann, which have been repeated by the author,[11] leave no doubt as to which result is the right one; so Mr. Neumann continues nevertheless:

[10] [Note by AKTA:] Original German text: *einfachen Stromumgang*, which is being translated here as a simple circulating current. In this work Neumann considers an inductor which has a sliding contact, so that the circuit has a stationary open circular component and another component which rotates relative to the laboratory.

[11] [Note by AKTA:] Weber's repetition of Neumann's experiments appear in Section 38 of his second paper on Electrodynamic Measurements of 1852, [Web52, Section 38, pp. 409-417 of Weber's *Werke*].

> "Weber's fundamental formula of electric action has proved itself in so many and different cases that the same cannot be made doubtful by the above remarks, rather the manner in which it is applied to the present case must be called into question."

This is now followed by the supplement to Neumann's calculation given by the author [that is, given by Weber] at the end of his treatise.[12] Mr. Neumann, as has been mentioned, has developed the formulas for two parts of the electromotive force from [Weber's] fundamental law of "Electrodynamic Measurements" which is exerted by a simple rotating current on a stationary conductor, in the case in which the circuit of the inducing current consists of a moving conductor section and a stationary conductor section. These two formulas are indeed quite correct and are in complete agreement with the formulas developed in the "Electrodynamic Measurements" for these two parts. After the author[13] has proved this, the same author shows that it is essentially a question of whether the two parts of the electromotive force to which these formulas apply complement each other in such a way that together they really represent the entire electromotive force in the case under consideration, or whether in this case there is still a third part for which Neumann has not yet developed the formula from the fundamental law of the "Electrodynamic Measurements". The author actually proves such a third part, then develops the formula for this part, too, from the fundamental law of "Electrodynamic Measurements" and shows how the whole sum which results in the formulas of all three parts, is in complete agreement with Neumann's law and thus also with experience.

In the case under consideration the circuit through which the inducing current flows decomposes into two parts that must be essentially distinguished from each other, namely, the moving conductor section and the dormant one. The first formula which Neumann developed from the fundamental law of "Electrodynamic Measurements" represents the part of the electromotive force which electricity exerts as it flows through the moving conductor piece. The second formula represents that part of the electromotive force which electricity exerts as it flows through those elements of the dormant conductor through which the current had not previously passed (or as it ceases to flow through those elements of the resting conductor through which the current had previously flowed).

Just as it is not enough, however, if the intensity of an inducing current suddenly changes, to take into account the movement of the electric fluids

[12] [Note by AKTA:] Weber is here referring to the calculations he presented in Section 39 of his paper published in 1852, [Web52, Section 39, pp. 417-427 of Weber's *Werke*].

[13] [Note by AKTA:] That is, Weber in his paper of 1852.

before and after this change, but also the transition of one movement into the other necessarily must be considered, it is also not enough that in the case under consideration, the movements of the electric fluids are taken into account both during the time in which they flow through the moving conductor section and also during the time when they are in the resting part, but the change of their movement must finally also be taken into account during the transition, and this gives the third part of the electromotive force in the case under consideration, for which the formula has not yet been developed by Neumann from the fundamental law of the "Electrodynamic Measurements".

If α is the current element at the transition point and u denotes the velocity at which the end of the moving conductor piece moves forward, it is clear that, for example, the positive electricity which passes from the moving conductor piece to the stationary one loses the velocity u in the element of time dt where it flows through α, which is as much as if it had received the velocity $-u$; and that the negative electricity which passes from the stationary conductor piece to the moving one receives the velocity $+u$ in the said element of time.

If the part of the movement which the electric fluids shared with their carrier is generally designated as v, then, if the speed of this carrier does not change, the part of the movement of the electric fluids also designated as v will not, as a rule, undergo any change either; in the case under consideration, however, this rule suffers an exception in the transition element α; because it follows from what has been said that while nothing at all changes in the movement of the moving conductor piece, the part of the movement of the positive electricity contained in α designated with v in the element of time dt, where it flows through α, suffers a decrease $-u$, and the part of the movement of the negative electricity contained in α also designated with v suffers an increase $+u$ in the same element of time.

In fact, the transition element α cannot be considered a current element at all, because the movements of the electric fluids in this element do not satisfy the conditions contained in the definition of galvanic currents.

The fundamental law laid down for electric actions in the "Electrodynamic Measurements" now applies in general, whatever movements the electric fluids may have, but the applications which have been made in the said place by this fundamental law refer, as expressly noted there, only to such electric fluids which are in real current movement. The general law of Volta induction[14] developed there also only expresses the electromotive force ex-

[14][Note by AKTA:] The expression utilized by Weber, *Volta-Induktion*, had been first suggested by Faraday himself in paragraph 26 of his first paper on electromagnetic induction published in 1832, see [Far32, §26, p. 267 of the *Great Books of the Western World*] and [Far11, p. 159]:

erted by a real current element and is therefore not directly applicable to the transition element α in the case under consideration here.

However, since there was no other reason for this restriction of the application of the general fundamental law than that most other movements of the electric fluids, except in the current elements, were not yet sufficiently determined precisely for such an application, it is self-evident that as soon as this determination is given for any other case which does not occur in current elements, as has just happened in the transition element of the case under consideration, there is nothing to prevent the application of the general fundamental law in this case either.

Actually the author[15] now develops the complete expression of the electromotive force exerted by the transition element from the aforementioned fundamental law, and the result is an expression composed of three parts, the first two of which are identical to the expression given in the "Electrodynamic Measurements" for a current element. In the case considered by Neumann, the third part that is added is the same as the second of the two parts, the value of which is doubled as a result, and it is this doubling that Neumann has already recognized as necessary.

In the "Electrodynamic Measurements" the following general expression of the electromotive force exerted by an inducing element α on an induced element α' is given:[16,17]

$$-\frac{\alpha\alpha'}{r^2}i\left(\sin\vartheta\sin\eta\cos\varepsilon-\frac{1}{2}\cos\vartheta\cos\eta\right)av\cos\vartheta'-\frac{1}{2}\frac{\alpha\alpha'}{r}a\cos\vartheta\cos\vartheta'\cdot\frac{di}{dt}\ .$$

This expression only applies to real inducing current elements. If α should also encompass the transition element mentioned above, the following expression is obtained using the same designations as stated in Article 30 at the place indicated:[18]

$$-\frac{\alpha\alpha'}{r^2}i\left(\sin\vartheta\sin\eta\cos\varepsilon-\frac{1}{2}\cos\vartheta\cos\eta\right)av\cos\vartheta'-\frac{1}{2}\frac{\alpha\alpha'}{r}a\cos\vartheta\cos\vartheta'\cdot\frac{di}{dt}$$
$$+\frac{1}{4}\frac{\alpha\alpha'}{r}a^2e\cdot\cos\vartheta\cos\vartheta'\left(\frac{dv}{dt}-\frac{dw}{dt}\right)\ ,$$

For the purpose of avoiding periphrasis, I propose to call this action of the current from the voltaic battery, *volta-electric induction.*

This phenomenon of Volta-induction is nowadays called Faraday's law of induction.

[15][Note by AKTA:] That is, Weber.

[16][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 202.

[17][Note by AKTA:] [Web46, p. 202 of Weber's *Werke*] and [Web07, p. 132].

[18][Note by AKTA:] Weber is here referring to Article 30 of his 1846 paper, [Web46, Article 30, pp. 196-207 of Weber's *Werke*] and [Web07, pp. 128-136].

where dv/dt and dw/dt denote the change, distinguishable for positive and negative electricity of that part of its speed, which it shares with its carrier. For any real current element α is now

$$\frac{dv}{dt} = \frac{dw}{dt} ,$$

which makes this expression the same as the previous one. However, for the transition element considered above

$$\frac{dv}{dt} = -\frac{dw}{dt} ,$$

and indeed for the duration of dt in which the electricity flows through the length of the current element α, that is for $dt = \alpha/u$,

$$dv = -dw = v ,$$

which gives the following expression consisting of three terms:

$$-\frac{\alpha\alpha'}{r^2} i \left(\sin\vartheta \sin\eta \cos\varepsilon - \frac{1}{2}\cos\vartheta\cos\eta\right) av\cos\vartheta' - \frac{1}{2}\frac{\alpha\alpha'}{r} a \cos\vartheta\cos\vartheta' \cdot \frac{di}{dt} + \frac{1}{2}\frac{\alpha'}{r} a^2 e \cdot \cos\vartheta\cos\vartheta' \cdot uv .$$

For all those elements α from which the first part of the induction calculated by Neumann originates, the second term is omitted in addition to the third term, because $di/dt = 0$, and only the first term remains:

$$-\frac{\alpha\alpha'}{r^2} i \left(\sin\vartheta \sin\eta \cos\varepsilon - \frac{1}{2}\cos\vartheta\cos\eta\right) av\cos\vartheta' .$$

For all those elements α from which the second part of the induction calculated by Neumann originates, the first term, besides the third term, also falls away, because $v = 0$, and only the second term remains:

$$-\frac{1}{2}\frac{\alpha\alpha'}{r} a \cos\vartheta\cos\vartheta' \cdot \frac{di}{dt} ,$$

and indeed for the duration of dt, in which the current i is generated in element α, that is for $dt = -\alpha/v$,

$$di = aeu ,$$

the electromotive force thus [becomes:]

$$-\frac{1}{2}\frac{\alpha\alpha'}{r}\cdot a^2e\cdot\cos\vartheta\cos\vartheta'\cdot u\ ,$$

or, because $\alpha=-vdt$,

$$+\frac{1}{2}\frac{\alpha'}{r}\cdot a^2e\cdot\cos\vartheta\cos\vartheta'\cdot uvdt\ .$$

Finally for the transition elements α, for which Neumann has not calculated the induction, one obtains when their length is reduced, whereby the first two terms disappear, the limiting value

$$+\frac{1}{2}\frac{\alpha'}{r}\cdot a^2e\cdot\cos\vartheta\cos\vartheta'\cdot uv\ ,$$

or the amount of the electromotive force for the duration of the element of time dt,

$$+\frac{1}{2}\frac{\alpha'}{r}\cdot a^2e\cdot\cos\vartheta\cos\vartheta'\cdot uvdt\ ,$$

according to which the third part is the same as the second part calculated by Neumann, which was to be proved.

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