On a Simple Formulation of the General Fundamental Law of Electric Action

Wilhelm Weber

Editor’s Note: An English translation of Wilhelm Weber’s 1869 paper “Ueber einen einfachen Ausspruch des allgemeinen Grundgesetzes der elektrischen Wirkung”, [Web69].

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By Wilhelm Weber

In these Annalen 1848, Vol. 73, p. 193 and following, where I have given an excerpt from my first treatise on “Electrodynamic Measurements,” I have added on p. 229, that the expression given in that treatise for the general fundamental law of electrical action could be simplified by specifying the expression of the potential instead of the expression of the force, that is, the function of the coordinates $x$, $y$, $z$, whose negative partial differential coefficients with respect to $x$, $y$, $z$, correspond to the components of the force parallel to these coordinates. If we denote by $e$, $e'$ two electric particles, by $r$ their distance from one another, and by $c$ a certain constant, then the expression of the force was

$$\frac{ee'}{r^2} \left( 1 - \frac{1}{c^2} \frac{dr^2}{dt^2} + \frac{2r}{c^2} \frac{d^2r}{dt^2} \right),$$

while the expression of the potential was

$$V = \frac{ee'}{r} \left[ 1 - \frac{1}{c^2} \left( \frac{dr}{dt} \right)^2 \right].$$

Neumann’s Lagrangian energy $W$, on the other hand, is expressed nowadays as:

$$W = \frac{ee'}{r} \left[ 1 + \frac{1}{c^2} \left( \frac{dr}{dt} \right)^2 \right].$$
Between every two electrical particles there is partly mutual motion, partly stimulation to mutual motion. If one calls the following values, namely that of mutual motion when no stimulation takes place, and that of mutual stimulation when no motion takes place, limit values, then the fraction missing from one limit value is always represented by an equal fraction of the other limit value.

The latter of the two limit values is the well-known electrostatic potential $\frac{ee'}{r}$, while the former limit value is always the same, namely the value of a mutual motion with the velocity $c$, which can be represented by $ac^2$. — If there is now a mutual motion between $e$ and $e'$ with the velocity $dr/dt < c$, the value of which is $a\frac{dr^2}{dt^2}$, and therefore the following fraction is missing at the first limit value

$$\frac{ac^2 - a\frac{dr^2}{dt^2}}{ac^2} = 1 - \frac{1}{c^2}\frac{dr^2}{dt^2},$$

then this missing fraction is represented by an equal fraction of the other limit value $ee'/r$, i.e. by $(ee'/r)(1 - (1/c^2)(dr^2/dt^2))$, which is the general expression of the potential, as given above.
If $e$ and $e'$ indicate the masses\footnote{\[Note by AKTA:] Weber is utilizing the symbols $e$ and $e'$ to indicate not only the values of the electric charges of the two particles, but also the values of their inertial masses. In 1871 he will represent the charges by $e$ and $e'$, while the inertial masses will be represented by $\varepsilon$ and $\varepsilon'$, [Web71] with English translation in [Web72].} of the electric particles and $\alpha$, $\beta$ the velocities of $e$ in the direction $r$ and perpendicular to it, $\alpha'$, $\beta'$ the same velocities for $e'$, after which $\alpha - \alpha' = dr/dt$ is the relative velocity of both particles, then we have

\[ \frac{1}{2} e (\alpha^2 + \beta^2) + \frac{1}{2} e' (\alpha'^2 + \beta'^2) \]

as the living force\footnote{\[Note by AKTA:] In German: lebendige Kraft, or vis viva in Latin. Term coined by G. W. Leibniz (1646-1716). Originally the vis viva of a body of mass $m$ moving with velocity $v$ relative to an inertial frame of reference was defined as $mv^2$, that is, twice the modern kinetic energy. For some years there was confusion in nomenclature and some authors called $mv^2/2$ by vis viva. What Weber calls here the living force (lebendige Kraft) of a particle should be understood as the modern kinetic energy, namely, $mv^2/2$.} or work belonging to the two particles, which expresses their motion, according to size, proportional to the moving masses and to the squares of their velocities. If we now set

for $\alpha$, \[ \frac{e\alpha + e'\alpha'}{e + e'} + \frac{e'\beta}{e + e'}, \]

for $\alpha'$, \[ \frac{e\alpha + e'\alpha'}{e + e'} + \frac{e\beta'}{e + e'}, \]

and note that $\alpha - \alpha' = dr/dt$, then one can represent this living force or work of the two masses $e$ and $e'$ in the following two parts, namely

\[ = \frac{1}{2} ee' \cdot \frac{dr^2}{dt^2} + \frac{1}{2} \left( \frac{(e\alpha + e'\alpha')^2}{e + e'} + e\beta^2 + e'\beta'^2 \right). \]

The former may be called the internal work, the latter the external work, because for the former the knowledge of the particles $e$ and $e'$ and the increase or decrease of their distance from one another is sufficient, while for the latter apart from the particles $e$ and $e'$, a fixed coordinate system must be given in order to be able to observe and measure the velocities $(e\alpha + e'\alpha')/(e + e')$, $\beta$ and $\beta'$.

It is now evident that this internal work \((1/2)[ee'/(e + e')] \cdot [dr^2/dt^2]\) is the exact value of the mutual motion of both particles, which was denoted above with \(a[dr^2/dt^2]\), so that \(a = (1/2)[ee'/(e + e')]\).
This internal work and the potential of the two particles $e$, $e'$ at the distance $r$ can have very different values, but if one value increases, the other decreases, and the increase and decrease are always in the same proportion. If the potential has decreased by $ee' / r$, the internal work has increased by $(1/2)[ee'/(e + e')]c^2 = ac^2$. If this internal work, which has taken the place of the vanished potential, is called the work equivalent of that potential, the work equivalent of an arbitrary potential $V$ results from the same relationship $= [rc^2 / 2(e + e')] \cdot V$.

The existing internal work and the work equivalent of the existing potential form together the sum of the existing internal work values. Understood in this way, the following simple formulation of our law results, namely:

For two electrical particles $e$ and $e'$, at any distance from each other, the sum of the existing internal work values is always the same, equal to $(1/2)[ee'/(e + e')] \cdot c^2$.

Since the existing internal work is $(1/2)[ee'/(e + e')] \cdot [dr^2 / t^2]$, the existing potential is $V$ and its work equivalent is $[rc^2 / 2(e + e')] \cdot V$. Consequently, the sum of the existing internal work values is equal to

$$\frac{1}{2} \frac{ee'}{e + e'} \cdot \frac{dr^2}{dt^2} + \frac{rc^2}{2(e + e')} \cdot V = \frac{1}{2} \frac{ee'}{e + e'} \cdot c^2,$$

or, divided by the last term,

$$\frac{1}{c^2} \cdot \frac{dr^2}{dt^2} + \frac{r}{ee'} \cdot V = 1,$$

from which the potential $V = (ee' / r)(1 - [1/c^2][dr^2 / dt^2])$ is obtained, as above.

The formulation of the law discussed here is only intended to show in the simplest way the dependence of two particles on each other in their motions, especially the dependence of their mutual stimulation on their existing motion. Quite different needs emerge when it comes to the task to find the complete mathematical development of all the consequences of this law in connection with the general principles of mechanics in the case of larger electrical masses connected in various ways with other bodies. For this task the principles of electrodynamics are to be brought into other forms, which was not the objective of this work.

\[14\] Note by AKTA: In German: Arbeitsäquivalent.
References


