Weber’s Electrodynamics

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Maxwell’s equations (1861-1873):

Gauss’s law:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

There are no magnetic monopoles:

$$\nabla \cdot \vec{B} = 0$$

Faraday’s law:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

“Ampère’s” circuital law with displacement current:

$$\nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

Lorentz’s force (1895):

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Wilhelm Weber (1804 – 1891)

J. C. Maxwell (1831 – 1879)

Professor of physics at Göttingen University working in collaboration with Gauss

Coulomb (1785): $$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2}$$

Ampère (1822): $$\vec{F} = -\frac{\mu_0}{4\pi} I_1 I_2 \frac{\hat{r}}{r^2} f(\alpha, \beta, \gamma)$$

Faraday (1831): $$emf = -M \frac{dI}{dt}$$

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Weber's hypothesis: $$Id\vec{\ell} \Leftrightarrow q\vec{v}$$

Weber's force (1846): $$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left[1 + K_1 v_1 v_2 + K_2 (a_1 - a_2)\right]$$

Weber's force (1846):

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2}\right)$$

$$\dot{r} = \frac{dr}{dt}, \quad \ddot{r} = \frac{d^2 r}{dt^2}, \quad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Weber measured *c* in 1856: ***c*** = 3 x 10^8 m/s.

Therefore, connection between electromagnetism and optics before Maxwell!

Properties of Weber's force

- In the static case ($dr/dt = 0$ and $d^2r/dt^2 = 0$) we return to the laws of Coulomb and Gauss.
- Action and reaction: Conservation of linear momentum.
- Force along the straight line connecting the particles: Conservation of angular momentum.
- It can be derived from a velocity dependent potential energy:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}\left(1 - \frac{\dot{r}^2}{2c^2}\right)$$

- Conservation of energy:

$$\frac{d(K+U)}{dt} = 0$$

- Faraday’s law of induction can be deduced from Weber’s force (see Maxwell, *Treatise*, Vol. 2, Chap. 23).
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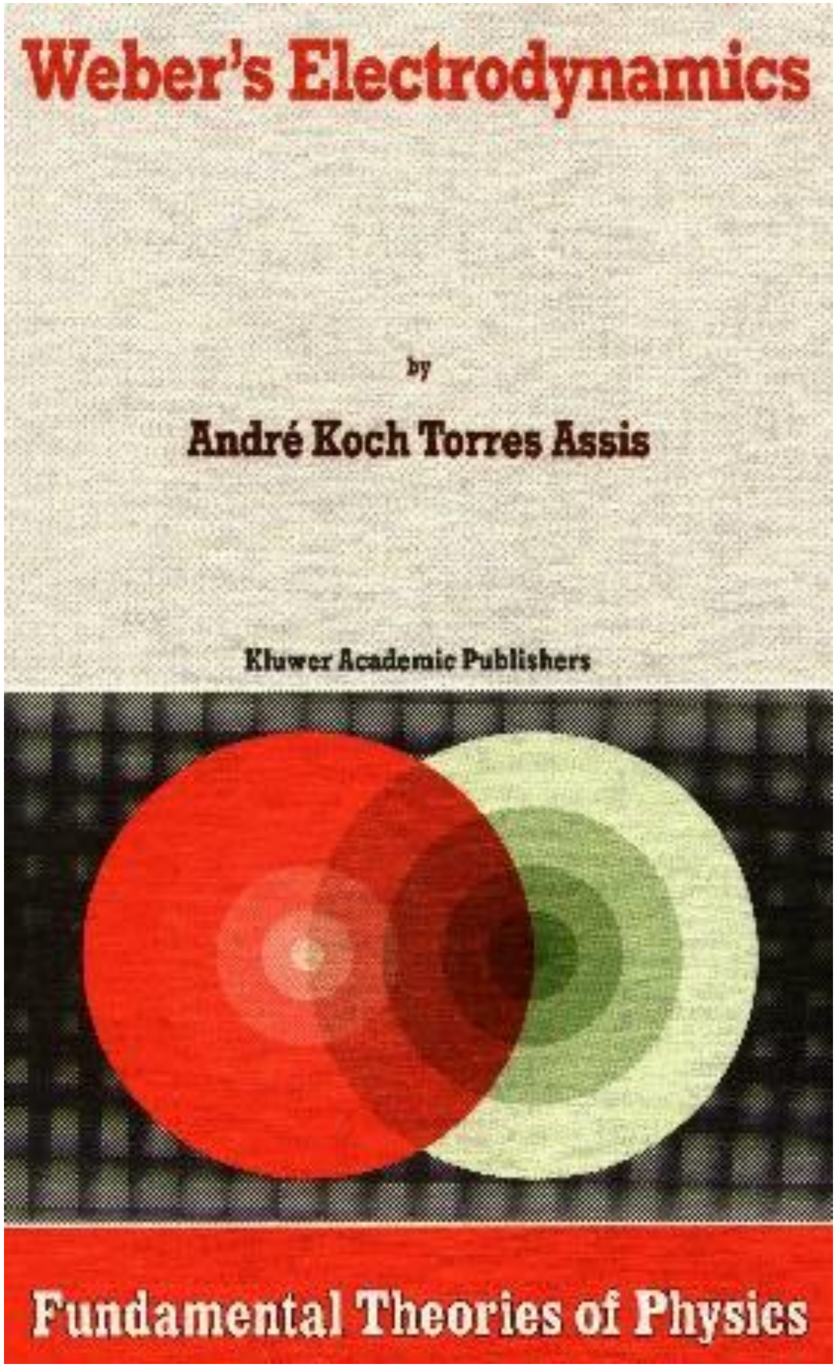
- Faraday's law of induction can be deduced from Weber's force (see Maxwell, *Treatise*, Vol. 2, Chap. 23).
- "Ampère's" circuital law can be deduced from Weber's force.
- Weber's force is completely **relational**. It depends only on r, dr/dt and d^2r/dt^2. It has the same value for all observers and in all systems of reference. It depends only on magnitudes intrinsic to the system of interacting charges. It depends only on the relation between the bodies.



1994
Kluwer
(Springer)

Weber → Ampère's force (1822-1826):

$$\vec{F} = -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}\left[2(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - 3(\hat{r}\cdot d\vec{\ell}_1)(\hat{r}\cdot d\vec{\ell}_2)\hat{r}\right]$$

Lorentz → Biot-Savart and Grassmann's force (1845):

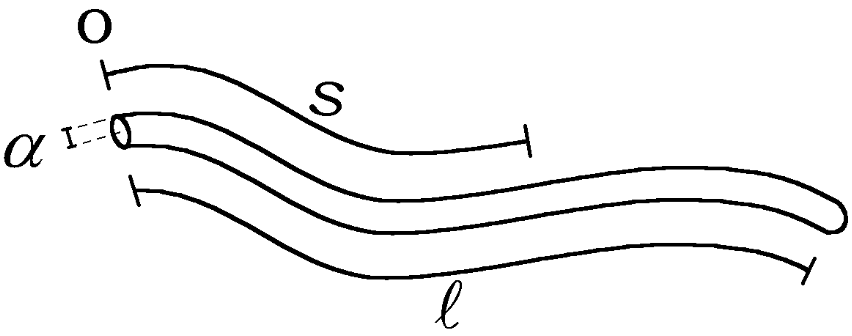
$$\vec{F} = Id\vec{\ell}_1 \times d\vec{B}_2 = I_1 d\vec{\ell}_1 \times \left(\frac{\mu_0}{4\pi}\frac{I_2 d\vec{\ell}_2 \times \hat{r}}{r^2}\right)$$

$$= -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}\left[(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - (d\vec{\ell}_1 \cdot \hat{r})d\vec{\ell}_2\right]$$

The propagation of electromagnetic signals was first obtained by Weber and Kirchhoff in 1857 utilizing Weber's electrodynamics, before Maxwell. In particular, they obtained the telegraphy equation:

$$\vec{J} = g\vec{E} = -g\left(\nabla\phi + \frac{\partial\vec{A}}{\partial t}\right)$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$



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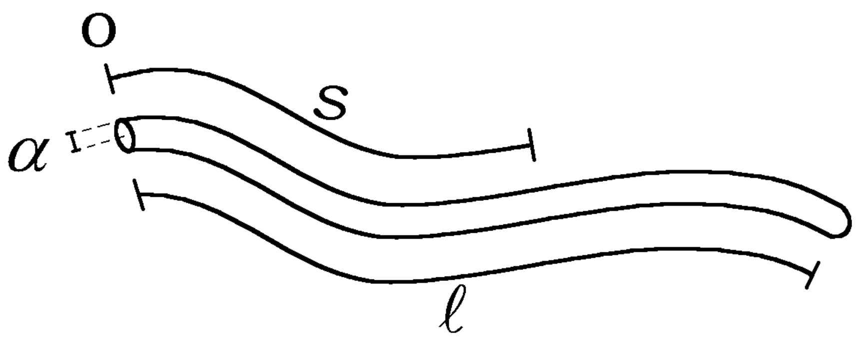


$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\frac{\partial^2 \xi}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} = \frac{2\pi\varepsilon_0 R}{\ell \ln \frac{\ell}{\alpha}}\frac{\partial \xi}{\partial t}$$

with $\xi = I, \sigma, \phi, A$

Maxwell introduced the displacement current in “Ampère’s” circutal law in 1864-1873:

$$\nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

Maxwell introduced the displacement current in “Ampère’s” circutal law in 1864-1873:

$$\nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

However:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Maxwell utilized the constant ***c*** which had been introduced by Weber in 1846.

$$c = 3 \times 10^8 \frac{m}{s}$$

Maxwell knew the value of this constant which had been first measured by Weber in 1856.

$$\frac{\partial^2 \xi}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} = 0$$

Maxwell knew that Weber and Kirchhoff, utilizing Weber’s force, had obtained the wave and telegraphy equations in 1857.

Main difference between the forces of Weber and Lorentz:

Weber's force depends on the position, velocity and acceleration a_1 of the test body 1:

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2} \right)$$

Lorentz's force depends only on the position and velocity of the test body, but does not depend on its acceleration a_1:

$$\vec{F}_{2\,\mathrm{em}\,1}^{\,\mathrm{Lorentz}} = q_1 \vec{E} + q_1 \vec{v}_1 \times \vec{B}$$

Weber versus Lorentz

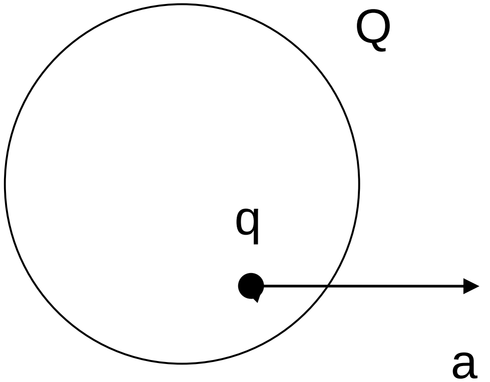
$$\vec{F}_{2\,\mathrm{in}\,1}^{Weber} = \vec{F}(r_1, r_2, v_1, v_2, a_1, a_2) =$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left\{1 + \frac{(\vec{v}_1 - \vec{v}_2)\cdot(\vec{v}_1 - \vec{v}_2)}{c^2} - \frac{3\left[\hat{r}\cdot(\vec{v}_1 - \vec{v}_2)\right]^2}{2c^2} + \frac{\vec{r}\cdot(\vec{a}_1 - \vec{a}_2)}{c^2}\right\}$$

$$\vec{F}_{2\,\mathrm{in}\,1}^{\mathrm{Lorentz}} = q_1\vec{E} + q_1\vec{v}_1 \times \vec{B} = \vec{F}(r_1, r_2, v_1, v_2, a_2) =$$

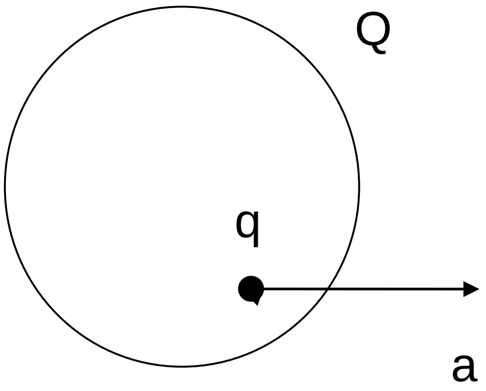
$$= q_1\left\{\frac{q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\left[\left(1 + \frac{v_2^2}{2c^2} - \frac{3(\hat{r}\cdot\vec{v}_2)^2}{2c^2} - \frac{\vec{r}\cdot\vec{a}_2}{2c^2}\right)\hat{r} - \frac{r\vec{a}_2}{2c^2}\right]\right\} + q_1\vec{v}_1 \times \left\{\frac{q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\frac{\vec{v}_2 \times \hat{r}}{c^2}\right\}$$

Force exerted by a charged spherical shell acting on an internal test charge accelerated relative to the shell:



$$\vec{F}^{\,Lorentz} = 0$$

Force exerted by a charged spherical shell acting on an internal test charge accelerated relative to the shell:

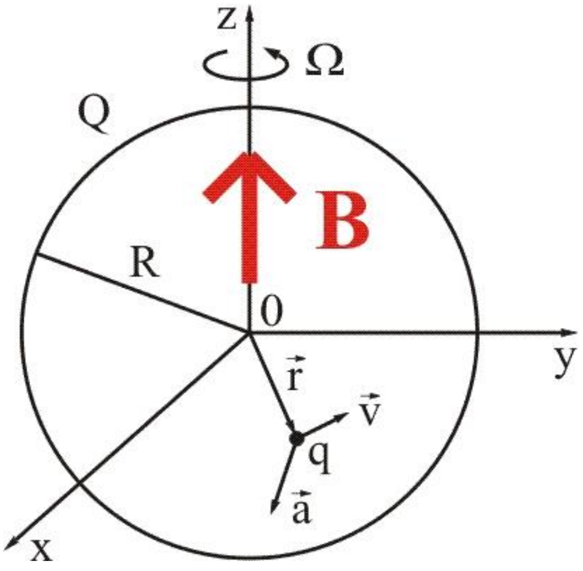


$$\vec{F}^{Lorentz} = 0$$

$$\vec{F}^{Weber} = \frac{\mu_0 q Q}{12\pi R}\vec{a}$$

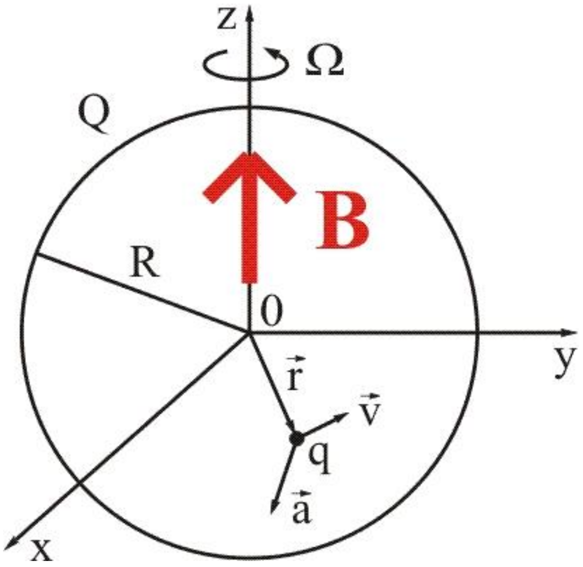
According to Weber’s electrodynamics, the test charge should behave as if it had an effective inertial mass which depends on the surrounding charges.

Force exerted by a spinning charged spherical shell acting on an internal test charge moving relative to the shell:



$$\vec{F}^{\,Lorentz} = q\vec{E} + q\vec{v}\times\vec{B} = q\vec{v}\times\frac{\mu_0 Q\vec{\Omega}}{6\pi R}$$

Force exerted by a spinning charged spherical shell acting on an internal test charge moving relative to the shell:



$$\vec{F}^{Lorentz} = q\vec{E} + q\vec{v}\times\vec{B} = q\vec{v}\times\frac{\mu_0 Q\vec{\Omega}}{6\pi R}$$

$$\vec{F}^{Weber} = \frac{\mu_0 qQ}{12\pi R}\left[\vec{a} + \vec{\Omega}\times\left(\vec{\Omega}\times\vec{r}\right) + 2\vec{v}\times\vec{\Omega}\right]$$

Weber's force (1846)

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2}\right)$$

$$\dot{r} = \frac{dr}{dt}$$

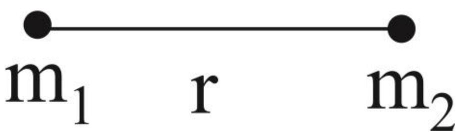
$$\ddot{r} = \frac{d^2 r}{dt^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times 10^8\,\frac{m}{s}$$

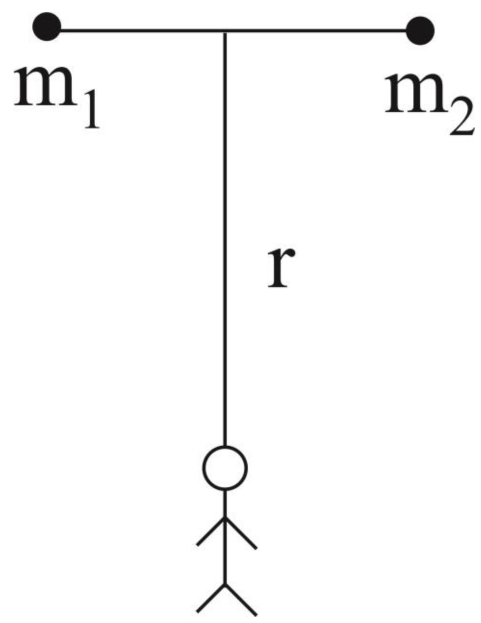
Two different gravitational theories, although the force law is expressed by the same equation:

$$F = G\frac{m_1 m_2}{r^2}$$

Newtonian theory

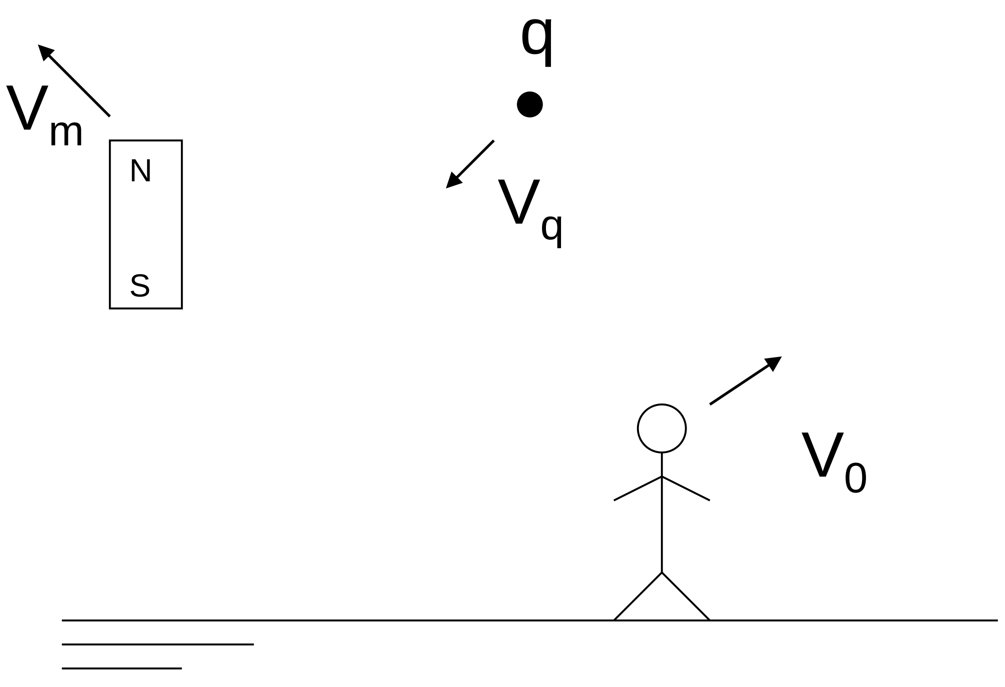


New gravitational theory



Lorentz's force (1895): $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

$\vec{v}$ is the velocity of the charge relative to what?



Different theories, although the force is expressed by the same equation:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

- Maxwell (1873): Velocity of the charge relative to the magnetic field.
- Thomson (1881) and Heaviside (1889): Velocity of the charge relative to the medium with dielectric constant ε and magnetic permeability μ.
- Lorentz (1895): Velocity of the charge relative to the ether (reference frame of the fixed stars).
- Einstein (1905): Velocity of the charge relative to the observer or frame of reference.

My next project:

To publish an English translation of Weber’s main works on electrodynamics.

I am looking for volunteers to help translate any of the articles.

Conclusion

Weber's electrodynamics is extremely powerful.

In the last few years there has been a renewed interest in Weber's electrodynamics due to novel experiments and new theoretical developments.

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