Wilhelm Weber's Main Works on Electrodynamics Translated into English

Volume II: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday

Edited by Andre Koch Torres Assis

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Apeiron
Montreal

Front cover of Volume II: The picture on the cover of Volume II comes from an 1856 lithograph by Rudolph Hoffmann (1820-1882) based on a photograph of Wilhelm Weber made by Bernhard Petri.

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Contents

Chapter 1

Introduction to Volume II

A. K. T. Assis[1]

The picture on the cover of Volume 2 comes from a 1856 lithograph by Rudolph Hoffmann (1820-1882) based on a photograph by Bernhard Petri.[2]

This second Volume begins with the text of the Gauss-Weber correspondence of 1845. It is related to Ampère's force between current elements and Weber's ideas about the unification of the electrostatic and electrodynamic laws. It is followed by a paper by Fechner published in 1845 in which he presented some qualitative ideas in the same direction. That is, to unify Ampère's force and Faraday's law of induction with electrostatics. To this end he suggested a force depending not only on the distance between the interacting electrified particles, but also on their velocities. At the end of his paper Fechner mentioned that his work might be seen as a forerunner of Weber's investigations.

Then comes Weber's First major Memoir on Electrodynamic Measurements, published in 1846. I consider this work Weber's most important publication. He introduced his bifilar electrodynamometer with which he could measure currents with high precision. Initially he utilized this instrument in order to prove Ampère' force. He then utilized Ampère's force between current elements in order to deduce his own force law between electrified particles. Weber's force between two particles electrified with charges e and e' depends not only on the distance r between the particles, but also on their *relative* velocity dr/dt and *relative* acceleration d^2r/dt^2. He showed that it was possible to unify the laws of Coulomb, Ampère and Faraday with his force law.

This Volume contains also Weber's 1848 paper in which he introduced his velocity dependent potential energy. Kirchhoff's 1849 paper on a deduction of Ohm's law in connection with the theory of electrostatics is also included in this Volume.

This Volume finishes with Weber's Second major Memoir on Electrodynamic Measurements (1852). The main focus of this work was the absolute measure of resistance. Weber's Memoir contains also his pioneering calculation of the distribution of charges along the surfaces of resistive conductors carrying steady currents. In particular, he considered a cylindrical straight conductor and a resistive ring.

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[1]Homepage: www.ifi.unicamp.br/~assis
[2]It appears, for instance, in [Wie60, p. 170] and [Wie67, p. 118].

Chapter 2

Text of the Gauss-Weber 1845 Correspondence

Carl Friedrich Gauss and Wilhelm Weber[3,4]

Editor's Note:[5] *The letters from Weber to Gauss, numbered 29 to 31, come from the Gauss manuscripts in the Manuscripts and Rare Books Division of the State and University Library of Lower Saxony, in Göttingen. They were transcribed from the German script by Karl Krause and Alexander Hartmann. The letter from Gauss to Weber of 19 March appears in Carl Friedrich Gauss, Werke, Vol. V, pages 627-629.[6] All the letters were translated into English by Susan P. Johnson.[7] The words in brackets are added by the translator; the footnotes are by the editor [Laurence Hecht].[8]*

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[3][GW96].

[4]The Notes by Laurence Hecht, the editor of this text of the Gauss-Weber 1845 correspondence, are represented by [Note by LH:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[5][Note by AKTA:] Text appearing on page 41 of [GW96].

[6][Note by AKTA:] [Gau45].

[7][Note by AKTA:] See [Joh97].

[8][Note by AKTA:] The original letters and transcriptions of some of them can be found in "The complete correspondence of Carl Friedrich Gauß", [Gau].

2.1   Weber to Gauss, Letter No. 29, 18 January 1845

Highly honored Herr Hofrath:[9]

...For some time now, I have occupied myself with a treatise, which I would like to present to the Royal Society in Göttingen; now that I am finished, however, I do not dare to venture a sound judgment, either about its correctness in your eyes, or about whether it is worthy of being presented to the Society, and therefore I would by far prefer to leave both to your benevolent decision. Hence I submit them to you with the request, that you will be good enough to look at them at your convenience, when your time permits...

With heartfelt affection and respect.

Leipzig, 1845, January 18

Your devoted,
Wilhelm Weber

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[9][Note by LH:] The title by which Weber addressed Gauss is approximately translated as "Mr. Court Councillor."

2.2 Weber to Gauss, Letter No. 30, 1 February 1845

Highly honored Herr Hofrath:

I have just noticed, that in the manuscript I recently sent to you, there is apparently missing a note regarding Ampère's formula, which would be necessary in order to understand it. Namely, Ampère has given a more general expression,[10] for the interaction of two current elements, than I introduce there, which I seek to justify, by means of the consideration that the empirically derived determination of the coefficient[11] of the second term, which I have discarded, seems completely untrustworthy, because of the unreliability of the method, and hence that coefficient, so long as it lacks a more precise quantitative determination, by the same reasoning would have to be set $= 0$. If I am not in error, you yourself earlier expressed certain thoughts about discarding the negative value which Ampère assumed for that coefficient by means of which two current elements, one following the other, would have to mutually repel one another.[12]

With heartfelt respect.

Leipzig, 1845, February 1

Yours most devoted,
Wilhelm Weber

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[10][Note by AKTA:] André-Marie Ampère (1775-1836). His masterpiece was published in 1826, [Amp26] and [Amp23]. There is a complete Portuguese translation of this work, [Cha09] and [AC11]. Partial English translations can be found at [Amp65] and [Amp69]. Complete and commented English translations can be found in [Amp12] and [AC15].

A huge material on Ampère and his force law between current elements can be found in the homepage *Ampère et l'Histoire de l'Électricité*, [Blo05].

[11][Note by AKTA:] In German: *aus der Erfahrung hergeleitete Bestimmung des Coefficienten*. This expression can also be translated as "determination of the coefficient derived from experience".

[12][Note by AKTA:] See Sections 7.5 (Obtaining the value $k = -1/2$) and 7.6 (Two remarkable results obtained by Ampère) of [AC15].

2.3   Gauss to Weber, 19 March 1945

Esteemed friend:

Since the beginning of this year, my time has been incessantly taken up and frittered away in so many ways, and on the other hand, the state of my health is so little favorable to sustained work, that up to now, I have not been in any position to go through the little treatise you were so good as to send me, and to which I just now have been able to give a first quick glance. This, however, has shown me that the subject belongs to the same investigations with which I very extensively occupied myself some 10 years ago (I mean especially in 1834-1836), and that in order to be able to express a thorough and exhaustive judgment upon your treatise, it does not suffice to read through *it*, but I would have to first plunge into study of my own work from that period, which would require all the more time, since, in the course of a preliminary survey of papers, I have found only some fragmentary snatches, although probably many more will be extant, even if not in completely ordered form.

However, if, having been removed from that subject for several years, I may permit myself to express a judgment based on recollection, I would think, to begin with, that, were Ampère still living, he would decidedly protest, when you express Ampère's law by means of the formula

$$-\frac{\alpha\alpha'}{rr}ii'\sin\theta\sin\theta'\cos\varepsilon \qquad\qquad (I)$$

since that is contained in a wholly different formula, namely

$$-\frac{\alpha\alpha'}{rr}ii'\left(\frac{1}{2}\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos\varepsilon\right) \qquad (II)^{[13]}$$

Nor do I believe that Ampère would be satisfied by the appended note, which you mention in a later letter, namely, where you cast the difference in such a way, that Ampère's formula would be a *more general one*, just like

$$-\frac{\alpha\alpha'}{rr}\left(F\cos\theta\cos\theta' + G\sin\theta\sin\theta'\cos\varepsilon\right)$$

where Ampère experimentally derived $F = \frac{1}{2}G$, while, because Ampère's experiments may not be very exact, you think that with equal correctness, you can claim that $F = 0$. In any other case than the present one, I would concede that in this discordance between you and Ampère, a third party would perhaps clarify the matter as follows, that:

whether one (with you) views this as merely a modification of Ampère's law, or

whether (as, in my estimation, Ampère would have to view the matter), this is nothing less than a complete overturning of Ampère's formula, and the introduction of an essentially different one,

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[13][Note by LH:] This seems to be Gauss's only error of memory: The epsilon should be an omega.

is at bottom little more than idle work-play. As I said, in any other case I would gladly grant this, since no one can be *in verbis facilior* [more easy-going in matters of verbal formulation] than I. However, in the present case the difference is a vital question, for Ampère's entire theory of the interchangeability of magnetism with galvanic currents depends absolutely on the correctness of Formula II and is wholly lost, if another is chosen in its place.

I cannot contradict you, when you pronounce Ampère's experiments to be not very conclusive, while, since I do not have Ampère's classic treatise at hand, nor do I recall the manner of his experiments at all, nonetheless I do not believe that Ampère, even if he himself were to admit the incompleteness of his experiments, would authorize the adoption of an entirely different formula (I), whereby his entire theory would fall to pieces, so long as this other formula were not reinforced by *completely decisive* experiments. You must have misunderstood the reservations which, according to your second letter, I myself have expressed. Early on I was convinced, and continued to be so, that the above-mentioned interchangeability *necessarily* requires the Ampère formula, and allows no other which is not identical with that one for a closed current, *if the effect is to occur in the direction of the straight lines connecting the two current elements*; that, however, if one relinquishes the just-expressed condition, one can choose countless other forms, which for a closed current, must always give the same end result as Ampère's formula. Furthermore, one can also add that, since for this purpose it is always a matter of effects at measurable distances, nothing would prevent us from presupposing that other components might possibly enter into the formula, which are only effective at immeasurably small distances (as molecular attraction takes the place of gravitation), and that thereby, the difficulty of the repulsion of two successive elements of the same current could be removed.

In order to avert misunderstanding, I will further remark, that the Formula II above can also be written

$$-\frac{\alpha\alpha'}{rr}ii'\left(-\frac{1}{2}\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos\varepsilon\right)$$

and that I do not know, whether Ampère (whose memoire, as I said, I do not have at hand) used the first or the second notation. Both of them signify the same thing, and one uses the first form, when one measures the angle θ, θ' with the same delimited straight line; thus, this line determines the side of the second angle in the opposite way, but determines the other form, when one is considering a straight line of indeterminate length, and, for the measurement of angle θ, θ', one resorts to that line twice, in one sense or another. And, likewise, one can place a $+$ sign in front of the whole formula instead of the $-$ sign, if one is considering as a positive effect, not repulsion, but attraction.

Perhaps I am in a position to again delve somewhat further into this subject, which has now grown so remote from me, by the time that you delight me with a visit, as you have given me hope that you will do at the end of April or the beginning of May. Without a doubt, I would have made my investigations public long ago, had it not been the case that at the point where I broke off, what I considered to be the actual keystone was lacking

Nil actum reputans si quid superesset agendum
[Discussions accomplish nothing, if work remains to be done]

namely, the *derivation* of the additional forces (which enter into the reciprocal action of electrical particles at rest, if they are in relative motion) from the action which is *not instantaneous*, but on the contrary (in a way comparable to light) propagates itself in time. At the time, I did not succeed; however, I recall enough of the investigation at the time, not to remain wholly without hope, that success could perhaps be attained later, although—if I remember correctly—with the subjective conviction, that it would first be necessary to make a constructible representation of the *way* in which the propagation occurs.

With hearty greetings to your brothers and sister and to Professor Möbius.[14]

Göttingen, 19 March 1845

Ever yours,
C. F. Gauss

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[14][Note by AKTA:] August Ferdinand Möbius (1790-1868).

2.4 Weber to Gauss, Letter No. 31, 31 March 1845

Highly honored Herr Hofrath:

Professor Buff[15] from Giessen, who is travelling from here to Göttingen, in order to visit Woehler,[16] his former colleague in Cassel, will have the goodness to bring you these pages. It has been of great interest to me to learn from what you were kind enough to write, that Ampère, in the definition[17] of the coefficient he calls k in his fundamental law, was guided by other reasons, than the ones from immediate empirical experience which he cites at the beginning of his treatise, and that hence the derivation, which I first gave, because it seemed somewhat simpler, is inadmissible, because it does not reproduce Ampère's law with exactness; yet, by means of what seems to me to be a slight modification in my premise, I have easily obtained the exact expression of Ampère's law.

Through the interest taken in the matter, and through the encouragement of Fechner[18] and later Möbius, I have been induced to occupy myself up to a point, with a subject which I conceived from the start might well be beyond me; I am all the happier that you are inclined to turn your attention once more to this arduous subject, and to give a complete development of it. Certainly, the explanation derived from a gradual propagation of the effect would be the most beautiful solution of the riddle. In response to your kind invitation, I will certainly not fail to come to Göttingen by the end of this spring.

In conformity with your instructions, I will send to the Royal Society in London a copy of the five last annual summaries of the *Resultate*,[19] by way of the book dealer, since it will be difficult for me to pursue the invitation to Cambridge. Whence the Royal Society has obtained a copy of the first annual summary, I do not know, since they did not buy it.

Möbius, who is now celebrating his silver wedding anniversary, and my sister, remember themselves to you and your daughter with the greatest regard.

With the most heartfelt respect.

Leipzig, 1845, March 31

Your most devoted,
Wilhelm Weber

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[15][Note by AKTA:] Johann Heinrich Buff (1805-1878).

[16][Note by AKTA:] Friedrich Wöhler (1800-1882).

[17][Note by AKTA:] The word in German seems to be: *Bestimmung*, [Wie60, p. 68]. If this is the case, the translation here should be "determination" instead of "definition".

[18][Note by AKTA:] Gustav Theodor Fechner (1801-1887).

[19][Note by AKTA:] *Resultate aus den Beobachtungen des magnetischen Vereins.*

Chapter 3

[Fechner, 1845] On the Connection between Faraday's Induction Phenomena and Ampère's Electrodynamic Phenomena

Gustav Theodor Fechner[20,21,22,23]

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[20][Fec45] with English translation in [Fec21].

[21]Translated by H. Härtel, haertel@astrophysik.uni-kiel.de and http://www.astrophysik.uni-kiel.de/~hhaertel/index_e.htm. Edited by A. K. T. Assis. We thank Frederick David Tombe and João Paulo Martins de Castro Chaib for relevant suggestions.

[22]The Notes by G. T. Fechner are represented by [Note by GTF:]; the Notes by Johann Christian Poggendorff (1796-1877), the Editor of the *Annalen der Physik und Chemie* where Fechner's paper was published, are represented by [Note by JCP:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[23][Note by JCP:] All who shared in the remarkable fate of the talented author, who miraculously regained the strength of his eyes after many years of blindness, will surely receive this first test of his renewed activity in science with the most sincere joy. But unfortunately this joy must be clouded very much by the letter with which the author accompanied the sending of his essay to me.

> "It is the first time for a long time," it says, "that I can relate to you again; and Heaven would like it not to be the last for a long time either. The condition of my eyes had miraculously improved for a short period of time so that I hoped for a complete recovery, but since then the situation has gotten so bad that these lines, as well as some parts of the accompanying treatise, had to be written by a strange hand and I see myself condemned again to almost complete inactivity."

Let us hope, however, that these gloomy prospects do not turn into reality, but that the force of the years, which the author enjoys, overpowers the threatening evil once again and forever. In this wish, I am certain that his numerous friends, near and far, to whom these lines are dedicated, will all agree with me! P.

Until now, Faraday's induction phenomena[24] have only been related to Ampère's electro-dynamic phenomena[25] by means of an empirical rule. The connection between them arises, at least partially as a consequence of the following two fundamental propositions, which are the generally accepted conclusions of experiments:

1) Every action of a current element consists of the actions of equal positive and negative particles of electricity passing each other simultaneously through the same spatial element in opposite directions.

2) The action of two current elements on each other are such that electricities of the same kind have an attractive action on one another if they go in the same direction or towards a common angular apex, but for electricities of opposite kind [an attraction takes place] if they go in the opposite direction, or so that one approaches the common angular apex while the other moves away from it.[26]

So far, however, only the interaction of the complete current elements on one another has been considered; but we can still analyze the interaction between the individual components of the current elements as described above, provided that on the one hand it reflects observation, and on the other hand it offers a means of analyzing the combination.

Incidentally, the interactions of the moving electricity considered above are indisputably not their actual full interactions, but rather only their net interactions. This is the only thing that needs to be taken into account here. For it cannot be assumed that the repulsive forces which two particles of electricity of the same kind exert on one another when at rest, will immediately turn into attraction if they start moving in the same direction, however slowly. The only thing that can be assumed is, that the repulsive forces will be reduced, either absolutely, or, if it should depend only on relative motions, in relation to the case

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[24][Note by AKTA:] Michael Faraday (1791-1867). See [Far32a] with German translation in [Far32c] and [Far89], and Portuguese translation in [Far11].

[25][Note by AKTA:] See footnote 10 on page 13.

[26][Note by AKTA:] Consider two infinitesimal elements of sizes ds and ds', carrying currents i and i'. According to Ampère's force between current elements, there are situations in which they attract one another when both currents flow towards a common angular apex P. Exceptions to this rule have been studied by Bertrand, [Ber74] with Portuguese translation in [Cha19]. One of these situations of attraction is shown in case (a) of the Figure of this footnote. According to Fechner, it might be possible to deduce Ampère's force in these situations assuming three conditions: (1) positive and negative particles of the same magnitude flowing in opposite directions in each current element; (2) particles of the same kind (both positive or both negative) in both elements attracting one another when both move towards a common angular apex P, as in case (b) of the next Figure; and (3) the positive particle of one element attracting the negative particle of the other element when one of them moves towards P, while the other moves away from P, as in case (c) of the Figure.

(a)   (b)   (c)
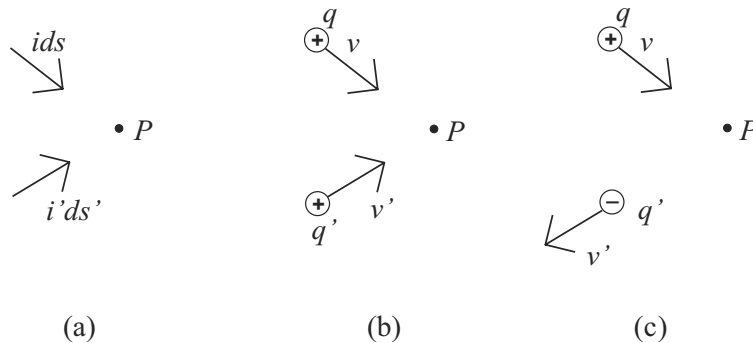
where the movement occurs in the opposite sense.[27],[28] But as regards the interaction of complete currents, however, as in natural electricity, where all the forces of static electricity cancel each other,[29] it will always be seen, as the analysis of the phenomena itself shows, as if electricities of the same kind attract each other when they move in the same direction, and repel each other when they move in the opposite direction. This analysis will be the fundamental basis of the following case scenario.

Let us now consider the first main case of induction. A wire $a'b'$, in which no current is flowing, is brought closer and in a parallel orientation to another wire ab, which is carrying an electric current.
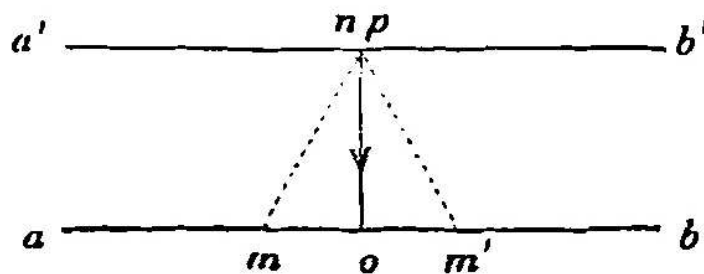
In this case the opposite electricities of the neutral wire, connected to natural electricity, are both simultaneously moved perpendicularly towards the current carrying wire. If it makes no difference to the nature of the motion and its consequences by what means it is produced, then it does not matter whether this motion is due to the influence of peculiar galvanic forces or mechanically caused by us.

So we have two currents of equally strong opposite electricity moving together *in the same sense* at right angles against a two-way current.[30]

In order to discover the inducing action which the wire $a'b'$ suffers due to the wire ab, we need to consider the action which any double particle of natural electricity np[31] experiences

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[27][Note by GTF:] It emerges from Weber's investigations mentioned later that one must stop at the latter assumption.

[28][Note by AKTA:] Fechner was referring to Wilhelm Eduard Weber (1804-1891).

[29][Note by AKTA:] That is, each current element can be considered as composed of equal and opposite charges moving in opposite directions relative to the wire. There is no net charge in each current element. Therefore, there is no net electrostatic force between two current elements.

[30][Note by AKTA:] In German: *eine doppelsinnige Strömung*. That is, a current of positive particles moving in one direction relative to the conductor, together with a current of negative particles moving in the opposite direction. Usually the direction of the current was understood as the direction of motion of the positively charged particles.

In this first case of induction considered by Fechner, the neutral wire ab is at rest relative to the ground and carries a constant current, let us say from a to b. This current can be considered as a flow of positive particles from a to b, coupled with a flow of negative particles from b to a. Initially there is no current in the stationary neutral wire $a'b'$. However, when $a'b'$ moves with a constant velocity u towards ab, with $a'b'$ remaining always parallel to ab, a current is induced in $a'b'$, flowing from b' to a'. This motion of the neutral wire $a'b'$ towards ab can be considered as a motion of a positively charged wire $a'b'$ towards ab, together with an equal motion of a negatively charged wire $a'b'$ towards ab. It is necessary to show that the positively and negatively electrified particles moving in opposite direction in ab will exert a force on the positive particles of $a'b'$ making them move from b' to a', exerting also a force on the negative particles of $a'b'$ making them move from a' to b'. That is, inducing a current in $a'b'$ directed from b' to a'.

[31][Note by AKTA:] This double particle is composed of a negatively charged particle n and a positively charged particle p.

from any two current particles m and m' that are situated to both sides of the vertical npo. Thereby it is sufficient only to pay attention to one kind of electricity in the particles m and m', since, as it is easy to see, the other will cause the same action.[32]

Therefore the total action of the particles m and m' on the positive particle p and on the negative particle n is composed of four individual forces which we have to decompose according to the direction of the wire $a'b'$ in order to find the inducing action on this wire.[33] If we just use Ampère's assumption that the forces between two current elements follow the direction of their connecting line, and consider the law of angular currents according to proposition 2), we find that the inducing lateral forces[34] of these four individual forces agree to drive p in the opposite direction from n, resulting in a two-way current, or current par excellence in the ordinary sense of the word, and this in a direction corroborated with experience.[35] On the other hand, the lateral forces which are oriented perpendicular to the wire $a'b'$, tend to drive n in the same direction as p. Therefore, in the case that m and m' are taken as symmetrical against the vertical npo, they both neutralize each other and subtract from each other with respect to the generation of current.

If one should doubt that the manner in which the motion of electricity has arisen does

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[32][Note by AKTA:] That is, the joint force of the negative particles of m and m' acting on the positive particle p will be equal to the joint force of the positive particles of m and m' acting on p. Likewise, the joint force of the negative particles of m and m' acting on the negative particle n will be equal to the joint force of the positive particles of m and m' acting on n.

[33][Note by AKTA:] These four individual forces acting on p are, (1) the force of the positive particle of m on p, (2) the force of the positive particle of m' on p, (3) the force of negative particle of m on p, and (4) the force of the negative particle of m' on p. Likewise there will be four individual forces acting on n due to the positive and negative particles of m and m'.

[34][Note by AKTA:] That is, forces decomposed along the direction of the wire $a'b'$.

[35][Note by AKTA:] These forces are illustrated on the Figure of this footnote. There is a current from a to b. The positive charges of m and m' move from a to b with velocities v. The positive charge p moves towards ab with a velocity u. The boldface arrows indicate the forces. The positive charge of m attracts p, as both of them move towards the apex point o in the middle of ab. The positive charge of m' repels p, as p moves towards the apex point o while the positive charge of m' moves away from it. The sum of these two forces will yield a net force on p pointing from b' to a'.

The forces of the negative charges of m and m' moving from b to a will also yield a net force on p from b' to a'. On the other hand, the forces of the positive charges of m and m' will yield a net force on the negative charge n pointing from a' to b'. Likewise, the forces of the negative charges of m and m' on n will also yield a net force force pointing from a' to b'. These total forces on p pointing from b' to a', coupled with the total forces on n pointing from a' to b', will induce a current from b' to a'.
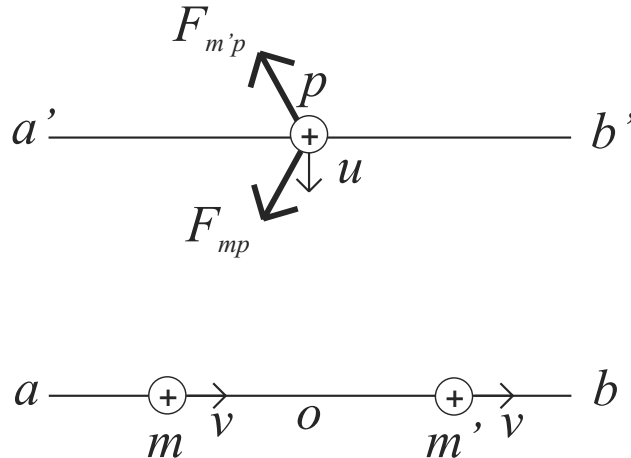
not have any influence on its action, the agreement with experiment would undoubtedly be one of the best proofs that the above conclusions are correct. It turns out to be irrelevant whether I cause the flow of electricity by a mechanical motion — with my hands — or whether it has received the impulse of its motion from galvanic contact.

And the same result occurs whether the wire $a'b'$ is moved towards the wire ab at rest, or vice-versa.[36] Experimental evidence confirms that only relative motion matters in order to apply the given principle in the given form.[37]

In the case so far considered, a two-way current acted on a one-way[38],[39] current *parallel* to it. Another case can be considered where the motion of one of the two currents is oriented *perpendicularly* to that of the other, as for instance, when an excited circular conductor or its equivalent, the cross-section of a magnet, rotates in its plane, while a neutral conductor at rest is positioned relative to it as shown in the Figure. In this case, too, one finds the experimental result according to the principles given, taking into account the law of relative motions.
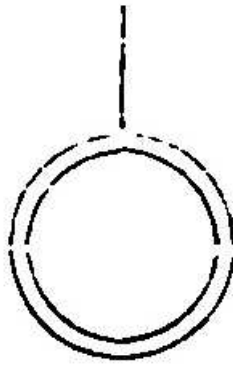
Lenz' general rule about the reciprocity between Ampère's and Faraday's phenomena

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[36][Note by AKTA:] That is, if $a'b'$ remains at rest in the laboratory and ab moves towards it, the same induction will take place as in the previous case, provided the relative motion between ab and $a'b'$ is the same in both cases.

[37][Note by AKTA:] In his paper read in 1831 Faraday showed that induction depended only on the *relative* motion between two interacting bodies A and B. These interacting bodies A and B might be a magnet and a closed circuit where induction took place. These interacting bodies A and B might also be a closed circuit carrying a steady current and another closed circuit where induction took place. In one experiment, for instance, he kept A at rest in the laboratory and moved B towards A and detected an induced current. In another experiment he kept B at rest in the laboratory and moved A towards B, detecting once more an induced current. Provided the relative motion between A and B was the same in both experiments, then the observed induced currents were also the same. See, for instance, [Far32a], [Far11], [Ass13] and [Ass14, Section 15.1: Electromagnetic induction].

[38][Note by GTF:] For a short description of the contrast it may be allowed to use the latter word for moving natural electricity.

[39][Note by AKTA:] Fechner is here distinguishing the German words *doppelsinnige* and *einsinnige* when referring to the current. A two-way current would be the typical galvanic current, as understood at that time, in which positive and negative particles move in opposite directions relative to the conductor. An one-way current, on the other hand, might be the motion of a body charged with only one kind of electricity. If the body is neutral as a piece of wire, then when it moves relative to the ground there will be an one-way current of positive electricity and another one-way current of negative electricity, both moving together with the body.

Fechner has just shown that in order to explain Faraday's law of induction in this case, a force parallel to $a'b'$ must act on the positive particles of $a'b'$ when $a'b'$ moves towards ab. A force in the opposite direction must act on the negative particles of $a'b'$ when $a'b'$ moves towards ab.

can be related to the above-mentioned principles through the well-known theorem of the parallelogram of forces, that, if P and Q arise as lateral forces from R, then conversely, R and Q appear as lateral forces from the decomposition of P, when Q is applied in the opposite direction from before.[40]

If the established principles are correct, a means may probably be found of determining the real or translational velocity of electricity,[41,42] by establishing a relationship between the easily determinable velocity at which *we* move the natural electricity in the conductor to be induced, and the velocity with which electricity moves itself under the influence of peculiar forces.

At first it seemed that it would be difficult to find a method by which this determination could be made with accuracy. But sometime later, Prof. W. Weber suggested a very promising method.

There are, however, still some conclusions which result from the above:

1) When a rod charged with one kind of electricity is rotated about its axis, then as well as the usual electrical phenomena, we should expect to observe also magnetic phenomena or something completely analogous, which should in turn induce currents in approaching conductors.

2) If an electrically charged rod, free to rotate on an axis, but not actually rotating, is approached by a magnet, such that if it were an iron rod it would be magnetized longitudinally, this will cause the rod to rotate.

When the two previous conclusions are combined, but not directly deducible from the previous principles, then a strange supposition arises, that when a non-electrically conductive rod rotating around its axis[43] approaches a magnet under the appropriate conditions, the same would show the phenomena of free electricity, and indeed of only one kind of

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[40][Note by AKTA:] Heinrich Friedrich Emil Lenz (1804-1865). See [Len34] with partial English translation in [Len69]. Lenz' rule, [Len69, p. 513]:

> If a metallic conductor moves in the neighborhood of a galvanic current or of a magnet, a galvanic current will be produced in it which will have such a direction that it would have occasioned in the wire, if it were at rest, a motion which is exactly opposite to that here given to the wire, provided that the wire when at rest is movable only in the direction of the motion and in the opposite direction.

[41][Note by GTF:] It is worth noting that what has hitherto been referred to as the velocity of electricity is not the real velocity of its particles, but merely the velocity of its wave propagation, an hitherto neglected, but yet quite a notable difference, on which to my knowledge W. Weber as the first drew attention.

[42][Note by AKTA:] That is, according to Fechner, Wilhelm Weber was the first to distinguish between the drift velocity of the particles composing the current, from the wave velocity of an electric perturbation in a wire. Weber believed that the drift velocity would be much lower than the wave velocity. In 1857 Weber and Kirchhoff deduced independently from one another, although both works were based on Weber's force of 1846, that an electric wave propagates along a wire of negligible resistance with light velocity, [Kir57b] with English translation in [Kir57a], [Pog57] with English translation in [Pog21], and [Web64] with English translation in [Web21a].

[43][Note by AKTA:] In German: *wenn man einem, um seine Axe gedrehten, nicht elektrischen leitenden Stabe*. That is, an insulating rod. Probably Fechner was referring here to a *charged* insulating rod spinning around it axis.

electricity.[44,45]

It will undoubtedly be difficult to prove the above conclusions by experiment, for if one remembers that, according to the experiments of Faraday and Gauss,[46] enormous quantities or [huge] velocities of machine electricity[47] are required to produce only moderate current actions, and that considerable currents are required to produce distinct magnetic or induction actions, it can be foreseen that only extraordinarily high velocities of rotation or strong electrification can lead to success in the indicated experiments. This also follows from the fact that a magnet or a galvanically excited conductor can be regarded as completely filled with currents, while a spinning electrical conductor is only covered by a single layer of electricity. Therefore I was not surprised, that I was not able to obtain any results with the few corresponding experimental means I had available to me. Meanwhile, others who have more powerful means at their disposal, may consider what has been said as an invitation to return to these experiments.

It cannot be denied that our concatenation of ideas leave something to be desired, namely, the proposition that it is only a matter of the relative motion. This can in fact only be presented as an empirical proposition, but not as a consequence of the principles mentioned above. The same is true of the additional proposition, which we must add, in order to cover the complete field of induction phenomena, namely, that the emergence or intensification of the current has a similar action on approaching,[48] as the disappearance or weakening of the current has when the distance is increased. In the meantime, this incompleteness of our conclusions is not a reason to drop what we have learned by them for the sake of what we did not learn.

In fact, the inadequacy that still shows up here, does not lie in a fault of the method of interpreting the action of the electricity in motion in the case of both electrical components. The progress made in the foregoing is based solely and exclusively on this method. The problem rather lies in an inadequacy in how we have phrased the action of electricity in motion up until now. It can easily be shown that the propositions and wording that we have used in the theory of electricity really do not really cover the *possibility* of all scenarios of electricity in motion, and that new assumptions must therefore be made.

Indeed, both classes of phenomena still to be explained prove irrefutably that moving electricity can have an influence on electricity at rest. This influence, as it arises in those phenomena, can neither be contained in the propositions which concern static electricity, because positive and negative electricity always act with the same strength from the same distance (therefore, according to these propositions, the result will always be zero in respect

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[44][Note by GTF:] According to this, a magnetic rod rotated about its axis would have to show the phenomena of free electricity by itself, of the opposite kind depending on its direction of rotation. That this is really the case seems to be confirmed by the following: if one connects by a wire a point of the axis and a point of the circumference of a rotated magnet, a current start flowing. According to the analogy with the galvanic apparatus, it can be assumed that after removing this connecting wire, free electricity will appear at the separation points either of a different nature or of a different magnitude. This could also be detectable by means of a capacitor if the rotation is sufficiently rapid.

[45][Note by AKTA:] The experiment that if one connects a point of the axis and a point of the circumference of a spinning cylindrical magnet by a metal wire, a current start flowing, was first performed by Faraday in 1832, [Far32b] with German translation in [Far32d].

[46][Note by AKTA:] Due to a misprint in the original, we have here *Gaus*. Fechner was referring to Carl Friedrich Gauss (1777-1855).

[47][Note by AKTA:] In German: *Maschinen-Elektricität*. That is, electricity produced by friction in electrostatic machines when a glass globe spins quickly relative to the ground.

[48][Note by AKTA:] That is, on bringing together the two interacting conductors.

to other electricities), nor is this influence contained in Ampère's propositions, because these allow no action whatsoever to be found between moving and stationary electricity.

Perhaps an attempt could be made to derive an extension of the principles, which would be able to satisfy what still has to be explained, from an analysis of the phenomena still to be explained themselves. However, it is now unnecessary to start such an activity, since, as I am pleased to announce, Prof. W. Weber, through investigations carried out from general points of view, has arrived at a principle whereby not only all the actions of moving electricity, but also of static electricity among themselves, as well as in mutual relationship to one another, can be deduced from a general law. Therefore, the phenomena of static electricity, Ampère's law and all induction phenomena come under this law only as special cases. I therefore hope that this little piece of work will only be seen as a forerunner of the investigations which we can expect to be published shortly.[49]

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[49][Note by AKTA:] Weber's work was published in 1846, [Web46] with a partial French translation in [Web87] and a complete English translation in [Web07]. Weber quotes Fechner's 1845 paper in Section 26 of his work, see Section 5.26 on page 171.

Chapter 4

Editor's Introduction to Weber's First Memoir on Electrodynamic Measurements

A. K. T. Assis[50]

This is the first of Weber's 8 major Memoirs on Electrodynamic Measurements.[51] It is probably Weber's most important work.

In the first part of this work Weber introduced his famous bifilar electrodynamometer.[52] Weber's original instrument belongs to the Historical Collection of the Physics Institute of Göttingen University, Figure 4.1. Weber utilized his electrodynamometer to confirm the force between current elements developed by André-Marie Ampère (1775-1836) in the period 1820-1827.[53] Friedrich Kohlrausch (1840-1910) discussed current measurement with Weber's electrodynamometer.[54]

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[50]Homepage: www.ifi.unicamp.br/~assis
[51][Web46] with partial French translation in [Web87] and a complete English translation in [Web07].
[52]See http://physicalisches-cabinet.uni-goettingen.de/phycab/main.php
[53]See footnote 10 on page 13.
[54][Koh83, Chapter 66a, p. 192].

Figure 4.1: Weber's bifilar electrodynamometer.

James Clerk Maxwell (1831-1879) discussed the bifilar suspension introduced by Gauss and Weber in Article 459 of his *Treatise on Electricity and Magnetism*, while Weber's bifilar electrodynamometer was discussed in Articles 725 to 729. He made the following comments on this instrument, [Max54, Vol. 2, Article 725, p. 371]:

> The instrument originally constructed by Weber is described in his *Elektrodynamische Maasbestimmungen*. It was intended for the measurement of small currents, and therefore both the fixed and the suspended coils consisted of many windings, and the suspended coil occupied a larger part of the space within the fixed coil than in the instrument of the British Association, which was primarily intended as a standard instrument, with which more sensitive instruments might be compared. The experiments which he made with it furnish the most complete experimental proof of the accuracy of Ampère's formula as applied to closed currents, and form an important part of the researches by which Weber has raised the numerical determination of electrical quantities to a very high rank as regards precision.

> Weber's form of the electrodynamometer, in which one coil is suspended within another, and is acted on by a couple tending to turn it about a vertical axis, is probably the best fitted for absolute measurements. A method of calculating the constants of such an arrangement is given in Art. 700.

Weber also utilized this instrument to verify the law of induction of currents due to Michael Faraday (1791-1867) which had been published in 1832.[55]

In the fourth Part of this work Weber succeeded in unifying all branches of electromagnetism into a single formula. Charles Augustin de Coulomb (1736-1806) obtained in 1785 a force describing the interaction between two electrified particles at rest relative to one another.[56] Weber was able to unify Coulomb's force with Ampère's force between current elements and also with Faraday's law of induction. To this end he introduced a central force pointing along the straight line connecting the two electrified particles and complying with the law of action and reaction due to Isaac Newton (1642-1727) as presented in his book *Mathematical Principles of Natural Philosophy*, usually known by its first Latin name, *Principia*.[57] Moreover, Weber's force depended not only on the distance r between the electrified particles, but also on their radial *relative* velocity, dr/dt, and on their radial *relative* acceleration, d^2r/dt^2. These are intrinsic properties of the system. The observer (or frame of reference) does not matter for the values of r, dr/dt and d^2r/dt^2. These magnitudes have the same value in all frames of reference, even for non-inertial frames. Later on I called them *relational magnitudes*.[58]

In a book published in 1994, I discussed Maxwell's points of view related to Weber's electrodynamics.[59] It is worth while to present a few quotations here. Since his first paper on electromagnetism of 1855 (published in 1858), Maxwell always praised Weber's theory. For instance, after presenting Faraday's ideas which he was trying to follow, Maxwell said:[60]

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[55]See footnote 24 on page 20.

[56][Cou88] with a partial English translation in [Cou35] and a complete English translation in [Cou12]. See also [Pot84]; [Gil71b] and [Gil71a].

[57]Isaac Newton (1642-1727). See [New34] and [New99]. Portuguese translation in [New90], [New08] and [New10].

[58][Ass98], [Ass99], [Ass13] and [Ass14].

[59][Ass94, Section 3.6, pp. 73-77].

[60][Max58, p. 207-9 of Niven's book].

There exists however a professedly physical theory of electro-dynamics, which is so elegant, so mathematical, and so entirely different from anything in this paper, that I must state its axioms, at the risk of repeating what ought to be well known. It is contained in M. W. Weber *Electro-dynamic Measurements*, and may be found in the Transactions of the Leibnitz-Society, and of the Royal Society of Sciences in Saxony.[61],[62] The assumptions are [...]. From these axioms are deducible Ampère's laws of the attraction of conductors, and those of Neumann and others, for the induction of currents. Here then is a really physical theory, satisfying the required conditions better perhaps than any yet invented, and put forth by a philosopher whose experimental researches form an ample foundation for his mathematical investigations.

In the Introduction of his paper of 1864 in which Maxwell completed his electromagnetic theory of light, he presented a similar point of view:[63]

The most obvious mechanical phenomenon in electrical and magnetical experiments is the mutual action by which bodies in certain states set each other in motion while still at sensible distance from each other. The first step, therefore, in reducing these phenomena into scientific form, is to ascertain the magnitude and direction of the force acting between the bodies, and when it is found that this force depends in a certain way upon the relative position of the bodies and on their electric and magnetic condition, it seems at first sight natural to explain the facts by assuming the existence of something either at rest or in motion in each body, constituting its electric and magnetic state, and capable of acting at a distance according to mathematical laws.

In this way mathematical theories of statical electricity, of magnetism, of the mechanical action between conductors carrying currents, and of the induction of currents have been formed. In these theories the force acting between the two bodies is treated with reference only to the condition of the bodies and their relative position, and without any express consideration of the surrounding medium.

These theories assume, more or less explicitly, the existence of substances the particles of which have the property of acting on one another at a distance by attraction or repulsion. The most complete development of a theory of this kind is that of M. W. Weber,[64],[65] who has made the same theory include electrostatic and electromagnetic phenomena.

In doing so, however, he has found it necessary to assume that the force between two electric particles depends on their relative velocity, as well as on their dis-

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[61][Note by Maxwell:] When this was written, I was not aware that part of M. Weber's Memoir is translated in Taylor's *Scientific Memoirs*, Vol. V. Art. XIV. The value of his researches, both experimental and theoretical, renders the study of his theory necessary to every electrician.

[62]Maxwell was referring in his footnote to the 1848 excerpt of Weber's First major Memoir on Electrodynamic Measurements, [Web48a] with English translation in [Web52c], [Web66d] and [Web19]. See Chapter 7.

[63][Max65, pp. 526-527 of Niven's book].

[64][Note by Maxwell:] Electrodynamische Maassbestimmungen. Leipzic Trans. vol. i. 1849, and Taylor's Scientific Memoirs, vol. v. art. xiv.

[65]Maxwell was referring to [Web46] with English translation in [Web07]; and [Web48a] with English translation in [Web52c], [Web66d] and [Web19]. See Chapters 5 and 7.

tance. This theory, as developed by MM. W. Weber and C. Neumann,[66,67] is exceedingly ingenious, and wonderfully comprehensive in its application to the phenomena of statical electricity, electromagnetic attractions, induction of currents and diamagnetic phenomena; and it comes to us with the more authority, as it has served to guide the speculations of one who has made so great an advance in the practical part of electric science, both by introducing a consistent system of units in electrical measurement, and by actually determining electrical quantities with an accuracy hitherto unknown.

The last Chapter of Maxwell's *Treatise* is devoted to Weber's electrodynamics.[68] He showed once more, as Weber had already done, that Ampère's force between current elements and Faraday's law of induction can be deduced from Weber's law. He also finally acknowledged that Weber's law complies with the conservation of energy.

Although many modern textbooks mention that Maxwell was the first scientist to present a mathematical formulation of Faraday's law of induction, this had already been accomplished by Weber in 1846, many years before Maxwell.

Weber's force between two electrified particles complies with Newton's action and reaction law in the strong form, always pointing along the straight line connecting them. It also complies with the conservation of linear and angular momentum. In 1848 he presented a potential energy from which he could deduce his force. He also showed that his force was compatible with the principle of conservation of energy. The laws of Coulomb, Ampère and Faraday are only particular cases of Weber's law.

I consider Weber's force one of the main achievements in the whole history of physics.

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[66][Note by Maxwell:] "Explicare tentatur quomodo fiat ut lucis planum polarizationis per vires electricas vel magneticas declinetur." — Halis Saxonum, 1858.

[67][Neu58]. See also [Neu63].

[68]Vol. 2, Chapter XXIII: Theories of Action at a Distance, [Max73] and [Max54]. German translation in [Max83]. Portuguese translation in [Ass92].

Chapter 5

[Weber, 1846, EM1] Electrodynamic Measurements, First Memoir, relating specially to a General Fundamental Law of Electric Action

Wilhelm Weber[69,70,71,72]

I - Introduction

The electrical fluids, when they are *moved* in ponderable bodies, cause interactions of the *molecules of these ponderable bodies*, from which all galvanic and electrodynamic phenomena arise. These interactions of the *ponderable bodies*, which are dependent upon the *motions* of the electrical fluids, are to be divided into two classes, whose differentiation is essential to the more precise investigation of the laws, namely,

1. such interactions which those molecules exert upon one other, when the distance between them is immeasurably small, and which one can designate galvanic or electrodynamic *molecular forces*, because they occur in the interior of the bodies through which the galvanic current flows; and

2. such interactions which those molecules exert upon one another, if the distance between them is measurable, and which one can designate galvanic or electrodynamic forces acting *at a distance* (in inverse proportion to the square of the distance). These latter forces also operate between the molecules which belong to two different bodies, for instance, two conducting wires.

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[69][Web46].

[70]The English version presented in this book is based on the translation by the late Susan P. Johnson, [Web07], see also [Joh97]. Edited by Laurence Hecht and A. K. T. Assis.

[71]Treatise at the founding of the Royal Scientific Society of Saxony on the day of the 200th anniversary celebration of Leibniz's birthday, published by the Prince Jablonowski Society, Leipzig 1846, pp. 211-378.

[72]Wilhelm Weber's Notes are represented by [Note by WW:], the Notes by H. Weber, the editor of the third volume of Weber's *Werke*, are represented by [Note by HW:], the Notes by L. Hecht are represented by [Note by LH:], while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

One may easily see, that for a complete investigation of the laws of the *first* class of interactions, a more precise knowledge is required of molecular relationships inside the ponderable bodies than we currently possess, and that without it, one could not hope to bring the investigation of this class of interactions to a full conclusion by establishing complete and general laws. The case is different, on the other hand, with the *second* class of galvanic or electrodynamic interactions, whose laws can be sought in the forces which two ponderable bodies, through which the electrical fluids are moving, exert upon each other in a *precisely measurable position and distance with respect to one another*, without it being necessary to presuppose that the *internal molecular relationships* of those ponderable bodies are known.

From these two classes of interactions, which were discovered by Galvani and Ampère,[73] a *third* class must meanwhile be fully distinguished, namely, the *electromagnetic* interactions, discovered by Oersted,[74] which take place between the molecules of two *ponderable* bodies at a measurable distance from each other, when in the one [molecule] the electrical fluids are put into motion, while in the other [molecule] the magnetic fluids are separated. This distinction between *electromagnetic* and *electrodynamic* phenomena is necessary for presenting the laws, so long as Ampère's conception of the essence of magnetism has not fully supplanted the older and more customary conception of the actual existence of magnetic fluids. Ampère himself gave expression to the essential distinction to be made between these two classes of interactions in the following way on page 285 of his Treatise:[75,76]

> As soon as Mr. Oersted had discovered the force which the conducting wire exerted on the magnet, one could in fact suspect that an interaction might exist between two conducting wires. But this was not a necessary consequence of that famous physicist's discovery: for a soft iron bar also acts upon a magnetic needle, without, however, any interaction occurring between two soft iron bars. As long as one knew simply the fact of the deflection of the magnetic needle by the conducting wire, could one not assume, that the electrical current simply imparted to this conducting wire the property of being influenced by the magnetic needle, in a way similar to that in which the soft iron was influenced by the same needle, for which it sufficed that it [the wire] acted on the needle, without any sort of effect resulting thereby between two conducting wires, if they were withdrawn from the influence of magnetic bodies? Simple experimentation could answer the question: I carried it out in September 1820, and the interaction of the voltaic conductors was proven.

Ampère rigorously develops this distinction in his Treatise, declaring that it is necessary for the laws of interaction discovered by himself and Oersted to be separately and completely derived, each by itself, from experimental evidence. After he has spoken of the difficulties of precisely observing the interaction of the conducting wires, he says on page 183, *loc. cit.*:[77]

> It is true that one meets with no such difficulties, when one measures the effect of a conducting wire on a magnet; however, this method cannot be used when it is

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[73][Note by AKTA:] Luigi Galvani (1737-1798) and André-Marie Ampère (1775-1836). See footnote 10 on page 13.

[74][Note by AKTA:] Hans Christian Ørsted (1777-1851). See [Oer20b], [Oer20a], [Oer20c], [Oer65], [Ørs86] and [Ørs98]. See also [Fra81] and [Rei13].

[75][Note by WW:] Mémoire sur la théorie mathématique des phénomènes électrodynamiques uniquement déduite de l'expérience. Mémoires de l'académie royale des sciences de l'institut de France, 1823.

[76][Note by AKTA:] Weber is referring to [Amp23, pp. 285-286] and [Amp26, pp. 113-114]. See, in particular, [AC15, p. 418].

[77][Note by AKTA:] See [Amp23, p. 183], [Amp26, p. 11] and [AC15, pp. 346].

a matter of determining the forces which two voltaic conductors exert upon each other. In fact, it becomes clear, that if the action of a conducting wire on a magnet, proceeds from a cause other than that which occurs between two conducting wires, the experiments made on the former would prove nothing at all with respect to the latter.

From this, it becomes clear, that even if many fine experiments have been conducted more recently in further pursuit of Oersted's discovery, nothing has directly occurred yet toward further pursuit of Ampère's discovery, and that this requires specific and unusual experiments which hitherto have been sorely lacking.

Ampère's classic work itself is concerned only in a lesser way with the phenomena and laws of the interaction of the conducting wires vis-à-vis each other, while the larger part is devoted to the development and application of his conception of magnetism, based on those laws. Nor did he consider his work on the interaction between two conducting wires as in any way complete and final, either from an experimental or theoretical standpoint, but on the contrary, repeatedly drew attention to what remained to be done in both connections.

He states on page 181 of the cited Treatise,[78] that in order to derive the laws of interaction between two conducting wires *from experimental evidence*, one can proceed in two different ways, of which he could pursue only one, and presents the reasons which kept him from attempting the other way, the most essential being the lack of precise *measuring instruments*, free of indeterminable foreign influences. He says the following on page 182 f., *loc. cit.*:[79]

> There is, moreover, a far more decisive reason, namely, the limitless difficulties of the experiments, if, for example, one intended to *measure* these forces by means of the number of vibrations of a body subjected to their influence. These difficulties arise from the fact that, when one causes a fixed conductor to act on a moveable part of a voltaic circuit, those parts of the apparatus, which are necessary to connect it to the dry battery, act on this moveable part simultaneously with the fixed conductor, and thus destroy the results of the experiments.

Likewise, Ampère repeatedly drew attention to what remains to be done from the *theoretical* standpoint. For example, he says on page 299, after showing that it is impossible to account for the interaction of the conducting wires on each other, by means of a certain distribution of static electricity in the conducting wires:[80]

> If one assumes, on the contrary, that the electrical particles in the conducting wires, set in motion by the influence of the battery, continually change their position, at every moment combining in a neutral fluid, separating again, and immediately recombining with other particles of the fluid of the opposite kind, then there exists *no contradiction* in assuming that from the influences which each particle exerts in inverse proportion to the square of the distance, a force could result, which did not depend solely upon their distances, but also on the alignments of the two elements, along which the electrical particles move, combine with molecules of the opposite kind, and instantly separate, in order to combine again with others. The force which then develops, and for which the experiments and calculations discussed

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[78][Note by AKTA:] See [Amp23, p. 181], [Amp26, p. 9] and [AC15, pp. 344].

[79][Note by AKTA:] See [Amp23, pp. 182-183], [Amp26, pp. 10-11] and [AC15, pp. 345].

[80][Note by AKTA:] See [Amp23, p. 299], [Amp26, p. 127] and [AC15, pp. 424].

in this Treatise have given me the quantitative data, depends, however, directly and indeed exclusively, on this distance and these alignments.

> If it were *possible* to prove on the basis of this consideration, that the interaction of two elements were in fact proportional according to the formula with which I have described it, then this account of the fundamental fact of the entire theory of electrodynamic phenomena would obviously have to be preferred to every other theory; it would, however, require investigations with which I have had no time to occupy myself, any more than with the still more difficult investigations which one would have to undertake in order to ascertain whether the opposing explanation, whereby one attributes electrodynamic phenomena to motions imparted to the ether by the electrical currents, could lead to the same formula.

Ampère did not continue these investigations, nor has anyone else published anything to date, from either the experimental or theoretical side, concerning further investigations, and since Ampère, science has come to a halt in this area, with the exception of Faraday's discovery of the phenomena of galvanic currents induced in a conducting wire when a nearby galvanic current is increased, weakened, or displaced.[82] This neglect of electrodynamics since Ampère, is not to be considered a consequence of attributing less importance to the fundamental phenomenon discovered by Ampère, than to those discovered by Galvani and Oersted, but rather it results from dread of the great difficulty of the experiments, which are very hard to carry out with present equipment, and no experiments were susceptible of such manifold and exact determinations as the electromagnetic ones. To remove these difficulties for the future, is the purpose of the work to be presented here, in which I will chiefly confine myself to the consideration of purely galvanic and electrodynamic interactions *at a distance.*

Ampère characterized his mathematical theory of electrodynamic phenomena in the title of his Treatise as *derived solely from experimental results*, and one finds in the Treatise itself the simple, ingenious method developed in detail, which he used for this purpose. In it, one finds the experiments he selected and their significance for the theory discussed in detail, and the instruments for carrying them out fully and precisely described; but an exact description of the experiments themselves is missing. With such fundamental experiments, it does not suffice to state their purpose and describe the instruments with which they are conducted, and add a general assurance that they were accompanied by the expected results, but it is also necessary to go into the details of the experiments more precisely, and to state how often each experiment was repeated, what changes were made, and what influence those changes had, in short, to communicate in protocol form, all data which contribute to establishing a judgment about the degree of reliability or certainty of the result. Ampère did not make these kinds of more specific statements about the experiments, and they are still missing from the completion of an actual direct proof of the fundamental electrodynamic law. The fact of the interaction of conducting wires has indeed been generally placed beyond doubt through frequently repeated experiments, but only with such equipment and under such conditions, that *quantitative* determinations are out of the question, not to speak of the possibility that these determinations could achieve the rigor required to consider the law of those phenomena as empirically proven.

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[81][Note by AKTA:] See [Amp23, p. 301], [Amp26, p. 129] and [AC15, p. 425].
[82][Note by AKTA:] See footnote 24 on page 20.

Now, Ampère, of course, more frequently made use of the *absence* of electrodynamic effects which he observed, similar to the use of measurements which yield the result $= 0$, and, by means of this expedient, he attempted, with great acuity and skill, to obtain the most necessary basic data and means of testing for his theoretical conjectures, which, in the absence of better data, was the only method possible; we cannot, however, in any way ascribe to such *negative* experimental results, even if they must temporarily take the place of the results of *positive* measurements, the entire value and the full force of proof which the latter possess, if the negative results are not obtained with the use of such techniques, and under such conditions, where true measurements can also be carried out, which was not possible with the instruments used by Ampère.

One may consider more precisely, for example, the experiment which Ampère describes on page 194 ff. of his Treatise as the third case of equilibrium, where a metal arc lies on two trays filled with mercury, from one of which the current is introduced and from the other drawn off, and where, additionally, the arc is fastened by a hinge to an arm which connects

it with a vertical shaft pivoting between the ends.[83,84]

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[83][Note by WW:] Ampère gives in another location the following description of his instrument: On a frame TT' (Figure 1) in the form of a table two vertical poles EF and EF' protrude, bound together by crosspieces LL' and FF'; an axle GH is held in a vertical position between these two crosspieces. Its two ends G and H are sharpened and are seated in conical depressions, one of which is in the lower crosspiece LL', the other on the end of a screw KZ which passes through the upper crosspiece FF' and which serves to steady the axle GH without fixing it. An arm QO is fastened at C to the axle. The end of the arm is equipped with a hinge, into which fits the middle of an arc AA', which is formed from a conductor. The arm, whose radius is equal to the distance from O to the axle GH, is always in a horizontal position. This arc is balanced with a counterpoise at Q, in order to decrease the friction at points GH where it is seated in the conical depressions.
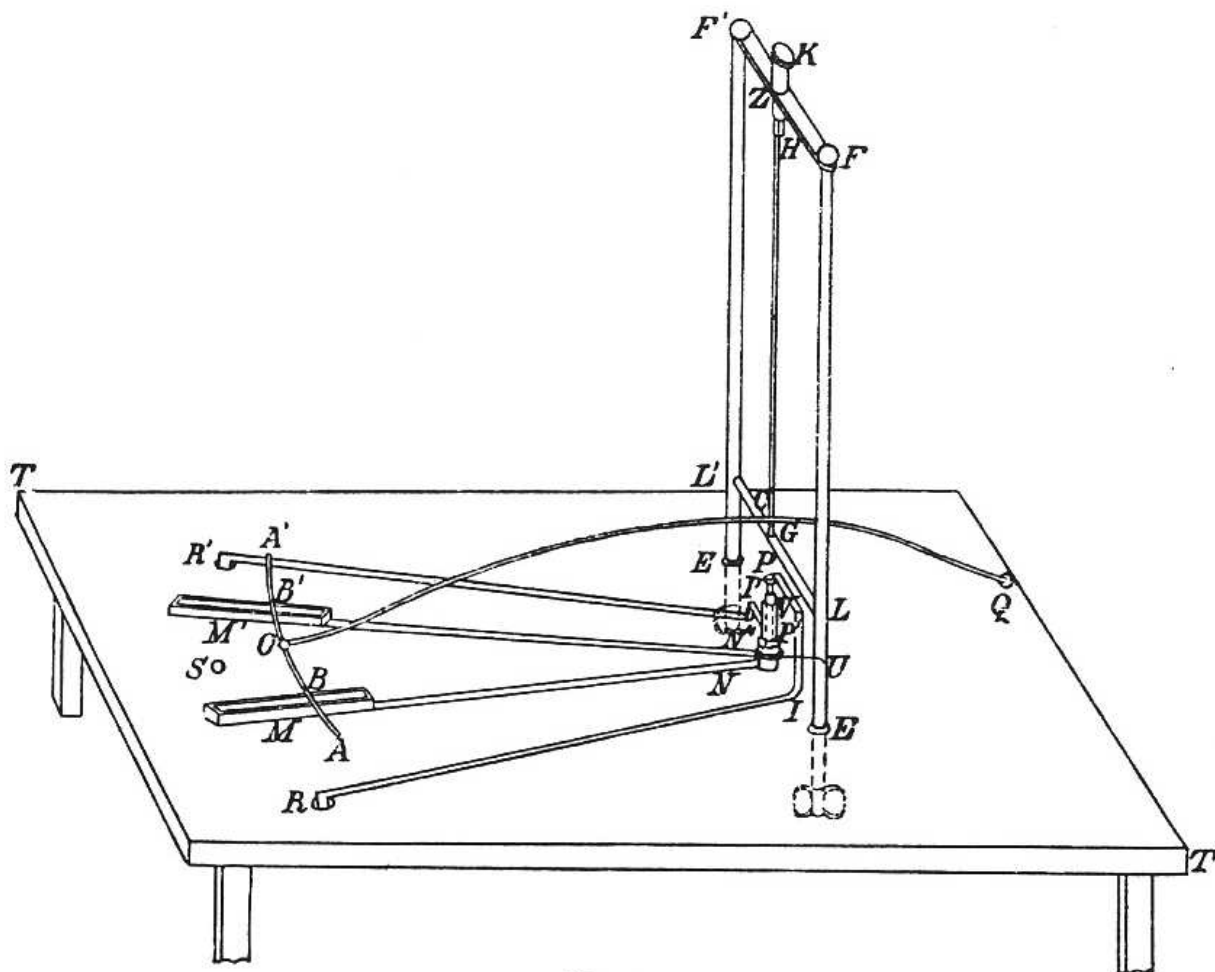
Fig. 1.

Under the arc AA' are two trays M and M' filled with mercury, so that the surface of the mercury, which rises above the rim of the tray, just touches the arc AA' at B and B'. These two trays communicate through the metallic conductors MN and $M'N'$ with the mercury-filled cups P and P'. The cup and the wire MN which connects it with the tray M are fastened on a vertical axle, which sits on the table so that it can turn

Ampère now observed that, while a galvanic current is passing through that arc, it is not displaced from its supports, if a closed circuit of current is made to act upon the arc, provided that the center of the arc falls on the axis of the shaft to which the arc is attached. However, one easily sees that, in order to put the arc into motion, a fourfold friction must be overcome, namely, the friction on the two supports on which the arc is lying (arc AA' on B and B' in Figure 1), and the friction on the two ends G and H, on which the vertical shaft pivots. Further, it is known that the electrodynamic forces which are produced with the strongest imaginable galvanic current in a simple wire, like the section of the arc BB' with current flowing through it, are so weak, that the wire must be extremely mobile, in order to show any perceptible effect at all. One would accordingly be inclined to expect, that that arc would not be displaced in the case where its center lay in the axis of rotation, but also that in the opposite case, where its center did not coincide with the axis of rotation, no displacement would occur, because the just-cited fourfold friction would counterpose far too great a resistance. Ampère now says, nevertheless, on page 196, *loc. cit.*:[85]

> When, by means of the hinge O, the arc is placed in such a position that its center lies outside the GH axis, the arc takes on motion and slides on the mercury of the little troughs M and M', in virtue of the action of the closed curvilinear current which goes from R' to S. If, on the contrary, its center is on the axis, it remains immobile.

It is regrettable, that Ampère did not mention the obvious problem of that fourfold friction, and never explicitly says that he himself saw and observed the movement of the eccentric arc. However, aside from the doubt that could therefore be raised about the actual observation of the datum, and assuming that Ampère himself saw the displacement of the arc under the conditions described, and also made certain that it had actually been the effect of *electrodynamic* forces, which were strong enough, to overcome all opposing obstacles, it is still in no way stated at what eccentricity of the arc this motion began, and within which *limits* it did *not* occur. Yet without determining such limits, no full force of proof can be attributed to this experiment. It is not known to me whether, since that time, this experiment has been successfully repeated and more precisely described by other physicists, yet this much can be

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freely. This axle passes through the cup P', with which the wire $M'N'$ is connected, in order that it may turn independently from the other cup. The axle is insulated by a little glass tube which surrounds it, and is kept separate by a little glass disc from the conductor of the tray M, so one may form an arbitrary angle with the conductors MN and $M'N'$.

Two other conductors, JR and $J'R'$, fastened to the table, are submerged respectively in cups P and P', and connect these with mercury-filled depressions in the table R and R'. Finally, between these two depressions, there is a third, S, also filled with mercury.

The apparatus is used in the following way: One battery lead, for instance, the positive, is dipped into depression R, and the negative into depression S, and the latter is connected with depression R' through an appropriately bent conductor. The current goes through the conductor RJ to cup P, from there through the conductor NM to tray M, through the conductor $J'R'$, and finally from the depression R' through the arbitrarily curved conductor to depression S, in which the negative battery lead is dipped.

The voltaic circulation is accordingly formed: (1) from arc BB' in contact with conductors MN and MN'; (2) from a circuit, which from the part RJP and $P'J'R'$ of the device, out of the curved conductor which goes from R' and S and from the pillar itself originates. The latter circuit works like a closed one because it is only interrupted by the thickness of the glass plate which separates the cups P and P'; hence, it suffices to observe its action on the arc BB' in order to experimentally confirm the effect of a closed current on an arc at the different positions which one can set up with respect to it.

[84][Note by AKTA:] See [Amp23, pp. 194-199], [Amp26, pp. 22-27] and [AC15, pp. 351-355].

[85][Note by AKTA:] See [Amp23, p. 196], [Amp26, p. 24] and [AC15, pp. 353-354].

summarily stated, that even in the most favorable of cases, the displacement occurred only at great eccentricities, from which, however, it cannot be concluded that the electrodynamic force acts precisely at right angles to the elements of the arc.

By means of these remarks about Ampère's experiments, I have only wished to demonstrate that the electrodynamic laws have found no sufficient proof in these experiments, communicated as they are without more precise details, and why I believe that such a proof could not be given by means of observations with Ampère's instruments, but instead, observations are required with precise measuring instruments which have not previously been used. If, despite the lack of direct factual proof, one remains convinced of the correctness of the laws advanced by Ampère, this conviction is based on grounds which in no way make direct proof superfluous. Electrodynamic measurements hence remain desirable in order to provide the direct proof which is lacking.

In fact, amidst the universal attempt to determine all natural phenomena according to number and measure, and thereby to obtain a basis for theory which is independent of either sensory perception or mere estimation, it seems amazing that in electrodynamics, no attempt of this kind has been made; nevertheless, I am aware of neither refined nor gross measurements of the interactions of two conducting wires vis-à-vis each other. All the more do I consider myself authorized to present here the first attempts which I have made toward such measurements. I hope thereby to prove, that these electrodynamic measurements possess importance and significance in quite other respects than as proof of the fundamental electrodynamic laws, namely, by becoming the source of entirely new investigations for which they are uniquely suited, and which, indeed, cannot be conducted without them.

II - Proof of Ampère's Law for the Interaction Between Electric Currents

5.1 Description of an Instrument for the Measurement of the Interaction of Two Conducting Wires — Electrodynamometer

The instruments Ampère used for his electrodynamic experiments, are not of the sort that allow the probative force of more rigorous measurements to be ascribed to the experiments made with them. The reason for this lies in the *friction* which often annuls the entirety of the electrical force to be observed, or a large part of it, and eliminates it from observation. Neither is it possible with those instruments, even under favorable conditions, to overcome this adverse friction by means of the weak electrodynamic forces, while by any more rigorous measurement it must be presupposed that the friction is a negligible fraction in comparison with the force to be measured.

Already, twelve years ago, for the purpose of excluding friction and introducing truer measurements, I equipped a wire wound on a thin wooden frame, through which a galvanic current was to be conducted, and which then was to be set into motion by the electrodynamic attraction and repulsion of a multiplier, with a *bifilar* suspension of two fine metal wires (in future, I will call these wire spirals with bifilar suspension the *bifilar coils*) and used one of these suspension wires for supplying the galvanic current, the other for drawing it off. I first came to know the full significance of this apparatus for the purpose of measurement, however, by way of the bifilar magnetometer of Gauss,[86] from whom I then borrowed the use of a mirror fastened to the bifilar coil. In the summer of 1837, I made such an instrument and carried out a series of experiments with it, all of which prove, that one can achieve the greatest refinement in the observation of electrodynamic phenomena with currents so weak, that previously, no one ever succeeded in eliciting these phenomena with them.

The instrument to be described here firstly, was constructed by Inspector Meyerstein in Göttingen in 1841,[87] yet it was in Leipzig that I first found occasion to provide a suitable setup for a greater series of measurements.

This instrument consists essentially of two parts: the *bifilar coil* with a mirror, and the *multiplier*.[88]

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[86][Note by AKTA:] [Gau38b] with English translation in [Gau41c]. See also [Web38a] with English translation in [Web41c] and [Web66a]; and [Web94].

[87][Note by AKTA:] Moritz Meyerstein (1808-1882). See [Hen04], [Hen05], [Hen07] and [Hen20].

[88][Note by AKTA:] A photograph of Weber's original instrument which belongs to the Historical Collection of the Physics Institute of Göttingen University can be found in Figure 4.1 on page 28.
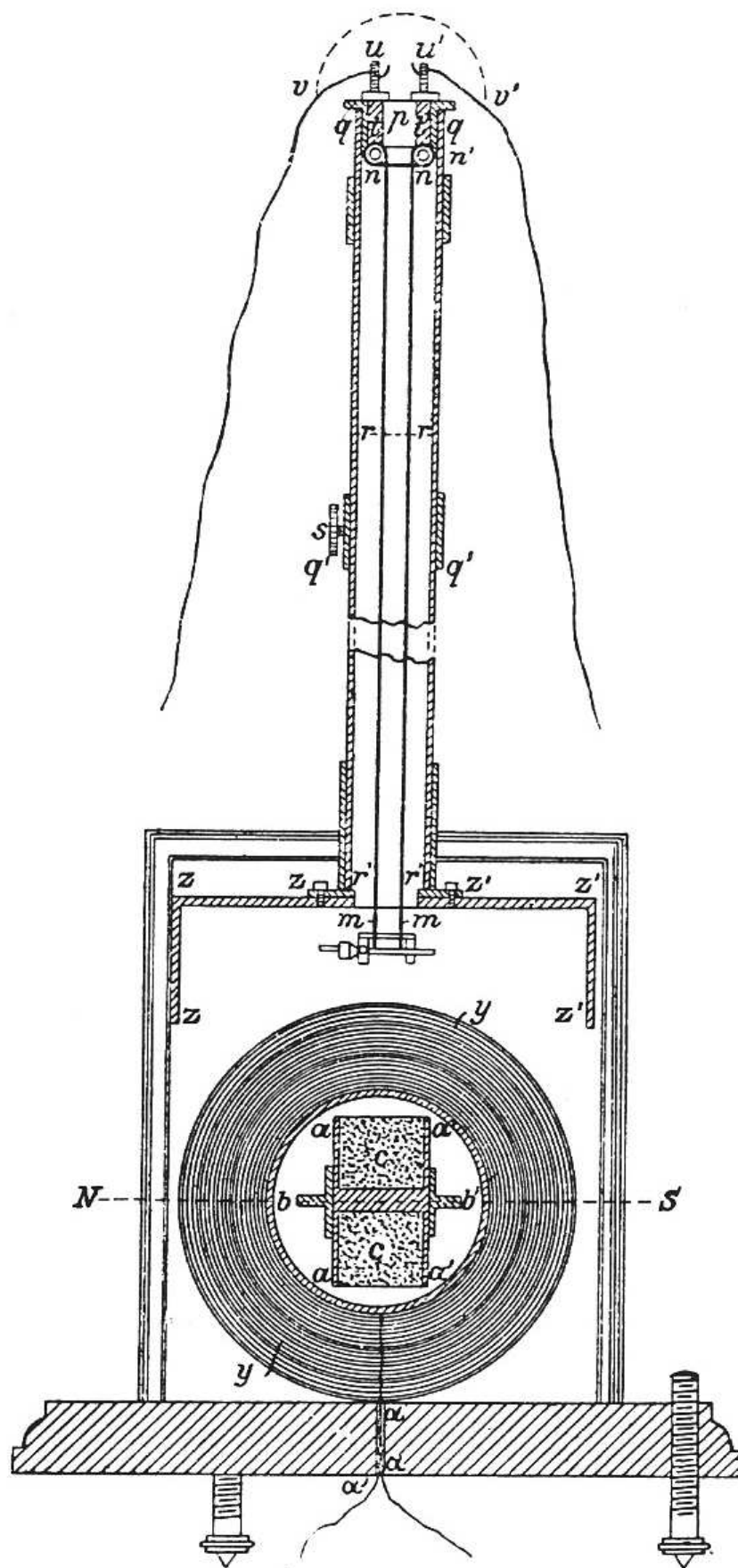
Fig. 2.
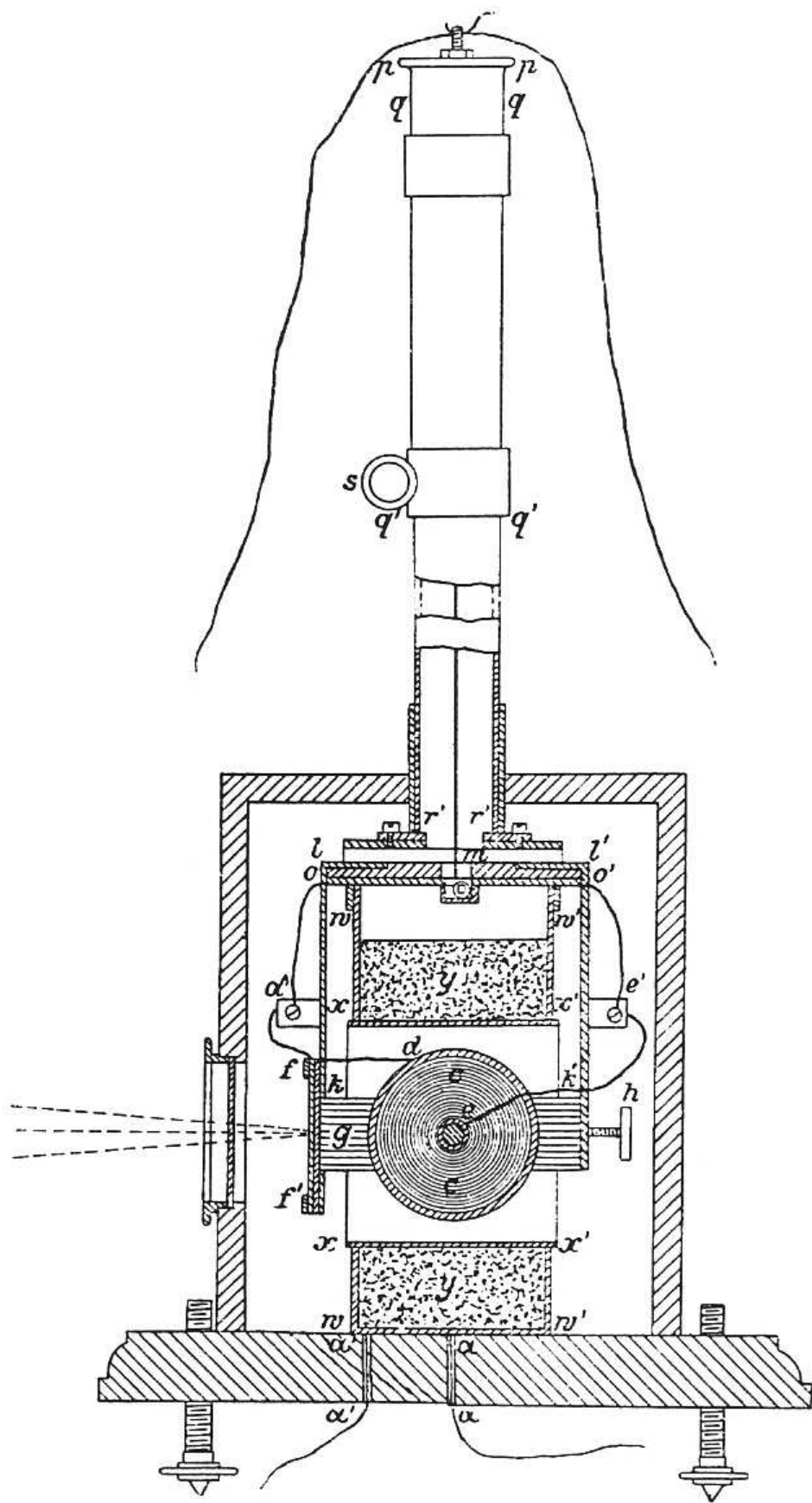
Fig. 3.

The *bifilar coil*, which is presented in a vertical cross-section in Figure 2, consists of two thin brass discs aa and $a'a'$ of 66.8 mm diameter, which are held in a fixed position by a 3-mm thick brass axis bb' at a distance of 30 mm apart. Around this axis between these discs is wound roughly 5 000 times a copper wire cc of 0.4 mm diameter, sheathed in silk, which completely fills up the space between the two discs. Figure 3 presents this coil in a vertical cross-section perpendicular to the previous one. One end of the wire is led, close to the brass axis, through a small opening lined with ivory in one disc at e (Figure 3) outward from e to e'; the other end is fastened at d on the periphery of the cylinder formed by the wire windings with silk thread. A plane mirror ff' (Figure 3) is now attached to this wire coil, and fixed by three screws to a small brass plate; the brass plate is equipped with two right-angled extensions g and g', of which in Figure 3 only the rear one, g, is visible.

Figure 4, which gives the horizontal cross-section, shows both extensions connected with the brass plate holding the mirror ff'. These two extensions are screwed at their ends to the outsides of the two brass discs aa and $a'a'$. The mirror ff' is located in a plane parallel to the axis bb' of the wire coil close to the periphery of the coil; diametrically opposite to it, a counter-weight h is mounted. I use now a square plane mirror ground in Berlin by Oertling;[89] its sides are 40 millimeters long.
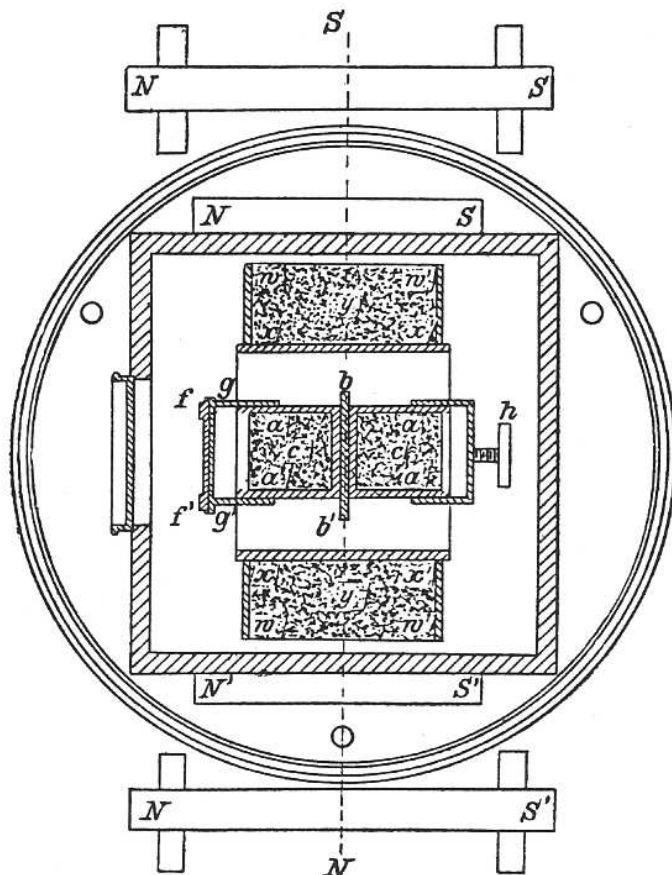
Fig. 4.

The *bifilar suspension* of this wire coil consists of three parts: the halter fastened onto the coil, the two suspension wires, and finally the immovable support from which the wires hang. The *halter* consists of a forked brass bracket or bow (Figure 3), ll', with two 100-mm-long

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[89][Note by AKTA:] Johann August Daniel Oertling (1803-1866).

parallel vertical arms lk and $l'k'$, 100-mm apart. The ends of both arms are screwed fast at k and k' to the brass plate which holds up the mirror, and, diametrically opposite, to the holder of the counter-weight. Figure 5 in particular shows this halter; at d and d', the two wires coming from b and c pass under two ivory plates which can be adjusted by means of screw a, and pass through two grooves in the ivory plates, which are in contact with each other at the center, and vertically upward through the opening e.
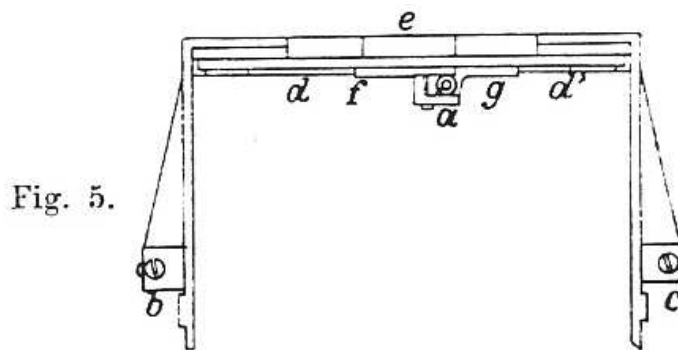
Fig. 5.

Figure 6 gives the view of the halter from below; at f and g, the connection of the screw a with the two ivory plates d and d' is represented. The vertical going through the center of gravity of the coil passes through the middle of the area between the two grooves. At each arm of the bow, finally, is located at d' and e' (Figure 3) a clamp insulated with ivory for fastening and connecting one of the silk-coated wires from each end of the coil with the lower end of one of the two uncoated suspension wires. The suspension wire is led from this clamp d' or e' through a small opening lined with ivory o or o', along the underside of the bow, to one of the two already mentioned grooves on the ivory plates which meet each other at the center, whence the wire goes upwards to the little brass cylinder at n and n' (Figure 2). The two suspension wires are copper, 1 meter long, and 1/6 meter thick; their distance apart, to be regulated by screw a (Figure 6), is usually 3 to 4 millimeters.
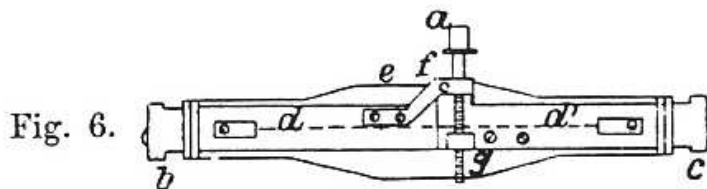
Fig. 6.

The *support*, to which both upper ends of the two suspension wires are fastened, consists of a strong piece of ivory p (Figure 2), which is fitted tight like a lid on the upper end of a 30-mm-wide brass tube qq'. This brass tube is 150 mm long and allows a second brass tube, rr', to pass through it, be rotated, and be adjusted by a set-screw s (Figure 3). These two tubes surround the two suspension wires along their entire length, and protect them from the influence of the air. On the underside of the piece of ivory, are attached two little movable brass rollers t and t' (Figure 2) of 10 mm diameter, fastened to the ivory with screw clamps u and u'; over each of these little rollers is led a suspension wire, which terminates in an eyelet. Both eyelets of the two wire ends are bound together with a strong silk thread between t and t', without touching each other. By means of these two little rollers and the binding together of the two wires, the two suspension wires are made to always have the same tension. To each of the two clamps u and u', which fasten the two little rollers to the

ivory, is, finally, fastened a coated copper wire, of which *uv* (Figure 2) serves to supply the galvanic current, *u′v′* to draw it off.

The *multiplier*, finally, consists of two square copper plates *ww* and *w′w′* (Figures 3 and 4), with sides of 140 mm, with a circular hole of 76 mm diameter. These two copper plates stand parallel and vertical, and are connected by a horizontal brass tube *xx′* of 76 mm diameter, by means of which they are kept at a 70-mm distance from one another. In the space *yy* above these tubes between those two parallel plates is wound approximately 3 500 times the 0.7-mm-thick multiplier wire. The upper side of the multiplier is closed off with a brass cover *zzz′z′* (Figure 2), which is screwed tight onto it, and has a circular opening in the center of its upper side, above which the brass tubes surrounding the suspension wires stand. On both sides of this cover, slots are placed, through which the bow of the bifilar coil can pass and swing freely. The space between the uppermost windings of the multiplier wire and the cover is also wide enough, that each arm of the bow finds sufficient room for its movements. The bow is first stuck through without the bifilar coil and fastened to the suspension wires, and only then is it screwed to the bifilar coil. The protruding lower edges of the two brass plates on the multiplier stand on a wooden plate, which can be made level by means of three screws. In this wooden plate are two holes *aa* and *a′a′* (Figure 3), through which the two ends of the multiplier wire are led toward the outside. The whole instrument, with the exception of the brass tubes in which the suspension wires are located, is contained in a mahogany casing, for protection against the influence of the air. This mahogany casing has no floor, but is placed with the level edges of the side walls flush with the wooden plate, by means of which it is closed off from below. On the upper side is placed a round opening, through which the already mentioned brass tube passes. A second opening is made on the front side of the casing and can be closed with a plane of glass. Through it the light of the scale falls on the mirror of the *bifilar coil* and is thrown back to the telescope. The entire casing is vertically divided in two halves, of which an individual half can be taken away. The arrangement of the telescope and the scale is exactly the same as in the magnetometer. In future I will designate the instrument described here with the name *electrodynamometer*, or *dynamometer* for short, because its most immediate destiny is to measure the electrodynamic forces discovered by Ampère.[90]

5.2 The Electrodynamic Force of Two Components of a Circuit Is Proportional to the Square of the Current Intensity

The *intensity* of a constant current is determined by the *amount* of electricity, which during the time-unit (during a second) goes through a *cross-section* of the circuit. This determination of the intensity of the current is, however, not suitable as the basis of a practical method for *measurement of the intensity of the current*; for that, two measurements would be required, of which one cannot be performed at all, the other not with precision: namely, a definite amount of electricity cannot be precisely measured under the prevailing conditions, and the length of time in which it flows through the cross-section of the conducting wire can not be measured at all. For actual practical application, it is necessary to make use of another method of measuring the current-intensity. Such a method, wholly conforming to re-

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[90][Note by AKTA:] Chapter 4 presented Maxwell's comments on Weber's bifilar electrodynamometer.

quirements, is offered by the *magnetic effects* of the currents, and will always be the standard method here. Accordingly, two currents, conducted successively through the same multiplier, that exert the same force on the same permanent magnet at the same distance and in the same position, have the same intensity; if the force they exert differs, then their intensities are related as these forces, and can be measured with the help of the usual *galvanometer*.

If different currents are now put through the same circuit successively, whose intensities, according to this measurement, are in the ratio 1 : 2 : 3 and so forth, then the electrodynamic interactions of two components of the circuit, through which these different currents are passing, are in the ratio of the series of the square of those intensities, i.e., 1 : 4 : 9 and so forth. The correctness of this law is now to be proven by means of the following electrodynamic measurements, which, even if the above law required no proof, would have their own interest, as the first example of the general rigor which it is possible to achieve in electrodynamic measurements.

The *dynamometer* described in the previous Section was placed on a stone ledge, without any iron or magnets in its immediate surroundings, in such a way that the plane of the fixed coil, or multiplier, was parallel to the magnetic meridian, and the plane of the bifilar coil was also vertical, but formed a right angle with the plane of the multiplier. The position of the multiplier could easily be adjusted, since it was possible to examine the vertical placement with sufficient exactness by means of a level, which was set on the cover of the multiplier, and the orientation was regulated by means of a compass also placed on the cover of the multiplier. The bifilar coil assumed a vertical position on its own when it was hung up, but whether the plane of the bifilar coil formed a right angle with the magnetic meridian, had to be tested by means of special experiments.

That is, it is a proof of the correct position of the latter, if it remains unchanged even when an arbitrarily strong positive or negative current is put through the bifilar coil alone, because in the event of any appreciable deviation from that position, the terrestrial magnetism had to either increase or decrease this deviation. In this way, the magnitude of the deviation can also be determined. Such a test came about when the western radius of the bifilar coil was to be turned by 14 minutes toward the north, in order to place the plane of the bifilar coil exactly perpendicular to the magnetic meridian. The instrument offered no suitable way to carry out this small correction with precision, and apart from the fact that such a small deviation did not appreciably affect the results, doing away with it would have had no lasting utility, because continued observations showed, that hanging the bifilar coil at the upper end of a one-meter-high free-standing brass cylinder offered no security against rotations on the part of the bifilar coil which began gradually and increased for a few minutes. Suspension from an isolated fixed stone column was the only way to provide complete security from such small deviations.

The mirror fastened to the western radius of the bifilar coil stood vertically, and in the vertical plane, its horizontal normal was placed about 6 meters distant from a telescope equipped with crosshairs. A scale, as used in the magnetometer, was mounted on the fixed base of the telescope, just as in the magnetometer. Measurement showed the horizontal distance of the mirror from the scale:

$$= 6018.6 \text{ scale divisions,}[91]$$

from which the arc measure of a scale division resulted:

$$= 17.136".$$

After this setup of the dynamometer for measuring the electrodynamic interaction of the

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[91][Note by AKTA:] In German: *Skalentheile.*

multiplier and the bifilar coil when a galvanic current is put through them, an *electromagnetic* apparatus for measuring the intensity of the current was now required for the investigation in hand.

5.3 Description of an Electromagnetic Apparatus for Measuring the Intensity of Galvanic Currents, Which Are Conducted Through the Dynamometer

Measuring the intensity of galvanic currents, which are conducted through the dynamometer, would have been easily effected by means of a so-called sine- or tangent-galvanometer adapted for fine measurements,[92] if it had been installed at a greater distance from the dynamometer, and the same current which passed through the dynamometer, had also been conducted through the multiplier of that galvanometer. This deduction[93] of the galvanic current can be dispensed with when one places a small (transportable) magnetometer in the magnetic meridian of the dynamometer at a distance from the dynamometer such that the dynamometer's fixed coil itself educes a deflection of the magnetometer which can be measured by fine fractions. It is obvious that at such a slight distance, the use of a large magnetometer (with a 600-millimeter-long needle) would be unsuitable, since in the case in question, it was a fundamental advantage to confine the distribution of the magnetism in the magnetometer to the smallest space possible. This occurred with the small or transportable magnetometer, which I have described in the *Resultaten aus den Beobachtungen des magnetischen Vereins im Jahre 1838* (*Results from observations by the Magnetic Society in 1838*).[94]

I have, however, contrived another instrument which suited this purpose still more fully, and will describe it here, because not only can it often replace the transportable magnetometer to advantage, but it provides for other purposes, especially thermo-magnetic measurements, an instrument which is often more precise than those previously applied. The advantages are well-known of using for such measurements a needle equipped with a mirror, along with a telescope and scale, instead of the compass with pointer and graduated scale. Yet using the mirror with small needles is risky, because the mirror is an inertial mass, which must be drawn along with the needle, and consequently, when a small needle has to draw a larger mirror along with it, the force of acceleration is greatly weakened, which is just as disadvantageous for the precision of the measurements to be made with the needle, as if a weakly magnetized needle were used. This disadvantage, however, can be removed from the outset if a *magnetic mirror* is employed, and this mirror is suspended by a silk thread as itself a magnetic needle. I have obtained such a mirror from the instrument-maker Mr. Oertling in Berlin. It consists of a tempered round steel plate *ab* (Figure 7), 33 millimeters in diameter and 6 millimeters thick.

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[92][Note by AKTA:] In German: *Sinus-Boussole* and *Tangenten-Boussole*. The tangent galvanometer was invented by Johan Jakob Nervander (1805-1848) and the sine galvanometer by Claude Servais Mathias Pouillet (1790-1868), [Ner33], [Pou37] and [Sih21]. Friedrich Kohlrausch discussed measurement of currents with the tangent and sine galvanometers, [Koh83, Chapters 64 and 65, pp. 188-192].

[93][Note by AKTA:] In German: *Ableitung*: deduction or derivation.

[94][Note by AKTA:] [Web39a] with English translation in [Web41d] and [Web66b].
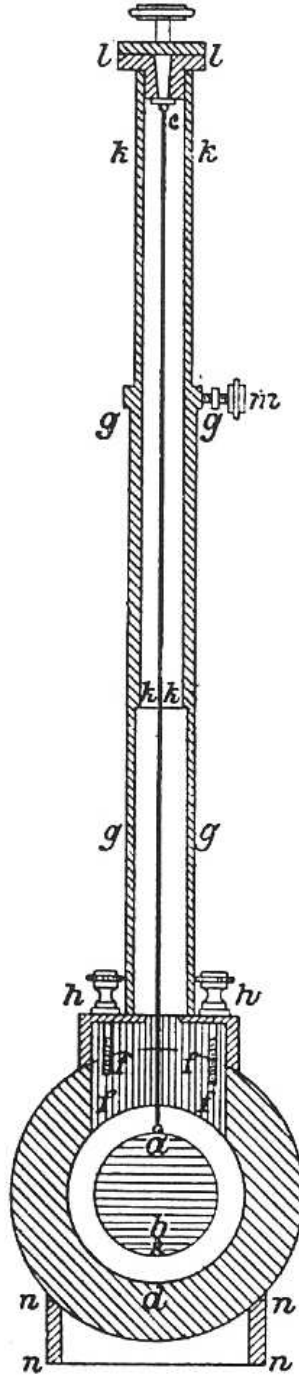
Fig. 7.

This steel plate is ground so completely smooth, that the mirror image of a scale appears very bright and clear through a telescope of tenfold magnification, and is little inferior to the image in a glass mirror. At the edge of this circular disc, small screw-threads are tapped at two diametrically opposed points, *a* and *b*, into each of which a little brass eyelet can be screwed, from which the mirror is hung with a silk thread. Only one of these threads is actually used, but soon the one, soon the other, according to whether the steel plate is to turn its mirror surface toward east or west. Now, I magnetized this tempered steel plate, by

placing two 25-pound magnetic rods in a straight line one behind the other, but in such a way, that there remained an interval of space, equal to the diameter of the mirror, between the south and north poles of the two rods, the poles being turned toward each other. The mirror was placed in this space, so that that diameter of the mirror which was perpendicular to the line connecting the two eyelets, a and b, linked the two magnets. Given the strength of the magnets and the smallness of the mirror, this sufficed to impart to the mirror the maximum magnetism it was capable of assuming.

This magnetic mirror was suspended by a silk thread ac (Figure 7) and made to oscillate. The arc of the oscillation decreased only very slowly, so that the oscillations could still be observed after a quarter of an hour, without the mirror having received any new impetus in the meantime. The period of its oscillations, however, was too small for the observational series to be carried out according to the rules for larger magnetometers, as the maximum and minimum of the curve of oscillation was repeatedly observed right after one another. In order to make precise observations of the average position of the mirror, it was an essential requirement that the oscillations of the mirror be powerfully damped and the mirror brought to a complete halt in the shortest possible time, without exerting any sort of influence on the position of the mirror itself. I completely satisfied this essential requirement for using this sort of magnetic mirror, by constructing a solid copper sphere ddd (Figure 8) of 90-millimeter diameter.
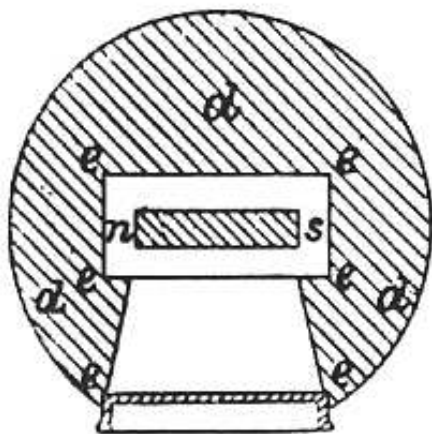
Fig. 8.

Into one side, a hole $eeee$ of 40-millimeter diameter was drilled 70 millimeters deep into this sphere, and this hole could be closed with a flat glass pane. This hole was somewhat enlarged at its back end for the magnetic mirror, and was also enlarged in the form of a funnel toward the exterior, in order to give the mirror more access to the light. In the enlarged back space $eeee$ was suspended the magnetic mirror, which can be seen in Figure 8 in the horizontal rectangular cross-section ns. From above, an 8-millimeter-wide, 40-millimeter-long opening $ffff$ (Figure 7) led to this enlarged space, through which the mirror, suspended from a silk thread, could be let down to the center of the sphere. The silk thread was led through a brass tube $gggg$, whose lower end was screwed to the sphere, with the help of a brass plate hh, which covered the mouth of the opening ff to the sphere. Inside this brass tube was a second exit-tube $kkkk$, which bore on its upper end a rotatable torsion circle ll with a hook,

at c, to which the silk thread was tied. The thread could be raised through the exit tube, until the mirror swung freely in the center of the copper sphere. Then the exit tube was fixed in place by means of a push-screw m. To fix this copper sphere in place, a simple copper ring $nnnn$ was installed, 20 millimeters high, of 70-millimeter diameter, and 2 millimeters thick, which formed a base into which the copper sphere was put. To level the instrument, a small box-level was placed on the torsion circle and the copper sphere was rotated in the ring until the level showed the correct alignment, which had to be carried out with great delicacy and precision. Owing to its great weight, the copper sphere lay so tightly in the ring, that no disturbance was ever noted.

The effect of this strong copper sphere on the oscillating mirror now consists in a *magneto-electric* damping, in virtue of which the preceding arc of oscillation was in the ratio to the succeeding one of 11 : 7 (the *decrementum logarithmicum* was = 0.19697),[95] so that after 16 oscillations or about 1 minute (the period of oscillation[96] was 3.78 seconds for this damping), the arc of oscillation was only about 1/1400 its original size, thus negligible. As a rule, given constant currents, it thus suffices to let 1 minute pass after the current begins, before one observes the deflected position of the mirror.

If such deflection experiments possess not merely a relative, but an absolute, value, then, according to the instructions given by Gauss in the *Intensitas vis magneticae terrestris ad mensuram absolutam revocata*,[97] the deflecting magnet or current must at most be placed at a distance 3 or 4 times that of the needle's length, for which in our case the triple or quadruple of the mirror diameter will be 105 to 140 mm, at which slight distance even very weak currents of the multiplier suffice to elicit sharp measurable deflections of the mirror. If now 105 or 140 mm would be a distancing of the multiplier, sufficient to give an absolute value to the measurement of the deflection, then this would take place still more, by far, at a distance of 583.5 mm, at which the multiplier was placed from the mirror in our experiments.

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[95][Note by AKTA:] The logarithmic decrement is defined as the logarithm of the ratio of any two successive peaks. Therefore, if the logarithmic decrement is $\log(x_{n-1}/x_n) = 0.19697$, then $x_{n-1}/x_n = 10^{0.19697} = 1.573874141 \approx 11/7$, as $11/7 = 1.571428571$.

[96][Note by AKTA:] In German: *Schwingunsdauer*.

Gauss and Weber utilized the old French definition of the period of oscillation t which is half of the English definition of the period of oscillation T, that is, $t = T/2$, [Gil71a, pp. 154 and 180]. For instance, the period of oscillation for small oscillations of a simple pendulum of length ℓ is $T = 2\pi\sqrt{\ell/g}$, where g is the local free fall acceleration due to the gravity of the Earth, while $t = T/2 = \pi\sqrt{\ell/g}$.

[97][Note by AKTA:] Gauss' work on the intensity of the Earth's magnetic force reduced to absolute measure was announced at the Königlichen Societät der Wissenschaften zu Göttingen in December 1832, [Gau32] with English translation in [Gau33a] and [Gau37a], see also [Rei02, pp. 138-150].

The original paper in Latin was published only in 1841, although a preprint appeared already in 1833 in small edition, [Gau41b] and [Rei19]. Several translations have been published. There are two German versions, one by J. C. Poggendorff in 1833 and another one in 1894 translated by A. Kiel with notes by E. Dorn; a French version by Arago in 1834; two Russian versions, one by A. N. Drašusov of 1836 and another one by A. N. Krylov in 1952; an Italian version by P. Frisiani in 1837; an English extract was published in 1935, while a complete English translation by S. P. Johnson was published in 2003; and a Portuguese version by A. K. T. Assis in 2003: [Gau33b], [Gau34], [Gau36], [Gau37b], [Gau94], [Gau35], [Gau52], [Gau75], [Gau03] and [Ass03].
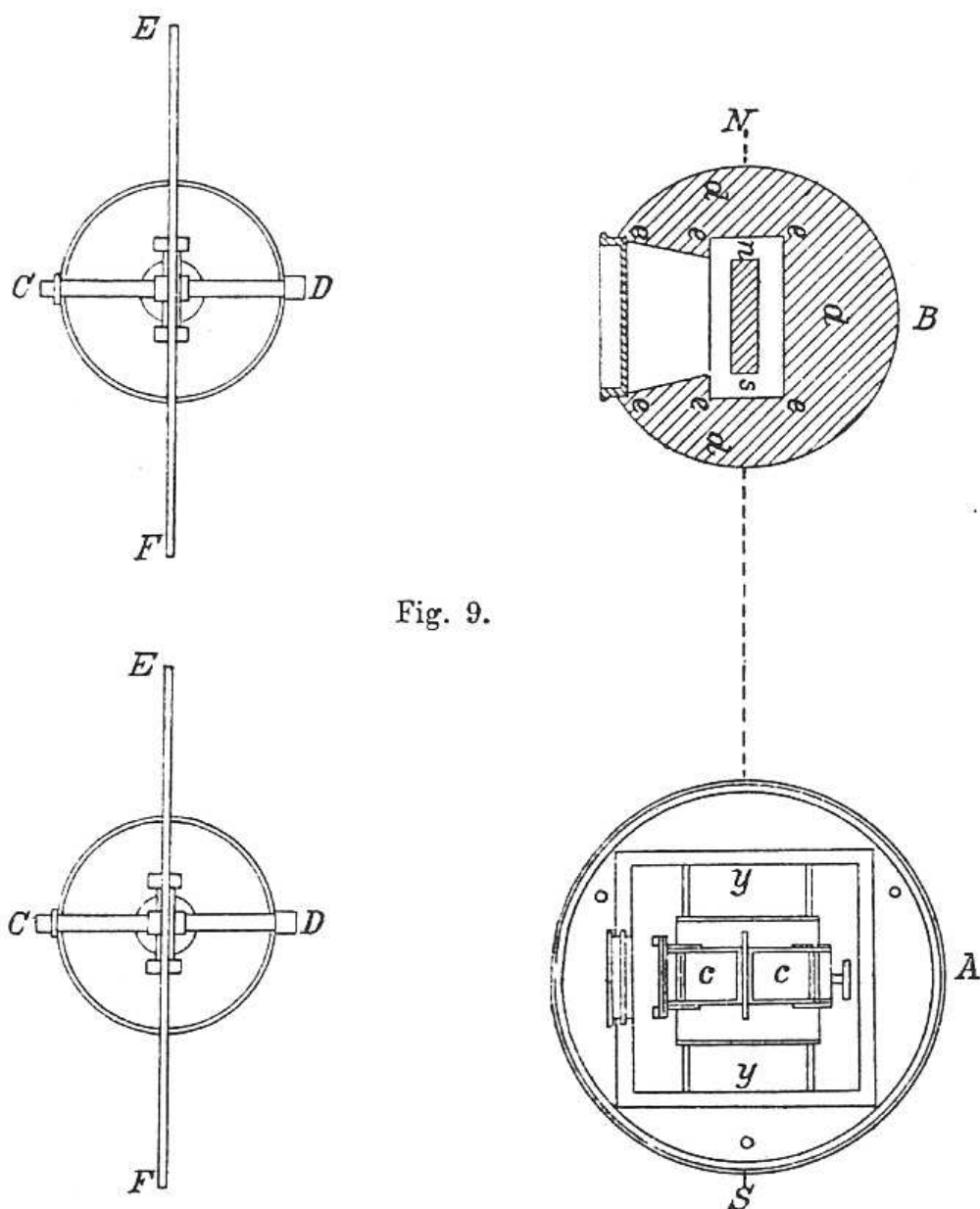
Fig. 9.

The mutual position of the two instruments, the dynamometer and the mirror magnetometer, is presented in Figure 9, where the dotted line NS is the magnetic meridian, which goes through both instruments; A is the horizontal cross-section of the dynamometer, as in Figure 4; B is the horizontal cross-section of the mirror magnetometer, as in Figure 8, CD are the telescopes for readings, aimed at the mirrors of both instruments; EF are the attached scales, whose mirror-image is observed. The use of the mirror magnetometer for thermo-magnetic observations, in which some additional devices come into play, will be treated on another occasion.

5.4 Experiments

After this description of the essential equipment, which was designed for *electromagnetic* measurement of the intensity of currents and for *electrodynamic* measurement of the interaction of two portions of the circuit, and before we proceed to a description of the experiments

themselves, we wish to make a prefatory remark about the elicitation and regulation of the currents which were used in those experiments.

Three small Grove's elements[98] from the instrument-maker Mr. Kleinert from Berlin were used, which were brought into the circuit, either all three, or only two, connected column-wise in series, or individually. Despite the fact that the currents were conducted through a very long, thin wire circuit, which formed the bifilar coil and the multiplier of the dynamometer, and which was even further extended by means of a long auxiliary wire, these currents, even given the great weakening which they underwent because of the great resistance of such a circuit, remained much too strong and deflected the dynamometer from its equilibrium position much too far, for this deflection to be measured by means of the 1-meter long scale. On the other hand, the intensity of these currents in the multiplier was quite suitable for eliciting a rigorously measurable deflection of the mirror magnetometer. Hence the deflection of the bifilar coil had to be diminished at a constant rate, without decreasing the intensity of the current in the dynamometer's multiplier. There were two ways for this to occur, either by increasing the separation of the suspension wires of the bifilar coil from each other, which would decrease the sensitivity of the dynamometer at a constant rate, or, the current could be apportioned so that only a small fraction of the current passing through the dynamometer, would be conducted through the bifilar coil. I preferred the latter method, in order to maintain the dynamometer's sensitivity, which was necessary for other experiments. A path or bridge was constructed for the current, by means of a short, thick copper wire, designated vv' in Figure 2, by which the current, outside the bifilar coil, was conducted directly to the wire returning back out of the bifilar coil before the current entered the bifilar coil. A precise comparison of the resistance of this connecting wire with that of the bifilar coil, yielded the ratio

$$1 : 245.26,$$

from which it follows, according to Ohm's law,[99] that the current intensity in the bifilar coil after this apportionment stood in the constant ratio of[100]

$$1 : 246.26$$

to the current intensity in the multiplier of the dynamometer, by means of which, thusly, without decreasing the deflection of the mirror magnetometer by the dynamometer's multiplier, the deflection of the dynamometer itself was diminished 246.26-fold. This 246.26-fold diminished deflection of the dynamometer could then be rigorously measured on the scale; the current might come from 3, 2, or only 1 Grove's element.

The measurements contained in the following Table were made in that way.

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[98][Note by AKTA:] In German: *Grove'sche Becher*. The Grove voltaic cell or Grove element was named after its inventor, William Robert Grove (1811-1896), [Gro39].

[99][Note by AKTA:] Georg Simon Ohm (1789-1854). Ohm's law is from 1826: [Ohm26a], [Ohm26c], [Ohm26d], [Ohm26b] and [Ohm27] with French translation in [Ohm60] and English translation in [Ohm66].

[100][Note by WW:] For if a denotes the intensity of the completely unapportioned current, as it passes through the multiplier, b and c the intensity of the two currents, into which each divides itself, b passing through the bifilar coil, c through the auxiliary wire vv' which connects the beginning and the end of the bifilar coil; then $a = b + c$, and according to Ohm's law, the intensities $b : c$ are related inversely to the measured resistances, that is,

$$b : c = 1 : 245.26;$$

consequently

$$b : a = b : (b + c) = 1 : 246.26.$$

Table of corresponding positions of the Mirror Magnetometer and the Dynamometer under the influence of currents of different intensity			
No.	Number of Grove's elements	Observed Position of Magnetometer	Observed Position of Dynamometer
1.	3	388.17	650.88
2.	0	279.74	209.79
3.	3	388.30	650.66
4.	0	279.68	209.47
5.	3	388.37	650.07
6.	0	280.05	209.70
7.	3	388.73	649.84
8.	0	279.95	209.55
9.	3	388.35	649.78
10.	0	279.78	209.53
11.	3	388.30	649.71
Average deflection	3 − 0	108.566	440.508
12.	0	279.54	209.25
13.	2	352.15	407.52
14.	0	280.00	208.99
15.	2	352.35	407.35
16.	0	280.00	208.82
17.	2	352.50	407.18
18.	0	280.15	208.87
19.	2	352.60	407.15
20.	0	280.17	208.92
21.	2	352.95	406.89
22.	0	280.40	208.80
Average deflection	2 − 0	72.438	198.305
23.	0	280.40	208.80
24.	1	316.77	259.68
25.	0	280.50	208.72
26.	1	216.93	259.53
27.	0	280.60	208.68
28.	1	316.90	259.50
29.	0	280.50	208.45
30.	1	316.85	259.38
31.	0	280.60	208.43
32.	1	216.90	259.35
33.	0	280.55	208.33
Average deflection	1 − 0	36.332	50.915

The following explanations should be appended to this Table: 1.) During all these experiments, the relationships of the conductors always remained the same, so that the conditions of current intensity in all portions of the circuit were always the same. 2.) The corresponding observations on the magnetometer and dynamometer were always carried out simultaneously by two different observers at both instruments. The observers were, in addition to myself, Dr. Stähelin from Basel, and my assistant Mr. Dietzel. 3.) Every

single dynamometer observation shown in the Table is not a simple reading, but each such observation is based on 7 readings: namely, as the oscillation occurred, the highest and lowest position were alternately read and the 6 averages from each two successive readings taken to begin with; the 5 second averages, taken in turn from two such successive averages, were considered as partial results, and the average value of these 5 partial results entered in the Table. 4.) Between every two observations of the deflected position, the circuit was broken, in order to observe the natural position without galvanic influence, because this position changes appreciably, though very slowly, over time. This breaking of the circuit is indicated by a zero in the column which shows the number of elements. 5.) The average values of the deflection for the observations in the Table from 1 to 11 are derived from the 11 preceding observations, by taking the 10 differences from each two successive observations during the closed and broken circuit, and the 9 averages were taken from every second such initial successive difference, of which, as a partial result, the overall average is given in the Table. 6.) Finally, as for the magnetometer, the horizontal distance of the mirror from the scale is to be noted during the experiments contained in this Table, because it later had to be frequently altered: it amounted to 1251 scale units. 7) The 11 observations, from which the average deflections of the magnetometer and dynamometer were calculated, give a proof of the exactness of the measurement; for one sees, that the 5 or 6 repetitions of the experiments, with the circuit closed and broken, which the 11 observations comprise, always agree, up to a fraction of a scale unit, in which connection it is to be noted, that even these small differences originate for the most part in the actual changes in current intensity; further, in the case of the magnetometer, they originate in the variations in declination appearing during the experiment; and, in the case of the dynamometer, from a placement which was not perfectly fixed and invariable.

The results of all these experiments can be briefly surveyed in the correlated average values of the deflection of the magnetometer and dynamometer by the current of 3, 2, and 1 Grove's elements, namely:

	Average deflection of magnetometer	Average deflection of dynamometer
for 3 elements	108.566	440.508
for 2 elements	72.438	198.305
for 1 element	36.332	50.915

According to the optical law of reflection, these numbers are proportional to the tangent of the doubled angle of deflection and are to be reduced to the tangents of the simple angle of deflection, which give the measure of the deflecting forces, in which a small influence on the part of the eccentricity of the mirror is to be taken into consideration. The corrections emerging from this are:

0.14	0.47
0.04	0.05
0.00	0.00,

from which, if these corrections are taken into account, the following corrected values are yielded, i.e., for the deflecting force

of the magnetometer	of the dynamometer
108.426	440.038
72.398	198.255
36.332	50.915 .

Now, according to the measure of *electromagnetic* intensity taken above as the foundation, the numbers in the *first* column are proportional to the *current intensity*, while the numbers in the *second* column give the corresponding *electrodynamic forces*, according to which, therefore, the dependency of the electrodynamic forces on the current intensity can be determined, which was the chief purpose of these experiments. Before this occurs, however, let it be noted, that it could seem as though a small extraneous influence must still be excluded from the numbers in the first column, i.e., that which originates from the influence of the *bifilar coil* on the magnetometer. That is, those numbers, then, could only hold true as a measure of current intensity, if the magnetometer is always deflected by that same fixed, uninterrupted segment of the circuit. This segment of the circuit was the uninterruptedly fixed multiplier of the dynamometer. In point of fact, this multiplier was located in a position vis-à-vis the magnetometer, such that it exerted the greatest deflecting force, while the bifilar coil suspended in the multiplier was brought originally into such a position, where, even if a stronger current was conducted through it, it was able to exert no deflecting force whatever. Now, however, in the foregoing experiments the bifilar coil was appreciably deflected or twisted, and after this twisting, it had to exert a deflecting force on the magnetometer, wherefore the numerical values above required a correction, in order to make them correspond to the exclusive influence of the multiplier. This correction is, however, merely very small, because the intensity of the current passing through the bifilar coil amounted to only a 246.26th of the current intensity in the multiplier, because of the separation [of the current] mentioned above. I have ascertained for myself, that even in the case where the correction was greatest, it still remained below 1/500 scale units, and hence can be disregarded.

If one now multiplies the square roots of the observed values for the electrodynamic interaction, namely, $\sqrt{440.038}$, $\sqrt{198.255}$, $\sqrt{50.915}$, by the constant factor

$$5.15534,$$

one obtains nearly the observed values for the electromagnetic effect, namely, the series:

$$108.144$$
$$72.589$$
$$36.786,$$

whose comparison with the observed values yields the following differences:

$$-0.282$$
$$+\,0.191$$
$$+\,0.454.$$

The greatest difference, which appears between these calculated values and the directly observed values for the electromagnetic force, thus amounts to less than half a scale unit, in virtue of which, the law underlying the calculation can be considered as proven, *that the electrodynamic force of two portions of a circuit is proportional to the square of the electromagnetic force, consequently proportional to the square of the current intensity.*

This experiment also makes it evident, that the method of electrodynamic measurement utilized here permits a rigor and precision almost equal to that permitted by the method of magnetic measurement with the magnetometer.

5.5 Proof of the Fundamental Electrodynamic Law by Means of Measurement

After this first test of the precision to be achieved with the described instruments of electro-dynamic measurement, I proceed at once to a system of the measurements carried out then, which is suited to a complete examination of the fundamental electrodynamic law.

Ampère, in his Treatise cited above, page 181 f., presents two methods of deriving the law of interaction of two conducting wires from experimentation:[101]

> The one way, consists of first measuring with the utmost precision the values of the interaction of two pieces of finite size, by successively bringing them at different distances and positions vis-à-vis each other; then one must make a hypothesis about the value of the interaction of two infinitely small parts, conclude from this the value of the [reciprocal] action, which emerges for the conductors of finite size, with which one has operated, and modify the hypothesis, until the results of the calculation agree with those of observation. [...] The other consists of experimentally confirming, that a moveable conductor stays in perfect equilibrium between equal forces or equal rotational moments,[102] when these forces or moments come from parts of fixed conductors, whose shape and size can be altered in any way whatever, under conditions, determined by the experiment, without disturbing the equilibrium, and from this to directly conclude by means of calculation, what the value of the interaction of two infinitely small parts must be, in order that the equilibrium may be actually independent of all changes of form or size, which are compatible with those conditions.

Ampère preferred the latter method for reasons among which one was already sufficient, namely that he did not possess the instruments indispensable for the former method. Of course, under such conditions the second method had to be preferred, which does not require the performance of actual measurements. Yet Ampère seems to have overvalued the latter method, when he expressed the view that it deserved an absolute preference over the former. An instrument for precise measurements has two prerequisites: 1.) a great refinement and sensitivity, which allows us to recognize the effects to be measured clearly and independently of extraneous, uncontrollable influences; 2.) a measuring apparatus suited to these effects. It is clear, however, that this latter requirement can always be easily fulfilled, if the former is satisfied, thus the former must be regarded as the main requirement. Fulfillment of this main requirement, however, is just as essential for the second method as for the first, because otherwise it will be quite illusory. The essential difference between these methods, in relation to experimentation, is thus simply that according to the former method, one holds the equilibrium of the electrodynamic forces by means of other known and measurable forces of nature, while according to the second method, one seeks conditions in which the electrodynamic forces will mutually maintain the equilibrium between themselves. There can be no doubt, that the latter method, if it is to lead to reliable and precise results, is less direct and less simple, in the *experimental* connection, than the former. Hence, at most, the fact can be brought to bear in favor of the second method, that in the *theoretical* connection,

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[101][Note by AKTA:] See [Amp23, pp. 181-182], [Amp26, pp. 9-10] and [AC15, pp. 344-345].

[102][Note by LH and AKTA:] In German: *Drehungsmomenten*. This can also be translated as rotatory actions or torques.

the fundamental law can be more easily and more directly derived from the results achieved by this method, which, however, is no longer a consideration, if the fundamental laws to be tested are already fully in hand, as occurs, through Ampère's services, in the foregoing case. In virtue of this, we are in a position to carry out a very simple system of measurements, which meets the requirements.

The two conducting wires, which act reciprocally upon each other, should form circles, or systems of parallel circles, which have a common axis and are called *conducting coils*. These two axes should have a position horizontal and right-angled to each other, specifically, so that the extension of the one axis goes through the center of the other coil. One of these coils is fixed, the other can be rotated around its vertical diameter. Now, either the extended axis of the fixed coil can go through the center of the moveable coil, or, vice versa, the extended axis of the moveable coil can go through the center of the fixed coil. In both cases, one can make measurements at different distances of the centers from each other. It is easily seen, that these two ways of ordering electrodynamic measurements fully correspond to those of the magnetic measurements, which Gauss has presented in the *Intensitas vis magneticae terrestris ad mensuram absolutam revocata* (Commentationes Soc. regiae Scient. Gottingensis recentiores, Vol. VIII, page 33).[103,104] For the electrodynamic interactions, we can add still a third ordering of measurements, where the centers of the two coils coincide, as occurs in the dynamometer described above. In all these cases, Ampère's law can be applied, and the results calculated, in order to compare them with the results of observation.

If the fixed coil acts at a distance on the moveable coil, then the two coils can have arbitrarily like or unlike diameters; if, however, the centers of the two coils are to coincide, as was the case with the measuring instruments described above, then the interior diameter of the one, ring-shaped coil must be larger than the exterior diameter of the other, so that the former can contain the latter. In the dynamometer described above, the moveable coil was the smaller, and was contained by the fixed coil. If, finally, the three series of experiments just indicated are to be carried out, by simply placing the fixed coil in different places in succession, without changing the suspension of the moveable coil, which is advantageous for more precise comparison of all the measurement results with each other, then the moveable coil must be larger, so that it can contain the fixed coil, because that is the only way the latter, unimpaired by the suspension wires, can be introduced through the moveable coil. This is the reason why, for this system of measurements, a special measuring apparatus was constructed by the instrument-maker Mr. Leyser[105] in Leipzig, which will be described here.

The *bifilar coil aaa* in Figure 10 consists of a thin brass ring of 100.5 mm diameter and 30 mm height, which lies between two parallel brass discs or washers of 122.7 mm exterior diameter and 100.5 mm interior diameter, and holds them at a distance of 30 mm from one another. A copper wire of 1/3 mm diameter, coated with silk, is wound around that brass ring about 3 000 times, between these two discs, so that it entirely fills up the space between the two discs. After the wire is wound, the two brass discs are held together by a fixed brass clamp *bb*, which encloses the wound wire and holds up the torsion-circle *cc* at its center. The torsion-circle consists of two horizontal (when the bifilar coil is situated vertically) discs, of which the lower is rigidly connected by means of the brass clamp to the bifilar coil, while the upper can rotate on the lower around a vertical axis. The upper disc is furnished with a

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[103][Note by HW:] Gauss, *Werke*, Vol. V, page 107.
[104][Note by AKTA:] See footnote 97 on page 51.
[105][Note by AKTA:] Georg Moritz Ludwig Leyser (1816-1881).

circular scale of units,[106] the lower with an index. On the upper disc rests a wooden peg d, which at the upper end holds the prong ee of a very moveable roller of 20 mm diameter. A silk thread ff is led around this roller, passes vertically upward over both sides of the roller, and is fastened on both sides, a few millimeters above the roller, to the two suspension wires fg and fg. To these connection-points f and f, the two ends of the wire wound around the bifilar coil are also brought, in such a way that the galvanic current can be conducted through the one suspension wire to one end of the bifilar coil, and out the other end of the bifilar coil into the second suspension wire. The two suspension wires go upward from these connection-points vertically toward the ceiling, where they are fastened to two brass hooks insulated from one another. From these two hooks, two other wires are led away, one to a commutator, the other to the galvanic battery.
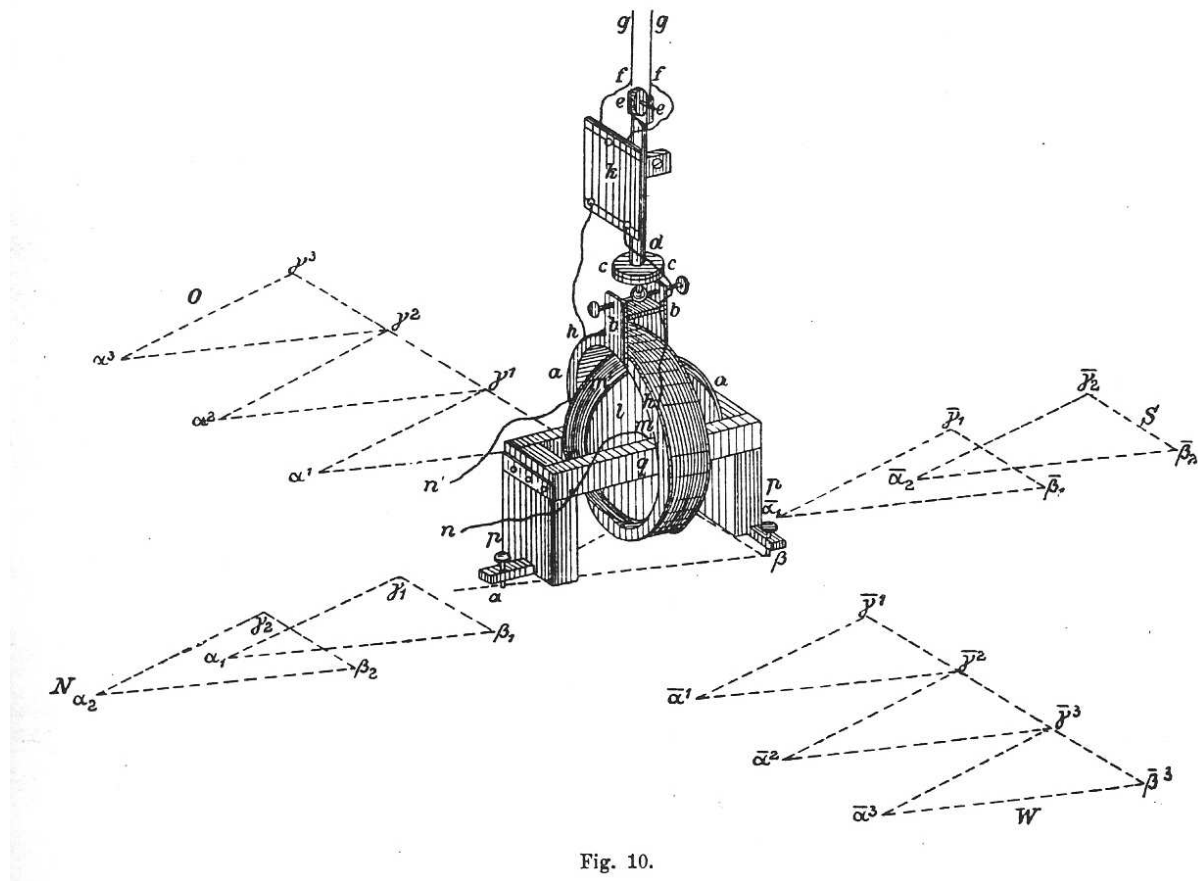
Fig. 10.

With the help of the torsion-circle, one can give the horizontal axis of the bifilar coil any arbitrary position, while the suspension wires maintain their natural parallel position. The torsion-circle was adjusted in such a way, that the axis of the bifilar coil coincided with the magnetic meridian NS, so that the terrestrial magnetism did not alter the position of the bifilar coil, when a galvanic current passed through the coil.

To the wooden peg on the torsion-plate a vertical flat mirror k was fastened, at which at about a 3.3 meter distance, a telescope with crosshairs was directed, in order to observe the image of a horizontal scale set up near the telescope.

The *fixed coil lll* in Figure 10 consists of two thin parallel brass plates of 88.8 mm diameter,

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[106][Note by AKTA:] In German: *Kreistheilung*. This expression can also be translated as "circular division". For instance, a circle divided into degrees.

which are held in a fixed position 300 mm distant from each other by a 5.5 mm thick brass axle m. This brass axle goes through both plates and extends 10 mm on both sides. Around the same axle between the two discs, a copper wire of 1/3 mm diameter, sheathed in silk, is wound roughly 10 000 times, so that it entirely fills the space between the two discs. One end of this wire is led outward, close to the axle, through a small opening lined with ivory at m in the one disc, from m to n; the other end is fastened to the periphery of the coil at m' with silk thread and goes outward from m' to n'. The one wire end $n'n'$ is brought to the commutator A (Figure 11), the other nn to the multiplier B (Figure 11) of a galvanometer.
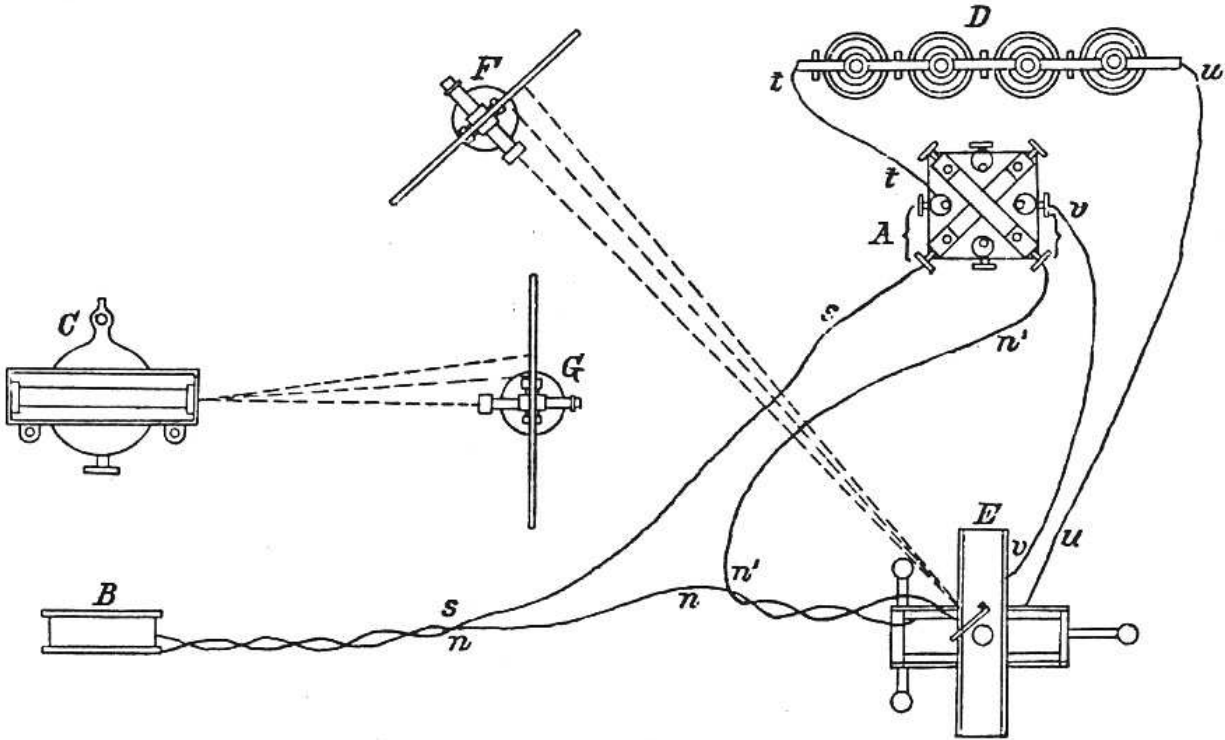
Fig. 11.

A small wooden frame[107] pp serves to keep this coil fixed (Figure 10), which presents two sockets q, into which the two protruding parts of the axle are laid. This frame stands on three feet which are fitted with screw-tips α, β, γ for levelling. One of these feet is fitted with a hinge r, and can be pushed open in such a way that one can freely move it, along with a part of the frame and of the fixed coil, through the bifilar coil, and then can push it down again. The fixed coil then comes to stand in the center of the bifilar coil, and the frame then rests with two feet on this side of the bifilar coil, and with the third foot on that side, on the immovable table, which is close below the bifilar coil.

On the flat horizontal table-top, the positions are precisely drawn ahead of time, at which the fixed coil is to be successively placed. Namely, the three screw-tips, which, with concentric placement of the two coils, stand on points α, β, γ of the table-top, are shifted in such a way that they come to stand either north at points $\alpha_1\beta_1\gamma_1$ or $\alpha_2\beta_2\gamma_2$ and so on, or south at points $\bar{\alpha}_1\bar{\beta}_1\bar{\gamma}_1$ or $\bar{\alpha}_2\bar{\beta}_2\bar{\gamma}_2$ and so on, or east at points $\alpha^1\beta^1\gamma^1$ or $\alpha^2\beta^2\gamma^2$ and so on, or west at points $\bar{\alpha}^1\bar{\beta}^1\bar{\gamma}^1$ or $\bar{\alpha}^2\bar{\beta}^2\bar{\gamma}^2$ or $\bar{\alpha}^3\bar{\beta}^3\bar{\gamma}^3$ and so on. For protection against the influence of the air, the bifilar coil is surrounded with a wooden casing, in which a glass sheet is inserted,

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[107][Note by AKTA:] In German: *Gestell*. This word can also be translated as stand or support.

through which the light can fall from the scale onto the mirror, and thence back into the telescope. The casing consists of two parts, one of which can be removed when the fixed coil is to be placed in the center of the movable coil.

In order, now, to make the system of electrodynamic measurements carried out with this instrument comparable among themselves, it was necessary to measure, independently of the system, the intensity of the current which was conducted through the two coils during each measurement. For this purpose, the apparatus described in Section 5.3 could not be applied, because of the adjustment of the fixed coil to be undertaken from one measurement to another. Hence the one end nn of the wire wound around the fixed coil was connected to a third wire coil B (Figure 11), which consisted of 618 parallel windings, enclosing an area of 8 313 440 square millimeters, and was placed 217 mm west of a transportable magnetometer, C, 8 meters away from the dynamometer (Figure 11), and which with the magnetometer formed a *galvanometer*. With its other end ss, this third wire coil was, finally, connected with the commutator A (Figure 11), to which one conducting wire tt of the galvanic battery D also led.

Figure 11 gives a clear representation of the ordering and connection of the different parts of the apparatus. It may be noted in this connection, that the two wire ends of the fixed coil, when they were located close to the bifilar coil, were wound around each other in such a way that the opposite currents passing through them had no influence on the bifilar coil. E represents the dynamometer in outline, F the accompanying telescope for the readings, along with the scale; C represents the magnetometer in outline, and G the accompanying telescope for the readings, along with the scale; B is the multiplier coil through which the same galvanic current is conducted as through the dynamometer, and which acts at a distance on the needle of the magnetometer C, whose deflection from the magnetic meridian is measured, in order to determine the intensity of the applied current and its variations during the experiments.

The galvanic battery, which was used for these experiments, consisted of 8 Bunsen carbon elements.[108] The direction of this current always remained the same in the wire of the bifilar coil of the dynamometer E, and was, as is clear from the placement of the commutator A, reversed in the fixed coil of the dynamometer E and in the third coil B, which took the place of the multiplier in the *galvanometer*, simply by means of the alternation of the commutator. It was necessary for the current in the bifilar coil to maintain its constant direction, in order to eliminate the influence of terrestrial magnetism. The reversal of the current in the fixed coil was necessary, in order to deflect the north end of the axis of the bifilar coil alternately eastward and westward by means of the effect of this fixed coil on the bifilar coil, and, through repeated measurement of these positive and negative deflections, to determine this effect with greater rigor. The reversal of the current in the third coil had the same purpose, in relation to the deflection of the magnetometer, which served to determine the current intensity. This purpose is attained by means of the equipment described, with the help of the commutator A; because the direction of the current constantly remained the same in the battery D and in all those portions of the circuit which the battery D connects with the commutator A, namely, in the wire tt, in the battery D, in the wire uu, in the *bifilar coil* of the dynamometer E and in the wire vv; on the other hand, the direction of the current can be changed by the commutator A in all those portions of the circuit which are separated by the commutator A from the battery D, namely, in the wire $n'n'$, in the fixed

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[108][Note by AKTA:] In German: *Bunsen'schen Kohlenbechern*. The Bunsen voltaic cell or element was named after its inventor, Robert Wilhelm Eberhard Bunsen (1811-1899).

coil of the dynamometer E, in the wire nn, in the multiplier coil B, and in the wire ss.

The period of oscillation of the bifilar coil without current was $= 13.3259$". The horizontal distance of the mirror of the bifilar coil from the scale was $3\,306.3$ scale units; the horizontal distance of the magnetometer's mirror from the scale was 1103 scale units. The results of these measurements are contained in the following Table, in the same order in which they were made.

A	Dynamometer			Galvanometer		
	516.27			250.47		
		26.41			321.49	
	542.68			571.96		
		26.74			321.48	
600 westerly	515.94			250.48		
		26.37	26.35		321.12	320.14
	542.31			571.60		
		26.24			319.41	
	516.07			252.19		
		26.00			317.22	
	542.07			569.41		
	506.37			254.05		
		44.47			314.65	
	550.84			568.70		
		44.87			314.22	
500 westerly	505.97			254.48		
		43.89	44.31		314.77	314.32
	549.86			569.25		
		44.50			314.33	
	505.36			254.92		
		43.84			313.63	
	549.20			568.55		
	517.27			566.80		
		20.34			312.08	
	537.61			254.72		
		20.43			312.98	
500 northerly	517.18			567.70		
		20.19	20.30		312.82	312.48
	537.37			254.88		
		20.36			312.63	
	517.01			567.51		
		20.19			311.89	
	537.20			255.62		
	505.06			257.92		
		43.04			308.39	
	548.10			566.31		
		43.09			308.98	
500 easterly	505.01			257.33		
		42.53	42.89		308.05	308.80
	547.54			565.38		
		42.32			309.09	
	505.22			256.29		
		43.46			309.50	
	548.68			567.79		

A	Dynamometer			Galvanometer		
	517.96			564.05		
		19.51			306.09	
	537.47			257.96		
		19.80			306.07	
500 southerly	517.67			564.03		
		19.19	19.49		305.14	305.56
	536.86			258.89		
		19.79			305.47	
	517.07			564.36		
		19.17			305.03	
	536.24			259.33		
	514.31			260.23		
		24.19			304.46	
	538.50			564.69		
		23.65			305.02	
600 easterly	514.85			259.67		
		24.06	23.72		304.58	304.92
	538.91			564.25		
		23.72			305.36	
	515.19			258.89		
		23.85			305.17	
	539.04			564.06		
	568.21			562.50		
		81.67			303.54	
	486.54			258.96		
		81.85			304.67	
400 easterly	568.39			563.63		
		81.77	81.64		303.35	303.79
	486.62			260.28		
		81.57			303.32	
	568.19			563.60		
		81.35			304.08	
	486.84			259.52		
	546.32			261.44		
		36.27			300.95	
	510.05			562.39		
		36.25			302.42	
400 northerly	546.30			259.97		
		36.14	36.15		302.73	302.07
	510.16			562.70		
		35.96			301.58	
	546.12			261.12		
		36.12			302.69	
	510.00			563.81		

A	Dynamometer			Galvanometer		
	488.36			261.99		
		79.71			300.99	
	568.07			562.98		
		79.78			301.45	
400	488.29			261.53		
westerly		79.60	79.60		300.97	300.80
	567.89			562.50		
		79.49			300.80	
	488.40			261.70		
		79.40			299.83	
	567.80			561.53		
	510.23			561.18		
		35.34			298.95	
	545.57			262.23		
		35.53			299.67	
400	510.04			561.90		
southerly		35.45	35.43		299.40	299.30
	545.49			262.50		
		35.56			299.37	
	509.93			561.87		
		35.28			299.11	
	545.21			262.76		
	566.29			263.73		
		79.45			298.81	
	486.84			562.54		
		79.39			300.31	
300	566.23			262.23		
southerly		78.13	78.85		300.30	299.89
	488.10			562.53		
		78.64			300.30	
	566.74			262.23		
		78.62			299.71	
	488.12			561.94		
	431.18			263.96		
		192.57			298.05	
	623.75			562.01		
		192.40			298.25	
300	431.35			263.76		
westerly		192.02	192.17		297.99	297.81
	623.37			561.75		
		191.96			297.30	
	431.41			264.45		
		191.91			297.45	
	623.32			561.90		

A	Dynamometer			Galvanometer		
	566.96			265.93		
		78.30			297.12	
	488.66			563.05		
		78.37			299.13	
300	567.03			263.92		
northerly		77.93	78.08		299.12	298.33
	489.10			563.04		
		77.98			298.15	
	567.08			264.89		
		77.80			298.14	
	489.28			563.03		
	433.52			266.49		
		190.26			296.69	
	623.78			563.18		
		109.43			298.16	
300	433.35			265.02		
easterly		190.23	190.08		296.98	297.30
	623.58			562.00		
		189.89			297.09	
	433.69			264.91		
		189.59			297.60	
	623.28			562.51		

The following explanations are to be appended to the Table. In column A, the distance between the centers of both coils of the dynamometer is given in millimeters, and it is noted, in which direction, taking the bifilar coil as the point of origin, the fixed coil was placed; under north and south, the direction is to be understood as aligned with the magnetic meridian; under east and west, the direction is to be understood as perpendicular to the magnetic meridian. — In the second column, headed *Dynamometer*, the position of the bifilar coil is given in scale units, alternating between the direct and reversed direction of the current in the fixed coil. Each of these numbers is based on 7 readings, in which from oscillation to oscillation, the maximum and minimum of the oscillation arc were alternately taken 7 times after one another, and from this, according to recognized rules, the average state of equilibrium of the oscillating coil was calculated. With the reversal of the current in the fixed coil, a procedure was applied which did not increase the arc of oscillation of the bifilar coil. In the Table, next to the observations of position, which relate alternately to the direct and reversed current in the fixed coil, are noted the differences for every second immediately successive observation, which provide in scale units the double deflection of the bifilar coil by means of the influence of the fixed coil. Finally, next to these particular values of the double deflection, their average value for each placement of the fixed coil is noted. — In the third column, headed *Galvanometer*, the position of the galvanometer is given, alternately with direct and reverse current direction in the coil B serving as multiplier. This position has been observed and calculated in the same way as with the dynamometer, and next to it are noted the differences and the average value of the double deflection of the galvanometer. The corresponding observations at the dynamometer and at the galvanometer were always made simultaneously by two observers at the two instruments.

All the observations assembled in the Table above were made in the order presented, on

one day, immediately after one another, and, since all external conditions remained exactly the same, all the results are directly comparable with one another. On this day, it had not been possible to carry out as well those observations, whereby the fixed coil received its placement in the center of the bifilar coil, because the re-positioning of the fixed coil required several time-consuming precautionary measures. This last series of experiments was hence postponed to the next day. However, because it was then no longer possible to be confident that all external conditions remained exactly the same as in the earlier experiments, on this second day, for comparison, two series of experiments, which had already been made on the first day, were repeated, namely, at a 300-mm east and west distance of the fixed coil from the bifilar coil, which could be used to reduce the last series of experiments in such a way, that the results became comparable with the results of the earlier experiments, independently of the small variations which might have occurred in the external conditions in the meantime. Also, the fact that on the next day, another galvanic battery was used, namely of 2 Grove (platinum-zinc) elements instead of 8 Bunsen carbon elements, had no influence on this comparison. This was necessary because otherwise, the deflection of the dynamometer when the fixed coil was placed in the center of the bifilar coil would have been too large to be measured on the scale. Finally, it may be noted that the constant direction of the current in the bifilar coil was the opposite on the next day from the first, which likewise had no influence on the reduced results. The results of this second series of experiments are contained in the following Table.

A	Dynamometer			Galvanometer		
	48.05			359.78		
		905.69			64.51	
	953.74			424.29		
		904.84			64.46	
	48.90			359.83		
0		904.00	903.97		64.47	64.45
	952.90			424.30		
		903.01			64.40	
	49.89			359.90		
		902.31			64.39	
	952.20			424.29		
	485.70			329.30		
		27.58			125.08	
	513.28			454.38		
		27.18			124.99	
300	486.10			329.39		
easterly		27.25	27.54		124.89	125.08
	513.35			454.28		
		28.26			125.10	
	485.09			329.18		
		27.43			125.35	
	512.52			454.53		
	512.37			454.50		
		25.65			125.18	
	486.72			329.32		
		27.77			125.29	
300	514.49			454.61		
westerly		27.43	27.20		125.35	125.23
	487.06			329.26		
		27.60			125.30	
	514.66			454.56		
		27.55			125.05	
	487.11			329.51		

Herewith it is to be noted that the current of 2 Grove's elements also elicited a larger deflection of the dynamometer than could be measured with the 1000-unit scale, when the fixed coil was placed in the center of the bifilar coil, and that therefore in this case the current was weakened through increasing the resistance of the circuit by inserting a long, thin conducting wire, which was removed again when the coils were placed 300 mm apart, because otherwise the deflection of the dynamometer would turn out to be too small for an exact measurement. This is discerned from the difference in the magnetometer deflection, which measures the current intensity, and in the latter case amounted to almost double that of the former.

The results of this series of experiments can easily be surveyed in the following compilation of all the average values of the simultaneous deflections of the dynamometer and galvanometer, namely:

Distance in mm	Dynamometer	Galvanometer
0	903.97	64.45
300 easterly	27.54	125.08
300 westerly	27.20	125.23.

These numbers are, according to the optical law of reflection, proportional to the tangents of the doubled angles of deflection, and are to be reduced to the tangents of the simple angles of deflection, because these will give the measure of the deflecting force. Here a slight influence of the eccentricity of the mirror is still to be taken into consideration. One obtains from this the following reduced values:

0	899.79	64.44
300 easterly	27.54	124.98
300 westerly	27.20	125.13.

We take the average from the last two series, which differ very little from one another, because they should be almost equal if the current intensity is the same and the position of the fixed coil easterly and westerly of the bifilar coil is totally symmetric, whereby we obtain the following values:

0	899.79	64.44
300	27.37	125.055

The results of the foregoing series of experiments can be surveyed in the compilation of all the average values for the dynamometer and galvanometer deflections in the following Table:

Distance	Easterly		Westerly		Southerly		Northerly	
Milli-meter	Dyna-mo-meter	Galva-no-meter	Dyna-mo-meter	Galva-no-meter	Dyna-mo-meter	Galva-no-meter	Dyna-mo-meter	Galva-no-meter
300	190.08	297.30	192.17	297.81	78.85	299.89	78.08	298.33
400	81.64	303.79	79.60	300.81	35.43	299.30	36.15	302.07
500	42.89	308.80	44.31	314.32	19.49	305.56	20.30	312.48
600	23.89	304.92	26.35	320.14	—	—	—	—

I have convinced myself, that the influence of the reduction of these numbers to the tangents of the simple deflection angles for the dynamometer is so slight, that it can be left out of consideration; that is, it is smaller than the unavoidable observational errors. This correction also comes into consideration in the galvanometer observations, because no large differences emerge in the deflection of the galvanometer.

5.6   Reduction of the Observations

The observed electrodynamic forces in the foregoing Section can not immediately be used for the intended test of the dependence of these forces, determined by Ampère's law, on the mutual position of the conducting wires acting upon each other, because they are based on different current intensities. Hence these observations are first to be reduced to *the same*

current intensities, for which the law proven in Section 5.4 comes into application, according to which the dynamometer deflections are proportional to the square of the galvanometer deflections. The application of this law to the observations at hand, however, itself presupposes another reduction, namely, the reduction *to the same directive force*[109] of the bifilar coil, which underwent appreciable changes during these experiments. In the observational results presented in Section 5.4, through which the cited law was proven, the correction resulting from this was negligible and hence did not need to be taken into account, because there the current which passed through the fixed coil of the dynamometer, was divided, and only a small part, namely 1/246 of the entire current, was conducted through the *bifilar coil*, which had no appreciable influence on the directive force of this coil. With regard to the present observational results, on the contrary, this reduction cannot be ignored, because here the entirety of the current conducted through the fixed coil, went further through the bifilar coil.

The *directive force of the bifilar coil* separates into a *constant* and a *variable* component. The *constant* component, which is called the *static moment*, depends on the weight of the bifilar coil and on the length and distance of separation of the suspension wires, and can be calculated from the observed *period of oscillation* and from the *inertial moment* of the bifilar coil. The *period of oscillation* of the bifilar coil, if no current is passing through it, was determined by means of special observations,

$$t = 13.3259".$$

The *inertial moment* K was found according to the formula given by Gauss in the *Intensitas*,[110]

$$K = 864\,800\,000,$$

in which millimeters and milligrams are the basis for measuring length and mass. The *static moment* S is obtained from[111]

$$S = \frac{\pi^2 K}{t^2} = 48\,064\,000 \ .$$

The *variable* component of the directive force of the bifilar coil, which is called the *electromagnetic moment*, depends on the intensity of the horizontal component of the *terrestrial magnetism* T, on the intensity of the current of the bifilar coil, χ, and on the size of the area, λ, which is demarcated by the wire windings of the bifilar coil, and is to be set equal to the product of these three magnitudes. The intensity of the horizontal component of the terrestrial magnetism was found to be, at the location of the bifilar coil,

$$T = 1.83 \ .$$

The size of the *area*, which was demarcated by the wire windings of the bifilar coil, could not be determined by direct measurement, because the number of wire windings was not precisely known. Hence this area was indirectly determined by comparison of the electromagnetic effect of this coil with that of another of known area on a distant galvanometers, by which

$$\lambda = 29\,314\,000 \text{ square mm}$$

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[109][Note by AKTA:] In German: *auf gleiche Direktionskraft*. The concept of "directive force" (or directional force) was introduced by Gauss in 1838, [Gau38b, p. 4] with English translation in [Gau41c, p. 254].

Consider, for instance, a compass needle of magnetic moment m. Utilizing Gauss and Weber's terminology, let T be the horizontal component of the Earth's magnetic force. The torque τ exerted by the Earth on the needle when it is deflected by an angle θ relative to the local magnetic meridian is given by $\tau = mT \sin\theta$. The so-called magnetic directive force is here given by mT.

[110][Note by AKTA:] See footnote 97 on page 51.

[111][Note by AKTA:] See footnote 96 on page 51.

was obtained. The *current intensities*, finally, were given for all individual experiments by means of the galvanometer observations in scale units, which nevertheless are to be reduced for present purposes to the *fundamental electromagnetic measure* of current intensity. To do this, the observed number of scale units is multiplied by a constant factor, which in accordance with the proof to be given in Section 5.9 is to be
$$= 0.000\,361\,4.$$

Thus, if y denotes the number of scale units observed on the galvanometer, then the current intensity is

$$\chi = 0.000\,361\,4 \cdot y \ .$$

From these elements results the *electromagnetic moment* of the bifilar coil

$$\chi \lambda T = 19\,400 \cdot y \ .$$

This value for the electromagnetic moment is to be subtracted, in the first series of experiments, from the *static moment*, in the second series of experiments, however, it is to be added to the static moment, in order to obtain the *directive force of the bifilar coil*, because, as already noted on page 67,[112] the direction of the current in the bifilar coil in the last series was opposite to that in the first. For the first series of experiments, the directive force in units of static moment resulted as

$$= 1 - \frac{19400}{48064000} \cdot y \ ,$$

for the second series of experiments

$$= 1 + \frac{19400}{48064000} \cdot y \ .$$

The observed *dynamometer deflections* are accordingly reduced to a constant directive force equal to the *static moment*, if one multiplies the number of scale units observed on the dynamometer x in the *first* series of experiments by $(1 - 194 \cdot y/480640)$, in the *second* by $(1 + 194 \cdot y/480640)$.

After performing this reduction, one obtains for the *first series* the values of the dynamometer and galvanometer deflections, assembled in the following Table.

Distance	Easterly		Westerly		Southerly		Northerly	
Milli- meter	Dyna- mo- meter	Galva- no- meter	Dyna- mo- meter	Galva- no- meter	Dyna- mo- meter	Galva- no- meter	Dyna- mo- meter	Galva- no- meter
300	167.26	297.30	169.06	297.81	69.30	299.89	68.67	298.33
400	71.63	303.79	69.93	300.81	31.15	299.30	31.74	302.07
500	37.54	308.80	38.69	314.32	17.09	305.56	17.74	312.48
600	20.95	304.92	22.94	320.14	—	—	—	—

For the *second series* one obtains the following correlated values:

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[112][Note by LH and AKTA:] [Web46, p. 62 of Weber's *Werke*].

Distance	Easterly or Westerly	
Millimeter	Dynamometer	Galvanometer
0	923.19	64.44
300	28.75	125.055

The *sensitivity* of an instrument is inversely proportional to its *directive force*, i.e. the force to be measured generates an all the greater deflection, the smaller its directive force is. The foregoing observational data, reduced to *the same directive force*, are thus equivalent to those obtained under the condition of *equal sensitivity* of the dynamometer.

After this reduction of the dynamometer observations *to the same directive force*, it is now possible to bring into application the law proven in Section 5.4 and to reduce all observations, for better comparison among themselves, *to the same current intensity*. For this, it is only necessary to determine more closely the normal current intensity for which the reduced observational data are supposed to be valid. Since it is not necessary to utilize for both series of experiments *equal* normal current intensities, that intensity may be chosen for the *first series* which corresponds to a galvanometer deflection in scale units, whose square = 100 000, for the *second series* a five-times-smaller one, for which this square = 4 000. According to the law proven in Section 5.4, one then obtains from the dynamometer deflection x, given in the Table, which corresponds to the galvanometer deflection y, likewise given in the Table, the reduced value for the *first* series

$$= 100\,000 \cdot \frac{x}{y^2} \,,$$

for the *second* series

$$= 4000 \cdot \frac{x}{y^2} \,.$$

In the following Table, the values of the *first* series reduced by this method are assembled:

Distance	Easterly	Westerly	Southerly	Northerly
300	189.24	190.62	77.06	77.16
400	77.61	77.28	34.77	34.78
500	39.37	39.16	18.30	18.17
600	22.53	22.38	—	—

The reduced values of the *second* series are as follows:

Distance	Easterly or Westerly
0	889.29
300	7.35.

From this latter it results that the electrodynamic force of the fixed coil on the bifilar coil, when the centers coincide, was

$$\frac{88929}{735} = 120.9$$

times greater, than when the centers were 300 mm distant from one another in the west-east direction.

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In the Table for the first series, we see that the corresponding values very closely agree, in east and west as well as in south and north, which is a proof for the precision of the measurement, as well as for the symmetrical placement of the fixed coil on both sides of the bifilar coil. If we now take the mean of these already closely agreeing values, and introduce for 0 distance, in conformity with the results just taken from the *second* series, the 120.9-fold value for the effect for 300-mm distance perpendicular to the magnetic meridian, we obtain the following Table:

Distance	Perpendicular to Magnetic Meridian	In the direction of Magnetic Meridian
0	22960	22960
300	189.93	77.11
400	77.45	34.77
500	39.27	18.24
600	22.46	—

5.7   Comparison with the Law of Magnetic Interaction

Before we use this system of measurements of the interaction of two conducting wires in order to directly test Ampère's law, we wish to make an interesting, if only indirect and partial, prefatory test. Namely, it is known, that one of the most important consequences of Ampère's law for the interaction of two current elements, is, that the interaction of two magnets, given all the differences in their respective positions, would also be elicited by means of constant galvanic currents, which occur in a specific way on the surface or in the interior of the magnets, and, conversely, that the interactions of two galvanic coils, like those with which our measurements were carried out, given all the differences in their respective positions, would also be elicited by two constant magnets, contained in areas surrounded by the wire windings of those coils, if the free magnetism were distributed in a specific way in the interior or on the surface. Accordingly, all the results which Gauss has proven for such magnets in the *Intensitas vis magneticae...,*[113] can be carried over to our two coils, and this can happen all the more easily, because we have ordered our measurements of the interactions of the two coils precisely as Gauss determined the measurements of the interactions of the two magnets. Gauss, *ibid.*, gave the distance of the two magnets in meters, and instead we use millimeters; further, Gauss determined the *simple* deflections, calculated with the natural state of rest of the needle as the starting-point, in degrees, minutes, and seconds, while we have stated the *doubled* tangents of the simple angle of deflection in scale units (that is, multiplied by the constant coefficient 6612.6). If, therefore, we wish to bring our measurements of the interaction of the two conductor coils into the same form as those magnetic measurements, we obtain the following Table of *measured* deflections:

R	v	v'
0.3 m	0°49′22″	0°20′3″
0.4 m	0°20′8″	0°9′2″
0.5 m	0°10′12″	0°4′44″
0.6 m	0°5′50″	—

[113][Note by AKTA:] See footnote 97 on page 51.

The tangents of v and v' should then, here as there, be developed according to the descending odd powers of R, and specifically,

$$\tan v = aR^{-3} + bR^{-5}$$

$$\tan v' = \frac{1}{2}aR^{-3} + cR^{-5}$$

where a, b, c are to be experimentally determined. If, now, in our case

$$\tan v = 0.0003572R^{-3} + 0.000002755R^{-5}$$

$$\tan v' = 0.0001786R^{-3} - 0.000001886R^{-5}$$

so the following Table of *calculated* deflections is yielded, to which the differences from the *measured* deflections is adjoined:

R	v	Difference	v'	Difference
0.3 m	0°49′22″	0	0°20′4″	−1
0.4 m	0°20′7″	+1	0°8′58″	+4
0.5 m	0°10′8″	+4	0°4′42″	+2
0.6 m	0°5′49″	+1	—	

A better agreement between observed and calculated values can not be hoped for, and accordingly, Ampère's law finds itself experimentally confirmed in one of its most general and most important consequences.

5.8   Comparison of Ampère's Law with the Observations

The Ampère formula for the interaction of two current elements, which is to be tested with respect to the present system of measuring this interaction, itself consists essentially of the following: The interaction of two current elements is the inverse of the square of their distance from one another, and directly proportional to the current intensity and the length of each current element, and moreover to a factor, which depends on the angle which the directions of the two current elements form with each other, and on the two angles, which the two current elements form with their straight connecting-line. Let r be the distance of the two current elements from each other, i and i' the two current intensities, ds and ds' the lengths of the two current elements, ε the angle which the directions of the two current elements form with one another, and finally ϑ the angle of the one current element with the line r, and ϑ' the angle of the other current element with the extended line r, then[114]

$$-\frac{ii'}{rr}\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right)dsds'$$

is an expression for the *magnitude* of the interaction of both elements; the *direction* of this action for both current elements coincides with their connecting-line, and for the two

[114][Note by AKTA:] These angles appear in the Figure of this footnote, [AC15, Section 2.8]:

opposed current elements it is repulsive, when the above expression has a positive value, in the opposite case attractive.

From this law it is now possible first of all to find the expression for the combined effect which a number of current elements, which together form a *closed* line, exert on any other current element.

This effect can be separated according to three rectilinear coordinate axes. If these three components are denoted X, Y, Z, and the angles are denoted λ, μ, ν, which the current element ds' which is acted upon forms with the three coordinate axes, and the center of the element ds' is the point of origin of the coordinates, Ampère has already proven that

$$X = -\frac{1}{2}ii'ds'\left(\cos\mu\int\frac{xdy - ydx}{r^3} - \cos\nu\int\frac{zdx - xdz}{r^3}\right)$$

$$Y = -\frac{1}{2}ii'ds'\left(\cos\nu\int\frac{ydz - zdy}{r^3} - \cos\lambda\int\frac{xdy - ydx}{r^3}\right)$$

$$Z = -\frac{1}{2}ii'ds'\left(\cos\lambda\int\frac{zdx - xdz}{r^3} - \cos\mu\int\frac{ydz - zdy}{r^3}\right)$$

(see Mémoires de l'acad. roy. des sc. de l'Institut de France, 1823, page 214).[115] If now, the closed line is a circular line of radius m, and the x-axis is parallel to the projection onto the plane of the circle of the straight line connecting the center of the circle with the point of origin of the coordinates, and the y-axis [is parallel] to the diameter of the circle perpendicular to that projection; further, if p denotes the distance projected onto the plane of the circle, of the center of the circle from the point of origin of the coordinates, and ω denotes the angle which the line p forms with the radius of a circle element ds; finally, if q denotes the perpendicular from the point of origin of the coordinates to the plane of the circle, than in this case, in the foregoing values of X, Y, Z [we have:]

$$z = q, \quad y = m\sin\omega, \quad x = p - m\cos\omega \ ,$$

therefore, since $r^2 = x^2 + y^2 + z^2$,

$$\int\frac{xdy - ydx}{r^3} = mp\int\frac{\cos\omega d\omega}{r^3} - m^2\int\frac{d\omega}{r^3}$$

(a) (b)
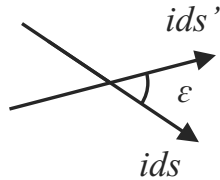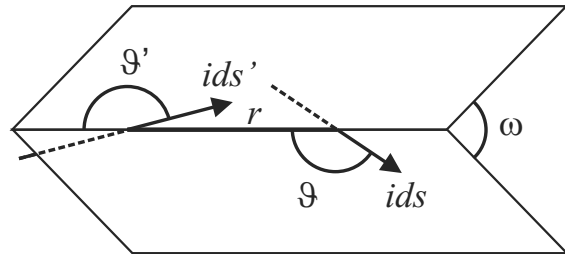
Here ω is the angle between the planes drawn through each element and the straight line joining their midpoints. These angles have the following values in radians: $0 \leq \varepsilon \leq \pi$, $0 \leq \vartheta \leq \pi$, $0 \leq \vartheta' \leq \pi$ and $0 \leq \omega \leq \pi$. Moreover, for Ampère $i \geq 0$ and $i' \geq 0$.

[115][Note by AKTA:] See [Amp23, p. 214], [Amp26, p. 42] and [AC15, pp. 366].

$$= mp \left(\frac{\sin \omega}{r^3} + 3 \int \sin \omega \cdot \frac{dr}{r^4} \right) - m^2 \int \frac{d\omega}{r^3}$$

$$\int \frac{zdx - xdz}{r^3} = mq \int \frac{\sin \omega d\omega}{r^3}$$

$$\int \frac{ydz - zdy}{r^3} = -mq \int \frac{\cos \omega d\omega}{r^3} = -mq \left(\frac{\sin \omega}{r^3} + 3 \int \sin \omega \cdot \frac{dr}{r^4} \right) .$$

If, finally, we substitute for dr the value yielded by the equation for r, namely:

$$r^2 = x^2 + y^2 + z^2 = m^2 + p^2 + q^2 - 2mp \cos \omega ,$$

that value being

$$dr = \frac{mp \sin \omega d\omega}{r} ,$$

and extend the integral value to the entire perimeter of the circle, then we obtain

$$\int \frac{xdy - ydx}{r^3} = 3m^2 p^2 \int \frac{\sin^2 \omega d\omega}{r^5} - m^2 \int \frac{d\omega}{r^3}$$

$$\int \frac{zdx - xdz}{r^3} = 0$$

$$\int \frac{ydz - zdy}{r^3} = -3m^2 pq \int \frac{\sin^2 \omega d\omega}{r^5} ;$$

consequently

$$X = -\frac{1}{2} ii' ds' \cdot m^2 \cos \mu \left(3p^2 \int \frac{\sin^2 \omega d\omega}{r^5} - \int \frac{d\omega}{r^3} \right)$$

$$Y = +\frac{1}{2} ii' ds' \cdot m^2 \left(3pq \cos \nu \int \frac{\sin^2 \omega d\omega}{r^5} \right.$$

$$\left. + \ 3p^2 \cos \lambda \int \frac{\sin^2 \omega d\omega}{r^5} - \cos \lambda \int \frac{d\omega}{r^3} \right) .$$

If the element ds' is part of a circle, whose radius is n, and whose plane is parallel to the coordinate axis z, and a denotes the perpendicular from the center of the circle m to the plane of circle n, c denotes the perpendicular from the center of circle n to the plane of circle m, b the distance between the two perpendiculars, and, as was the case in the previous experiment,

$$b = 0 ,$$

so we obtain the following equations for the angle α, β, γ, which the perpendicular to the plane of circle n forms with the coordinate axes:

$$\gamma = 90^\circ$$

$$\cos^2 \alpha + \cos^2 \beta = 1$$

$$\cos \alpha \cos \lambda + \cos \beta \cos \mu = 0 \ .$$

Since, moreover, it is posited that

$$\cos^2 \lambda + \cos^2 \mu + \cos^2 \nu = 1$$

we obtain

$$\cos \alpha = \frac{\cos \mu}{\sin \nu} \ , \quad \cos \beta = -\frac{\cos \lambda}{\sin \nu} \ .$$

For p and q we further obtain the following equations:

$$p \cos \beta = n \cos \nu$$

$$p^2 = a^2 + n^2 \cos^2 \nu$$

$$q = c + n \sin \nu \ .$$

If we now multiply the components X, Y, Z respectively with the cosine of angles α, β, γ, which the perpendicular makes on the plane of the circle with the coordinate axes, then the sum of these products yields the component in the direction perpendicular to the plane of the circle,[116] namely:

$$= X \cos \alpha + Y \cos \beta + Z \cos \gamma \ ,$$

or, if one substitutes the derived values for X, Y, $\cos \alpha$, $\cos \beta$, and γ, and eliminates p and q,

$$= -\frac{1}{2} ii'm^2 ds' \cdot \left[3 \left(a^2 \sin \nu - cn \cos^2 \nu \right) \int \frac{\sin^2 \omega d\omega}{r^5} - \sin \nu \int \frac{d\omega}{r^3} \right] \ ,$$

in which

$$r^2 = a^2 + c^2 + m^2 + n^2 + 2cn \sin \nu - 2m \cos \omega \cdot \sqrt{\left(a^2 + n^2 \cos^2 \nu \right)} \ .$$

If one writes in the foregoing expression for the length of the circle elements ds' its value expressed in terms of arc value and radius $= nd\nu$, and then multiply by the distance of the elements of the vertical diameter of the circle $= n \sin \nu$, one obtains the rotational moment of the force, in relation to the vertical diameter of the circle as rotational axis,

$$= -\frac{1}{2} ii' \cdot m^2 n^2 \sin \nu \cdot d\nu \left[3 \left(a^2 \sin \nu - cn \cos^2 \nu \right) \int \frac{\sin^2 \omega d\omega}{r^5} - \sin \nu \int \frac{d\omega}{r^3} \right] \ .$$

If this expression is now integrated between the limits $\nu = 0$ to $\nu = 2\pi$, then one obtains the rotational moment which the circular current m exerts on circular current n.

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[116][Note by AKTA:] That is, the component of the total force along the direction perpendicular to the plane of the circle.

For the given placement of the two circles with respect to each other (namely, where their planes are perpendicular to one another, and the perpendiculars to their centers intersect one another), three principal cases can be differentiated, which occur solely in the foregoing experiments, namely, either

1. the plane of circle m halves the plane of circle n, or $c = 0$; or

2. the plane of circle n halves the circle m, or $a = 0$; or, lastly,

3. both elements mutually halve each other, or $a = 0$ and $c = 0$.

For the *first* case, there results the following expression for the rotational moment acting on circle n, namely:

$$-\frac{1}{2}ii' \cdot m^2 n^2 \int_0^{2\pi} \sin^2 \nu d\nu \left(3a^2 \int \frac{\sin^2 \omega d\omega}{r^5} - \int \frac{d\omega}{r^3} \right) ;$$

in which

$$r^2 = a^2 + m^2 + n^2 - 2m \cos \omega \cdot \sqrt{(a^2 + n^2 \cos^2 \nu)} .$$

For the *second* case, the following rotational moment results:

$$+\frac{1}{2}ii' \cdot m^2 n^2 \int_0^{2\pi} \sin \nu d\nu \left(3cn \cos^2 \nu \int \frac{\sin^2 \omega d\omega}{r^5} + \sin \nu \int \frac{d\omega}{r^3} \right) ,$$

in which

$$r^2 = c^2 + m^2 + n^2 + 2cn \sin \nu - 2mn \cos \nu \cos \omega .$$

For the *third* case the following rotational moment results:

$$+\frac{1}{2}ii'm^2 n^2 \int_0^{2\pi} \sin^2 \nu d\nu \int \frac{d\omega}{r^3} ,$$

in which

$$r^2 = m^2 + n^2 - 2mn \cos \nu \cos \omega .$$

The first integration of the foregoing expressions, namely, with respect to ω, can only be carried out by developing $1/r^3$ and $1/r^5$ in series of rising powers of $\cos \omega$. Since r^2 has the form:

$$l^2 (1 - k \cos \omega) ,$$

the result is:

$$\frac{1}{r^3} = \frac{1}{l^3} \left(1 + \frac{3}{2}k \cos \omega + \frac{15}{8}k^2 \cos^2 \omega + \frac{35}{16}k^3 \cos^3 \omega + \frac{315}{128}k^4 \cos^4 \omega + ... \right)$$

$$\frac{1}{r^5} = \frac{1}{l^5} \left(1 + \frac{5}{2}k \cos \omega + \frac{35}{8}k^2 \cos^2 \omega + \frac{105}{16}k^3 \cos^3 \omega + \frac{1155}{128}k^4 \cos^4 \omega + ... \right) .$$

Since, further

$$\pi = \frac{1}{2}\int_0^{2\pi} d\omega = \int_0^{2\pi} \sin^2\omega\, d\omega = \int_0^{2\pi} \cos^2\omega\, d\omega = 4\int_0^{2\pi} \sin^2\omega \cos^2\omega\, d\omega$$

$$= \frac{4}{3}\int_0^{2\pi} \cos^4\omega\, d\omega = 8\int_0^{2\pi} \sin^2\omega \cos^4\omega\, d\omega = etc.$$

$$0 = \int_0^{2\pi} \cos\omega\, d\omega = \int_0^{2\pi} \sin^2\omega \cos\omega\, d\omega = \int_0^{2\pi} \cos^3\omega\, d\omega$$

$$= \int_0^{2\pi} \sin^2\omega \cos^3\omega\, d\omega = etc. \ ,$$

we obtain

$$\int_0^{2\pi} \frac{\sin^2\omega\, d\omega}{r^5} = \frac{\pi}{l^5}\left(1 + \frac{35}{32}k^2 + \frac{1155}{1024}k^4 + \dots\right)$$

$$\int_0^{2\pi} \frac{d\omega}{r^3} = \frac{2\pi}{l^3}\left(1 + \frac{15}{16}k^2 + \frac{945}{1024}k^4 + \dots\right) .$$

If we substitute these values, we obtain for the *first* principal case, where $c = 0$, the value of the electrodynamic rotational moment

$$= -\frac{\pi}{2}\frac{m^2 n^2}{l^3} ii' \cdot \Sigma \ ,$$

where Σ denotes the following integral value:

$$\int_0^{2\pi} \sin^2\nu\, d\nu \left[3\frac{a^2}{l^2}\left(1 + \frac{35}{32}k^2 + \frac{1155}{1024}k^4 + \dots\right)\right.$$

$$\left. - 2\left(1 + \frac{15}{16}k^2 + \frac{945}{1024}k^4 + \dots\right)\right] .$$

Therein,

$$a^2 + m^2 + n^2 = l^2 \quad \text{and} \quad 4\left(a^2 + n^2\cos^2\nu\right)\cdot\frac{m^2}{l^4} = k^2 \ .$$

If we substitute this value of k^2, and integrate the expression ordered according to the powers of $\cos^2\nu$, then we obtain the electrodynamic rotational moment

$$= -\frac{\pi^2}{2}\frac{m^2 n^2}{l^3} ii' \left[3\frac{a^2}{l^2} - 2 + \frac{15}{32}\left(7\frac{a^2}{l^2} - 4\right)\left(4 + \frac{n^2}{a^2}\right)\frac{a^2 m^2}{l^4} + \dots\right] .$$

Thus this expression yields for the first principal case under consideration the measure of the rotational moment, which a ring of radius $= m$ exerts on a ring of radius $= n$. For a system of rings, whose radii progress arithmetically from 0 to m, one obtains as measure of the rotational moment, which the system exerts on the ring of radius $= n$, the integral of

the above expression multiplied by dm, taken between the limits $m = 0$ to $m = m$. If, for the sake of brevity, we set

$$\frac{m^2}{a^2 + n^2} = v^2; \quad \frac{n^2}{a^2 + n^2} = w^2; \quad \frac{4a^2 + n^2}{16\left(a^2 + n^2\right)} = f; \quad \frac{8a^4 + 4a^2n^2 + n^4}{64\left(a^2 + n^2\right)^2} = g ,$$

then the sought-for electrodynamic rotational moment

$$= -\frac{\pi^2}{2} v^3 n^2 i i' \cdot S ,$$

where S denotes the following series:

$$S = + \left[\frac{1}{3} - w^2\right]$$

$$-\frac{3}{2}\left[\frac{3}{5} - w^2 - \left(3 - 7w^2\right) f\right] v^2$$

$$+\frac{15}{8}\left[\frac{5}{7} - w^2 - 2\left(5 - 9w^2\right) f + 3\left(5 - 11w^2\right) g\right] v^4$$

$$-\frac{35}{16}\left[\frac{7}{9} - w^2 - 3\left(7 - 11w^2\right) f + 11\left(7 - 13w^2\right) g\right] v^6$$

$$+\frac{315}{128}\left[\frac{9}{11} - w^2 - 4\left(9 - 13w^2\right) f + 26\left(9 - 15w^2\right) g\right] v^8$$

$$- \text{ etc.}$$

A precise comparison with the observational results requires a determination of the rotational moment which a system of such ring-systems with a common axis would exert on another similar system, for which still further integrations would be necessary. Meanwhile, it is easily seen that, if one proceeds from the most central of these ring systems located on an axis, its effect must be taken as the median value for each two symmetrical systems lying on both sides of it, because the effect of one of the two latter nearly exceeds by just as much that median value, as the effect of the other remains below it. This is all the more true, the smaller the fractions which the radii m and n are of the distance a of the centers of both systems. Hence we can hold to the last given expression as the measure of the effect.

If we now insert the values known from observation of m and n, specifically, in millimeters:

$$m = 44.4$$

$$n = 55.8 ,$$

and the successively following different values for a:

$$1. \quad a' = 300$$

$$2. \quad a'' = 400$$

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$$3. \quad a''' = 500 \,,$$

then one obtains the following values of the rotational moment, to be multiplied by $\pi^2 i^2$:

$$1. \quad -1.454\,4$$

$$2. \quad -0.654\,7$$

$$3. \quad -0.345\,2 \,.$$

If a similar procedure is applied to the *second* principal case, where $a = 0$, then one obtains the value of the electrodynamic rotational moment

$$= +\pi^2 v^3 n^2 i i' \cdot S \,,$$

in which, for brevity's sake,

$$\frac{m^2}{c^2 + n^2} = v^2; \quad \frac{c^2}{c^2 + n^2} = f; \quad \frac{n^2}{c^2 + n^2} = 4gv^2$$

is formulated, and S denotes the following series:

$$S = + \left[\frac{1}{3}\right]$$

$$- \frac{3}{2}\left[\frac{1}{5} - \frac{10}{3}fg\right] v^2$$

$$+ \frac{15}{8}\left[\frac{1}{7} + \frac{2}{5}(1 - 14f)g + 42f^2g^2\right] v^4$$

$$- \frac{35}{16}\left[\frac{1}{9} + \frac{3}{7}(2 - 18f)g - \frac{54}{5}(1 - 11f)fg^2 - 572f^3g^3\right] v^6$$

$$+ \frac{315}{128}\left[\frac{1}{11} + \frac{4}{9}(3 - 22f)g + \frac{12}{7}(1 - 22f + 143f^2)g^2\right.$$

$$\left. + \frac{1144}{5}(1 - 10f)f^2g^3 + \frac{24310}{3}f^4g^4\right] v^8$$

$$- \text{ etc.}$$

If one now inserts in this expression the values known from observation of m and n, specifically in millimeters:

$$m = 44.4$$

$$n = 55.8 \,,$$

and the successively following different values for c:

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1. $c' = 300$

2. $c'' = 400$

3. $c''' = 500$

4. $c'''' = 600$,

then one obtains the following values of the rotational moment, to be multiplied by $\pi^2 i^2$:

1. $+ 3.562\,5$

2. $+ 1.466\,1$

3. $+ 0.742\,0$

4. $+ 0.426\,7$.

For the *third* principal case, finally, where $a = c = 0$ and m/n is a proper fraction, it does not suffice for our purpose to assume for n a median value, but rather one must multiply any value found for n by dn, and take the integral of this product between the limit values of n given by observation, which we choose to denote by n' and n''. The resulting expression is then to be divided by $n'' - n'$, in order to reduce its value to the measure of the expressions given for the first and second cases, which have not been integrated with respect to n. One then obtains for this third case, where $a = 0$ and $c = 0$, the following expression for the rotational moment:

$$+\frac{\pi^2 m^3}{n'' - n'} ii' \left[\frac{1}{3} \ln \frac{n''}{n'} + \frac{9}{160} \left(\frac{1}{n''^2} - \frac{1}{n'^2} \right) m^2 - \frac{225}{14336} \left(\frac{1}{n''^4} - \frac{1}{n'^4} \right) m^4 \right.$$
$$\left. + \frac{6125}{884736} \left(\frac{1}{n''^6} - \frac{1}{n'^6} \right) m^6 + \frac{694575}{184549376} \left(\frac{1}{n''^8} - \frac{1}{n'^8} \right) m^8 + \ldots \right]$$

If one inserts in this expression the values of m, n', and n'' known from observation, specifically in millimeters:

$$m = 44.4$$

$$n' = 50.25$$

$$n'' = 61.35 \ ,$$

one then obtains the following value of the rotational moment, to be multiplied by $\pi^2 i^2$:

$$442.714 \ .$$

Given the proximity of the coils in this case, heed must be taken, finally, that the combined windings of each coil do not lie in one plane. Hence, if for the midpoint of the central cross-section of both coils, the distances are also $a = 0$ and $c = 0$, this does not hold true for the remaining cross-sections. From this results, as one easily sees, a diminution of the effect. In what relation this diminution stands to the total effect, can be determined with sufficient rigor, if, in the general formula given on page $77,$[117] after substitution of the values of $1/r^3$ and $1/r^5$, one simply keeps to the first term, independent of χ, and integrates its integral, taken between the limiting values $\omega = 0$ to $\omega = 2\pi$, after it is multiplied by $n \sin \nu$ and with $dm\,dn\,da\,dc$, and $nd\nu$ is substituted for ds', between the limits $\nu = 0$ to $\nu = 2\pi$, $m = 0$ to $m = 44.4$, $n = 50.25$ to $n = 61.35$, $a = 0$ to $a = 15$ and $c = 0$ to $c = 15$. If this calculation is carried out, an expression of the following form is obtained

$$A \left(1 - \frac{\alpha^2}{5000} + \frac{\gamma^2}{22000} \right) \cdot \alpha\gamma \ ,$$

in which A is simply dependent on i and i' and depends on the limiting values of m and n, and α and γ denote the greatest values for a and c. The sought-for reduction, expressed in components of the total effect, is accordingly

$$= \frac{1}{5000}\alpha^2 - \frac{1}{22000} \cdot \gamma^2 \ ,$$

and amounts to, according to the given numerical values, $\alpha = \gamma = 15$,

$$\frac{1}{29} \ .$$

Thus if $\frac{1}{29} \cdot 442.714$ is subtracted from the foregoing values, one obtains the following value, to be multiplied by $\pi^2 i^2$, for the electromagnetic rotational moment, corresponding to the *third case*,

$$= 427.45 \ .$$

If, by analogy with the observations, one compiles the results of the calculations, one obtains the following Table for the calculated values of the electrodynamic rotational moment:

Distance	Perpendicular to the magnetic meridian	In the direction of the magnetic meridian
0	+ 427.45	+ 427.45
300	+ 3.5625	− 1.4544
400	+ 1.4661	− 0.6547
500	+ 0.7420	− 0.3452
600	+ 0.4267	—

These values, if Ampère's law is correct, must be proportional to the observed values. In fact, if all the values are multiplied by the constant factor

$$53.06 \ ,$$

then one obtains values very closely approaching the observed ones, which are contained together with the differences from the latter in the following Table.

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[117][Note by LH and AKTA:] [Web46, p. 72 of Weber's *Werke*].

Distance	Perpendicular to the magnetic meridian	Difference	In the direction of the magnetic meridian	Difference
0	+ 22680	+ 280	+ 22680	+ 280
300	+ 189.03	+ 0.90	− 77.17	− 0.06
400	+ 77.79	− 0.34	− 34.74	+ 0.03
500	+ 39.37	− 0.10	− 18.31	− 0.07
600	+ 22.64	− 0.18	—	—

Here the first calculated value, namely, +22680, is compared with the 120.9-fold values of those which were obtained at distances of 300 millimeters east or west, because this value, in accordance with the results drawn from the second series of experiments in Section 5.6, corresponds to the effect of the fixed coil, when its center coincides with that of the bifilar coil. The indicated difference of 280 units appears exaggerated, and corresponds to an observational error of 1/3 scale units, which was made in the second series of experiments (Section 5.5) in the determination of the dynamometer deflection at a 30-mm distance.

This complete agreement between the values calculated according to Ampère's formula and the observed values (namely, the differences never exceed the possible amount contributed by unavoidable observational error) is, under such diverse conditions, a full proof of the truth of Ampère's law.

From the foregoing Table, one sees that the calculated values of the electrodynamic rotational moment result partly as positive, partly as negative. The significance of the difference in signs is as follows. The planes of the two wire coils were presumed to be at right angles to one another. The electrodynamic rotational moment, which the fixed coil exerts on the moveable one (the bifilar coil), endeavors to make the plane of the latter parallel to the plane of the former, which can occur in a double fashion, starting from the original right-angled position, namely, by means of rotation toward both sides. The one rotation leads to the kind of parallelism of the planes, in which the currents go around an axis perpendicular to both planes in the same way; the other rotation, in contrast, leads to the kind of parallelism in which the currents go around such an axis in opposite ways. The electrodynamic rotational moment, according to whether it effects the former or the latter rotation, will be designated in the calculation as positive or negative. The signs of the calculated values in the foregoing Table thus teach us, that if the fixed coil acts on the bifilar coil at a distance from north or south, a rotation of the bifilar coil results, which, if it amounts to 90 degrees, will cause the currents to go in *opposite* directions around axes aligned in the same way. The latter then takes place, according to calculation, if the centers of both coils coincide.

These results of calculation also find themselves fully confirmed by the results of all observations. The conditions to be therefore taken into consideration are not extensively discussed in the description given above, simply because to completely state the direction of the current in all particular parts of the conducting circuit and the direction of the observed rotations would have expended too much space. Moreover, since no exact measurements are necessary for testing these results of calculation, it was also possible to achieve their confirmation, by the methods used up to now, and that confirmation has already been thereby obtained, for which reason it suffices here, to take note merely in a general way of the agreement of the observations communicated with the foregoing results of calculation.

5.9 Reduction to Absolute Measure

Ampère's law gives the calculated rotational moments expressed in *absolute measures*,[118] assuming that, for the values of the current intensity i, an absolute measure of intensity is taken as the basis; specifically, in so doing, the fundamental measure of current intensity is to be considered as that current intensity, with which two equal, parallel current elements perpendicular to the connecting-line, at the distance equal to the unit of length, exert a force on each other, which forms the same fraction of the unit of force established in mechanics, as the *square of the length of those current elements* does of the *unit of area*. Then we put into Ampère's formula for the magnitude of the electrodynamic force of two current elements of length α and of equal intensity, namely:

$$-\frac{\alpha^2}{r^2}i^2\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right) ,$$

1.) the angle ε, which the two current elements form with each other, $= 0^\circ$ or $= 180^\circ$; 2.) the angles ϑ and ϑ' which both current elements form with the connecting line, $= 90^\circ$ or $= 270^\circ$; 3.) the distance $r = 1$; and so the value of the electrodynamic force obtained for the *unit* of current intensity is

$$\pm\alpha^2 ,$$

i.e. in Ampère's formula, a unit of current intensity is presupposed, in which the *electrodynamic force in the designated case* is in the ratio to the *unit of force*, as

$$\alpha^2 : 1 ,$$

that is, as the *square of the length of those current elements* is to the *unit of area*.[119] Thus the *electrodynamic principle* itself underlies this fundamental measure of current intensity.

For the purpose of our measurements, we have, however, based the *electromagnetic principle* on the unit of current intensity, according to which the fundamental unit of current intensity is to be posited as that current intensity, which must occur in a conductor limiting the unit area, in order to elicit the same effects on a *distant magnet*, as a magnet in the same place, whose magnetic moment is equal to that specified by Gauss in the *Intensitas...*,[120] and whose axis has the same direction as the normal of the plane of the current.

These two fundamental measures may now be compared with each other according to the relationship given by Ampère between *electrodynamics* and *electromagnetism*. For, according to this relationship, the other *distant magnet* can also be replaced, in the same way as the first, by a closed current.

The rotational moment of a magnet on another distant magnet, when their magnetic moments $= m$ and m' according to the absolute measure, as easily results from Gauss' given prescriptions (*Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1840*, pp. 26-34),[121,122,123]

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[118][Note by AKTA:] In German: in *absoluten Maassen*.
[119][Note by AKTA:] In German: *Flächenmaass*.
[120][Note by AKTA:] See footnote 97 on page 51.
[121][Note by HW:] Gauss' *Werke*, Vol. V, page 427 to 435.
[122][Note by AKTA:] See [Gau41d].
[123][Note by AKTA:] In the original this equation is written as

$$= \frac{mm'}{r^3} \sin \delta \cdot \sqrt{1 + 3\cos^2 \psi}$$

will be found, where ψ designates the angle which the axis of the first magnet makes with the connecting line r, and δ the angle, which the axis of the second magnet defines with that alignment, for which the rotational moment is $= 0$.

If a current of intensity χ is now applied at the location of the first magnet, which bounds the small plane λ, whose normal has the same alignment as the magnet's axis, then according to the electromagnetic law (which gives the strength of the electromagnetic force of a current element of length α and intensity χ on an element of magnetic fluid μ at distance r, when r and α enclose the angle φ, as $= \alpha\chi\mu \sin \varphi / r^2$, specifically normal to the plane which is parallel to α and r) the rotational moment exerted by this current on the distant magnet

$$= \frac{\chi\lambda \cdot m'}{r^3} \sin \delta \cdot \sqrt{1 + 3\cos^2 \psi} \ ,$$

in which for the current intensity χ, the *electromagnetic measure* given above is the basis. Thus, according to this measure, it must be the case that

$$\chi\lambda = m$$

if this rotational moment is to equal the preceding one.

According to the relationship given by Ampère, without changing the effect, in the same way *the second magnet can be replaced by a closed circuit*, for which

$$\chi'\lambda' = m' \ ,$$

and this yields the magnitude of the rotational moment, which the first current exerts on the second,

$$= \frac{\chi\chi'\lambda\lambda'}{r^3} \sin \delta \cdot \sqrt{1 + 3\cos^2 \psi} \ ,$$

in which the *electromagnetic measure* given above is the basis for the current intensities χ and χ'.

If, however, we now calculate according to Ampère's formula (page 74)[124] the rotational moment, which such a small plane-current exerts on another at a great distance, the result is[125]

$$= \frac{mm'}{r^3} \sin \delta \cdot \sqrt{1 + 3\cos \psi^2} \ .$$

The expression $\cos \psi^2$ should be understood as $(\cos \psi)^2$. I replaced everywhere in this translation the general formula $\cos \theta^2$ by its modern expression, $\cos^2 \theta$, where θ is an arbitrary angle.

[124][Note by LH and AKTA:] [Web46, p. 70 of Weber's *Werke*].

[125][Note by WW:] The case in which $\delta = \psi = 90°$, and consequently the electrodynamic rotational moment

$$= -\frac{1}{2} \frac{ii'\lambda\lambda'}{r^3} \ ,$$

corresponds to the *first major case* considered above, for which the strength of the rotational moment on page 79, [Note by LH and AKTA: [Web46, p. 74 of Weber's *Werke*].], was found

$$= -\frac{\pi^2}{2} \frac{m^2 n^2}{l^3} ii' \left[3\frac{a^2}{l^2} - 2 + \frac{15}{32}\left(7\frac{a^2}{l^2} - 4\right)\left(4 + \frac{n^2}{a^2}\right)\frac{a^2 m^2}{l^4} + ... \right]$$

$$= -\frac{1}{2} \frac{ii'\lambda\lambda'}{r^3} \sin\delta \cdot \sqrt{1 + 3\cos^2\psi} \ ,$$

in which the current intensities i and i' are based on the *electrodynamic measure* given above.

Now, from this it follows that, if the latter value, according to the *electrodynamic* measure, is to be identical with the former, according to the *electromagnetic* measure, the electrodynamic and electromagnetic measures of current intensity defined above must stand in

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For large distances, as assumed here, m and n become negligible vis-à-vis l, and r can be substituted for a and l; thus for this case, the rotational moment will be

$$= -\frac{\pi^2}{2} \frac{m^2 n^2}{r^3} ii' \ ,$$

which is identical with the values derived for this case from the formula above, because πm^2 and πn^2 designate the areas λ and λ'.

The above-cited *analogous* laws of magnetism, electromagnetism, and electrodynamics, from which the simple association of these different classes of phenomena can easily be perceived, which does not immediately appear from the *fundamental laws*, can be derived from the latter in the following way.

1. Derivation of the law of magnetic action, which one magnetic rod exerts on another at a distance.

Gauss, in the *Resultaten, etc. 1840*, p. 26 ff., [[Note by HW:] Gauss, *Werke*, Vol. V, page 427. [Note by AKTA:] See [Gau41d].], derived from the *fundamental law of magnetism*, the law of magnetic action, which one magnetic rod exerts on the unit of north-magnetic fluid, conceived of as concentrated in a distant point. This law is the following:
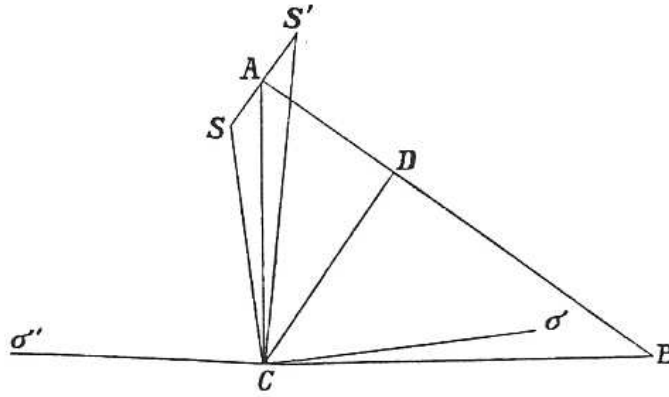
Fig. 12.

If (Figure 12) A is the midpoint of the magnetic rod, whose magnetic moment is designated as m, n an arbitrary other point on the north pole side of its magnetic axis, which goes through A, C the point, for which the magnetic action of the magnetic rod on the unit of north magnetic fluid conceived of as concentrated in that very place, is to be determined, and if CB is a normal with respect to CA in the plane in which n, A, C lie, and B its point of intersection with the magnetic axis, and if, finally, D cuts from AB the segment $AD = AB/3$: then the *strength* of the force, which the magnetic rod exerts on the unit of north magnetic fluid, conceived of as concentrated at point C,

$$= \frac{CD}{AD} \cdot \frac{m}{AC^3} \ .$$

The *direction* of this force is, if nAC is an obtuse angle, CD, and, if nAC is an acute angle, DC.

Now, in triangle ABC, because $ACB = 90°$,

$$AC = AB \cos BAC = 3AD \cos DAC \ .$$

relation to one another such that χ and χ', according to the latter measure, designate the same current intensities as $i\sqrt{1/2}$ and $i'\sqrt{1/2}$ according to the former. Consequently, all determinations of current intensity made according to the fundamental *electromagnetic* measure are to be multiplied by the constant factor $\sqrt{2}$, in order to reduce them to the measure of *electrodynamic* intensity underlying the Ampère formula.

Assuming this, the *constant factor*, by which all calculated values are to be multiplied, in

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Further, in triangle ACD,

$$CD = \sqrt{AC^2 + AD^2 - 2AC \cdot AD \cdot \cos DAC} = AD \cdot \sqrt{1 + 3\cos^2 DAC} \,,$$

and therefore,

$$\frac{CD}{AD} = \sqrt{1 + 3\cos^2 DAC} \,.$$

If we make $AC = r$ and $nAC = \psi$, then, because $DAC^2 = \cos^2 nAC = \cos^2 \psi$, the *strength* of the force

$$\frac{CD}{AD} \cdot \frac{m}{AC^3} = \frac{m}{r^3} \cdot \sqrt{1 + 3\cos^2 \psi} \,.$$

If, in a steel rod, the north magnetic mass $+\mu$ and the south magnetic mass $-\mu$ are divided at C by line α, which is infinitely small with respect to r, then $\alpha\mu = m'$ is the magnetic moment of the steel rod and $+\frac{m\mu}{r^3}\sqrt{1 + 3\cos^2 \psi}$ and $-\frac{m\mu}{r^3}\sqrt{1 + 3\cos^2 \psi}$ are the two forces, which act on it in the direction CD or DC. If n' is the end-point of the small line α, in which the mass $+\mu$ is thought of as concentrated, and C its midpoint, and δ designates the angle which Cn' forms with the direction CD or DC of the force given above, then $\alpha \sin \delta$ is the distance of the points of action of both forces, estimated perpendicular to their direction. The product of this distance into the value of the force above, then yields the rotational moment, which the magnetic rod at A exerts on the magnetic rod at C,

$$= \alpha \sin \delta \cdot \frac{m\mu}{r^3}\sqrt{1 + 3\cos^2 \psi} = \frac{mm'}{r^3}\sin \delta \sqrt{1 + 3\cos^2 \psi} \,.$$

The magnet at C is thereby rotated in the plane ACD in the way in which Cn approaches the direction CD or DC of the force given above.

2. Derivation of the law of electromagnetic action, which a closed plane current exerts on a magnetic rod.

From the *electromagnetic law*, the action of a closed current on the north magnetic mass $+\mu$ of the magnetic rod, which is thought of as concentrated in a point at C (Figure 12), can first of all be determined. One puts through C and through the midpoint A of the plane delimited by the current, a plane ACB perpendicular to the latter, CB being perpendicular to CA; s and s' are the intersection-points of the current with this plane. Further, each current element is separated into three elements perpendicular to each other, the first directed to C, and the second perpendicular to the direction CB. The elements directed toward C do not act on the magnetism at C, and thus can remain entirely out of consideration, because, in the general expression for the force strength, $= \alpha\chi\mu\sin\varphi/r^2$, the value for them of φ is $= 0$. To the *second* class belong the two perpendicular elements in s and s' on the plane ACB, whose length is designated as ds. The force which the former exerts on the magnetism at C has, according to the electromagnetic law, the direction $C\sigma$ perpendicular to Cs; the force of the latter has the direction $C\sigma'$ perpendicular to Cs', and the strength of this force is, if χ designates the current intensity according to the basic electromagnetic measure,

$$\frac{\chi\mu ds}{Cs^2} \quad \text{and} \quad \frac{\chi\mu ds}{Cs'^2} \,.$$

If one now separates these forces parallel to CA and perpendicular to it, then we obtain

the component parallel with $CA = \frac{\chi\mu ds}{Cs^2}\cos AC\sigma + \frac{\chi\mu ds}{Cs'^2}\cos AC\sigma'$

the component perpendicular to $CA = \frac{\chi\mu ds}{Cs^2}\sin AC\sigma - \frac{\chi\mu ds}{Cs'^2}\sin AC\sigma'$.

If we now use ψ to designate the angle which the normal of the current plane AB forms with $AC = r$, and take note that As and As' will become negligible with respect to r, then we obtain

order to give the observed values, can also be derived from the galvanometer observations, and the comparison of the factors thusly determined with those utilized above, namely, with

$$53.06 \ ,$$

then finally there results a touchstone for the correctness of the *absolute* values calculated

$$Cs = r - As\cos\psi \ , \quad Cs' = r + As'\cos\psi$$

or

$$\frac{1}{Cs} = \frac{1}{r}\left(1 + \frac{As}{r}\cos\psi\right) \ , \quad \frac{1}{Cs'} = \frac{1}{r}\left(1 - \frac{As'}{r}\cos\psi\right) \ ;$$

$$\cos AC\sigma = \sin ACs = ACs$$

$$\cos AC\sigma' = \sin ACs' = ACs'$$

$$sCs' = \frac{(ss')}{r}\cos\psi \ .$$

Substituting these values, and designating the distance ss' as x, we obtain the component parallel to CA

$$= \frac{\chi\mu}{r^3}\cos\psi \cdot xds \ .$$

Since all current elements surround A very closely, the factor $\frac{\chi\mu}{r^3}\cos\psi$ can be regarded as constant, and we thus obtain the component parallel to CA for all current elements of the second class:

$$= \frac{\chi\mu}{r^3}\cos\psi \cdot \int xds \ .$$

But the integral $\int xds$ represents the area $= \lambda$ delimitated by the current; accordingly the component parallel to CA for all current elements of the *second* class is given by

$$= \frac{\chi\lambda\mu}{r^3}\cos\psi \ .$$

Likewise, the component perpendicular to CA for all current elements of the *second* class is

$$= \frac{\chi\lambda\mu}{r^3}\sin\psi \ .$$

In a similar way we further find the component parallel to CA for all current elements of the *third* class

$$= \frac{\chi\lambda\mu}{r^3}\cos\psi \ ,$$

the component perpendicular to CA for all current elements of the *third* class

$$= 0 \ .$$

The resultant of all these forces is thus

$$= \frac{\chi\lambda\mu}{r^3}\sqrt{4\cos^2\psi + \sin^2\psi} = \frac{\chi\lambda\mu}{r^3}\sqrt{1 + 3\cos^2\psi} \ .$$

The direction of this resultant falls on the plane ACB and forms with CA an angle, whose tangent is equal to the components perpendicular to AC, $= \chi\lambda\mu\sin\psi/r^3$, divided by the component parallel to AC, $= 2\chi\lambda\mu\cos\psi/r^3$, that is,

$$= \frac{1}{2}\tan\psi \ .$$

from Ampère's formula, or for the correctness of the relation posited between electrodynamics and electromagnetism.

For this purpose, there is a triple requirement: 1. determining the factor with which all dynamometer effects observed by us are to be multiplied, in order to reduce them to the absolute measure of *rotational moment*; 2. determining the factor with which all galvanometer effects observed by us are to be multiplied, in order to reduce them to the *fundamental electromagnetic measure of current intensity*; 3. determining the *areas* which are demarcated

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Since $CAB = \psi$ and $ACB = 90^o$, then, if $AD = AB/3$,

$$\sin ACD : \sin \psi = \frac{1}{3} AB : CD$$

$$\cos ACD : \cos \psi = \frac{2}{3} AB : CD \, ,$$

therefore

$$\tan ACD = \frac{1}{2} \tan \psi \, ,$$

whose result is that CD is the direction of the resultant. It is assumed in these derivations, that if one thinks of oneself as perpendicular to the current plane standing on A, with the head at B, the current circulates in the manner of the apparent daily motion of the sun. If the opposite is the case, then the force direction DC is to be substituted for CD. Accordingly, the closed circuit at A has the same action on the magnetism at C, as according to [derivation] (1) a magnetic rod at A, whose magnetic moment is

$$m = \chi \lambda \, ,$$

and whose magnetic axis coincides with the normal of the current plane, specifically, the south pole on that side of the current plane, taking which as the point of observation, the current flows in the direction of the apparent daily motion of the sun. It follows from this, that if we place, as in (1), a magnetic rod at C, whose magnetic moment $= m'$, and whose magnetic axis forms the angle δ with CD, the rotational moment, which the closed circuit at A exerts on this magnetic rod, is equal to that rotational moment found in (1), if one substitutes therein $\chi \lambda$ for m, thus

$$= \frac{\chi \lambda m'}{r^3} \sin \delta \sqrt{1 + 3 \cos^2 \psi} \, ,$$

which was to be proven.

3. Derivation of the law of electrodynamic action, which a closed plane current exerts upon another at a distance.

The law of the action exerted by a closed plane-current on a current element at a distance, was already derived by Ampère on pp. 214 and 227 of his Treatise from the fundamental law of electrodynamics, [[Note by AKTA:] See [Amp23, pp. 214 and 227], [Amp26, pp. 42 and 55] and [AC15, pp. 366, 376 and 377].]. It can be expressed in the following way: If the current element is located at C (Figure 12) and the closed planar current at A, AB is the normal to the current plane, CB is perpendicular to CA, and $AD = AB/3$, then the force, which the current at A exerts on the current element at C, is perpendicular to the two directions of the current element itself and of the line CD; and if, according to the basic electrodynamic measure we designate the intensity of the closed current as i, and that of the current element as i', and further, the length of the current element as ds', $r = AC$, and $\psi = CAD$, the strength of the force is then

$$= \frac{1}{2} i i' ds' \frac{\lambda}{r^3} \sqrt{1 + 3 \cos^2 \psi} \, .$$

If, now, a closed planar current is also located at C, and the normal to its plane forms with CD the angle δ, then each element of this current can be separated into two elements, one parallel to the line in which a plane normal to CD cuts the plane of the current, the other perpendicular to this cutting-line. The first elements can be ordered pairwise in the same length ds' and connected to that cutting-line by means of a

by the bifilar coil and by the fixed coil of the dynamometer.

1. Determination of the factors for reduction of the observed dynamometer effects to absolute measure.

The observed dynamometer deflections are measured according to *scale units* and hence, in order to transform them into absolute *angular measurement*, given the smallness of the angle, are to be divided simply by the doubled horizontal displacement of the scale mirror ($=$ $6\,612.6$ scale units). Further, the given number of scale units corresponds to the difference between the positive and negative displacement, and hence, moreover, is still to be divided by 2, in order to reduce it to the simple deflection. Thus if x designates the *number of scale units* in the foregoing Tables, then

$$\frac{x}{13\,225.2}$$

yields the simple *angular displacement* in radius units.[126] Further, if S designates the *static moment* of the bifilar coil given in Section 5.6, to which the deflections have been reduced, then, if x designates the reduced value, one need only multiply the angular displacement $= x/13225.2$ with that value of S, to obtain the *electrodynamic rotational moment*, which the deflection elicited, expressed according to the fundamental measure specified by statics. Thus, this moment is

$$= \frac{x}{13\,225.2} \cdot S = 3\,634 \cdot x \ .$$

Consequently, $3\,634$ is the constant factor, with which the dynamometer deflections stated at the end of Section 5.6 are to be multiplied, in order to be reduced to absolute measure.

2. Determination of the factors for reduction of observed galvanometer effects to absolute measure.

perpendicular. If the length of this perpendicular is designated as x, then the result is that the effect of the closed circuit at A on such a pair consists of a rotational moment, which is equal to the product of $x \sin \delta$ in the force cited above, that is,

$$= \frac{1}{2} i i' \frac{\lambda}{r^3} \sin \delta \sqrt{1 + 3 \cos^2 \psi} \cdot x ds' \ .$$

The current at A thus exerts on all current elements parallel with the cutting-line above the rotational moment

$$= \frac{1}{2} i i' \frac{\lambda}{r^3} \sin \delta \sqrt{1 + 3 \cos^2 \psi} \cdot \int x ds \ ,$$

where the integral $\int x ds'$ designates the area $= \lambda'$ demarcated by the current at C; therefore this rotational moment is

$$= \frac{1}{2} i i' \frac{\lambda \lambda'}{r^3} \sin \delta \cdot \sqrt{1 + 3 \cos^2 \psi} \ .$$

If one considers in a similar way the action of the closed circuit at A on the elements perpendicular to the foregoing cutting-line, then the rotational moment will be $= 0$, from which it follows, that the just-given rotational moment is the entire action, which the closed current at A exerts on the closed current at C, which was to be proven, Q.E.D.

[126][Note by AKTA:] In German: *in Theilen des Halbmessers*. This expression can also be translated as "in parts of the radius".

The galvanometer effects are likewise stated above in *scale units*, and specifically, the given number y corresponds to the difference between the positive and negative deflection. Since the horizontal distance of the mirror from the scale amounts to 1 103 scale units on the galvanometer, the simple *angular displacement* according to absolute angular measure, i.e. in radius units, will be

$$= \frac{y}{4\,412} \ .$$

This angular displacement is elicited by means of a wire coil, through which the current to be determined is passing, and which was placed at a 217-millimeter distance west of the small magnetometer.

If the sine of this angular displacement is multiplied by the directive force $= m'T$,[127] which the terrestrial magnetism $= T$ exerts on the compass with magnetic moment $= m'$, then we obtains the rotational moment with which the terrestrial magnetism drives the deflected compass back to the magnetic meridian,

$$= m'T \cdot \sin \frac{y}{4\,412} \cdot \frac{180º}{\pi} \ .$$

According to absolute measure, in this formula the value of

$$T = 1.91$$

is to be used, as it was found at the location of the compass.[128]

The compass was now kept in equilibrium in that deflected position, by means of that rotational moment, which the current in the 217-mm-distant wire coil exerted on it, and consequently the strength of this latter rotational moment was

$$= 1.91 \cdot m' \sin \frac{y}{4\,412} \cdot \frac{180º}{\pi} \ .$$

According to the law demonstrated on page 88 in item 2 of the footnote,[129] if the current had acted from a *great* distance r, this latter rotational moment would be

$$= \frac{\chi \lambda m'}{r^3} \sin \delta \cdot \sqrt{1 + 3\cos^2 \psi} \ ,$$

in which the value of ψ for our case $= 0$, and δ is the expansion of the observed displacement angle to 90º, by means of which this expression becomes

$$= 2\frac{\chi \lambda m}{r^3} \cos \frac{y}{4\,412} \cdot \frac{180º}{\pi} \ .$$

Now, however, the distance of 217 millimeters becomes far too small to bring this law into immediate application. Hence, in order to facilitate this application, I carried out special experiments to compare the effect of the coil at the 217-mm distance with its effect at greater distances r, for which the foregoing law is applicable, and found the ratio of these effects to be

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[127][Note by AKTA:] See footnote 109 on page 70.

[128][Note by WW:] The compass stood near the wall of an adjoining room, in which large magnets were installed; if these magnets were removed, the value of T sank to 1.83, which is approximately the present value for the horizontal component of terrestrial magnetism in Leipzig.

[129][Note by LH and AKTA:] See item 2 of footnote 125 on page 86, [Web46, p. 84 of Weber's *Werke*].

$$1 : 1388 \cdot \frac{10^4}{r^3} \ .$$

The observed rotational moment $= 1.91 \cdot m' \sin \frac{y}{4\,412} \cdot \frac{180°}{\pi}$ must thus be multiplied by the factor

$$1388 \cdot \frac{10^4}{217^3}$$

if it is to be made equivalent for the expression which is valid for large distances; thus we obtain

$$1388 \cdot \frac{10^4}{217^3} \cdot 1.91 \cdot m' \sin \frac{y}{4\,412} \cdot \frac{180°}{\pi} = 2 \frac{\chi \lambda m'}{217^3} \cdot \cos \frac{y}{4\,412} \cdot \frac{180°}{\pi} \ ,$$

and from this results, by small arcs, the value

$$\chi \lambda = 3004 \cdot y \ .$$

By means of precise measurement, however, it was found that

$$\lambda = 8\,313\,440 \text{ square millimeters .}$$

From this results

$$\chi = 0.000\,361\,4 \cdot y \ ,$$

from which it follows that

$$0.000\,361\,4$$

is the factor for reduction of the observed galvanometer effects to the fundamental *electrodynamic* measure of current intensity. This is the factor already introduced above in Section 5.6 for the purpose of reducing the observations to equal directive force of the bifilar coil. The current intensity i according to the fundamental *electrodynamic* measure underlying Ampère's formula is finally obtained by means of multiplying the effects observed in scale units by the factor $0.0003614 \cdot \sqrt{2}$. It is to be noted, however, that this reduction factor rests on empirical data, which were in part obtained only in approximation, and therefore make claim to no great precision.

3. Determination of the area, which is demarcated by the bifilar coil and by the fixed coil of the dynamometer.

The area of the bifilar coil is already given in Section 5.6 as

$$= 29\,314\,000 \text{ square millimeters.}$$

In the same way, the area of the other fixed coil of the dynamometer was also determined, namely,

$$= 31\,327\,000 \text{ square millimeters.}$$

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It is evident that, in view of the indirect method by which it was made, this determination, too, can make claim to no great precision.

With the help of these three determinations, it is finally possible to also subject to empirical proof the *absolute* value of electrodynamic effects, as yielded by Ampère's law. Namely, from (2)[130] is obtained the value of i^2, which corresponds to the *normal* current intensity, to which the observations are reduced. That is, if, in accordance with page 72,[131] for [the intensity] we make

$$y^2 = 100\,000 \;,$$

then

$$i^2 = 2\chi^2 = 2 \cdot 0.000\,361\,4^2 \cdot y^2 = 0.026\,12 \;.$$

Further, one easily sees that in the calculation of electrodynamic rotational moment on page 80 made according to Ampère's formula,[132] the area of the bifilar coil was taken into account only as

$$\pi \cdot 55.8^2 \text{ square millimeters}$$

instead of, according to (3)[133]

$$= 29\,314\,000 \text{ square millimeters,}$$

and that in the same way, the area of the fixed coil of the dynamometer (in the place cited) was calculated only as

$$\frac{1}{3}\pi \cdot 44.4^3 \text{ square millimeters}$$

instead of, according to (3)

$$= 21\,327\,000 \text{ square millimeters} \;.$$

From this it follows that the *calculated values* presented in the Table on page[134] 83 are to be multiplied by

$$\frac{29\,314\,000 \cdot 21\,327\,000}{\frac{1}{3}\pi^2 \cdot 55.8^2 \cdot 44.4^3} \cdot \pi^2 i^2 = 180\,000$$

in order to determine the electrodynamic rotational moment according to Ampère's law in absolute measure. From (1),[135] however, one sees that the dynamometer effects *observed* in scale units in the Table on page[136] 73 are to be multiplied by the factor $3\,634$, in order to reduce them to absolute rotational moments. Thus if the previous factor is divided by this latter one, the factor 49.5 is obtained, with which the *calculated* values presented in

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[130][Note by AKTA:] That is, from item "2." on page 91.
[131][Note by LH and AKTA:] [Web46, p. 67 of Weber's *Werke*].
[132][Note by LH and AKTA:] [Web46, p. 76 of Weber's *Werke*].
[133][Note by AKTA:] That is, from item "3." on page 93.
[134][Note by LH and AKTA:] [Web46, p. 78 of Weber's *Werke*].
[135][Note by AKTA:] That is, from item "1." on page 91.
[136][Note by LH and AKTA:] [Web46, p. 68 of Weber's *Werke*].

the Table on page[137] 83 are to be multiplied, in order to be compared with the *observed* values presented in the Table on page[138] 73. This factor is about 6% smaller than the factor 53.06 above, which was immediately derived from the comparison of the calculated and observed values, a difference of the kind that had to be expected with so many elements taken from experimental evidence which are necessary for the determination of factors, among which several were only approximately determined (see [2] and [3]). Thus, the correctness of the *absolute* values calculated from Ampère's formula, or the correctness of the relation between electrodynamics and electromagnetism, is thereby confirmed only in so far as the experiments which were made can be validated. This test of the *absolute values* or of the stated relationship between *electrodynamics* and *electromagnetism* was not originally among the purposes of the experiments communicated here, which merely concerned the dependence of the electrodynamic force on the mutual position and distance of the conducting wires which acted on each other, otherwise arrangements would have been made to determine the galvanic current with greater precision also according to its *absolute* intensity, as well as to *directly* ascertain the number of windings on the part of the two coils of the dynamometer; that test, however, is presented at the same time in approximate fashion, because the experiments described placed the essential data at our disposal. Yet, because not *all* these data possess the requisite precision, a more rigorous execution of this test must be reserved for a future occasion. It is readily evident which arrangements and alterations are to be made in the experiments in order to lend greater precision to the data determined here with less exactness, and requires no further discussion.

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[137][Note by LH and AKTA:] [Web46, p. 78 of Weber's *Werke*].
[138][Note by LH and AKTA:] [Web46, p. 68 of Weber's *Werke*].

III - Volta-Induction with the Electrodynamometer

5.10 Observations

Up to now we have considered the first class of electrodynamic phenomena, namely, those discovered by Ampère, having to do with the forces with which the *conductors* seek to move one another at a given current intensity, and we have confirmed the law established by Ampère for this class of phenomena. Ten years later, Faraday's discovery adds to this first class of electrodynamic phenomena a second class, where the electrodynamic effects consist of forces which seek to move, not the conductors, but the *electricity in the conductors.* For these phenomena, comprehended under the name *Volta-induction,*[139] we can distinguish two fundamental experiments, both of which originate from Faraday.

At the very beginning of his *Experimental Researches in Electricity* (Poggendorff's *Annalen* 1832, Vol. XXV, page 93, Article 10),[140] Faraday describes the *first* fundamental experiment in voltaic induction, where two insulated copper wires were wound close to another on a block of wooden, and one was connected with the galvanometer, the other with a voltaic battery, and where the generation of a current in the first wire was observed each time at the moment when the circuit involving the second wire, was either broken or closed again. The *second* fundamental experiment follows in Article 18, where he fastened two copper wires bent in the same zigzags, separated from each other, to two boards, and connected one with the galvanometer, the other with the voltaic battery, and where the generation of a current in the first wire was observed each time at the moment when the board with this wire was either suddenly brought close from far away and laid on the board with the second wire, or the board lying on the other board was suddenly lifted and taken away from that other board.

After Faraday, Nobili[141] and Lenz[142] in particular occupied themselves with this kind of induction, and the latter established a simple law, by means of which the induction of a current in a conductor which is moved, can be reduced to Ampère's law of electrodynamic motions. Says Lenz (Poggendorff's *Annalen* 1834, Vol. XXXI, page 484 f.)[143]

> Immediately upon reading Faraday's Treatise, it seemed to me as though it must be possible to very simply reduce the collective experimental results of electrodynamic distribution[144] to the formulations for electrodynamic motions, so that if one assumes

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[139][Note by LH and AKTA:] The expression utilized by Weber, *Volta-induktion,* had been first suggested by Faraday himself in paragraph 26 of his first paper on electromagnetic induction of 1831, [Far32a, § 26, page 267 of the *Great Books of the Western World*] with German translation in [Far32c] and Portuguese translation in [Far11, p. 159]:

> For the purpose of avoiding periphrasis, I propose to call this action of the current from the voltaic battery, *volta-electric induction.*

In this English translation of Weber's work we utilized the expressions *Volta-induction* and *voltaic induction* for this class of phenomena which is nowadays called Faraday's law of induction.

[140][Note by AKTA:] See [Far32a], with German translation in [Far32c] and [Far89], and Portuguese translation in [Far11].

[141][Note by AKTA:] Leopoldo Nobili (1784-1835). See [Nob33] and [LA98].

[142][Note by AKTA:] See footnote 40 on page 24.

[143][Note by AKTA:] [Len34] with partial English translation in [Len69].

[144][Note by AKTA:] In German: *elektrodynamischen Vertheilung.* In the English translation of Lenz' work in Magie's book this expression was translated as "electrodynamic induction" because Lenz was referring to

these to be established, those too are determined, and since this view confirmed itself for me through multiple experiments, I will discuss them in what follows, and test them partly with familiar experiments, partly with experiments carried out expressly for this purpose. The law, according to which the reduction of magneto-electric phenomena to electromagnetic phenomena occurs, is as follows:

If a metallic conductor moves in the vicinity of a galvanic current or a magnet, a galvanic current will be induced in it, whose direction is such that it would have elicited motion in the wire at rest, which would be directly opposite to that given to it, assuming that the wire were movable only in the direction of the movement or opposite to it.[145]

For confirmation of this law, insofar as it concerns the induction of a current in a conductor which is put into motion, Lenz now introduces the following three experiments by Faraday, by himself, and by Nobili.

"a) When, of two straight-line conductors parallel to one another, one has a galvanic current flowing through it, and if the other conductor is brought closer to it in a parallel direction, during the movement, an opposite current will be induced in the conductor which is moved from that in the unmoved conductor; however, if it is moved away, the induced current is in the same direction as the arousing one." (Faraday.)

"b) When, of two vertical circular conductors of approximately the same diameter, which stand with their planes perpendicular to each other, the one, standing fixed, has a galvanic current flowing through it, and if then the other, which is rotatable around the common vertical diameter as its axis, is suddenly brought out of the perpendicular into a position lying parallel, then a current is generated in it, which is opposite to the current in the other conductor. I carried out this last experiment," says Lenz, "with two circular conductors, each consisting of 20 windings of covered copper wire; one was connected to a 2-square-foot large zinc-copper couple, the other to a sensitive Nobili multiplier."

"c) If a bounded conductor, standing perpendicular on an unbounded conductor which has a galvanic current flowing through it, moves along this unbounded conductor and in the direction of its current, then a current is generated in it, which is directed toward the bounded conductor; however, if the bounded conductor moves against the direction of the current in the unbounded conductor, the direction of the current induced in it by means of distribution is toward the unbounded current." (Nobili, Poggendorff's *Annalen* 1833, No. 3, page 407).[146]

Faraday's law of induction, [Len69, p. 512].

[145][Note by AKTA:] This sentence has been translated as follows in Magie's book, [Len69, p. 513]:

If a metallic conductor moves in the neighborhood of a galvanic current or of a magnet, a galvanic current will be produced in it which will have such a direction that it would have occasioned in the wire, if it were at rest, a motion which is exactly opposite to that here given to the wire, provided that the wire when at rest is movable only in the direction of the motion and in the opposite direction.

[146][Note by AKTA:] [Nob33].

By means of the above law, first stated by Lenz, the induced currents are determined, to begin with, only according to their direction: Lenz did not give a quantitative determination for the intensity of the induced currents. This, however, was provided by Neumann in a still unpublished Treatise, of which an excerpt has just appeared in Poggendorff's *Annalen*, 1846, Vol. LXVII, page 31.[147] The quantitative determinations thereby obtained, however, require a test by experiment, for which the necessary measurements are still lacking.

Henry reported original experiments on the induction of currents in a stationary conductor by *breaking* the circuit of a nearby voltaic battery (Poggendorff's *Annalen* 1842, supplementary volume, page 282),[148] putting the induced wire at varying distances and in varying positions. He also re-used the induced current itself, in order to induce a current in a third conductor, and so forth. After these experiments, he directed these induced currents in parallel wires alternately in opposite directions; the first, however, being the same direction as the voltaic battery's current which was lost due to the *breaking* of the circuit.

In this Section, it will now be shown, *first*, how the phenomena of voltaic induction can also be observed with the electrodynamometer, then some *measurements* for the second of Faraday's fundamental experiments will be reported.

In representing the phenomena of voltaic induction, two different things must be essentially distinguished, namely, *first*, the apparatus for inducing currents, and *second*, because the induced current is not immediately perceptible, an apparatus for observing a perceptible effect of the induced current. In the second Faraday fundamental experiment, for example, the two zigzag-shaped bent copper wires, of which one is attached to a galvanic circuit, along with the device by which both wires are suddenly brought closer to each other or farther distant from each other, constitutes the first apparatus, which is for *inducing* the current; the *galvanometer*, on the other hand, which is connected with the other wire, constitutes the second apparatus, for observing a *visible effect* of the induced current. Thus the two essential pieces of apparatus for the experiment are distinguished and separated from each other.

Now, however, an essential simplification of the experiment can be achieved by means of the *electrodynamometer*, where it is possible to use the same apparatus which serves to induce the current, for observing a visible effect of the current as well. That is, the bifilar coil of the electrodynamometer is made to *oscillate*, and this motion is used for induction; then the *decrease of the arc of the oscillations* of this bifilar coil is observed, which, as will be shown forthwith, is the result of the electrodynamic interaction of the inducing and induced currents. The lawfulness of the induction mediated oscillations, as well as of the decrease of the arc of the oscillations, observed as the visible effect of the induced current, permits us to carry out precise *measurements* for these phenomena of induction.

Namely, if one connects the wire of the *one* coil of the dynamometer, while the bifilar coil is *swinging*, with a voltaic current, then, in order to *induce* a current in the *other* coil, one need only join the two wire ends to one another. This induced current, which in itself is of course imperceptible in the second coil, now immediately exerts in the dynamometer itself a *perceptible electrodynamic force* on the current of the first coil, and thereby changes the oscillation of the bifilar coil. Thus, if one observes this change, one can come to know the electrodynamic force which causes it, and in turn, from the electrodynamic force, to know the *induced current*, to which it is proportional, without it being necessary to conduct

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[147][Note by AKTA:] Franz Ernst Neumann (1798-1895). See [Neu46], [Neu47], [Neu48] and [Neu49].

[148][Note by AKTA:] Joseph Henry (1797-1878). See [Hen38] with partial German translation in [Hen42]. See also [Smi17].

the induced current through the multiplier of a *galvanometer*. The *dynamometer* itself thus serves for *inducing* the current as well as for observing a *visible and measurable effect* of the induced current.

If the bifilar coil is stationary, no current will be induced; consequently the electrodynamic force = 0, and the bifilar coil will then not be moved by the fixed coil. However, if the bifilar coil is swinging, there are two cases to distinguish: i.e., either the fixed coil is connected to the voltaic battery and the bifilar coil is a closed circuit in itself; in that case a current is induced in the swinging bifilar coil: or the swinging bifilar coil itself is connected by its two suspension wires to the voltaic battery, and the fixed coil is a closed circuit in itself; in that case, a current is induced in the fixed coil. In both cases, an electrodynamic force is yielded which changes the oscillation of the bifilar coil in the same way.

The *observation* of these changes in oscillation, as a result of an induced current, and of the *electrodynamic* interaction between the induced and inducing wire coils, which is dependent on that induced current, according to Ampère's law, must be carried out, however, in an altogether *different* way from the observations with the dynamometer described in the preceding Section. *Observations of the decrease of the arc of the oscillations* must replace the previous *observations of position* on the dynamometer. The necessity of this altered method of observation is easily shown, as follows.

The electrodynamic interaction of the two coils, which is to be observed with the electro-dynamometer, consists, according to Ampère's law, of a rotational moment, which acts on the swinging bifilar coil and corresponds to an altered *rest position* of this coil. However, this *rest position* of the bifilar coil can not be directly observed when the coil is swinging, but can only be determined from several observations, which diverge from one another as to period of oscillation, that is, only under the precondition, that in the interim, the external forces acting on the coil, have remained *constant*, or have changed continuously and *proportionally* over *time*. Therefore, if the electrodynamic influence which occurs on the swinging coil as a result of the induced current, *reverses itself from oscillation to oscillation*, then the rest position of the coil, as determined from a system of observations during the oscillation, will be *unchanged* despite the presence of the electrodynamic influence. Observation shows, in fact, that the latter occurs, that the electrodynamic influence, if it actually exists as a result of an induced current, would thus have to reverse itself from oscillation to oscillation, and cannot be investigated by means of mere *observations of position* on the dynamometer.

Now, if such an electrodynamic influence on the swinging coil now actually occurs, which reverses itself from oscillation to oscillation: then it will certainly not be discernible by means of determining the rest position of the coil; it must, however, be recognizable in the *arcs of oscillation* of the coil; namely, the size of the arcs of oscillation must *change* from oscillation to oscillation, either always increasing, or always decreasing.

In actuality, empirical results show that, while the calculated rest position of the swinging coil always remains the same, the arc of oscillation always *decreases*, and it emerges from the succeeding experiments, that this decrease actually originates from *electrodynamic* influences and not from extraneous external causes, if the ordinary influence of the resistance of the air is taken into account.

Therefore, in order to observe this second class of phenomena with the electrodynamometer, it will be necessary for precise measurement of the decrease of the arcs of oscillation, to make *oscillation experiments* with the bifilar coil of the dynamometer, while for purposes of the Ampère electrodynamic phenomena, we could confine ourselves to *deflection experiments* or *observations of position*.

For our purposes, it is of primary importance to indicate that the *observations of oscillation* can be carried out on the dynamometer by the same method, and with just as great precision, as on a magnetometer. First, I wish to present a prefatory series of oscillation experiments which I made with the dynamometer, in which *no* electrodynamic influence occurred, given that no galvanic current whatever was conducted through the instrument and the wire ends even remained unconnected.

The method of setting up these experiments is the same method presented by Gauss in the *Resultaten aus den Beobachtungen des magnetischen Vereins in Jahre 1837*, page 58 ff.,[149,150] and thus it is not necessary to fully cite the original protocols; it will suffice to cite the extract which is derived from these protocols, just as in other locations.

The Meyerstein dynamometer,[151] depicted in Figures 2, 3 and 4, served for the following observations, where the swinging coil was suspended at the center of the fixed coil, and the telescope was placed about 6 meters from the instrument. The distance of the mirror from the scale was $6\,018.6$ scale units, and

$$1 \text{ scale unit} = 17.135\,6" \, .$$

The observations were made in alternation by different observers, namely, by Dr. Stähelin from Basel, by my assistant Mr. Dietzel, and by me. Each made a set of observations according to the formulation given in *loc. cit.*, page 61,[152,153] which comprises six instances of going past a designated scale point lying close to the midpoint of the arc of oscillation and seven elongation points. In the following Table, each horizontal line gives the results of such a set of observations, namely, the numerical ordering of the oscillation,[154] the corresponding time, the corresponding rest position in scale units, the corresponding arc of oscillation in scale units, and the logarithm of the latter.

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[149][Note by HW:] Gauss, *Werke*, Vol. V, page 374.

[150][Note by AKTA:] [Gau38a].

[151][Note by AKTA:] See footnote 87 on page 41.

[152][Note by HW:] Gauss, *Werke*, Vol. V, page 376.

[153][Note by AKTA:] [Gau38a].

[154][Note by AKTA:] In German: *die Bezifferung der Schwingung*. This expression can also be translated as "the numbering of the oscillation" or "the numbering of the vibration".

Oscillation Number	Time	Position	Arcs of Oscillation	Log.
0.	5^h 16' 28.53"	457.10	650.80	2.813 448
14.	20' 10.20"	457.38	601.43	2.779 185
25.	23' 4.39"	457.15	564.90	2.751 972
52.	30' 12.50"	457.19	485.28	2.685 992
82.	38' 8.02"	457.29	409.62	2.612 381
109.	45' 16.16"	457.15	353.08	2.547 873
134.	51' 25.08"	457.65	306.70	2.486 714
163.	59' 31.80"	457.41	261.08	2.416 774
189.	6^h 6' 23.90"	457.56	226.33	2.354 742
212.	12' 28.22"	457.69	198.68	2.298 154
232.	17' 45.45"	457.63	178.26	2.251 054
254.	23' 33.89"	457.78	157.98	2.198 602
284.	31' 29.30"	457.73	134.17	2.127 655
309.	38' 5.53"	456.55	116.30	2.065 580
328.	43' 6.90"	458.02	105.25	2.022 222
369.	53' 56.24"	457.81	83.68	1.922 622
387.	58' 41.96"	457.90	75.45	1.877 659

Observations for determining the period of oscillation and the decrease of the arcs of oscillation of the bifilar coil of the dynamometer in open circuits.

If the difference between the first and last time is divided by the number of oscillations, one obtains a rather precise determination of the period of oscillation of the swinging coil, because the correction by reduction to infinitely small arcs contributes only a little in the case of such small arcs of oscillation as occurred here. This approximated period of oscillation is

$$= 15.848\,65" \, .$$

If this approximated period of oscillation is used to reduce all the times in the Table, by deducting the product of the oscillation number[155] times the period of oscillation, from the first time, then the values contained in the third column of the following Table are obtained:

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[155][Note by AKTA:] In German: *Zahl der Schwingung.* See the first column in the previous Table.

Oscillation Number	Time	Reduced time	Variation from the mean
0.	5^h 16' 28.52"	5^h 16' 28.53"	+ 0.13"
14.	20' 10.20"	28.32"	−0.08"
25.	23' 4.39"	28.17"	−0.23"
52.	30' 12.50"	28.37"	−0.03"
82.	38' 8.02"	28.43"	+ 0.03"
109.	45' 16.16"	28.66"	+ 0.26"
134.	51' 52.08"	28.36"	−0.04"
163.	59' 31.80"	28.47"	+ 0.07"
189.	6^h 6' 23.90"	28.50"	+ 0.10"
212.	12' 28.22"	28.31"	−0.09"
232.	17' 45.45"	28.56"	+ 0.16"
254.	23' 33.89"	28.33"	−0.07"
284.	31' 29.30"	28.28"	−0.12"
309.	38' 5.53"	28.30"	−0.10"
328.	43' 6.90"	28.54"	+ 0.14"
369.	53' 56.24"	28.07"	−0.33"
387.	58' 41.96"	28.53"	+ 0.13"

From the agreement of these reduced values, whose variation from the mean values always remains less than 1/3 second, it is obvious, that the determination of the *period of oscillation* of the dynamometer's bifilar coil is capable of the same rigor and precision as in the case of the magnetometer, in which connection it is to be taken into consideration, that that variation appears to be magnified by the constant variation which is known to always occur between two observers. The determinations of the *rest position* of the swinging coil from the elongation observations in the third column of the first Table show great agreement, as proven by the following overview of their deviations from the mean values, expressed according to their arcs:

− 6.3"	+ 3.1	+ 4.5
− 1.5"	− 1.0	− 15.8
− 5.5"	+ 1.5	+ 9.4
− 4.8"	+ 3.8	+ 5.8
− 3.1"	+ 2.7	+ 7.4
− 5.5"	+ 5.3	

One could not wish for greater agreement of all observations of position, particularly when one takes into consideration, that the telescope stand was placed on the wooden floor of the room, where, it is evident, the orientation of the telescope could easily be somewhat altered by footsteps on the floor.

Lastly, it remains for us to consider the *decrease in the arcs of oscillation.* The individual sets of observations succeed each other, in part, in such a short time, that the decrease in the arcs of oscillation in the interim is not large enough to give a precise determination of the ratio between two successive arcs of oscillation. Hence the logarithms of this ratio may be determined, by, instead of dividing the difference of each two immediately successive logarithm of the arcs of oscillation, dividing the difference between the first and fifth, the

second and sixth, and so forth, by the number of oscillations in between. One then obtains from the above 17 sets of observation, instead of 16 values, only 13 values, but more precise values of the *logarithmic decrement*, namely, the following. Before each value the oscillation number is noted, to whose mean it appertains.

Oscillation No.	Logarithmic decrement	Variation from the mean
41.	0.002 452	+ 0.000 038
$61\frac{1}{2}$.	0.002 435	+ 0.000 021
$79\frac{1}{2}$.	0.002 433	+ 0.000 019
$107\frac{1}{2}$.	0.002 425	+ 0.000 011
$135\frac{1}{2}$.	0.002 408	− 0.000 006
$160\frac{1}{2}$.	0.002 424	+ 0.000 010
183.	0.002 405	− 0.000 009
$208\frac{1}{2}$.	0.002 397	− 0.000 017
$236\frac{1}{2}$.	0.002 390	− 0.000 024
$260\frac{1}{2}$.	0.002 398	− 0.000 016
280.	0.002 384	− 0.000 030
$311\frac{1}{2}$.	0.002 400	− 0.000 014
$335\frac{1}{2}$.	0.002 427	+ 0.000 013
Mean = 0.002 414.		

Thus there results a mean *decrease in the arcs of oscillation*, according to which the size of the arc, after 124.7 swings, or after 32 minutes $56\frac{1}{3}$ seconds, declines by half. The agreement of the partial values proves, that even these small decreases in the arcs of oscillation can be rigorously measured.

On the same day, immediately before the series of observations just described, another similar series of observations was made under altogether the same external conditions, merely with the difference, that the two ends of the fixed coil were connected with a battery of three small Grove's elements, exactly as in Section 5.4, and that the free ends of the suspension wires of the bifilar coil were linked to each other. More precise information about the current conducted through the fixed coil, was provided by observation of the deflection which this coil itself produced on the *mirror magnetometer* (described in Section 5.3), which was placed 583.5 mm north of the coil. This observed deflection of the mirror magnetometer is noted in the last column of the following Table. The value of the scale unit of this magnetometer depends on the horizontal distance of the mirror from the scale, which was = 1301 scale units. The observers and the methods of observation were the same. The following Table gives the extract of this series of observations exactly as the preceding Table gives the other.

Oscillation Number	Time	Position	Arcs of oscillation	Log.	Deflection of mirror magnetometer
Observations for determining the period of oscillation and the decrease of the arcs of oscillation of the bifilar coil of the dynamometer when a current from three Grove's elements is passing through the fixed coil, while the conducting wire of the bifilar coil was closed.					
0.	3^h 29' 44.88"	464.05	764.10	2.883 150	108.50
9.	32' 7.03"	464.44	679.15	2.831 966	
18.	34' 29.58"	464.23	604.05	2.781 073	
35.	38' 50.17"	464.07	484.15	2.684 980	108.60
47.	42' 9.10"	464.20	414.60	2.617 629	
57.	44' 47.66"	464.25	365.50	2.562 887	
74.	49' 16.79"	464.22	292.27	2.465 784	109.10
85.	52' 10.80"	464.30	253.30	2.403 635	
103.	56' 56.11"	464.40	200.80	2.302 764	
118.	4^h 0' 53.43"	464.25	165.56	2.218 955	108.95
130.	4' 3.26"	464.37	141.37	2.150 357	
143.	7' 28.90"	465.23	119.33	2.076 750	
157.	11' 11.11"	464.96	100.49	2.002 123	109.20
179.	16' 59.23"	465.20	75.59	1.878 464	
196.	21' 28.65"	464.88	60.58	1.782 329	190.40
210.	25' 10.23"	464.96	50.08	1.699 664	

For this series of observations, which is otherwise very similar to the preceding one, I confine myself to considering the decrease in the arcs of oscillation. The logarithm of the ratio of two successive arcs of oscillation, or the logarithmic decrement, is to be determined here by dividing the difference between the first and fourth, the second and fifth, and so forth, logarithm by the number of the oscillations in between. From the 16 sets of observation above, this yields 13 values of the logarithmic decrement, as contained in the following Table, with the addition of the oscillation number to whose mean each appertains.

Oscillation No.	Logarithmic decrement	Variation from the mean
$17\frac{1}{2}$.	0.005 662	+ 0.000 042
28.	0.005 640	+ 0.000 020
$37\frac{1}{2}$.	0.005 595	− 0.000 025
$54\frac{1}{2}$.	0.005 620	0.000 000
66.	0.005 631	+ 0.000 011
80.	0.005 655	+ 0.000 035
96.	0.005 610	− 0.000 010
$107\frac{1}{2}$.	0.005 628	+ 0.000 008
123.	0.005 650	+ 0.000 030
$137\frac{1}{2}$.	0.005 560	− 0.009 060
$154\frac{1}{2}$.	0.005 549	− 0.000 071
$169\frac{1}{2}$.	0.005 555	− 0.000 065
$183\frac{1}{2}$.	0.005 707	+ 0.000 087
Mean = 0.005 620.		

Thus there results a mean *decrease in the arcs of oscillation*, according to which the size of the arc, after 53 564 swings, or after 14 minutes 8.187 seconds, declines by half. Here too the agreement of the partial values attests to the rigor of the measurement, and it is not remarkable that at the end, where the arcs of oscillation had become very small, the differences appear somewhat larger.

The difference which occurs between this latter determination of the logarithmic decrement and the previous one, is based, not on different external conditions influencing the swinging coil, for these remain completely the same, but on the *inducing* influence of the fixed coil on the swinging coil, which constitutes the sole difference between the first and second series of experiments. Both series of experiments were repeated on several days, and not only yielded almost exactly the same difference in values of the logarithmic decrement, but also yielded nearly equal absolute values for both decrements, whereby no doubt remains, that an induction of galvanic currents actually takes place in the closed bifilar coil by means of the galvanic current in the fixed coil: in fact, an induction whose strength was such that the effect of the induced current which is visible in the decrease of the arcs of oscillation, is susceptible of precise measurement.

5.11   Law of Damping Produced by Volta-Induction

After this demonstration of the *practical* usefulness of the electrodynamometer for displaying the phenomena of voltaic induction, *secondly*, we proceed to derive some *lawful determinations* for these phenomena from the observations of the oscillations and of the decrease of the arcs of oscillation of the bifilar coil.

First, it has already been noted, that the *changing magnitude* of the arcs of oscillation as a result of the induced currents, given an unchanged mean position of the bifilar coil, proves that the direction of the induced current changes with the *direction of motion* of the swinging bifilar coil, and that consequently opposite currents are induced by means of opposite motions, as is also the case in magnetic induction.

Second, the *decrease* in the arcs of oscillation proves, that as parallel elements of the inducing wire *approach*, a current opposite to that inducing wire is induced; as parallel elements *withdraw*, a current *in the same direction* as that in the inducing wire is induced. If the opposing relation of the current directions of the inducing and induced currents existed, there would have to result a continuous *increase* in the arcs of oscillation. This determination as well is analogous to the one empirically established for magnetic induction.

Third, the *geometric law* of the decrease in the arcs of oscillation due to the induced current, proves that the intensity of the induced current is proportional to the *velocity* of the inducing motion; for the geometric law of the decrease of arcs of oscillation proves that the force which produces this decrease, i.e. the intensity of the induced currents, always remains proportional to the magnitude of the arcs of oscillation: it is known, however, that the magnitude of the arcs of oscillation of an *isochronous* swinging body is always proportional to the velocity it reaches in the corresponding moments of its period of oscillation.

Fourth, as for the lawful determination of the *absolute* strength of the voltaic induction, we wish lastly to derive the following principle from observations on the dynamometer.

> The *voltaic induction* is equal to the *magnetic induction* in the oscillating closed circuit bifilar coil, when the former is elicited by a galvanic current conducted through the fixed coil, the latter elicited by magnets which are located in a position with

respect to the bifilar coil such that when a current goes through the bifilar coil, the *electrodynamic* rotational moment of that current is equal to the *electromagnetic* rotational moment of these magnets.

By means of this principle, as can easily be seen, the determination of voltaic induction with the help of known *electromagnetic* and *electrodynamic* forces is reduced to the laws of magnetic induction, which have already been investigated more precisely by other means. At the moment, admittedly, in order to prove this principle, I can only give a few measurements carried out with the dynamometer, which were made under circumstances in which no precise determinations down to small fractions were possible; however, these measurements may be regarded as sufficient for the present, because, if the foregoing principle were incorrect, there would be no basis for that approximate agreement which undoubtedly arose from the observations. For a more refined test of the foregoing principle, all the involved measurements would have to be carried out with greater precision. However, in order to set up all the conditions fully appropriately for attaining this uniform precision, it would be necessary to prepare special instruments simply for this purpose, which up to now it has not been possible for me to do.

Here I will briefly assemble the results of the observations, without going into the details of the observations themselves, which generally conformed to those of the preceding observations.
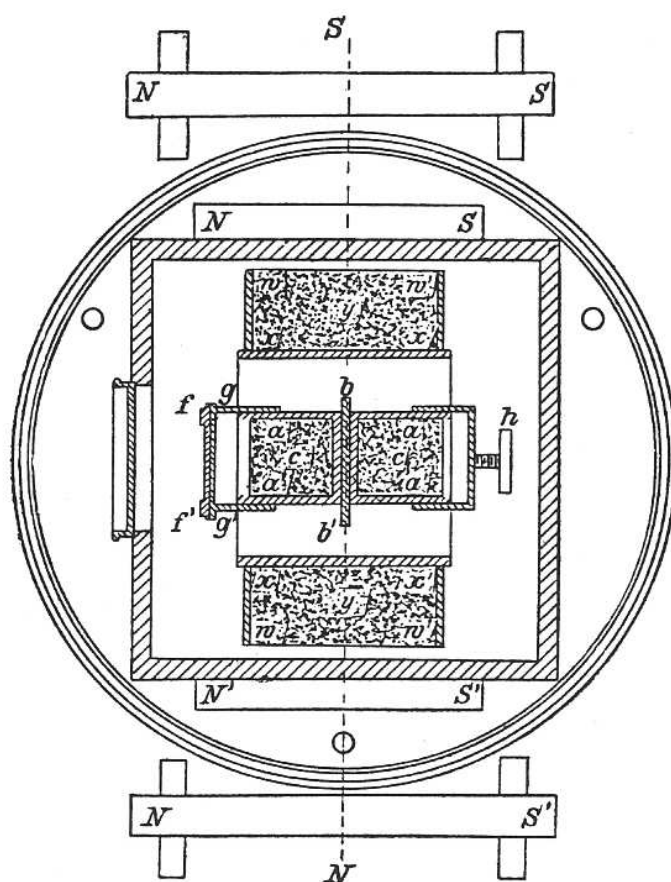
Fig. 4.

The *first* series of observations concerned measurement of magnetic induction. This is exactly the series for which conditions could be shaped least favorably, and which accordingly

set narrower limits to the precision of the entire measurement, which under somewhat more favorable conditions could have easily been significantly extended. That is, the bifilar coil of the dynamometer described in Section 5.1 and depicted in Figures 2, 3 and 4, was made a closed circuit and put into oscillation, while outside the casing which protected the swinging bifilar coil from the air, several small magnets NS, $N'S'$ (Figure 4) were fixed in the position in which they induced the strongest *magneto-electric currents* in the swinging bifilar coil. Namely, all of these small magnets together lay perpendicular to the magnetic meridian passing through the axis of the bifilar coil, i.e., symmetrically north and south of the bifilar coil, and their corresponding poles were thereby turned toward the same side, as the diagram shows, in which N and N' denote the north poles, S and S' the south poles. Then the oscillations of the bifilar coil, as earlier, were observed, starting from the moment when they could be measured by means of the scale, until they became too small for precise determination of the decrease of the arcs of oscillation. These observations were calculated in the same way as above, and yielded the *logarithmic decrement* for the decrease of the arcs of oscillation

$$= 0.002\,638 \ .$$

The same series of experiments was once again repeated, with the sole difference, that the bifilar coil was unclosed, and there resulted for the *logarithmic decrement* of the decrease of the arcs of oscillation the following somewhat smaller value:

$$= 0.002\,541 \ .$$

The slight difference between these two values,

$$= 0.000\,097 \ ,$$

is the effect of the *magneto-electric currents*, which were induced in the swinging, closed-up bifilar coil by means of the fixed magnets. The greatest care was taken to determine this small difference with the utmost possible precision, and the experiment leaves nothing more to be desired in that respect; nevertheless, it is in the nature of the smallness of the difference, that, as the repetitions of the experiments showed, it must be considered as 6 to 8 percent uncertain.

The *second* series of experiments concerned the *electromagnetic* rotational moment. The small magnets remained undisturbed in their place, while a weak current was conducted from a constant voltaic battery; this battery's current passed as well through a *galvanometer*, by means of which its intensity was measured. Now the *rest position* of the bifilar coil was observed, alternately, when the voltaic battery was closed and when it was open. From a series of repetitions, after the reduction of the results to the same current intensity (which had varied only very little), there resulted, with great agreement, the difference

$$= 19.1 \text{ scale units.}$$

This difference is a measure of the *electromagnetic* rotational moment, which the magnetic rods mentioned above exerted on the current in the bifilar coil.

The *third* series of experiments concerned the *electrodynamic* rotational moment. The small magnets were distanced, and the two wire ends of the fixed coil of the dynamometer were connected to a strong voltaic battery, while the same weak current of a constant voltaic battery was conducted through the bifilar coil, as in the previous series. The intensity of

both currents was measured by means of a *galvanometer*.[156] Now, as in the previous series of experiments, the *rest position* of the bifilar coil was observed, alternately when the voltaic battery was closed and when it was open. From a series of repetitions, after the reduction of the results to the same current intensity, there resulted, with great agreement, the difference

$$= 101.9 \text{ scale units.}$$

This difference is a measure of the *electrodynamic* rotational moment, which the strong current in the fixed coil exerts on the weak current in the bifilar coil.

Lastly, the *fourth* series of experiments concerned *voltaic induction*. The bifilar coil was closed up and put into oscillation, while the current of the same voltaic battery was conducted through the fixed coil of the dynamometer, as in the previous series of experiments. Then the oscillations of the bifilar coil were observed just as in the first series of experiments, and from this the *logarithmic decrement* of the decrease in the arcs of oscillation was calculated. After reduction to that current intensity in the fixed coil, on which the value of the *electrodynamic* rotational moment found by means of the preceding series of experiments is based, this decrement resulted

$$= 0.005\,423 \ .$$

The same series of experiments was once again repeated with the sole difference, that the bifilar coil was open, and the following smaller value resulted for the *logarithmic decrement* of the decrease in the arcs of oscillation:[157]

$$= 0.002\,796 \ .$$

The difference between these two values,

$$= 0.002\,627 \ ,$$

is the effect of the *voltaic induction*, which took place in the closed and oscillating bifilar coil, by means of the current in the fixed coil.

Since, therefore, the *electrodynamic* force of our current in the fixed coil, after the *third* series of experiments, was not equal to the *electromagnetic* force of our magnets in the *second* series of experiments, but were in the ratio of

$$101.9 : 19.1 \ ,$$

the forces of the two *induced currents*, induced under altogether the same conditions in the bifilar coil, should also not be equal, but should likewise be in the ratio of

$$101.9 : 19.1 \ .$$

If, however, the intensities of the currents induced in the swinging bifilar coil stand in the given ratio, then, from the interaction of these currents with those generating them (and

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[156][Note by WW:] Both currents originated from the same constant battery, and their differing intensity in the two coils was effected by means of a division of the current.

[157][Note by WW:] This value would be still smaller, if at the same time, the current in the fixed coil were interrupted, because this current, even given an open bifilar coil, still induced currents in the brass mounting of the coil during the oscillation, exactly as was the case in the first series of experiments with the magnets, which, however, worked far more weakly.

therefore galvanic and magnetic forces proportional to them) must result in an attenuation of the oscillations of the bifilar coil, whose *logarithmic decrements* are in the ratio of the squares of 101.9 : 19.1, i.e., like

$$28.5 : 1 .$$

Instead of that, from the observations of the decrease of the arcs of oscillation, we have found in both cases the ratio of the portions of the logarithmic decrement stemming from the induced current according to the *fourth* and *first* series of experiments to be

$$0.002\,627 : 0.000\,097 = 27.1 : 1 ,$$

a ratio differing from the calculated one by about 5 percent; the small logarithmic decrements proceeding from the *magneto-electric* currents in the observed cases, as already mentioned above[158] on page[159] 107, can no longer be relied upon.

5.12 An Induced Current of Equal Strength to the Inducing Current

The *constancy* of the logarithmic decrement of the swinging bifilar coil under the influence of a constant current in the fixed coil, and of the currents thereby induced in the swinging bifilar coil, already resulted[160] on page[161] 105 in the law for induction, that the intensity of the induced current is proportional at any moment to the *velocity* of the swinging coil at that moment. If hereby this law is now placed beyond doubt, then it follows that, in the case of a given *constant* inducing current, one could arbitrarily strengthen the current *induced* by it, if one increased that *velocity*, and that there would have to be a velocity, at which the *intensity of the induced current would be just as strong as that of the inducing current*. It may not be uninteresting to give a more precise *determination* of this velocity. This determination can easily be obtained, if one

1. calculates from the measured arcs of oscillation of our coil and from its periods of oscillation, likewise measured according to known laws, the *velocity* which the coil possesses in the center of its oscillation;

2. if one calculates, from the likewise measured values of the logarithmic decrement caused by the voltaic induction, the *deflection* of the coil, which would be produced by the force which retards the velocity of the swinging bifilar coil at the moment when it finds itself in the center of its oscillation, if it continues uniformly in the same direction; and

3. lastly, if a current is put through the bifilar coil, and the intensity of this current is varied until the electrodynamic deflection of the coil as a result of the interaction of this current and of the constant current in the fixed coil is equal to that deflection, and if one then determines the *ratio* of the intensities of both currents.

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[158][Note by LH and AKTA:] Regarding the 6-8% uncertainty.

[159][Note by LH and AKTA:] [Web46, p. 105 of Weber's *Werke*].

[160][Note by LH and AKTA:] Third law at the beginning of Section 5.11.

[161][Note by LH and AKTA:] [Web46, p. 103 of Weber's *Werke*].

It is then clear that, when the velocity of the swinging coil is increased according to the ratio of these intensities, the induced current will be equal in strength to the inducing current at the moment when the coil finds itself in the center of its arc of oscillation. In this way it turned out that the bifilar coil of the dynamometer described in Section 5.1 would have to be turned around its perpendicular axis of rotation

<div align="center">31 times</div>

in one second, in order for the current *induced* by the arbitrarily strong or weak current of the fixed coil of this instrument to have the *intensity of the original current* at the moment when both coils were perpendicular to one another. At this velocity of the coil's rotation, the greatest linear velocity of the current elements would amount to 6.5 meters or about 20 feet in one second, since, according to page[162] 44, the radius of the bifilar coil was 33.4 millimeters.

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[162][Note by LH and AKTA:] [Web46, p. 36 of Weber's *Werke*].

IV - Applications of the Electrodynamometer

5.13   Determination of the Duration of Momentary Currents with the Dynamometer, Along with Application to Physiological Experiments

In order to portray and to measure the interaction of two conducting wires with the help of the dynamometer, no strong currents are required, as the data presented prove; on the contrary, weak currents suffice, which, if other devices are used, are barely perceptible, such as, for example, the induced currents produced by the oscillations of the bifilar coil, according to Section 5.10, which were barely visible without an optical device. This circumstance is of practical importance, because these experiments thereby received a far greater expansion, and the way was paved for the most manifold applications of the dynamometer, especially to *galvanometric* determinations as well. A compass or a magnetometer is called a *galvanometer* when it is equipped with a multiplier, because it serves to measure the intensity of the galvanic currents which are led through the multiplier wire. The measurement of the intensity of galvanic currents is hereby based, *not on purely galvanic*, but on *electromagnetic* effects. By the same right a *Voltameter*,[163] too, deserves the name of a *galvanometer*, because it likewise serves to measure the intensity of galvanic currents which are conducted through the Voltameter; it is merely that the latter is an *electrochemical* galvanometer, the former an *electromagnetic* one. Now, the *electrodynamometer* is also a *galvanometer*, because it serves to measure the intensity of galvanic currents which are conducted through it; it is, however, a *purely galvanic* or *electrodynamic* one, because it is the interaction of the galvanic currents itself which is used therein for measuring the current intensity, and hence it merits even preferentially the name *galvanometer*.

Nevertheless, it does not seem possible to ascribe any great practical importance to the *electrodynamometer*, if it is no longer a matter of testing the fundamental electrodynamic laws, but merely of *galvanometric* determinations, because the multifarious apparatus of the Voltameters and of the electromagnetic galvanometers for measuring the intensity of galvanic currents already performs such good and convenient service, that no reason presents itself to replace this already utilized instrument with a new one. As long as it is simply a matter of objectives which have already been attained with the latter instruments, or can be attained with them, a new instrument like the dynamometer can, in fact, have no great practical importance attached to it. However, things are different in those cases, where the existing devices are inadequate, as, for example, when it is a matter of determining the current intensity *at particular moments*.

That is, the sine or the tangent of the deflection of the magnetic needle in the sine or tangent-galvanometer only gives a correct measure of the current intensity in the multiplier *at a definite moment*, if the current in the multiplier acting on the needle is *constant*; if, on the contrary, its intensity is *variable*, then the intensity of the current for a particular moment cannot be derived at all from the deflection of the magnetic needle, or only by means of calculating with the help of a given law designed for those variations. Of course, we may

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[163][Note by AKTA:] A Voltameter, Volta-electrometer or Coulometer is an instrument used for measuring quantity of electricity through electrolytic action. Faraday used an apparatus that he termed a Volta-electrometer, [Far34b, Article 565, Note 1] and [Far34a, article 704] with German translation in [Far34c]. Later on this term was shortened to Voltameter. The present-day term is Coulometer.

then let the current act on the needle for only a *moment*, but the deflection of the needle elicited by this momentary influence, even if it is large enough for precise observation and permits refined measurement, in no way suffices per se for the determination of the current intensity at that moment; on the contrary, knowledge of another element is required, namely, knowledge of the *duration* of that momentary influence, which cannot be achieved with the instrument. Only when one knows the *amount* of electricity which the momentary current puts through, and the *time* in which this electricity has gone through a cross-section, can the intensity be determined by dividing the former by the latter. However, from the deflection of the needle elicited by that momentary influence, only a determination of that amount of electricity can be derived; the time remains undetermined.

Now, the *dynamometer* serves in such cases essentially to *supplement* the *electromagnetic* galvanometer, for both instruments give us *two intrinsically different*, mutually independent determinations, from which *the two unknown elements*, on which the current intensity depends, can be derived. The *difference* of the determinations obtained with the two instruments already manifests itself, when one conducts continuous *constant* currents of *differing intensity* through a circuit, in which the usual *galvanometer*, and also the *dynamometer*, is included, and the *angle of deflection* is observed, at which for each of these currents, equilibrium exists on these instruments. These angles of deflection increase on both instruments with the intensity, but according to different laws; for the tangents of the angles of deflection of the *dynamometer* are, as was proven in Section 5.2, proportional to the *squares* of the tangents of the angle of deflection of the *magnetometer*.

That *difference* in the determinations provided by the two instruments shows itself even more remarkably, if a *constant* current, as just described, is put through both instruments, and the corresponding deflections of both are observed, and then, without changing the current intensity, simply the *direction* of the current in all the conducting wires of the two instruments is *reversed* with the help of a commutator; it is known, that after this reversal of the direction of the current direction in the multiplier, the multiplier's *magnetic needle* is deflected just as far as before the reversal, but to the *opposite* side. In the *dynamometer* this does not take place, but rather, the deflection occurring before the reversal of the current remains *unchanged* even after the reversal of the current, so that, provided that the reversal of the current has actually been momentary, without interruption, *no influence at all* on the dynamometer is to be perceived from this reversal. The dynamometer in this case acts as an *electromagnetic* galvanometer would act, if, at the moment when the current in the multiplier were reversed, the *poles of the needle were also switched*, assuming that the needle, like the bifilar coil of the dynamometer, possessed a definite directive force independent of the state of its poles. This sameness of the effects of positive and negative currents in the dynamometer in this easily executed experiment, ought to arouse all the more attention, the more one is accustomed to see opposite currents correspond to opposite effects.

This experimentally proven *difference* in the determinations provided by the two instruments can now easily be more precisely *defined*. The direct effect of the current passing through the conducting wires of both instruments is a *rotational moment*, which strives to put the compass or bifilar coil on which it acts, into a rotational motion. This rotational moment is proportional in the *magnetic galvanometer* to the intensity i of the current acting on the needle, and to the magnetic moment m of the needle, which is acted upon, and is thus represented by the formula

$$a \cdot mi$$

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in which, if we confine ourselves to small angles of deflection, a is to be considered as a constant to be determined once and for all for each instrument. The effect of this rotational moment in the time element dt is then expressed by the product

$$ami \cdot dt$$

and is equal to the product of the rotational velocity, in which the rotatable body is thereby put, with the moment of inertia of this body.

In the *dynamometer*, on the contrary, the rotational moment is proportional to the intensity i of the current in the fixed coil, which acts on the bifilar coil, and also to the intensity i of the current in the bifilar coil itself, which is acted upon, and is thus represented by the formula

$$b \cdot i^2 \ ,$$

where b, if we confine ourselves to small angles of displacement, denotes a constant to be determined once and for all for each dynamometer. The effect of this rotational moment in the time element dt is thus expressed by the product

$$bi^2 \cdot dt \ ,$$

and is likewise equal to the product of the rotational velocity produced with the moment of inertia of the rotatable body.

Now, if this current persists uniformly during the short time from $t = 0$ to $t = \vartheta$, and if the inertial moment of the needle and of the bifilar coil are denoted with p and q, then the *angular velocity* which is thereby produced is

$$\text{for the needle} = \int_0^\vartheta \frac{a}{p} \cdot midt = \frac{am}{p} \cdot i\vartheta$$

$$\text{for the bifilar coil} = \int_0^\vartheta \frac{b}{q} \cdot i^2 dt = \frac{b}{q} \cdot i^2\vartheta \ .$$

If both instruments were previously at rest, then they are put into oscillation by the transmission of this angular velocity, and if s and ζ denote the *periods of oscillation* of the two instruments, then, according to well-known laws of oscillation, if no attenuation takes place, and if the time interval ϑ, in which the needle and the bifilar coil receive that angular velocity, is so small, that the *disturbance* itself during this small time interval, as with a *shock*, does not need to be taken into consideration, then the *rotational velocity* for any moment at the end of time t is expressed by

$$\frac{e\pi}{s} \cdot \cos\frac{\pi}{s}(t - \vartheta) \quad \text{and} \quad \frac{\varepsilon\pi}{\zeta} \cdot \cos\frac{\pi}{\zeta}(t - \vartheta) \ ,$$

where e and ε denote the *elongation lengths*, which can be determined for both instruments by *observation*. If now the first moment after the cessation of the current is substituted for t, that is, $t = \vartheta$, then the velocities originally transmitted by the current to the two instruments are obtained:

$$\frac{am}{p} \cdot i\vartheta = \frac{e\pi}{s} \ , \qquad\qquad \frac{b}{q} \cdot i^2\vartheta = \frac{\varepsilon\pi}{\zeta} \ ,$$

or one has two equations for determining the *current intensity* i and the *duration of the current* ϑ, by means of which they can be calculated from the *measured deflections* of both instruments e and ε, namely:

$$i\vartheta = \frac{\pi p}{ams} \cdot e \ , \qquad\qquad i^2\vartheta = \frac{\pi q}{b\zeta} \cdot \varepsilon \ ,$$

where $\pi p/ams$ and $\pi q/b\zeta$ denote constants to be determined once and for all. The sought-for *current intensity i* results from this:

$$i = \frac{am}{b} \cdot \frac{q}{p} \cdot \frac{s}{\zeta} \cdot \frac{\varepsilon}{e} \ ,$$

and the sought-for *duration* of this current:

$$\vartheta = \frac{\pi b p^2 \zeta}{a^2 m^2 q s^2} \cdot \frac{e^2}{\varepsilon} \ .$$

Since the *periods of oscillation* of the two instruments s and ζ can be directly determined, it is merely necessary for the complete determination of the *constants* of both instruments, to put through both instruments a constant standard current, whose *intensity* $= 1$, and to observe the tangents of the *angles of deflection* e' and ε', for which equilibrium then exists. The tangents of the angles of deflection are then, according to well-known laws, to be equated with the ratios of the deflecting *rotational moments* for the current intensity $= 1$, namely

$$am \qquad \text{and} \qquad b \ ,$$

to the *directive forces* of the compass and the bifilar coil, namely

$$\frac{\pi^2 p}{s^2} \qquad \text{and} \qquad \frac{\pi^2 q}{\zeta^2} \ ,$$

thus:

$$e' = am \cdot \frac{s^2}{\pi^2 p} \ , \qquad\qquad \varepsilon' = b \cdot \frac{\zeta^2}{\pi^2 q} \ .$$

If these values are substituted in the above equations, one obtains

$$i\vartheta = \frac{s}{\pi} \cdot \frac{e}{e'} \ , \qquad\qquad i^2\vartheta = \frac{\zeta}{\pi} \cdot \frac{\varepsilon}{\varepsilon'} \ ,$$

and consequently

$$i = \frac{\zeta}{s} \cdot \frac{e'}{\varepsilon'} \cdot \frac{\varepsilon}{e}$$

$$\vartheta = \frac{1}{\pi} \cdot \frac{s^2}{\zeta} \cdot \frac{\varepsilon'}{e'^2} \cdot \frac{e^2}{\varepsilon} \ ,$$

in which by means of a one-time observation of the deflections e' and ε' as well as the period of oscillation of the compass and the bifilar coil s and ζ, the constant coefficients ζ/s, e'/ε', s^2/ζ and ε'/e'^2 are determined for ever. Thus it follows from this, that the observations simultaneously made on both instruments of the deflections e and ε supplement each other, by jointly providing complete data for determining the *intensity* and the *duration* of a momentary current, while each one, considered individually, acquaints us with neither the one nor the other.

The cases where this complete determination of momentary currents, attainable by means of simultaneous use of both instruments, finds useful *applications*, are not far to seek; they

present themselves in a manifold way. *Momentary* currents, for example, are frequently used for *physiological* experiments, in order to investigate the *influence of galvanism on the nervous system*; for it turns out that a continued action of the galvanic current very quickly deadens the nerves through which it goes, particularly when it is a *sensory nerve*, so that no extended series of rapidly successive experiments can be carried out in this manner, which becomes possible, if the current is always allowed to go through the nerve only for a moment. These highly interesting observations, however, can lead to no definite results, without a *knowledge of the currents* which elicit those effects, especially of their *intensity* and their *duration*. A thorough investigation of the physiological effects of galvanic currents on the nervous system hence requires the complete determination of these two elements, which, however, can only be achieved according to the just-developed method of simultaneous observations of the *galvanometer* and *dynamometer*. In any case, it is an interesting task for the physiology of nerves, to establish the time *limits* for how long a current must act on the nerves, in order to elicit a definite effect, and how this necessary time interval varies with the strength of the current. I venture to hope that the electrodynamometer will be used for the purpose presented, especially since in the local *Physiological Institute*, some experimental tests have already been made with good results, which will be communicated on another occasion. At present, I will confine myself above all to applications which can be made in the realm of physics itself, specifically, above all in the field of pure *electrical theory*.

5.14 Repetition of Ampère's Fundamental Experiment with Common Electricity, and Measurement of the Duration of the Electrical Spark During Discharge of a Leyden Battery

Ampère's fundamental experiment regarding the interaction of two conducting wires at a distance had up to now been carried out with a single form of galvanic currents, namely, currents which originated from a voltaic battery. If one now finds oneself justifiably moved to the conjecture, that all galvanic currents, from whatever source they might originate, are subject to the same laws, and that therefore Ampère's law regarding the interaction of two conducting wires would be confirmed for all kinds of galvanic or electrical currents, this confirmation itself is in no way superfluous. Thus far, it already seems important that according to the experiments communicated in the preceding Sections, the Ampère interaction has been proven as well for *magneto-electric* currents and by means of currents induced by means of *voltaic induction*. However, it seems still more important to repeat Ampère's fundamental experiment with *common electricity*,[164] as it occurs in the discharge of a Leyden jar or battery by means of the applied discharge wire, since there are such considerable differences between this current of common electricity and all other galvanic currents, that only empirical experimentation can show, whether Ampère's fundamental experiment can hold good, or not. In particular, so long as empirical experimentation had not decided this question, one could easily conjecture, that either the extremely *short duration* of a current of common electricity, or, given a longer duration, the *discontinuity* of the current might be inherently obstructive to the interaction of two long conducting wires, like those of the two coils of the dynamometer, because it would be possible that the current in one wire

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[164][Note by AKTA:] In German: *gemeiner Electricität*.

had already stopped again while it was just beginning in the other. Experimentation with the *electrodynamometer* has proven, however, that the fundamental Ampère experiment also succeeds with common electricity, of which I will now give a more precise account here.

It is known, that the repetition of Oersted's fundamental experiment[165] with the *common electricity* collected in a Leyden jar is made most reliably, when one end of a wet string is fastened to the discharging rod, the other end to the conducting wire which forms the multiplier of the *galvanometer*, and the wire's other end is in a conducting connection with the outer coating of the Leyden jar. If the Leyden jar is then discharged with the discharging rod while the wet string hangs on it, a deflection of the magnetic needle is observed in that *direction* which can be predetermined by means of the *electromagnetic* laws. The use of a wet string is, however, not absolutely necessary to this fundamental experiment, but seems to be advantageous only when one wants to directly connect the wire ends of the multiplier of a sensitive galvanometer with the positive and negative conductor of an electrical machine. It is also not necessary to insulate the wires better than for other galvanic circuits. In the first case the use of a wet string was advantageous, because without it, the intensity of the discharge entails the danger of a confluence of the divided electricities which are collected in the battery, by other paths than through all the windings of the conducting wires. This danger is prevented by means of inserting a wet string, which diminishes the intensity of the discharge and nevertheless permits very large masses of electricity to unite with each other in a very short time through the conducting wire.

Now, while the main point of performing Oersted's fundamental experiment with common electricity is simply to conduct very large masses of electricity through the multiplier, whereas *the time* in which the electricity goes through the wire, comes less into consideration, the successful execution of Ampère's fundamental experiment essentially rests instead on leading large masses of electricity in *the shortest possible time* through the conducting wire, for which, therefore, the collection of electricity in batteries and the discharge of the batteries by means of a wet string seems especially suited. The effect of equal masses of electricity is always the *same* in the *first* experiment, the amount of time of flow may be smaller or larger, as long as it does not become so large that it requires a considerable portion of the period of oscillation; with the *latter* experiment, however, in conformity with the preceding Section, the effect is to be *inversely proportional* to the transit time. Accordingly, it seems that the use of the Leyden jar along with a wet string must be considered as especially favorable, if not necessary, for our experiment, and hence I have in fact used both in my first experiments.

Thus I joined together for this purpose two wire ends of the two coils of the dynamometer, and led one of the two other wire ends to the outer lining of a Leyden jar, the other to a wet string which was fastened onto the insulated discharging rod. The battery was charged, and lastly, the discharging rod was brought toward the metal knob which was connected with the interior lining of the battery. At the moment when the discharge of the battery through the wet string and through the dynamometer coils took place, the dynamometer, which had previously been at rest, was put into an *oscillation*, which often comprised an arc of several hundred scale units, of which several examples are to be presented forthwith. The observer standing at the telescope could easily determine the *magnitude* of the first elongation and the *side* toward which it occurred.

If thereupon the experiment was repeated, by re-charging the Leyden jar or battery in the same manner, but with the difference, that the wire which was previously connected to the outer lining, was fastened to the end of the wet string of the discharging rod, and the

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[165][Note by AKTA:] See footnote 74 on page 34.

other wire end was instead disconnected from the string and connected to the outer lining of the battery, then the effect was the *same*, not only with respect to the magnitude, but also with respect to its direction, so that, as with ordinary currents, *no difference whatever* took place in the effect of the *positive* and *negative* currents. And this *direction* of the deflection of the dynamometer as a result of the current of common electricity passing through, turned out to be like that which was already *predetermined* by means of Ampère's law. It is hereby proven, that the fundamental Ampère experiment can also be made with a current of common electricity.

It was also of further interest, however, to test whether the use of the wet string was necessary or dispensable for the success of this experiment, as well as whether there might exist any cases at all, where the current of common electricity would yield Oersted's fundamental experiment, but not Ampère's, or whether with respect to common electricity, both kinds of effects are always associated. For this purpose, more extensive series of experiments are required, than I have undertaken up to now; yet a few preliminary experiments may meanwhile be relevant.

The earlier experiments were repeated, both utilizing the wet string and excluding it, and also in association with the *electromagnetic* experiments, by inserting the multiplier of a magnetic galvanometer into the same circuit which included the two coils of the dynamometer. The latter effect then served as a criterion and a measure,[166] *whether* and *how much* electricity went through the wire circuit when the Leyden jar was discharged. When the wet string was excluded, in order to replace by other means the large resistance which it provided, a fine German silver [Argentan] wire of 0.3-millimeter diameter was wound around two glass columns 3.75 meters distant from each other, in such a way that the individual 7.5-meter-long windings were approximately 40 millimeters from each other, whereby they were completely insulated from one another. The German silver wire formed 32 such windings, and the end of this wire was now led freely through the air to the charged battery. In the following Table I assemble the results of two series of experiments for comparison, namely, one in which the current went through the wet string, the other in which the wet string was excluded from the circuit. The electric battery consisted of 4 jars each of about 2 square feet coated surface, which were charged moderately strongly and so uniformly in all experiments as could be discerned on the quadrant electrometer. The string was made of hemp, 320 millimeters long, 4 millimeters thick, and was dipped in water before each experiment.

1. Discharge using the wet string:		
No.	Elongation of galvanometer	Elongation of dynamometer
	$= e$	$= \varepsilon$
1.	51.75	206.99
2.	56.26	214.94
3.	61.36	236.98
4.	52.68	216.63
5.	55.31	223.88
2. Discharge using the wire circuit, without string:		
6.	7.06	0.85
7.	7.04	0.85

The observations on the *galvanometer* show that, if when using the string, all the electricity went through the circuit, without the string only 7 to 8 parts of it went through,

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[166][Note by AKTA:] In German: *Maassstab*. This word can also be translated as standard or scale.

according to which, under the presupposition, that the discharge without the string resulted faster, or at least not slower, than with the string, an effect would be expected of at least 50 parts of the previous amount of electrodynamic effect. This did not occur, however, but instead, as the comparison of the observations presented in the third column under ε shows, the effect was almost six times smaller. Although this latter effect was so small, it was nevertheless clearly perceptible.

The influence which the water exerted when the electricity was conducted through it, seemed to be susceptible of more precise investigation when the wet string was replaced with a glass tube filled with water. Hence a 1200-millimeter-long, 13-millimeter wide empty glass tube was bent into a U shape and filled with water, inserted between the discharging rod and the rest of the circuit, and the earlier experiments repeated, yielding the following results, with the same charge in the battery as earlier, which prove that water contained in a glass tube could not replace a wet string in this case.

No.	Elongation of galvanometer	Elongation of dynamometer
	$= e$	$= \varepsilon$
1.	4.68	3.23
2.	4.50	1.57

Discharge with a glass tube filled with water:

In vain were all precautionary measures which were taken in this experiment and in the preceding one with the exclusion of the wet string, in order to compel the electricity to take its path through the water in the tube, and thence through the German silver wire, in order to diminish the intensity of the discharge by means of the resistance of these bodies, and to make all the electricity take its path through the instrument's conducting wires; only a slight portion of the electricity seemed to actually adopt the latter path. If, on the contrary, the glass tube was exchanged for a string of *glass threads*, this string, when it was externally moistened, performed comparable service to the wet string. The discharge through such a 500-millimeter-long string dampened with ammonia gave the following elongations on the galvanometer and dynamometer respectively:

100.55	70.35 .

The electricity coming out of a Leyden jar seems to especially spread on the surface of the body, and therefore a moist conductor seems to have more effect, when it externally covers the surface of this body, than when it is enclosed.

Lastly, the results of a series of experiments performed with the wet string may be relevant, in which a battery of 8 jars just like the ones used earlier, and a hemp string of 7 millimeters thickness and 2 000 millimeters length was inserted; this length, however, was gradually shortened to 125 millimeters.

Length of string	Elongation of galvanometer $= e$	Elongation of dynamometer $= \varepsilon$	$\frac{e^2}{\varepsilon}$
2 000 mm	79.9	65.6	97.3
1 000 mm	76.6	153.0	38.3
500 mm	82.3	293.8	23.0
250 mm	87.3	682.0	11.2
125 mm	93.2	out of scale	
250 mm	82.9	609.1	11.3
500 mm	95.6	422.8	21.6
1 000 mm	95.8	210.1	43.7
2 000 mm	101.5	98.0	105.0

It may be further noted, that, when the string was dipped into *dilute sulphuric acid*, a discharge of the battery gave a deflection of 83 scale units on the galvanometer, while the deflection on the dynamometer itself was too large, when the string length was 2 000 millimeters, to be measured on the scale.

It is easily seen, that a broad field of interesting experiments stands open here, which I have not further pursued, because of the need to subject the amount of electricity in the battery used for the experiments to a direct precise measurement, according to the model given by Ries[167] in his electrical experiments, for which I do not currently have at my disposal the appropriate equipment, and therefore I am postponing this work to a more favorable time.

Meanwhile, however, the last series of experiments performed already shows, apart from the *strength* of the effects, such a degree of *regularity*, that it becomes probable that, in discharging the Leyden battery by means of a wet string, *all the electricity* in fact goes through the conducting wire and forms a current in it which might be comparable in continuity to the current of a galvanic battery.[168] Were this the case, one could make an important application of the preceding observations, by applying to them the rules developed in Section 5.13, in order to determine the *duration* of the current which may be considered as equal to the duration of the *discharge spark*, according to an *absolute time measure*. It is well known that Wheatstone effected the determination of the duration of the discharge spark in a completely different manner,[169] and it would be interesting to compare with each other the results found in such different ways. In order to reduce to an *absolute time measure* the *relative time measure* which we have already included for the above experiments themselves in the column headed $e^2\varepsilon$, it requires, according to page[170] 114, simply an experiment with a constant current passing through both instruments, which I have carried out for this purpose, and have found that the values for e^2/ε in the Table above are to be divided by

$$1\,188 \, ,$$

in order to obtain the duration of the current in *seconds*. The following Table is calculated accordingly:

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[167][Note by AKTA:] Peter Theophil Rieß (1804-1883).

[168][Note by WW:] Electrodynamic experiments can be arranged with *two dynamometers* in such a way that the electricity is conducted *successively* in one, *simultaneously* in the other, through the fixed and suspended coils. By comparing the results for both instruments, when a battery is discharged through them, it would be possible to investigate more precisely the continuity or discontinuity of the current.

[169][Note by AKTA:] Charles Wheatstone (1802-1875). See [Whe34].

[170][Note by LH and AKTA:] [Web46, p. 114 of Weber's *Werke*].

Length of string	Duration of spark
Millimeters	Seconds
2 000	0.081 9
1 000	0.032 2
500	0.019 3
250	0.009 4
250	0.009 5
500	0.018 2
1 000	0.036 8
2 000	0.088 3

or in mean values:

Length of string	Duration of spark
Millimeters	Seconds
2 000	0.085 1
1 000	0.034 5
500	0.018 7
250	0.009 5

It follows that the *duration of the spark is almost proportional to the length of the string*, as the following overview of the thus *calculated* values and their difference from the *observed* values proves:

Length of string	Calculated duration of spark	Difference from observed value
2 000	0.081 6	− 0.003 5
1 000	0.040 8	+ 0.006 3
500	0.020 4	+ 0.001 7
250	0.010 2	+ 0.000 7

If one compares the results found by Wheatstone, according to which the duration of the spark by discharges merely through a metallic conductor, is negligibly small in relation to the duration found here, then this stands in complete accord with the proportionality found here between the duration of the spark and the length of the wet discharge string. In any case, the fact that the *motion of the electricity in water* occurs so *slowly*, that the time it requires for the short path of 2 meters amounts to roughly 1/12 seconds, merits particular attention. Apart from the objection derived from the discontinuity of the currents of common electricity (which was already discussed above, and which may be largely diminished or entirely eliminated by means of the influence of the water), it could, of course, be objected against the application of the rule by which these time determinations were made, that the current is most intense in the first moments, and will *gradually decline*, while the above rule can only be applied with precision, when the current always possesses the same intensity during its short duration. If, however, one empirically finds, in this case as well, not the true duration, but that duration which would correspond to a *mean current strength*, the value of the determination may lose little on this account, because it will generally be of more interest to know the latter duration, than the former. It is also noteworthy that for the same reasons, a similar difference was occasioned in Wheatstone's determination of the duration of the spark, because the spark was extended in a line which as a result of that decrease gradually dispersed without sharp delimitation.

5.15 Velocity of Current Distribution and Electromotive Force of a Circuit

Two investigations in the field of *pure electrical theory* are still to be presented here, for which the use of the dynamometer opens a new path; however, I will not go more precisely into these investigations for the present, because the necessary experiments are still lacking, in order to demonstrate conjointly the method and the results obtained by it. These two investigations concern:

1. the determination of the velocity of current distribution,[171] for which up to now merely a few experiments by Wheatstone are in hand,[172] which, however, according to Wheatstone's own statement, have still not led to any certain results;

2. the determination of the electromotive force[173] of a galvanic circuit, independent of the polarization of its plates.

The *first* application requires that the bifilar coil is separated from the fixed coil by means of long conducting wires, and in this long circuit a current is produced, whose direction changes as fast as Wheatstone's mirror is rotated. The use of the dynamometer, by comparison with Wheatstone's method, provides the advantage of using galvanic currents instead of common electricity, and the circuit is never interrupted, which was necessary for Wheatstone for producing the spark. The *latter* application is based on the measurement of momentary currents according to Section 5.13.

5.16 Application of the Dynamometer to Measurement of the Intensity of Sound Vibrations

It still remains, to describe an application of the dynamometer to researches in another domain of physics, which seems to have a special interest associated with it, because it casts a bright light on a specific aspect of what can be done with this instrument. We possess extraordinarily refined *galvanoscopes*, with which we are in a position to discover and investigate even the weakest currents found in Nature. We need merely recall the fine work of Melloni,[174] in order to place the greatest weight for science in general upon the use of these refined instruments and the traces of electrical motions which we find by means of them. Despite this refinement in the instruments, however, in many cases success has not been achieved in demonstrating electrical currents everywhere we surmise that they exist, perhaps because those instruments, despite their refinement, were not suited to the purpose. This reason deserves all the more consideration, in that *one* sort of current can be demonstrated and exactly described, to which even the finest instruments are insensitive, in the nature of the case. This occurs, when we are dealing with an alternating current, which in

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[171][Note by AKTA:] In German: *Geschwindigkeit der Stromverbreitung*. This expression can also be translated as *velocity of current propagation*. Weber is referring here to the velocity of propagation along the circuit of a perturbation in the current, or the velocity of propagation along the circuit of the distribution of free electricity. See also footnote 42 on page 24.

[172][Note by AKTA:] See footnote 169 on page 119.

[173][Note by AKTA:] In German: *der elektromotorischen Kraft*.

[174][Note by AKTA:] Macedonio Melloni (1798-1854).

very short sequential time intervals *constantly changes its direction.* The alternating opposite actions of the current on the most sensitive magnetic needle must cancel each other, if the magnetism of the needle always remains the same. The phenomena observed by Poggendorff (*Annalen* 1838, Vol. LXV, page 355 ff.),[175] in which this does not seem to occur, originate from a changeability in the needle's magnetism, and, given a very accelerated change in current, would disappear again. Such currents, whose direction changes very rapidly, can thus exist to a great extent in Nature, without us having an inkling of their existence, because we possess no way to discover them. And it is not at all improbable, that such currents exist, for the movement of electricity in them would differentiate itself from the movement of electricity in the usual currents, only by the fact that the former consists of an *oscillation*, while in the latter, the motion of the electricity is *progressive.* Since the progressive motion of electricity occurs so abundantly in Nature, it is not obvious why, given such great mobility occasional conditions should not also occur, which favor a vibrating movement. If, e.g., light undulations exert an action on the electrical fluids, and have the power to disturb their equilibrium, it would certainly be expected that these *effects* of light undulations would be structured in time with the same periodicity as the *light undulations themselves*, so that the result would consist of an *electrical vibration*, which, however, we are unable to discover with our instruments. Now, the undulations of light occur so rapidly, that, if the vibrations they elicit follow an equally rapid alternation, we could scarcely hope to observe their effects with any instrument. Slower vibrations also occur in Nature, however, e.g. acoustical ones, and hence the question arises, whether there are not electrical motions in Nature whose origin is due to them, and if there are such motions, in what way we could discover and investigate them.

I want to give at least one example here of such *electrical vibrations*, elicited by sound vibrations, and provide the actual proof of how such electrical vibrations can be observed and investigated with the help of the *dynamometer*, and how the measurable effects of these electrical vibrations can in turn be used to elucidate the sound vibrations from which they originate, and thereby to open a new path for many acoustical investigations, for which we still altogether lack suitable means of measuring the *intensity of sound vibrations.*

In fact, the peculiarity of the dynamometer, which most characterizes it and distinguishes it from all other galvanometers, consists in the fact, that it is *indifferent* to the *direction* of the current acting upon it, while other galvanometers undergo opposite effects, given opposite directions of the currents. Attention was already called to this in Section 5.13 above. We can express this succinctly by saying that the dynamometer with respect to constant currents gives a measure for the *square of the current intensity*, while other galvanometers provide a measure for the current intensity itself.

From this characteristic property of the dynamometer, it is now obvious, that the rapidly successive actions of the opposed currents do not, as in an electromagnetic galvanometer, cancel each other, but rather must be additive; and that in virtue of its nature the dynamometer consequently finds its true purpose in bringing to light such otherwise unobservable currents.

Now, the *sound vibrations* are mostly, of course, contained in such narrow, almost microscopic boundaries, that we can scarcely hope to use them to elicit electrical vibrations whose boundaries have the breadth necessary to register an effect upon the dynamometer. If, however, we calculate the absolute velocities, with which the resonating bodies move in the middle of their vibrations, it turns out that these velocities, considering the short duration of the vibrations, are not entirely inappreciable, despite the small oscillation curves,

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[175][Note by AKTA:] [Pog38].

but often amount to a foot or more in one second. Building upon this, I have carried out an experiment so that it seemed the first to be capable of producing results.
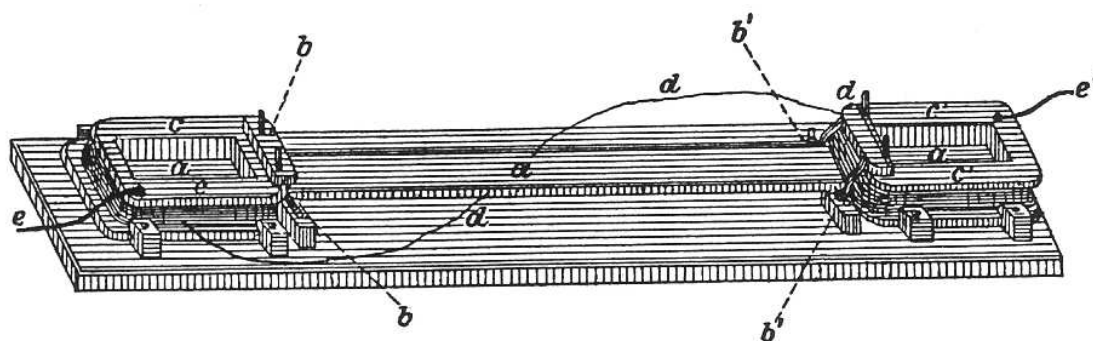
Fig. 13.

I prepared a sounding-rod of steel (*aaa*, Figure 13) and let it harden, magnetized it, and fastened its nodal lines at the end-points b, b, b', b' between screw-heads as axes of rotation, as I have described in Poggendorff's *Annalen* 1833, Vol. XXVIII, page 4,[176,177] and divided it into three sections simultaneously vibrating toward opposite sides. Hence the two end sections made their vibrations simultaneously in the same direction, alternately upward and downward. The free magnetism, which is disseminated in these rods, can be thought of as disseminated on the surface of the rod, according to Gauss' ideal distribution, which represents the actual distribution in all outward effects;[178] and, specifically in the case of strong magnetization, the free north magnetism must be thought of as almost entirely on the surface of one vibrating end-section, the free south magnetism almost entirely on the surface of the other vibrating end-section, and indeed the closer to the end, the greater the concentration, i.e., exactly the most [concentrated] where the sound vibrations are the greatest. I wound these two vibrating end-sections with strong inductors ccc and $c'c'c'$ made of fine copper wire, which, however, never touched the rod, so that its vibrations would not be inhibited. In addition, there was a gap in the windings on the sides of the inductors turned toward each other, through which the ends of the rod were inserted into the inductors. The windings of the inductors were parallel to each other and lay in a plane perpendicular to the sound vibrations of the sounding-rod. The two inductors were connected to each other with two of their wire ends $dddd$, so that they formed spirals wound in opposite directions. Their two wire ends ee and $e'e'$ were connected with two wire ends of the fixed and moveable coils of the dynamometer, whose other two wire ends were connected to each other. The dynamometer was completely at rest. After everything had been prepared in this way, the sounding-rod was made to vibrate strongly by means of a sharp rap at its midpoint with a soft clapper. There appeared at once a deflection of the bifilar coil of 20 to 30 scale units, and thereupon, when the maxima and minima of the vibration curve of the bifilar coil, which was vibrating from then on, were recorded, it was seen, that the state of rest calculated from this, around which the vibration occurred, was altered, but that it quickly returned again to its original state as the sound vibrations decreased in strength. I would note that I elongated the bifilar coil by several hundred scale units, letting the sounding-rod vibrate only so long, as the elongation was increasing, while on the other hand I deadened the sounding-rod, while the bifilar coil swung back again, and struck the sounding-rod again as soon as the bifilar

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[176][Note by HW:] Wilhelm Weber's *Werke*, Vol. I, page 367.

[177][Note by AKTA:] [Web33].

[178][Note by AKTA:] [Gau39] with English translations in [Gau41a] and [GT14].

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coil began to move in the original direction, and so forth.

It hardly need be mentioned, that if more precise determinations of the intensity of sound vibrations are really to be obtained according to the method presented, the sounding-rod cannot be put into vibration by means of a clapper stroke, because the intensity of the vibrations elicited in this way decrease very quickly and almost entirely disappear; but they must be maintained in a state of constant vibration for a longer time by means of a continually adjusted intervention.

It can safely be presumed, that the electrical vibrations which are actually demonstrated, take place under the conditions in which we made our observations; hence it was only a question of testing the method by which such vibrations are made *observable*. After this method has been found to be proven, however, one can further build upon it, and it is certain that using this method will lead to the discovery of electrical vibrations under previously unthought-of conditions. To illustrate the manifoldness of these phenomena, the following experiment may be cited here. If a strong galvanic current is introduced close to a vibrating string, which forms a component of a wire circuit running back into itself, then as a result of that vibration, alternating positive and negative currents are induced in the circuit, whose intensity can be measured with the dynamometer, similarly to the way it is induced by the vibrating magnetized rod.

5.17   On Various Constructions of the Dynamometer

There are essentially *three* different constructions which can be given to the dynamometer, all of which are suited to exact measurements, and provide special advantages under differing conditions. In addition to the *first* construction, which has been applied up to now, a *second* presents itself almost automatically to begin with, since it is already frequently used, with regard to its most essential components, for observing the effects of the Earth's magnetism on a conductor. Specifically, for this purpose, a conductor wound in circles, together with the battery from which the current issued, was hung up on a thread or wire, like a magnet, and the rotational moment which the Earth exerts on this kind of closed circuit, was observed in the same way as on a suspended magnetic needle. In fact, this apparatus provides a rotatable conductor, whose oscillations and deviations can be observed with as great a refinement as those of our bifilar coil, and it is only necessary to surround the suspended battery with a fixed multiplier, through which a current likewise flows, in order to complete the dynamometer. Now, add to this the fact that the discovery of *constant* batteries by Daniell[179] and Grove has paved the way for more refined applications of such an instrument, which was previously blocked by the variability of the currents. For this a small Grove's element is particularly suited, which, considering its small dimensions and low weight, provides a fairly strong and constant current. If mirror, telescope, and scale are added, the most refined observations can be carried out with this instrument. Figure 14 depicts such an instrument, as used by me for this purpose. A is the wire wound around in a ring, whose ends are connected by means of brass couplings ab and $a'b'$ to the platinum and zinc poles of a small Grove's element B from the instrument-maker Kleinert in Berlin. This element rests on a wooden stand, whose upper part is equipped with a torsion ring C, to which the suspension filaments are fastened at D.

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[179][Note by AKTA:] John Frederic Daniell (1790-1845).
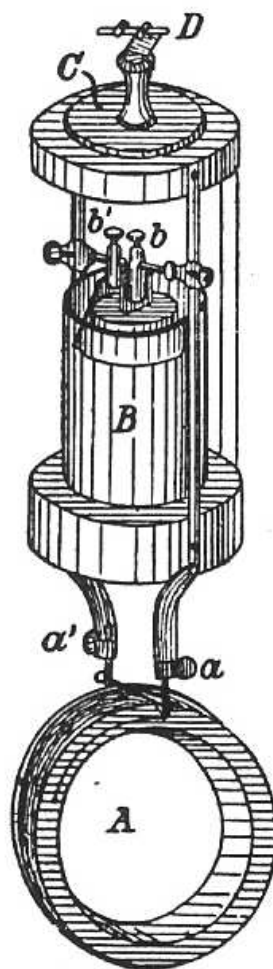
Fig. 14.

However suitable this construction of the dynamometer may be for a few special purposes, nonetheless it is far from able to replace the first construction, because it lacks *two* properties which the dynamometer with the *bifilar coil* possesses, and which are based on the fact that the current going through the bifilar coil can be further conducted, both through the fixed coil serving as multiplier, and through any other conductor as well. The *first* property consists in the fact that this *dynamometer* can be used together with a *galvanometer*, by means of which an independent measurement of the intensity of the current in the bifilar coil can be obtained, which is not the case with the other instrument, because there the current from the suspended battery can not be drawn off through the multiplier of a *galvanometer*. However, the simultaneous observations on the *galvanometer* and *dynamometer* permit us to reduce the electrodynamic effects to the *same current intensity*, as repeatedly occurred in the previous undertakings. The lack of this property is not completely overcome by the use of constant batteries, because the current intensity even in such batteries is still always subject to considerable variations, which can in no way be disregarded in the course of more precise determinations.

The *second* property consists in the fact that, by letting the currents to be investigated with the *dynamometer* pass through *both* coils, the fixed coil as well as the rotatable one, one can determine the *square of the current intensity*, which is independent of the *direction* of the current. On this was based the peculiar characteristic of the instrument, which made it

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possible, in association with the electromagnetic galvanometer, to provide the elements necessary for knowledge of *momentary* currents. See Section 5.13 above. The other instrument whose rotatable coil formed a suspended, self-contained battery, also lacked this property; for here the different currents to be *investigated* can simply be brought through the conducting wire of the *fixed* coil, while the current in the *rotatable* coil remains unchanged, whereupon the effect of the current intensity, as with an electromagnetic galvanometer, is proportional to the current intensity itself, and consequently the instrument is capable of simply serving the role of an electromagnetic galvanometer, but not of supplementing it.

I now proceed to the *third* construction of the dynamometer, which, in that it shares the most essential properties of the first, is suited to give to electrodynamic measurements a still greater expansion, especially in cases where the *first* fails us because of the necessary fineness of the suspension wires through which the current is conducted.

This third construction is based on the same principle, which I have developed in the *Commentat. Soc. Reg. Sc. Gottingensis recentiores*, Vol. VIII,[180,181] for the purpose of describing a perfectly rotatable, friction-free *balance scale*,[182] namely, on the principle of compensation between the *force of gravity* and *elastic force*. There I hung the horizontal balance beam on two elastic vertical springs. These springs bent, of course, when the balance beam was turned, and thus, the more the beam was turned, the more they sought by means of their *elastic force* to *inhibit* the rotation; but if the rotation of the balance beam took place around an axis, which lay lower than its center of gravity, then, when the balance beam was rotated, the more the balance beam was rotated, the more the *force of gravity* sought to *accelerate* the rotation, and it turned out that, in this construction, the *inhibiting* influence of *elasticity* and the *accelerating* influence of *gravity* balanced each other, and consequently the beam remained firmly in equilibrium not merely in a horizontal position, but also in an inclined position, and, without becoming hampered by friction, was able to switch from one of these positions to the other at the slightest impulsion.

I now used this kind of *compensated* balance beam for the dynamometer, and thereby replaced the rotatable coil, by making the same use of the two suspension springs to feed in and draw off the current, as I make of the two suspension wires. These springs are especially preferable to those fine wires, when it is a question of high-intensity currents, which should not be conducted through fine wires. It is sufficient to put the current through the strongest and shortest possible circuit; then the balance beam, through which this current is to pass, consists of a moderately long bar, held up by one of those two springs, to which bar, however, a mirror for more refined observation is attached. Finally, the *fixed* coil is replaced for the same reason with another moderately long *fixed* bar, by means of which the galvanic current is likewise conducted, and which then acts on that *rotatable* bar, and deflects it, like a balance scale. The sensitivity of this instrument primarily depends on the two bars (the fixed one and the rotatable one) being placed parallel to each other at a slight distance apart. I have designed this instrument above all to give a greater range to electrodynamic experiments with *common* electricity, by rendering dispensable the special conditions which were necessary to achieve a truly reliable discharge in a Leyden jar through the many windings of the two coils of the *first* dynamometer. As yet, this latter instrument has not been perfected to the degree necessary for such a series of experiments.

Before I conclude this Section on the construction of the dynamometer, I wish to add

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[180][Note by HW:] Wilhelm Weber's *Werke*, Vol. I, page 497.
[181][Note by AKTA:] [Web41a].
[182][Note by AKTA:] In German: *Wage.*

another remark about its transformation into a *magnetic galvanometer*. I have already mentioned, that the wholly self-contained, *suspended* battery used for the *second* construction described above, was used earlier in *electromagnetic* experiments, specifically in order to observe the influence of Earth magnetism on a current conductor. With this self-contained suspended battery, if one were able to fully rely on the constancy of its current, all experiments on, and measurements of, Earth magnetism could be carried out exactly as with the magnetometer, and to that extent it would warrant the name of a *galvanic magnetometer*. Our first dynamometer, on the other hand, could be used as a *magnetic galvanometer*, which offers great advantages, even in comparison with a magnetometer equipped with a multiplier, if it is a question of *absolute*, not merely relative, determination of current intensity. The current conductor is in a fixed position with respect to the magnetometer equipped with a multiplier, and the magnet is rotatable; however, there is no essential influence on the effect, when one reverses this relationship and fixes the magnet, while the conductor is rotatable. The coil of our dynamometer, suspended by two wires, can now serve as the rotatable conductor, and the Earth itself can be used as the fixed magnet (which substitutes here for the fixed coil). However, if the Earth is now to actually perform this role, the bifilar coil must be oriented in a different way, namely, instead of being oriented like a *declination magnetometer*, as it was earlier, so that its axis is parallel to the magnetic meridian, it must be oriented, like the *intensity magnetometer*, so that its axis is perpendicular to the magnetic meridian. It can then be called a *magnetic bifilar galvanometer*. This simple instrument then presents great advantages for the *absolute* determination of current intensity, precisely because the position and distance apart of the individual components of the conducting wire compared with the individual components of the magnets no longer need be taken into account, because of the great distance at which the Earth magnetism acts, and hence, what is required for the purpose of this absolute determination of current intensity, in addition to the knowledge of the Earth magnetism, the deflection, the period of oscillation, and the inertial moment, in terms of absolute measure, is only the knowledge of one single element, namely, knowledge of the *area* surrounded by the wire, as I have already discussed in the *Resultaten aus den Beobachtungen des Magnetischen Vereins im Jahre 1840*, page 93,[183,184] where I have communicated several such determinations of intensity according to absolute measure, which were made with this instrument.

Hitherto, the investigation primarily had the purpose of leading to *experimental* paths to measurements for electrodynamic forces, and to expressing those forces according to the *absolute* measure, reduced to measure of space, time, and mass. This was the motivation for the construction given to the instruments, which, as in the case of Gauss' magnetometer, lays claim to a more solid arrangement and a greater scope than is called for by other physical apparatus, in which the scale of measurement is directly mounted on the instrument to be observed. Given the appropriate construction, it was possible to carry out larger individual series of experiments with precision; this construction, however, is not so easily altered again and adjusted to different kinds of purposes. In this connection I must acknowledge, as an especially favorable circumstance, that the spaciousness of the Leipzig Physics Institute was on the whole advantageous for this construction; nevertheless, as mentioned several times, I had to confine myself for the present to preliminary experimental tests, because not all the constructions could be adequately manufactured in the same way. In consideration of these external constraints, present elsewhere still more than here, and because many experimenters

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[183][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, page 15.

[184][Note by AKTA:] [Web41e] with English translation in [Web20a]. See also [Web42].

are less accustomed to make observations with such instruments, I commissioned the local instrument-maker Mr. Leyser to complete smaller portable instruments for easier and more convenient manual use, without catoptric equipment, in the usual simple manner with pointer and subdivided circular scale, which suffice for conducting most experiments and for ordinary measurements. I call these smaller instruments to the attention of those who wish to engage in similar experiments, under conditions which do not permit the use of the instruments described.

V - On the Connection between Electrostatic and Electrodynamic Phenomena with Application to Electrodynamic Measurements

5.18 On the Meaning of a General Fundamental Law of Electrical Action

Since the fundamental law of electrodynamics put forward by Ampère is found to be fully confirmed by precise measurements, the *foundations of electrodynamics* could perhaps be considered as definitively established. This would be the case, if all further research consisted of nothing but developing the applications and results which can be based on that law. For, granted that we could inquire into the *connection*, which exists between the fundamental laws of *electrodynamics* and *electrostatics*, yet, however interesting it may be, and however important for a more precise acquaintance with the *nature of bodies*, to have investigated this connection, nothing further would have been yielded for the explanation of *electrodynamic phenomena*, if these phenomena have really found their complete explanation in Ampère's law. In short, essential progress for electrodynamics itself would not be achieved by reducing its fundamentals to the fundamentals of electrostatics, however important and interesting such a reduction might be in other respects.

This view of the conclusions which the fundamentals of electrodynamics has reached through Ampère's basic law and its confirmation, essentially presupposes, however, that *all* electrodynamic phenomena are actually explained by that law. If this were not the case, if there existed any class of electrodynamic phenomena, which it does not explain, then that law would have to be considered merely as a provisional law, to be replaced in future by a truly universally valid, definitive law applicable to all electrodynamic phenomena. And in that case it could well occur, that this definitive law would be arrived at, by first seeking to reduce Ampère's law to a more general one, encompassing electrostatics. Namely, it would be possible that, under different conditions, the law of the remaining electrodynamic phenomena, which could not be directly traced to Ampère's law, would emerge out of the same sources from which both the electrostatic law and Ampère's law were derived, and that the foundation of electrodynamics in its greatest generality, would then be represented, not in isolation per se, but solely as dependent on the most general law of electricity, subsuming the foundation of electrostatics.

Now, in fact, there does exist such a class of electrodynamic phenomena, which, as we assume throughout this Treatise, depend on the interactions which electrical charges exert on each other *at a distance*, and which are not included in Ampère's law and cannot be explained by it, namely, the phenomena of *Volta-induction* discovered by Faraday, i.e., the *generation of a current* in a conducting wire through the influence of a current to which it is brought near; or the *generation of a current* in a conducting wire, when the intensity of the current in another nearby conducting wire increases or decreases.

Ampère's law leaves nothing to be desired, when it deals with the interactions of conducting wires, whose currents posses a *constant intensity*, and which are *fixed in their positions* with respect to one another; as soon as changes in the intensity of the current take place, however, or the conducting wires are moved with respect to one another, Ampère's law gives no complete and sufficient account; namely, in that case, it merely makes known the actions

which take place on the *ponderable* wire element, but not the actions which take place on the *imponderable* electricity contained therein. Therefore, from this it follows, that this law holds only as a particular law, and can be only provisionally taken as a fundamental law; it still requires a definitive law with truly general validity, applicable to all electrodynamic phenomena, to replace it.

We are now in a position, to also predetermine in part the phenomena of *Volta-induction*; however, this determination is based, not on Ampère's law, but on the law of magnetic induction, which can be directly derived from experience, and which up to now has had no intrinsic connection with Ampère's law. And that predetermination of Volta-induction is in fact able to proceed, not through a strict deduction, but according to a mere analogy. Since such an analogy can indeed give an excellent guideline for scientific investigations, but as such must be deemed insufficient for a theoretical explanation of phenomena, it follows that the phenomena of Volta-induction are still altogether lacking theoretical explanation, and in particular have not received such explanation from Ampère's law. In addition, that predetermination of the phenomena of Volta-induction merely extends to those cases, where the inductive operation of a current, by analogy with its electrodynamic operation, can be replaced by the operation of a magnet. This, however, presupposes *closed currents* whose form is invariable. We can, however, claim, with the same justification as Ampère did for his law with respect to the interaction of constant current elements, that the law of Volta-induction holds true for all cases, in that it gives a general determination for the interaction of any two smallest elements, out of which all measurable effects are composed and can be calculated.

Thus, if we take up the connection between the *electrostatic* and *electrodynamic* phenomena, we need not simply be led by its general scientific interest to delve into the existing relations between the various branches of physics, but over and above this, we can set ourselves a more closely defined goal, which has to do with the *measurement of Volta-induction by means of a more general law of pure electrical theory.* These measurements of Volta-induction then belong to the *electrodynamic measurements* which form the main topic of this Treatise, and which, when they are complete, must also include the phenomena of *Volta-induction.* It is self-evident, however, that establishing such measurements is most profoundly connected with establishing the *laws,* to which the phenomena in question are subject, so that the one can not be separated from the other.

5.19   Development of a General Fundamental Law of Electrical Action

In order to obtain for this investigation the most reliable possible guideline based on experience, the foundation will be *three special facts*, which are in part based indirectly on observation, in part contained directly in Ampère's law, which is confirmed by all measurements.

The *first fact* is, that two current elements lying in a straight line which coincides with their direction, *repel* or *attract* each other, according to whether the electricity flows through them in the *same* or *opposite* way.

The *second fact* is, that two parallel current elements, which form right angles with a line connecting them, *attract* or *repel* each other, according to whether the electricity flows through them in the *same* or *opposite* way.

The *third fact* is, that a current element, which lies together with a wire element in a straight line coinciding with the directions of both elements, induces a *like-* or *opposite-directed* current in the wire element, according to whether the intensity of its own current *decreases* or *increases.*

These three facts are, of course, not directly given through experience, because the effect of one *element* on another can not be directly observed; yet they are so closely connected with directly observed facts, that they have almost the same validity as the latter. The first two facts were already comprehended under Ampère's law; the third was added by Faraday's discovery.

The three adduced facts are considered as *electrical,* viz., we consider the indicated forces as *actions of electrical masses on each other.* The *electrical law* of this interaction is still unknown, however; for, even if the first two facts are comprehended under Ampère's law, nevertheless, even apart from the third fact, which is not comprehended by it, Ampère's law is itself, in the strict sense, *no electrical law,* because it identifies *no electrical force,* which an electrical mass exerts on the other. Ampère's law merely provides a way to identify a force acting on the *ponderable mass of the conductor.* Ampère did not deal with the *electrical forces* which the *electrical fluids* flowing through the conductor exert on one another, though he repeatedly expressed the hope that it would be possible to explain the reciprocal effect of the *ponderable conductors* identified by his law, in terms of the interactions of the *electric fluids* contained in them.

If we now direct our attention to the *electrical fluids* in the two current elements themselves, we have in them like amounts of positive and negative electricity, which, in each element, are in motion in an opposing fashion. This simultaneous opposite motion of positive and negative electricity,[185] as we are accustomed to assume it in all parts of a linear conducting wire, admittedly can not exist in reality, yet can be viewed for our purposes as an *ideal* motion, which, in the cases we are considering, where it is simply a matter of actions *at a distance,* *represents* the actually occurring motions in relation to all the actions to be taken into account, and thereby has the advantage, of subjecting itself better to calculation. The actually occurring lateral motion through which the particles encountering each other in the conducting wire (which latter forms *no mathematical line*) *avoid each other,* must be considered as without influence on the actions *at a distance,* hence it seems permissible for our purpose, to adhere to the foregoing simple view of the matter (see Section 5.31).

We have, then, in the *two* current elements we are considering, *four interactions* of electrical masses to consider, *two repulsive,* between the two positive and between the two negative masses in the current elements, and *two attractive,* between the positive mass in the first and the negative mass in the second, and between the negative mass in the first and the positive mass in the second.

Every two *repulsive* forces would have to be *equal* to these two *attractive* forces, if the recognized laws of *electrostatics* had an *unconditional application to our case,* because the like, repulsive masses are equal to the unlike, attractive masses, and act on one another at the same distance. Whether those recognized *electrostatic* laws, however, find an *unconditional application* to our case, can not be decided a priori, because these laws chiefly refer

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[185][Note by AKTA:] According to the context of the discussion presented in this work, we can conclude that these opposite motions of the positive and negative electricities should be understood as the drift velocities of these electrified particles relative to the matter of the conductor. That is, Weber assumes that a positively electrified particle moves in one direction relative to the conductor, while a negatively electrified particle moves in the opposite direction relative to the conductor.

only to such electrical masses, which are situated in *equilibrium* and at *rest* with respect to one another, while our electrical masses are in motion with respect to one another. Consequently, only *experience* can decide, whether that electrostatic law permits such an *enlarged application* to our case as well.

The two first *facts* adduced above refer, of course, chiefly to forces, which act on the *ponderable current carriers*; we can, however, consider these forces as the *resultants* of those forces, which act on the *electrical masses* contained in the ponderable carrier. Strictly speaking, that way of considering these forces is, to be sure, only permissible, when these electrical masses are bound to their common ponderable carrier in such a way, that they cannot be put in motion without it, and because this is not the case in the galvanic circuit, but on the contrary, the electrical masses are also in motion when their carrier is at rest, Ampère, as is stated in the introduction on page 35,[186] particularly called attention to this circumstance, with the consideration that the force acting on the ponderable carrier could thereby be essentially modified. Although, however, the electrical masses are susceptible of being displaced in the direction of the conducting wire, they are in no way *freely moveable* in this direction; otherwise they would have to *persist* in the motion once it were transmitted to them in this direction, without a new external impetus (that is, without ongoing *electromotive force*), which is not the case. For no galvanic current persists *of itself*, even with a persistent closure of the circuit. Rather, its intensity at any moment corresponds only to the existing *electromotive force*, as determined by Ohm's law; thus it stops by itself, as soon as this force disappears. From this it follows, that not simply those forces, which act on the electrical masses in such directions (perpendicular to the conducting wire) that the masses can only be moved in tandem with the ponderable carrier, have to be transmitted to the latter, but that this very fact also holds true even of such forces, which act in the direction of the conducting wire and which move the electrical masses in the carrier, only with the difference, that the latter transmission requires an interval of time, although a very short one, which is not the case for the former. The *direct* action of the forces parallel to the conducting wire consists, to be sure, simply of a motion of the electrical masses in this direction; the effect of this motion is, however, a *resistance* in the ponderable carrier, by means of which, in an immeasurably short time, it is canceled once more. Through this *resistance*, during the time interval in which this motion is canceled, all forces, which had previously induced this motion, are *indirectly* transmitted to the ponderable bodies which exercise the resistance. Finally, since we are dealing with the effects of forces, which have the capacity to communicate a *measurable* velocity to the *ponderable* carrier itself, then on the other hand, those effects of forces, which only momentarily disturb the *imponderable* masses a little, can be disregarded with the same justification with which we disregard the *mass of the electricity* compared with the mass of its ponderable carrier. From this, however, it follows, that the force acting on the *current carrier* acts, as stated above, as the *resultant* of all forces acting on the *electrical masses* contained in the current carrier.

This presupposes, as shown by the *first two facts stated above*, that the *resultant* of those four interactions of the electrical masses contained in the two current elements under consideration, which, according to the *electrostatic* laws, ought to be zero, *departs* more from zero, the greater the *velocity*, with which the electrical masses flow through both current elements, that is, the greater the current intensities.

From this it follows, therefore, that the *electrostatic* laws have *no unconditional application* to electrical masses which are *in motion* with respect to one another, but on the contrary,

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[186][Note by LH and AKTA:] [Web46, p. 29 of Weber's *Werke*].

they merely provide for the forces, which these masses reciprocally exert upon each other, a *limiting value*, to which the *true value* of these forces approximates more closely, the slighter the reciprocal motions of the masses, and from which, on the contrary, the *true value* is more divergent, the greater the reciprocal motions. To the values, which the *electrostatic* laws give for the force exerted by *two electrical masses* upon one another, must thus be added a *complement dependent upon their reciprocal motion*, if this force is to be correctly determined, not simply for the case of mutual rest and equilibrium, but universally, including any arbitrary *motion* of the two masses with respect to one another. This *complement*, which would confer upon the electrostatic laws a more general applicability than they presently possess, will now be sought.

The *first fact* stated above further shows, not simply that the sum of the repulsive forces of like electrical masses in the current elements under consideration *diverges* from the sum of the attractive forces of unlike masses, but also shows, when the first sum is *greater* and when it is *smaller* than the latter, and all determinations resulting therefrom can be unified in the simple statement,

> *that the electrical masses, which have an opposite motion, act upon one another more weakly, than those which are moved in the same direction.*

For, 1) if the direction of the current is *the same* in the two elements, then *repulsion* occurs, consequently the *attractive force of the unlike masses* must be *weaker* than the repulsive forces of the like masses. In this case, however, it is the unlike masses, which are in *opposite motion*. If, however, 2) the direction of the current in the two elements is *opposite*, then *attraction* occurs; consequently the *repulsive forces of the like masses* must be *weaker* than the attractive forces of the unlike masses. In this case, however, it is the like masses, which are put into *opposite motion*. In both cases it is thus the masses *in opposite motion*, which act *more weakly* upon one another, confirming the statement above.

The *first fact*, to which the statement above was referred, further permits the following, more precise, determination to be added,

> *that two electrical masses (repulsive or attractive, according to whether they are like or unlike) act more weakly upon one another, the greater the square of their relative velocity.*

The *relative velocity* of two electrical masses can, if r denotes the distance between the two masses, be expressed as dr/dt, and is positive or negative, according to whether the two masses are withdrawing from or approaching one another; since, however, this difference between approach and withdrawal, or, in short, the difference of the sign for dr/dt, has no influence upon the magnitude of the force, it was necessary in the just-stated rule to introduce, instead of the relative velocity itself, its *square*.

If we denote by e and e' the positive electrical masses in both elements, and by u and u' their *absolute* velocities, which have a positive or negative value according to the direction of the current, then $-e$ and $-e'$ will be the negative masses, and $-u$ and $-u'$ their *absolute* velocities.[187] In the cases subsumed by the *first fact*, where all electrical masses are in motion

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[187][Note by AKTA:] According to the context of the discussion presented in this work, these absolute velocities should be understood as the drift velocities of the electrical masses relative to the matter of the conductors.

in one and the same straight line, the *relative* velocities, however, result from the *absolute* by means of simple subtraction, namely, for the *like* masses:

$$+e \quad \text{and} \quad +e' \quad \text{the relative velocity} \quad \frac{dr}{dt} = u - u' \, ,$$

$$-e \quad \text{and} \quad -e' \quad \text{the relative velocity} \quad \frac{dr}{dt} = -u + u' \, ;$$

for the unlike masses:

$$+e \quad \text{and} \quad -e' \quad \text{the relative velocity} \quad \frac{dr}{dt} = u + u' \, ,$$

$$-e \quad \text{and} \quad +e' \quad \text{the relative velocity} \quad \frac{dr}{dt} = -u - u' \, .$$

From this results, according to the foregoing principle of the interaction of *like* (two positive, as well as two negative) masses, a diminution dependent upon[188]

$$\frac{dr^2}{dt^2} = (u - u')^2 \, ,$$

in comparison with the case considered in electrostatics, of rest and equilibrium; for the interaction of *unlike* masses, on the contrary, a decrease dependent upon

$$\frac{dr^2}{dt^2} = (u + u')^2 \, .$$

The simplest form, which the law of this decrease can have, is that in which the value of the force for the case of rest and equilibrium is multiplied by the factor

$$\left(1 - a^2 \frac{dr^2}{dt^2}\right) \, ,$$

whereby the following expression would therefore serve for the *complete determination of the force*:

$$\frac{ee'}{r^2} \left(1 - a^2 \frac{dr^2}{dt^2}\right) \, ,$$

in which e and e' have positive or negative values, according to whether the electrical masses which they denote are part of the positive or negative fluids. a^2 is a constant.

For our case, when we try to make use of this simplest form, there result the following four interactions between the electrical masses in the two current elements:

$$1. \text{ between} \quad +e \quad \text{and} \quad +e' \quad \text{the force} \quad +\frac{ee'}{r^2} \left(1 - a^2 (u - u')^2\right) \, ,$$

$$2. \text{ between} \quad -e \quad \text{and} \quad -e' \quad \text{the force} \quad +\frac{ee'}{r^2} \left(1 - a^2 (u - u')^2\right) \, ,$$

$$3. \text{ between} \quad +e \quad \text{and} \quad -e' \quad \text{the force} \quad -\frac{ee'}{r^2} \left(1 - a^2 (u + u')^2\right) \, ,$$

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[188] [Note by LH and AKTA:] The notation $\frac{dr^2}{dt^2}$ should be understood as $\left(\frac{dr}{dt}\right)^2$.

4. between $\quad - e \quad$ and $\quad + e' \quad$ the force $\quad - \dfrac{ee'}{r^2} \left(1 - a^2 \left(u + u' \right)^2 \right)$.

The sum of the first two forces, that is, the sum of the *repulsions of like masses*, is thus

$$= +2\frac{ee'}{r^2} \left(1 - a^2 \left(u - u' \right)^2 \right) \; ;$$

the sum of the two latter forces, that is, the sum of the *attractions of unlike masses*, is

$$= -2\frac{ee'}{r^2} \left(1 - a^2 \left(u + u' \right)^2 \right) \; .$$

These two sums are thus, apart from their signs (distinguishing repulsion and attraction), distinguished according to their magnitude. Their algebraic sum, which yields *the resultants* of all four interactions, and consequently the force, which is transmitted from the electrical masses to the *current carrier* itself, and on which Ampère's law is based, is accordingly

$$= +8\frac{ee'}{r^2} a^2 \cdot uu' \; ,$$

i.e., it follows that this force, in complete agreement with Ampère's law, is directly proportional to the current intensity in both current elements, and inversely proportional to the square of the distance between the two current elements.

We further observe, that the foregoing expression is *positive*, and consequently denotes a *repulsion of the current-elements*, if u and u' *both* have either a positive or negative value, i.e, if the electricity flows through both current elements in the same sense; and that if *only one of the two is positive, the other negative*, the foregoing expression becomes *negative*, which denotes an *attraction of current-elements*, if the electricity is flowing through them *in the opposite sense*. All these results precisely correspond to the *first fact* stated above.

If we now proceed to the *second fact stated above*, it is clear that the supplement to the electrostatic law just provided will no longer suffice here, because for all cases included under this second fact, it yields the value of the relative velocity of the electrical masses

$$\frac{dr}{dt} = 0 \; .$$

That is to say, if we follow two electrical particles in their paths, the result is that their relative distance decreases up to the moment in question, and from then on increases again, and therefore, at the moment in question itself, neither increase nor decrease in the distance takes place; consequently, for all these cases, the electrostatic law itself, would be brought into application in order to determine the four interactions of the electric masses in both current elements, without applying a supplement to the law, according to which the two current elements ought to have no effect at all upon one another, which is not the case.

It is easily proven, however, that for this *second* class of cases, where the value of the relative velocity dr/dt disappears, the value of the *relative acceleration d^2r/dt^2* stands out all the more significantly, while for the *first* class, where the latter value d^2r/dt^2 disappears, the first dr/dt stood out all the more significantly.

Thus we assume, that the magnitude of the interaction of electrical masses in motion, as determined by the electrostatic law, requires a supplement, which depends, however, not simply on the *square of the relative velocity* of both masses $= dr^2/dt^2$, but also on their *relative acceleration* $= d^2r/dt^2$; the simplest form, which the general law of interaction of

two electrical masses can have, is that in which the value of the force for the case of rest and equilibrium is multiplied by the factor

$$\left(1 - a^2 \frac{dr^2}{dt^2} + b \frac{d^2r}{dt^2}\right)$$

and in which, therefore, the following expression would serve for the complete determination of the force:[189]

$$\frac{ee'}{r^2}\left(1 - a^2 \frac{dr^2}{dt^2} + b \frac{d^2r}{dt^2}\right) ,$$

in which e and e' have positive and negative values, accordingly as the electrical masses which they denote, are part of the positive or negative electrical fluid. a^2 is the same constant as before; b is another magnitude independent of velocity and acceleration, whose value and sign remain to be more closely determined.

If, as before, e and e' now denote the positive electrical masses in both current elements, u and u' their *absolute* velocities, $-e$ and $-e'$, the negative masses, and $-u$ and $-u'$ their absolute velocities, and R denotes the distance between the current elements, r the distance of the two *positive* electrical masses, then for the first moment $r = R$, but because the electrical masses are in motion, r soon changes, while R remains unchanged, and after the time-interval t has occurred, the following equation is yielded for determining the value of r, calculated from that moment on:

$$r^2 = R^2 + (u - u')^2 t^2 ,$$

consequently, because R, u and u' are constant,

$$r dr = (u - u')^2 t dt$$

and

$$r d^2r + dr^2 = (u - u')^2 dt^2 ,$$

which yields the values of the *relative velocity* and *relative acceleration* at the end of time-interval t, namely:

$$\frac{dr}{dt} = \frac{(u - u')^2}{r} t$$

$$\frac{d^2r}{dt^2} = \frac{(u - u')^2}{r}\left(1 - \frac{(u - u')^2}{r^2} t^2\right) .$$

If we apply these general determinations to the considered moment, for which $t = 0$, we will obtain the values for the *relative velocity and acceleration of both positive masses* to be introduced into our expression:

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[189][Note by AKTA:] The following equation should be understood as:

$$\frac{ee'}{r^2}\left[1 - a^2 \left(\frac{dr}{dt}\right)^2 + b \frac{d^2r}{dt^2}\right] .$$

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$$\frac{dr}{dt} = 0$$

$$\frac{d^2r}{dt^2} = \frac{(u - u')^2}{r} \ ,$$

consequently, for the first of the four interactions we obtain:

1. between $+e$ and $+e'$ the force $+\dfrac{ee'}{r^2}\left(1 + \dfrac{b}{r}(u - u')^2\right)$.

It is self-evident, that the remaining interactions can be derived from this first one, through substitution of the corresponding masses and velocities; then we obtain

2. between $-e$ and $-e'$ the force $+\dfrac{ee'}{r^2}\left(1 + \dfrac{b}{r}(u - u')^2\right)$.

3. between $+e$ and $-e'$ the force $-\dfrac{ee'}{r^2}\left(1 + \dfrac{b}{r}(u + u')^2\right)$.

4. between $-e$ and $+e'$ the force $-\dfrac{ee'}{r^2}\left(1 + \dfrac{b}{r}(u + u')^2\right)$.

The sum of the first two forces, that is, the sum of the *repulsions of like masses*, is thus

$$= +2\frac{ee'}{r^2}\left(1 + \frac{b}{r}(u - u')^2\right) \ .$$

The sum of the last two forces, that is, the sum of the *attractions of unlike masses*, is, however,

$$= -2\frac{ee'}{r^2}\left(1 + \frac{b}{r}(u + u')^2\right) \ .$$

These two sums are, therefore, apart from their signs (distinguishing repulsion and attraction), *distinguished* by their magnitude. Their algebraic sum, which yields *the resultant* of all four forces, consequently the force which is transmitted from the electrical masses to the *current carrier* itself, and on which Ampère's law is based, is accordingly

$$= -8\frac{ee'}{r^2} \cdot \frac{b}{r} \cdot uu' \ ,$$

i.e., this force accordingly emerges in complete agreement with Ampère's law, directly proportional to the current intensity in both current elements, and inversely proportional to the square of the distance between the two current elements.

We further observe, that if b is positive, the above expression would be *negative*, and consequently would denote an *attraction of current elements*, if u and u' both have either a positive or a negative value, i.e., if electricity flows through both current elements in *the same* way; if, however, only one of the two is positive, the other negative, then the above expression will be *positive*, which denotes a *repulsion* of the current elements, if the electricity flows through them in an *opposite* way. All these results precisely correspond to the *second fact* stated above.

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If, finally, we return to Ampère's formula itself, which includes both facts as special cases, according to which the repulsion of two current elements is the following:

$$\frac{ii'}{r^2}\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right)dsds' \; ,$$

wherein the letters have the significance given on page[190] 74, then, for the cases included under the *first fact*,

$$\varepsilon = 0° \quad \text{or} \quad = 180° \; ,$$

according to whether ϑ and ϑ' *both*

$$= 0° \quad \text{or} \quad = 180° \; ,$$

or only *one* of the two
$$= 0°, \quad \text{the } other = 180°.$$

Consequently, the sought-for value for the force in the cases included under the *first fact* is, according to Ampère's law

$$= \mp\frac{1}{2}\cdot\frac{ii'}{r^2}dsds' \; .$$

For the cases included under the *second fact*,
$$\varepsilon = 0° \quad \text{or} \quad 180°,$$

according to whether ϑ and ϑ' *both*
$$= 90° \quad \text{or} \quad = 270°,$$

or only *one* of the two
$$= 90°, \quad \text{the } other = 270°.$$

Consequently, the sought-for value for the force in the cases included under the *second fact* is, according to Ampère's law

$$= \pm\frac{ii'}{r^2}dsds' \; .$$

According to Ampère's fundamental law, we also obtain (apart from signs) a value for the latter case double that of the first.

This also results from our own determinations, if we make

$$a^2 = \frac{1}{2}\frac{b}{r}$$

whereby the value and the sign of b are more closely determined, namely:

$$b = 2ra^2 \; .$$

If we substitute this value of b in our general expression for the interaction of two electrical masses, the resulting *repulsive force* is

$$= \frac{ee'}{r^2}\left(1 - a^2\frac{dr^2}{dt^2} + 2a^2\cdot r\frac{d^2r}{dt^2}\right) \; .$$

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[190][Note by LH and AKTA:] [Web46, p. 70 of Weber's *Werke*].

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The *third fact stated above* is ultimately based, not, like the two previous ones, on forces, which merely act on the *current carrier*, but rather on forces which act on the *electrical masses* themselves and move them in their carrier, seeking to separate unlike masses; that is, on *electromotive* forces, which are exerted by electrical masses in motion in a galvanic conductor on electricity at rest. These forces, however, are not only *not determined* by the *electrostatic* law, but also *not determined* by Ampère's *electrodynamic* law, because the *latter* relates merely to the forces transmitted to the current carrier, and the *former*, were it to be applicable, would yield the value of the electromotive force = 0. Thus these forces form an essentially *new class*, with which Faraday's *discovery* has first acquainted us.

If we consider once more simply the *electrical masses* in the current element as well as in the element without current, we again have in each one, equal masses of positive and negative electricity; specifically, at any time in the current element these two masses are in motion with equally great velocity in opposed directions, and these velocities increase or decrease simultaneously by equal amounts; in the element without current, on the other hand, both masses are still at rest and in equilibrium. Further, among these four masses, four interactions are now to be distinguished, namely, two repulsive and two attractive, the former between the *like* masses, the latter between the *unlike*.

Now, from the *fact*, that a current *is produced* in the element, in which previously there was no current, we must conclude, that *another force*, than the one acting on the *negative* mass, must be acting on the *positive* electrical mass in this element, in the direction of the latter, because those masses can only receive that *opposite* motion through such a *difference* in the forces acting upon them, of which motion the current which manifests itself essentially consists. We thus express the fact initially in this way,

> *that the sum of the two forces, which are exerted by the positive and negative electrical masses in the current element on the positive mass at rest in the element without current, in the direction of the latter, is different from the sum of those two forces, which those masses exert in the cited current element on the negative mass at rest in the element without current, in the direction of the latter; that, however, the difference of the two sums, that is, the electromotive force itself, is dependent on the change in velocity of the two electrical masses in the given current element, and increase or decrease and disappear with this change.*

Thus we are led by this *third fact*, as well, to add to the electrical forces determined by the *electrostatic* law, *a supplement contingent upon their motion*, and the question is merely, whether this justifies exactly the same supplement, as that which was established on the basis of the first two facts. This *third fact* therefore yields a *criterion* for testing the results already obtained, and is especially suited to their rejection or their firmer substantiation.

If we now denote, as above, e and e' the positive electrical masses in both wire elements, u and 0 their *absolute* velocities, and R the distance between the wire elements, r the distance between the two positive electrical masses: then for the first moment of time, $r = R$, but because mass e distances itself from, or approaches, the mass at rest e' with variable velocity u, r soon changes, while R remains unchanged, and we have for the determination of the value of r, after time-interval t has occurred, and calculated from that moment forward,

$$r = R \pm \int_0^t u\,dt \; ,$$

where the *upper* sign is in effect, if mass e lies on the positive side of mass e', and consequently is still further distanced from it with a positive velocity; conversely, if mass e lies on the negative side of mass e', and consequently approaches it with a positive velocity, the lower sign is in effect.

By means of differentiation, we obtain:

$$dr = \pm u\,dt$$

$$d^2r = \pm du\,dt \ .$$

According to this, the values of *relative velocity* and *relative acceleration* of both masses at the end of time-interval t are thus:

$$\frac{dr}{dt} = \pm u$$

$$\frac{d^2r}{dt^2} = \pm\frac{du}{dt} \ ;$$

in which u and du are functions of t. If we now apply these general determinations to the considered moment under consideration, and denote the values which u and du assume if $t = 0$, as u_0 and du_0, then, according to the general law of interaction of two electrical masses, to which the *two first facts* led, we obtain as the first of four interactions:

1. between $+e$ and $+e'$ the force $+\frac{ee'}{r^2}\left(1 - a^2 u_0^2 \pm 2a^2 r\frac{du_0}{dt}\right)$.

It also becomes clear, that the remaining interactions can be derived from this first one, through substitution of the corresponding masses, velocities, and accelerations; we then obtain:

2. between $-e$ and $+e'$ the force $-\frac{ee'}{r^2}\left(1 - a^2 u_0^2 \mp 2a^2 r\frac{du_0}{dt}\right)$.
3. between $+e$ and $-e'$ the force $-\frac{ee'}{r^2}\left(1 - a^2 u_0^2 \pm 2a^2 r\frac{du_0}{dt}\right)$.
4. between $-e$ and $-e'$ the force $+\frac{ee'}{r^2}\left(1 - a^2 u_0^2 \mp 2a^2 r\frac{du_0}{dt}\right)$.

The sum of the two first forces, that is, the sum of the forces acting on the *positive mass* $+e'$ *in the element without current*, is therefore

$$= \pm 4\frac{ee'}{r}a^2\frac{du_0}{dt} \ .$$

The sum of the two latter forces, that is, the sum of the forces acting on the *negative mass* $-e'$ *in the element without current*, is, however,

$$= \mp 4\frac{ee'}{r}a^2\frac{du_0}{dt} \ .$$

These two sums are differentiated by their *opposing signs* (distinguishing repulsion and attraction). Their *difference* yields the *electromotive force*, which seeks to separate the positive and negative masses in the element without current,

$$= \pm 8\frac{ee'}{r}a^2\frac{du_0}{dt} \ ,$$

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i.e., the *electromotive force* is directly proportional to the self-initiated change in the drift velocity[191] at the moment under consideration, and inversely proportional to the distance of the current element from the element without current.

Further, as for the double signs in our expression for the *electromotive force*, they can be eliminated, if we base them on the distance r and thus impute to it positive and negative values, calculating r from the locus of the mass at rest e' as the initial point, and specifically as a positive magnitude, when the mass e calculated from this initial point lies on the positive side (toward which the positive velocities are directed), and as a negative magnitude, when the mass e lies on the negative side from this initial point. If, for example, in Figure 15, A denotes the locus of the mass at rest e', BAC the given line of direction, and the side on which C lies is established as the positive side, then r is positive, if mass e is at point C, negative, when mass e is at point B.

Fig. 15.

If, therefore, two like current elements are located at B and C, through which electricity is flowing *in the same sense*, and the intensity of their current increases or decreases by the same amount, then these two current elements will exert opposite electrical forces on the electrical masses at rest at A, such that that mass, which is repulsed from C, is attracted by B, and vice versa; the force which seeks to separate the positive and negative masses at A, is thus *doubled* by means of the combined operation of the two current elements at B and C.

Finally, if r is positive, if, e.g., the current element is located at C, and if, further, u and du both have either negative or positive values, i.e., if the absolute drift velocity at C increases, regardless of its direction, then the foregoing expression has a positive or negative value, according to whether u has a positive or negative value, i.e., therefore, under *increasing* current intensity, an *electromotive force* acts from C repulsively or attractively on the positive electrical mass at A, according to whether the current at C itself is directed forwards or backwards, and thus excites at A a current *opposite* to the one present at C, fully corresponding to the determinations contained in the *third fact* stated above.

From this it follows, that this *third fact* confirms the result derived from the first two, in that *the same complement* of the electrostatic law into a general law, which served to explain the first two facts, also suffices to explain the third.

5.20   Comparison with Other Fundamental Laws

In the foregoing Section, following the guideline of experience, we have sought to add to the electrostatic formulation for the repulsive or attractive force, with which two like or unlike electrical masses act upon one another at a distance, in such a way, that the formulation is applicable, not simply when both masses are at rest with respect to one another, but also when they are in motion with respect to one another. We have tested and confirmed this

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[191][Note by AKTA:] In German: *Stromgeschwindigkeit*. This expression can also be translated as *current velocity*. Weber is referring here to the velocity of each electrified particle relative to the matter of the conductor. See also footnotes 41 and 42 on page 24.

expansion on particular facts, and in the following Section, will present this test with greater generality.

Assuming the correctness of the results which we achieved, a case would arise here, in which the force, with which two masses act upon one another, would depend, not simply upon *the magnitude of the masses and their distance* from one another, but also on their *relative velocity and relative acceleration.* The calculation of these forces will thus in many cases come up against greater mathematical difficulties, than the calculation of such forces which simply depend upon the magnitude of the masses and their distances. It should also be expected, if this dependency of the electrical forces, not simply on the magnitude of the electrical masses and their distances, but also on their relative velocities and accelerations, were firmly established, that this very dependency, even if to a lesser extent, would exist in other forces, according to more exact investigation.

Thereby a completely new element would be introduced into the dependency of forces on given physical relationships, and the domain of forces, whose determination would require taking this new element into account, would form a specific class, requiring a special investigation.

As, however, it must also appear highly desirable, for the purpose of simplifying and facilitating our investigations, that the domain of those forces which depend simply on the magnitude of the masses and their distances, be extended as widely as possible, then, *only experience* can decide whether other forces, which are also dependent on the mutual velocities and accelerations of the masses, must be assumed to be present, or not. This question cannot be decided a priori, because formally, the assumption of such forces contains neither a contradiction, nor anything unclear or indeterminate.

The law of the dependence of forces upon given physical relationships is called the *fundamental law of physics*, and, in accordance with the goals of physics, it is not supposed to provide an *explanation* of the forces based on their true causes, but only a clearly demonstrated and useful general method for *quantitative* determination of forces, according to the fundamental measures[192] established in physics for space and time. Hence, from the standpoint of physics, one can not take offense at the fact that a force is made into a function *of a relationship dependent on time*, any more than one can take offense at the fact that it is made into a function of *distance*, because a relationship dependent on time is just as measurable a magnitude as a distance; therefore, in virtue of their nature, both are suited to more rigorous *quantitative* determination, even if it is not appropriate to seek in them the *inherent reason* for a force.

At most, accordingly, against the introduction of a time-dependent relationship in the general expression for a force, the *analogy with another fundamental law of physics*, e.g. with the law of gravitation, may be asserted, where this time-dependent relationship does not occur. Yet such an analogy can only be viewed as binding, when it offers ways and means to achieve the goal; where the analogy with known cases does not suffice, in the nature of the case new paths must be sought.

If, therefore, the introduction of such *time-dependent relationships* in the general expression for a force cannot be rejected in general, then all the less so, if those relationships are an essential part of the complete determination of the *existing condition* of masses acting upon one another, since in any case the force, which two masses exert upon one another, since it *does not always* remain the same, must be thought of subject to the *condition existing at the time.* Complete determination of the present condition of two masses, however, essentially

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[192][Note by AKTA:] In German: *Grundmaassen.*

involves, in addition to the determination of their *relative position* by means of their mutual distance r, the determination of their *relative movement* by means of their relative velocity dr/dt. For, according to the principle of inertia, one has no choice but to calculate the velocity of a body essentially in its present condition, because the reason for the inertia lies, according to that principle, in the body itself, and consequently the persistence in *different* motion must correspond to *different* internal conditions of the body, which, themselves inaccessible to our observation, can only be distinguished by means of their effects emerging over time.

5.21 Deduction from Ampère's Law for the Interaction of Electric Currents — Transformation of Ampère's Law

What was proven in the foregoing Sections for a few special facts, is now to be proven more generally and more precisely for all facts contained under Ampère's law. Ampère's law determines the *total effect* which one current element exerts on the other, depending on the *distance* of the two elements from each other, on their two *current intensities*, and on the three *angles* which the directions of the current elements make with each other and with the straight lines connecting them. Now, if it is to be possible to reduce this *total effect*, thus determined, to *elementary electrical forces*, then *first* Ampère's formula must be able to be broken down into several parts, which correspond to the effects of *each pair of electrical masses* in both current elements, in particular to the effect of the positive mass of the one element on the positive mass of the other, of the negative mass of the one element on the negative mass of the other, of the positive mass of the former on the negative of the latter, and finally of the negative mass of the former on the positive of the latter. *Secondly*, each of these parts, as *elementary electrical force*, must be wholly dependent on such magnitudes, which exclusively appertain to the nature and the mutual relations of the two electrical masses, to which the part refers, and be completely determined thereby, independently of other conditions. *Thirdly* and finally, all these *elementary electrical forces* would have to be susceptible of being brought under a *general law*. It is, however, not necessary, to make any sort of hypothesis in advance about this general law; rather, Ampère's law, under such a transformation, would have to lead directly to the statement of this general law and decide on the admissibility or inadmissibility of such a hypothesis posed in advance. At the outset, the following question is to be answered:

> whether Ampère's formula permits a transformation, such that the current intensities contained therein, i and i', and the angles ε, ϑ, and ϑ', which the two current elements form with each other and with the straight line connecting the two elements, vanish from the formula, and instead of these, only such new magnitudes are introduced, which fully and exclusively refer to the electrical masses themselves and their mutual relations.

This transformation is now actually to be carried out here, and then it will be examined whether the expression for the electrodynamic force, transformed in this way, permits the requisite decomposition into four parts, corresponding to four partial effects, of which the total effect would be composed.

Ampère's formula for the repulsive force of two current elements is as follows:

$$-\frac{ii'}{r^2}\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right)\cdot dsds' \ ,$$

in which the letters have the signification given in Section 5.8, page[193] 74.
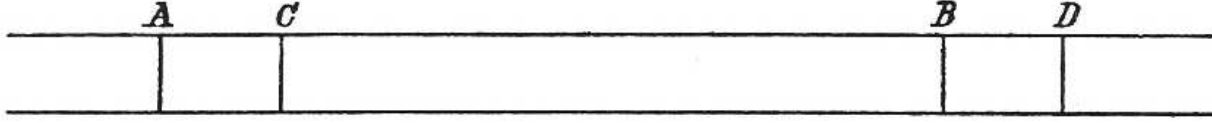
Fig. 16.

In Figure 16, AB is a segment of the one conducting wire of length $= 1$, and the quantity of the uniformly distributed positive electricity in it is denoted by e, so that eds is the mass of positive electricity which the current element contains, whose length $= ds$.

With the constant velocity u, which all positive electrical components possess in the conducting wire AB when a constant current passes through, in one second the one farthest forward traverses the path BD, the one farthest back the path AC, and the electrical mass e, which at the beginning of the second was uniformly distributed in the segment $AB = 1$, is located at the end of the second in segment $CD = 1$. Hence, during one second, all the electricity which, at the end of the second, is contained on the other side from B in the segment of the conducting wire $BD = u$, has passed through the cross-section of the conducting wire at B. This electricity, in conformity with the definition of current intensity given at the beginning of Section 5.2 (according to which it is proportional to the amount of electricity passing through a cross-section of the circuit in one second), can now be set $= i/a$, where a denotes a constant. There then results

$$\frac{i}{a} : e = u : 1 \ ,$$

consequently $i = aeu$. The value of a is different from that in Section 5.19.

It likewise results that, if u' denotes the drift velocity[194] of the electricity in another conducting wire,

$$i' = ae'u' \ .$$

If one substitutes these values in Ampère's formula, the formula will be

$$-\frac{eds\cdot e'ds'}{r^2}a^2uu'\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right) \ ,$$

where, therefore, the first factor $eds\cdot e'ds'/r^2$ denotes the product of two *electrical masses* acting on one another in the two current elements, divided by the square of their *distance*.

Further, Ampère has already shown on page 207 of his Treatise,[195] that it would be the case that

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[193][Note by LH and AKTA:] [Web46, p. 70 of Weber's *Werke*].

[194][Note by AKTA:] In German: *Strömungsgeschwindigkeit*. See also footnote 191 on page 141.

[195][Note by AKTA:] See [Amp23, p. 207], [Amp26, p. 35] and [AC15, p. 360].

$$\cos \vartheta = \frac{dr}{ds}, \qquad \cos \vartheta' = -\frac{dr}{ds'}$$

and

$$\cos \varepsilon = -r \frac{d^2 r}{ds \, ds'} - \frac{dr}{ds} \frac{dr}{ds'}.$$

If one substitutes these values, the Ampère formula takes the following form:

$$-\frac{eds \cdot e'ds'}{r^2} \cdot a^2 uu' \left(\frac{1}{2} \frac{dr}{ds} \frac{dr}{ds'} - r \frac{d^2 r}{ds \, ds'} \right).$$
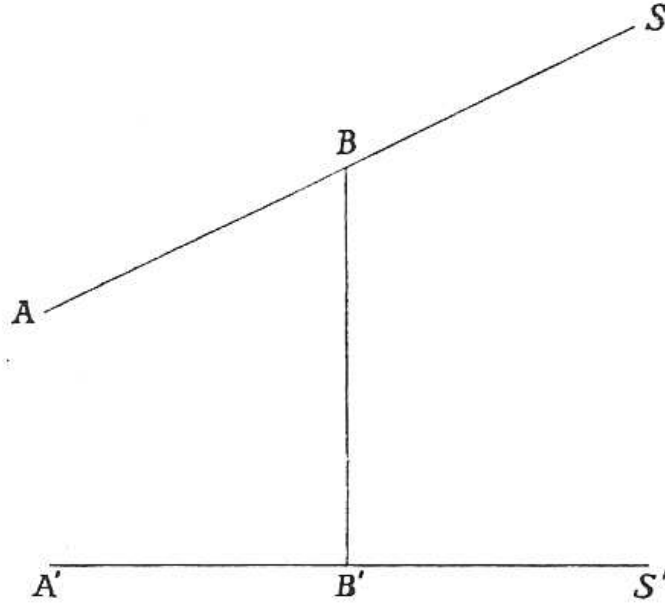
Fig.17.

Let the element ds of the conducting wire ABS be located at B in Figure 17; the initial point of the conducting wire would be put at A, consequently $AB = s$. Let the element ds' of the conducting wire $A'B'S'$ lie at B, A' be the initial point of this wire, $A'B' = s'$ and $BB' = r$. The last magnitude r, if the conducting wires ABS and $A'B'S'$ are given, is a function of s and s', and the following expressions obtain for dr and $d^2 r$:

$$dr = \frac{dr}{ds} ds + \frac{dr}{ds'} ds'$$

$$d^2 r = \frac{d^2 r}{ds^2} ds^2 + 2 \frac{d^2 r}{ds \, ds'} ds \, ds' + \frac{d^2 r}{ds'^2} ds'^2.$$

If s and s' now denote the lengths of the conducting wires from their initial points to the current elements themselves which are under consideration, then s and s' have constant values for two given current elements. However, s and s' can also signify the length of the conducting wires from their initial points to the *electrical masses* just now existing in

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the current elements under consideration, but flowing through them further. In this last signification, s and s' are variable with the time t, and then one has

$$\frac{dr}{dt} = \frac{dr}{ds} \cdot \frac{ds}{dt} + \frac{dr}{ds'} \cdot \frac{ds'}{dt} \ ,$$

$$\frac{d^2r}{dt^2} = \frac{d^2r}{ds^2} \cdot \frac{ds^2}{dt^2} + 2\frac{d^2r}{dsds'} \cdot \frac{dsds'}{dt^2} + \frac{d^2r}{ds'^2} \cdot \frac{ds'^2}{dt^2} \ .$$

Here ds/dt is the path element of the electrical mass divided by the time element in which it is passed through, i.e., the velocity of the electrical mass, and therefore $ds/dt = u$, when the *positive* mass is considered first. Likewise, then $ds'/dt = u'$. If these values are substituted, then

$$\frac{dr}{dt} = u\frac{dr}{ds} + u'\frac{dr}{ds'} \ ,$$

$$\frac{d^2r}{dt^2} = u^2\frac{d^2r}{ds^2} + 2uu'\frac{d^2r}{dsds'} + u'^2\frac{d^2r}{ds'^2} \ .$$

From the latter equation, and from the one derived from the first

$$\frac{dr^2}{dt^2} = u^2\frac{dr^2}{ds^2} + 2uu'\frac{dr}{ds}\frac{dr}{ds'} + u'^2\frac{d^2r}{ds'^2}$$

the following values obtain for $2uu'\frac{d^2r}{dsds'}$ and $2uu'\frac{drdr}{dsds'}$:

$$2uu'\frac{d^2r}{dsds'} = \frac{d^2r}{dt^2} - u^2\frac{d^2r}{ds^2} - u'^2\frac{d^2r}{ds'^2} \ ,$$

$$2uu'\frac{drdr}{dsds'} = \frac{dr^2}{dt^2} - u^2\frac{dr^2}{ds^2} - u'^2\frac{dr^2}{ds'^2} \ ,$$

from which it follows:

$$uu'\left(\frac{1}{2}\frac{drdr}{dsds'} - r\frac{d^2r}{dsds'}\right) = \left(\frac{1}{4}\frac{dr^2}{dt^2} - \frac{1}{2}r\frac{d^2r}{dt^2}\right) - \left(\frac{1}{4}\frac{dr^2}{ds^2} - \frac{1}{2}r\frac{d^2r}{ds^2}\right)u^2$$

$$- \left(\frac{1}{4}\frac{dr^2}{ds'^2} - \frac{1}{2}r\frac{d^2r}{ds'^2}\right)u'^2 \ .$$

If these values are substituted, then Ampère's formula takes the following form:

$$-\frac{eds \cdot e'ds'}{r^2}a^2\left\{\left(\frac{1}{4}\frac{dr^2}{dt^2} - \frac{1}{2}r\frac{d^2r}{dt^2}\right) - \left(\frac{1}{4}\frac{dr^2}{ds^2} - \frac{1}{2}r\frac{d^2r}{ds^2}\right)u^2\right.$$

$$\left. - \left(\frac{1}{4}\frac{dr^2}{ds'^2} - \frac{1}{2}r\frac{d^2r}{ds'^2}\right)u'^2\right\} \ .$$

In this transformation of Ampère's formula, there are first introduced merely the *positive electrical masses*, which move in their trajectories with the velocities u and u'. It is clear that one can also introduce the *negative* electrical masses instead of the positive ones. It then results, if this occurs for both current elements alike, that both of the masses introduced are therefore again *of the same kind*, but their velocities, in accordance with the determinations

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given for galvanic currents on page[196] 133, both maintain the opposite values, namely $-u$ and $-u'$, in turn in the same expression. Then if r_1, ζ and ζ' denote for the *negative* masses the same thing that r, s, and s' denote for the *positive*, Ampère's formula would be obtained at first in the following form:

$$-\frac{eds \cdot e'ds'}{r_1^2} \cdot a^2 \left\{ \left(\frac{1}{4}\frac{dr_1^2}{dt^2} - \frac{1}{2}r_1\frac{d^2r_1}{dt^2} \right) - \left(\frac{1}{4}\frac{dr_1^2}{d\zeta^2} - \frac{1}{2}r_1\frac{d^2r_1}{d\zeta^2} \right) u^2 \right.$$
$$\left. - \left(\frac{1}{4}\frac{dr_1^2}{d\zeta'^2} - \frac{1}{2}r_1\frac{d^2r_1'}{d\zeta'^2} \right) u'^2 \right\} .$$

For the moment under consideration, where those *positive* masses (to which r, s, and s' refer) and these *negative* masses (to which r_1, ζ, and ζ' refer) go through the same current elements, however,

$$r = r_1 , \qquad s = \zeta , \qquad s' = \zeta' .$$

Further, it is also the case that

$$\frac{dr_1}{d\zeta} = \frac{dr}{ds} , \qquad \frac{d^2r_1}{d\zeta^2} = \frac{d^2r}{ds^2} , \qquad \frac{dr_1}{d\zeta'} = \frac{dr}{ds'} , \qquad \frac{d^2r_1}{d\zeta'^2} = \frac{d^2r}{ds'^2} ,$$

because all these values are simply dependent upon the position of the current elements through which those *positive* and these *negative* masses flow, but independent of the motion of the masses in these current elements. Finally,

$$\frac{d\zeta}{dt} = -u = -\frac{ds}{dt} , \qquad \frac{d\zeta'}{dt} = -u' = -\frac{ds'}{dt} ,$$

consequently,

$$\frac{dr_1}{dt} = \frac{dr_1}{d\zeta} \cdot \frac{d\zeta}{dt} + \frac{dr_1}{d\zeta'} \cdot \frac{d\zeta'}{dt} = - \left(\frac{dr}{ds} \cdot \frac{ds}{dt} + \frac{dr}{ds'} \cdot \frac{ds'}{dt} \right) = -\frac{dr}{dt} ,$$

which yields[197]

$$\frac{dr_1^2}{dt^2} = \frac{dr^2}{dt^2}$$

Likewise one finds

$$\frac{d^2r_1}{dt^2} = \frac{d^2r}{dt^2} .$$

By substitution of these values, the latter expression changes into the former.

It is a different case when a *positive* and a *negative* mass are introduced, viz., with *unlike kinds* of masses. If one keeps the *positive* mass in the first current element, the *negative* in

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[196][Note by LH and AKTA:] [Web46, p. 139 of Weber's *Werke*].

[197][Note by AKTA:] Due to a misprint, the next equation appeared in the original text as:

$$\frac{dr_1}{dt^2} = \frac{dr^2}{dt^2} .$$

The correct equation should be $\frac{dr_1^2}{dt^2} = \frac{dr^2}{dt^2}$, as we corrected it, that is, $(dr_1/dt)^2 = (dr/dt)^2$.

the second, and denotes their distance with r_2, then Ampère's formula is obtained in the following form:

$$+\frac{eds \cdot e'ds'}{r_2^2} \cdot a^2 \left\{ \left(\frac{1}{4}\frac{dr_2^2}{dt^2} - \frac{1}{2}r_2\frac{d^2r_2}{dt^2}\right) - \left(\frac{1}{4}\frac{dr_2^2}{ds^2} - \frac{1}{2}r_2\frac{d^2r_2}{ds^2}\right)u^2 \right.$$
$$\left. - \left(\frac{1}{4}\frac{dr_2^2}{d\zeta'^2} - \frac{1}{2}r_2\frac{d^2r_2'}{d\zeta'^2}\right)u'^2 \right\}.$$

On the other hand, if one keeps the *negative* mass in the first current element, the *positive* in the second, and denotes their distance with r_3, then Ampère's formula is obtained in the following form:

$$+\frac{eds \cdot e'ds'}{r_3^2} \cdot a^2 \left\{ \left(\frac{1}{4}\frac{dr_3^2}{dt^2} - \frac{1}{2}r_3\frac{d^2r_3}{dt^2}\right) - \left(\frac{1}{4}\frac{dr_3^2}{d\zeta^2} - \frac{1}{2}r_3\frac{d^2r_3}{d\zeta^2}\right)u^2 \right.$$
$$\left. - \left(\frac{1}{4}\frac{dr_3^2}{ds'^2} - \frac{1}{2}r_3\frac{d^2r_3'}{ds'^2}\right)u'^2 \right\}.$$

Here too, if it is now the case that $r_2 = r_3 = r$, then

$$\frac{dr_2}{ds} = \frac{dr_3}{d\zeta} = \frac{dr}{ds}, \qquad \frac{d^2r_2}{ds^2} = \frac{d^2r_3}{d\zeta^2} = \frac{d^2r}{ds^2},$$

$$\frac{dr_2}{d\zeta'} = \frac{dr_3}{ds'} = \frac{dr}{ds'}, \qquad \frac{d^2r_2}{ds'^2} = \frac{d^2r_3}{ds'^2} = \frac{d^2r}{ds'^2};$$

however, it results that

$$\frac{dr_2}{dt} = \frac{dr_2}{ds}\cdot\frac{ds}{dt} + \frac{dr_2}{d\zeta'}\cdot\frac{d\zeta'}{dt} = +\frac{dr}{ds}\cdot\frac{ds}{dt} - \frac{dr}{ds'}\cdot\frac{ds'}{dt},$$

$$\frac{dr_3}{dt} = \frac{dr_3}{d\zeta}\cdot\frac{d\zeta}{dt} + \frac{dr_3}{ds'}\cdot\frac{ds'}{dt} = -\frac{dr}{ds}\cdot\frac{ds}{dt} + \frac{dr}{ds'}\cdot\frac{ds'}{dt} = -\frac{dr_2}{dt},$$

consequently $dr_2^2/dt^2 = dr_3^2/dt^2$ is different from dr^2/dt^2. Likewise, one finds $d^2r_2/dt^2 = d^2r_3/dt^2$ to be different from d^2r/dt^2. By substituting these values, in both cases where one introduces masses of a *different kind*, one obtains the same expression for Ampère's formula, namely:

$$+eds \cdot e'ds' \cdot a^2 \left\{ \frac{1}{r_2^2}\left(\frac{1}{4}\frac{dr_2^2}{dt^2} - \frac{1}{2}r_2\frac{d^2r_2}{dt^2}\right) - \left(\frac{1}{4}\frac{dr^2}{ds^2} - \frac{1}{2}r\frac{d^2r}{ds^2}\right)\frac{u^2}{r^2} \right.$$
$$\left. - \left(\frac{1}{4}\frac{dr^2}{ds'^2} - \frac{1}{2}r\frac{dr^2}{ds'^2}\right)\frac{u'^4}{r^2} \right\}.$$

Now, since both expressions, the earlier one, which was obtained by introducing masses *of the same kind*, as well as the later one, obtained by introducing masses *of a different kind*, represent the force with which two current elements act upon each other, are both identical with Ampère's formula, yet a third will be derived from them for the same force, likewise identical with Ampère's formula, if one takes their half-sums, that is,

$$-\frac{a^2}{2}\frac{eds \cdot e'ds'}{r^2}\left(\frac{1}{4}\frac{dr^2}{dt^2} - \frac{1}{2}r\frac{d^2r}{dt^2}\right) + \frac{a^2}{2}\frac{eds \cdot e'ds'}{r_2^2}\left(\frac{1}{4}\frac{dr_2^2}{dt^2} - \frac{1}{2}r_2\frac{d^2r_2}{dt^2}\right).$$

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This last expression, equivalent to Ampère's formula, is the sought-for *transformation*. For thereby the magnitudes i, i', ε, ϑ and ϑ' are eliminated, and only such magnitudes introduced in their place which have to do with, partly the *same*, partly the *different kind* of electrical masses themselves and their mutual relations.

This *transformed* expression for Ampère's formula can now be represented as a sum of four parts, which can be considered as the *elementary electrical forces*, namely, in the following way:

$$+\frac{eds \cdot e'ds'}{r^2}\left(1 - \frac{a^2}{16}\frac{dr^2}{dt^2} + \frac{a^2}{8}r\frac{d^2r}{dt^2}\right) \text{, as action of } +eds \text{ on } +e'ds' \text{ ;}$$

$$+\frac{eds \cdot e'ds'}{r_1^2}\left(1 - \frac{a^2}{16}\frac{dr_1^2}{dt^2} + \frac{a^2}{8}r_1\frac{d^2r_1}{dt^2}\right) \text{, as action of } -eds \text{ on } -e'ds' \text{ ;}$$

$$-\frac{eds \cdot e'ds'}{r_2^2}\left(1 - \frac{a^2}{16}\frac{dr_2^2}{dt^2} + \frac{a^2}{8}r_2\frac{d^2r_2}{dt^2}\right) \text{, as action of } +eds \text{ on } -e'ds' \text{ ;}$$

$$-\frac{eds \cdot e'ds'}{r_3^2}\left(1 - \frac{a^2}{16}\frac{dr_3^2}{dt^2} + \frac{a^2}{8}r_3\frac{d^2r_3}{dt^2}\right) \text{, as action of } -eds \text{ on } +e'ds' \text{ ;}$$

Each of these four partial actions reduces itself, for the case of rest, where $dr/dt = dr_1/dt = dr_2/dt = dr_3/dt = 0$ and likewise $d^2r/dt^2 = d^2r_1/dt^2 = d^2r_2/dt^2 = d^2r_3/dt^2 = 0$, to the same values, as are given for this case by the fundamental law of *electrostatics*; for these four forces are expressed in that case by the product of the masses acting upon each other, divided by the square of their distances. Accordingly as each product has a positive or negative value, the force acts to repel or attract.

If, as in electrostatics, the electrical masses are denoted simply by e and e', and these masses themselves are given positive or negative values, according to whether they belong to the positive or negative fluid, then all those partial effects can be brought under the *general law*, in which the repulsive force of those masses is represented by[198]

$$\frac{ee'}{r^2}\left(1 - \frac{a^2}{16}\frac{dr^2}{dt^2} + \frac{a^2}{8}r\frac{d^2r}{dt^2}\right) \ .$$

Therefore, from this analysis of Ampère's law, which is a more precise expression of a very extensive class of facts, there follows the same *fundamental electrical law*, which in the preceding Sections was established merely by introducing particular facts, and this was demonstrated without hypothesis.

5.22   Theory of Two Constant Current Elements

Having attained the *fundamental electrical law* expressed in the previous Section, we can place it at the head of the theory of electricity, and from it synthetically derive a system of consequences, which is the ultimate purpose of such a law.

The consequences which can be derived from it for static electricity, are found in Poisson's classic Treatise in the *Mémoires de l'academie des sciences de l'institut de France*, for the year 1812.[199] For the foregoing fundamental law is, for the case of statics, identical with that law which Poisson, in the cited location, placed at the head of electrostatics.

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[198][Note by AKTA:] The following equation should be understood as:

$$\frac{ee'}{r^2}\left[1 - \frac{a^2}{16}\left(\frac{dr}{dt}\right)^2 + \frac{a^2}{8}r\frac{d^2r}{dt^2}\right] \ .$$

[199][Note by AKTA:] Siméon Denis Poisson (1781-1840). See [Poi12a], [Poi12b] and [Poi13].

For *moving* electricity, first the *uniform* motion of the electricity of galvanic currents in conductors at rest is to be considered, to which Ampère's law relates. Now, since the above fundamental electrical law was developed analytically from Ampère's law, Ampère's law must in turn follow synthetically from this fundamental law. This derivation is actually to be given here.

In two current elements α and α', which, with the straight line connecting them, lie in planes which make the angle ω with one another,[200] four *electrical masses* are given, namely, one *positive* and one *equally large negative* in each current element.

For element α, $+\alpha e$ would denote the *positive* mass, which moves with constant velocity $+u$ in the direction of element α, which forms the angle ϑ with the straight line r directed from the first element to the second; for the same element, $-\alpha e$ would denote the *negative* mass, which moves in the same direction with the constant velocity $-u$, viz., backwards.

The letters with primes $\pm\alpha'e'$, $\pm u'$ and ϑ' denote the same thing for the other element α', as the letters without primes denote for the first element α.

Among these four masses, the following four actions are to be considered:
$$\text{of } +\alpha e \text{ on } +\alpha'e' \ ,$$
$$\text{of } -\alpha e \text{ on } -\alpha'e' \ ,$$
$$\text{of } +\alpha e \text{ on } -\alpha'e' \ ,$$
$$\text{of } -\alpha e \text{ on } +\alpha'e' \ .$$

The four distances of these masses acting upon each other at a distance are equal at the moment under consideration, when all these masses are located in the two given elements α and α', to the given distance of these two elements r. These four distances, because they do not always remain equal, on account of the differing motions of the masses, are denoted by r_1, r_2, r_3, r_4, and therefore, at the moment under consideration

$$r_1 = r_2 = r_3 = r_4 = r \ .$$

The application of the fundamental law given at the end of the previous Section then directly yields the values for these four partial actions, in succession,

$$+\frac{\alpha e \cdot \alpha'e'}{r_1^2}\left(1 - \frac{a^2}{16}\frac{dr_1^2}{dt^2} + \frac{a^2}{8}r_1\frac{d^2r_1}{dt^2}\right) \ ,$$

$$+\frac{\alpha e \cdot \alpha'e'}{r_2^2}\left(1 - \frac{a^2}{16}\frac{dr_2^2}{dt^2} + \frac{a^2}{8}r_2\frac{d^2r_2}{dt^2}\right) \ ,$$

$$-\frac{\alpha e \cdot \alpha'e'}{r_3^2}\left(1 - \frac{a^2}{16}\frac{dr_3^2}{dt^2} + \frac{a^2}{8}r_3\frac{d^2r_3}{dt^2}\right) \ ,$$

$$-\frac{\alpha e \cdot \alpha'e'}{r_4^2}\left(1 - \frac{a^2}{16}\frac{dr_4^2}{dt^2} + \frac{a^2}{8}r_4\frac{d^2r_4}{dt^2}\right) \ .$$

These four forces are transferred from the electrical masses $+\alpha'e'$ and $-\alpha'e'$, on which they directly act, according to Section 5.19, page[201] 132, to the ponderable mass of the element α', and combine therein into a resultant, which is equal to the algebraic sum of those forces. This sum is, with respect to the already mentioned equality of the distances,

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[200][Note by AKTA:] See footnote 114 on page 74.
[201][Note by LH and AKTA:] [Web46, p. 137 of Weber's *Werke*].

$$-\frac{a^2}{16} \cdot \frac{\alpha e \cdot \alpha' e'}{r^2} \left\{ \left(\frac{dr_1^2}{dt^2} + \frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) \right.$$

$$\left. - 2r \left(\frac{d^2 r_1}{dt^2} + \frac{d^2 r_2}{dt^2} - \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) \right\} .$$

If the mass $+\alpha e$ now progresses in its path in the time element dt with velocity $+u$ by the element of displacement $+udt$, which path forms the angle ϑ with the straight line r_1, while the mass $+\alpha' e'$ progresses in its path in the same time element dt with the velocity $+u'$ by the element of displacement $+u'dt$, which path forms the angle ϑ' with the extended straight line r_1, and if these small displacements are projected onto the direction r_1, then

$$r_1 + dr_1 = r_1 - udt \cdot \cos \vartheta + u'dt \cdot \cos \vartheta' ,$$

in which dr_1 denotes the change of length of the straight line connecting the two positive masses during the time element dt. From this follows

$$\frac{dr_1}{dt} = -u \cos \vartheta + u' \cos \vartheta' .$$

Likewise there results for the two negative masses $-\alpha e$ and $-\alpha' e'$:

$$\frac{dr_2}{dt} = +u \cos \vartheta - u' \cos \vartheta' ;$$

further, for the positive $+\alpha e$ and for the negative $-\alpha' e'$:

$$\frac{dr_3}{dt} = -u \cos \vartheta - u' \cos \vartheta' ;$$

finally for the negative $-\alpha e$ and for the positive $+\alpha' e'$:

$$\frac{dr_4}{dt} = +u \cos \vartheta + u' \cos \vartheta' .$$

Hence,

$$\left(\frac{dr_1^2}{dt^2} + \frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) = -8uu' \cos \vartheta \cos \vartheta' .$$

Now, since, further, the velocities u and u' are constant, when the *changes* in the angles ϑ and ϑ' (which themselves of course have the same value at the moment under consideration for all four pairs of masses, but which values change with time and become unequal) during the time element dt, are denoted

for the first pair of masses, $d\vartheta_1$ and $d\vartheta'_1$
for the second pair of masses, $d\vartheta_2$ and $d\vartheta'_2$
for the third pair of masses, $d\vartheta_3$ and $d\vartheta'_3$
for the fourth pair of masses, $d\vartheta_4$ and $d\vartheta'_4$,

there results through differentiation of the first differential coefficients:

$$\frac{d^2 r_1}{dt^2} = +u \sin \vartheta \cdot \frac{d\vartheta_1}{dt} - u' \sin \vartheta' \cdot \frac{d\vartheta'_1}{dt} ,$$

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$$\frac{d^2 r_2}{dt^2} = -u \sin \vartheta \cdot \frac{d\vartheta_2}{dt} + u' \sin \vartheta' \cdot \frac{d\vartheta_2'}{dt} \ ,$$

$$\frac{d^2 r_3}{dt^2} = +u \sin \vartheta \cdot \frac{d\vartheta_3}{dt} + u' \sin \vartheta' \cdot \frac{d\vartheta_3'}{dt} \ ,$$

$$\frac{d^2 r_4}{dt^2} = -u \sin \vartheta \cdot \frac{d\vartheta_4}{dt} - u' \sin \vartheta' \cdot \frac{d\vartheta_4'}{dt} \ .$$

Hence

$$\left(\frac{d^2 r_1}{dt^2} + \frac{d^2 r_2}{dt^2} - \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) = +u \sin \vartheta \left(\frac{d\vartheta_1}{dt} - \frac{d\vartheta_2}{dt} - \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt} \right)$$

$$- u' \sin \vartheta' \left(\frac{d\vartheta_1'}{dt} - \frac{d\vartheta_2'}{dt} + \frac{d\vartheta_3'}{dt} - \frac{d\vartheta_4'}{dt} \right) \ .$$

Now, let AB in Figure 18 represent the line r.
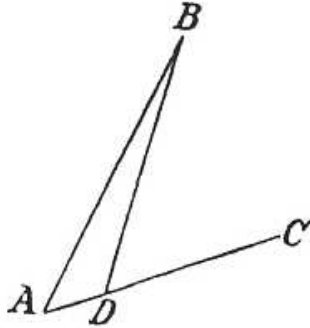
Fig. 18.

Let the mass $+\alpha e$ be located at A and move in the direction AC with the velocity $+u$ in the time element dt through $AD = +udt$. The angle which the current direction AC forms with AB, is $BAC = \vartheta$. As a result of the motion of A to D, the angle BAC becomes BDC, and

$$BDC = BAC + ABD = \vartheta + \frac{udt}{r} \sin \vartheta \ .$$

The line AB in Figure 19, which again represents r, is extended to B'.

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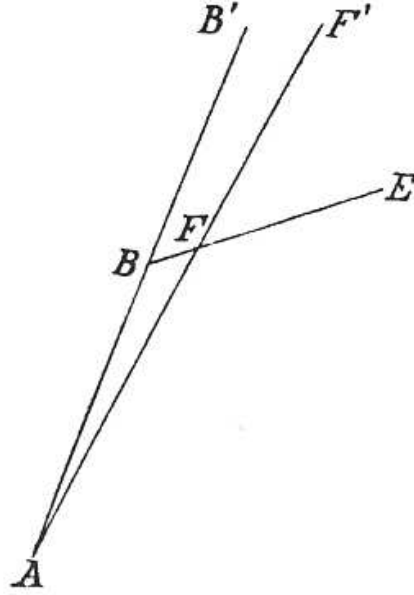
Fig. 19.

The mass $+\alpha' e'$ is located at B and moves in the direction BE with velocity $+u'$ in the time element dt through $BF = +u'dt$. The angle which the current direction BE forms with BB', is $B'BE = \vartheta'$. As a result of the motion of B to F, the angle $B'BE$ becomes $F'FE$, and

$$\vartheta' = B'BE = AFB + BAF = F'FE + \frac{u'dt}{r}\sin\vartheta' \; ,$$

accordingly is

$$F'FE = \vartheta' - \frac{u'dt}{r}\sin\vartheta' \; .$$

Finally, if, through the center of a sphere, lines are drawn parallel to the direction AB and to the two current directions AC and BE in Figures 18 and 19, which cut the sphere's surface at R, U, and U' in Figure 20, and R is connected with U and U' by the arcs of great circles, then the plane of the arc $UR = \vartheta$ is parallel to the plane BAC in Figure 18, and the plane of the arc $U'R = \vartheta'$ is parallel to the plane $B'BE$ in Figure 19, and the angle formed by the two planes at R is the angle denoted ω.

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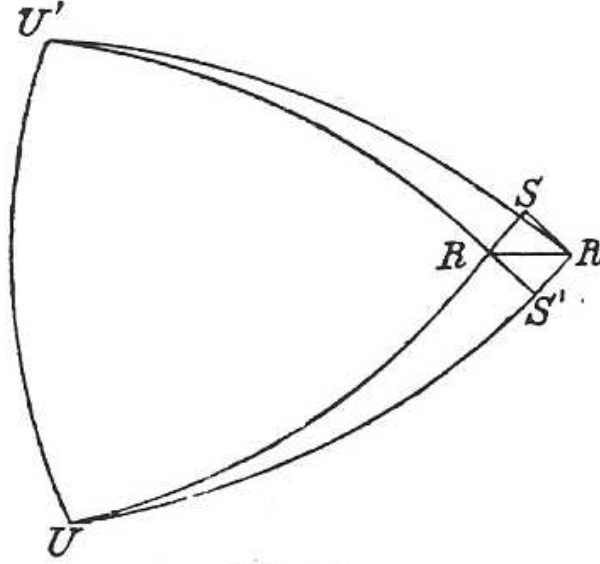
Fig. 20.

Let arc UR be extended to S, $U'R$ to S', and make

$$RS = +\frac{udt}{r}\sin\vartheta'\,, \qquad\qquad RS' = -\frac{u'dt}{r}\sin\vartheta'\,.$$

Then US is the arc of the angle BDC in Figure 18, and $U'S'$ is the arc of the angle $F'FE$ in Figure 19. The element of the surface of the sphere, in which R, S, and S' lie, can also be considered as an element of the plane touching the surface of the sphere at R, and the arc elements RS and RS' as straight lines in this plane. If the parallelogram $RSR'S'$ is completed in this plane, then a line drawn through the center of the sphere parallel to the straight line connecting both masses at the end of the time element dt, goes through the point R'. From this it follows that the direction of this straight line is changed by the simultaneous motion of both masses exactly as it would change, if the one mass were at rest and its motion, taken as being opposite, were attributed to the other mass. Both motions, transferred to a point in this way, can then be combined according to the law of parallelograms, and the cited result is obtained.

Finally, if R' is connected with U and U' by means of the great circle arcs, then

$$UR' = \vartheta + d\vartheta_1 = UR + d\vartheta_1$$

$$U'R' = \vartheta' + d\vartheta'_1 = U'R + d\vartheta'_1\,.$$

It follows that:

$$d\vartheta_1 = UR' - UR = RS + RS'\cos\omega$$

$$d\vartheta'_1 = U'R' - U'R = RS' + RS\cos\omega\,.$$

Now, since $RS = +\frac{udt}{r}\sin\vartheta$, $RS' = -\frac{u'dt}{r}\sin\vartheta'$, it follows that:

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$$d\vartheta_1 = +\frac{udt}{r}\sin\vartheta - \frac{u'dt}{r}\sin\vartheta'\cos\omega$$

$$d\vartheta_1' = -\frac{u'dt}{r}\sin\vartheta' + \frac{udt}{r}\sin\vartheta\cos\omega \ .$$

Accordingly,

$$r\frac{d\vartheta_1}{dt} = +u\sin\vartheta - u'\sin\vartheta'\cos\omega$$

$$r\frac{d\vartheta_1'}{dt} = -u'\sin\vartheta' + u\sin\vartheta\cos\omega \ .$$

In the same way, there results for the two *negative* masses $-\alpha e$ and $-\alpha' e'$:

$$r\frac{d\vartheta_2}{dt} = -u\sin\vartheta + u'\sin\vartheta'\cos\omega$$

$$r\frac{d\vartheta_2'}{dt} = +u'\sin\vartheta' - u\sin\vartheta\cos\omega \ ,$$

further, for the positive mass $+\alpha e$ and for the negative $-\alpha' e'$:

$$r\frac{d\vartheta_3}{dt} = +u\sin\vartheta + u'\sin\vartheta'\cos\omega$$

$$r\frac{d\vartheta_3'}{dt} = +u'\sin\vartheta' + u\sin\vartheta\cos\omega \ ,$$

finally, for the negative mass $-\alpha e$ and for the positive $+\alpha' e'$:

$$r\frac{d\vartheta_4}{dt} = -u\sin\vartheta - u'\sin\vartheta'\cos\omega$$

$$r\frac{d\vartheta_4'}{dt} = -u'\sin\vartheta' - u\sin\vartheta\cos\omega \ .$$

If these values are now substituted, the following equation is obtained:[202]

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[202][Note by WW:] This equation can also be derived from the equations of motion of the four electrical masses. Let a plane be laid parallel with α' through the element α. Let O be that point in this plane, at which direction α is cut by direction α', which is projected on this plane. Let O be the origin of the coordinates, direction α as the x axis, and the z axis be perpendicular to the above-mentioned plane. Further, imagine that both masses always move forward uniformly in the same directions, and choose that moment as the initial point of time t, for which the coordinates of the mass later considered in α' are

$$x' = 0, \qquad y' = 0, \qquad z' = c \ .$$

If ε then denotes the angle which the directions α and α' form with each other, x, y, z the coordinates of the mass later considered in α, and u and u' the velocities of both masses, then the equations of motion are

<table>
<tr><td>for the one mass:</td><td>for the other mass:</td></tr>
<tr><td>$x = b + ut$</td><td>$x' = u't \cdot \cos\varepsilon$</td></tr>
<tr><td>$y = 0$</td><td>$y' = u't \cdot \sin\varepsilon$</td></tr>
<tr><td>$z = 0$</td><td>$z' = c$</td></tr>
</table>

where b and c are given constants. Accordingly,

$$r\left(\frac{d^2r_1}{dt^2} + \frac{d^2r_2}{dt^2} - \frac{d^2r_3}{dt^2} - \frac{d^2r_4}{dt^2}\right) = -8uu'\sin\vartheta\sin\vartheta'\cos\omega \ .$$

If these values and those found for $\left(\frac{dr_1^2}{dt^2} + \frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2}\right)$ are substituted in the above expression for the resultant of the four partial actions, then one obtains the following value for it:

$$x' - x = (u'\cos\varepsilon - u)\cdot t - b$$

$$y' - y = u't\cdot\sin\varepsilon$$

$$z' - z = c$$

and, since $r_1^2 = (x'-x)^2 + (y'-y)^2 + (z'-z)^2$,

$$r_1^2 = [(u'\cos\varepsilon - u)\cdot t - b]^2 + u'^2t^2\sin^2\varepsilon + c^2 \ .$$

If this equation is differentiated with respect to r_1 and t, one obtains:

$$\frac{dr_1}{dt} = \frac{1}{r_1}\cdot[(u'\cos\varepsilon - u)\cdot t - b](u'\cos\varepsilon - u) + u'^2t\cdot\sin^2\varepsilon \ ,$$

and, through repeated differentiation,

$$r_1\frac{d^2r_1}{dt^2} + \frac{dr_1^2}{dt^2} = u^2 + u'^2 - 2uu'\cos\varepsilon \ .$$

Now, for the moment where the two masses have reached α and α', if ϑ denotes the angle which the direction from α to α' forms with the *first* coordinate axis,

$$x' - x = r_1\cos\vartheta \ .$$

If lines are drawn parallel with the three coordinates, further with the direction from α to α', and finally with the direction α' itself, through the center of a sphere, whose surface is cut into

$$X,\ Y,\ Z,\ R \text{ and } P \ ,$$

then RY is the arc of the angle, which the line from α to α' forms with the *second* coordinate axis, and hence for the moment, where the two masses reach α and α',

$$y' - y = r_1\cos RY \ .$$

Now, however, in the spherical triangles PRX and PRY, because the radius P (which is parallel to the direction α') lies in the same greatest circle with the radii X and Y (which are parallel to the plane of the coordinate axes x and y),

$$\cos RX\sin PY + \cos RY\sin PX = \cos PR\sin XY \ ,$$

and further,

$$XY = 90° \ , \quad PX = \varepsilon \ , \quad RX = \vartheta \ , \quad PR = \vartheta' \ ,$$

where ϑ' denotes the angle which the line from α to α' forms with the direction of α' itself. If these values are substituted, there results

$$\cos RY = \frac{\cos\vartheta' - \cos\vartheta\cos\varepsilon}{\sin\varepsilon} \ ,$$

hence

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$$-\frac{\alpha\alpha'}{r^2} \cdot aeu \cdot ae'u' \left(\sin\vartheta \sin\vartheta' \cos\omega - \frac{1}{2}\cos\vartheta\cos\vartheta' \right) .$$

If one substitutes here, according to page[203] 144,

$$aeu = i , \qquad\qquad ae'u' = i' ,$$

then, according to this derivation from the established fundamental electrical law, there results for the repulsive force of two current elements the same value as according to Ampère's law, namely:

$$-\frac{\alpha\alpha'}{r^2}ii' \left(\sin\vartheta \sin\vartheta' \cos\omega - \frac{1}{2}\cos\vartheta\cos\vartheta' \right) ,$$

or, when ε denotes the angle which the two elements α and α' themselves make, and where then $\cos\varepsilon = \sin\vartheta \sin\vartheta' \cos\omega + \cos\vartheta\cos\vartheta'$,

$$-\frac{\alpha\alpha'}{r^2}ii' \left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta' \right) .$$

The actions at a distance of *uniform* electrical currents in conducting wires *at rest* are hereby fully determined. The derivations of the established fundamental law carried out up to now are all empirically confirmed.

$$y' - y = r_1 \cdot \frac{\cos\vartheta' - \cos\vartheta\cos\varepsilon}{\sin\varepsilon} .$$

If now t in the above equations for $x' - x$ and $y' - y$ denotes that value, which correspond to the moment at which the two masses reach α and α', then the above values of $x' - x$ and $y' - y$ are to be set equal to the ones just found, or

$$(u'\cos\varepsilon - u)t - b = r_1 \cos\vartheta$$

$$u't \cdot \sin\varepsilon = r_1 \cdot \frac{\cos\vartheta' - \cos\vartheta\cos\varepsilon}{\sin\varepsilon} .$$

If these values are substituted in the expression for $\frac{dr_1}{dt}$, the result is:

$$\frac{dr_1}{dt} = + u'\cos\vartheta' - u\cos\vartheta .$$

If from this is subtracted the square of the value found for $r_1\frac{d^2r_1}{dt^2} + \frac{dr_1^2}{dt^2}$, then it remains the case that

$$r_1\frac{d^2r_1}{dt^2} = u^2\sin^2\vartheta + u'^2\sin^2\vartheta' - 2uu'(\cos\varepsilon - \cos\vartheta\cos\vartheta')$$

or, if the angle ω is introduced, in accordance with the equation $\cos\varepsilon = \sin\vartheta \sin\vartheta' \cos\omega + \cos\vartheta\cos\vartheta'$,

$$r_1\frac{d^2r_1}{dt^2} = u^2\sin^2\vartheta + u'^2\sin^2\vartheta' - 2uu'\sin\vartheta\sin\vartheta'\cos\omega .$$

The corresponding differential coefficients of the other pairs of masses are found in the same way, which then together give the above equation.

[203][Note by LH and AKTA:] [Web46, p. 152 of Weber's *Werke*].

5.23   Theory of Volta-Induction

It still remains to develop, from the established fundamental electrical law, the effects of *variable* electrical currents in *moving* conductors, which development comprises the *theory of voltaic induction.*

Voltaic induction[204] differentiates itself from Ampère's *electrodynamics* in that it has to do with the *generation* of currents, which is wholly excluded from the latter.

The following is *empirically* known about voltaic induction. We know, first, that it can be elicited in two essentially different ways: namely, currents can be induced by means of *constant* currents and by means of *variable* ones. Induction occurs by means of *constant* currents, either when a conducting wire, through which the constant current is passing, approaches the conducting wire in which a current is to be induced, or is moved away from it, or when, vice versa, the latter approaches the former or is moved away from it. It seems to be a matter of indifference for the effect, whether only the one, or only the other wire, or both alike are moved, provided that their *relative* motion is the same. If the two wires are parallel to each other, then a current of opposite direction will be induced by bringing them closer, a current of like direction will be induced by drawing them apart. Induction occurs by means of *variable* currents, even when the conducting wire, through which the variable current passes, remains immovable[205] with respect to the wire in which a current is to be induced. If the two wires are parallel to each other, increasing current intensity induces a current of opposite direction, decreasing intensity a current of like direction.

We empirically know, *secondly*, that the induction caused by a constant current in a conducting wire moving toward it is the same as the induction caused by a magnet in the same conducting wire, if the electrodynamic force of repulsion or attraction, which that current would exert on this conducting wire when a determined current passed through the latter, is equal to the electromagnetic force, which the magnet would exert on the same wire under the same conditions. See Section 5.11, page[206] 105.

These empirical findings can serve to test the correctness of the laws of voltaic induction which are to be established.

Moreover, it should be noted, that the theory of *voltaic induction* is a theory of *electromotive forces*, by means of which the *induced currents* themselves are still not completely determined. In order to completely determine the *induced currents* themselves, also according to their *intensity*, as well as the electrodynamic forces of repulsion and attraction and secondary inductions which they themselves further elicit, it requires, besides the determination of the *electromotive force* to be drawn from the theory of *voltaic induction*, a statement of the *resistance* of the entire circuit to which the induced conducting wire belongs, as is obvious from the dependency given by Ohm's law of the current intensity on the electromotive force and the total resistance of the circuit.

The complete development of the actions of *non-uniform* electrical currents in *moving* conductors comprises, finally, not merely the *theory of voltaic induction*, that is to say, it not merely accounts for the generation, strengthening, and weakening of currents in the ponderable conductors, but it also encompasses all *electrodynamic* forces of repulsion and attraction, which are effects of the above-cited currents, and which move the ponderable conductors themselves.

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[204][Note by AKTA:] See footnote 139 on page 96.

[205][Note by AKTA:] In German: *unverrückt.* This word can also be translated as undisturbed or unmoved.

[206][Note by LH and AKTA:] [Web46, p. 103 of Weber's *Werke*].

In the following Sections, we intend *first* to begin with a prefatory consideration of particular cases, and *then* to follow with the general development of the actions of electrical currents which are *not uniform*, as they take place in *galvanic currents* of variable intensity, while the ponderable conductors are *in motion*.

5.24 Law of Exciting a Current in a Conductor, which Approaches a Constant Current Element at Rest, or is Distanced from It

The simplest case of *voltaic induction* to which the established fundamental law can be applied, is the one in which, of the two elements, only one, namely, the inducing one, already contains a current, specifically, a current of constant intensity, and the distance between the two elements is altered simply by means of the motion of the other element, namely, the induced one.

If α now denotes the length of the inducing element, α' the length of the induced element, then four electrical masses are to be differentiated in these two elements, namely:

$$+\alpha e \ , \quad -\alpha e \ , \quad +\alpha' e' \ , \quad -\alpha' e' \ .$$

The *first* of these masses, $+\alpha e$, moves with *constant* velocity $+u$ in the direction of the element at rest α, which forms the angle ϑ with the straight line drawn from α to α';[207] the *second*, $-\alpha e$, moves in the same direction with velocity $-u$, viz., backwards; the *third*, $+\alpha' e'$, which indeed rests in the element α', is *carried forward* by it with velocity $+u'$ in that direction which forms the angle ϑ' with the extended straight line drawn from α to α'; and with the same straight line, lies in a plane, which, with the plane containing element α and that straight line, forms the angle ω; the *fourth*, finally, $-\alpha' e'$, which likewise rests in element α', is carried forward by this element with the same velocity $+u'$ in the same direction as the third mass. The distances of the first two masses from the second two are all equal at the moment in question to distance r, at which the elements α and α' are found at that moment; since, however, they do not remain equal, they are denoted,[208] as on page[209]

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[207][Note by AKTA:] The angles ϑ, ϑ', ω, ε and φ for this case are represented in the Figure of this footnote:

(a)   (b)
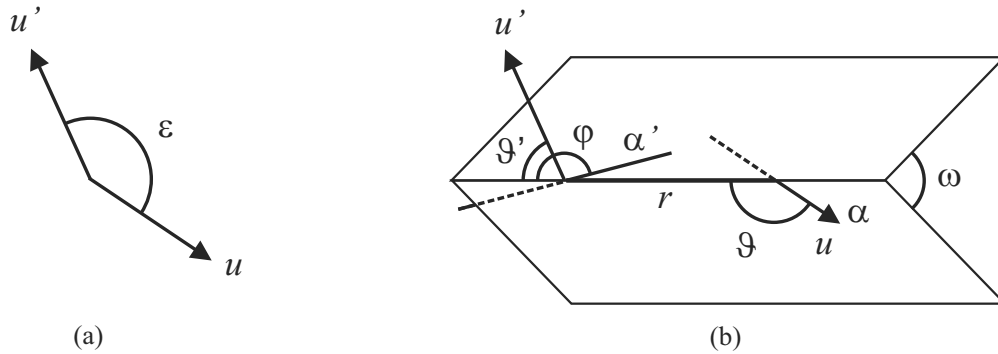
Here ε is the angle between the directions u and u'. Moreover, the directions u' and r form a plane. Likewise, the directions u and r form another plane. The angle between these two planes is denoted by ω.

[208][Note by LH and AKTA:] See beginning of Section 5.22.

[209][Note by LH and AKTA:] [Web46, p. 158 of Weber's *Werke*].

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, r_1, r_2, r_3, r_4.

The application of the fundamental law then yields, as on page[210] 150, the following four partial actions among these four masses:

$$+\frac{\alpha e \cdot \alpha' e'}{r_1^2}\left(1 - \frac{a^2}{16}\frac{dr_1^2}{dt^2} + \frac{a^2}{8}r_1\frac{d^2r_1}{dt^2}\right)$$

$$+\frac{\alpha e \cdot \alpha' e'}{r_2^2}\left(1 - \frac{a^2}{16}\frac{dr_2^2}{dt^2} + \frac{a^2}{8}r_2\frac{d^2r_2}{dt^2}\right)$$

$$-\frac{\alpha e \cdot \alpha' e'}{r_3^2}\left(1 - \frac{a^2}{16}\frac{dr_3^2}{dt^2} + \frac{a^2}{8}r_3\frac{d^2r_3}{dt^2}\right)$$

$$-\frac{\alpha e \cdot \alpha' e'}{r_4^2}\left(1 - \frac{a^2}{16}\frac{dr_4^2}{dt^2} + \frac{a^2}{8}r_4\frac{d^2r_4}{dt^2}\right)$$

These four partial actions can now first be combined into two forces, of which one is the action of the two masses of the inducing elements $+\alpha e$ and $-\alpha e$ on the *positive* mass $+\alpha' e'$ of the induced element, the other [force is] the action of the same masses on the *negative* mass $-\alpha' e'$ of the induced element. The former force is the sum of the first and fourth, the latter is the sum of the second and third. The former force is thus, with regard to the equality of r_1, r_2, r_3 and r_4 with r at the moment in question,

$$= -\frac{a^2}{16} \cdot \frac{\alpha e \cdot \alpha' e'}{r^2}\left\{\left(\frac{dr_1^2}{dt^2} - \frac{dr_4^2}{dt^2}\right) - 2r\left(\frac{d^2r_1}{dt^2} - \frac{d^2r_4}{dt^2}\right)\right\} \; ;$$

the latter force is

$$= -\frac{a^2}{16} \cdot \frac{\alpha e \cdot \alpha' e'}{r^2}\left\{\left(\frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2}\right) - 2r\left(\frac{d^2r_2}{dt^2} - \frac{d^2r_3}{dt^2}\right)\right\} \; .$$

Now, insofar as the motions elicited by these forces in both electrical masses, $+\alpha' e'$ and $-\alpha' e'$, in their ponderable carrier α' are cancelled almost instantaneously by the *resistance* of the carrier, and thereby all the forces acting on those masses are immediately transferred to this carrier, the *sums* of the above two forces, as on page[211] 151, gives the force which moves the carrier α' itself,

$$-\frac{a^2}{16} \cdot \frac{\alpha e \cdot \alpha' e'}{r^2}\left\{\left(\frac{dr_1^2}{dt^2} + \frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2}\right)\right.$$

$$\left. - 2r\left(\frac{d^2r_1}{dt^2} + \frac{d^2r_2}{dt^2} - \frac{d^2r_3}{dt^2} - \frac{d^2r_4}{dt^2}\right)\right\} \; .$$

Before the transference to their carriers of those forces which originally acted on the electrical masses, the electrical masses themselves are, however, somewhat displaced in their carriers, and when this displacement is *different* for the positive mass $+\alpha' e'$ and the negative mass $-\alpha' e'$, the two thus being thereby *separated* from each other, then a galvanic current is produced in carrier α', and the force which causes this separation, is called the *electromotive force*. It is clear, that this *electromotive force* depends upon the *difference* of the above two forces, i.e., on

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[210][Note by LH and AKTA:] [Web46, p. 158 of Weber's *Werke*].
[211][Note by LH and AKTA:] [Web46, p. 159 of Weber's *Werke*].

$$-\frac{a^2}{16} \cdot \frac{\alpha e \cdot \alpha' e'}{r^2} \left\{ \left(\frac{dr_1^2}{dt^2} - \frac{dr_2^2}{dt^2} + \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) \right.$$

$$\left. - 2r \left(\frac{d^2 r_1}{dt^2} - \frac{d^2 r_2}{dt^2} + \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) \right\} .$$

According to the determinations given in Section 5.22 for *two constant* current elements *at rest* in relation to the motion of their electrical masses, the value obtained there for that former *sum* was equal to the force determined by Ampère's law,

$$= -\frac{\alpha \alpha'}{r^2} i i' \left(\cos \varepsilon - \frac{3}{2} \cos \vartheta \cos \vartheta' \right) ;$$

there the value of this latter *difference* would then, however, be

$$= 0 .$$

According to the determinations given in this Section for a *constant* current element *at rest* and for a *moving* wire element *without current* with respect to their electrical masses, the value of that former *sum*, however,

$$= 0 ,$$

and the value of this latter *difference*

$$= -\frac{\alpha \alpha'}{r^2} a e' u' i \left(\cos \varepsilon - \frac{3}{2} \cos \vartheta \cos \vartheta' \right) ,$$

as is to be proven in what follows.

It is merely necessary for this purpose, in the differential coefficients determined on page[212] 151, to put $+u'$ instead of $-u'$ for the velocity of the *negative* mass; one then obtains:

$$\frac{dr_1}{dt} = \frac{dr_3}{dt} = -u \cos \vartheta + u' \cos \vartheta'$$

$$\frac{dr_2}{dt} = \frac{dr_4}{dt} = +u \cos \vartheta + u' \cos \vartheta' .$$

Hence, then

$$\frac{dr_1^2}{dt^2} + \frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} = 0 .$$

On the other hand:

$$\frac{dr_1^2}{dt^2} - \frac{dr_2^2}{dt^2} + \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} = -8uu' \cos \vartheta \cos \vartheta' .$$

Further, one obtains:

$$\frac{d^2 r_1}{dt^2} = +u \sin \vartheta \cdot \frac{d\vartheta_1}{dt} - u' \sin \vartheta' \cdot \frac{d\vartheta_1'}{dt}$$

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[212][Note by LH and AKTA:] [Web46, p. 159 of Weber's *Werke*].

$$\frac{d^2 r_2}{dt^2} = -u \sin \vartheta \cdot \frac{d\vartheta_2}{dt} - u' \sin \vartheta' \cdot \frac{d\vartheta'_2}{dt}$$

$$\frac{d^2 r_3}{dt^2} = +u \sin \vartheta \cdot \frac{d\vartheta_3}{dt} - u' \sin \vartheta' \cdot \frac{d\vartheta'_3}{dt}$$

$$\frac{d^2 r_4}{dt^2} = -u \sin \vartheta \cdot \frac{d\vartheta_4}{dt} - u' \sin \vartheta' \cdot \frac{d\vartheta'_4}{dt} \ ,$$

hence:

$$\frac{d^2 r_1}{dt^2} + \frac{d^2 r_2}{dt^2} - \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} = +u \sin \vartheta \left(\frac{d\vartheta_1}{dt} - \frac{d\vartheta_2}{dt} - \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt} \right)$$

$$-u' \sin \vartheta' \left(\frac{d\vartheta'_1}{dt} + \frac{d\vartheta'_2}{dt} - \frac{d\vartheta'_3}{dt} + \frac{d\vartheta'_4}{dt} \right) \ .$$

In contrast is

$$\frac{d^2 r_1}{dt^2} - \frac{d^2 r_2}{dt^2} + \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} = +u \sin \vartheta \left(\frac{d\vartheta_1}{dt} + \frac{d\vartheta_2}{dt} + \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt} \right)$$

$$-u' \sin \vartheta' \left(\frac{d\vartheta'_1}{dt} - \frac{d\vartheta'_2}{dt} + \frac{d\vartheta'_3}{dt} - \frac{d\vartheta'_4}{dt} \right) \ .$$

Further, according to page[213] 155 and the following, if one also attributes the velocity $+u'$ to the *negative* mass of the induced element $-\alpha' e'$, it follows that

$$r \frac{d\vartheta_1}{dt} = r \frac{d\vartheta_3}{dt} = +u \sin \vartheta - u' \sin \vartheta' \cos \omega$$

$$r \frac{d\vartheta_2}{dt} = r \frac{d\vartheta_4}{dt} = -u \sin \vartheta - u' \sin \vartheta' \cos \omega$$

$$r \frac{d\vartheta'_1}{dt} = r \frac{d\vartheta'_3}{dt} = -u' \sin \vartheta' + u \sin \vartheta \cos \omega$$

$$r \frac{d\vartheta'_2}{dt} = r \frac{d\vartheta'_4}{dt} = -u' \sin \vartheta' - u \sin \vartheta \cos \omega \ ,$$

from which it results that:

$$r \left(\frac{d\vartheta_1}{dt} - \frac{d\vartheta_2}{dt} - \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt} \right) = r \left(\frac{d\vartheta'_1}{dt} + \frac{d\vartheta'_2}{dt} - \frac{d\vartheta'_3}{dt} - \frac{d\vartheta'_4}{dt} \right) = 0 \ ;$$

however, on the other hand,

$$r \left(\frac{d\vartheta_1}{dt} + \frac{d\vartheta_2}{dt} + \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt} \right) = -4u' \sin \vartheta' \cos \omega$$

$$r \left(\frac{d\vartheta'_1}{dt} - \frac{d\vartheta'_2}{dt} + \frac{d\vartheta'_3}{dt} - \frac{d\vartheta'_4}{dt} \right) = +4u \sin \vartheta \cos \omega \ .$$

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[213][Note by LH and AKTA:] [Web46, p. 162 and the following of Weber's *Werke*].

From this it follows that:

$$r\left(\frac{d^2r_1}{dt^2}+\frac{d^2r_2}{dt^2}-\frac{d^2r_3}{dt^2}-\frac{d^2r_4}{dt^2}\right)=0$$

$$r\left(\frac{d^2r_1}{dt^2}-\frac{d^2r_2}{dt^2}+\frac{d^2r_3}{dt^2}-\frac{d^2r_4}{dt^2}\right)=-8uu'\sin\vartheta\sin\vartheta'\cos\omega\ .$$

Substituting these values, it is obtained the *sum* of both forces, which act at the *positive* and *negative* masses of the induced element,

$$=0\ ,$$

in contrast, their *difference* is

$$=-\frac{\alpha\alpha'}{r^2}aeu\cdot ae'u'\left(\sin\vartheta\sin\vartheta'\cos\omega-\frac{1}{2}\cos\vartheta\cos\vartheta'\right)\ ,$$

or, since, according to page[214] 157, $\cos\varepsilon=\sin\vartheta\sin\vartheta'\cos\omega+\cos\vartheta\cos\vartheta'$ and according to page[215] 144, $aeu=i$,

$$=-\frac{\alpha\alpha'}{r^2}i\cdot ae'u'\left(\cos\varepsilon-\frac{3}{2}\cos\vartheta\cos\vartheta'\right)\ ,$$

which was to be proven.

Now, the force hereby determined seeks to separate from each other the *positive* and *negative* electricities in the induced element α' in the direction of the straight line r. In reality, however, this separation can only ensue in the direction of α', because in a linear conductor, a galvanic current can only take place in the direction of the conductor. Hence, if one takes the components of the above force in the direction of element α' and perpendicular to it, then only the first part comes under consideration as *electromotive* force, and, if φ denotes the angle which the element α' makes with the extended straight line r,[216] this term is

$$=-\frac{\alpha\alpha'}{r^2}i\left(\cos\varepsilon-\frac{3}{2}\cos\vartheta\cos\vartheta'\right)\cdot ae'u'\cos\varphi\ .$$

Ordinarily, by *electromotive force* is understood the accelerating force which the given absolute force exerts on the electrical mass e' contained in the unit of length of the induced conducting wire, which is obtained by division of the above value by e'. Finally, the *electromotive force* of a constant current element at rest on a moving wire element would hence be maintained as

$$=-\frac{\alpha\alpha'}{r^2}i\left(\cos\varepsilon-\frac{3}{2}\cos\vartheta\cos\vartheta'\right)\cdot au'\cos\varphi\ .$$

Now, accordingly as this expression has a *positive* or *negative* value, the inducing current is *positive* or *negative*, where by positive currents is understood one whose positive electricity

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[214][Note by LH and AKTA:] [Web46, p. 164 of Weber's *Werke*].
[215][Note by LH and AKTA:] [Web46, p. 152 of Weber's *Werke*].
[216][Note by AKTA:] See footnote 207 on page 159.

moves in that direction of element α' which forms the angle φ with the extended straight line r.

If, for example, the elements α and α' are parallel to each other, and the direction in which the latter moves with velocity $+u'$ is in the plane of both elements and perpendicular to them, then, when α' *distances itself* from α by means of its motion,

$$\vartheta = \varphi \ , \qquad\qquad \cos \vartheta' = \sin \vartheta \ , \qquad\qquad \cos \varepsilon = 0 \ ,$$

hence the *electromotive force*

$$= +\frac{3}{2}\frac{\alpha\alpha'}{r^2} i \sin \vartheta \cos^2 \vartheta \cdot au' \ .$$

This value is always *positive*, when $\vartheta < 180°$, and this *positive* value here denotes an induced current of the same direction as the inducing, in accord with what empirical experience has yielded for this case.

Under the same conditions, with the mere difference that the element α' *approaches* the element α by means of its motion,

$$\vartheta = \varphi \ , \qquad\qquad \cos \vartheta' = -\sin \vartheta \ , \qquad\qquad \cos \varepsilon = 0 \ ,$$

hence the *electromotive force*

$$= -\frac{3}{2} i \sin \vartheta \cos^2 \vartheta \cdot au' \ .$$

The *negative* value of this force denotes an induced current of opposite direction from the inducing one, likewise in accord with what empirical experience has yielded for this case.

5.25   Comparison with the Empirical Propositions in Section 11

The experiments communicated in Sections 5.10 and 5.11 relate to the case of *voltaic induction* considered in the previous Section. For *quantitative* determination of *voltaic induction* in this case, the proposition has been set forth and empirically tested there,

> that the induction by a constant current at rest in a conducting wire in motion toward it is the same, as the induction in the same conducting wire by a magnet, if the electrodynamic force, which that constant current would exert on that conducting wire with a current flowing through it, were equal to the electromagnetic force, which the magnet would exert on the wire through which the same current were flowing.

In order to empirically establish this proposition, the following experiments were made:

1. the *electrodynamic* force was measured, which a closed circuit A did exert on another closed circuit B;

2. the closed circuit A was replaced with a magnet C, and the electromagnetic force which C did exert on B was measured;

3. the closed conductor B, without current, was put into a specific motion, and the current was measured, which was then produced by current A in the moving conductor by means of *voltaic induction*;

4. given the same motion of the closed conductor B, the current produced by means of *magnetic induction* by the magnet C, which had been substituted for the current A was measured.

In conformity with these four experiments, the following four laws are now to be listed for comparison:

1. the law of the *electrodynamic* action of a closed circuit on a current element;

2. the law of the *electromagnetic* action of a magnet on a current element;

3. the law of *voltaic induction* by a closed circuit in an element of a moving conductor;

4. the law of *magnetic induction* by a magnet in an element of a moving conductor.

5.25.1 The Law of the Electrodynamic Action of a Closed Circuit on a Current Element

This law is developed on page[217] 90 in item 3 of the footnote, for the case where the closed circuit delimits a plane and acts at a distance. Instead of returning to this special law, here I shall return to the more general one which Ampère has given on page 214 of his Treatise,[218] and which is presented on page[219] 75 of this Treatise. According to this law, the electrodynamic force acting on the current element α' is decomposed along three right-angled coordinate axes, whose origin lies in the center of element α', into the components X, Y, Z, which are defined as follows:

$$X = -\frac{ii'}{2}\alpha'\left(C\cos\mu - B\cos\nu\right)$$

$$Y = -\frac{ii'}{2}\alpha'\left(A\cos\nu - C\cos\lambda\right)$$

$$Z = -\frac{ii'}{2}\alpha'\left(B\cos\lambda - A\cos\mu\right)\ ,$$

in which $A = \int\frac{ydz-zdy}{r^3}$, $B = \int\frac{zdx-xdz}{r^3}$, $C = \int\frac{xdy-ydx}{r^3}$, α' denotes the length of the current element which is acted upon, λ, μ, ν the angles which α' forms with the three coordinate axes, and i and i' the intensities of the closed current and of the current element.

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[217][Note by LH and AKTA:] See item 3 of the footnote 125 on page 86, [Web46, p. 86 of Weber's *Werke*].
[218][Note by AKTA:] See [Amp23, p. 214], [Amp26, p. 42] and [AC15, p. 366].
[219][Note by LH and AKTA:] [Web46, p. 70 of Weber's *Werke*].

5.25.2 The Law of the Electromagnetic Action of a Magnet on a Current Element

According to the fundamental law of electromagnetism, the electromagnetic force which a mass of north or south magnetic fluid $\pm\mu$ exerts on a current element of length α' and of current intensity i' at distance r, when φ denotes the angle which α' forms with r, is represented by

$$\pm\frac{i'\alpha'}{\sqrt{2}}\cdot\frac{\mu\sin\varphi}{r^2}$$

in which $i'\sqrt{\frac{1}{2}}$ replaces χ' according to page[220] 88, and this force seeks to move the current element in a direction perpendicular to α' and r. It follows from this the magnitude and direction of both forces, which the two masses of north and south magnetic fluid contained in a *small* magnet exert on the current element. These two forces can be combined according to the law of parallelograms, and from this results the *magnitude* of the resultant, when m'[221] denotes the magnetic moment and ψ denotes the angle which the magnetic axis makes with the straight line r, and ε the angle which direction α' makes with the direction D lying in the plane of the magnetic axis and of line r, and the sine of this angle with line r is to $\sin\psi$ as $1:\sqrt{1+3\cos^2\psi}$, and finally, if for the sake of brevity, $\frac{1}{r^3}\sqrt{1+3\cos^2\psi}$ is denoted by $\mathfrak{d}$,

$$=\frac{i'}{\sqrt{2}}\alpha'm'\mathfrak{d}\sin\varepsilon\ .$$

The *direction* of this resultant is perpendicular to the directions α' and D. If, now, one denotes by

$$a,\quad b,\quad c$$

the cosines of the angles which the resultant, thus determined, forms with three right-angled coordinate axes, whose origin lies in the center of element α', and decomposes the resultant according to the direction of the latter, then the following three components are obtained:

$$\frac{i'}{\sqrt{2}}\cdot\alpha'm'\cdot a\mathfrak{d}\sin\varepsilon$$

$$\frac{i'}{\sqrt{2}}\cdot\alpha'm'\cdot b\mathfrak{d}\sin\varepsilon$$

$$\frac{i'}{\sqrt{2}}\cdot\alpha'm'\cdot c\mathfrak{d}\sin\varepsilon$$

and for a, b, c the following equations are obtained, when the angles which the direction of element α' forms with the coordinate axes are denoted

$$\lambda\ ,\quad\mu\ ,\quad\nu\ ,$$

and the cosines of the angles which the direction D forms with the same coordinate axes are denoted

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[220][Note by LH and AKTA:] [Web46, p. 86 of Weber's *Werke*].
[221][Note by LH and AKTA:] In the original it appears m instead of m'. We replaced the m by m'.

$$\frac{\mathfrak{a}}{\mathfrak{d}}, \quad \frac{\mathfrak{b}}{\mathfrak{d}}, \quad \frac{\mathfrak{c}}{\mathfrak{d}},$$

namely:

$$a\mathfrak{a} + b\mathfrak{b} + c\mathfrak{c} = 0$$

$$a \cos \lambda + b \cos \mu + c \cos \nu = 0$$

$$aa + bb + cc = 1$$

$$\frac{\mathfrak{a}}{\mathfrak{d}} \cos \lambda + \frac{\mathfrak{b}}{\mathfrak{d}} \cos \mu + \frac{\mathfrak{c}}{\mathfrak{d}} \cos \nu = \cos \varepsilon \ .$$

These equations, by elimination of b and c, yield the value of a as

$$a = \frac{\mathfrak{b} \cos \nu - \mathfrak{c} \cos \mu}{\sqrt{1 - \left(\frac{\mathfrak{a}}{\mathfrak{d}} \cos \lambda + \frac{\mathfrak{b}}{\mathfrak{d}} \cos \mu + \frac{\mathfrak{c}}{\mathfrak{d}} \cos \nu \right)^2}} = \frac{\mathfrak{b} \cos \nu - \mathfrak{c} \cos \mu}{\mathfrak{d} \sin \varepsilon} \ ,$$

and in the same way, the following values of b and c:

$$b = \frac{\mathfrak{c} \cos \lambda - \mathfrak{a} \cos \nu}{\mathfrak{d} \sin \varepsilon}$$

$$c = \frac{\mathfrak{a} \cos \mu - \mathfrak{b} \cos \lambda}{\mathfrak{d} \sin \varepsilon} \ .$$

If these expressions are substituted into those for the three components of the electromagnetic force, the following values are obtained for the latter:

$$-\frac{i'}{\sqrt{2}} \cdot \alpha' m' \left(\mathfrak{c} \cos \mu - \mathfrak{b} \cos \nu \right)$$

$$-\frac{i'}{\sqrt{2}} \cdot \alpha' m' \left(\mathfrak{a} \cos \nu - \mathfrak{c} \cos \lambda \right)$$

$$-\frac{i'}{\sqrt{2}} \cdot \alpha' m' \left(\mathfrak{b} \cos \lambda - \mathfrak{a} \cos \mu \right) \ .$$

For a large magnet, which is composed of many small ones, the three components X', Y', Z' of the electromagnetic force it exerts on the current element α' are hereafter determined as follows:

$$X' = -\frac{i'}{\sqrt{2}} \cdot \alpha' \left(C' \cos \mu - B' \cos \nu \right)$$

$$Y' = -\frac{i'}{\sqrt{2}} \cdot \alpha' \left(A' \cos \nu - C' \cos \lambda \right)$$

$$Z' = -\frac{i'}{\sqrt{2}} \cdot \alpha' \left(B' \cos \lambda - A' \cos \mu \right) \ ,$$

in which $A' = S(\mathfrak{a}m')$, $B' = S(\mathfrak{b}m')$, $C' = S(\mathfrak{c}m')$.[222]

5.25.3 The Law of Voltaic Induction by a Closed Circuit in an Element of a Moving Conductor

The elementary law of induction developed in the previous Section, which holds for any inducing element α, yields the following value for the *electromotive* force with which one such element α seeks to separate from each other the positive and negative electrical masses in the induced element α' in the direction of the straight line r:

$$-\frac{\alpha\alpha'}{r^2}i\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right)\cdot au' \,,$$

in which $+u'$ denotes the velocity with which the induced element α' is moved, and ε and ϑ' the angles which the direction of this motion forms with the direction in which the positive electricity flows in the inducing current element α, and with the extended straight line r. ϑ denotes, as in the theory of two constant current elements in Section 5.22, the angle which the direction in which the positive electricity flows in the first element α, forms with the straight line r.

If this value for the *electromotive* force is compared with the value found on page[223] 157 for the *electrodynamic force* in the theory of two constant current elements, in accordance with Ampère's law, then the following simple relation results between the two, namely, that the former force is obtained from the latter by multiplication with the constant factor au'/i', provided that the direction, in which the positive electricity flows in element α', in the latter force, were the same as the direction in which the induced element α' itself moves, in the former force, that is

$$\beta = \lambda \,, \quad \gamma = \mu \,, \quad \delta = \nu \,,$$

when the angles formed by both directions with three right-angled coordinate axes are respectively denoted

$$\lambda \,, \quad \mu \,, \quad \nu \qquad\qquad \text{and} \qquad\qquad \beta \,, \quad \gamma \,, \quad \delta$$

for then the values of ε and ϑ' are equal in both expressions.

From this it is now obvious, under the presupposition made, that the values stated under (1)[224] for the *electrodynamic force* X, Y, Z also need only to be multiplied by the constant factor au'/i', in order to obtain the components $\mathfrak{X}$, $\mathfrak{Y}$, $\mathfrak{Z}$ of the *electromotive* force which a *closed circuit* exerts on the induced element α'. From this it follows that

$$\mathfrak{X} = -\frac{au'}{2}\cdot i\alpha'\left(C\cos\gamma - B\cos\delta\right)$$

$$\mathfrak{Y} = -\frac{au'}{2}\cdot i\alpha'\left(A\cos\delta - C\cos\beta\right)$$

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[222][Note by LH and AKTA:] The symbol S here means summation. That is, $A' = \int(\mathfrak{a}m')$, $B' = \int(\mathfrak{b}m')$ and $C' = \int(\mathfrak{c}m')$.

[223][Note by LH and AKTA:] [Web46, p. 164 of Weber's *Werke*].

[224][Note by AKTA:] That is, in Subsection 5.25.1.

$$3 = -\frac{au'}{2} \cdot i\alpha' \left(B \cos \beta - A \cos \gamma \right) ,$$

in which A, B, C have the same signification as under (1).[225]

5.25.4 The Law of Magnetic Induction by a Magnet in an Element of a Moving Conductor

From the *elementary electromagnetic force*, determined according to the basic law of electromagnetism, which a mass of north or south magnetic fluid, $\pm\mu$, exerts on a current element of length α' and of current intensity i' at distance r, when φ denotes the angle which the direction of flow of the positive electricity in α' forms with the straight line r, namely, from the active force cited under (2),[226] normal to the plane parallel with r and α'[227]

$$\pm \frac{i'\alpha'}{\sqrt{2}} \cdot \frac{\mu \sin \varphi}{r^2} ,$$

we obtain, by multiplication with the constant ku'/i', according to the basic law of magneto-induction, the *elementary electromotive force* with which that magnetic mass seeks to separate the positive and negative electricity in the induced element α', in a direction normal to the plane parallel with r and α', when the induced element α' is moving here with the velocity u in the same direction that the positive electricity flows there in element α'. Therefore this *electromotive force* is

$$= \pm \frac{k\alpha'u'}{\sqrt{2}} \cdot \frac{\mu \sin \varphi}{r^2} .$$

Here k denotes a constant factor independent of u', whose value, however, has thus far not been more closely determined by any measurement.

If one denotes the angles, which in the one case the direction in which the positive electricity in element α' is moved, in the other case the direction in which the induced element α' itself is moved, form with three right-angled coordinate axes, as respectively

$$\lambda , \quad \mu , \quad \nu \qquad \text{and} \qquad \beta , \quad \gamma , \quad \delta ,$$

then under the just-presupposed identity of the directions specified,

$$\beta = \lambda , \quad \gamma = \mu , \quad \delta = \nu .$$

Here too, it is obvious that, under the presupposed identity of the two directions mentioned, the values of X', Y', Z' stated under (2)[228] need only be multiplied by the constant factor

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[225][Note by AKTA:] That is, in Subsection 5.25.1.

[226][Note by AKTA:] That is, in Subsection 5.25.2.

[227][Note by AKTA:] Due to a misprint in the original German text, the next equation appeared with a' instead of α', namely

$$\pm \frac{i'a'}{\sqrt{2}} \cdot \frac{\mu \sin \varphi}{r^2} .$$

[228][Note by AKTA:] That is, in Subsection 5.25.2.

ku'/i' in order to obtain the components $\mathfrak{X}'$, $\mathfrak{Y}'$, $\mathfrak{Z}'$ of the electromotive force, which a *whole magnet* exerts on the induced element α'. From this it follows that

$$\mathfrak{X}' = -\frac{ku'}{\sqrt{2}} \cdot \alpha' \left(C' \cos \gamma - B' \cos \delta \right) \ ,$$

$$\mathfrak{Y}' = -\frac{ku'}{\sqrt{2}} \cdot \alpha' \left(A' \cos \delta - C' \cos \beta \right) \ ,$$

$$\mathfrak{Z}' = -\frac{ku'}{\sqrt{2}} \cdot \alpha' \left(B' \cos \beta - A' \cos \gamma \right) \ ,$$

in which A', B', C' have the same signification as under (2).[229]

The relations will now be examined between the laws set forth here and the empirical proposition mentioned in the beginning. Now, from the foregoing laws there results, when the *electrodynamic* forces stand to the *electromagnetic* forces in the ratio $1 : n$, viz., when

$$\frac{X'}{X} = \frac{Y'}{Y} = \frac{Z'}{Z} = n$$

or, if for X, Y, Z, and X', Y', Z', their values found above are substituted, when

$$\frac{C' \cos \mu - B' \cos \nu}{C \cos \mu - B \cos \nu} = \frac{A' \cos \nu - C' \cos \lambda}{A \cos \nu - C \cos \lambda} = \frac{B' \cos \lambda - A' \cos \mu}{B \cos \lambda - A \cos \mu} = \frac{i}{\sqrt{2}} \cdot n \ ,$$

hence

$$A' = \frac{i}{\sqrt{2}} \cdot nA \ , \quad B' = \frac{i}{\sqrt{2}} \cdot nB \ , \quad C' = \frac{i}{\sqrt{2}} \cdot nC \ ,$$

the following ratio of the *electromotive* force obtained by means of *voltaic induction* and by means of *magnetic induction*:

$$\frac{\mathfrak{X}'}{\mathfrak{X}} = \frac{k\sqrt{2}}{ai} \cdot \frac{C' \cos \gamma - B' \cos \delta}{C \cos \gamma - B \cos \delta} = \frac{k}{a} \cdot n \ ,$$

$$\frac{\mathfrak{Y}'}{\mathfrak{Y}} = \frac{k\sqrt{2}}{ai} \cdot \frac{A' \cos \delta - C' \cos \beta}{A \cos \delta - C \cos \beta} = \frac{k}{a} \cdot n \ ,$$

$$\frac{\mathfrak{Z}'}{\mathfrak{Z}} = \frac{k\sqrt{2}}{ai} \cdot \frac{B' \cos \beta - A' \cos \gamma}{B \cos \beta - A \cos \gamma} = \frac{k}{a} \cdot n \ .$$

This, finally, yields the following result:

$$\frac{X'}{X} : \frac{\mathfrak{X}'}{\mathfrak{X}} = \frac{Y'}{Y} : \frac{\mathfrak{Y}'}{\mathfrak{Y}} = \frac{Z'}{Z} : \frac{\mathfrak{Z}'}{\mathfrak{Z}} = a : k \ ,$$

which is in agreement with the empirical proposition mentioned at the beginning, because the ratio $a : k$ is *constant*. That empirical proposition, however, shows us still more than the comparison of the above laws, in that it makes this constant ratio equal to *unity*, by means of which the constant factor in the fundamental law of magnetic induction, k, a factor still undetermined by any measurement as yet, becomes equal to the constant factor a in the

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[229][Note by AKTA:] That is, in Subsection 5.25.2.

fundamental electrical law. Specifically, that would also have to take place, if there existed no magnetic fluid in the magnet, but, in accord with Ampère, all the effects of the magnets were produced by electrical currents in them.

5.26 Comparison with the Theorems Established by Fechner and Neumann

Fechner has been the first to attempt, by developing their intrinsic connection, an explanation of the Faraday phenomena of induction in terms of the Ampère electrodynamic phenomena, which Lenz[230] previously put into relation with one another merely by means of an empirical rule; Fechner has published the explanation in Poggendorff's *Annalen*, 1845, Vol. LXIV, page 337.[231] In so doing, Fechner has confined himself to that form of *voltaic induction*, with which the foregoing Section dealt, namely, to that by a constant current at rest in a conducting wire moving toward it. For this form of *voltaic induction*, Fechner has actually succeeded in discovering its *intrinsic connection* with Ampère's electrodynamic phenomena, and in basing an explanation of it on a somewhat *more generalized* form of Ampère's law which holds for the latter phenomena. — That *intrinsic connection* consists essentially in the fact that, with regard to that induction, apart from the current first elicited by the induction, one is dealing, just as in the Ampère phenomena, with *interactions of electrical currents*, hence the explanation of both kinds of phenomena would have to rest on the laws of these interactions. The electricity in the induced conducting wire, Fechner says specifically, would also begin to flow, as soon as this conducting wire were *moved*, specifically because it participates in the motion of its carrier. The electrical currents in such induced conducting wires are only differentiated from the galvanic currents in the inducing wires in that equal masses of *positive* and *negative* electricity move simultaneously with the same velocity in *opposite* directions in the latter, in *the same* directions in the former. — The *generalization* which Fechner has given to Ampère's law, consists *first* in the fact that the force which, according to Ampère, acts on the ponderable carrier, would originally act with the same strength and in the same direction on the electrical masses located in the carrier, and would first be communicated from them to the carrier; *secondly*, in the fact that Ampère's law does not merely hold for the total action of a galvanic current on another, but also for the two partial actions, which the first current would exert on the *positive* and *negative* electricity of the second.

This explanation accords with the theory of this induction developed in the previous Section; for one finds there the justification of the right to generalize Ampère's law, on which that explanation is founded. This can be proven, if one considers in particular the two forces acting on the *positive* or *negative* electricity, as stated on page[232] 150, where one finds that Ampère's law holds not merely for all four forces, but also for any two of them.

Moreover, Fechner himself has already remarked that the standpoint from which he has interpreted the connection of Faraday's induction phenomena with the Ampère electrodynamic phenomena is not so general that it could be extended over all of Faraday's induction phenomena. As soon as the induced wire is *at rest*, the induction phenomena cannot be grasped from this standpoint, because then the motion of electricity in the induced wire is

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[230][Note by AKTA:] See footnote 40 on page 24.

[231][Note by AKTA:] [Fec45] with English translation in [Fec21]. See Chapter 3.

[232][Note by LH and AKTA:] [Web46, p. 158 of Weber's *Werke*].

out of the question. On this point, Fechner says, *loc. cit.*, page 341:[233]

> In the induction experiments, instead of moving the (neutral) wire $a'b'$ *away from* the (excited) wire ab at rest, one could do the opposite, and the induction would always still occur. This must be accepted as an *empirical datum*, for proving that what matters here is simply the relation of the motions, and that it is permissible to substitute the converse for motion of the excited wire and rest in the neutral wire, in order to be able to apply the principle in the stated form.

Neumann has based his investigation on the empirical rule by which Lenz linked the Faraday induction phenomena to the Ampère electrodynamic phenomena, and has found a supplement to it in the proposition, that the strength of the induction is proportional to the velocity of the motion of the induced wire, when the induction was elicited by a motion of the latter. These two empirical rules complement each other in such a way, that Neumann has been able to derive from them the *general laws of induced currents*, since the laws immediately following from them for the case in which the induction is elicited by a motion of the induced conductor, are of the kind that can immediately find application in wider domains without undergoing modification, and can be extended to all forms of induction. These *general laws of induced currents* admit of virtually no doubt, with respect to their intrinsic connection or also to the empirical rules implied in them, and for that reason it is interesting to compare the results of the theory developed above with these laws which Neumann derived in completely different ways.

Since Neumann's Treatise, submitted to the *königliche Akademie der Wissenschaften* in Berlin, has not yet been printed, I can only refer to the excerpt just now appearing in Poggendorff's *Annalen*, in this year's first issue, from which I take the following passage:[234]

> From Lenz's theorem that the action which the inducing current or magnet exerts on the induced conductor, always produces, when the induction is elicited by a motion of the latter, an inhibiting influence on this motion, conjointly with the theorem that the strength of the momentary induction is proportional to the velocity of this motion, is derived the general law of linear induction:

$$Eds = -\varepsilon v C ds \ .$$

> Here ds signifies an element of the induced wire, and Eds the electromotive force induced in the element ds; v is the velocity, with which ds is moved, C is the action of the inductor on ds, resolved according to the direction in which ds is moved, this element being thought of as having the unit of current flowing through it. The magnitude ε, independent of the nature of the induced conductor, can be treated as a constant in the case of linear induction, but is a function of time, such that it very quickly decreases, when its argument has an appreciable value, and be treated as such in the case of surface induction and of induction in bodies.

From the theory developed above, has resulted the following expression, at the end of Section 5.24, for the *electromotive* force induced in element α', in which u' denotes the velocity with which α' is moved:

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[233][Note by AKTA:] [Fec45, p. 341].
[234][Note by AKTA:] [Neu46] and [Neu48].

$$-\frac{\alpha\alpha'}{r^2}i\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right)\cdot au'\cos\varphi\ .$$

This expression was the value, resolved in the direction of element α', of the total separating force[235] exerted by the inductor α in the direction of the connecting straight line r, from which, by elimination of the factor $\cos\varphi$, the total force is once more obtained. In Section 5.25, at (3),[236] this total force is compared with the *electrodynamic* force, determined by Ampère's law, which the inductor α would exert on element α', when α' were parallel to the direction in which the element α' were moved for purposes of induction, and through which a current flowed in this direction, whose intensity were $= i'$. Namely, one obtains that total *electromotive* force exerted in the direction of the connecting straight line r by multiplying this *electrodynamic* force by the factor au'/i'. The above expression itself is obtained by multiplying the same force, *resolved in the direction of the induced element α'*, by the factor au'/i'. If, therefore, this electrodynamic force, resolved in the direction of the induced element α', is denoted

$$i'\alpha'\cdot D\ ,$$

then the above expression is to be made

$$= -au'D\alpha'\ .$$

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[235][Note by AKTA:] In German: *scheidenden Kraft.* This expression can also be translated as "force of separation" or "segregating force".

I present here a simple example of a separating force. Consider a metal plate AB insulated from the ground by a dielectric support I as in Figure (a) of this footnote:

(a)　　　　　　　(b)
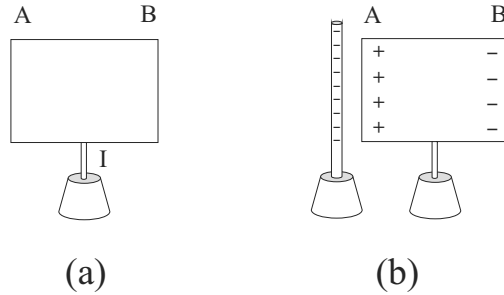
If a negatively charged straw is placed close to side A of the plate, the charges on the plate become separated as illustrated in Figure (b). Side A of the plate becomes positively electrified, while side B becomes negatively electrified. This polarization of the plate is caused by the electric force of the negatively electrified straw acting on the free electrons of the plate. I presented several interesting experiments on this topic made with simple material, together with many quotes from original sources, in the 2 volumes of the book *The Experimental and Historical Foundations of Electricity* which is available in English, Portuguese, Italian and Russian: [Ass10a], [Ass10b], [Ass15], [Ass17], [Ass18a], [Ass18b] and [Ass19].

Another effect of a separating force takes place in electrolysis. The electric forces in general are proportional to the charge q of the test particle on which they are acting. A positively electrified particle with $q > 0$ experiences a force in one direction, while a negatively electrified particle with $q < 0$ will be forced in the opposite direction. If these particles are free to move as in electrolysis, a double current will be produced due to this separating electric force. That is, the positive particles will move in one direction and the negative particles will move in the opposite direction.

[236][Note by AKTA:] That is, in Subsection 5.25.3.

Here, u' and α' are to be written v and ds, in accordance with Neumann's notation; hence the theory developed above, yields the equation, in this notation:

$$Eds = -avDds ,$$

in which a denotes a constant factor independent of the nature of the induced conductor, like ε in Neumann's equation, because here it is a matter of linear induction. Both equations are thus in agreement with each other up to the factors C and D. These factors also have in common their ability, multiplied by ds, to express the *electrodynamic* force, resolved in a definite direction, which the inductor would exert on an element ds, thought of as located in the place through which the induced unit of current flows. Yet the two factors are differentiated from one another

1. by the direction, which the element ds, thought of as at the point of induction, would be given, and

2. through the direction in which the electrodynamic force exerted on this element is to be resolved. Specifically, these two directions are *exchanged* in Neumann's law.

Neumann's law would, as can be seen from this, *contradict* ours, if one wanted to apply it to an individual current element as inductor, because factors C and D would then have entirely different values. It is obvious, however, that Neumann's law, in accordance with its derivation, holds first of all not for that individual inducing current element, but only for a closed circuit or for a magnet as inductor, specifically because Lenz's theorem, from which it is derived, can, being experimentally based, hold merely for closed circuits and magnets. That *apparent contradiction* now automatically dissolves, as soon as the application of Neumann's law is confined to closed circuits, interchangeable with magnets, as inductors, in which case the identity of factors C and D can then be proven in the following way.

According to Ampère, the three components X, Y, Z of that force which a closed circuit of intensity i, for which the position of the elements is defined by the coordinates x, y, z, exerts on any other current element ds' of current intensity i', whose direction makes the angles λ, μ, ν with the coordinate axes, when the origin of the coordinates lies in the center of the element ds', are

$$X = -\frac{1}{2}ii'ds' \left(\cos\mu \cdot \int \frac{xdy - ydx}{r^3} - \cos\nu \cdot \int \frac{zdx - xdz}{r^3} \right)$$

$$Y = -\frac{1}{2}ii'ds' \left(\cos\nu \cdot \int \frac{ydz - zdy}{r^3} - \cos\lambda \cdot \int \frac{xdy - ydx}{r^3} \right)$$

$$Z = -\frac{1}{2}ii'ds' \left(\cos\lambda \cdot \int \frac{zdx - xdz}{r^3} - \cos\mu \cdot \int \frac{ydz - zdy}{r^3} \right) .$$

From this the values for the factors C and D can now be derived for closed circuits as inductors.

For, *first*, factor C in Neumann's law is obtained, if X_1, Y_1, Z_1 denote the values taken on by X, Y, Z when we make $i' = 1$ and λ, μ, ν are the angles which the induced element forms with the coordinate axes. Namely, if α, β, γ are the angles which the direction in which the induced element is moved, forms with the three coordinate axes, then

$$Cds' = X_1 \cos\alpha + Y_1 \cos\beta + Z_1 \cos\gamma \ .$$

This expression is simplified, if a coordinate system is chosen in which the direction of the x axis coincides with the direction *in which the induced element is moved*. Namely, then

$$\cos\alpha = 1 \ , \quad \cos\beta = 0 \ , \quad \cos\gamma = 0 \ ,$$

hence

$$Cds' = X_1 = -\frac{1}{2}ids'\left(\cos\mu\int\frac{xdy - ydx}{r^3} - \cos\nu\int\frac{zdx - xdz}{r^3}\right) \ .$$

Secondly, factor D is obtained, if the values assumed by X, Y, Z are denoted X', Y', Z', when we make $i' = 1$, and $\lambda = \alpha'$, $\mu = \beta'$, $\nu = \gamma'$, where α', β', γ' are the angles which the direction in which the induced element is moved, forms with the three coordinate axes (which would thus be identical with α, β, γ, if the same coordinate system were chosen). Namely, if, according to the present coordinate system, λ', μ', ν' are the angles which the induced element forms with the three coordinate axes (which would thus be identical with λ, μ, ν, if the present coordinate system were identical with the former one), then:

$$Dds' = X'\cos\lambda' + Y'\cos\mu' + Z'\cos\nu' \ .$$

This expression is simplified, if one chooses a different coordinate system, as earlier, namely, one in which the direction of the x-axis coincides with the direction *of the induced element itself*, because then

$$\cos\lambda' = 1 \ , \quad \cos\mu' = 0 \ , \quad \cos\nu' = 0$$

hence:

$$Dds' = X' = -\frac{1}{2}ids'\left(\cos\beta'\int\frac{xdy - ydx}{r^3} - \cos\gamma'\int\frac{zdx - xdz}{r^3}\right) \ .$$

Now the two coordinate systems, namely, that in which the x-axis is parallel to the direction *in which the induced element is moved*, and that in which the x-axis is parallel to the direction *of the induced element itself*, can have in common the y-axis, if it is normal to both directions, that of the induced element and its motion. Assuming this, it will be the case that

$$\cos\mu = 0 \ , \quad \cos\beta' = 0 \ , \quad \cos\nu = \cos\gamma' \ ,$$

and since, moreover, it can be proven that

$$\int\frac{zdx - xdz}{r^3}$$

would have an equal value according to both coordinate systems, then

$$C = D \ ,$$

which was to be proven. That $zdx - xdz$ would have the same value for all right-angled coordinate systems in which, as in the two above, the origin coincides with the y-axis, is

evident from the fact that $\frac{1}{2}(zdx - xdz)$ represents the area projected on a plane normal to the common axis y, which is formed by the common coordinate-origin, and by the current element in question. The straight line r, which connects the current element in question with the induced element, has a value altogether independent of the coordinate system chosen. From this it results that the value of the quotient[237] $(zdx - xdz)/r^3$ for the two coordinate systems employed above is always the same, hence also is [equal] the value of the integral extended over the entire closed circuit $\int \frac{zdx - xdz}{r^3}$.

It follows from this that Neumann's law for the domain of phenomena to which, in virtue of its derivation, it refers, namely, where all inductors are either magnets or closed circuits, concurs with the law derived from the theory developed above, but that the application of Neumann's law outside that domain to non-closed circuits as inductors is not permitted.

5.27 Law of Excitation of a Current in a Conductor at Rest, when a Constant Current Element Approaches or Withdraws from It

The law of *voltaic induction* for this case, where the induced conductor is at rest, and the inducing current element is in motion, can be derived just as it was for the first case, from the established fundamental electrical law. It is, however, not necessary to give this derivation, because a simple consideration shows that, for the second case, it would have to lead back to the same law as for the first.

Namely, the fundamental electrical law, from which all laws of *voltaic induction* are to be derived, makes the action of one electrical mass on another dependent merely upon their *relative* distance, velocity, and acceleration. These, however, remain unchanged by a *common* motion attributed to both masses; hence, the action of one electrical mass on another is also not changed by such a *common* motion. Consequently, such a *common* motion can be attributed to all electrical masses without changing their actions, hence also without changing the induction dependent upon them. Therefore, if one has an inducing current element α, which is in motion with the *absolute* velocity u' in any direction, while the induced element α' is at *absolute* rest, then, without changing the induction, one can attribute to both elements, along with the electrical masses contained in them, a *common* motion of velocity u' in that direction which is diametrically opposite to the direction in which current element α actually is in motion. By adding this common motion, the inducing element α is brought to rest, while now the induced element α' moves with the same velocity, but in the opposite direction, as the current element is actually moving. Therefore, from the established fundamental law, the same induction must result for the same *relative* motion of both elements, independently of whether, during this relative motion, one or the other or neither of the two elements is at *absolute* rest. As is well known, empirical experience accords with this result.

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[237][Note by AKTA:] Due to a misprint, the next equation appeared in the original as $zdx - xdz/r^3$.

5.28 Law of Excitation of a Current in a Conductor by Changing the Current Intensity in an Adjacent Conductor

If α and α' denote the lengths of the inducing and induced elements, then in both elements four electrical masses can be further distinguished:

$$+\alpha e \,, \quad -\alpha e \,, \quad +\alpha' e' \,, \quad -\alpha' e' \,.$$

The *first* of these masses $+\alpha e$ would move with the *variable* velocity u in the direction of the element at rest α, which makes the angle ϑ with the straight line drawn from α to α', and du would denote the change in u during time-element dt; the *second*, $-\alpha e$, would move, in accordance with the determinations relating to a galvanic current, in the same direction with velocity $-u$, that is backwards, and $-du$ would denote the change in this velocity during time-element dt; the *third*, $+\alpha' e'$, would move with constant velocity $+u'$ in the direction of the element at rest α', which makes the angle ϑ' with the straight line drawn and elongated from α to α'; the *fourth*, $-\alpha' e'$, would, finally, move, again according to the determinations relating to a galvanic current, in the same direction with velocity $-u'$, viz., backwards. The distances of the first two masses from the second two are themselves all the same at the moment in question as distance r between the two elements α and α'; since, however, they do not remain equal, they are to be denoted r_1, r_2, r_3, r_4.

For the *sum* of the forces which are acting on the *positive* and *negative* electricity in element α', i.e., for the force, which moves element α' itself, one obtains the same expression as in Section 5.24, namely:

$$-\frac{a^2}{16} \cdot \frac{\alpha e \cdot \alpha' e'}{r^2} \left\{ \left(\frac{dr_1^2}{dt^2} + \frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) \right.$$
$$\left. - 2r \left(\frac{d^2 r_1}{dt^2} + \frac{d^2 r_2}{dt^2} - \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) \right\} \,.$$

However, for the *difference* of those forces, on which the *induction* depends,

$$-\frac{a^2}{16} \cdot \frac{\alpha e \cdot \alpha' e'}{r^2} \left\{ \left(\frac{dr_1^2}{dt^2} - \frac{dr_2^2}{dt^2} + \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) \right.$$
$$\left. - 2r \left(\frac{d^2 r_1}{dt^2} - \frac{d^2 r_2}{dt^2} + \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) \right\} \,.$$

Further, the same values hold here for the first differential coefficients as were found in Section 5.22, namely:

$$\frac{dr_1}{dt} = -\frac{dr_2}{dt} = -u \cos \vartheta + u' \cos \vartheta' \,,$$

$$\frac{dr_3}{dt} = -\frac{dr_4}{dt} = -u \cos \vartheta - u' \cos \vartheta' \,.$$

Hence

$$\left(\frac{dr_1^2}{dt^2} + \frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) = -8uu' \cos \vartheta \cos \vartheta' \,,$$

$$\left(\frac{dr_1^2}{dt^2} - \frac{dr_2^2}{dt^2} + \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2}\right) = 0 \ .$$

Since the velocity u is now variable, however, there result values for the second differential coefficients other than those in Section 5.22, where it was constant, namely:

$$\frac{d^2r_1}{dt^2} = +u\sin\vartheta \cdot \frac{d\vartheta_1}{dt} - u'\sin\vartheta' \cdot \frac{d\vartheta_1'}{dt} - \cos\vartheta \cdot \frac{du}{dt} \ ,$$

$$\frac{d^2r_2}{dt^2} = -u\sin\vartheta \cdot \frac{d\vartheta_2}{dt} + u'\sin\vartheta' \cdot \frac{d\vartheta_2'}{dt} + \cos\vartheta \cdot \frac{du}{dt} \ ,$$

$$\frac{d^2r_3}{dt^2} = +u\sin\vartheta \cdot \frac{d\vartheta_3}{dt} + u'\sin\vartheta' \cdot \frac{d\vartheta_3'}{dt} - \cos\vartheta \cdot \frac{du}{dt} \ ,$$

$$\frac{d^2r_4}{dt^2} = -u\sin\vartheta \cdot \frac{d\vartheta_4}{dt} - u'\sin\vartheta' \cdot \frac{d\vartheta_4'}{dt} + \cos\vartheta \cdot \frac{du}{dt} \ .$$

Therefore, there results for

$$\left(\frac{d^2r_1}{dt^2} + \frac{d^2r_2}{dt^2} - \frac{d^2r_3}{dt^2} - \frac{d^2r_4}{dt^2}\right) = +u\sin\vartheta \left(\frac{d\vartheta_1}{dt} - \frac{d\vartheta_2}{dt} - \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt}\right)$$

$$- u'\sin\vartheta' \left(\frac{d\vartheta_1'}{dt} - \frac{d\vartheta_2'}{dt} + \frac{d\vartheta_3'}{dt} - \frac{d\vartheta_4'}{dt}\right)$$

the same value as in Section 5.22, namely, when one substitutes the values of $d\vartheta_1/dt$, $d\vartheta_1'/dt$, and so forth, developed there on page[238] 155,

$$r\left(\frac{d^2r_1}{dt^2} + \frac{d^2r_2}{dt^2} - \frac{d^2r_3}{dt^2} - \frac{d^2r_4}{dt^2}\right) = -8uu'\sin\vartheta\sin\vartheta'\cos\omega \ .$$

On the other hand,

$$\left(\frac{d^2r_1}{dt^2} - \frac{d^2r_2}{dt^2} + \frac{d^2r_3}{dt^2} - \frac{d^2r_4}{dt^2}\right) = +u\sin\vartheta \left(\frac{d\vartheta_1}{dt} + \frac{d\vartheta_2}{dt} + \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt}\right)$$

$$- u'\sin\vartheta' \left(\frac{d\vartheta_1'}{dt} + \frac{d\vartheta_2'}{dt} - \frac{d\vartheta_3'}{dt} - \frac{d\vartheta_4'}{dt}\right) - 4\cos\vartheta \cdot \frac{du}{dt} \ .$$

Since, however, according to page[239] 155, the values

$$\frac{d\vartheta_1}{dt} + \frac{d\vartheta_2}{dt} = \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt} = \frac{d\vartheta_1'}{dt} + \frac{d\vartheta_2'}{dt} = \frac{d\vartheta_3'}{dt} + \frac{d\vartheta_4'}{dt} = 0$$

then

$$\left(\frac{d^2r_1}{dt^2} - \frac{d^2r_2}{dt^2} + \frac{d^2r_3}{dt^2} - \frac{d^2r_4}{dt^2}\right) = -4\cos\vartheta \cdot \frac{du}{dt} \ .$$

If these values are substituted, one obtains the *sum* of the forces acting on the *positive* and *negative* electricity in element α', as [in] Section 5.22

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[238][Note by LH and AKTA:] [Web46, p. 162 of Weber's *Werke*].
[239][Note by LH and AKTA:] [Web46, p. 162 of Weber's *Werke*].

$$= -\frac{\alpha\alpha'}{r^2} \cdot aeu \cdot ae'u' \left(\sin\vartheta \sin\vartheta' \cos\omega - \frac{1}{2}\cos\vartheta \cos\vartheta' \right) ,$$

viz., the force acting on element α' is, when the current intensity is variable, determined just as it is when the current is constant, and Ampère's law is applicable to variable currents as well.

The *difference* between those two forces acting on the *positive* and *negative* electricity in element α', on which the *induction* depends, results, on the other hand, as

$$= -\frac{1}{2}\frac{\alpha\alpha'}{r} \cdot a^2 ee' \cdot \cos\vartheta \cdot \frac{du}{dt} ,$$

or, since according to page[240] 144, $aeu = i$, hence u is variable, and $ae \cdot du = di$,

$$= -\frac{1}{2}\frac{\alpha\alpha'}{r} \cdot ae' \cdot \cos\vartheta \cdot \frac{di}{dt} .$$

The force determined in this way tries to separate the *positive* and *negative* electricities in the induced element α' in the direction of the straight line r. But in this direction the separation can not succeed, it can only happen in the direction of the induced element α' itself, which forms the angle ϑ' with the extended straight line r. Decomposing then this total force, which tries to separate both electricities in α', along this direction, that is, multiplying the *difference* above with $\cos\vartheta'$, we obtain the force which produces the real separation,

$$= -\frac{1}{2}\frac{\alpha\alpha'}{r} \cdot ae' \cdot \cos\vartheta \cos\vartheta' \cdot \frac{di}{dt} .$$

If this value is divided by e', there results the *electromotive* force, in the ordinary sense, exerted by the inducing element α on the induced element α' (see Section 5.24, page[241] 163):

$$= -\frac{a}{2} \cdot \frac{\alpha\alpha'}{r} \cdot \cos\vartheta \cos\vartheta' \cdot \frac{di}{dt} .$$

The *induction* during the time element dt, viz., the product of this time element with the acting electromotive force, is therefore

$$= -\frac{a}{2} \cdot \frac{\alpha\alpha'}{r} \cdot \cos\vartheta \cos\vartheta' \cdot di ,$$

hence the *induction* for any period of time, in which the intensity of the inducing current increases to i, while r, ϑ and ϑ' remain unchanged,

$$= -\frac{a}{2} \cdot \frac{\alpha\alpha'}{r} i \cos\vartheta \cos\vartheta' .$$

The *positive* value of this expression denotes an induced current in element α' in the direction of α', which makes the angle ϑ' with the extended straight line r; the *negative* value denotes an induced current of opposite direction.

If both elements α and α' are parallel to each other, and $\vartheta = \vartheta'$, the above expression has a *negative* value for *increasing* current intensity, or for a positive value of i, viz., when the current intensity increases in α, a current in the opposite direction from the inducing current

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[240][Note by LH and AKTA:] [Web46, p. 152 of Weber's *Werke*].
[241][Note by LH and AKTA:] [Web46, p. 170 of Weber's *Werke*].

is excited in α'. The reverse takes place when the current intensity decreases. Both results agree with known facts. The proportionality of the induction to the change in intensity i of the inducing current also corresponds to empirical experience, to the degree that estimates suffice without precise measurement.

5.29 Comparison of Induction Effects of Constant Currents on a Moving Conductor with Those of Variable Currents on Conductors at Rest

In the previous Section, the laws of *voltaic induction* have been derived from the fundamental electrical law, in agreement with empirical experience, not only for the case where the voltaic induction is elicited by *constant* currents in *moving* conductors, but also for the case, where it is elicited by *variable* currents in conductors *at rest*. The *laws of induction* for these two cases are very different, and on that account it is very interesting, that nevertheless they yield very simple relationships between the effects of both inductions.

One such simple relationship between the induction effect of *constant* currents on a *moving* conductor and the induction effect of *variable* currents on a conductor at rest, results from the laws already developed in Sections 5.24 and 5.28 for individual inducing and induced elements, when the motion of the induced element occurs, in the first case, in the direction of the straight line r. For if one calculates under this assumption the total induction effect, which a current element of *constant* intensity i elicits, while the induced element is withdrawn from a given position infinitely far in the direction of the straight line r, or, from an infinite distance, approaches that position, then one finds that this total induction effect is equal to that which the inducing element would elicit, if its current intensity were to decrease or increase by i, in the induced element, if it continued in the given position. Therefore this yields the rule, for this special case, to begin with, *that, by means of the appearance or disappearance of a current in the proximity of a conductor, the same current would be induced in this conductor, as if that current would have uniformly persisted, but were either transferred from a great distance into that proximity to the conductor, or, conversely, transferred from that proximity to a great distance.*

For the cited special case, this theorem easily results, as follows. The expression found at the end of Section 5.24 for the electromotive force is to be multiplied by the time element dt, in order to obtain the *induction effect* corresponding to this time element dt, or corresponding to the element of displacement $u'dt$ traversed during this time element. The value of the integral of this product between definite time or displacement limits then yields the total *induction effect* corresponding to the time interval or to the displacement traversed in that time interval

$$= -ai \int \frac{\alpha\alpha'}{r^2} \left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta' \right) \cos\varphi \cdot u'dt \ .$$

In our case, where the motion occurs in the straight line r, is now

$$u'dt = dr \ , \qquad \text{and} \qquad \cos\vartheta' = 1 \ .$$

According to Section 5.24, $\cos\varepsilon = \sin\vartheta\sin\vartheta'\cos\omega + \cos\vartheta\cos\vartheta'$, therefore here:

$$\cos \varepsilon = \cos \vartheta \ .$$

Since, finally, the angles ϑ and φ have constant values during the motion in the direction of straight line r of the element α' constantly parallel to itself, that *induction effect* is

$$= + \frac{ai}{2} \cdot \alpha\alpha' \cos \vartheta \cos \varphi \cdot \int \frac{dr}{r^2} \ .$$

The value of this integral between the limits $r = r$ to $r = \infty$, viz., the *induction effect*, while the induced element is infinitely distant from a given position, is

$$= + \frac{ai}{2} \frac{\alpha\alpha'}{r} \cos \vartheta \cos \varphi \ ;$$

between the limits $r = \infty$ to $r = r$, viz. the *induction effect*, while the induced element, from an infinite distance, *reaches* a given position, is, on the contrary,

$$= - \frac{ai}{2} \frac{\alpha\alpha'}{r} \cos \vartheta \cos \varphi \ .$$

If it is taken into consideration that φ denotes here, in accordance with Section 5.24, the same angle which is ϑ' in Section 5.28, namely, the angle which the induced element α' makes with the prolonged straight line r, then it is seen that the induction effect is equal to that which, according to the law given in Section 5.28, is obtained when the induced element α' persists in the given position, and the current intensity i in the inducing element α vanishes or arises.

The relation found for both induction effects can be expressed more generally, not, of course, for individual elements, but for *closed currents and conductors*. The case may first of all be considered, where all elements of the induced closed conductor have the same, parallel, motion.

The *induction effect* of current element α on the induced element α' is, as before,

$$= -ai \int \frac{\alpha\alpha'}{r^2} \left(\cos \varepsilon - \frac{3}{2} \cos \vartheta \cos \vartheta' \right) \cos \varphi \cdot u'dt \ .$$

If β and β' now denote the angles, which the two elements α and α' make with the plane produced by the straight line r by the motion of the element α', and further, if γ and γ' denote the angles, which the projections of α and α' make in the plane with the direction of the motion, then

$$\cos \vartheta = \cos \beta \cos(\vartheta' - \gamma) \ ,$$

$$\cos \varphi = \cos \beta' \cos(\vartheta' - \gamma') \ ,$$

$$\cos \varepsilon = \cos \beta \cos \gamma \ .$$

The projection of the displacement element $u'dt$ on the straight line r yields the value of dr for the time-element dt,

$$dr = u'dt \cdot \cos \vartheta' \qquad \text{or} \qquad u'dt = \sec \vartheta' \cdot dr \ .$$

If these values are substituted, the *induction effect* of α on α' becomes

$$= -\int ai\alpha\alpha' \cos\beta \cos\beta' \left(\cos\gamma \sec\vartheta' - \frac{3}{2}\cos(\vartheta' - \gamma)\right) \cos(\vartheta' - \gamma') \cdot \frac{dr}{r^2} \ ,$$

or, when $\cos(\vartheta' - \gamma)$ and $\cos(\vartheta' - \gamma')$ are developed,

$$= +\frac{ai}{2}\int \alpha\alpha' \cos\beta \cos\beta' \cdot dR \ ,$$

in which, for the sake of brevity, the following expression is denoted by dR:

$$\left(\cos\gamma \cos\gamma' - 2\cos\gamma \sin\gamma' \tan\vartheta' - 3\cos(\gamma + \gamma')\sin^2\vartheta'\right.$$

$$\left. + 3\sin(\gamma + \gamma')\sin\vartheta' \cos\vartheta'\right) \cdot \frac{dr}{r^2} \ .$$

If it is taken into consideration, that in the like and parallel motion of all elements, each of them is displaced *parallel with itself*, hence the angles β, β', γ, γ' are constant, and if one makes

$$\sin\vartheta' = \frac{b}{r} \ , \quad \cos\vartheta' = \frac{\sqrt{r^2 - b^2}}{r} \ , \quad \tan\vartheta' = \frac{b}{\sqrt{r^2 - b^2}} \ ,$$

in which b denotes the perpendicular from α to the path of induced element α', then the integration can be carried out, and the following expression is obtained as an *indefinite integral*:

$$-\frac{ai}{2}\frac{\alpha\alpha'}{r}\cos\vartheta \cos\varphi - \frac{ai}{2}\frac{\alpha\alpha'}{r}\cos\beta \cos\beta' \sin(\gamma' - \gamma)\cot\vartheta' \ .$$

The sought-for *induction effect* is the *definite integral* or the difference between the two values, which the expression receives, when the two limiting values for r, ϑ, φ, and ϑ' are substituted in it.

If the same expression as that for elements α and α' is formed for *all* combinations of inducing and induced elements, which are contained in the closed circuit and conductor, and if the *summation* of all of them is denoted

$$-\frac{ai}{2}\mathbf{S}\frac{\alpha\alpha'}{r}\cos\vartheta \cos\varphi - \frac{ai}{2}\mathbf{S}\frac{\alpha\alpha'}{r}\cos\beta \cos\beta' \sin(\gamma' - \gamma)\cot\vartheta' \ ,$$

then the induction effect of the closed circuit on the closed conductor is equal to the difference between the two values, which this summation receives, when the values for r, ϑ, φ and ϑ', corresponding to those at the beginning and end of the induction, are substituted in it.

Now, the above *summation* consists of two terms, and it will be proven, that the *latter term* is null for all values of r and ϑ'. Then the *induction effect* of a *closed* circuit on a *closed* conductor reduces itself to the difference between the two values, which the *first term* of the above summation assumes, when the values for r, ϑ, φ, corresponding to the beginning and end of the induction are substituted in it.

That the *latter term* of the above summation is, namely,

$$-\frac{ai}{2}\mathbf{S}\frac{\alpha\alpha'}{r}\cos\beta \cos\beta' \sin(\gamma' - \gamma)\cot\vartheta' = 0$$

can easily be proven, if one analyzes the inducing and induced elements according to the law that, for determining the interaction of two elements, for any one of them, three others can be put in, which form the three edges of a parallelepiped, whose diagonal is taken up by the given elements. On this theorem, see Section 5.31 below.

Accordingly, if the elements α and α' are each decomposed into three elements, of which the *first* would be parallel to the direction of the motion, the *second* perpendicular to r, in the plane produced by r when α' is in motion, the *third* perpendicular to the two others, and if they are denoted

$$\alpha_1 , \quad \alpha_2 , \quad \alpha_3 , \quad \text{and} \quad \alpha'_1 , \quad \alpha'_2 , \quad \alpha'_3 ,$$

then $[\alpha\alpha'/r]\cdot\cos\beta\cos\beta'\sin(\gamma'-\gamma)\cot\vartheta'$ becomes a summation of 9 terms. For the two terms proportional to $\alpha_3\alpha'_1$ and to $\alpha_3\alpha'_2$, the factor is $\cos\beta = 0$; for the two terms proportional to $\alpha_1\alpha'_3$ and to $\alpha_2\alpha'_3$, the factor is $\cos\beta' = 0$; for the term proportional to $\alpha_3\alpha'_3$ the two factors are $\cos\beta = \cos\beta' = 0$; finally, for the 6th and 7th terms, which are proportional to $\alpha_1\alpha'_1$ and to $\alpha_2\alpha'_2$, the factor is $\sin(\gamma'-\gamma) = 0$. Hence there remain only two more terms, namely, those proportional to $\alpha_1\alpha'_2$ and to $\alpha_2\alpha'_1$, for which $\cos\beta = 1$, $\cos\beta' = 1$, $\sin(\gamma' - \gamma) = \mp\cos\vartheta'$; these two terms are thus:

$$\pm\frac{ai}{2}\cdot\frac{\alpha_1\alpha'_2}{r}\cos\vartheta'\cot\vartheta' \qquad \text{and} \qquad \pm\frac{ai}{2}\cdot\frac{\alpha_2\alpha'_1}{r}\cos\vartheta'\cot\vartheta' ,$$

and for the sake of brevity, may be denoted A and B. If one now proceeds in like manner with each two elements of the closed circuit and conductor, then one finds that, among the remaining terms formed in just this way, two terms exist, by which A and B are cancelled, and which are to be denoted A' and B'. If this holds true in general, then it follows that

$$-\frac{ai}{2}\mathbf{S}\frac{\alpha\alpha'}{r}\cos\beta\cos\beta'\sin(\gamma' - \gamma)\cot\vartheta' = 0 ,$$

which was to be proven.

Now, the element A', by which A was cancelled, is found in the following way. Through the center of the *inducing* element α as apex, let two cones be put, whose common axis would be parallel to the direction of motion, i.e., to α_1. Let these two cones delimit the *induced* element α'. It is evident, that at least a *second* element $\mathfrak{a}'$ of the *closed circuit* would still have to be delimited. And specifically, a current, which goes into α' *from the outer cone to the inner*, must go into $\mathfrak{a}'$ conversely *from the inner to the outer*. The value of ϑ' is the same for both elements. If one now decomposes the second element $\mathfrak{a}'$ in just the same way as the first α', and denotes as $\mathfrak{a}'_2$ that lateral element which, perpendicular to the r' connecting $\mathfrak{a}'$ with α, lies in the plane produced by r' by the motion of $\mathfrak{a}'$, then the term proportional to $\alpha_1\mathfrak{a}'_2$ will be the term A', by means of which A is cancelled. However,

$$A' = \mp\frac{ai}{2}\cdot\frac{\alpha_1\mathfrak{a}'_2}{r'}\cdot\cos\vartheta'\cot\vartheta' ,$$

and $\alpha'_2 : \mathfrak{a}'_2$ are in the ratio of their distances from the common apex of the two cones, i.e., the ratio $r : r'$, hence

$$\frac{\mathfrak{a}'_2}{r'} = \frac{\alpha'_2}{r} .$$

If these values are substituted, then

$$A' = \mp \frac{ai}{2} \frac{\alpha_1 \alpha_2'}{r} \cdot \cos \vartheta' \cot \vartheta' \ ,$$

and is, irrespective of the sign, equal to the value of A. From the *opposite direction* in which, as stated above, the elements α' and $\mathfrak{a}'$, or α_2' and $\mathfrak{a}_2'$, have the same current flowing through them, it can be easily recognized, that if in A, $\sin(\gamma' - \gamma) = \mp \cos \vartheta'$, and in A', $\sin(\gamma' - \gamma) = \pm \cos \vartheta'$, that therefore the values of A and A' always have opposite signs; hence the two cancel each other out.

It can occur, that in addition to α' and $\mathfrak{a}'$, yet a *third* element of the conductor is defined by the same cones; then, however, there must necessarily exist, if the conductor is *closed*, yet a *fourth* as well, and the same is true of the third and fourth as of the first and second, and so forth.

In a similar way, B', which cancels B, is found, when the center of the *induced* element α' is made the apex of two cones, whose common axis is parallel to the direction of the motion, and which delimit the *inducing* element α. The same cones then delimit, from the *closed inductor*, yet a second element, from whose decomposition B' results, as A' did previously from the decomposition of element $\mathfrak{a}'$.

From the mutual cancellation of all terms denoted A, A', B, B', and so forth, it now follows that for *closed currents and conductors*, the equation is valid:

$$-\frac{ai}{2} \mathbf{S} \frac{\alpha \alpha'}{r} \cos \beta \cos \beta' \sin(\gamma' - \gamma) \cot \vartheta' = 0 \ .$$

Now, from this it follows, *first*, when a closed conductor with all its parts is moved identically and parallel *always in the same direction*, the *induction effect* is

$$= \frac{ai}{2} \mathbf{S} \frac{\alpha \alpha'}{r_0} \cos \vartheta_0 \cos \varphi_0 - \frac{ai}{2} \mathbf{S} \frac{\alpha \alpha'}{r_1} \cos \vartheta_1 \cos \vartheta_1 \ ,$$

in which the values of r, ϑ, φ are denoted r_0, ϑ_0, φ_0 for the beginning of the induction, and r_1, ϑ_1, φ_1 for the end. If one makes $r_1 = \infty$, viz., the closed conductor, from a given position, is removed infinitely far distant from the inducing current, then the total induction effect elicited thereby is

$$= \frac{ai}{2} \mathbf{S} \frac{\alpha \alpha'}{r_0} \cos \vartheta_0 \cos \varphi_0 \ ,$$

the same, which results, according to the preceding Section, for the same *inducing* current conductor and for the same *induced* conductor, when they *persist* in their initial mutual positions and the current i *vanishes* in the former.

Secondly, when a closed conductor with all its parts is only slightly displaced identically and parallel in any definite direction, and then displaced again in a somewhat *changed* direction, and so forth, and when the values of r, ϑ, φ are denoted r_0, ϑ_0, φ_0 at the start of the induction, at the end of the first or beginning of the second displacement are denoted r_1, ϑ_1, φ_1, at the end of the second or beginning of the third displacement r_2, ϑ_2, φ_2, and so forth, it follows that the total *induction effect* is

$$= +\frac{ai}{2} \mathbf{S} \frac{\alpha \alpha'}{r_0} \cos \vartheta_0 \cos \varphi_0 - \frac{ai}{2} \mathbf{S} \frac{\alpha \alpha'}{r_1} \cos \vartheta_1 \cos \varphi_1$$

$$+ \frac{ai}{2} \mathbf{S} \frac{\alpha \alpha'}{r_1} \cos \vartheta_1 \cos \varphi_1 - \frac{ai}{2} \mathbf{S} \frac{\alpha \alpha'}{r_2} \cos \vartheta_2 \cos \varphi_2$$

<div align="center">+ and so forth.</div>

If r_n, ϑ_n, φ_n denote the values of r, ϑ, φ at the end of all these motions effected successively in *different* directions, then, because all terms with the exception of the first and last cancel each other out, the indicated value of the total induction effect reduces itself to

$$\frac{ai}{2}\mathbf{S}\frac{\alpha\alpha'}{r_0}\cos\vartheta_0\cos\varphi_0 - \frac{ai}{2}\mathbf{S}\frac{\alpha\alpha'}{r_n}\cos\vartheta_n\cos\varphi_n \ ,$$

from which one sees, when $r_n = \infty$, that the *induction effect* is the same, when a closed conductor is removed, from a given position with respect to a closed current, infinitely far from the inducing current through an arbitrarily *curved* trajectory, but in such a way that all parts always remain parallel to each other, as if the same thing would occur through a *straight* trajectory, or as if the closed conductor would *persist* in its original position and the current i in the inducing conductor would *vanish*, namely

$$= \frac{ai}{2}\mathbf{S}\frac{\alpha\alpha'}{r_0}\cos\vartheta_0\cos\varphi_0 \ .$$

If, *thirdly* and finally, the closed conductor moves with complete arbitrariness, then the motion of any one of its elements at any moment can be resolved into a *rotation* around its center, and into a parallel *displacement* of the whole element. The induction effect of the *rotation* of an element around its center is $= 0$, because r remains unchanged thereby, hence $dr = 0$. The *displacement* of each element can be decomposed into three displacements in the directions of three coordinate axes. For the parallel displacement of all elements of the closed conductor in *any* of these directions, then,

$$\mathbf{S}\frac{\alpha\alpha'}{r}\cos\beta\cos\beta'\sin(\gamma'-\gamma)\cos\vartheta' = 0 \ ,$$

from which it can easily be seen that even in *arbitrary motion* of the closed conductor, it follows that the *induction effect*

$$= \frac{ai}{2}\mathbf{S}\frac{\alpha\alpha'}{r_0}\cos\vartheta_0\cos\varphi_0 - \frac{ai}{2}\mathbf{S}\frac{\alpha\alpha'}{r_n}\cos\vartheta_n\cos\varphi_n$$

in which r_0, ϑ_0, φ_0 and r_n, ϑ_n, φ_n denote the values of r, ϑ, φ at the beginning and end of the induction.

The relationship discussed here between the induction effect of a closed *constant* current on a closed conductor *in motion*, and between the induction effect of a closed *variable* current on a closed conductor *at rest*, has already been presented with greater generality by Neumann, *loc. cit.* Namely, Neumann bases on the *empirical* foundation cited in Section 5.26, the conclusion that the total induction effect corresponding to the transference of the induced conductor from one position to another, is independent of the intermediate positions, which it passes through, and merely depends upon the difference in the *potential values* of the inductor at the start and end of the trajectory. After Neumann has stated this theorem for the induction effect of *constant* currents on *moving* conductors, he continues on page 39, *loc. cit.*:[242]

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[242][Note by AKTA:] [Neu46, p. 39].

From the independence of the induced electromotive force from the motion per se, it is inferred, that *any cause*, which elicits a change in the value of the *potential* of a closed current with respect to a closed conductor, induces a current, whose electromotive force is expressed by means of the *change* which the *potential* has undergone.

With the help of this theorem, Neumann has reduced the determination of the second kind of *voltaic induction*, namely, that of a *variable* current on a conductor at rest, to that of the first kind, namely, of a *constant* current on a conductor *in motion*. The above-mentioned relationship between both induction effects follows self-evidently. The final basis of all these relationships can now be directly proven according to the above, in the *fundamental electrical law*, according to which every two electrical masses act on each other at a distance.

5.30 General Law of Volta-Induction

After considering the two main cases of *voltaic induction*, namely, where either the current is *constant*, but the conductor is *in motion*, or where the current is *variable*, but the conductor is *unmoved*, the general law of determination of the effects of *arbitrarily moving* conductors *through which a current flows according to the laws of galvanism* can easily be developed.

α and α' denote once again the lengths of two elements, of which the first, α, is assumed to be at rest. In accordance with Section 5.27, this assumption does not restrict the generality of the treatment, because each motion of element α can be carried over to α', by attributing to it the opposite direction in α'. In these two elements, as earlier, the following four electrical masses are distinguished:

$$+\alpha e \, , \quad -\alpha e \, , \quad +\alpha' e' \, , \quad -\alpha' e' \, .$$

The *first* of these masses, $+\alpha e$, would move with velocity $+u$ in the direction of the element at rest α, which makes the angle ϑ with the straight line drawn from α to α'. This velocity would change during time-element dt by $+du$. The *second* mass $-\alpha e$, in conformity with the determinations given for a galvanic current, would move in the same direction, with velocity $-u$, viz., backwards, and this velocity would change during time-element dt by $-du$. The *third* mass $+\alpha' e'$ would move with velocity $+u'$ in the direction of element α', which makes the angle ϑ' with the straight line drawn and extended from α to α'. This velocity changes in time-element dt by $+du'$. However, this electrical mass also shares the motion of element α' itself, which occurs with velocity v in a direction which makes the angle η with the straight line drawn and extended from α to α', and is contained in a plane laid through this straight line, which forms the angle ϖ with the plane laid through the same straight line parallel to element α. Velocity v would change during the time-element dt by dv. The *fourth* mass $-\alpha' e'$ would move, in conformity with the determinations for a galvanic current, in the same direction as element α' with velocity $-u'$, which changes in time-element dt by $-du'$; additionally, however, it would share with the preceding mass the velocity v of element α' itself in the already signified direction. The distances of the two former masses from the two latter ones are all, at the moment in question, equal to the distance r of the two elements themselves; however, since they do not remain equal, they are to be denoted r_1, r_2, r_3, r_4. If two planes are laid through the straight line drawn from α to α', the one parallel to α, the other with α', then ω denotes the angle formed by these two planes.

For the *sum* of the forces which act on the *positive* and *negative* electricity in element α', that is, for the force, which moves element α' itself, one then obtains the same expression as in Section 5.24, namely:

$$-\frac{a^2}{16} \cdot \frac{\alpha e \cdot \alpha' e'}{r^2} \left\{ \left(\frac{dr_1^2}{dt^2} + \frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) \right.$$
$$\left. - 2r \left(\frac{d^2 r_1}{dt^2} + \frac{d^2 r_2}{dt^2} - \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) \right\} .$$

for the *difference* of those forces, however, on which *induction* depends,

$$-\frac{a^2}{16} \cdot \frac{\alpha e \cdot \alpha' e'}{r^2} \left\{ \left(\frac{dr_1^2}{dt^2} - \frac{dr_2^2}{dt^2} + \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) \right.$$
$$\left. - 2r \left(\frac{d^2 r_1}{dt^2} - \frac{d^2 r_2}{dt^2} + \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) \right\} .$$

Further, when, along with the motion of the electrical masses in their conductors, one also takes into calculation the motion they share with their conductors, the first differential coefficients are found in the way presented in Section 5.22, by adding to the values found there the velocity of element α', resolved in the direction of straight line r. One then obtains:

$$\frac{dr_1}{dt} = -u \cos \vartheta + u' \cos \vartheta' + v \cos \eta$$

$$\frac{dr_2}{dt} = +u \cos \vartheta - u' \cos \vartheta' + v \cos \eta$$

$$\frac{dr_3}{dt} = -u \cos \vartheta - u' \cos \vartheta' + v \cos \eta$$

$$\frac{dr_4}{dt} = +u \cos \vartheta + u' \cos \vartheta' + v \cos \eta .$$

Therefore:

$$\left(\frac{dr_1^2}{dt^2} + \frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) = -8uu' \cos \vartheta \cos \vartheta' ,$$

$$\left(\frac{dr_1^2}{dt^2} - \frac{dr_2^2}{dt^2} + \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) = -8uv \cos \vartheta \cos \eta .$$

The second differential coefficient is obtained as in Section 5.22, when, in addition, the variability of velocities u, u', v is considered, namely:

$$\frac{d^2 r_1}{dt^2} = +u \sin \vartheta \cdot \frac{d\vartheta_1}{dt} - u' \sin \vartheta' \cdot \frac{d\vartheta_1'}{dt} - v \sin \eta \frac{d\eta_1}{dt} - \cos \vartheta \frac{du}{dt} + \cos \vartheta' \frac{du'}{dt} + \cos \eta \frac{dv}{dt}$$

$$\frac{d^2 r_2}{dt^2} = -u \sin \vartheta \cdot \frac{d\vartheta_2}{dt} + u' \sin \vartheta' \cdot \frac{d\vartheta_2'}{dt} - v \sin \eta \frac{d\eta_2}{dt} + \cos \vartheta \frac{du}{dt} - \cos \vartheta' \frac{du'}{dt} + \cos \eta \frac{dv}{dt}$$

$$\frac{d^2r_3}{dt^2} = +u\sin\vartheta \cdot \frac{d\vartheta_3}{dt} + u'\sin\vartheta' \cdot \frac{d\vartheta_3'}{dt} - v\sin\eta\frac{d\eta_3}{dt} - \cos\vartheta\frac{du}{dt} - \cos\vartheta'\frac{du'}{dt} + \cos\eta\frac{dv}{dt}$$

$$\frac{d^2r_4}{dt^2} = -u\sin\vartheta \cdot \frac{d\vartheta_4}{dt} - u'\sin\vartheta' \cdot \frac{d\vartheta_4'}{dt} - v\sin\eta\frac{d\eta_4}{dt} + \cos\vartheta\frac{du}{dt} + \cos\vartheta'\frac{du'}{dt} + \cos\eta\frac{dv}{dt} \ .$$

Hence

$$\left(\frac{d^2r_1}{dt^2} + \frac{d^2r_2}{dt^2} - \frac{d^2r_3}{dt^2} - \frac{d^2r_4}{dt^2}\right) = +u\sin\vartheta\left(\frac{d\vartheta_1}{dt} - \frac{d\vartheta_2}{dt} - \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt}\right)$$

$$-u'\sin\vartheta'\left(\frac{d\vartheta_1'}{dt} - \frac{d\vartheta_2'}{dt} + \frac{d\vartheta_3'}{dt} - \frac{d\vartheta_4'}{dt}\right) - v\sin\eta\left(\frac{d\eta_1}{dt} + \frac{d\eta_2}{dt} - \frac{d\eta_3}{dt} - \frac{d\eta_4}{dt}\right)$$

and

$$\left(\frac{d^2r_1}{dt^2} - \frac{d^2r_2}{dt^2} + \frac{d^2r_3}{dt^2} - \frac{d^2r_4}{dt^2}\right) = +u\sin\vartheta\left(\frac{d\vartheta_1}{dt} + \frac{d\vartheta_2}{dt} + \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt}\right)$$

$$-u'\sin\vartheta'\left(\frac{d\vartheta_1'}{dt} + \frac{d\vartheta_2'}{dt} - \frac{d\vartheta_3'}{dt} - \frac{d\vartheta_4'}{dt}\right) - v\sin\eta\left(\frac{d\eta_1}{dt} - \frac{d\eta_2}{dt} + \frac{d\eta_3}{dt} - \frac{d\eta_4}{dt}\right)$$

$$- 4\cos\vartheta \cdot \frac{du}{dt} \ .$$

For the determination of the differential coefficients $d\vartheta_1/dt$, $d\vartheta_1'/dt$, $d\eta_1/dt$, and so forth, one now proceeds as on page[243] 151 and the following, or as in the footnote on page[244] 155. Namely, the resulting changes in the direction of straight line r_1

in the plane of angle $\vartheta = +\frac{udt}{r_1} \cdot \sin\vartheta$
in the plane of angle $\vartheta' = -\frac{u'dt}{r_1} \cdot \sin\vartheta'$
in the plane of angle $\eta = -\frac{vdt}{r_1} \cdot \sin\eta$.

If one now draws lines parallel to line r, and with the directionalities of velocities u, u' and v, through the center of a sphere, which cut the surface (Figure 21) at R, U, U', and V, and connects R with U, U' and V through the greatest arcs, then the plane containing the arc $UR = \vartheta$, forms the angle designated ω, with the plane of the arc $U'R = \vartheta'$, and forms the angle designated ϖ with the plane of the arc $VR = \eta$.

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[243][Note by LH and AKTA:] [Web46, p. 159 and the following of Weber's *Werke*].
[244][Note by LH and AKTA:] See footnote 202 on page 155, [Web46, p. 162 of Weber's *Werke*].
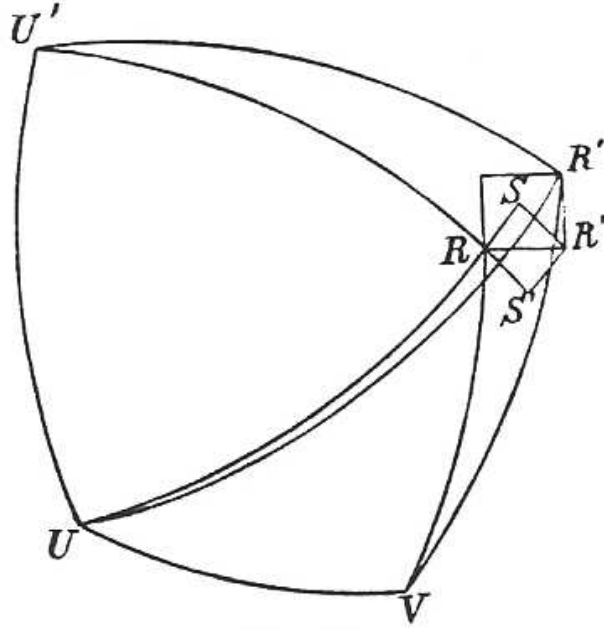
Fig. 21.

Let the arc UR be extended to S, $U'R$ to S', and VR to T,[245] and let

$$RS = +\frac{udt}{r_1}\sin\vartheta \,, \quad RS' = -\frac{u'dt}{r_1}\sin\vartheta' \,, \quad RT = -\frac{vdt}{r_1}\sin\eta \,.$$

The element of the sphere's surface in which R, S, S' and T lie, can now, as on page[246] 154, be considered as an element of the plane touching the sphere at R, and the arc elements RS, RS' and RT as straight lines in this plane. If the parallelogram $RSR'S'$ is completed in this plane, the diagonal RR' is drawn, and the second parallelogram $RR'R''T$ is completed, then a line drawn through the center parallel to straight line r_1, which connects the two *positive* masses $+\alpha e$ and $+\alpha'e'$ at the end of time element dt, goes through point R''.

Finally, if R'' is connected with U, U' and V by the greatest arc, then

$$UR'' = \vartheta + d\vartheta_1 = UR + d\vartheta_1$$

$$U'R'' = \vartheta' + d\vartheta'_1 = U'R + d\vartheta'_1$$

$$VR'' = \eta + d\eta' = VR + d\eta_1 \,.$$

From this follows that

$$d\vartheta_1 = UR'' - UR = RS + RS'\cos\omega + RT\cos\varpi$$

$$d\vartheta'_1 = U'R' - U'R = RS' + RS\cos\omega + RT\cos(\omega + \varpi)$$

$$d\eta_1 = VR'' - VR = RT + RS\cos\varpi + RS'\cos(\omega + \varpi) \,.$$

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[245][Note by AKTA:] The point T, in the extension of the arc VR, was not represented on Figure 21.
[246][Note by LH and AKTA:] [Web46, p. 161 of Weber's *Werke*].

If the values presented above of RS, RS' and RT are substituted, then one obtains:

$$r_1 \frac{d\vartheta_1}{dt} = +u\sin\vartheta - u'\sin\vartheta'\cos\omega - v\sin\eta\cos\varpi$$

$$r_1 \frac{d\vartheta_1'}{dt} = -u'\sin\vartheta' + u\sin\vartheta\cos\omega - v\sin\eta\cos(\omega+\varpi)$$

$$r_1 \frac{d\eta_1}{dt} = -v\sin\eta + u\sin\vartheta\cos\varpi - u'\sin\vartheta'\cos(\omega+\varpi) \ .$$

In the same way, the result for the two *negative* masses $-\alpha e$ and $-\alpha'e'$ is:

$$r_2 \frac{d\vartheta_2}{dt} = -u\sin\vartheta + u'\sin\vartheta'\cos\omega - v\sin\eta\cos\varpi$$

$$r_2 \frac{d\vartheta_2'}{dt} = +u'\sin\vartheta' - u\sin\vartheta\cos\omega - v\sin\eta\cos(\omega+\varpi)$$

$$r_2 \frac{d\eta_2}{dt} = -v\sin\eta - u\sin\vartheta\cos\varpi + u'\sin\vartheta'\cos(\omega+\varpi) \ ;$$

further for the *positive* mass $+\alpha e$ and for the *negative* mass $-\alpha'e'$:

$$r_3 \frac{d\vartheta_3}{dt} = +u\sin\vartheta + u'\sin\vartheta'\cos\omega - v\sin\eta\cos\varpi$$

$$r_3 \frac{d\vartheta_3'}{dt} = +u'\sin\vartheta' + u\sin\vartheta\cos\omega - v\sin\eta\cos(\omega+\varpi)$$

$$r_3 \frac{d\eta_3}{dt} = -v\sin\eta + u\sin\vartheta\cos\varpi + u'\sin\vartheta'\cos(\omega+\varpi) \ ;$$

finally, for the *negative* $-\alpha e$ and for the *positive* $+\alpha'e'$:

$$r_4 \frac{d\vartheta_4}{dt} = -u\sin\vartheta - u'\sin\vartheta'\cos\omega - v\sin\eta\cos\varpi$$

$$r_4 \frac{d\vartheta_4'}{dt} = -u'\sin\vartheta' - u\sin\vartheta\cos\omega - v\sin\eta\cos(\omega+\varpi)$$

$$r_4 \frac{d\eta_4}{dt} = -v\sin\eta - u\sin\vartheta\cos\varpi - u'\sin\vartheta'\cos(\omega+\varpi) \ .$$

Now, since for the moment under consideration, $r_1 = r_2 = r_3 = r_4 = r$, from this one obtains

$$r\left(\frac{d\vartheta_1}{dt} - \frac{d\vartheta_2}{dt} - \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt}\right) = -4u'\sin\vartheta'\cos\omega$$

$$r\left(\frac{d\vartheta_1}{dt} + \frac{d\vartheta_2}{dt} + \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt}\right) = -4v\sin\eta\cos\varpi \ ;$$

further:

$$r\left(\frac{d\vartheta_1'}{dt} - \frac{d\vartheta_2'}{dt} + \frac{d\vartheta_3'}{dt} - \frac{d\vartheta_4'}{dt}\right) = +4u\sin\vartheta\cos\omega$$

$$r\left(\frac{d\vartheta'_1}{dt} + \frac{d\vartheta'_2}{dt} - \frac{d\vartheta'_3}{dt} - \frac{d\vartheta_4}{dt}\right) = 0 ,$$

finally:

$$r\left(\frac{d\eta_1}{dt} + \frac{d\eta_2}{dt} - \frac{d\eta_3}{dt} - \frac{d\eta_4}{dt}\right) = 0$$

$$r\left(\frac{d\eta_1}{dt} - \frac{d\eta_2}{dt} + \frac{d\eta_3}{dt} - \frac{d\eta_4}{dt}\right) = +4u\sin\vartheta\cos\varpi .$$

If one substitutes these values into the aggregates of the second differential coefficients given above, then one obtains

$$r\left(\frac{d^2r_1}{dt^2} + \frac{d^2r_2}{dt^2} - \frac{d^2r_3}{dt^2} - \frac{d^2r_4}{dt^2}\right) = -8uu'\sin\vartheta\sin\vartheta'\cos\omega$$

$$r\left(\frac{d^2r_1}{dt^2} - \frac{d^2r_2}{dt^2} + \frac{d^2r_3}{dt^2} - \frac{d^2r_4}{dt^2}\right) = -8uv\sin\vartheta\sin\eta\cos\varpi - 4r\cos\vartheta\cdot\frac{du}{dt} .$$

These values, finally, yield the *sum* of the forces which act on the *positive* and *negative* electricity in element α',

$$-\frac{\alpha\alpha'}{r^2}\cdot aeu\cdot ae'u'\left(\sin\vartheta\sin\vartheta'\cos\omega - \frac{1}{2}\cos\vartheta\cos\vartheta'\right) ,$$

viz., the electrodynamic force acting on the ponderable element α' is determined for moving conductors and variable current intensities, as well as for conductors at rest and constant current intensities, and Ampère's law finds general application with regard to these forces for given positions of the current elements and given current intensities. The application of this law only requires that the current intensities *for each individual moment* be given, with inclusion of the portion added as a result of *induction.*

The *difference* of the forces acting on the *positive* and *negative* electricity in element α' results in the same way,

$$-\frac{\alpha\alpha'}{r^2}\cdot aeu\cdot ae'u'\left(\sin\vartheta\sin\eta\cos\varpi - \frac{1}{2}\cos\vartheta\cos\eta\right) - \frac{1}{2}\frac{\alpha\alpha'}{r}a^2ee'\cdot\cos\vartheta\cdot\frac{du}{dt} ,$$

or, since, in accordance with page[247] 144, $aeu = i$, and, because u is variable, $ae\cdot du = di$,

$$= -\frac{\alpha\alpha'}{r^2}i\left(\sin\vartheta\sin\eta\cos\varpi - \frac{1}{2}\cos\vartheta\cos\eta\right)\cdot ae'v$$

$$-\frac{1}{2}\frac{\alpha\alpha'}{r}ae'\cdot\cos\vartheta\cdot\frac{di}{dt} .$$

Now, the force determined in this way seeks to separate the *positive* and *negative* electricity in the induced element α' in the direction of straight line r. The separation cannot succeed in this direction, but only in the direction of the induced element α' itself, which makes the angle ϑ' with the extended straight line r. If, therefore, one resolves that entire force in this

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[247][Note by LH and AKTA:] [Web46, p. 152 of Weber's *Werke*].

direction, viz., if one multiplies the above value by $\cos\vartheta'$, then one obtains the force which actually brings about the separation,

$$= -\frac{\alpha\alpha'}{r^2}i\left(\sin\vartheta\sin\eta\cos\varpi - \frac{1}{2}\cos\vartheta\cos\eta\right)\cdot ae'v\cos\vartheta'$$

$$-\frac{1}{2}\frac{\alpha\alpha'}{r}ae'\cdot\cos\vartheta\cos\vartheta'\cdot\frac{di}{dt}\ .$$

If this value is divided by e', then the result is the *electromotive* force, in the usual sense (see Section 5.24, page[248] 163), exerted by the inducing element α on the induced element α'

$$= -\frac{\alpha\alpha'}{r^2}i\left(\sin\vartheta\sin\eta\cos\varpi - \frac{1}{2}\cos\vartheta\cos\eta\right)\cdot av\cos\vartheta' - \frac{1}{2}\frac{\alpha\alpha'}{r}a\cos\vartheta\cos\vartheta'\cdot\frac{di}{dt}\ .$$

If the change in the current intensity is made

$$\frac{di}{dt} = 0\ ,$$

then once more we find the same law which was found in Section 5.24 for the induction of a *constant* current element on the *moving* element of a conductor, and then the *electromotive* force is

$$= -\frac{\alpha\alpha'}{r^2}i\left(\sin\vartheta\sin\eta\cos\varpi - \frac{1}{2}\cos\vartheta\cos\eta\right)\cdot av\cos\vartheta'\ ,$$

in which the same angles, which were denoted ϑ', ω, φ in Section 5.24, are named η, ϖ and ϑ', and the velocity, which was called u', is denoted v.

On the other hand, if, in the general value, one makes

$$v = 0\ ,$$

one obtains the same law which was found in Section 5.28 for the induction of a *variable* current element on the element of a conductor *at rest*, and then the *electromotive* force is

$$= -\frac{1}{2}\frac{\alpha\alpha'}{r}a\cos\vartheta\cos\vartheta'\cdot\frac{di}{dt}\ .$$

The electromotive force of a *variable* current element on the *moving* element of a conductor is therefore the sum of the electromotive forces which would take place,

1. if the element of the conductor were *not in motion* at the moment under consideration,

2. when the element of the conductor were indeed in motion, but the *current intensity* of the inducing element at the moment under consideration were *unchanged*.

The general law of determining the effects of *arbitrarily moving* conductors with a current flowing through them *according to the galvanic laws*, is herewith completely given, if it may be assumed, that all electrical motions in linear conductors comprised under the name *galvanic currents*, actually conform precisely to the determinations given on page[249] 131 and

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[248][Note by LH and AKTA:] [Web46, p. 170 of Weber's *Werke*].
[249][Note by LH and AKTA:] [Web46, pages 135 and 139 of Weber's *Werke*].

page 133. However, even if it is not to be doubted that all *galvanic currents* come close to those determinations, small deviations can nevertheless rightly be expected, given the great dissimilarity in the *sources of galvanism*. These deviations and their influence on the *electrodynamic measurement* will be further discussed here.

According to the determinations given on page 131 and page 133,[250] each current element should contain the *same amount* of positive and negative electricity, and both should flow through the element with *the same velocity*, but in opposite directions. If a *constant* current were to consist of nothing but such elements, whose respective positions remained unchanged, then they would mutually exert no *electromotive* force whatever on each other. See Section 5.24, page[251] 161. The *electromotive* forces, which would overcome the resistance of the individual elements, and would thereby, according to page[252] 132, bring about the continuation of the current in all elements simultaneously, would then have to exist *independently of the current elements*, and would be distributed on all current elements in proportion to their resistance, if the current is to uniformly continue to exist in all elements.

Depending on the nature of the *sources of galvanism* generating the *original* electromotive forces, which are independent of the interaction of the current elements themselves, that equal relation between the forces and the resistance to be overcome by them in all elements of the conductor will sometimes occur, sometimes not. Serving an example of the first case, is a homogeneous, circularly shaped conductor, in which a galvanic current is induced by the motion of a magnet in the normal passing through the center of the circle to the plane of the circle. In this case an *electromotive* force acting uniformly on all the elements of the circle would be obtained by means of *magneto-induction*, and, since the *resistance* is likewise the same for all elements, the conditions are hereby fulfilled for the uniform presence of the current in all segments. Given the nature of things, however, such a case seldom occurs; as a rule, no equal relation between the *original* electromotive forces and the *resistance* in all the elements will occur, and the inequalities must then be equalized by means of the *interaction* of the elements. Now, if such an interaction of the elements of a constant current, an interaction consisting of electromotive forces, is not to be excluded, then the definition of galvanic currents must be broadened.

By a *galvanic current*, as opposed to other electrical motions not comprised under this name, should be understood a motion of the electricity in a closed conductor, such that the same amounts of positive and negative electricity flow through all its cross-sections simultaneously in the opposite directions. This equality of the positive and negative electricity *flowing through* does not necessarily presuppose the equality of the *moving* positive and negative masses, which was previously assumed, but rather, it can exist even when the latter are of unequal magnitudes, if the *larger* mass flows *slower*, the *smaller* one *faster*. In a galvanic current of the latter kind, *new* electromotive forces arise from the interaction of the elements, by means of which forces the unequal relationship of the *original* electromotive forces can be equalized. For as soon as the *positive* amount of electricity in an element is not equal to the *negative*, viz., as soon as the element, because of an excess of one electricity, is charged with *free electricity*, this *free electricity* itself, *in accordance with the laws of the excitation of electricity by means of separation*, becomes a source of *electromotive* forces for all other elements, which, through intensifying that charge, can be increased such that, added to the *original* electromotive forces, they become proportional to the resistance in all elements, for

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[250][Note by LH and AKTA:] [Web46, pages 135 and 139 of Weber's *Werke*].

[251][Note by LH and AKTA:] [Web46, p. 168 of Weber's *Werke*].

[252][Note by LH and AKTA:] [Web46, p. 136 of Weber's *Werke*].

which, in the galvanic circuits with which we are familiar, a very low degree of electrical charge suffices.

The investigation of how this charge in the individual elements in a closed galvanic circuit *arises* spontaneously in virtue of the initial inequality of the current in the different parts of the circuit, and increases until the given condition of a current uniform in all parts of the circuit is satisfied, leads to the *internal mechanics of the galvanic circuit* and is outside the scope of this Treatise, because there the action of electrical masses on *adjacent* masses must be taken into calculation, while here, merely the actions exerted *at a distance* need be considered. Independently of the investigation of the generation of these charges, and the resulting laws of their strength and distribution, here we will only discuss the influence which they have, *when they are present*, on the electrodynamic measurements. The discussion of this influence is important in this connection, because the presence of such charges is to be viewed as a rule having only infrequent exceptions. Even if this influence is so slight that, even without taking it into consideration, the calculation accords with empirical experience in most cases, nevertheless, it can be useful to know what this influence consists of and how it can become appreciable.

Under the conditions stated on page[253] 186, think of the *positive* mass $+\alpha e$ in the element α as increased by $m\alpha e$, where m denotes a small fraction, while the velocity $+u$ of this mass, however, is thought of as decreasing by the small magnitude $+mu$; likewise think of the *positive* mass[254] $+\alpha'e'$ as increased by $n\alpha'e'$, its velocity $+u'$ as decreased by nu'. The forces acting on both electrical masses in element α' are to be determined, which come about through these changes.

The two forces which the *positive* mass $+\alpha e$ in element α exerted on the *positive* and *negative* masses $+\alpha'e'$ and $-\alpha'e'$ in element α', were

$$+\frac{\alpha e \cdot \alpha'e'}{r^2}\left(1 - \frac{a^2}{16}\frac{dr_1^2}{dt^2} + \frac{a^2}{8}r\frac{d^2r_1}{dt^2}\right)$$

$$-\frac{\alpha e \cdot \alpha'e'}{r^2}\left(1 - \frac{a^2}{16}\frac{dr_3^2}{dt^2} + \frac{a^2}{8}r\frac{d^2r_3}{dt^2}\right) ,$$

in which, in accordance with page[255] 187, we are to make

$$\frac{dr_1}{dt} = -u\cos\vartheta + u'\cos\vartheta' + v\cos\eta$$

$$\frac{dr_3}{dt} = -u\cos\vartheta - u'\cos\vartheta' + v\cos\eta ,$$

and, in accordance with page 187 and page 190:[256]

$$r\frac{d^2r_1}{dt^2} = +u^2\sin^2\vartheta + u'^2\sin^2\vartheta' + v^2\sin^2\eta$$

$$- 2\left(uu'\sin\vartheta\sin\vartheta'\cos\omega + uv\sin\vartheta\sin\eta\cos\varpi - u'v\sin\vartheta'\sin\eta\cos(\omega+\varpi)\right)$$

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[253][Note by LH and AKTA:] [Web46, p. 196 of Weber's *Werke*].

[254][Note by AKTA:] Due to a misprint in the original German text, the next mathematical expression appeared as $+\alpha'e$ instead of $+\alpha'e'$.

[255][Note by LH and AKTA:] [Web46, p. 198 of Weber's *Werke*].

[256][Note by LH and AKTA:] [Web46, pages 198 and 200 of Weber's *Werke*].

$$- r \left(\cos \vartheta \frac{du}{dt} - \cos \vartheta' \frac{du'}{dt} - \cos \eta \frac{dv}{dt} \right)$$

$$r \frac{d^2 r_3}{dt^2} = + u^2 \sin^2 \vartheta + u'^2 \sin^2 \vartheta' + v^2 \sin^2 \eta$$

$$+ 2 \left(uu' \sin \vartheta \sin \vartheta' \cos \omega - uv \sin \vartheta \sin \eta \cos \varpi - u'v \sin \vartheta' \sin \eta \cos(\omega + \varpi) \right)$$

$$- r \left(\cos \vartheta \frac{du}{dt} + \cos \vartheta' \frac{du'}{dt} - \cos \eta \frac{dv}{dt} \right) \ .$$

The *difference* between the above two forces, on which the *electromotive* force depends, can be made

$$= 2 \frac{\alpha e \cdot \alpha' e'}{r^2} \ ,$$

because the remaining terms are very small in comparison with this first one. Now, if $(1+m)e$ is substituted for e and multiplied by $\cos \vartheta'/e'$, and the original value multiplied by $\cos \vartheta'/e'$ is subtracted, one obtains, in accordance with page 163 and page 192,[257] the *electromotive* force which arises from the charging of element α with free electricity and which acts on element α'

$$= 2m \frac{\alpha \alpha'}{r^2} e \cos \vartheta' \ .$$

Charging element α' itself, which is acted upon, does not change the electromotive force; for if, in the above difference, $(1 + n)e'$ is substituted for e' and multiplied by $\cos \vartheta'/(1+n)e'$, and the original value multiplied by $\cos \vartheta'/e'$ is subtracted, there is no remainder.

The *sum* of the above two forces, on which the *electrodynamic* force acting on the ponderable carrier depends, is obtained by substitution of the values arrived at

$$= -\frac{1}{2} \frac{\alpha \alpha'}{r^2} \cdot ae \cdot ae' \left[uu' \sin \vartheta \sin \vartheta' \cos \omega - u'v \sin \vartheta \sin \eta \cos(\omega + \varpi) \right.$$

$$\left. - \frac{1}{2} uu' \cos \vartheta \cos \vartheta' + \frac{1}{2} u'v \cos \vartheta' \cos \eta - \frac{1}{4} r \cos \vartheta' \cdot \frac{du}{dt} \right] \ .$$

From this is obtained

1. the portion arising from the increase in the mass $+\alpha e$, of the force with which the elements α and α' repel each other, when $(1+m)e$ is substituted for e, and the original value is subtracted,

$$= -\frac{m}{2} \frac{\alpha \alpha'}{r^2} \cdot ae \cdot ae' \left[uu' \sin \vartheta \sin \vartheta' \cos \omega - u'v \sin \vartheta' \sin \eta \cos(\omega + \varpi) \right.$$

$$\left. - \frac{1}{2} uu' \cos \vartheta \cos \vartheta' + u'v \cos \vartheta' \cos \eta - \frac{1}{4} r \cos \vartheta' \frac{du}{dt} \right] \ ;$$

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[257][Note by LH and AKTA:] [Web46, pages 170 and 202 of Weber's *Werke*].

2. the portion of the force arising from the decrease in velocity $+u$, when $(1-m)u$ is substituted for u, and the original value is subtracted,

$$= +\frac{m}{2}\frac{\alpha\alpha'}{r^2} \cdot ae \cdot ae' \left[uu' \sin\vartheta \sin\vartheta' \cos\omega - \frac{1}{2}uu' \cos\vartheta \cos\vartheta' \right] \ ;$$

3. the portion of the force arising from the increase in the mass $+\alpha'e'$, when $(1+n)e'$ is substituted for e', and the original value is subtracted,

$$= -\frac{n}{2}\frac{\alpha\alpha'}{r^2} \cdot ae \cdot ae' \left[uu' \sin\vartheta \sin\vartheta' \cos\omega - u'v \sin\vartheta' \sin\eta \cos(\omega+\varpi) \right.$$

$$\left. - \frac{1}{2}uu' \cos\vartheta \cos\vartheta' + \frac{1}{2}u'v \cos\vartheta' \cos\eta - \frac{1}{4}r \cos\vartheta' \frac{du'}{dt} \right] \ ;$$

4. the portion of the force arising from the decrease in the velocity $+u'$, when $(1-n)u'$ is substituted for u', and the original value is subtracted,

$$= +\frac{n}{2}\frac{\alpha\alpha'}{r^2} \cdot ae \cdot ae' \left[uu' \sin\vartheta \sin\vartheta' \cos\omega - u'v \sin\vartheta' \sin\eta' \cos(\omega+\varpi) \right.$$

$$\left. - \frac{1}{2}uu' \cos\vartheta \cos\vartheta' + \frac{1}{2}u'v \cos\vartheta' \cos\eta \right] \ .$$

If all these portions which arise are conjoined, one obtains the influence which the charging of elements α and α' with free *positive* electricity (if m and n have positive values) or *negative* electricity (if m and n have negative values) has on the *electrodynamic* repulsive force which α and α' exert; to be precise, it is the resulting increase in this repulsive force, when one makes $aev = \chi$, $ae'u' = i'$ and $ae'du' = di'$,

$$= + \frac{m}{2}\frac{\alpha\alpha'}{r^2}\chi i' \left(\sin\vartheta' \sin\eta \cos(\omega+\varpi) - \frac{1}{2}\cos\vartheta' \cos\eta \right)$$

$$+ \frac{m+n}{8}\frac{\alpha\alpha'}{r}ae \cos\vartheta' \cdot \frac{di'}{dt} \ .$$

This influence, therefore, wholly vanishes, when the action on a *constant* current element *at rest* is considered, for which $v=0$ and $di'=0$. Further, this influence also vanishes in a *constant* current element *in motion* α', when the element α acting upon it possesses no free electricity, because in that case $m=0$ and $di'=0$. Finally, if free electricity is present in element α, there exists that influence in a force which is equal to that force which would be exerted on current element α' by another current element in the place of α, when the masses contained in it, $+\frac{1}{2}m\alpha e$ and $-\frac{1}{2}m\alpha e$ were to flow with velocities $-v$ and $+v$ in the direction in which current element α' is moved with velocity $+v$. The necessity of this influence can also be examined from Fechner's viewpoint in Section 5.16, page[258] 171. For the case where a change occurs in current intensity i' in current element α', which is acted upon, there is

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[258][Note by LH and AKTA:] [Web46, p. 179 of Weber's *Werke*].

added to the above, finally, an influence proportional to this change di', and with the sum of the free electricity present in both elements α and α', which determines the last term in the formula.

5.31 About the Influence of Changing Speed and Direction of the Electricity Moving in the Current

In the method for determining galvanic current given in Section 5.19, on which the law describing two electrical masses acting on one another at a distance is based, instead of the *actual* current, in which the velocity of the flowing electricity probably fluctuates in its passage from one ponderable particle to the other in a steady alternation, an *ideal* current of *uniform* velocity is assumed. This substitution was necessary to simplify the treatment, and it seems permissible because it is simply a question of an action *at a distance*. It now remains to prove this initial assumption about the electrical law.

Let there be two electrical masses, e and e', which at the end of time t are found at a distance r from one another. Let their relative velocity up to this instant be a constant $= \gamma$. The repulsive force of the two masses in the last moment of the given time period t, would thus be, according to the fundamental electrical law:

$$\frac{ee'}{r^2}\left(1 - \frac{a^2}{16}\gamma^2\right) \ .$$

In the following element of time, ε, an acceleration

$$\frac{d^2r}{dt^2} = \alpha$$

occurs, whereby the repulsive force for the duration of the time period will be

$$= \frac{ee'}{r^2}\left(1 - \frac{a^2}{16}\gamma^2\right) + \frac{a^2}{8}\cdot\frac{ee'}{r}\alpha \ .$$

We now multiply the increase in force, which has occurred from the previous moment to the present one, by the time element ε itself. We thus obtain, as the amount by which the repulsive action has grown by this acceleration over the path dr, in which the masses e and e' have distanced themselves in the time ε,

$$= \frac{a^2}{8}\cdot\frac{ee'}{r}\cdot\alpha\varepsilon \ .$$

The relative velocity of the two masses, which before the time element ε was $= \gamma$ is then, after this time element,

$$= \gamma + \alpha\varepsilon \ .$$

Let this now remain unchanged, then the repulsive force of the two masses, when they have arrived at the distance ρ,

$$= \frac{ee'}{\rho^2}\left(1 + \frac{a^2}{16}(\gamma + \alpha\varepsilon)^2\right) \ ,$$

whereby, when $\alpha\varepsilon$ is very small in comparison to γ, it becomes

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$$= \frac{ee'}{\rho^2} \left(1 - \frac{a^2}{16}\gamma^2 - \frac{a^2}{8}\alpha\gamma\varepsilon \right) .$$

Multiplying this expression by the time

$$\frac{d\rho}{\gamma + \alpha\varepsilon} ,$$

in which both masses have distanced themselves from one another by the line element $d\rho$, and integrating between the limits $\rho = r$ to $\rho = r_1$, we get the repulsive action of the two masses over the distance $r_1 - r$, as

$$= \frac{ee'}{\gamma + \alpha\varepsilon} \left(1 - \frac{a^2}{16}\gamma^2 - \frac{a^2}{8}\alpha\gamma\varepsilon \right) \left(\frac{1}{r} - \frac{1}{r_1} \right) .$$

Finally at the instant when the two masses are at the distance r_1, a deceleration

$$\frac{d^2r}{dt^2} = -\alpha$$

occurs, which just as the earlier acceleration lasted only during the time element ε, so now the relative velocity of the two masses again returns to its original value

$$= \gamma ,$$

and in the path traveled in the time element ε there takes place a decrease in the repulsive action

$$= -\frac{a^2}{8} \cdot \frac{ee'}{r_1} \cdot \alpha\varepsilon .$$

One then gets as the sum of the repulsive action over the entire path $r_1 - r$, including the time elements ε, in which both the acceleration and deceleration took place,

$$= +\frac{a^2}{8}\frac{ee'}{r}\alpha\varepsilon + \frac{ee'}{\gamma + \alpha\varepsilon} \left(1 - \frac{a^2}{16}\gamma^2 - \frac{a^2}{8}\alpha\gamma\varepsilon \right) \left(\frac{1}{r} - \frac{1}{r_1} \right) - \frac{a^2}{8} \cdot \frac{ee'}{r_1}\alpha\varepsilon ,$$

or, when $\alpha\varepsilon$ is very small in comparison to γ,

$$= \frac{ee'}{\gamma + \alpha\varepsilon} \left(1 - \frac{a^2}{16}\gamma^2 \right) \left(\frac{1}{r} - \frac{1}{r_1} \right) .$$

The time for which this sum applies is, however

$$= \frac{r_1 - r}{\gamma + \alpha\varepsilon} .$$

If one divides the sum by this time, the average repulsive force during this time is obtained:

$$= \frac{ee'}{rr_1} \left(1 - \frac{a^2}{16}\gamma^2 \right) ,$$

that is, the same value as would occur if the path $r_1 - r$ had been traversed at the original velocity γ. It thus follows that if the relative velocity of two electrical masses, arriving successively at two different distances of separation is the same, their average repulsive force

over the time interval is the same as the average repulsive force which they would have achieved, if they had traveled with the initial relative velocity from the first distance to the latter.

This theorem may now be applied to the proof of the above assumption. For, when a particle of electricity moves in a galvanic current from one ponderable molecule to another, it will arrive in places both before and behind the molecule, where its velocity is the same as that of another electrical particle moving in another current. The average repulsive force of both particles for the duration of the passage of the first particle out of the first position into the next, is then the same, as it would have been if both particles had moved through the space with their initial relative velocities, that is, as if no change had taken place in the velocity of the electricity flowing from one molecule of the ponderable conductor to the other.

Besides the change in velocity of the electrical particles as they move from one molecule of the ponderable conductor to the next, we must also consider the changes of direction by which approaching particles avoid each other. One easily sees that within the measurable distances of the current element under consideration, no significant variation in the distances would occur, and accordingly only periodic variations in the relative velocity produced by these changes of direction would remain, which variations have already been included in the foregoing.

It stands to reason, that in place of a current in which the velocity and direction of the flowing electricity are subjected to a periodic change, a *uniform* current can rightfully be substituted, as is done in Section 5.19.

It is also permitted, that, in place of a straight current element, a bent one be substituted, so long as the beginning and end points remain unaltered, and no perceptible difference from the straight line joining them is allowed. Finally, as happens in Section 5.29, in place of one element, three elements may be considered, which behave in respect to the one like the edges of a parallelepiped to its diagonal.

5.32   Different Formulations of the General Fundamental Law of Electrical Action

The discovered fundamental electrical law can be expressed in different ways, which will be illustrated by a few examples.

1) Because distance r is always a positive magnitude, it can be written as ρ^2. This yields[259]

$$dr = 2\rho d\rho \ , \qquad d^2 r = 2\rho d^2 \rho + 2d\rho^2$$

hence[260]

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[259][Note by LH and AKTA:] The next equations should be understood as:

$$dr = 2\rho d\rho \ , \qquad d^2 r = 2\rho d^2 \rho + 2(d\rho)^2 \ .$$

[260][Note by LH and AKTA:] These equations should be understood as

$$r = \rho^2 \ , \quad \left(\frac{dr}{dt}\right)^2 = 4\rho^2 \left(\frac{d\rho}{dt}\right)^2 \ , \quad \frac{d^2 r}{dt^2} = 2\rho\frac{d^2\rho}{dt^2} + 2\left(\frac{d\rho}{dt}\right)^2 \ .$$

$$r = \rho^2 , \quad \frac{dr^2}{dt^2} = 4\rho^2\frac{d\rho^2}{dt^2} , \quad \frac{d^2r}{dt^2} = 2\rho\frac{d^2\rho}{dt^2} + 2\frac{d\rho^2}{dt^2} .$$

If these values are substituted in the formula $\frac{ee'}{r^2}\left(1 - \frac{a^2}{16}\frac{dr^2}{dt^2} + \frac{a^2}{8}r\frac{d^2r}{dt^2}\right)$, the following *shorter* formula is obtained:

$$\frac{ee'}{\rho^4}\left(1 + \frac{a^2}{4}\rho^3\frac{d^2\rho}{dt^2}\right) .$$

2) By *reduced relative velocity* of the masses e and e' should be understood that relative velocity, which those masses, reaching at the end of time t the distance r, the relative velocity dr/dt, and the relative acceleration d^2r/dt^2 would possess, *if the last-named were constant*, at the moment $(t - \vartheta)$, at which both, according to this premise, would meet at one point. If v denotes this *reduced relative velocity*, then according to the well-known law of *uniform acceleration*:

$$\frac{dr}{dt} - v = \frac{d^2r}{dt^2}\cdot\vartheta$$

$$r = v\vartheta + \frac{1}{2}\frac{d^2r}{dt^2}\cdot\vartheta^2 .$$

By elimination of ϑ, these two equations yield:

$$\frac{1}{2}v^2 = \frac{1}{2}\frac{dr^2}{dt^2} - r\frac{d^2r}{dt^2} .$$

If these values are substituted in the formula $\frac{ee'}{r^2}\left(1 - \frac{a^2}{16}\frac{dr^2}{dt^2} + \frac{a^2}{8}r\frac{d^2r}{dt^2}\right)$, the following *shorter* formula is obtained:

$$\frac{ee'}{r^2}\left(1 - \frac{a^2}{16}v^2\right) ,$$

which can be verbally expressed in the following way:

> *The decrease, caused by the motion, in the force with which two electrical masses would act upon each other, if they were not in motion, is proportional to the square of their reduced relative velocity.*

3) If $\frac{ee'}{r^2}\left(1 - \frac{a^2}{16}\frac{dr^2}{dt^2} + \frac{a^2}{8}r\frac{d^2r}{dt^2}\right)$ is the *absolute force* with which the mass e acts on and repels the mass e', and conversely, e' acts on and repels e, then there follows from this the *accelerative force* for mass e[261]

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[261] [Note by LH and AKTA:] What Weber calls here the *accelerative force for mass e* (beschleunigende Kraft für die Masse e) is the acceleration of the particle with charge e relative to an inertial system of reference when we suppose a system of units for which the inertial mass of this particle is equal to e. In his Sixth major Memoir published in 1871, which has already been translated to English (W. Weber, Philosophical Magazine, Vol. 42, pp. 1-20 and 119-149 (1872), *Electrodynamic measurements — Sixth Memoir, relating specially to the principle of the conservation of energy*), [Web72], Weber generalized this result considering the inertial masses of the particles with charges e and e' as given by, respectively, ε and ε'. In this case he was considering a system of units for which the unit of mass is one milligram; see especially pages 2 and 3

$$= \frac{e'}{r^2} \left(1 - \frac{a^2}{16} \frac{dr^2}{dt^2} + \frac{a^2}{8} r \frac{d^2 r}{dt^2} \right) \; ,$$

for mass e',

$$= \frac{e}{r^2} \left(1 - \frac{a^2}{16} \frac{dr^2}{dt^2} + \frac{a^2}{8} r \frac{d^2 r}{dt^2} \right) \; .$$

The following *relative acceleration* results for both masses:

$$= \frac{e + e'}{r^2} \left(1 - \frac{a^2}{16} \frac{dr^2}{dt^2} + \frac{a^2}{8} r \frac{d^2 r}{dt^2} \right) \; .$$

If to this is added that *relative acceleration* which results for the same masses, partly from the persistence of their motion in their present trajectories, partly from the influence of other bodies, which would be conjointly denoted as f, then the following equation is obtained for the *total relative acceleration*, i.e., for d^2r/dt^2:

$$\frac{d^2 r}{dt^2} = \frac{e + e'}{r^2} \left(1 - \frac{a^2}{16} \frac{dr^2}{dt^2} + \frac{a^2}{8} r \frac{d^2 r}{dt^2} \right) + f \; .$$

With the help of this equation, the differential coefficient d^2r/dt^2 can be determined and its value put into the formula $\frac{ee'}{r^2} \left(1 - \frac{a^2}{16} \frac{dr^2}{dt^2} + \frac{a^2}{8} r \frac{d^2 r}{dt^2} \right)$, which then becomes the following expression, representing the force with which two electrical masses act upon each other, independent of their *relative acceleration*:[262]

$$\frac{ee'}{r^2 - \frac{a^2}{8}(e + e')r} \cdot \left(1 - \frac{a^2}{16} \frac{dr^2}{dt^2} + \frac{a^2}{8} r f \right) \; .$$

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of this English translation of 1872. In this case the acceleration of the particle with charge e would be given by, according to Newton's second law of motion:

$$a = \frac{ee'}{\varepsilon r^2} \left(1 - \frac{a^2}{16} \frac{dr^2}{dt^2} + \frac{a^2}{8} r \frac{d^2 r}{dt^2} \right) \; .$$

By the same reasoning the acceleration a' of the particle with charge e' would be given by

$$a' = \frac{ee'}{\varepsilon' r^2} \left(1 - \frac{a^2}{16} \frac{dr^2}{dt^2} + \frac{a^2}{8} r \frac{d^2 r}{dt^2} \right) \; .$$

[262][Note by LH and AKTA:] In the paper of 1871 quoted in footnote 261, this expression takes the following more generalized form (see pages 3, 4 and 147 of W. Weber, Philosophical Magazine, Vol. 43, pp. 1-20 and 119-149 (1872), *Electrodynamic measurements — Sixth Memoir, relating specially to the principle of the conservation of energy*), [Web72, pp. 3, 4 and 147]:

$$\frac{ee'}{rr - \frac{2r}{cc} \cdot \frac{\varepsilon + \varepsilon'}{\varepsilon \varepsilon'} ee'} \cdot \left(1 - \frac{1}{cc} \frac{dr^2}{dt^2} + \frac{2rf}{cc} \right) \; .$$

In this equation e and e' are the charges of the particles with inertial masses ε and ε', and Weber replaced $4/a$ by c. This constant c had already been measured by Weber and Kohlrausch in 1854-5, who found it as $439450 \times 10^6 \, mm/s$. That is, it is essentially $\sqrt{2}$ times light velocity in vacuum. It should not be confused with the present day constant c, which is equal to the light velocity in vacuum. Weber and Kohlrausch published three main works related to this measurement: [Web55] with English translation in [Web21b]; [WK56] with English translation in [WK03] and Portuguese translation in [WK08]; and [KW57] with English translation in [KW21].

Accordingly, this force depends on the magnitude of the masses, on their distance, on their relative velocity, and, finally, on that relative acceleration f, which belongs to them partly as a result of the persistence of their already existing motion, partly as a result of the forces acting on them from *other bodies.*

It seems to follow from this, that the *direct interaction of two electrical masses* would not exclusively depend on these masses themselves and their relations to one another, but would also depend on the presence of *third* bodies. Now, it is well known that Berzelius[263] has already supposed the possibility of the *dependency of the direct interaction of two bodies on the presence of a third*, and has given the name *catalytic* to the forces resulting from this. If we avail ourselves of this name, then it can be said hereafter that the *electrical phenomena* also originate in part from *catalytic forces.*

This demonstration of *catalytic* forces for *electricity* is, however, no *strict* inference from the discovered fundamental electrical law. That would be the case only if one necessarily had to associate this fundamental law with the *idea* that only such forces would thereby be determined which electrical masses *directly* exerted upon one another at a distance. It is, however, possible to *conceive* that the forces included under the discovered fundamental law are also the kind of forces which two electrical masses *indirectly* exert upon one another, and which hence must depend, *first of all* upon the *transmitting medium*, and *further* upon all *bodies, which act on this medium.* It can easily occur, that such *indirectly* exerted forces, when the transmitting medium evades our observation, *appear as catalytic forces*, although they are not. In order to speak of *catalytic* forces in such cases, the concept of *catalytic* force would have to be fundamentally modified. That is, by catalytic force one would have to understand the kind of *indirectly* exerted force, which can be determined by a *general rule*, by means of a positive knowledge of the bodies to whose influence the *transmitting* medium is subjected, without knowledge, however, of this *medium itself.* The discovered fundamental electrical law yields a general rule for determination of *catalytic* forces in this sense.

Another still undecided question is, however, whether the knowledge of the *transmitting* medium, even if it is not necessary for the determination of forces, would nevertheless be *useful.* That is, the general rule for determination of forces could perhaps be expressed still more *simply*, when the transmitting medium were taken into consideration, than was otherwise possible in the *fundamental electrical law presented here.* However, *investigation of the transmitting medium*, which perhaps would elucidate many other things as well, is itself necessary in order to decide this question.

The *idea of the existence* of such a transmitting medium is already found in the *idea of the all-pervasive neutral electrical fluid*, and even if this *neutral fluid*, apart from conductors, has up to now almost entirely evaded the physicists' observations, nevertheless there is now hope that we can succeed in gaining more direct elucidation of this all-pervasive fluid in several new ways. Perhaps in other bodies, apart from conductors, no currents appear, but only *vibrations*, which can be observed more precisely for the first time with the methods discussed in Section 5.16. Further, I need only recall Faraday's latest discovery of the influence of *electrical currents on light vibrations*,[264] which make it not improbable, that the all-pervasive neutral electrical medium is itself that all-pervasive ether, which creates and propagates light vibrations, or that at least the two are so intimately interconnected, that observations of light vibrations may be able to explain the behavior of the neutral electrical medium.

Ampère has already called attention to the possibility of an *indirect* action of electrical

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[263][Note by AKTA:] Jöns Jacob Berzelius (1779-1848). See [Ber36c], [Ber36a] and [Ber36b].
[264][Note by AKTA:] [Far46a].

masses on each other, as cited in the introduction on page[265],[266] 36,

namely, according to which, the *electrodynamic phenomena*

would be ascribed

to the *motions communicated to the ether* by electrical currents.

Ampère himself, however, pronounced the examination of this possibility an extraordinarily difficult investigation, which he would have no time to undertake.

If, in addition, new empirical data, such as, for example, those which will perhaps emerge from further pursuit of the experiments to be carried out in accordance with Section 5.16 on *electrical vibrations*, and from Faraday's discovery, should appear to be particularly appropriate for gradually eliminating the difficulties not overcome by Ampère, then the fundamental electrical law in the form given here, independent of the transmitting medium, may afford a not insignificant basis for expressing this law in other forms, dependent upon the transmitting medium.

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[265][Note by LH and AKTA:] [Web46, p. 30 of Weber's *Werke*].
[266][Note by AKTA:] See [Amp23, p. 301], [Amp26, p. 129] and [AC15, p. 425].

Chapter 6

Introduction to the Excerpt of Weber's First Memoir on Electrodynamic Measurements

A. K. T. Assis[267]

Here I present the English translation of the excerpt of Weber's First major Memoir on *Electrodynamic Measurements.* This excerpt was published originally in 1848.[268] Its English translation has been published in 1852, 1866 and 2007.[269] I am maintaining the original title of this English translation as published during Weber's lifetime.

The First major Memoir had been published in 1846,[270] see Chapter 5.

The great importance of this excerpt is that Weber presented here for the first time a velocity-dependent potential energy from which he could deduce his force law which he had introduced in 1846.

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[267]Homepage: `www.ifi.unicamp.br/~assis`
[268][Web48a].
[269][Web52c], [Web66d] and [Web19].
[270][Web46] with a partial French translation in [Web87] and a complete English translation in [Web07].

Chapter 7

[Weber, 1848a] On the Measurement of Electro-dynamic Forces

Wilhelm Weber[271],[272]

A quarter of a century has elapsed since Ampère laid the foundation of electro-dynamics,[273] a science which was to bring the laws of magetism and electro-magnetism into their true connexion and refer them to a fundamental principle, as has been effected with Kepler's laws by Newton's theory of gravitation.[274] But if we compare the further development which electrodynamics have received with that of Newton's theory of gravitation, we find a great difference in the fertility of these two fundamental principles. Newton's theory of gravitation has become the source of innumerable new researches in astronomy, by the splendid results of which all doubt and obscurity regarding the final principle of this science have been removed. Ampère's electro-dynamics have not led to any such result; it may rather be considered, that all the advances which have since been really made have been obtained independently of Ampère's theory, — as for instance the discovery of induction and its laws by Faraday. If the fundamental principle of electro-dynamics, like the law of gravitation, be a true law of nature, we might suppose that it would have proved serviceable as a guide to the discovery and investigation of the different classes of natural phenomena which are dependent upon or are connected with it; but if this principle is not a law of nature, we should expect that, considering its great interest and the manifold activity which during the space of the last twenty-five years that peculiar branch of natural philosophy has experienced, it would have long since been disproved. The reason why neither the one nor the other has been effected, depends upon the fact, that in the development of electro-dynamics no such combination of observation with theory has occurred as in that of the general theory of gravitation. Ampère, who was rather a theorist than an experimenter, very ingeniously applied the most trivial experimental results to his system, and refined this to such an extent, that the crude observations immediately concerned no longer appeared to have any direct relation

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[271][Web48a] with English translation in [Web52c], [Web66d] and [Web19].

[272]Wilhelm Weber's Notes are represented by [Note by WW:]; the Notes by H. Weber, the editor of the third volume of Weber's *Werke*, are represented by [Note by HW:], while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[273][Note by AKTA:] See footnote 10 on page 13.

[274][Note by AKTA:] Johannes Kepler (1571-1630) and Isaac Newton (1642-1727). See footnote 57 on page 29.

to it. Electro-dynamics, whether for their more secure foundation and extension, or for their refutation, require a more perfect method of observing; and in the comparison of theory with experiment, demand that we should be able accurately to examine the more special points in question, so as to provide a proper organ for what might be termed the spirit of theory in the observations, without the development of which no unfolding of its powers is possible.

The following experiments will show that a more elaborate method of making electro-dynamic observations is not only of importance and consideration in proving the fundamental principle of electro-dynamics, but also because it becomes the source of new observations, which could not otherwise have been made.

7.1   Description of the Instrument

The instrument about to be described is adapted for delicate observation on, and measurements of, electro-dynamic forces; and its superiority over those formerly proposed by Ampère depends essentially upon the following arrangement.

The two galvanic conductors, the reciprocal action of which is to be observed, consist of two thin copper wires coated with silk, which, like multipliers, are coiled on the external part of the cavities of two cylindrical frames. One of these two coils incloses a space which is of sufficient size to allow the other coil to be placed within it and to have freedom of motion.

When a galvanic current passes through the wires of both coils, one of them exerts a rotatory action upon the other, which is of the greatest intensity when the centres of both coils correspond, and when the two planes to which the convolutions of the two coils are parallel form a right angle with each other. The common diameter of both coils is the axis of rotation. This respective position of the two coils constitutes the normal position, which they obtain in the instrument. Hence also the common diameter of the two coils, or their axis of rotation, has a vertical position, in order that the rotation may be performed in a horizontal plane.

That coil which is to be rotated, to allow of the onward transmission and return of the current, must be brought into connexion with two immoveable conductors; and the main object of the instrument is to effect these combinations in such a manner that the rotation of the coil is not in the least interfered with even when the impulse is the least possible, as occurs when these connexions are effected by means of two points dipping into two metallic cups filled with mercury in which the two immoveable conductors terminate, as in Ampère's arrangement. Instead of these combinations, which on account of the unavoidable friction do not allow of the free rotation of the coil, in the present arrangement two long and thin connecting wires are used, which are fastened at their upper extremities to two fixed metallic hooks, in which the two immoveable conductors terminate, and at their lower extremities to the frame of the coil, and are there firmly united to the ends of the wires of the coil. The coil hangs freely suspended by these two connecting wires, and each wire supports half the weight of the coil, whereby both wires are rendered equally tense.

These two connecting wires thus effect the transmission of the galvanic current from one of the immoveable conductors to the coil, and back to the other immoveable conductor; and they effect this without the least friction interfering with the rotation of the coil.

These connecting wires are also of service, because each rotation of the coil through a certain angle corresponds to a definite rotatory momentum, which tends to diminish the angle, and is proportional to the sine of the angle of rotation; whence a standard is formed for all rotatory momenta, by the aid of which any other rotatory momentum acting upon

the coil may be measured. This is effected according to those simple laws which Gauss has developed in the case of the bifilar magnetometer.[275] Lastly, this measure may be made more or less delicate at pleasure, or as occasion require, by the approximation or separation of the two connecting wires. This method of suspension not being accompanied with any friction, allows of increase in the weight of the suspended coil, which may be any amount provided it is not more than the connecting wires are capable of supporting. Hence a very long wire may be wound many times around the coil, and thus a very strong multiplication of the galvanic force be obtained. Moreover, this rotating coil may without injury be loaded with a speculum, which also rotates, and here, as in Gauss's magnetometer, may be used for the delicate measurement of angles; for provided friction be excluded, the application of delicate optical instruments in this case also does not conform any impediment. Regarding the construction of the instrument in detail, as this has been described very perfectly by M. Leyser,[276] the instrument-maker in Leipzig, I shall insert the explanation which he has given, and which refers to the figures sketched by him, Figures 1-10. The instrument is called an *Electro-dynamometer*.

7.2 Description of the Electro-dynamometer

Figure 1 represents the little frame for supporting the reel which vibrates in the multiplier, seen diagonally.
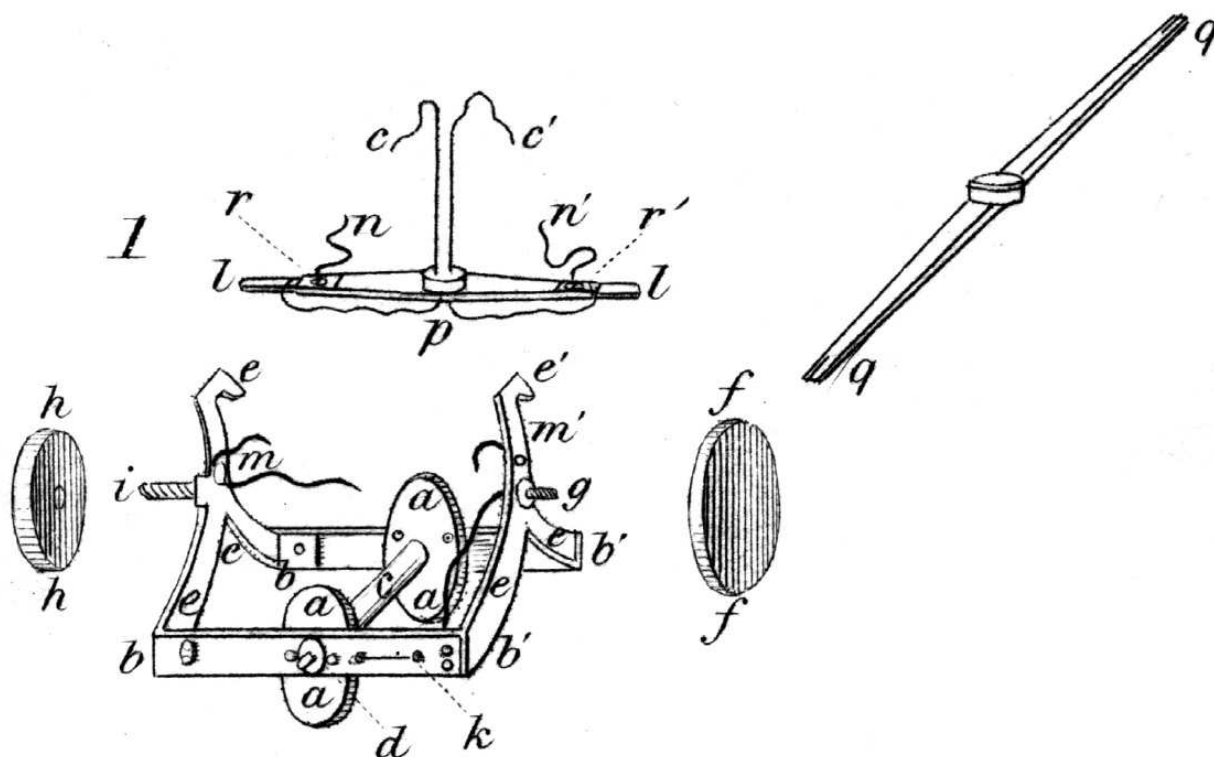
This frame consists of two round ivory discs, *aa* and *aa*, which are riveted to two ivory plates, *bb'* and *bb'*; their distance apart is regulated by a small ivory roller, *c*. The latter is hollow, so that a metallic rod can be passed through it, and by means of a screw each

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[275][Note by AKTA:] [Gau38b] with English translation in [Gau41c].
[276][Note by AKTA:] See footnote 105 on page 58.

of the discs with its plate can be fixed to the ends of the roller; and thus a reel is formed for the reception of the wire. One end of the wire to be coiled passes through the small role d, and projects from it. When the wire is placed upon the reel and the end fixed by means of silk, the metallic supports, eee and eee', of the reel are fixed to the ends of the plates above mentioned; thus, one support, eee', to which the speculum ff is screwed at g, is riveted at $b'b'$; whilst the other support, eee, to which the counterpoise hh is fixed by the screw i, is fastened by screws at bb; so that this support, near the screws bb, may be thrown back in the direction bb', in order that the entire reel may be conveniently placed in the multiplier. The commencement of the reel, which was left projecting through the hole at d, is now placed lengthwise along a portion of the plate bb' towards b', until the circumference of the reel admits at k of its being again placed within the frame and then ascending to the support of the speculum, where by means of a small screw m' above the point at which the speculum is fixed, it comes into metallic contact with the support. The end of the reel is also brought into metallic contact with the other support by means of the screw m; this end must however be long enough not to stand in the way of the support when it is thrown back. When the speculum ff is now placed at g, and its counterpoise hh at i, the reel is prepared for suspension in the multiplier by the metallic threads. For this purpose both the supports of the reel terminate at e and e' in hooks or pieces in the form of Υ, and the bifilar metallic threads are furnished below with a small ivory beam, ll, which towards each end terminates in a metallic plate, and this again in a small metallic cylinder; the latter fit into the above hooks or upsila of the support, and thus receive the reel. The bifilar metallic threads no and $n'o'$ are united to the cross-beam ll in the following manner. The commencement n of the thread no is fastened by means of a screw to the metallic plate r, proceeds a short distance towards l, and then returns through a small hole at the end of the plate beneath the beam ll to its centre p, where it runs through a small hole again above the beam, and can then be continued to o and further. The thread $n'o'$ is arranged in the same manner, its direction however being reversed; in the centre p of the beam ll each has a separate aperture, through which it passes; these lie very near each other, but are separated and kept isolated by the ivory. The index qq is placed upon the centre of the beam before the metallic threads no and $n'o'$ are inserted.

Figure 2 exhibits the lateral view of the vibrating reel, with the coil as placed upon the beam, and the mirror and counterpoise adapted and vibrating on the bifilar metallic threads. Only the very narrow anterior portion of the index is perceptible.
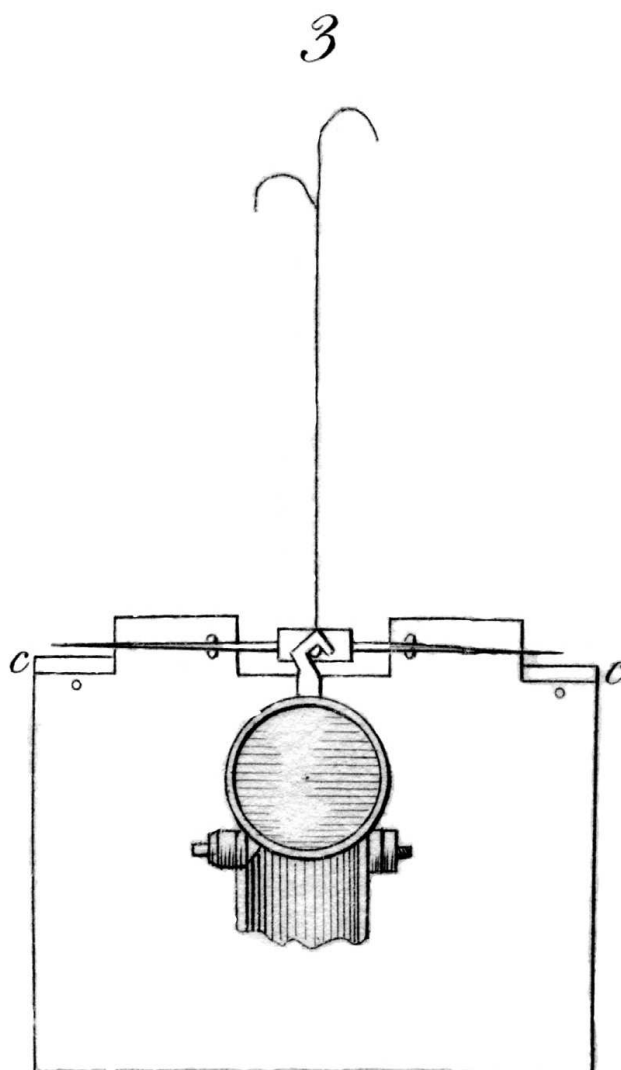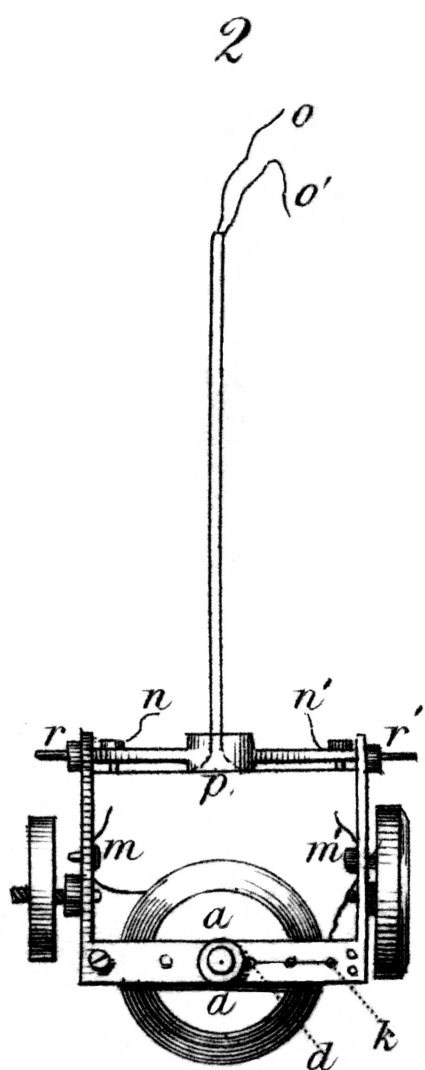
Figure 3 represents the reel seen at right angles to the surface of the speculum; the hooks or upsila, as also the index vibrating above the scale-plate *cc*, are very distinctly seen.

Figure 4 presents the view from above, in which the beam and the index form a right-angled cross.
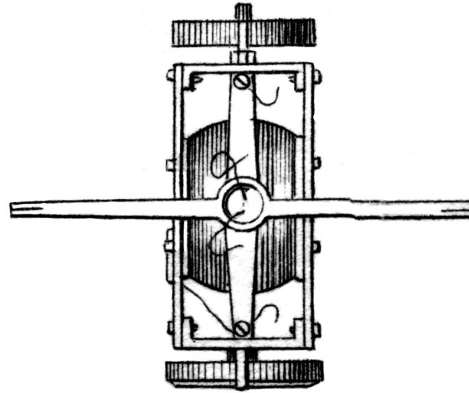
Figure 5 serves to illustrate the further course of the bifilar metallic thread to its termination; for the sake of distinctness it is represented of twice the size of the other figures, and as seen in a vertical section.

The bifilar metallic threads continue to ascend from o and o', inclosed in a brass tube; they are wound round the moveable rollers a and a', and are finally fixed to the ivory roller B at b and b' round rotating pegs. The threads can be wound up or unwound on these pegs or small rollers by means of a small key, according as the weight of the vibrating reel may render this requisite; the small rollers a and a' are also necessarily turned round at either of these operations. The ivory cylinder itself, B, with the prong and the screw ee, can also be screwed up or down by means of the nut, ff; and thus the vibrating reel may be arranged in the proper position as regards the multiplier, in the centre of which it should oscillate. At the same time the roller B, which is moveable in the prong ee around the peg m, assumes a state of equilibrium as soon as the vibrating reel is suspended freely from the bifilar metallic wires, since these wires act at band b' as it were at the ends of a lever, the centre of motion of which is at m. Thus the load of the vibrating cylinder is equally divided between the two threads.
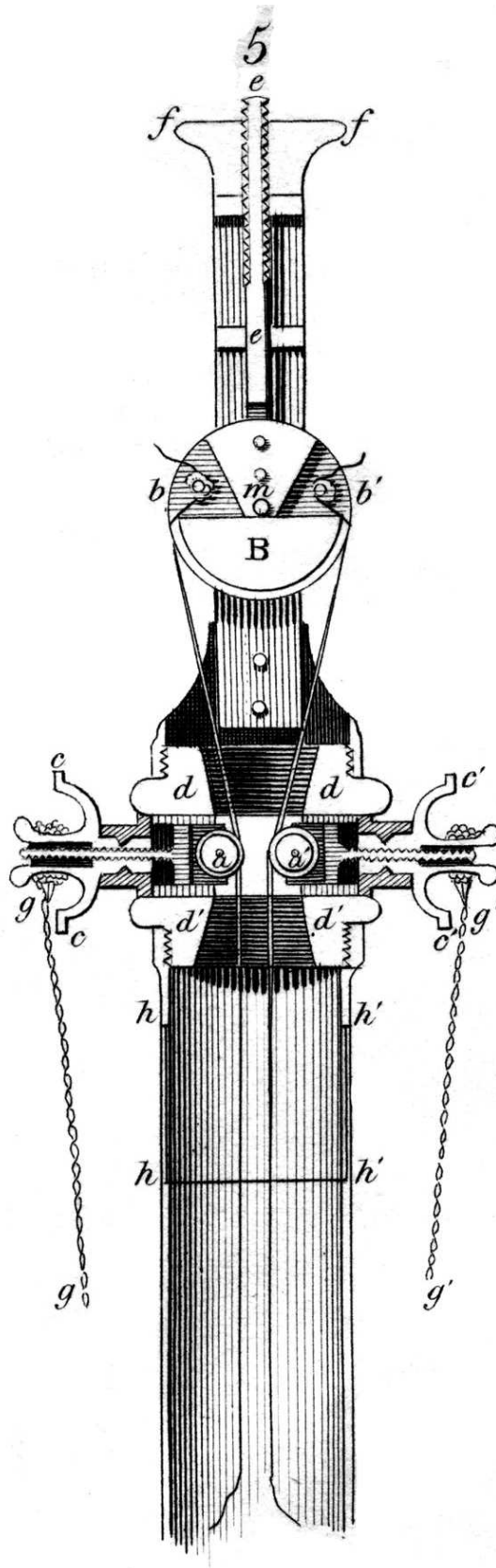
To allow of the approximation or separation of the two bifilar wires, the rollers a and a' are set in broad prongs, which, as seen in the figure, terminate in screws, by means of which they can be approximated or separated between two metallic plates (indicated by the

lines engraved perpendicularly) with the nuts cc and $c'c'$. The latter are fitted into a kind of case, indicated in the figure by lines drawn obliquely, in which they are fixed by a peg, but are not impeded as regards their rotation. The roller a, with its prong and screw, plate and nut cc, is isolated from the roller a', with its prong and screw, plate and nut $c'c'$, because the circular discs dd and $d'd'$, which are perforated in the centre, and which connect them above and below, are made of ivory. To allow of the bifilar metallic wires being brought out conveniently, the nuts cc and $c'c'$ terminate in trumpet-shaped projections, as shown in the figure, from which hangs a wire gg and $g'g'$ thrice wound round. Hence a galvanic current takes the following course: — If it enters at g, it ascends to g, is communicated to the nut cc, and the roller a (it also ascends to b, but as b is isolated it returns), and runs down the threads to o; from o it proceeds (Figure 2) further down through the centre p of the transverse beam, then to its extremity r, where by the metallic contact with the support it runs down it, and at m enters the extremity of the reel itself, through the coils of which it continues, again making its exit at d, but again passing to the other support at m' through k, from r' along the transverse beam to its centre, and from this up to o'; from o' the current (Figure 5) again runs over the other roller a' into the nut $c'c'$, and finally arrives at the other conducting wire, $g'g'$. Thus the current, to arrive at one conducting wire $g'g'$ from the other gg, must necessarily run through the vibrating reel, inasmuch as the wire from g to g' is perfectly isolated. To do away with the torsion of the bifilar metallic wires, the whole of the upper portion of the instrument as far as hh and $h'h'$ rotates horizontally, and is furnished with a torsion circle and an index, as is distinctly seen in Figures 6 and 7 at hh'.
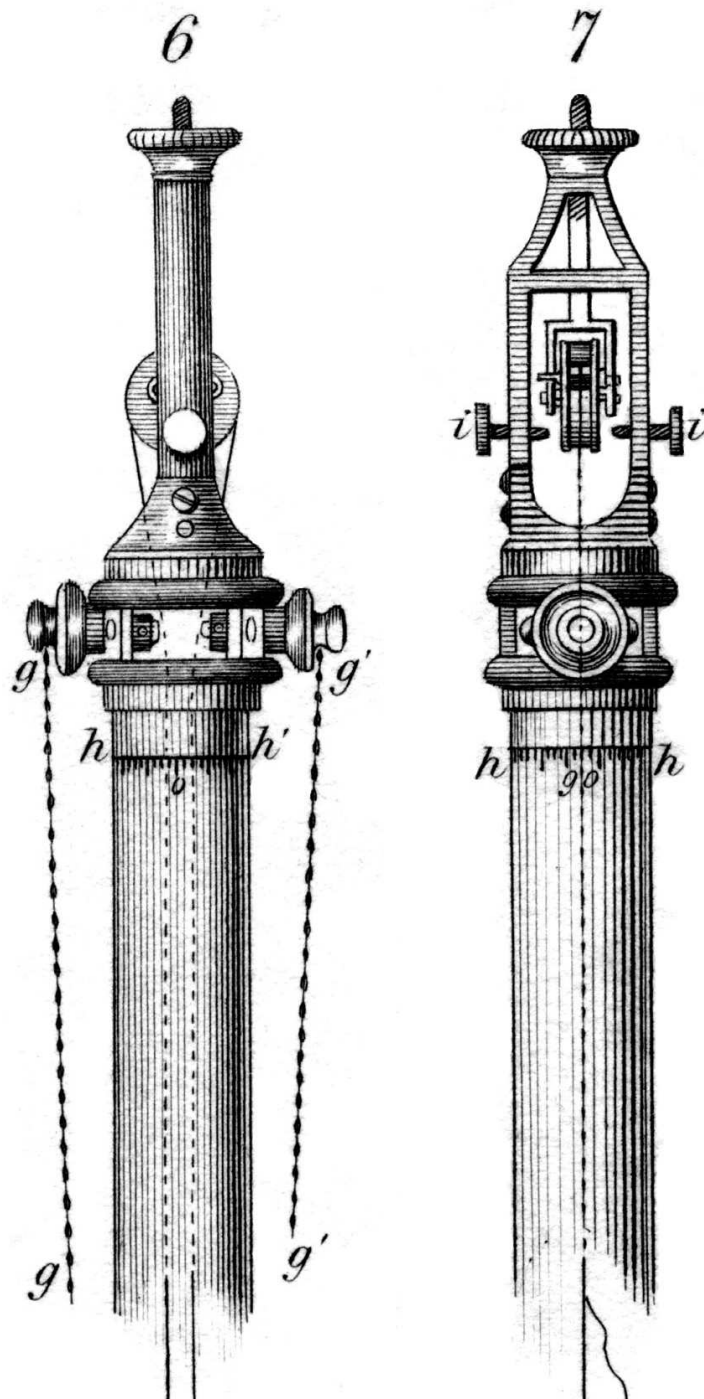
Figures 6 and 7 are not sectional, and Figure 6 belongs to Figure 2. Figure 7 exhibits the roller B with the prong and the screw ee' of Figure 5 more distinctly; ii here represent two screws, to fix the roller B on moving the instrument, without which precaution the bifilar threads would be easily injured.

We now pass to Figure 8, which exhibits in a vertical section the lower part of the instrument, with the multiplier and the pedestal, which is constructed of serpentine.
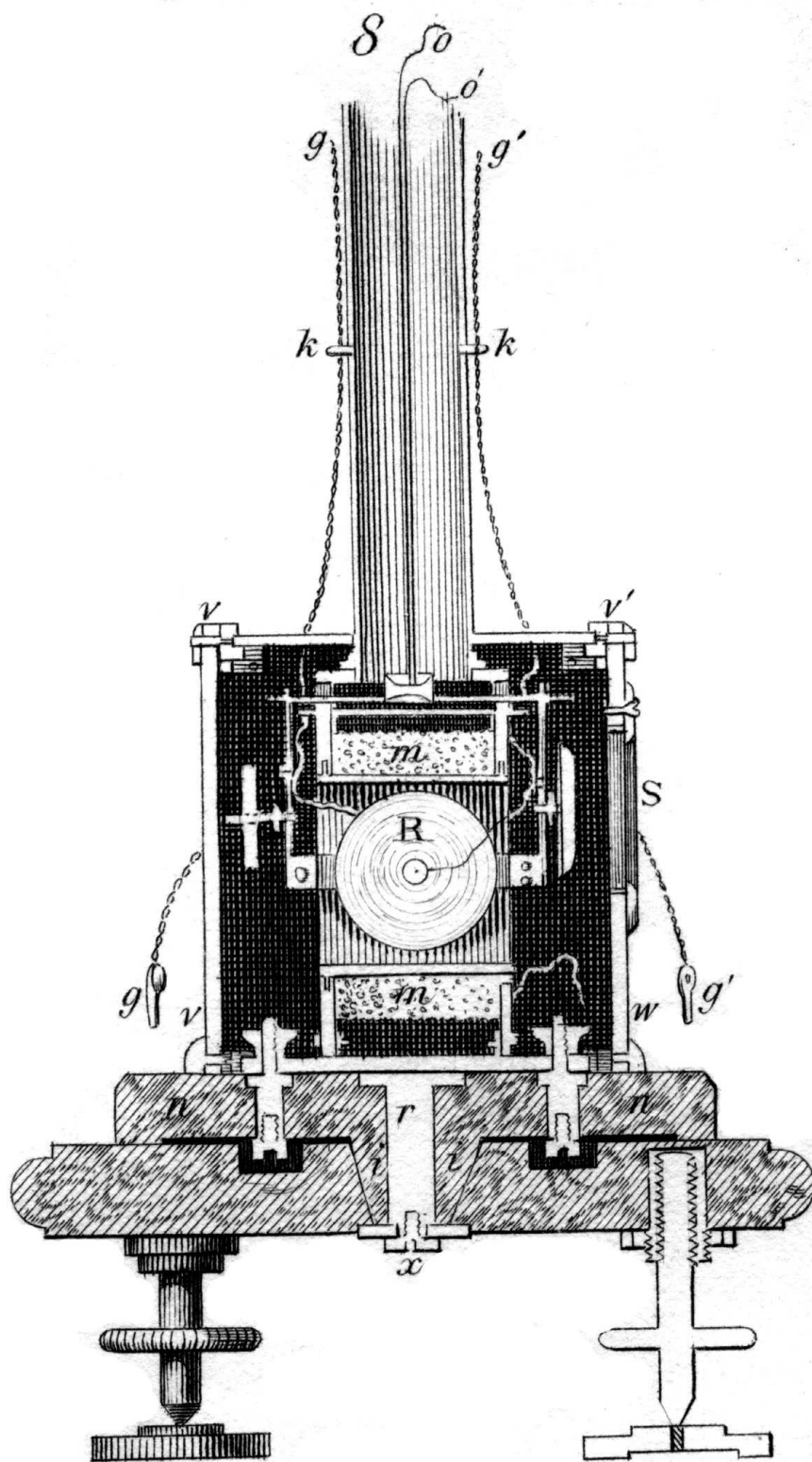
In it we first recognise Figure 2, suspended by the bifilar metallic wires o and o', also as seen on a vertical section. The letters mm exhibit a section of the multiplier, would round a brass drum furnished with wooden sides, in the interior of which the vibrating cylinder R is placed. These wooden sides support the tubes, within which the bifilar threads descend; the two scales for the index are also fixed to them.

Figure 10, a view of the instrument as seen from above, exhibits more accurately the

scale and the metallic plates, to which the tube is fastened.
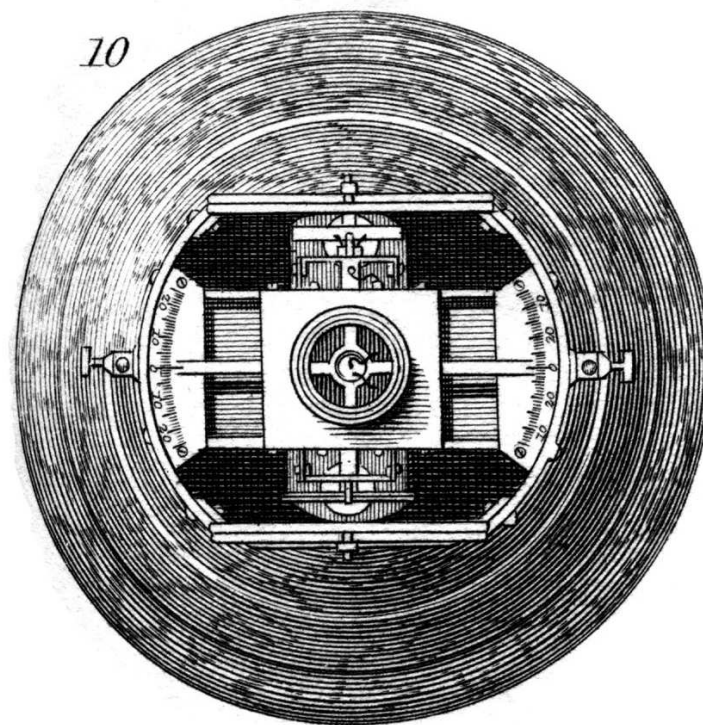
The sides of this multiplier are in connexion with a strip of copper, which by means of two cap-screws can be connected with the upper part nn of the foot of serpentine. This portion, nn, with its cone ii, is capable of rotation in the lower part of the serpentine foot, and by means of the metallic bolt r is kept in connexion with it by the screw x. Since, as shown in Figure 8, both the speculum and the counterpoise project towards the wooden sides of the multiplier, the whole is protected from the influence of a current of air by a cylindrical wooden cover, which is fixed to the upper corners of the wooden sides of the multiplier. In the direction of the speculum to the counterpoise, however, this cylindrical cover is flattened, so as to allow of a free view through the cavity of the multiplier. The flat side of the cover next the speculum can be opened or closed at pleasure by a wooden plate, which however, to enable us to use the mirror, is furnished with a flat parallel glass, S. The whole of the other flat side of the cover, which is turned towards the counterpoise, may be closed or opened by a glass plate. Thus the vibrating reel, when the sides of the cover are closed, can still be seen, and its free oscillation in the cavity of the multiplier be observed and regulated by means of the three screws in the serpentine pedestal. Moreover, from above downwards, above the graduation, the cover is closed by two glass plates, which are moveable towards each other in metallic grooves, and excavated in a semicircular form in the centre, to allow the tube in which the bifilar wire are suspended to pass through them. In Figure 8, vv exhibits the glass plate at the side; $v'w$ is the wooden plate, with the flat parallel glass S at the other side; vv' is one of the upper glass plates. The letters kk are loops, through which the conducting wires gg and $g'g'$ in Figure 6 descend; these wires are fixed in these loops to avoid their lying loosely throughout their entire length; they terminate in pegs, or small cylinders.

Figure 9 also exhibits a vertical section, but at right angles to that of Figure 8; m is the multiplier, and R a section of the reel vibrating within it.

217
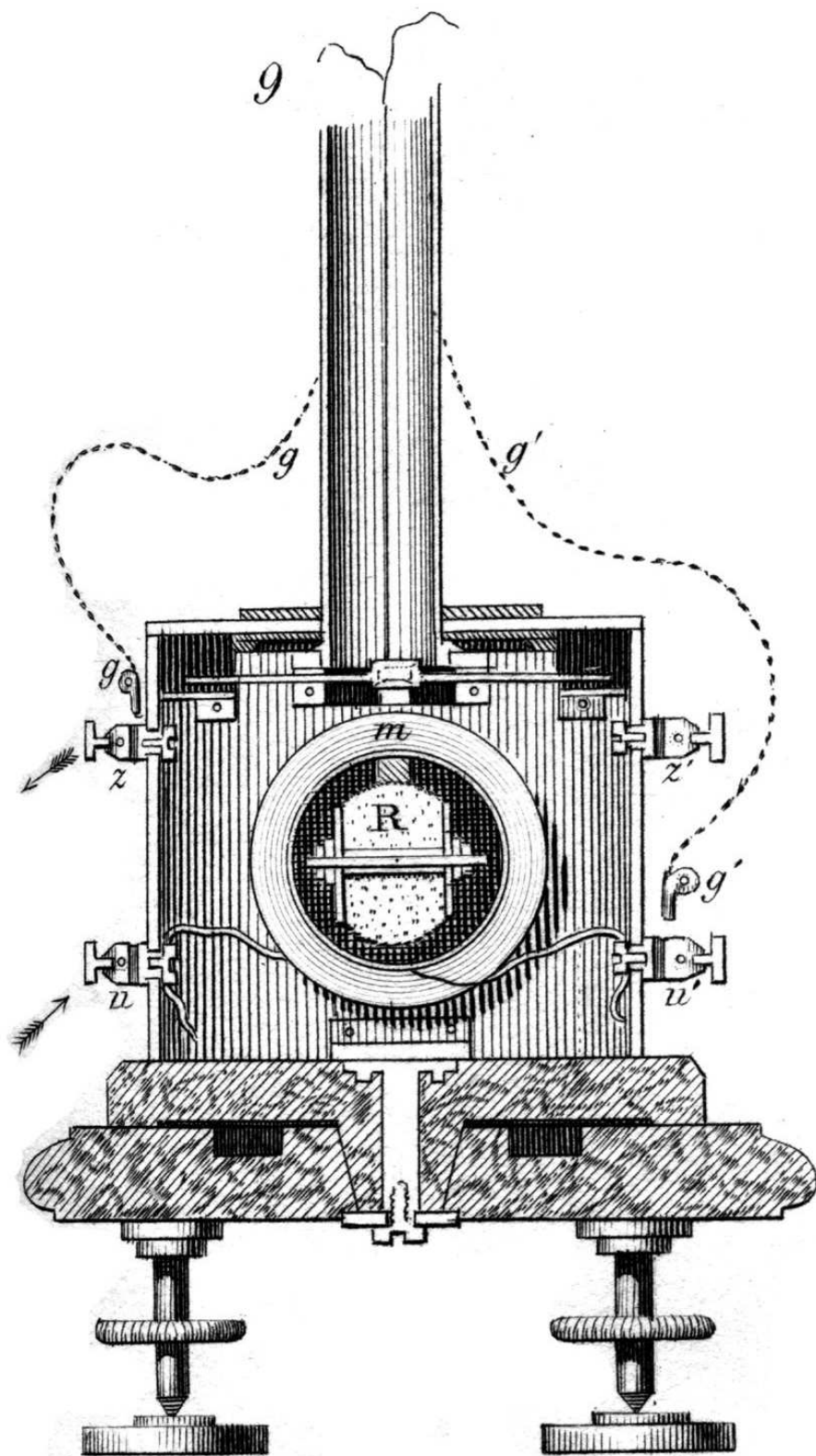
At the side of the case we perceive four metallic knobs, marked $uu'zz'$. These are perforated crucially, and the perforation most distant from the case is furnished with a screw; on the inner side of the case it is fixed to it by another screw. Two of these knobs, u and u', are in metallic contact with the commencement and termination of the multiplier, so that

a current from the knob u can run through the multiplier into the knob u', and *vice versa*. The other two knobs, z and z', are perfectly isolated; but all four of the knobs are very useful for reversing the current, and for effecting various combinations. In this figure also we see the index vibrating above the scale-plate, as also in Figure 3, where the case is supposed to be removed.

Let us now trace the course of a galvanic current, which enters the instrument at the knob u; it passes from u through the multiplier m and towards u'; if the conducting wire $g'g'$ with its metallic cylindrical extremity be now inserted into this knob, the current ascends in $g'g'$, and (Figure 5) towards the nut $c'c'$ above the roller a', then down within the tube to o'; whence (Figure 2) from o' through the centre p of the transverse beam to $r'm'kd$, through the vibrating reel to $mrpo$, and (Figure 5) to o, ascending above the roller a in the nut cc, to the second conducting wire gg and (Figure 9) through gg down into the knob z, whence it runs into the other of the two exciting surfaces.

By means of the upper rotating part of the serpentine pedestal, the instrument may be arranged in any part of a hall or room as required. All the figures are drawn one-fourth of the linear magnitude of the electro-dynamometer, excepting Figure 5, which is one-half the real magnitude.

The wire on the vibrating reel is 200 metres in length, that of the multiplier 300; the first forms about 1200 coils, the latter about 900. The length of the bifilar wires, (which are very fine; composed of silver, and were heated to redness,) from the transverse beam to the small rollers aa', was half a metre.

The price of the instrument is 10 guineas.

7.3   Observations Demonstrating the Fundamental Principle of Electro-dynamics

The following observations were not made with the instrument which has just been described. However, it is unnecessary to describe separately the instrument made use of on this occasion, because it merely differs from the former in minor points of arrangement, which were less convenient than those in the latter. One important modification only requires to be mentioned, viz. that the multiplier, which in the above description assumes an invariable position, in which its centre coincides with the centre of the bifilarly-suspended reel, was left moveable, so that it could be placed in any position as regards the vibrating reel, for the purpose of extending the observations to all relative positions of the two galvanic conductors, which act upon each other. Now as these two conductors form two coils, one of which can enclose the other, and in the instrument described above the inner and smaller coil was suspended by two threads, to serve as it were as a galvanometer-needle, whilst the outer and larger coil, was fixed and formed the multiplier; it was requisite for the object in question to reverse the arrangement, and to suspend the outer and larger coil by two threads so as to use the inner and smaller coil as a multiplier, because it was only by this means that the position of the multiplier could be altered at pleasure without interfering with the bifilar suspension. It is at once seen that the external reel, on account of its size, has a greater momentum from inertia, which produces a longer duration of its vibration; this influence however may be easily compensated for when necessary by altering the arrangement of the bifilar suspension.

As regards the observations themselves, it remains to be remarked, that to render the

results comparable, the intensity of the current transmitted by the two conductors of the dynamometer was, simultaneously with the observation of the dynamometer, accurately measured by a second observer with a galvanometer. This was requisite, because no reliance can be placed upon the constancy of the intensity of the current during a continued series of experiments, even when the so-called constant battery of Grove or Bunsen is used.[277]

The first experiment was made by passing three currents of different intensity, *i.e.* from 3, 2 and 1 of Grove's elements, through the two conductors of the dynamometer, and observing the simultaneous deflections of the dynamometer and galvanometer. After making the necessary reductions, the following means were obtained as the deflections:

Number of Grove's elements	Deflections	
	Of the Dynamometer	Of the Galvanometer
3	440.038	108.426
2	198.255	72.398
1	50.915	36.332

If we denote the dynamometric observations by δ, and the galvanometric observations by γ, we obtain

$$\gamma = 5.155\,34\sqrt{\delta}$$

for if we calculate the values of γ from the values found by observation for δ according to this formula, we obtain in the order of the series,

108.144
72.589
36.786,

which exhibit less differences from the values of γ found by observation than could be anticipated, thus:

-0.282
$+0.191$
$+0.454.$

The electro-dynamic force of the reciprocal action of two conducting wires, through which currents of equal intensity are transmitted, is therefore in proportion to the square of this intensity, which is exactly what is required by the fundamental principle of electro-dynamics.

A more extended series of experiments was then made for the purpose of ascertaining the dependence of the electro-dynamic force, with which the two conducting wires of the dynamometer react upon each other, upon the relative position and distance of these wires.

For this purpose the arrangement was effected in such a manner, that one conducting wire, *i.e.* the multiplier, could be placed in any position as regards the other, *i.e.* as regards the bifilarly-suspended coil, the latter forming the larger coil, which inclosed the former smaller one.

Both coils were always placed in such a position that their axes were in the same horizontal plane, and formed a right angle with each other.

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[277][Note by AKTA:] See footnotes 98 and 108 on pages 53 and 61.

The distance of the two coils was determined by the distance of their centres from each other, and was thus assumed as = 0 when the centres of the two coils coincided.

When the latter was not the case, in addition to the magnitude of the distance of the two centres, it was also requisite to measure the angle which the line uniting the two central points formed with the axis of the bifilarly-suspended coil, whereby the direction in which the centre of the multiplier was removed from the centre of the bifilarly-suspended coil was defined. For this purpose the four cardinal directions were selected at which the former angle had the value 0°, 90°, 180° and 270°, *i.e.* when the axis of the bifilarly-suspended coil, like the axis of the needle of a magnet, was arranged in the magnetic meridian, the centre of the multiplier was removed from the centre of the above coil, sometimes in the direction of the meridian, *north* or *south*, and sometimes in the direction at right angles to the magnetic meridian, *east* or *west*. In each of these different directions the multiplier was placed successively at different distances from the suspended coil.

This arrangement of different positions and distances of the two conducting wires of the dynamometer accurately corresponds, as is seen at a glance, to the arrangement of different positions and distances of the two magnets, upon which Gauss based his measurements, in demonstrating the fundamental principle of magnetism. The bifilarly-suspended coil here occupied the place of Gauss's magnetic needle and the multiplier the place of Gauss's deflection-rod. The only important difference is, that the mutual action of the magnets could only be observed from a distance; consequently in the magnetic observations that case was excluded in which the centres of the two magnets coincided; whilst in the electro-dynamic measurements of which we are now speaking, the system could moreover be rendered complete by the case, in which the centre of the two coils coincided.

Simultaneously with the observations made on the dynamometer, the intensity of the current which was transmitted through the two coils of the dynamometer was measured by another observer with a galvanometer. By the auxiliary observations I was enabled to reduce all the observations made on the dynamometer in accordance with the law shown above, (that the electro-dynamic force is in proportion to the square of the intensity of the current,) to an equal intensity of the current, and thus to render the results obtained comparable.

The following Table gives the reduced mean values which were obtained in the different instances. The first vertical column shows the distance of the two coils of the dynamometer; above the other columns, the direction formed by the line uniting the two centres with the axis of the bifilarly-suspended coil directed towards the magnetic meridian is given:

Distance in mm	North, 0°	East, 90°	South, 180°	West, 270°
0	22 960	22 960	22 960	22 960
300	77.16	189.24	77.06	190.62
400	34.78	77.61	34.77	77.28
500	18.17	39.37	18.30	39.16
600	—	22.53	—	22.38

It is at once seen that when the centres of the two coils of the dynamometer coincide, or their distance apart is = 0, the difference dependent upon the change of the direction in which the multiplier is removed from the bifilarly-suspended coil, vanishes. The result obtained in this case therefore could only be repeated in the above Table in the various columns.

Moreover, the above Table shows that the results obtained for an equal distance in opposite directions varying 180°, agree together as far as the observations could be depended upon.

These values, when reduced by taking their means, after converting the divisions of the scale into degrees, minutes and seconds, yield the following Table:

R	v	v'
0.3	0°49′22″	0°20′3″
0.4	0°20′8″	0°9′2″
0.5	0°10′12″	0°4′44″
0.6	0°5′50″	—

in which the same notation is adopted as used by Gauss in his *Intensitas vis magneticae, etc.* (Ann. 1833, Vol. XXVIII, p. 604)[278,279] in the comparison of the magnetic observations.

According to the fundamental principle of electro-dynamics, we should be able to develop the tangents of the angle of deflection v and v' according to the diminishing odd powers of the distance R, and we should have

$$\tan v = aR^{-3} + bR^{-5}$$

$$\tan v' = \frac{1}{2}aR^{-3} + cR^{-5} \ ,$$

where a, b and c are constants to be determined from the observations. If now in the present instance we make

$$\tan v = 0.000\,3572R^{-3} + 0.000\,002\,755R^{-5}$$

$$\tan v' = 0.000\,1786R^{-3} - 0.000\,001\,886R^{-5} \ ,$$

we obtain the following Table of *calculated* deflections, and their difference from those *found by observation*:

R	v	Difference	v'	Difference
0.3	0°49′22″	0″	0°20′4″	−1″
0.4	0°20′7″	+1″	0°8′58″	+4″
0.5	0°10′8″	+4″	0°4′42″	+2″
0.6	0°5′49″	+1″	—	—

Thus in this agreement of the calculated values with those obtained by observation, we have a confirmation of one of the most universal and most important consequences of the fundamental principle of electro-dynamics, viz. *that the same laws apply to electro-dynamic forces exerted at a distance as to magnetic forces.*

In this application of the laws of magnetism to electro-dynamic observations, that case of the latter where the centres of the two coils of the dynamometer coincide must be excluded. Moreover, in this extension of the laws of magnetism to electro-dynamic observations, the

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[278][Note by HW:] Gauss' *Werke*, Vol. V, p. 109.
[279][Note by AKTA:] See footnote 97 on page 51.

values of three constants must be deduced from the observations themselves, which is unnecessary when we have recourse to the fundamental principle of electro-dynamics itself, and calculate directly from it the results which the observations should have yielded in accordance with it. Hence from the fundamental principle of electro-dynamics —

- 1. In that case in which the straight line uniting the centre of the two coils coincides with the axis of the bifilarly suspended coil,

when m designates the radius of the multiplying coil, n the radius of the bifilarly-suspended coil, and a the distance of the centres of the two coils, and for brevity we make

$$\frac{m^2}{a^2 + n^2} = v^2 \ ,$$

$$\frac{n^2}{a^2 + n^2} = w^2 \ ,$$

$$\frac{4a^2 + n^2}{16(a^2 + n^2)} = f \ ,$$

$$\frac{8a^4 + 4a^2n^2 + n^4}{64(a^2 + n^2)^2} = g \ ,$$

the electro-dynamic momentum of rotation which the multiplying coil exerts upon the bifilarly-suspended coil, when a current of the intensity i passes through both coils, is determined with sufficient accuracy to be

$$= -\frac{\pi^2}{2} v^3 n^2 i^2 S \ ,$$

S designating the following series:

$$S = + \left[\frac{1}{3} - w^2 \right]$$

$$- \frac{3}{2} \left[\frac{3}{5} - w^2 - \left(3 - 7w^2 \right) f \right] v^2$$

$$+ \frac{15}{8} \left[\frac{5}{7} - w^2 - 2 \left(5 - 9w^2 \right) f + 3 \left(5 - 11w^2 \right) g \right] v^4$$

$$- \frac{35}{16} \left[\frac{7}{9} - w^2 - 3 \left(7 - 11w^2 \right) f + 11 \left(7 - 13w^2 \right) g \right] v^6$$

$$+ \frac{315}{128} \left[\frac{9}{11} - w^2 - 4 \left(9 - 13w^2 \right) f + 26 \left(9 - 15w^2 \right) g \right] v^8$$

$$- \ etc.$$

If in this equation we substitute the values known from direct measurement, in millimetres,

$$m = 44.4 \ ,$$

$$n = 55.8 \ ,$$

and successively

$$a = 300, \ 400, \ 500,$$

we obtain as the rotating momentum sought, the following three values to be multiplied by $\pi^2 i^2$:

$$- \ 1.454\,4$$

$$- \ 0.654\,7$$

$$- \ 0.345\,2,$$

Moreover,

- 2. In that case where the right line uniting the centres of both coils is at right angles to the axis of the bifilarly suspended coil,

m, n and a having the same signification, and

$$\frac{m^2}{a^2 + n^2} = v^2 \ ,$$

$$\frac{a^2}{a^2 + n^2} = f \ ,$$

$$\frac{n^2}{a^2 + n^2} = 4gv^2 \ ,$$

the rotatory momentum required is

$$= +\pi v^3 n^2 i^2 S'$$

S' expressing the following series:

$$S' = + \ \frac{1}{3}$$

$$- \ \frac{3}{2} \left[\frac{1}{5} - \frac{10}{3} fg \right] v^2$$

$$+ \ \frac{15}{8} \left[\frac{1}{7} + \frac{2}{5}(1 - 14f)g + 42f^2 g^2 \right] v^4$$

$$- \ \frac{35}{16} \left[\frac{1}{9} + \frac{3}{7}(2 - 18f)g - \frac{54}{5}(1 - 11f)fg^2 - 572f^3 g^3 \right] v^6$$

$$+ \ \frac{315}{128} \left[\frac{1}{11} + \frac{4}{9}(3 - 22f)g + \frac{12}{7} \left(1 - 22f + 143f^2 \right) g^2 \right.$$

$$+ \frac{1144}{5}(1 - 10f)f^2 g^3 + \frac{24310}{3}f^4 g^4 \bigg] v^8$$

$$- \ etc.$$

If in this series we substitute for m and n the given values, and successively $a = 300, 400, 500$ and 600, we obtain as the rotating momentum required, the following values to be multiplied by $\pi^2 i^2$:

$$+ \ 3.562\,5$$

$$+ \ 1.466\,1$$

$$+ \ 0.742\,0$$

$$+ \ 0.426\,7 \ .$$

Lastly,

- 3. In that case where the centres of both coils coincide, when m designates the radius of the multiplier, and n' and n'' the least and greatest radius of the bifilarly-suspended coil, the rotatory momentum sought is

$$= \frac{\pi^2 m^3}{n'' - n'} i^2 \bigg[\frac{1}{3} \log \mathrm{nat} \frac{n''}{n'} + \frac{9}{160} \bigg(\frac{1}{n''^2 - n'^2} \bigg) m^2 - \frac{225}{14336} \bigg(\frac{1}{n''^4} - \frac{1}{n'^4} \bigg) m^4$$

$$+ \frac{6125}{884736} \bigg(\frac{1}{n''^6} - \frac{1}{n'^6} \bigg) m^6 + \frac{694575}{184549376} \bigg(\frac{1}{n''^8} - \frac{1}{n'^8} \bigg) m^8 + ... \bigg] \ .$$

If in this formula we substitute the values known from direct measurement in millimetres,

$$m = 44.4$$

$$n' = 50.25$$

$$n'' = 61.35,$$

we obtain as the rotatory momentum the following value to be multiplied by $\pi^2 i^2$:

$$+ \ 442.714 \ .$$

This value suffers a reduction of about 1/29th when we take into consideration that all the turns of the two coils do not lie in one plane, which in this case exerts greater influence on account of their proximity than in the other cases. The above value thus becomes reduced to

$$+ \ 427.45\pi^2 i^2 \ .$$

The numerical coefficients thus calculated should now be proportional to the observed values; and when multiplied by $\pi^2 i^2$, the intensity of the current i being expressed according to the dimensions upon which the above measurements were based, should be equal.

In fact, when all the calculated numerical coefficients are multiplied by 53.06, and then arranged according to the analogy of the observed values, we obtain the following Table of the calculated values, and their difference from those found by observation:[280]

Distance in mm	North or south, 0° or 180°	Difference	East or west, 90° or 270°	Difference
0	+ 22 680.00	+ 280.00	+ 22 680.00	+ 280.00
300	189.03	+ 0.90	77.17	−0.06
400	77.79	−0.34	34.74	+ 0.03
500	39.37	−0.10	18.31	−0.07
600	22.64	−0.18	—	—

In this comparison of theory and experiment, the single factor 53.06 was deduced from observations; and this was merely done because this factor could not be determined with sufficient accuracy by direct measurements. The direct determination of this factor is based upon the ascertainment of the proportion of that measure of the intensity of the current, upon which the scale of the galvanometer used is based, to that absolute measure to which the theoretical expression refers. The measurements necessary for ascertaining this proportion could not all be effected with the requisite accuracy, because separate measures were not taken for this purpose. In fact, however, the above factor was provisionally, as well as circumstances permitted, determined by direct measurement, and found = 49.5. This result also exhibits an agreement with that previously deduced from the observations, which under the circumstances could not have been expected to be greater.

7.4 Observations Tending to Enlarge the Domain of Electro-dynamic Investigations

7.4.1 A. Observation of Voltaic Induction

If the bifilarly-suspended coil of the dynamometer be made to oscillate whilst a current is transmitted through it, or through the coil of the multiplier, or through both simultaneously, this motion is *inductive*, and excites a current in the conductor, through which no current was passing, or alters the current passing through this conductor. This mode of excitation of the current is called *voltaic induction*.[281] The inducing motion, *i.e.* the velocity of the oscillating coil, is on each occasion diminished or *checked* by the antagonism of the currents excited by the voltaic induction and those conducted through the coil. This *check* to the vibrating coil *effected* by the voltaic induction may be accurately observed; and at the same time the motion of the oscillating coil itself, which *produces* the *voltaic induction*, may be accurately determined; and this twofold use of the dynamometer affords the data necessary for the more accurate investigation of the laws of voltaic induction.

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[280][Note by AKTA:] The fourth column of the first line appears in the original German text as: *Südl. od. westl.*, namely, *South or west*. Probably the word *Südl.* should be a misprint. The correct expression should be like in the English translation, namely, *East or west.* I also applied this correction here.

[281][Note by AKTA:] See footnote 139 on page 96.

The bifilarly-suspended coil closed in itself was made to oscillate to the greatest extent at which the scale permitted observations to be made, and its oscillations from 0 were counted until they became too minute for accurate observation. During the counting, the magnitude of the arc of oscillation was measured from time to time. These experiments were *first* made under the influence of voltaic induction, a current from three Grove's elements being conducted through the multiplying coil; the same experiments were *next* repeated, after the removal of the elements, without voltaic induction:

With voltaic induction.		Without voltaic induction.	
Enumeration of the oscillations	Arcs of oscillation	Enumeration of the oscillations	Arcs of oscillation
0	764.10	0	650.80
9	679.14	14	601.43
18	604.05	25	564.90
35	484.15	52	485.28
47	414.60	82	409.62
57	365.50	109	353.08
74	292.27	134	306.70
85	253.30	163	261.08
103	200.80	189	226.33
118	165.56	212	198.68
130	141.37	232	178.26
143	119.33	254	157.98
157	100.49	284	134.17
179	75.59	309	116.30
196	60.58	328	105.25
210	50.08	369	83.68
—	—	387	75.45

It is evident on comparison, that the diminution of the magnitude of the arc, which without the influence of induction from one oscillation to another amounted on an average to 1/180th, with the cooperation of the induction rose to 1/77th part.

When for the multiplying coil with the current transmitted through it, a magnet equivalent in an electro-magnetic point of view is substituted, the diminution of the arc is found to be equally great, *i.e.* the magnetic induction of this magnet is equal to the voltaic induction of the current in the multiplier.

The velocity which the inducing motion must possess for the intensity of the induced current to be equal to that of the inducing current, may also be deduced from these experiments.

7.4.2 B. Determination of the Duration of the Momentary Currents, as also Its Application to Physiological Experiments

When the intensity of a *continued* constant current is to be determined, both the galvanometer (the sine- or tangent-galvanometer)[282] and the dynamometer may be used; but if the

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[282][Note by AKTA:] In German: *Sinus- oder Tangenten-Bussole.* This expression was translated as "sine- or tangent-compass", [Web52c, p. 506]. I replaced it by the more common expression, namely, sine- or tangent-galvanometer", see footnote 92 on page 48.

current, the intensity of which is to be determined, is merely of *momentary* duration, observation made with either of these instruments is not sufficient, because the deflection observed does not depend merely upon the intensity of the current, but also upon the duration itself. It is therefore requisite, in experimentally investigating the intensity of the current, also to determine its duration.

The two instruments, *i.e.* the galvanometer and the dynamometer, are complementary to each other, so that when the same momentary current is transmitted through both, and the deflection of both instruments thus produced is observed, both the duration and the intensity of the momentary current can be determined from these two observations. This reciprocity is based upon the circumstance that the observed deflection of both instruments depends in the same manner upon the duration of the momentary current, *i.e.* it is proportional to it, whilst it is not dependent in the same manner upon the intensity of the current, because the deflection of the galvanometer is in proportion to the intensity of the current.

Let s and ς indicate the duration of the oscillations of the galvanometer and dynamometer;

e' and ε' the deflection at which both instruments remain when the same constant current of the intensity i' is transmitted through them;

Whilst e and ε indicate the extent of the deflection which both instruments attain in consequence of a momentary current of the duration Θ and of the intensity i; the following equation then gives the *duration* Θ:

$$\Theta = \frac{1}{\pi} \cdot \frac{s^2}{\varsigma} \cdot \frac{\varepsilon'}{e'^2} \cdot \frac{e^2}{\varepsilon}$$

and the following that of the *intensity* of the current i:

$$i = \frac{\varsigma}{s} \cdot \frac{e'}{\varepsilon'} \cdot i' \cdot \frac{\varepsilon}{e} \ .$$

s, ς, e', ε', i', e and ε in these formulae are magnitudes which can be determined by observation.

This combination of the dynamometer with the galvanometer is of special importance in physiology, to investigate accurately the excitation of the nerves by galvanic currents. For it is found that nerves of sensation especially are quickly deadened by continued currents, and hence that for such experiments momentary currents are frequently required to be used. But the observed impressions of sense depend less upon the duration of the current than upon its intensity; and it is essential to be acquainted with both.

7.4.3 C. Repetition of Ampère's Fundamental Experiment with Common Electricity and Measurement of the Duration of the Electric Spark on the Discharge of a Leyden Jar

It is evident from the preceding remarks, that the action of a current upon the dynamometer depends more upon the intensity of the current, to the square of which it is proportionate, than upon the duration of the current, to which it is simply proportional. Hence it follows that even a small quantity of electricity, when passed through the dynamometer within a very short period, so that it forms a current of very short duration but very great intensity, will produce a sensible effect. This is, in fact, the case when the small quantity of electricity which can be collected in a Leyden jar or battery is transmitted during its discharge through

the dynamometer. By this means it was found that Ampère's fundamental experiment, which had previously been made only with powerful galvanic batteries, could also be made with common electricity.

When the same electricity, collected in Leyden jars, after having been transmitted through the dynamometer, was also conducted through a galvanometer and the deflection thus produced in both instruments was measured, in accordance with the above rules, the duration of the current, *i.e.* the duration of the electric spark on the discharge of the Leyden jar, and at the same time the intensity of the current could be determined, admitting that the current might be considered as uniform during its brief duration.

It is well known that in experiments of this kind the discharge of the Leyden jar is effected by means of a wet string, to prevent its taking place through the air instead of through the fine wires of the two instruments. In this manner a series of experiments was made: a battery of eight jars being discharged through a wet hempen string, 7 millimetres in thickness and of different lengths, the following results were obtained:

Length of the string. Millimetres	Duration of the spark. Seconds
2 000	0.085 1
1 000	0.034 5
500	0.018 7
250	0.009 5

Hence the duration of the spark was nearly in proportion to the length of the string; for the observed duration of the spark is:

Seconds
0.081 6 + 0.003 5
0.040 8 − 0.006 3
0.020 4 − 0.001 7
0.010 2 − 0.000 7

The first part of the duration of the spark is thus exactly in proportion to the length of the string; but the second part is so small that it may be considered as arising from error of observation, which was unavoidable.

It is thus evident that the result obtained by Prof. Wheatstone,[283] according to which the duration of the spark on discharge by simple metallic conductors is infinitely short in comparison with that ascertained in the present case, is in direct accordance with this result.

7.4.4   D. Application of the Dynamometer to the Measurement of Sonorous Vibrations

When a rapid alternation of positive and negative currents ensues in a conducting wire, the continued motion of the electricity becomes converted into an *oscillation*. An oscillation of this kind cannot however be observed by means of a galvanometer (for instance, a sine- or tangent-galvanometer), because in this case the effects of the successive opposite oscillations destroy each other.

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[283][Note by AKTA:] See footnote 169 on page 119.

But the case is different with the dynamometer, in the two coils of which the direction of the vibration always changes simultaneously, and in which the deflection observed is in proportion to the square of the intensity of the current; for it is self-evident that the simultaneous change of the direction in both coils can exert no influence upon the action, because in the dynamometer a negative current transmitted through both coils produces a deflection towards the same side as a positive current transmitted through both coils. The occurrence of the deflection of the dynamometer to one side of the other does not, as in the galvanometer, depend upon the direction of the transmitted current, but merely upon the mode of connexion of the extremities of the wires of both coils.

But an electric vibration may be readily produced in a conducting wire by a magnetized steel bar vibrating so as to produce a musical sound, when one portion of the conducting wire, forming as it were the inducing coil, surrounds the free vibrating end of the bar, so that the direction of the vibration is at right angles to the plane of the coils of the wire. All vibrations of the bar on one side then produce positive currents in the wire, and all the vibrations on the other side produce negative currents, which follow each other as rapidly as the sonorous vibrations themselves.

When the ends of the wire of the inducing coil are united to the ends of that of the dynamometer, a deflection of the latter during the vibration of the bar is observed, which can be accurately measured. This deflection remains unaltered so long as the intensity of the sonorous vibrations remains unaltered, but speedily diminishes when the intensity of the sonorous vibrations diminishes; and when the amplitude of the sonorous vibrations has fallen to a half, it then amounts to the fourth part only.

The dynamometer thus presents a means of estimating the intensity of sonorous vibrations, which is of importance, because methods adapted to these measurements are still much required.

In addition to the investigations which we have hitherto considered, and which are based on the use of the dynamometer, there are others which will be subsequently treated of, when some modifications in the construction of the instrument for special objects will also be more accurately detailed.

7.5 On the Connexion of the Fundamental Principle of Electro-dynamics with that of Electro-statics

The fundamental principle of electro-statics is, that when two electric (positive or negative) masses, denoted by e and e', are at a distance r from each other, the amount of the force with which the two masses act reciprocally upon each other is expressed by

$$\frac{ee'}{r^2},$$

and that repulsion or attraction occurs accordingly as this expression has a positive or negative value.

On the other hand, the fundamental principle of electro-dynamics is as follows: When two elements of a current, the lengths of which are α and α' and the intensities i and i', and which are at the distance r from each other, so that the directions in which the positive electricity in both elements moves, form with each other the angle ε, and with the connecting

right line the angles Θ and Θ', the magnitude of the force with which the elements of the current reciprocally act upon each other is determined by the expression

$$-\frac{\alpha\alpha'ii'}{r^2}\left(\cos\varepsilon - \frac{3}{2}\cos\Theta\cos\Theta'\right)$$

and repulsion or attraction occurs according as this expression has a positive or negative value. The expressions of the rotatory momentum exerted by one coil of the dynamometer upon the other, developed at pages[284] 223 and 224, are all deduced from this fundamental principle.

The *former* of the two fundamental principles mentioned refers to two electric masses and their interaction, the *latter* to two elements of a current and their interaction. A more intimate connexion between the two can only be attained by recurring, likewise in the case of the elements of the current, to the consideration of the electric magnitudes existing in the elements of the current, and their interaction.

Thus the next question is, what electric magnitudes are contained in the two elements of a current, and upon what mutual relations of these masses their reciprocal actions may depend.

If the mass of positive electricity in a portion of the conducting wire equal to a unit of length be represented by e, and consequently the mass of the positive electricity contained in the elements of the current, the length of which is $=\alpha$, by αe, and if u indicates the velocity with which the mass moves, the product eu expresses that mass of positive electricity which in a unit of time passes through each section of the conducting wire, to which the intensity of the current i must be considered as proportional;[285] hence, when a expresses a constant factor,

$$aeu = i \ .$$

If now αe represent the mass of positive electricity in the element of the current α, and u its velocity, $-\alpha e$ represents the mass of negative electricity in the same element of the current, and $-u$ its velocity.

We have also, when

$$ae'u' = i' \ ,$$

$\alpha'e'$ as the mass of positive electricity in the second element of the current α', and u' its velocity, and lastly, $-\alpha'e'$ as the mass of negative electricity, and $-u'$ its velocity. If now for i and i', in the expression of the force which one element of a current exerts upon another, their values $i = aeu$, and $i' = ae'u'$ are substituted, we then obtain for them

$$-\frac{\alpha e\alpha'e'}{r^2}a^2uu'\left(\cos\varepsilon - \frac{3}{2}\cos\Theta\cos\Theta'\right) \ .$$

If now we *first* consider in this expression $\alpha e\alpha'e'$ as the product of the *positive* electric masses αe and $\alpha'e'$ in the two elements of the current, and uu' as the product of their velocities u and u', and if we denote by r the variable distance of these two masses in motion; and lastly, by $s_|$ and $s'_|$, the length of a portion of each of the two conducting wires,

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[284][Note by AKTA:] Pages 229 and 230 of Weber's *Werke*.

[285][Note by AKTA:] According to the context of the discussion presented in this work, this velocity u represents the motion of the electric mass e relative to the matter of the conducting wire.

to which the elements of the current α and α' just considered belong, estimated from a definite point of origin and proceeding in the direction of the *positive* electricity, as far as the element of the current under consideration, we then know that the cosines of the two angles Θ and Θ', which the two conducting wires in the situation of the elements of the current mentioned form with the connecting right line $r_|$, may be represented by the partial differential coefficients of $r_|$ with respect to $s_|$ and $s_|'$; thus

$$\cos \Theta = \frac{dr_|}{ds_|} , \quad \cos \Theta' = - \frac{dr_|}{ds_|'} ,$$

we then have

$$\cos \varepsilon = - \; r_| \frac{d^2 r_|}{ds_| ds_|'} - \frac{dr_|}{ds_|} \frac{dr_|}{ds_|'}$$

as the cosine of the angle ε which the directions of the two conducting wires form with each other. Moreover, if the differential coefficients above mentioned be substituted for the cosines of the three angles ε, Θ and Θ', we have

$$- \frac{\alpha e \alpha' e'}{r_|{}^2} a^2 u u' \left(\frac{1}{2} \frac{dr_|}{ds_|} \frac{dr_|}{ds_|'} - r_| \frac{d^2 r_|}{ds_| ds_|'} \right)$$

as the expression of the force with which one element of the current acts upon the other.

Secondly, if in the above expression, $-\alpha e \alpha' e'$ be considered as the product of the *positive* electric mass αe of one element of the current α into the *negative* electric mass $-\alpha' e'$ of the other element of the current α', and $-uu'$ as the product of their velocities u and $-u'$; moreover, if the variable distance of these two moving masses be denoted by $r_{||}$ and by $s_|$ and $s_{||}'$ the length of a portion of each of the two conducting wires, to which the elements of the current under consideration belong, taken from a definite point of origin, and proceeding in that direction in which, in the first the *positive*; in the second the *negative* electricity runs, as far as the element of the current mentioned, we obtain in the same manner

$$\cos \Theta = \frac{dr_{||}}{ds_|} , \quad \cos \Theta' = \frac{dr_{||}}{ds_{||}'} , \quad \cos \varepsilon = r_{||} \frac{d^2 r_{||}}{ds_| ds_{||}'} + \frac{dr_{||}}{ds_|} \frac{dr_{||}}{ds_{||}'} .$$

On substituting these values, we have the following expression for the force with which one element of the current acts upon the other:

$$+ \frac{\alpha e \alpha' e'}{r_{||}{}^2} a^2 u u' \left(\frac{1}{2} \frac{dr_{||}}{ds_|} \frac{dr_{||}}{ds_{||}'} - r_{||} \frac{d^2 r_{||}}{ds_| ds_{||}'} \right) .$$

If, *thirdly*, we consider in the original expression $\alpha e \alpha' e'$ as the product of the *negative* electrical masses $-\alpha e$ and $-\alpha' e'$ into the two elements of the current, and uu' as the product of their velocities $-u$ and $-u'$ and $r_{|||}$ denote the variable distance of these two moving masses, and lastly, $s_{||}$ and $s_{||}'$ denote the length of a portion of each of the two conducting wires to which the elements of the current under consideration belong, calculated from a definite point of origin, and proceeding in that direction in which the *negative* electricity runs, as far as the element of the current under consideration; we have

$$\cos \Theta = - \frac{dr_{|||}}{dr_{||}} , \quad \cos \Theta' = \frac{dr_{|||}}{ds_{||}'} ,$$

$$\cos \varepsilon = - \, r_{|||}\frac{d^2 r_{|||}}{ds_{||}ds'_{||}} - \frac{dr_{|||}}{ds_{||}}\frac{dr_{|||}}{ds'_{||}} \ .$$

On substituting these values, we have a third expression for the force with which one element of the current acts upon the other, namely,

$$-\frac{\alpha e \alpha' e'}{r_{|||}{}^2}a^2 uu' \left(\frac{1}{2}\frac{dr_{|||}}{ds_{||}}\frac{dr_{|||}}{ds'_{||}} - r_{|||}\frac{d^2 r_{|||}}{ds_{||}ds'_{||}} \right) \ .$$

In fine, if, *fourthly*, in the original expression we consider $-\alpha e \alpha' e'$ as the product of the *negative* electric mass $-\alpha e$ of the element of the current α into the *positive* electric mass $\alpha' e'$ of the element of the current α', and $-uu'$ as the product of their velocities $-u$ and u'; if, moreover, $r_{||||}$ designate the variable distance of these two moving masses, and $s_{||}$ and $s'_{|}$, the length of a portion of each of the two conducting wires to which the elements of the current under consideration belong, calculated from a defined point of origin, proceeding in that direction in which in the first the *negative*, in the second the *positive* electricity runs, we have

$$\cos \Theta = - \frac{dr_{||||}}{ds_{||}} \ , \quad \cos \Theta' = - \frac{dr_{||||}}{ds'_{|}} \ ,$$

$$\cos \varepsilon = r_{||||}\frac{d^2 r_{||||}}{ds_{||}ds'_{|}} + \frac{dr_{||||}}{ds_{||}}\frac{dr_{||||}}{ds'_{|}} \ .$$

If now these values be substituted, we have the fourth expression of the force with which one element of the current acts upon the other, viz.

$$+\frac{\alpha e \alpha' e'}{r_{||||}{}^2}a^2 uu' \left(\frac{1}{2}\frac{dr_{||||}}{ds_{||}}\frac{dr_{||||}}{ds'_{|}} - r_{||||}\frac{d^2 r_{||||}}{ds_{||}ds'_{|}} \right) \ .$$

Now at that moment in which the electric masses alluded to occur in the two elements α and α', the distances $r_{|}$, $r_{||}$, $r_{|||}$, $r_{||||}$, have all the same value, which is expressed by r. Hence the four expressions of the electro-dynamic force of the two elements of the current α and α' become converted into the following:

$$-\frac{\alpha e \alpha' e'}{r^2}a^2 uu' \left(\frac{1}{2}\frac{dr_{|}}{ds_{|}}\frac{dr_{|}}{ds'_{|}} - r\frac{d^2 r_{|}}{ds_{|}ds'_{|}} \right) \ , \qquad (1)$$

$$+\frac{\alpha e \alpha' e'}{r^2}a^2 uu' \left(\frac{1}{2}\frac{dr_{||}}{ds_{|}}\frac{dr_{||}}{ds'_{|}} - r\frac{d^2 r_{||}}{ds_{|}ds'_{|}} \right) \ , \qquad (2)$$

$$-\frac{\alpha e \alpha' e'}{r^2}a^2 uu' \left(\frac{1}{2}\frac{dr_{|||}}{ds_{||}}\frac{dr_{|||}}{ds'_{||}} - r\frac{d^2 r_{|||}}{ds_{||}ds'_{||}} \right) \ , \qquad (3)$$

$$+\frac{\alpha e \alpha' e'}{r^2}a^2 uu' \left(\frac{1}{2}\frac{dr_{||||}}{ds_{||}}\frac{dr_{||||}}{ds'_{|}} - r\frac{d^2 r_{||||}}{ds_{||}ds'_{|}} \right) \ , \qquad (4)$$

from which we can construct the fifth expression, viz. (5):

$$- \frac{\alpha e \alpha' e'}{r^2} \frac{a^2}{4} u u' \left[\frac{1}{2} \left(\frac{dr_|}{ds_|} \frac{dr_|}{ds'_|} - \frac{dr_{||}}{ds_|} \frac{dr_{||}}{ds'_{||}} + \frac{dr_{|||}}{ds_{||}} \frac{dr_{|||}}{ds'_{||}} - \frac{dr_{||||}}{ds_{||}} \frac{dr_{||||}}{ds'_|} \right) \right.$$
$$\left. - r \left(\frac{d^2 r_|}{ds_| ds'_|} - \frac{d^2 r_{||}}{ds_| ds'_{||}} + \frac{d^2 r_{|||}}{ds_{||} ds'_{||}} - \frac{d^2 r_{||||}}{ds_{||} ds'_|} \right) \right] .$$

The four variable distances $r_|$, $r_{||}$, $r_{|||}$, $r_{||||}$, are now respectively dependent upon the variable magnitudes of the paths $s_|$ and $s'_|$, $s_|$ and $s'_{||}$, $s_{||}$ and $s'_{||}$, $s_{||}$ and $s'_|$, through which the moveable masses to which they refer have passed in the two given conducting wires, and which consequently are again functions of the time t. On developing their complete differentials, we have

$$dr_| = \frac{dr_|}{ds_|} ds_| + \frac{dr_|}{ds'_|} ds'_| ,$$

$$dr_{||} = \frac{dr_{||}}{ds_|} ds_| + \frac{dr_{||}}{ds'_{||}} ds'_{||} ,$$

$$dr_{|||} = \frac{dr_{|||}}{ds_{||}} ds_{||} + \frac{dr_{|||}}{ds'_{||}} ds'_{||} ,$$

$$dr_{||||} = \frac{dr_{||||}}{ds_{||}} ds_{||} + \frac{dr_{||||}}{ds'_|} ds'_| ;$$

moreover,

$$d^2 r_| = \frac{d^2 r_|}{ds_|^2} ds_|^2 + 2 \frac{d^2 r_|}{ds_| ds'_|} ds_| ds'_| + \frac{d^2 r_|}{ds'^2_|} ds'^2_| ,$$

$$d^2 r_{||} = \frac{d^2 r_{||}}{ds_|^2} ds_|^2 + 2 \frac{d^2 r_{||}}{ds_| ds'_{||}} ds_| ds'_{||} + \frac{d^2 r_{||}}{ds'^2_{||}} ds'^2_{||} ,$$

$$d^2 r_{|||} = \frac{d^2 r_{|||}}{ds_{||}^2} ds_{||}^2 + 2 \frac{d^2 r_{|||}}{ds_{||} ds'_{||}} ds_{||} ds'_{||} + \frac{d^2 r_{|||}}{ds'^2_{||}} ds'^2_{||} ,$$

$$d^2 r_{||||} = \frac{d^2 r_{||||}}{ds_{||}^2} ds_{||}^2 + 2 \frac{d^2 r_{||||}}{ds_{||} ds'_|} ds_{||} ds'_| + \frac{d^2 r_{||||}}{ds'^2_|} ds'^2_| .$$

If these differentials are respectively divided by the elements of the time dt, and their squares dt^2, and admitting that

$$\frac{ds_|}{dt} = \frac{ds_{||}}{dt} = u , \qquad \frac{ds'_|}{dt} = \frac{ds'_{||}}{dt} = u' ,$$

we have

$$\frac{dr_|}{dt} = u \frac{dr_|}{ds_|} + u' \frac{dr_|}{ds'_|} ,$$

$$\frac{dr_{||}}{dt} = u \frac{dr_{||}}{ds_|} + u' \frac{dr_{||}}{ds'_{||}} ,$$

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$$\frac{dr_{|||}}{dt} = u\frac{dr_{|||}}{ds_{||}} + u'\frac{dr_{|||}}{ds'_{||}} \;,$$

$$\frac{dr_{||||}}{dt} = u\frac{dr_{||||}}{ds_{||}} + u'\frac{dr_{||||}}{ds'_{|}} \;;$$

moreover,

$$\frac{d^2 r_{|}}{dt^2} = u^2\frac{d^2 r_{|}}{ds_{|}^{\,2}} + 2uu'\frac{d^2 r_{|}}{ds_{|}ds'_{|}} + u'^2\frac{d^2 r_{|}}{ds'^{\,2}_{|}} \;,$$

$$\frac{d^2 r_{||}}{dt^2} = u^2\frac{d^2 r_{||}}{ds_{|}^{\,2}} + 2uu'\frac{d^2 r_{||}}{ds_{|}ds'_{||}} + u'^2\frac{d^2 r_{||}}{ds'^{\,2}_{||}} \;,$$

$$\frac{d^2 r_{|||}}{dt^2} = u^2\frac{d^2 r_{|||}}{ds_{||}^{\,2}} + 2uu'\frac{d^2 r_{|||}}{ds_{||}ds'_{||}} + u'^2\frac{d^2 r_{|||}}{ds'^{\,2}_{||}} \;,$$

$$\frac{d^2 r_{||||}}{dt^2} = u^2\frac{d^2 r_{||||}}{ds_{||}^{\,2}} + 2uu'\frac{d^2 r_{||||}}{ds_{||}ds'_{|}} + u'^2\frac{d^2 r_{||||}}{ds'^{\,2}_{|}} \;.$$

From the four last equations we get immediately:

$$2uu'\frac{d^2 r_{|}}{ds_{|}ds'_{|}} = +\frac{d^2 r_{|}}{dt^2} - u^2\frac{d^2 r_{|}}{ds_{|}^{\,2}} - u'^2\frac{d^2 r_{|}}{ds'^{\,2}_{|}} \;,$$

$$-2uu'\frac{d^2 r_{||}}{ds_{|}ds'_{||}} = -\frac{d^2 r_{||}}{dt^2} + u^2\frac{d^2 r_{||}}{ds_{|}^{\,2}} + u'^2\frac{d^2 r_{||}}{ds'^{\,2}_{||}} \;,$$

$$2uu'\frac{d^2 r_{|||}}{ds_{|}ds'_{||}} = +\frac{d^2 r_{|||}}{dt^2} - u^2\frac{d^2 r_{|||}}{ds_{||}^{\,2}} - u'^2\frac{d^2 r_{|||}}{ds'^{\,2}_{||}} \;,$$

$$-2uu'\frac{d^2 r_{||||}}{ds_{||}ds'_{|}} = -\frac{d^2 r_{||||}}{dt^2} + u^2\frac{d^2 r_{||||}}{ds_{||}^{\,2}} + u'^2\frac{d^2 r_{||||}}{ds'^{\,2}_{|}} \;.$$

Now the differential coefficients

$$\frac{d^2 r_{|}}{ds_{|}^{\,2}} \;,\quad \frac{d^2 r_{||}}{ds_{|}^{\,2}} \;,\quad \frac{d^2 r_{|||}}{ds_{||}^{\,2}} \;,\quad \frac{d^2 r_{||||}}{ds_{||}^{\,2}}$$

have the same value, which is dependent merely upon the position and form of the *first* conducting wire, and which we shall denote by $d^2 r/ds^2$. This applies also to the differential coefficients

$$\frac{d^2 r_{|}}{ds'^{\,2}_{|}} \;,\quad \frac{d^2 r_{||}}{ds'^{\,2}_{||}} \;,\quad \frac{d^2 r_{|||}}{ds'^{\,2}_{||}} \;,\quad \frac{d^2 r_{||||}}{ds'^{\,2}_{|}} \;,$$

all of which denote the same magnitudes, which are dependent merely upon the position and form of the *second* conducting wire, and which for brevity we shall denote by $d^2 r/ds'^2$. On summation, bearing this in mind, we have

$$2uu'\left(\frac{d^2r_|}{ds_|ds'_|} - \frac{d^2r_{||}}{ds_|ds'_{||}} + \frac{d^2r_{|||}}{ds_{||}ds'_{||}} - \frac{d^2r_{||||}}{ds_{||}ds'_|}\right)$$

$$= \frac{d^2r_|}{dt^2} - \frac{d^2r_{||}}{dt^2} + \frac{d^2r_{|||}}{dt^2} - \frac{d^2r_{||||}}{dt^2} \ .$$

But from the first four equations, after they have been squared, we have[286]

$$2uu'\frac{dr_|}{ds_|}\frac{dr_|}{ds'_|} = +\frac{dr_|^2}{dt^2} - u^2\frac{dr_|^2}{ds_|^2} - u'^2\frac{dr_|^2}{ds'^2_|} \ ,$$

$$-2uu'\frac{dr_{||}}{ds_|}\frac{dr_{||}}{ds'_{||}} = -\frac{dr_{||}^2}{dt^2} + u^2\frac{dr_{||}^2}{ds_|^2} + u'^2\frac{dr_{||}^2}{ds'^2_{||}} \ ,$$

$$2uu'\frac{dr_{|||}}{ds_{||}}\frac{dr_{|||}}{ds'_{||}} = +\frac{dr_{|||}^2}{dt^2} - u^2\frac{dr_{|||}^2}{ds_{||}^2} - u'^2\frac{dr_{|||}^2}{ds'^2_{||}} \ ,$$

$$-2uu'\frac{dr_{||||}}{ds_{||}}\frac{dr_{||||}}{ds'_|} = -\frac{dr_{||||}^2}{dt^2} + u^2\frac{dr_{||||}^2}{ds_{||}^2} + u'^2\frac{dr_{||||}^2}{ds'^2_|} \ .$$

Now the differential coefficients

$$\frac{dr_|^2}{ds_|^2} \ , \quad \frac{dr_{||}^2}{ds_|^2} \ , \quad \frac{dr_{|||}^2}{ds_{||}^2} \ , \quad \frac{dr_{||||}^2}{ds_{||}^2}$$

have also the same value, which shall be denoted by dr^2/ds^2, as have likewise

$$\frac{dr_|^2}{ds'^2_|} \ , \quad \frac{dr_{||}^2}{ds'^2_{||}} \ , \quad \frac{dr_{|||}^2}{ds'^2_{||}} \ , \quad \frac{dr_{||||}^2}{ds'^2_|} \ ,$$

which we shall denote by dr^2/ds'^2.

On summation, keeping this in view, we have

$$2uu'\left(\frac{dr_|}{ds_|}\frac{dr_|}{ds'_|} - \frac{dr_{||}}{ds_|}\frac{dr_{||}}{ds'_{||}} + \frac{dr_{|||}}{ds_{||}}\frac{dr_{|||}}{ds'_{||}} - \frac{dr_{||||}}{ds_{||}}\frac{dr_{||||}}{dr'_|}\right)$$

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[286][Note by AKTA:] The following four equations should be understood as:

$$2uu'\frac{dr_|}{ds_|}\frac{dr_|}{ds'_|} = +\left(\frac{dr_|}{dt}\right)^2 - u^2\left(\frac{dr_|}{ds_|}\right)^2 - (u')^2\left(\frac{dr_|}{ds'_|}\right)^2 \ ,$$

$$-2uu'\frac{dr_{||}}{ds_|}\frac{dr_{||}}{ds'_{||}} = -\left(\frac{dr_{||}}{dt}\right)^2 + u^2\left(\frac{dr_{||}}{ds_|}\right)^2 + (u')^2\left(\frac{dr_{||}}{ds'_{||}}\right)^2 \ ,$$

$$2uu'\frac{dr_{|||}}{ds_{||}}\frac{dr_{|||}}{ds'_{||}} = +\left(\frac{dr_{|||}}{dt}\right)^2 - u^2\left(\frac{dr_{|||}}{ds_{||}}\right)^2 - (u')^2\left(\frac{dr_{|||}}{ds'_{||}}\right)^2 \ ,$$

$$-2uu'\frac{dr_{||||}}{ds_{||}}\frac{dr_{||||}}{ds'_|} = -\left(\frac{dr_{||||}}{dt}\right)^2 + u^2\left(\frac{dr_{||||}}{ds_{||}}\right)^2 + (u')^2\left(\frac{dr_{||||}}{ds'_|}\right)^2 \ .$$

$$= \frac{dr_|^2}{dt^2} - \frac{dr_{||}^2}{dt^2} + \frac{dr_{|||}^2}{dt^2} - \frac{dr_{||||}^2}{dt^2} \ .$$

On substituting these values in the fifth expression found for the electro-dynamic force, it becomes

$$-\frac{\alpha e\alpha'e'}{r^2}\frac{a^2}{16}\left[\left(\frac{dr_|^2}{dt^2} - \frac{dr_{||}^2}{dt^2} + \frac{dr_{|||}^2}{dt^2} - \frac{dr_{||||}^2}{dt^2}\right)\right.$$

$$\left. - 2r\left(\frac{d^2r_|}{dt^2} - \frac{d^2r_{||}}{dt^2} + \frac{d^2r_{|||}}{dt^2} - \frac{d^2r_{||||}}{dt^2}\right)\right] \ ,$$

an expression which may be resolved into the four following members:

$$-\frac{\alpha e\alpha'e'}{r_|^2}\frac{a^2}{16}\left(\frac{dr_|^2}{dt^2} - 2r_|\frac{d^2r_|}{dt^2}\right) \ ,$$

$$+\frac{\alpha e\alpha'e'}{r_{||}^2}\frac{a^2}{16}\left(\frac{dr_{||}^2}{dt^2} - 2r_{||}\frac{d^2r_{||}}{dt^2}\right) \ ,$$

$$-\frac{\alpha e\alpha'e'}{r_{|||}^2}\frac{a^2}{16}\left(\frac{dr_{|||}^2}{dt^2} - 2r_{|||}\frac{d^2r_{|||}}{dt^2}\right) \ ,$$

$$+\frac{\alpha e\alpha'e'}{r_{||||}^2}\frac{a^2}{16}\left(\frac{dr_{||||}^2}{dt^2} - 2r_{||||}\frac{d^2r_{||||}}{dt^2}\right) \ .$$

Each of these four members refers exclusively to *two* of the four electric masses distinguished in the two elements of the current, viz. the *first* member to the two positive masses αe and $\alpha'e'$, the relative distance of which is $r_|$, velocity $dr_|/dt$, and acceleration $d^2r_|/dt^2$; the *second* to the positive mass αe in the first, and to the negative mass $-\alpha'e'$, in the second element the relative distance of which is $r_{||}$, velocity $dr_{||}/dt$, and acceleration $d^2r_{||}/dt^2$, and so on; and in fact all four are members of the masses to which they refer, the distance, velocity and acceleration of which are composed *in exactly the same manner.*

Hence it is evident that if the entire expression of the electrodynamic force of two elements of a current be considered as the sum of the forces, which each two of the four electric masses they contain exert upon each other, this sum would be decomposed into its *original constituents*, the four above members representing individually the four forces which the four electric masses in the two elements exert *in pairs* upon each other.

Hence also the force with which any positive or negative mass E acts upon any other positive or negative mass E', at the distance R, with a relative velocity of dR/dt, and acceleration d^2R/dt^2, may be expressed by[287]

$$-\frac{a^2}{16}\cdot\frac{EE'}{R^2}\left(\frac{dR^2}{dt^2} - 2R\frac{d^2R}{dt^2}\right) \ ;$$

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[287][Note by AKTA:] The following equation should be understood as

$$-\frac{a^2}{16}\cdot\frac{EE'}{R^2}\left[\left(\frac{dR}{dt}\right)^2 - 2R\frac{d^2R}{dt^2}\right] \ .$$

for this fundamental principle is necessary and at the same time sufficient to allow of the deduction of Ampère's electro-dynamic laws, which are confirmed by the above measurements.

However, this new fundamental principle of electro-dynamics is in its nature more *general* than that formerly laid down by Ampère; for the latter refers merely to the special case, in which four electric magnitudes are given at the same time, subject to the conditions premised for invariable and undisturbed elements of the current, whilst such a limitation to the above conditions does not occur in the former. This fundamental principle, consequently, admits of application in those cases where the former is inapplicable; hence its greater utility.

If, lastly, the newly-discovered fundamental principle of electrodynamics be compared with the fundamental principle of electro-statics mentioned at the commencement, we see that each estimates a force which *two electric masses* exert upon each other; but that in the cases hitherto considered, one of the two forces disappears each time, whence the other only requires consideration. This occurs *first* in all cases which belong to electrostatics, because here the force determined by the new principle of electro-dynamics always disappears; but it also occurs, *secondly*, in all cases belonging to electro-dynamics which have yet come under consideration, where relations are constantly pre-supposed to exist, in which all forces estimated by the principle of electro-statics are mutually checked.

Thus the two principles are complementary to each other, and hence they may be combined to form a general fundamental principle *for the whole theory of electricity*, which comprises both electro-statics and electro-dynamics.

By the fundamental principle of electro-statics, a force

$$= \frac{EE'}{R^2}$$

was found for two electric masses E and E' at the distance R; if this force be then added to that yielded by the new principle of electrodynamics,

$$= -\frac{a^2}{16} \cdot \frac{EE'}{R^2} \left(\frac{dR^2}{dt^2} - 2R\frac{d^2R}{dt^2} \right) \ ,$$

we obtain, as the general expression for the complete determination of the force which any electric mass E exerts upon another E', whether at rest or in motion,[288]

$$\frac{EE'}{R^2} \left(1 - \frac{a^2}{16} \cdot \frac{dR^2}{dt^2} + \frac{a^2}{8} \cdot R\frac{d^2R}{dt^2} \right) \ .$$

For a definite magnitude assumed for the purpose of measuring the time, in which $a = 4$, this expression becomes

$$\frac{EE'}{R^2} \left(1 - \frac{dR^2}{dt^2} + 2R\frac{d^2R}{dt^2} \right) \ .$$

Moreover, supposing that both R and dR/dt, are functions of t, consequently that dR/dt, is to be regarded as a funcion of R, which we shall denote by $[R]$, we may also say that the *potential* of the mass E, in regard to the situation of the mass E', is

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[288][Note by AKTA:] The following equation should be understood as

$$\frac{EE'}{R^2} \left[1 - \frac{a^2}{16} \cdot \left(\frac{dR}{dt}\right)^2 + \frac{a^2}{8} \cdot R\frac{d^2R}{dt^2} \right] \ .$$

$$= \frac{E}{R} \left(1 - [R]^2 \right) \; ;$$

for the partial differential coefficients of this expression, with respect to the three coordinate x, y, z, yield the components of the decomposed accelerating force in the direction of the three coordinate axes.

Lastly, if by the *reduced relative velocity* of the masses E and E', we understand that relative velocity which these magnitudes, — the distance of which apart at the moment supposed was R, the relative velocity dR/dt, and the acceleration d^2R/dt^2, *if the latter were constant,* — would possess at that instant in which both, in accordance with this supposition, met at one point, and if V denoted this *reduced relative velocity*, the above expression,

$$\frac{EE'}{R^2} \left(1 - \frac{dR^2}{dt^2} + 2R\frac{d^2R}{dt^2} \right) \; ,$$

becomes converted into the following,

$$\frac{EE'}{R^2} \left(1 - V^2 \right) \; ,$$

which may be verbally expressed as follows: *This diminution arising from motion of the force with which two electric masses would act upon each other when they are at rest, is in proportion to the square of their reduced relative velocity.*

Thus the expression given for the determination of the force which two electric masses exert upon one another are now confirmed

1. As regards the entire domain of electro-statics;

2. As regards that domain of electro-dynamics the object of which is the consideration of the forces of the elements of the current when invariable and undisturbed; hence

3. Its confirmation, as regards all that domain of electro-dynamics which is not limited to the invariable and undisturbed state of the elements of the current, is all that remains to be desired.

7.6   Theory of Voltaic Induction

It has already been mentioned that the principle of electrodynamics laid down by Ampère refers merely to the special case, where four electric masses occur under the conditions premised to exist where two invariable and undisturbed elements of a current are concerned. Under conditions where these principles do not exist, the new fundamental principle only can be applied for the *à priori* determination of the forces and phenomena; and it is exactly in this way that the greater advantage of the new principle, arising from its more general application, will be exhibited.

The case in which the principle of electro-dynamics laid down by Ampère is inapplicable, thus occurs even when one element of a current is disturbed or its intensity varies; in addition to which it may also happen, that instead of the other element of the current, one element only of the conductor of a current may be present, without however any current being present in it. In fact, we know from experience that currents are then excited or *induced*, and the

phenomena of these induced currents are comprised under the name of *voltaic induction*;[289] but none of these phenomena could be predicted or estimated *à priori* either from the principle of eletro-statics or the principle of electro-dynamics laid down by Ampère. It will now however be shown, that by means of the new fundamental principle as laid down here, the laws for the *à priori* determination of all the phaenomena of voltaic induction may be deduced. It is evident that the laws of voltaic induction deduced in this manner are correct, so far only as we are in possession of definite observations.

For the purpose of this deduction the magnitudes concerned may be denoted as follows: α and α' denote the length of two elements, the former of which, α, is supposed to be *at rest*. This supposition does not limit the generality of the consideration, because every movement of the element α may be transferred to α', by attributing the opposite direction to it in α'. The four following electric masses are distinguished in these two elements, viz.

$$+\alpha e \ , \quad -\alpha e \ , \quad +\alpha' e' \ , \quad -\alpha' e' \ .$$

The *first* of these masses $+\alpha e$ would move with velocity $+u$ in the direction of the quiescent element α, which forms the angle Θ with the right line drawn from α to α'. This velocity during the element of time dt would alter by $+du$.

The *second* mass $-\alpha e$ would move, in accordance with the determinations relating to a galvanic current, in the same direction as the velocity $-u$, *i.e.* backwards, and this velocity during the element of time dt would alter by $-du$.

The *third* mass $+\alpha' e'$ would move with the velocity $+u'$ in the direction of the element α', which with the right lines drawn from α to α', and produced, forms the angle Θ'. This velocity in the element of time dt would alter by $+du'$. Moreover, this electric mass would itself share the motion of the element α', which takes place with the velocity v in a direction which forms the angle η with the prolonged right line drawn from α to α', and is contained in a plane lying in this right line, which with the plane running parallel with the element α through the same right line, encloses the angle γ. The velocity v would alter during the element of time dt by dv.

The *fourth* mass $-\alpha' e'$ would move, in accordance with the determinations for a galvanic current, in the direction of the element α', with the velocity $-u'$ which during the element of time dt alters by $-du'$; but, moreover, like the previous mass, would itself acquire the velocity v of the element α' in the direction already indicated.

The distances of the two former masses from the two latter, at the moment under consideration, are equal to the distance r of the two elements themselves; but since they do not remain the same, they may be denoted by r_1, r_2, r_3, r_4.

Lastly, if two planes pass through the right line drawn from α to α', the one parallel to α, the other to α', ω would denote the angle enclosed by these two planes.

Then, on applying the new principle, we obtain as the *sum* of the forces which act upon the *positive* and *negative* electricity in the element α', *i.e.* as the force which moves the element α' itself, the following expression:

$$-\frac{a^2}{16} \cdot \frac{\alpha e \cdot \alpha' e'}{r^2} \left\{ \left(\frac{dr_1^2}{dt^2} + \frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) \right.$$

$$\left. - 2r \left(\frac{d^2 r_1}{dt^2} + \frac{d^2 r_2}{dt^2} - \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) \right\} \ .$$

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[289][Note by AKTA:] See footnote 139 on page 96.

But for the *difference* of these forces, upon which the *induction* depends, we have the following expression:

$$-\frac{a^2}{16} \cdot \frac{\alpha e \cdot \alpha' e'}{r^2} \left\{ \left(\frac{dr_1^2}{dt^2} - \frac{dr_2^2}{dt^2} + \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) \right.$$

$$\left. - 2r \left(\frac{d^2 r_1}{dt^2} - \frac{d^2 r_2}{dt^2} + \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) \right\} .$$

Moreover, when, in addition to the motions of the electric masses in their conductors, the motion common to them and their conductors is taken into account, we have the following expression for the first differential coefficients:

$$\frac{dr_1}{dt} = -u \cos \Theta + u' \cos \Theta' + v \cos \eta \;,$$

$$\frac{dr_2}{dt} = +u \cos \Theta - u' \cos \Theta' + v \cos \eta \;,$$

$$\frac{dr_3}{dt} = -u \cos \Theta - u' \cos \Theta' + v \cos \eta \;,$$

$$\frac{dr_4}{dt} = +u \cos \Theta + u' \cos \Theta' + v \cos \eta \;.$$

Hence

$$\left(\frac{dr_1^2}{dt^2} + \frac{dr_2^2}{dt^2} - \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) = -8uu' \cos \Theta \cos \Theta' \;,$$

$$\left(\frac{dr_1^2}{dt^2} - \frac{dr_2^2}{dt^2} + \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) = -8uv \cos \Theta \cos \eta \;.$$

We obtain the second differential coefficients when the variability of the velocity u, u', and v is also taken into account:

$$\frac{d^2 r_1}{dt^2} = +u \sin \Theta \frac{d\Theta_1}{dt} - u' \sin \Theta' \frac{d\Theta_1'}{dt} - v \sin \eta \frac{d\eta_1}{dt}$$

$$- \cos \Theta \frac{du}{dt} + \cos \Theta' \frac{du'}{dt} + \cos \eta \frac{dv}{dt} \;,$$

$$\frac{d^2 r_2}{dt^2} = -u \sin \Theta \frac{d\Theta_2}{dt} + u' \sin \Theta' \frac{d\Theta_2'}{dt} - v \sin \eta \frac{d\eta_2}{dt}$$

$$+ \cos \Theta \frac{du}{dt} - \cos \Theta' \frac{du'}{dt} + \cos \eta \frac{dv}{dt} \;,$$

$$\frac{d^2 r_3}{dt^2} = +u \sin \Theta \frac{d\Theta_3}{dt} + u' \sin \Theta' \frac{d\Theta_3'}{dt} - v \sin \eta \frac{d\eta_3}{dt}$$

$$- \cos \Theta \frac{du}{dt} - \cos \Theta' \frac{du'}{dt} + \cos \eta \frac{dv}{dt} \;,$$

$$\frac{d^2 r_4}{dt^2} = -u \sin\Theta \frac{d\Theta_4}{dt} - u' \sin\Theta' \frac{d\Theta'_4}{dt} - v \sin\eta \frac{d\eta_4}{dt}$$

$$+ \cos\Theta \frac{du}{dt} + \cos\Theta' \frac{du'}{dt} + \cos\eta \frac{dv}{dt} \; .$$

Consequently it becomes

$$\left(\frac{d^2 r_1}{dt^2} + \frac{d^2 r_2}{dt^2} - \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) = +u \sin\Theta \left(\frac{d\Theta_1}{dt} - \frac{d\Theta_2}{dt} - \frac{d\Theta_3}{dt} - \frac{d\Theta_4}{dt} \right)$$

$$- u' \sin\Theta' \left(\frac{d\Theta'_1}{dt} - \frac{d\Theta'_2}{dt} + \frac{d\Theta'_3}{dt} - \frac{d\Theta'_4}{dt} \right) - v \sin\eta \left(\frac{d\eta_1}{dt} + \frac{d\eta_2}{dt} - \frac{d\eta_3}{dt} - \frac{d\eta_4}{dt} \right)$$

and

$$\left(\frac{d^2 r_1}{dt^2} - \frac{d^2 r_2}{dt^2} + \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) = +u \sin\Theta \left(\frac{d\Theta_1}{dt} + \frac{d\Theta_2}{dt} + \frac{d\Theta_3}{dt} + \frac{d\Theta_4}{dt} \right)$$

$$- u' \sin\Theta' \left(\frac{d\Theta'_1}{dt} + \frac{d\Theta'_2}{dt} - \frac{d\Theta'_3}{dt} - \frac{d\Theta'_4}{dt} \right) - v \sin\eta \left(\frac{d\eta_1}{dt} - \frac{d\eta_2}{dt} + \frac{d\eta_3}{dt} - \frac{d\eta_4}{dt} \right)$$

$$- 4 \cos\Theta \frac{du}{dt} \; .$$

The differential coefficients $d\Theta_1/dt$, $d\Theta'_1/dt$, $d\eta_1/dt$, etc. are easily developed according to the well-known laws of trigonometry; and we thus obtain the following expressions, viz.

$$r_1 \frac{d\Theta_1}{dt} = +u \sin\Theta - u' \sin\Theta' \cos\omega - v \sin\eta \cos\gamma \; ,$$

$$r_1 \frac{d\Theta'_1}{dt} = -u' \sin\Theta' + u \sin\Theta \cos\omega - v \sin\eta \cos(\omega + \gamma) \; ,$$

$$r_1 \frac{d\eta_1}{dt} = -v \sin\eta + u \sin\Theta \cos\gamma - u' \sin\Theta' \cos(\omega + \gamma) \; ,$$

$$r_2 \frac{d\Theta_2}{dt} = -u \sin\Theta + u' \sin\Theta' \cos\omega - v \sin\eta \cos\gamma \; ,$$

$$r_2 \frac{d\Theta'_2}{dt} = +u' \sin\Theta' - u \sin\Theta \cos\omega - v \sin\eta \cos(\omega + \gamma) \; ,$$

$$r_2 \frac{d\eta_2}{dt} = -v \sin\eta - u \sin\Theta \cos\gamma + u' \sin\Theta' \cos(\omega + \gamma) \; ,$$

$$r_3 \frac{d\Theta_3}{dt} = +u \sin\Theta + u' \sin\Theta' \cos\omega - v \sin\eta \cos\gamma \; ,$$

$$r_3 \frac{d\Theta'_3}{dt} = +u' \sin\Theta' + u \sin\Theta \cos\omega - v \sin\eta \cos(\omega + \gamma) \; ,$$

$$r_3 \frac{d\eta_3}{dt} = -v \sin\eta + u \sin\Theta \cos\gamma + u' \sin\Theta' \cos(\omega + \gamma) \ ,$$

$$r_4 \frac{d\Theta_4}{dt} = -u \sin\Theta - u' \sin\Theta' \cos\omega - v \sin\eta \cos\gamma \ ,$$

$$r_4 \frac{d\Theta'_4}{dt} = -u' \sin\Theta' - u \sin\Theta \cos\omega - v \sin\eta \cos(\omega + \gamma) \ ,$$

$$r_4 \frac{d\eta_4}{dt} = -v \sin\eta - u \sin\Theta \cos\gamma - u' \sin\Theta' \cos(\omega + \gamma) \ .$$

Now since for the moment under consideration $r_1 = r_2 = r_3 = r_4 = r$, we thus get

$$r\left(\frac{d\Theta_1}{dt} - \frac{d\Theta_2}{dt} - \frac{d\Theta_3}{dt} + \frac{d\Theta_4}{dt}\right) = -4u' \sin\Theta' \cos\omega \ ,$$

$$r\left(\frac{d\Theta_1}{dt} + \frac{d\Theta_2}{dt} + \frac{d\Theta_3}{dt} + \frac{d\Theta_4}{dt}\right) = -4v \sin\eta \cos\gamma \ ;$$

again:

$$r\left(\frac{d\Theta'_1}{dt} - \frac{d\Theta'_2}{dt} + \frac{d\Theta'_3}{dt} - \frac{d\Theta'_4}{dt}\right) = +4u \sin\Theta \cos\omega \ ,$$

$$r\left(\frac{d\Theta'_1}{dt} + \frac{d\Theta'_2}{dt} - \frac{d\Theta'_3}{dt} - \frac{d\Theta'_4}{dt}\right) = 0 \ ;$$

lastly:

$$r\left(\frac{d\eta_1}{dt} + \frac{d\eta_2}{dt} - \frac{d\eta_3}{dt} - \frac{d\eta_4}{dt}\right) = 0 \ ,$$

$$r\left(\frac{d\eta_1}{dt} - \frac{d\eta_2}{dt} + \frac{d\eta_3}{dt} - \frac{d\eta_4}{dt}\right) = +4u \sin\Theta \cos\gamma \ .$$

These values by substitution become

$$r\left(\frac{d^2 r_1}{dt^2} + \frac{d^2 r_2}{dt^2} - \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2}\right) = -8uu' \sin\Theta \sin\Theta' \cos\omega \ ,$$

$$r\left(\frac{d^2 r_1}{dt^2} - \frac{d^2 r_2}{dt^2} + \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2}\right) = -8uv \sin\Theta \sin\eta \cos\gamma - 4r \cos\Theta \frac{du}{dt} \ .$$

With these values, the *sum* of the forces which act upon the positive and negative electricity in the element α' is[290]

$$= -\frac{\alpha\alpha'}{r^2} \cdot aeu \cdot ae'u' \left(\sin\Theta \sin\Theta' \cos\omega - \frac{1}{2}\cos\Theta \cos\Theta'\right) \ .$$

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[290][Note by AKTA:] Due to a misprint, the next equation appeared in the original text as:

$$= -\frac{\alpha\alpha'}{r^2} \cdot aeu \cdot a'e'u' \left(\sin\Theta \sin\Theta' \cos\omega - \frac{1}{2}\cos\Theta \cos\Theta'\eta\right) \ .$$

If in this equation the angle which the directions of the two elements α and α' form with each other be denoted by ε, and, as in page[291] 231, i and i' be substituted for aeu and[292] $ae'u'$, the above sum, with slight transposition, becomes

$$= -\frac{\alpha\alpha'ii'}{r^2}\left(\cos\varepsilon - \frac{3}{2}\cos\Theta\cos\Theta'\right) ,$$

the same expression at which Ampère arrived where the elements of the current are invariable and undisturbed, *i.e.* the electro-dynamic force acting upon the entire element α' is determined in the same manner when the conductors are in motion and the intensities of the current variable, as when the intensities of the current remain invariable and the conductors undisturbed. Hence Ampère's law is of general application in the determination of the forces which act upon the entire element of the current when the position of the elements of the current and the intensities of the current are given. The application of this law merely requires that the intensities of the current when variable, as also the position when variable, be given *for each individual moment*, and further, the intensities of the currents, including that part added at each moment in consequence of induction.

But as regards the *difference* of the forces which act upon the *positive* and *negative* electricity in the element α', by which these two electricities are separated from each other, and move in the conductor in opposite directions, this now becomes

$$= -\frac{\alpha\alpha'}{r^2}\cdot aeu \cdot ae'v\left(\sin\Theta\sin\eta\cos\gamma - \frac{1}{2}\cos\Theta\cos\eta\right) - \frac{1}{2}\frac{\alpha\alpha'}{r}a^2ee'\cos\Theta\frac{du}{dt} ,$$

or, because $aeu = i$ and $aedu = di$,

$$= -\frac{\alpha\alpha'}{r^2}i\left(\sin\Theta\sin\eta\cos\gamma - \frac{1}{2}\cos\Theta\cos\eta\right)\cdot ae'v - \frac{1}{2}\frac{\alpha\alpha'}{r}ae'\cos\Theta\frac{di}{dt} .$$

The force thus determined then tends to separate the *positive* and *negative* electricity in the induced element α' in the direction of the right line r. When the conductor is linear, however, separation cannot occur in this direction, but only in the direction of the induced linear element α' itself, which forms the angle Θ' with the right line r. By thus decomposing the whole of the above separating force[293] in this direction, *i.e.* by multiplying the above value by $\cos\Theta'$, we find the force, which effects the true separation,

$$= -\frac{\alpha\alpha'}{r^2}i\left(\sin\Theta\sin\eta\cos\gamma - \frac{1}{2}\cos\Theta\cos\eta\right)\cdot ae'v\cos\Theta'$$

$$- \frac{1}{2}\frac{\alpha\alpha'}{r}ae'\cos\Theta\cos\Theta'\frac{di}{dt} .$$

This expression, divided by e', gives the *electromotive* force exerted by the inducing element α, upon the induced element α', in the ordinary direction,

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[291][Note by AKTA:] Page 238 of Weber's *Werke*.

[292][Note by AKTA:] Due to a misprint, the next expression appeared as $a'e'u'$, while the correct expression should be $ae'u'$.

[293][Note by AKTA:] In German: *Scheidungskraft*. This expression can also be translated as "force of separation" or "segregating force", see footnote 235 on page 173.

$$= -\frac{\alpha\alpha'}{r^2}i\left(\sin\Theta\sin\eta\cos\gamma - \frac{1}{2}\cos\Theta\cos\eta\right)\cdot av\cos\Theta'$$

$$- \frac{1}{2}\frac{\alpha\alpha'}{r}a\cos\Theta\cos\Theta'\frac{di}{dt}\ .$$

This is therefore *the general law of voltaic induction, as found by deduction from the newly laid down fundamental principle of the theory of electricity.*

If we now, *first*, take the case *in which no alteration occurs, in the intensity of the current,* thus

$$\frac{di}{dt} = 0\ ,$$

we have *the law of the induction exerted by a constant element of a current upon the element of a conductor moved against it, i.e.* the *electromotive* force becomes

$$= -\frac{\alpha\alpha'}{r^2}i\left(\sin\Theta\sin\eta\cos\gamma - \frac{1}{2}\cos\Theta\cos\eta\right)\cdot av\cos\Theta'\ ,$$

or, when ε denotes the angle which the direction of the inducing element of the current forms with the direction in which the induced element itself is moved, by a transformation which is readily made it becomes

$$= -\frac{\alpha\alpha'}{r^2}i\left(\cos\varepsilon - \frac{3}{2}\cos\Theta\cos\eta\right)\cdot av\cos\Theta'\ .$$

The induced current is *positive* or *negative* according as this expression has a *positive* or *negative* value; by a positive current being understood one, the positive electricity of which moves in that direction of the element α', which with the produced right line r forms the angle Θ'.

Now if *e.g.* the elements α and α' are parallel to each other, and if the direction in which the latter is moved with the velocity v is contained within the plane of these two parallels, and at right angles to their direction, we have, *when α' by its motion recedes from α,*

$$\Theta = \Theta'\ ,\quad \cos\eta = \sin\Theta\ ,\quad \cos\varepsilon = 0\ ;$$

consequently the *electromotive* force is

$$= +\frac{3}{2}\frac{\alpha\alpha'}{r^2}i\sin\Theta\cos^2\Theta\cdot av\ .$$

This value is always *positive*, because we must consider $\Theta < 180°$; and this *positive* value here denotes an induced current of the same direction as the inducing current, in conformity with that which has been found by experiment for this case.

Under the same conditions, with the difference merely that *the element α' by its motion becomes approximated to the element α,* we have

$$\Theta = \Theta'\ ,\quad \cos\eta = -\sin\Theta\ ,\quad \cos\varepsilon = 0\ ;$$

consequently the *electromotive* force becomes

$$= -\frac{3}{2}\frac{\alpha\alpha'}{r^2}i\sin\Theta\cos^2\Theta\cdot av\ .$$

The *negative* value of this force denotes an induced current, in the opposite direction to that of the inducing current, also in conformity with that found by experiment for this case.

As is well known, voltaic induction may be produced in two essentially different ways; for currents may be induced by *constant* and by *variable* currents. It is produced by *constant* currents either when the conductor through which the current passes is moved towards that conductor in which a current is about to be induced, or *vice versa*. It may be induced by *variable* currents even when the conductor through which the variable current passes remains undisturbed as regards that conductor in which a current is about to be induced.

Just as the particular law of the first kind of voltaic induction was at once found from the *general laws of voltaic induction* deduced above by the conditional equation

$$\frac{di}{dt} = 0 \ ,$$

so we also find the peculiar law of the latter kind of voltaic induction by the conditional equation

$$v = 0 \ .$$

Thus if we take, *secondly*, the case in which *no motion of the conductors as regards each other takes place*, or where $v = 0$, the *law of the induction of a variable current upon that element of a current which is not moved as regards it*, or the value of the electromotive force becomes

$$= -\frac{1}{2} \frac{\alpha\alpha'}{r} a \cos\Theta \cos\Theta' \cdot \frac{di}{dt} \ .$$

Hence the induction, during the element of time dt, *i.e.* the product of this element of time into the acting *electromotive force*, becomes

$$= -\frac{a}{2} \cdot \frac{\alpha\alpha'}{r} \cos\Theta \cos\Theta' \cdot di \ ,$$

consequently the induction for any period of time in which the intensity of the induced current increases by i, whilst r, Θ and Θ' remain unchanged, is

$$= -\frac{a}{2} \cdot \frac{\alpha\alpha'}{r} i \cos\Theta \cos\Theta' \ .$$

The *positive* value of this expression denotes a current induced in the element α' in the direction of α', which with the produced right line r forms the angle Θ'; the *negative* value denotes an induced current in the opposite direction.

When the two elements α and α' are parallel, and $\Theta = \Theta'$, the above expression, when the intensity of the current is *increasing*, or where the value of i is positive, has a *negative* value, *i.e.* when the intensity of the current is on the increase in α, a current is excited in α' in an opposite direction to that of the inducing current. The reverse applies when the intensity of the current diminishes. Both results agree with well-known facts. The proportionality of the induction to the variation of the intensity i of the inducing current is also in accordance with experiment.

Lastly, if we return from the consideration of these two distinct kinds of *voltaic induction* to the general case, where at the same time the intensity of the inducing current is variable and the two conductors are in motion as regards each other, the *electromotive force* exerted

by the variable element of a current upon the *moved* element of a conductor is found to be simply as the *sum of the electromotive forces* which would occur:

1. If the element of the conductor *were not in motion* at the moment under consideration;

2. If the element of the conductor were in motion, but the *intensity of the current* of the induced element did *not* alter at the moment under consideration.

Chapter 8

[Weber, 1848b] On the Excitation and Action of Diamagnetism according to the Laws of Induced Currents

Wilhelm Weber[294,295,296297]

The repulsion of bismuth by a magnet, first observed by Brugmanns in 1778,[298] had remained almost unknown until Faraday discovered it anew and examined it more carefully,[299] and thus laid the foundation for the new doctrine of *diamagnetism*, the further development of which has become an important physical problem. To solve this question little can be expected from the more delicate processes of measurement, owing to the feebleness of the diamagnetic forces of bodies, even when very powerful electro-magnets act upon them, and it is therefore the more to be expected that we shall become acquainted with the nature of diamagnetism from the *various modifications* of its effects, the discovery of which is possible even in the case of the most feeble forces. The object of the following experiments is to establish with greater certainty and precision, from some peculiar modifications of the diamagnetic effects, a hypothesis already advanced by Faraday to explain the diamagnetic phenomena, and then to deduce this hypothesis required for the explanation of diamagnetic phenomena from known laws.

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Diamagnetic bismuth repels both the north and south pole of a magnet, and is repelled

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[294][Web48b] and [Web48c], with English translation in [Web52b] and [Web66c].

[295]Wilhelm Weber's Notes are represented by [Note by WW:]; the Notes by H. Weber, the editor of the third volume of Weber's *Werke*, are represented by [Note by HW:]; the Notes by Richard Taylor, the editor of the Scientific Memoirs where the English translation of this paper was published, are represented by [Note by RT:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[296][Note by HW:] According to the wording, these two treatises agree except for a final consideration, which is printed in a supplement on page 266.

[297][Note by AKTA:] H. Weber is referring here to [Web48b] and [Web48c]. The final consideration appears only in the paper published in the *Annalen der Physik und Chemie*.

[298][Note by AKTA:] Sebald Justinus Brugmans (1763-1819). He observed that bismuth was repelled by a magnetic pole.

[299][Note by AKTA:] [Far46b] and [Far46c].

by them. This *indifferent* repulsion of opposite poles might appear of little importance if the origin of the magnetic force were to be sought for in the unvarying metallic particles of the bismuth itself; for we are accustomed to assume generally of the ponderable bodies that they oppose without distinction equal resistance to the movements both of the two opposite magnetic as well as of the two electric fluids. But the action *at a distance* might appear more surprising than this indifferent effect, were we to admit that the diamagnetic force has its origin in the unvarying metallic particles of the bismuth itself, because it would be the first case in which the action of a ponderable upon an imponderable body *at a distance* had been observed. It appears therefore above all things important to decide the question, whether the source of the diamagnetic force acting at a distance is to be found in the unvarying ponderable constituents of bodies, or whether it arises from an *imponderable constituent*, and is connected with a certain *distribution* thereof.

To decide this question the experiment made by M. Reich[300],[301] is of the highest importance, according to which both north and south poles, when they act at the same time on the same side of a piece of bismuth, by no means repel it with the sum of the forces which they would individually exert, but only with the difference of these forces.

From this single experiment it might be concluded with the greatest probability, that the origin of the diamagnetic force is not to be sought for in the never-changing metallic particles of the bismuth, but in an imponderable constituent moving between them, which, on the approach of the pole of a magnet, is displaced and distributed differently according to the difference of this pole. The origin of the diamagnetic force is thus placed in the reciprocal action of two imponderable bodies, instead of in the reciprocal action between ponderable and imponderable bodies at a distance; and the similar effect upon opposite poles is then explained by the different distribution of the imponderable constituent in the bismuth which is produced by the antithesis of the poles. The simultaneous approach of two opposite poles on the same side must however have for result, that the imponderable constituent in the bismuth can neither assume the one or the other distribution upon which depends the appearance of the diamagnetic force, when the disappearance of the diamagnetic force in this case is self-evident.

But if it be now further asked, what is the nature of the imponderable constituent which is distributed in such a different manner in the bismuth on the approach of a north or south pole, and then with this distribution constantly re-acts with a repulsive force upon the approached pole, there present themselves only the two magnetic fluids, or the two electric fluids in the form of molecular currents. At all events, before any other assumption can appear admissible, the impossibility of explaining the phenomena in question by the known relations of the above imponderables must be shown.

From this it will be seen that Reich's experiment may be employed to establish more firmly a view already advanced by Faraday (Poggendorff's Annalen, Vol. 70, p. 48, Articles 2429, 2430).[302] Faraday there states that[303]

> "2429. Theoretically, an explanation of the movements of the diamagnetic bodies, and all the dynamic phenomena consequent upon the actions of magnets on them, might be offered in the supposition that magnetic induction caused in them a contrary

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[300][Note by RT:] Philosophical Magazine for February 1849, p. 127.

[301][Note by AKTA:] [Rei48] and [Rei49].

[302][Note by RT:] Experimental Researches, Articles 2429, 2430.

[303][Note by AKTA:] [Far46c, Articles 2429, 2430].

state to that which it produced in magnetic matter; i.e., that if a particle of each kind of matter were placed in the magnetic field both would become magnetic, and each would have its axis parallel to the resultant of magnetic force passing through it; but the particle of magnetic matter would have its north and south poles opposite, or facing towards the contrary poles of the inducing magnet, whereas with the diamagnetic particles the reverse would be the case; and hence would result approximation in the one substance, recession in the other.

2430. Upon Ampère's theory,[304] this view would be equivalent to the supposition, that as currents are induced in iron and magnetics parallel to those existing in the inducing magnet or battery wire; so in bismuth, heavy glass and diamagnetic bodies, the currents induced are in the contrary direction. This would make the currents in diamagnetics the same in direction as those which are induced in diamagnetic conductors at the *commencement* of the inducing current; and those in magnetic bodies the same as those produced at the *cessation* of the same inducing current. No difficulty would occur as respects non-conducting magnetic and diamagnetic substances, because the hypothetical currents are supposed to exist not in the mass, but round the particles of the matter."

I shall now submit this ingenious view, first proposed by Faraday, and which has obtained greater probability from Reich's experiment, to a still more direct criticism by the following experiments, which, in my opinion, scarcely leave a doubt of its correctness.

All the diamagnetic forces hitherto observed have exhibited a repulsive, never an attractive action; but from Faraday's assumption, it follows that diamagnetic forces must likewise occur which act *attractively* upon the pole of a magnet, and such cases may easily be determined more accurately and tested by experiment.

But for this purpose we must not observe the force which the diamagnetic bismuth exerts upon that pole by which it has been rendered diamagnetic, but those forces which this bismuth exerts upon other magnet-poles at a distance, and which have no influence upon its diamagnetic condition.

Now if a piece of bismuth is placed in the plane which is bisected at right angles by a small magnet-needle suspended by a silk thread and symmetrically magnetized, it is evident that the poles of the small needle can have no influence, or at least no perceptible influence, upon the diamagnetic state of the distant piece of bismuth, according to Reich's experiment. In fact it is easily seen that the needle experiences not the slightest deflection by the bismuth.

But if we arrange a powerful horse-shoe magnet of iron, so that the locality previously occupied by the bismuth is situated in the free space between its two poles, and the magnet is at the same time brought into such a position that its magnetic axis prolonged bisects the needle, this powerful magnet will exert a very great momentum of rotation upon the needle. But if this rotatory action exerted by the horse-shoe magnet is compensated by another equally powerful but opposite rotatory action of a bar-magnet brought to bear upon the needle from the opposite side, we can cause the needle to re-assume its original position and its original vibratory power (sensitiveness), so that with respect to the needle it is just the same as if no magnet acted upon it.

Now if, after these preliminary arrangements, the same piece of bismuth which previously had no action upon the needle is brought to the same position as before, i.e. between the

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[304][Note by AKTA:] See footnote 10 on page 13.

two poles of the horse-shoe magnet, a very perceptible and measurable effect is exhibited, viz. a deflection of the needle, owing to one pole being repelled and the other attracted.

If the poles of the magnets, the effects of which upon the needle are compensated, be reversed, and the experiment repeated, it is found that the same piece of bismuth brought to the same spot and in the very same position, now produces exactly the opposite deflection.

If, lastly, a piece of iron is substituted for the bismuth, it is found that the deflection produced by the latter is the opposite of that produced by the former.

These experiments may be variously modified, but in every case the force of the bismuth must be observed upon the other magnet-poles than that which determines the diamagnetic condition of the bismuth; they all confirm the assertion that bismuth constantly acts upon such poles in an opposite manner to iron in its place, that it consequently repels where iron attracts, and attracts where iron repels; in short, that at other magnet-poles than that which diamagnetizes the bismuth, we as frequently observe attractive as repulsive forces of the bismuth.[305,306] For instance, if the one extremity of the bar of bismuth was brought near the north end of a powerful magnet, while its other extremity was approached to the north end of the magnet-needle, the latter was attracted; but if the same extremity of the bar of bismuth was brought near to the south end of the powerful magnet, the north end of the magnet-needle was repelled by the other extremity of the bar of bismuth approached to it.

We may hence regard Faraday's supposition as proved, at least in so far as it places the origin of the diamagnetic force, not in the unvarying metallic particles of the bismuth itself, but in a variable distribution which occurs in the bismuth, and acts upon other magnets in the same manner as a definite distribution of magnetic fluids.

In order, lastly, to remove every doubt as to its being really nothing else than the magnetic fluids, or their equivalent, Ampère's currents, which are subject to this variable distribution in the bismuth, it may be required to be shown by experiment, not merely that the effects connected with the *presence* of the diamagnetic and of a certain magnetic state are equal, but likewise that the effects connected with the *origin* of the two states are so.

It is well known that, according to the laws of induction discovered by Faraday, the motion of the magnetic fluids in a body, or the rotation of the molecular currents of Ampère, is connected with an electrical action at a distance upon neighbouring conductors, owing to which an electric current is excited or induced in the latter.

Consequently, if the two magnetic fluids, or their equivalents, Ampère's currents, are really present in the diamagnetic bodies, which are set in motion or rotated under the influence of a powerful magnet, they must induce an electric current in a neighbouring conductor at the moment this change takes place.

Now to observe this induced current itself, it is requisite that no other current be induced in the same conductor, for instance by the powerful magnet to which the bar of bismuth is approached. For this purpose therefore the force of this magnet must be retained unaltered during the experiment, which presupposes in an electro-magnet a constant galvanic current. But on the other hand, the conductor upon which the bismuth is to act must have a fixed immutable position to that magnet, so that it encloses the space in an annular form, in which the bar of bismuth has to be brought in order to produce in it the diamagnetic distribution by the influence of the magnet. That, lastly, the current induced by the bismuth can be observed

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[305][Note by HW:] In the article published in the *Annalen der Physik und Chemie*, edited by J. C. Poggendorff, Vol. 73, Leipzig, pp. 241-256, you will find the following addition: ...].

[306][Note by AKTA:] This addition appears in the continuation of this paragraph, as the English translation made by Francis was taken from the *Annalen der Physik und Chemie*, [Web48b], [Web52b] and [Web66c].

by continuing the two extremities of the above annular conductor, and connecting them with the ends of the multiplier of a sensitive galvanometer, requires no further explanation.

But with respect to the power of this current induced by the bar of bismuth, it may readily be estimated *à priori* how small it will be if we consider how feeble the diamagnetic forces are in comparison to the magnetic forces of the iron substituted for the bismuth. On further examination, it results that the induced current must be so feeble that it can no longer be observed if all the conditions do not act together most favourably for the object.

The following arrangements were made to attain this end, viz. *to induce galvanic currents in a neighbouring conductor by the diamagnetization of the bismuth, and thus actually to observe the induced currents.*

An iron nucleus 600 millimetres in length, coated several times with thick copper wire, was used as electromagnet. To the circular terminal surface, 50 millimetres in diameter, of this iron nucleus was fixed the annular conductor, which consisted of copper wire 300 metres long and 2/3 millimetres thick, well spun with silk, and coiled upon wooden cylinders. The space included in this annular conductor, in which the bar of bismuth was to be placed, was 140 millimetres in length and 15 millimetres in breadth; the bar of pure precipitated bismuth was somewhat thinner. The extremities of the annular conductor were connected with a commutator, as were also the extremities of the multiplier of a very sensitive galvanometer, the magnet-needle of which was provided with a mirror in which the image of the distant scale was observed by a telescope directed towards it. The galvanometer was moreover provided with so effective a damper that it was scarcely possible to observe any vibration of the needle.

Now whilst a very powerful and constant galvanic current passed through the thick wire of the electro-magnet, the bar of bismuth was withdrawn from the annular conductor in which it was situated, the commutator changed, and the bar of bismuth again inserted, the commutator again changed, and the bar of bismuth withdrawn, etc. During this experiment, continued for about 1 minute, the state of the galvanometer was read off at intervals of about 10 seconds.

A second series of experiments was now made, but with this difference, that the commutator assumed that position on withdrawing the bar of bismuth which it had occupied in the first series on inserting the bismuth, and *vice versa*.

The third series was an accurate repetition of the first, and so forth.

Previous to commencing each series the state of the galvanometer was observed, without however waiting until the needle had attained a perfect state of rest. Each series was begun by withdrawing the bismuth.

In the following Table the whole of the readings made on the galvanometer are arranged together. The different series are distinguished by Roman numbers; the two states of the commutator which occurred in the different series on the withdrawal of the bar of bismuth are distinguished in the heading by *A* and *B*. The state of the galvanometer before commencing each series is also noticed in the heading.

I. *A.*	II. *B.*	III. *A.*	IV. *B.*	V. *A.*	VI. *B.*	VII. *A.*
512.3	517.4	515.9	517.2	517.0	523.0	524.7
513.3	513.0	519.5	517.1	518.2	522.0	526.0
514.1	512.9	520.7	517.5	518.7	519.0	528.0
514.5	512.8	519.1	516.2	525.0	518.5	530.0
515.3	514.2	519.2	516.7	525.1	519.0	530.7
515.6	515.2	518.3	517.7	523.0	521.0	530.0
516.7	516.0	515.5	—	—	—	528.5
514.92	514.02	518.72	517.04	522.00	519.90	528.87

Now if we compare the states of the galvanometer in the odd alternate series, where the commutator occupied the position *A* on withdrawing the bismuth from the annular conductor, with the mean value in the bottom-line, it is seen that the latter is always somewhat *greater*. For instance, the mean values are:

$$\text{1. Series } 514.92 = 512.3 + 2.62 \text{ ,}$$
$$\text{3. Series } 518.72 = 515.9 + 2.82 \text{ ,}$$
$$\text{5. Series } 522.00 = 517.0 + 5.00 \text{ ,}$$
$$\text{7. Series } 528.87 = 524.7 + 4.17 \text{ .}$$

The same comparison yields for the even series, where the commutator occupied the position *B* on removing the bismuth from the annular conductor, always a somewhat *smaller* mean value.

$$\text{2. Series } 514.02 = 517.4 - 3.38 \text{ ,}$$
$$\text{4. Series } 517.04 = 517.2 - 0.16 \text{ ,}$$
$$\text{6. Series } 519.90 = 523.0 - 3.10 \text{ .}$$

It should be borne in mind that the state of the galvanometer observed before the commencement of each series was not exactly that of rest. To avoid the uncertainty arising from this, the reading made previous to each series may be wholly excluded from the calculation, and the comparison restricted to the mean values of the several series. The comparison of the mean value of the 2nd to the 6th series, with the mean from the immediately preceding and succeeding series, then gives the following results:

$$\text{2. Series } 514.02 = 516.82 - 2.80 \text{ ,}$$
$$\text{3. Series } 518.72 = 515.53 + 3.19 \text{ ,}$$
$$\text{4. Series } 517.04 = 520.36 - 3.32 \text{ ,}$$
$$\text{5. Series } 522.00 = 518.47 + 3.53 \text{ ,}$$
$$\text{6. Series } 519.90 = 525.43 - 5.53 \text{ .}$$

We see then also from this, that in the uneven series, in which the commutator occupied the position *A* while the bismuth was withdrawn from the annular conductor, the state of the galvanometer was constantly somewhat higher, and that the reverse occurred in the even series in which the commutator had the position *B* on the removal of the bar of bismuth. The differences are somewhat greater for the last than for the first series, which is easily explained from the change of induction being gradually accelerated.

Corresponding experiments were now made for the purpose of direct comparison, the

bar of bismuth being exchanged for a slender bar of iron. The induced current was then so powerful that no repetition could be made as in the case of the bismuth, and that only the extreme end of the iron bar could be inserted in the annular conductor. And even then the induced current was so powerful that the deviation of the needle could not be observed on the galvanometer, but merely the direction, whether the position of the galvanometer rose, i.e. went from lower to higher divisions of the scale, or the reverse.

First Experiment.

Position of the commutator A:

Increasing numbers on inserting the iron bar in the annular conductor.
Decreasing numbers on withdrawing the iron bar from the annular conductor.

Second Experiment.

Position of the commutator B:

Decreasing numbers on inserting the iron bar in the annular conductor.
Increasing numbers on removing the iron bar from the annular conductor.

The position of the commutator A, and the case in which the iron bar was removed from the annular conductor, for which consequently a *decrease* in the deflection of the galvanometer was observed, will serve to compare this experiment made with iron with the former relative to bismuth. In the above experiments with the bismuth, this case corresponds to the uneven series, for which a *higher* state of the galvanometer resulted with the induction continued in the same direction. It results consequently that the bismuth induced a positive current under the same conditions that iron induced a negative one, and *vice versa*.

Hence the induction of electric currents by the diamagnetization of the bismuth is proved; and it is at the same time evident that the direction of these currents is constantly the reverse of those induced by iron under the same circumstances, precisely as it should be if bismuth contained magnetic fluids or their equivalent, Ampère's currents, which are set in motion or rotated under the influence of powerful magnets in exactly an opposite direction to that in iron. The view advanced by Faraday appears therefore to be placed beyond all doubt.

Now although a rule has been found according to which the variable diamagnetic conditions of bodies are determined for all cases in such a manner that the collective effects appear as a necessary consequence according to magnetic and electro-dynamic laws, the *cause* of this rule remains still unknown and unexplained according to magnetic and electro-dynamics laws. For if magnetic fluids are really contained in the diamagnetic bodies, on the approach of a magnet-pole, the one fluid must be attracted, the other repelled; and the direction of the separation of the two fluids is, according to this, necessarily determined by magnetic laws. But this direction is exactly the reverse of that stated in the above rule. Exactly the same, however, obtains upon the other assumption, which presupposes the existence of Ampère's molecular currents in diamagnetic bodies, instead of the magnetic fluids, which on the approach of a pole of a magnet should be rotated in a direction determined by electro-magnetic laws. But this rotation is exactly the reverse of that indicated by the above rule.

There exists consequently a contradiction between the above rule of *excitation* and the laws of the *activity* of the diamagnetic condition. Until this contradiction is removed, all the diamagnetic conditions of bodies continue to form a group of isolated facts without any connection with other phenomena, just as those of rotation-magnetism formed a similar

group until Faraday gave the key to their solution by his discovery of induction.

In the preceding observations which referred to the *effects*, it was indifferent whether separate magnetic fluids or Ampère's molecular currents of the same direction constitute the excited diamagnetic state of bodies.[307],[308] This is no longer the case in the following considerations which relate to the *causes*, i.e. to the forces exciting the diamagnetic state of bodies. For if it were a certain distribution of the magnetic fluids which constituted the diamagnetic condition of bodies, no account, as above shown, could be given of the forces producing them, at least this distribution could not be explained from the known *magnetic* forces which act upon these fluids. But the case is different if the diamagnetic condition of bodies is constituted by molecular currents of like direction; for a system of molecular currents of like direction can obtain in a two-fold manner. In the *first* place, it is possible that the molecular currents existed previously in the bodies, and that only one force acted upon these already existing currents which communicated the *same direction* to them; but, *secondly*, it is also possible that the currents of like direction, which form the diamagnetic condition of bodies, did not previously exist, but first *originated* or were *induced* on diamagnetizing the body. Now if one of these two possible cases falls to the ground for the same reasons as that of the above considered distribution of magnetic fluids, the other possible case for the molecular currents still remains, according to which they have been *produced by induction.*

Hitherto it has never been a question of *induced molecular currents*, but solely of fixed invariable molecular currents, according to Ampère's definition, to whom indeed the origin of currents by induction was unknown. But it is evident if the existence of molecular currents be admitted, we must further allow that their intensity may be increased or diminished, and that even new currents of this kind may be produced by the very forces which produce currents in larger circuits.

If we go back to *induction* in order to explain diamagnetism, it might at first sight be doubted whether it is really necessary to admit induced molecular currents for this purpose, or whether the currents induced in large circuits are not of themselves sufficient. These currents would, it is true, be able to produce all diamagnetic phenomena if they were *permanent*; but as these currents, which are subject to Ohm's laws,[309] are not permanent, but instantly disappear with the inducing force, and can only be maintained by continued induction, they can for this reason alone not serve to explain diamagnetism.

But if the rapid disappearance of these currents is the sole reason of its being impossible to deduce thence the diamagnetic condition of bodies, there appears to be no reason why the persistent diamagnetic state of bodies should not be ascribed to *induced molecular currents*, as these must behave in all other respects like those currents, and differ only in possessing that *permanency* which is wanting in the others. For the difference between those currents which move through conductors in large circuits and these molecular currents, consists solely in the circumstance that the circulating electricity of the former is so quickly deprived of its active force in passing to the molecules of the conductor, that it would come to rest in an immeasurably small time if the loss it sustained were not constantly replaced by continuous

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[307][Note by HW:] The following is replaced in the article published in the *Annalen der Phykik und Chemie*, edited by J. C. Poggendorff, Vol. 73, pp. 241-256, with the addendum printed at the end of this paper.

[308][Note by AKTA:] The sequence of this work in English follows the final version published by Weber, as the English translation made by Francis was taken from the *Annalen der Physik und Chemie*, [Web48b], [Web52b] and [Web66c].

[309][Note by AKTA:] See footnote 99 on page 53.

electro-motive forces, whence it results that currents of this kind are, according to the laws of Ohm, constantly proportional to the existing electro-motive force, and instantly disappear with the electro-motive force. The reverse applies to the molecular currents which do not pass through a conductor from molecule to molecule, but circulate around a single molecule, to which consequently the above reason, deprivation of their active force, does not apply. These currents therefore persist of equal intensity without any electro-motive force.

Now admitting an *inducing force* which acts upon the electricity of a conductor, the latter is set in motion, and this motion distributes itself according to laws in proportion to the capacity for conduction between all the paths which the conductor presents; consequently a portion of the motion must be likewise take its course around the individual molecules of the conductor, and form *induced molecular currents*, which because they find no resistance in their course around the molecules, by which they might be retarded, must continue in their full strength until, in consequence of a new opposite induction, other induced molecular currents are added which neutralize the previous ones.

If therefore, with Ampère, we admit *molecular currents* in the doctrine of electro-magnetism, we must at present, as a necessary consequence, after the discovery of induction, adopt *induced molecular currents* in the doctrine of magneto-electricity, and must ascribe permanence to all, whether they have always existed or been first produced by induction. Assuming this, it results that all bodies in which diamagnetic effects have been observed, must have been acted upon by forces which must have induced molecular currents, and indeed such as produce the effects designated by the name of diamagnetic.

The latter follows from the fact, that a magnetic force tends to give such a direction to an *existing current* that its course is exactly opposed to that of a *current induced by the increase of that magnetic force*. Consequently, if this induced current is a molecular current which is persistent, it will likewise have permanently the opposite effects of another molecular current which existed (for instance in iron) independently of the increase of that magnetic force, but has acquired its present direction by means of that force. The opposite behaviour of the diamagnetized bismuth and of the magnetized iron follows according to this from known laws. The essential difference between bismuth and iron would then be this, that molecular currents, whose direction however is not alterable, exist in iron independently of any external excitation, but subject to the influence of external forces, which is not the case in bismuth. However, bismuth and iron may in so far be rendered equivalent as a decreasing or increasing magnetic force induces in both fresh persistent molecular currents, which however must be much weaker in the iron than those existing in it already, independently of such induction.

Chapter 9

Editor's Introduction to Kirchhoff's 1849 Paper on a Deduction of Ohm's Laws

A. K. T. Assis[310]

Here I present the English translation of Gustav Kirchhoff's 1849 paper on a deduction of Ohm's laws in connexion with the theory of electrostatics.[311] Ohm had identified electroscopic force with volume-density of electric charge. Moreover, he had assumed that free electricity in a conductor was distributed uniformly throughout the volume of the conductor not only when the charges were at rest, but also when moving with constant velocity in the case of steady currents. The present paper has two main relevant aspects. (I) Kirchhoff was able to deduce Ohm's laws from principles which were in agreement with the theory of electrostatics. Kirchhoff showed, in particular, that free electricity can only exist at the surface of the conductor not only when electricity is at rest, but also when flowing steadily in closed circuits. (II) Moreover, Kirchhoff identified Ohm's electroscopic force with the electrostatic potential.[312]

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[310]Homepage: www.ifi.unicamp.br/~assis

[311][Kir49b] with English translation in [Kir50] and French translation in [Kir54].

[312][Whi73, pp. 90-93 and 224-226].

Chapter 10

[Kirchhoff, 1849] On a Deduction of Ohm's Laws, in Connexion with the Theory of Electrostatics

Gustav Kirchhoff[313],[314]

In a deduction of his laws of galvanic currents, Ohm set out with certain assumptions regarding electricity which are not in conformity with those it has been necessary to make in order to explain electrostatic phenomena;[315] he assumes that the electricity in a conductor is at rest when it is distributed throughout the latter in a state of uniform [volume] density. Now although it must always appear desirable to determine the laws to which electrical currents are subject, by considerations connected with the theory of electrostatics, this becomes absolutely requisite to enable us to produce a satisfactory theory of experiments, in which both electricity in motion and electricity at rest are concerned, — experiments similar to those made by M. Kohlrausch upon the closed circuit with the condenser and electrometer.[316],[317] My present object is to show how Ohm's formulas may be deduced from the electrostatic laws of the mutual repulsion of electrical atoms, when certain assumptions referring to questions in the theory of electrostatics, which have remained perfectly open, are brought to bear.

When electricity is communicated to a conductor, it will assume a state of equilibrium, when the forces exerted by the free electricity upon an electric atom existing in any part of the interior of a conductor mutually neutralize each other. This occurs when the potential of the total amount of free electricity in relation to a point within the conductor remains constant. Theory shows us that this can only be the case when the free electricity has become arranged in a particular manner upon the surface of the conductor.

When two conductors of different kinds, as a piece of copper and a piece of zinc, which separately contained no free electricity, are brought into contact with each other, one conductor becomes positively, while the other becomes negatively electrical. The electricity

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[313][Kir49b] with English translation in [Kir50] and French translation in [Kir54].

[314]Gustav Kirchhoff's Notes are represented by [Note by GK:], while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[315][Note by AKTA:] See footnote 99 on page 53.

[316][Note by GK:] Poggendorff's *Annalen*, vol. lxxviii, p. 1.

[317][Note by AKTA:] Rudolf Hermann Arndt Kohlrausch (1809-1858). See [Koh49].

excited at the point of contact soon assumes a state of equilibrium; in it the potential of the total amount of free electricity must necessarily remain constant with regard to all points of each of the two conductors: hence it follows that free electricity cannot exist within the conductor, and that it must be situated solely upon its surface; one portion of the electricity will remain at the surface of contact of the two conductors, whilst another covers its free surface.

The potential of all the free electricity is constant with regard to all parts of each of the conductors: its value, however, will be different in the case of the first conductor from that of the second; for theory teaches us, that if its value were the same in both conductors, there should be no free electricity present, inasmuch as the sum of all the free electricity is $= 0$. Now as regards the difference between the two values of the potential in the two conductors, this might depend upon the nature of the material of which the two conductors were composed, and their form. I shall assume that it is independent of the latter, and is that magnitude which is known as the tension of the two bodies.[318] Let u_1 denote the potential of the entire amount of free electricity in regard to a point in the first conductor, and u_2 the same in regard to a point of the second conductor; both u_1 and u_2 must then be constant; if the $U_{1,2}$ denote the tension of the two bodies, we must have

$$u_1 - u_2 = U_{1,2} \ .$$

If we imagine several conductors, say three, so placed in contact that the first conductor touches the second, and this the third conductor, the electricity in them may always assume a state of equilibrium. If we again denote the potential of the total amount of free electricity in any point of the first conductor by u_1, for one in the second by u_2, and for one in the third by u_3, and further the tension between one and two by $U_{1,2}$, that between two and three by $U_{2,3}$, it is essential to the existence of a state of equilibrium that each of the three magnitudes, u_1, u_2, and u_3 be constant, and that the equations

$$u_1 - u_2 = U_{1,2}$$

$$u_2 - u_3 = U_{2,3}$$

be satisfied. But if we assume that the conductors 1, 2 and 3 have been so placed in contact that each of them comes into contact with the two others, electric equilibrium cannot possibly always exist in them. Should equilibrium exist, each of the magnitudes u_1, u_2, and u_3 must be constant, and the equations

$$u_1 - u_2 = U_{1,2}$$

$$u_2 - u_3 = U_{2,3}$$

$$u_3 - u_1 = U_{3,1}$$

must be satisfied. These equations, on addition, produce

$$0 = U_{1,2} + U_{2,3} + U_{3,1} \ ;$$

[318][Note by AKTA:] Kirchhoff is here identifying the difference of the two values of the electrostatic potential in the two conductors as the tension between them.

thence the tensions of the three conductors must satisfy this condition to allow of the possibility of electric equilibrium; the condition is satisfied when the three conductors belong to the so-called tension series.

We have next to examine what will occur when this condition is not satisfied. At a particular moment the distribution of free electricity in the system will be a certain one; I leave it undetermined whether this free electricity exists only at the surface of the conductor, or whether it has penetrated into its interior. Let its potential in regard to any point of one of the conductors be u; this u is not constant, but a function of the coordinates of the point to which it relates; hence the forces which are exerted by the free electricity upon a particle of electricity existing at any spot within the conductor will not retain a state of equilibrium, but produce a definite resultant. Let us imagine the existence of an element of space, v, within the conductor, and let us denote the above resultant for any point in v by R. If v contains no free electricity, the neutral electric fluid contained in it becomes decomposed; the positive electricity will be moved in the direction of R, the negative in the opposite direction; the quantities of positive and negative electricity excited in the element v, and also their velocities, must therefore be the same.[319] I shall assume that the quantity of either fluid, which is moved in a unit of time through a section of v, perpendicular to the direction of R, and the magnitude of which may be denoted by dw, is $= dwkR$, in which k denotes the conducting power of the substance. To determine what takes place when v contains free electricity, I shall assume that no motion of the electric fluids can occur in a conductor, except when equal quantities of the two electricities pass in opposite directions through each surface-element of it simultaneously. Hence it follows, even when v contains free electricity, that as much positive electricity passes in a unit of time through dw in the direction of R, as negative electricity in the opposite direction. As regards the quantity of the electricities flowing through dw, I assume that it is again $= dwkR$.

If to these assumptions, most of which have been already put forth by Weber in his electrodynamic measurements,[320] we further assume that the difference in the values of the potential of the total amount of free electricity in the case of two points lying in immediate proximity to each other on the proximal and distal sides of the surface of contact of two conductors remains the same, whether a current flows through the conductor, or the electricity is at rest in them; we arrive, on the supposition that the electric condition of the system has become stationary, at the same equations for the potential of the free electricity as those given by Ohm's expression of the electroscopic force,[321] *i.e.* the [volume] density of the electricity.

In fact, if we denote the normal of the element dw, which has the direction of R, by N, then[322]

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[319][Note by AKTA:] Kirchhoff is here assuming implicitly that the positive and negative particles have the same mass. If they had different masses, the acquired velocities after a certain amount of time t would have different magnitudes due to an application of forces of the same magnitude and opposite directions acting in both of them.

[320][Note by AKTA:] In German: *elektrodynamischen Maassbestimmungen.* Kirchhoff is referring to W. E. Weber (1804-1891) and his First major Memoir on Electrodynamic Measurements, see Chapter 5.

[321][Note by AKTA:] In German: *die aus der Ohm'schen Vorstellung für die elektroskopisch Kraft.* This text was translated as, [Kir50, p. 466]: *as those given by Ohm's expression of the electric force.* I preferred to translate "elektroskopisch Kraft" as "electroscopic force" instead of "electric force".

[322][Note by AKTA:] In German: *In der That, nennen wir die Normale des Elementes dw, die die Richtung von R hat, N, so ist.* This text was translated as, [Kir50, p. 466]: *In fact, if we denote the normal of the element by dw, that having the direction of R by N, then.* Instead of this translation, I preferred the one presented in this work.

$$R = \frac{du}{dN} \ ,$$

hence the quantity of positive or negative electricity flowing through dw in a unit of time, is

$$= -kdw\frac{du}{dN} \ .$$

The same expression is obtained for this quantity by Ohm's method, if u be used to denote the electroscopic force.[323],[324] But we may conclude from this expression, without entering into the signification of u, that when the condition of the system has become stationary, u must satisfy the differential equation

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = 0 \ ;$$

and for each point of the free surfaces of the conductor, the boundary condition[325]

$$\frac{du}{dN} = 0 \ ;$$

and further, that the equation

$$k\frac{du}{dN} + k_1\frac{du_1}{dN} = 0 \ ;$$

applies in the case of every point of the surfaces of contact of two bodies.

To these conditions, both as regards Ohm's proposition and those we have enumerated, must be added, that in the case of every point of the same surface of contact, $u - u_1 =$ the tension of the two bodies. Thus the same equations are obtained for the magnitude u by both propositions. As regards the currents which are determined by the differential quotients of these magnitudes, we consequently obtain the same results from whichever we start. But different results are obtained in regard to the distribution of free electricity in the circuit. According to Ohm, the value of u at every part of the system directly gives the [volume] density of the electricity, which is not the case in the view we have developed, from which, on the contrary, it follows that even in the closed circuit free electricity can only exist at the surface of the conductor. Thus as u within one of the conductors satisfies the equation

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = 0 \ ,$$

u must be a potential of masses which are situated externally to this conductor. But u is a potential of all the free electricity; hence no part can be situated in the interior of any conductor.

The considerations we have laid down hold good whatever the number, the form, and the arrangement of the conductors may be which are placed in contact; they also hold good

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[323][Note by GK:] Poggendorff's *Annalen*, vol. lxxv, p. 191. We have used the word tension here to denote Ohm's electroscopic force.

[324][Note by AKTA:] See [Kir48, p. 191]. Kirchhoff is here identifying Ohm's electroscopic force with the electrostatic potential u.

[325][Note by AKTA:] In German: *der Gränzbedingung*. This text was translated as, [Kir50, p. 466]: *the limitary condition.* I preferred to translate "Gränzbedingung" as "boundary condition" instead of "limitary condition".

in that case in which one plate of a condenser is placed in contact with a point of a closed circuit, and hence afford the theory of experiments similar to those of M. Kohlrausch. The results which they yield agree perfectly with those of this experiment.

The considerations we have laid down are based upon the electrostatic law of the action of electric particles. Neither Ampère's electrodynamic phenomena,[326] nor the phenomena of induction, can be explained by this law. Weber has discovered a more general law, by which he has succeeded in explaining these phenomena;[327] a law, in the expression of which the relative velocity of the particles, whose action upon each other is under consideration, is introduced, and which passes into that of electrostatics, when this velocity disappears. In bringing the various fields of the theory of electricity under a single point of view, we must therefore aim at deducing the laws of the currents in the closed circuit from Weber's law. This deduction appears to be difficult; still it is easy to prove, *à posteriori*, that the idea regarding the currents, to which the admission of the electrostatic law has led, is also in conformity with Weber's law, when a certain hypothesis is called in aid, viz. that hypothesis, according to which, on calculating the force which a separation of the two electricities produces in the element of space v of one of the conductors, the electricities in v must be regarded as at rest. There is nothing opposed to this view, when we bear in mind, that the motion of the electricity in one conductor only passes from molecule to molecule; so that every particle of electricity finds a point of rest in a molecule which it reaches. Adopting this view, it may readily be granted that the quantity of electricity which is transferred from one molecule to a neighbouring one is only occasioned by the forces which are exerted upon the particles of electricity, whilst they are still in a state of rest in the former particle, but not by the forces, which act upon it, whilst they are passing to the following molecule. As regards Weber's theory of induction, it is unimportant whether this assumption is made or not. If it be made, and the currents in the circuit be regarded generally as in accordance with the view of the electrostatic law, it is a matter of indifference in regard to the magnitude and the direction of the force which tends to separate the electricities in the element v, and therefore in regard to the electromotive force,[328] as Weber calls it, whether we start from the eletrostatic or Weber's law. The difference which might possibly occur must therefore arise from the forces exerted by the electricities flowing in the other parts of the system; and these forces, according to what Weber has pointed out, do not contribute to this electromotive force, inasmuch as the currents are constant, and convey equal quantities of both electricities with the same velocity in opposite directions.

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[326][Note by AKTA:] See footnote 10 on page 13.

[327][Note by AKTA:] See Chapter 5.

[328][Note by AKTA:] In German: *elektromotorische Kraft*. This text was translated as, [Kir50, p. 468]: electro-motor force. I preferred to translate "elektromotorische Kraft" as "electromotive force".

Chapter 11

[Weber, 1851] On the Measurement of Electric Resistance according to an Absolute Standard

Wilhelm Weber[329] ,[330,331]

11.1 Explanation of the Absolute Unit of Measure for Electric Resistances

If there are measures for time and space, a special *fundamental measure* for *velocity* is not necessary;[332] and in like manner no special *fundamental measure for electric resistance* is needed if there are measures for electromotive force and for intensity of the current; for then *that resistance can be taken as unit of measure*, which *a closed conductor possesses in which the unit of measure of electromotive force produces the unit of measure of current intensity.* Upon this depends the reduction of the measurements of electric resistance to an absolute standard.

It might be thought that this reduction would be more simply effected by reverting to the spatial dimensions, length and cross-sectional area of the galvanic conductor, and adhering to that metal (*copper*) which is best fitted and is most frequently used for such conductors.[333]

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[329][Web51] with English translation in [Web61].

[330]Wilhelm Weber's Notes are represented by [Note by WW:]; the Notes by H. Weber, the editor of the third volume of Weber's *Werke*, are represented by [Note by HW:]; the Notes by the Editors of the *Philosophical Magazine* (namely, Sir David Brewster, Sir Robert Kane, William Francis and John Tyndall) which published originally this English translation are represented by [Note by EPM:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[331][Note by EPM:] Translated from Poggendorff's *Annalen*, vol. lxxxii, p. 337, by Dr. E. Atkinson. [From the great scientific and practical importance which the determination of electric resistance has of late acquired, it has been thought advisable to give a translation of Weber's original paper published in 1851, containing the method of referring these resistances to an absolute standard. — Eds.]

[332][Note by AKTA:] In German: "Wie für die *Geschwindigkeit* kein eigenes *Grundmaass* aufgestellt zu werden braucht, wenn Raum- und Zeitmaass gegeben sind, ..." This text can also be translated as: "If there are *fundamental units* for time and space, a special *fundamental unit* for *velocity* is not necessary, ..."

[333][Note by AKTA:] In German: "Man könnte glauben, dass sich diese Zurückführung noch einfacher ausführen liesse, wenn man auf die räumlichen Dimensionen (Länge und Querschnitt) der galvanischen Leiter zurückginge und sich dabei an dasjenige Metall hielte, welches zu solchen Leitern am geeignetsten ist

In that case the absolute unit of measure of resistance would be that resistance which a copper conductor possesses whose length is equal to the measure of length, and whose cross-section is equal to the measure of surface, in which, therefore, besides measure of length and surface, *the specific resistance of copper must be given as unit* for the specific resistance of conducting substances. Thus a special *fundamental measure for specific resistances* would be necessary, the introduction of which would be open to question. *First*, because there would be no saving in the number of the fundamental measures if, in order to do without a fundamental measure for the absolute resistance, another fundamental measure must be introduced which is otherwise superfluous. And *secondly*, neither copper nor any other metal is fitted for use in establishing a fundamental measure for resistances. Jacobi[334] says that there are differences in the resistances of even the chemically purest metals, which cannot be explained by a difference in the dimensions; and that, accordingly, if one physicist referred his rheostat and multiplier[335] to copper wire a metre in length and 1 millimetre thick, other physicists could not be sure that his copper wire and theirs had the same *coefficient of resistance*, that is, whether the *specific* resistance of all these wires was the same. The reduction of measurements of galvanic resistances to an absolute measure can therefore only have an essential importance, and find a practical application, if it takes place in the first mentioned way, in which no other measures are presupposed than those for *electromotive force* and for *current intensity*.

The question then arises, as to what are the measures[336] of *electromotive forces* and *intensities*? In measuring these magnitudes, no specific *fundamental measures* are requisite, but they can be referred to *absolute measure* if the magnetic measures for *bar magnetism* and *terrestrial magnetism*, as well as *measure of space and time*, are given.

As an absolute unit of measure of *electromotive force, may be understood that electromotive force which the unit of measure of the earth's magnetism exerts upon a closed conductor, if the latter is so turned that the area of its projection on a plane normal to the direction of the earth's magnetism increases or decreases during the unit of time by the unit of surface.* As an absolute unit of *current intensity, can be understood the intensity of that current which, when it circulates around a plane of the magnitude of the unit of measure, exercises, according to electro-magnetic laws, the same action at a distance as a bar-magnet which contains the unit of measure of bar magnetism.*[337] The absolute measures of *bar magnetism* and of *terrestrial magnetism* are known from the treatise of Gauss, *Intensitas vis magneticae terrestris ad mensuram absolutam revocata*, Göttingae, 1833 (Poggendorff's *Annalen*, vol. xxviii, pp. 241 and 591).[338,339]

From this statement it is clear that the measures of electric resistances can be referred to an absolute standard, provided measures of *space*, *time*, and *mass* are given as fundamental

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und am häufigsten dazu gebraucht wird, an das *Kupfer*." This text was translated as, [Web61, p. 226]: "It might be thought that this reduction would be more simply effected by reverting to the special dimensions, length and section, and adhering to that metal (*copper*) which is best fitted and is most frequently used for such conductors." I replaced this translation by the text included here.

[334][Note by AKTA:] Moritz Hermann von Jacobi (1801-1874). See [Jac51].

[335][Note by AKTA:] In German: *Multiplicatoren* or *Multiplikatoren*. This word was translated as "multiplicator", [Web61, p. 227]. I replaced this translation by "multiplier".

[336][Note by AKTA:] In German: *Maasse*. I preferred to translate this word here as "measures" instead of "measurements".

[337][Note by AKTA:] A magnetized bar with a unit measure of bar magnetism has a magnetic moment = 1.

[338][Note by HW:] Gauss' Werke, Vol. V, p. 79.

[339][Note by AKTA:] See footnote 97 on page 51.

measures; for the absolute measures of *bar magnetism* and of *terrestrial magnetism* depend simply on these three fundamental measures. A closer consideration shows that even of these three fundamental measures, the *measure of mass* does not come into consideration, as follows from the following summary of the simple relations which are established by the determination of the absolute measures of these various kinds of magnitude.

As fundamental measures, there are to be considered the *measure of length R*, and the *measure of time S*; as *absolute measures*, the surface measure F, and the units of measure of *bar magnetism M*, of *terrestrial magnetism T*, of *electromotive force E*, of *current intensity J*, and of *resistance W*.

Hence, *first*, if wW is the resistance of any closed circuit, eE the electromotive force acting upon this conductor, and iJ the intensity of the current produced by this electromotive force, we have the relation between the three numbers

$$w = \frac{e}{i} \ ,$$

from which it is clear that if the numbers e and i are determined, the number w is also indirectly obtained without needing a special determination.

Secondly, let eE stand for the electromotive force which acts upon any closed (plane) conductor, fF the area of the plane enclosed by this conductor, tT the earth's magnetism on which the electromotive force depends; and let sS express the space of time in which the plane of that conductor is moved by rotation from a position parallel to the direction of the earth's magnetism to a position at right angles to it, in such a manner that the limited surface produced by its projection on a plane at right angles to this direction of the earth's magnetism increases, proportional to the time, by the unit of surface measure during the unit of time. We shall then have between these four numbers e, f, t, s, the following relation:

$$e = \frac{ft}{s} \ ,$$

and hence it is clear that if the three numbers f, t, s are determined, the number e is also thereby directly given without necessitating a special measurement.

If, *thirdly*, iJ is the intensity of the current in any closed conductor, fF the area of the plane enclosed by this conductor, and mM the magnetism of a bar[340] which, when substituted for that conductor (its magnetic axis at right angles to the plane of the conductor), exercises the same actions at a distance, according to electro-magnetic laws, as that conductor, the following relation obtains between the three numbers i, f, and m,

$$i = \frac{m}{f} \ ,$$

from which it follows that if the numbers f and m are determined by measurement, i can be directly obtained without a special measurement.

From these three relations we get, finally,

$$w = \frac{e}{i} = \frac{fft}{sm} \ ,$$

hence if the four numbers f, s, m, t are determined, the number w is also directly obtained. The number f is obtained by measuring the area of the plane embraced by the conductor; s is found by measuring the time; and there only remain the numbers m and t, which are

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[340][Note by AKTA:] That is, mM is the magnetic moment of this magnetized bar.

obtained by measuring the bar magnetism by the method described by Gauss in the above paper.[341] The unchangeability of the unit of measure for electric resistance can accordingly be guaranteed so long as the four given measures (space, time, and the units of measure for the earth's magnetism and for bar magnetism) are obtained unchanged. But it by no means follows that the maintenance of these four given measures is a necessary condition for the unchangeability of the unit of measure of electric resistances; the simple maintenance of that unit of measure for *velocities* is sufficient for the purpose.

For if tT is the earth's magnetism, on which the electromotive force depends, which acts upon the closed conductor whose resistance has been measured; if, further, $m'M$ is the magnetism of a bar (whose magnetic axis is parallel to the direction of the earth's magnetism, while the straight line drawn from its centre to the centre of the plane enclosed by the conductor is normal thereto) which, according to magnetic laws, would, from a great distance, exert the same action as tT the earth's magnetism; and, finally, if Rr is the length of the straight line drawn from the middle of this bar to the middle of the plane enclosed by the conductor, we have, according to the *"Intensitas"*, the simple relation

$$ t = \frac{m'}{r^3} \; . $$

Substituting this value of t in the equation for w, we have

$$ w = \frac{ff}{r^3} \cdot \frac{m'}{m} \cdot \frac{1}{s} \; . $$

If, finally, $r'R$ is the side of a square whose area is equal to the area of the plane enclosed by the conductor, from which is obtained the relation

$$ f = r'^2 \; , $$

and substituting this value of f in the above equation, we have

$$ w = \frac{r'^3}{r^3} \cdot \frac{m'}{m} \cdot \frac{r'}{s} \; . $$

It is self-evident that a change of the given measures has no influence on the value of the factor

$$ \left(\frac{r'^3}{r^3} \cdot \frac{m'}{m} \right) \; , $$

but a change of the given measures of time and space does influence the value of the factor f'/s, and accordingly the value of the number w, if both measures are not simultaneously increased or diminished in proportion. The value of the number w is hence quite independent of all alterations of the given measures, so long as there is no change in the *measure of velocity*. But if, by an alteration of the given measures, the standard of velocity is increased or diminished n times, an n times larger or smaller value is obtained for the factor r'/s, and therefore also for the number w, which is as much as to say that the resistance in this case is expressed according to an n times smaller or larger standard. The unchangeability of the unit of measure for resistance merely depends therefore on the unchangeability of the given measure of velocity. But if the measure of velocity is taken n times larger or smaller, the unit of measure for resistance becomes simultaneously n times larger or smaller.

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[341][Note by AKTA:] That is, by measuring the magnetic moment of the magnetized bar.

11.2 Method of Measuring Electric Resistance according to an Absolute Standard

The measurements of length and of time, which, according to the preceding paragraph, are adequate for the determination of electric resistance, presuppose circumstances on the convenient arrangement of which the practical execution and accuracy of such a determination depend. The following arrangement may serve as a simple summary of the essential circumstances.

Out of the galvanic conductor whose resistance is to be determined, two circular rings, A and B, are formed, which are connected in the manner represented in the figure.

The whole conductor, consisting of the two circles A, B, and the junctions form a continuous line, of which it may be assumed, for the sake of simplicity, that it is situated in one plane, and that the straight line connecting the centres of both circles coincides with the direction of the earth's magnetism. Let T be the force of the earth's magnetism as determined according to an absolute standard by magnetometric measurements; let r be the diameter of the circles, which, for simplicity sake, are assumed to be equal. If now the circle A is projected in the direction of the earth's magnetism AB on a plane normal to AB, the area of the projected plane is $= 0$. From the flexibility of the wires connecting the two circles, let it be supposed that the circle A is so twisted as to be at right angles to AB, in which case the area of the plane of the projection is πr^2. Let this rotation take place in a short time s, in such a manner that the area of the plane of the projection of the circle increases uniformly in this time from 0 to πr^2. From the *magneto-electrical laws*, an *electromotive force* results which the terrestrial magnetism T exerts upon the rotated circular conductor A during the time s, and which, according to the unit of measure explained in the preceding paragraph, is expressed by Ee, in which the number e is determined by the equation

$$e = \frac{\pi r^2}{s} \cdot T \ .$$

By this electromotive force a current is produced in the time s passing through the whole closed conductor, whose *intensity*, according to the unit explained in the preceding paragraph, is expressed by iJ. This current passes also through the circle B, and acts from here on a distant magnetic needle in C, whose axis of rotation lies in the plane of the circle at right angles to the direction of the earth's magnetism. Let C lie in the produced AB.[342] It follows now from *electro-magnetic* laws, that the momentum of rotation exerted on the needle at C by a current passing through the circle B, is equal to the moment of rotation exerted by a bar-magnet placed in the centre of the circle in such a manner that its magnetic axis is at right angles to the plane of the circle, if its magnetism M,[343] expressed according to absolute measure, is

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[342][Note by WW:] That is, the line joining the centres of the circles A and B.
[343][Note by AKTA:] That is, its magnetic moment M.

$$M = \pi r^2 i \ .$$

If, further, the magnetism of the needle in C expressed in the same measure $= m$, and $Bc = R$, and φ the angle which the magnetic axis of the needle in C makes with the direction of the earth's magnetism AB, the momentum of rotation exerted by the bar magnetism M on the bar magnetism m is expressed, according to known magnetic laws, by

$$\frac{Mm}{R^3} \cdot \cos \varphi = \frac{\pi r^2}{R^3} \cdot im \cos \varphi \ .$$

From which it follows that if K is the moment of inertia[344] of the needle, the *acceleration* of the rotation is

$$\frac{d^2\varphi}{ds^2} = \frac{\pi r^2}{R^3} \cdot \frac{im}{K} \cdot \cos \varphi \ ,$$

and therefore that if the needle were previously at rest, and $\varphi = 0$, the *velocity of rotation* at the end of the short time s is

$$\frac{d\varphi}{ds} = \frac{\pi r^2}{R^3} \cdot \frac{im}{K} \cdot s \ .$$

The greatest deflection α of the needle set in oscillation is known by *direct observation*; and the following expression is obtained for it from the above velocity, from known laws of oscillation, by multiplying by the period of oscillation[345] t and dividing by the number π:

$$\alpha = \frac{r^2}{R^3} \cdot \frac{im}{K} \cdot st \ .$$

For the period of oscillation we have the known equation[346]

$$mT = \frac{\pi^2 K}{t^2} \ ,$$

from which

$$\frac{mt}{K} = \frac{\pi^2}{tT} \ ,$$

and thus

$$\alpha = \frac{\pi^2 r^2}{R^3} \cdot \frac{is}{tT} \ .$$

Now α is obtained by direct observation; and hence for determining i we have

$$i = \frac{R^3}{\pi^2 r^2} \cdot \frac{t}{s} \cdot T\alpha \ .$$

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[344][Note by AKTA:] In German: *Trägheitsmoment.* Instead of being translated as "moment of inertia", this expression was translated simply as "inertia" in [Web61, p. 231].

[345][Note by AKTA:] In German: *Schwingungsdauer.* This expression was translated several times in [Web61] either as "length of oscillation" or "time of oscillation". I replaced all these translations by "period of oscillation". See also footnote 96 on page 51.

[346][Note by AKTA:] Weber is here utilizing the equation of motion of the needle as $\tau = -mT\varphi = K\ddot{\varphi}$, where K is the moment of inertia of the needle, $\tau = -mT\varphi$ is the torque or rotational moment acting on it when it suffers a small deflection φ, m is the magnetic moment of the needle and T is the earth's magnetism.

Remembering that the current passing through the circle B also traverses the circle A, we might also calculate the action of the circular current A upon the needle in C; but, for the sake of simplicity, it may be assumed that the distance AC is so great that this action vanishes in comparison with the action of the circular current B; in that case the *actually observed* deflection of the needle in C gives directly the value of α.

Consequently, by the *electromotive force eE*, expressed in an absolute measure, for which has been found the expression

$$ e = \frac{\pi r^2}{s} \cdot T \ , $$

a current is produced, in the whole closed conductor whose space is to be measured, the *intensity* of which is expressed in an absolute measure by iJ, in which

$$ i = \frac{R^3}{\pi^2 r^2} \cdot \frac{t}{s} \cdot T\alpha $$

has been found. But, according to the unit explained in the preceding paragraph, the *desired resistance* of the whole closed conductor is expressed by wW, in which w is determined by the relation of the numbers e and i, namely:

$$ w = \frac{e}{i} = \frac{\pi^3 r^4}{R^3 t\alpha} \ . $$

Hence the execution of the measurement of an electric resistance depends on the measurement of the magnitudes

$$ r, \quad R, \quad t, \quad \alpha \ , $$

in other words, the resistance of the whole closed conductor can be expressed in an absolute measure, if by observations, *first*, the number α has been found which gives the deflection of the needle in parts of the diameter; *secondly*, the number r/R, which gives the diameter of both circles in parts of the distance BC; *thirdly*, the velocity r/t, with which the diameter of those circles is traversed during one oscillation[347] of the needle. Hence it appears that the *measure of velocity* is the only measure which must be given if the resistance of a conductor is to be determined according to an absolute standard.

11.3   Observations

Of the four magnitudes which, according to the preceding paragraph, are to be found by *observation* for the purpose of determining electric resistances according to an absolute standard, three can readily be measured, namely, the diameter r of the two circles, the distance $BC = R$ of the circle B from the needle at C, and the period of oscillation of the needle t. There only remains the fourth magnitude, that is the deflection of the needle α expressed in parts of the diameter, and this is usually so small that it cannot be observed. This is the reason why, in actually making the observations, a slight deviation must be made from the arrangement described in the previous paragraph. For in order to obtain a value of α large enough for accurate observation, it is *first* necessary that the magnetic needle, upon which the circular current B is to act, instead of being at a great distance $BC = R$, be suspended

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[347][Note by AKTA:] In German: *Schwingung*. It was translated as "rotation" in [Web61, p. 233].

in the centre of the circular current itself, in which case the action is the greater the smaller is the diameter r in comparison with R. Care must also be taken that the length of the needle is much smaller than the diameter of the circle, in order that the peculiar distribution of the magnetism in the needle need not be taken into account, because the investigation of this distribution is attended with difficulties. It is further necessary that both circles, instead of one, shall consist of several windings of the conductor, by which they become changed into rings of large diameter. In that case, however, the influence of all the windings must be individually taken into account, because they have different diameters, and are not all on the same plane as the needle.

For the conductor whose resistance was to be measured, a very long thick copper wire was chosen which weighed 169 kilogrammes. Of this 16 kilogrammes were used for the ring A, which consisted of 145 windings; enclosing altogether a surface of nearly 105 square metres. This ring was placed vertically, and by means of a winch could be rapidly rotated in a semicircle, so that the perpendicular upon the plane of the ring at the commencement and at the end of the rotation coincided with the magnetic meridian. The other 153 kilogrammes were used for the ring B, which consisted of 1854 windings, giving together a cross-section 202 millimetres in breadth, and 70.9 millimetres in height: the internal diameter of this ring was 303.51, and the external 374.41 millimetres. This second ring was firmly fixed, and its plane coincided with that of the magnetic meridian. In the centre of this second ring B, a small magnetic needle 60 millimetres long, provided with a mirror, was suspended by a filament of silk, as in a small magnetometer; and the oscillations and deflections of the needle were observed with a telescope, directed to the mirror, on a scale about 4 metres from the mirror.

The observations were made in the following manner. The ring A was first so placed that its plane coincided with the magnetic meridian, and the needle in the middle of the ring was thereby brought to rest; thereupon the ring A was suddenly turned 90°. By this means the needle in the middle of the ring was set in oscillation,[348] and by means of the telescope the position of the needle was observed on the scale at its greatest (positive) deflection after half a period of oscillation. One period of oscillation later, that is $1\frac{1}{2}$ period of oscillation after the beginning, the needle attained its greatest deflection on the opposite side,[349] which was also observed on the scale. In the moment at which the needle passed its original position of rest, and therefore two oscillations after the beginning of the experiments, the ring A was rotated 180°. The oscillating needle was thereby arrested in the middle of its motion, and thrown backwards, upon which its greatest negative and greatest positive deflections were observed on the scale. After the expiration of four oscillations from the commencement, that is, at the moment at which the needle returning from its last deflection passed its original position of rest, the ring was again turned forwards by 180°, and then the same oscillation observed as in the first case, and in this manner the experiments were continued until a sufficient series of observations was obtained. For each series, in the *first* column of the following Table are given the deflections observed on the scale and arranged in order under one another; in the *second* column the mean between two successive positive or negative deflections are added. In the *third* column are the differences of the means referring to positive and negative deflections that is, the magnitude of the whole arc.

[348][Note by AKTA:] In German: *Schwingung*. It was translated as "rotation" in [Web61, p. 234].
[349][Note by AKTA:] See footnote 96 on page 51.

First series			Second series			Third series			Fourth series		
467.1			467.1			463.0			462.0		
540.7			540.5			536.7			534.7		
	543.70			543.65			539.65			538.20	
546.7			546.8			542.6			541.7		
		80.10			79.65			80.40			80.00
461.4			461.3			456.6			455.3		
	463.60			464.00			459.25			458.20	
465.8			466.7			461.9			461.1		
		79.75			79.55			80.35			79.75
540.6			540.8			537.6			535.1		
	543.35			543.55			539.60			537.95	
546.1			546.3			541.6			540.8		
		79.25			79.90			79.55			79.50
462.3			461.8			458.3			456.0		
	464.10			463.65			460.05			458.45	
465.9			465.5			461.8			460.9		
		79.45			80.00			79.70			79.50
541.4			542.1			537.7			535.3		
	543.55			543.65			539.75			537.95	
545.7			545.2			541.8			540.6		
		79.75			79.70			79.95			80.05
462.3			462.8			457.9			456.0		
	463.80			463.95			459.80			457.90	
465.3			465.1			461.7			459.8		
		79.70			79.85			79.85			79.85
542.0			542.3			537.6			536.1		
	543.50			543.80			539.65			537.75	
545.0			545.3			541.7			539.4		
		79.45			80.10			79.70			79.55
462.8			462.7			458.2			456.8		
	464.05			463.70			459.95			458.20	
465.3			464.7			461.7			459.6		
		79.45			79.80			80.10			79.65
542.0			542.3			537.6			536.0		
	543.50			543.50			540.05			537.85	
545.0			544.7			542.5			539.7		
		79.65			79.75			80.05			79.70
462.9			462.8			457.3			456.5		
	463.85			463.75			460.00			458.15	
464.8			464.7			462.7			459.8		
		79.85			79.60			79.50			79.60
542.7			541.9			536.6			535.8		
	543.70			543.35			539.50			537.75	
544.7			544.8			542.4			539.7		
		79.45			79.75			79.75			79.55
463.4			462.3			457.2			456.4		
	464.25			463.60			459.75			458.20	
465.1			464.9			462.3			460.0		
		79.70			79.85						79.55
542.6			541.3						535.7		
	543.95			543.45						537.75	
545.3			545.6						539.8		
		79.75									
462.8											
	464.20										
465.6											
Mean 79.64			Mean 79.79			Mean 79.90			Mean 79.69		

The mean value of these four series is 79.755 parts of the scale = 79.4 millimetres, which must be increased by 1/2 a millimetre if we are to take into account the influence of the fact that the rotation of the ring A cannot be effected in a time so small that it can be neglected in comparison with the period of oscillation of the needle. From this we obtain for α the value

$$\alpha = \frac{79.9}{8175},$$

275

inasmuch as double the horizontal distance of the mirror from the scale is exactly 8175 millimetres.

The period of oscillation of the needle was found from 300 oscillations to be

$$t = 10.281\,8'' \ ,$$

in which the part of the directive force, arising from the elasticity of the thread was the 1770th part of the magnetic directive force, and hence

$$\frac{1}{1+\varphi} = \frac{177\,0}{177\,1} \ .$$

Finally, on account of the great distance of the two rings in a room not free from iron, the period of oscillation of the same needle was compared for the position of both rings, and their ratio found to be as 2.9126 : 2.9095; from which it follows that if T' is the terrestrial magnetism for A, T'' for B, we have

$$T' : T'' = 470 : 471 \ .$$

These observations are sufficient for determining the resistance of the whole closed conductor; and by accurate calculation we get the value

$$\omega = 2166 \cdot 10^8 \ .$$

11.4 Application of the Principle of Deadening

Instead of using *terrestrial magnetism* to obtain an electromotive force which can be referred to an absolute measure, *bar magnetism* may be employed; in that case it is obvious that the most convenient position for the bar-magnet whose magnetism is to be used, will be in the *centre* of the ring formed by the closed conductor. The magnet may then either be fixed, and the ring turnable about its diameter at right angles to the magnetic axis of the bar; or inversely, the ring may be fixed and the magnet turned[350] back and forth about that diameter. In the latter case a strong *oscillating magnetic needle* may be used, suspended in the centre of the ring.

The current produced in the closed conductor by the electromotive force arising from the bar magnetism of a magnetic needle oscillating in the centre of the ring, itself reacts according to the principle of *deadening* on the oscillating needle, and produces a diminution in the amplitude of its oscillations which can be observed with great accuracy; and the *intensity* of this current may also, from these observations, be determined according to an absolute standard with great accuracy. It is then evident that the current does not need to be passed through a second ring serving as galvanometer, in order to measure the intensity of the current. Hence the whole conductor, whose resistance is to be measured, can be used to form a single ring which serves at once for indicator and multiplier.

According to this simplification, the *observation of the arcs of oscillation of a magnetic needle oscillating in the centre of the ring* is sufficient: by their *magnitude* the strength of the electromotive force, and by their *decrease* the intensity of the current produced in the closed conductor by that electromotive force, can be determined.

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[350][Note by AKTA:] In German: *Gedreht.*

In executing the observations according to this principle of *deadening*, it is of prime importance that the magnetism of the needle oscillating in the centre of the ring be very powerful; and also that the length of the needle be very small as compared with the diameter of the ring, in order that, in calculating the resistance, there shall be no necessity for an accurate knowledge of the distribution of the magnetism in the needle, the determination of which would be difficult. In the ring now solely used, which is that previously called B, and which has 303.51 millimetres internal, and 374.4l millimetres external diameter, and is 202 millimetres in height, a magnetic needle 90 millimetres long, and as strong as possible, was suspended. The experiment was commenced by *detaching* from each other the ends of the wire forming the ring. The needle was then set in oscillation, and its period of oscillation and the decrease of its amplitude, or the logarithmic decrement of this decrease, was determined according to the method given by Gauss in the "Results of the Observations of the Magnetic Association in the year 1837."[351,352,353,354] Thereupon the annular conductor was *closed* and the same observations repeated. The results of these observations are given in the following Table, in which the logarithmic decrement of the diminution of the arc of oscillation with a *closed* conductor, stands in the *first* column under A, the same with an *open* conductor stands under B, while in the *third* column under t is given the observed period of oscillation. The mean values are indicated underneath:

A	B	t
0.028 645	0.000 460	9.112 8
0.027 955	0.000 360	9.114 8
0.028 565	0.000 380	9.110 7
0.028 388	0.000 400	9.112 8

From this we obtain, according to Brigg's system,[355] for that part of the logarithmic decrement arising from the deadening,

$$= 0.028\,388 - 0.000\,400 = 0.027\,988 \; ,$$

or according to the natural system,

$$\lambda = 0.064\,445 \; .$$

The bar magnetism of the oscillating needle M, determined from magnetometric measurements, was found, according to absolute standard as compared with the horizontal part of the earth's magnetism T,

$$\frac{M}{T} = 20\,733\,000 \; .$$

That part of the directive force of the needle arising from the elasticity of the thread was found to be 68 times less than that arising from the magnetism, or

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351 [Note by HW:] Gauss' Werke, Vol. V, p. 374.

352 [Note by AKTA:] See [Gau38a].

353 [Note by EPM:] See Taylor's *Scientific Memoirs*, Part VI. Vol. II.

354 [Note by AKTA:] The Editors of the Philosophical Magazine may be referring to one of these two works of Gauss translated into English: [Gau41a] or [Gau41c].

355 [Note by AKTA:] Henry Briggs (1561-1630) introduced common (base 10) logarithms.

$$\frac{1}{1+\vartheta} = \frac{68}{69} \; .$$

For the calculation of the resistance from these observations, executed on the principle of deadening, we have the following rules.

According to the law of magnetic induction, the *electromotive force* of a small magnet oscillating in the centre of a circular conductor, whose magnetic axis makes the angle φ with the plane of the circle, is directly proportional to its magnetism M, to the cosine of the angle φ, and to the velocity of rotation $d\varphi/dt$, and inversely proportional to the diameter of the circle r; and if M is expressed according to an absolute measure, is determined by

$$e = \frac{2\pi M}{r} \cdot \cos\varphi \frac{d\varphi}{dt} \; .$$

On the contrary, according to electro-magnetic laws the *momentum of rotation* which the induced current in the circular conductor exerts upon the small magnet oscillating in the centre is directly proportional to the magnetism M, to the cosine of the angle φ, and to the current intensity, and is inversely proportional to the diameter r; and if i is expressed in absolute measure, is determined by

$$D\frac{d\varphi}{dt} = \frac{2\pi M}{r} \cdot i\cos\varphi \; .$$

For small oscillations in which φ differs little from 0, we have

$$e = \frac{2\pi M}{r} \cdot \frac{d\varphi}{dt} \; ,$$

$$D\frac{d\varphi}{dt} = \frac{2\pi M}{r} \cdot i \; .$$

If K is the moment of inertia[356] of the oscillating magnet, upon which the directive force MT, arising from the horizontal part of the terrestrial magnetism, acts, the equation of its motion becomes

$$0 = \frac{d^2\varphi}{dt^2} + \frac{MT}{K}\varphi + \frac{D}{K}\frac{d\varphi}{dt} \; ,$$

and hence by integration,

$$\varphi = p + Ae^{-Dt/2K}\sin(t-B)\sqrt{\frac{MT}{K} - \frac{1}{4}\frac{D^2}{K^2}} \; .$$

Here $D/2K$ is the logarithmic decrement on the natural system of the diminution of the amplitude of oscillation reduced to the unit of time: hence if τ is the period of oscillation under the influence of deadening,

$$\lambda = \frac{D\tau}{2K} = \frac{\pi M}{rK} \cdot \frac{dt}{d\varphi} \cdot \tau i \; ,$$

and the *intensity of the current* is

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[356][Note by AKTA:] In German: *Trägheitsmoment.*

$$i = \frac{rK\lambda}{\pi M\tau} \cdot \frac{d\varphi}{dt} \ .$$

From this we obtain for calculating the resistance,

$$\omega' = \frac{e}{i} = \frac{2\pi^2 M^2}{r^2 K\lambda} \cdot \tau \ .$$

From the above equation for φ we get for the determination of the period of oscillation under the influence of the deadening,

$$\tau\sqrt{\frac{MT}{K} - \frac{1}{4}\frac{D^2}{K^2}} = \pi = \tau\sqrt{\frac{MT}{K} - \frac{\lambda^2}{\tau^2}} \ ,$$

from which

$$\frac{M\tau}{K} = \frac{\pi^2 + \lambda^2}{\tau T} \ ,$$

hence

$$\omega' = \frac{2\pi^2}{r^2} \cdot \frac{\pi^2 + \lambda^2}{\lambda\tau} \cdot \frac{M}{T} \ .$$

From this, taking into account the correction arising from the deadener as being made up of several windings, and the correction for the elasticity of the thread, we find from the above observations

$$\omega' = 189\,8 \cdot 10^8 \ .$$

11.5 Comparison of the Resistance Determined according to Absolute Measure with Jacobi's Standard of Resistance

To compare the resistance of two conductors, there are different methods which need no explanation. The resistances considered in the preceding paragraphs have been compared according to the method examined in this memoir, and it has been found that

$$\omega : \omega' = 113\,8 : 1\,000 \ .$$

If the first resistance be reduced to the second according to this proportion, we obtain

$$\omega' = \frac{1\,000}{1\,138}\omega = 1903 \cdot 10^8 \ ,$$

while the direct determination in the preceding paragraph gave

$$\omega' = 189\,8 \cdot 10^8 \ .$$

From both these closely agreeing values, determined according to entirely different methods, the number

$$19 \cdot 10^{10}$$

will in future be assumed as the mean value of this resistance.

Jacobi has dwelt on the importance of introducing a definite measure for resistance to be accepted by all physicists, especially at the present time, when so many voltaic investigations are being made with the most varied instruments, the comparison of which is often of great importance. For this purpose he has proposed as a *standard measure* a copper wire, which he has sent to several physicists who are engaged with voltaic measurements, and has requested them to compare this standard with theirs, and for the future to give their measurements in this measure.[357]

This standard is a copper wire $7169\frac{3}{4}$ millimetres in length, and 2/3 millimetres in thickness, which weighs $22449\frac{3}{10}$ milligrammes.

The standard introduced by Jacobi, which, it is to be hoped, will find general acceptance is by no means supplanted by the *absolute measure* here discussed; for it is not possible to compare every resistance directly according to this measure, while every resistance can be directly compared with Jacobi's standard. But considering the importance which absolute determinations of measure have in many investigations, it is desirable to be able to reduce all the values, made according to Jacobi's standard, to an absolute measure, which can be easily effected by comparing the resistance determined as above according to an absolute measure with the resistance of Jacobi's standard.

Such a comparison has been made; and it has been found that the two resistances are nearly as 32 : 10, or, more accurately, as 19 000 : 5 980. But as the first resistance has been found in absolute measure to represent 19 000 million units, Jacobi's standard represents 5 980 million units; or the resistance determined according to Jacobi's measure can be reduced to absolute measure by multiplication by 6 milliards. By this determination it would be possible to reproduce approximately Jacobi's standard, even if it were lost.

11.6 On the Value of the Constants Found by Kirchhoff, on which the Intensity of Induced Electric Currents Depends

The *induction-constant* which Neumann,[358] in his development of the mathematical laws of induced electric currents, calls ε, has the following meaning. If W be the absolute unit of measure proposed as above for electric resistances, and W' that measure of resistance which is actually used; if, further, C be the measure of velocity which forms the basis in establishing the above absolute measure (1 millimetre in a second); if, on the contrary, C' be the measure of velocity actually used in measuring the induced motions and actions of the induced currents (1 Prussian inch = 26.154 millimetres in a second, according to Kirchhoff),[359] we have

$$\varepsilon = 2\frac{C'W}{CW'} \ .$$

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[357][Note by AKTA:] [Jac51].
[358][Note by AKTA:] See [Neu46], [Neu47], [Neu48] and [Neu49].
[359][Note by AKTA:] The Prussian inch is to the English as 1.03 to 1.

It follows from this, that if the value of this induction-constant is once determined, any resistance given according to the measure chosen can be referred to an absolute measure.

In the determination of the induction-constant ε given by Kirchhoff in the seventy-sixth volume of Poggendorff's *Annalen*,[360] the resistance of a *copper wire* has been chosen as a standard, the length of which was 1 Prussian inch = 26.154 millimetres, and the cross-section 1 Prussian square inch = 684 square millimetres. Here unfortunately there is no determinate measure of resistance; for different pieces of copper of the same dimensions have different resistance; and it follows, therefore, that the value of the induction-constant ε is left undetermined within the limits of that variability of the resistance of copper. Kirchhoff himself says,

> "Since the conductivity[361] of copper varies within certain limits, in giving the value of ε, only a limited accuracy is of interest."

Kirchhoff wished to give only an approximate value of ε, which would be sufficient for his purpose; and he was the more content therewith because the methods and instruments which he used would scarcely have permitted a better determination of ε if he proposed a perfectly definite measure of resistance.

The interest which an accurate determination of the value ε has, is lost in consequence of that uncertainty in the choice of the measure of resistance; and it is important to restore it by the removal of that uncertainty. This may be accomplished by keeping, not to *copper in general*, but to the piece of copper actually used by Kirchhoff in his investigations, and by choosing the resistance of a wire of this copper 26.154 millimetres in length, and with a cross-section of 684 square millimetres as a measure of resistance. It is thus only necessary to reduce the result found by Kirchhoff, as well as the measures made therewith or referred thereto, to the measure thus accurately determined in this manner. Kirchhoff took one Prussian inch in a second as a measure of velocity, and found in this way

$$\varepsilon = \frac{1}{192} \; ,$$

from which it follows (since $C' = 26.154C$) that that resistance which amounts to 52.308 units of the above absolute measure is the $1/192$ of the resistance of a wire of Kirchhoff's copper the length of which is 26.154 millimetres and the cross-section 584 square millimetres; in other words, that the measure of resistance chosen by Kirchhoff is 10043 times that of the above absolute measure.

Although this value of ε can only be considered as approximative, it is interesting to compare it with other values which have been found by entirely different methods and with different instruments, because an examination of the various natural laws brought thereby into operation is obtained. Kirchhoff's measurements refer to currents produced by *voltaic induction*, and hence in his case it is the laws of voltaic induction which have been used in determining the value of ε. My measurements, on the contrary, refer to currents produced by *magnetic induction*, and hence in this case it is the laws of magnetic induction which lead to the value of ε.

First of all, the value of ε shall be given which is obtained from my measurements. It is clear that the value of ε can be determined from these measurements, if only the resistance

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[360][Note by AKTA:] [Kir49a].
[361][Note by AKTA:] In German: *Leitungsfähigkeit*.

of Kirchhoff's copper wire is compared with the resistance of Jacobi's standard. I have made that comparison by means of the wire which Kirchhoff kindly sent to me, and can here give the result of the comparison: it is as follows.

A piece of Kirchhoff's wire which was 13.573 Prussian inches in length and 0.406 1 square line in cross-section, had a resistance which was to the resistance of Jacobi's standard as

$$1 : 106 .$$

From this we get the relation of the resistance of the measure chosen by Kirchhoff to that of Jacobi's standard as

$$1 : 106 \times 13.573 \times \frac{144}{0.406\,1} .$$

If J be the resistance of Jacobi's standard, and W' that of Kirchhoff's we have

$$\frac{J}{W'} = 510\,180 .$$

Now the resistance of Jacobi's standard is equal to 5980 million units of the absolute measure found above; hence, if W be the absolute resistance,

$$\frac{J}{W} = 5\,980\,000\,000 ,$$

hence

$$\frac{W'}{W} = 11\,720 .$$

But now

$$\frac{C'}{C} = 26.154 ,$$

hence

$$\varepsilon = 2\frac{C'W}{CW'} = \frac{1}{224} ,$$

that is, one-seventh less than Kirchhoff had found. A closer agreement was not to be expected, inasmuch as only an approximate value was claimed for Kirchhoff's statement.

I may give here a determination *of the specific resistance of the different kinds of copper* which have been used for Jacobi's standard, for Kirchhoff's wire, and for the damper which I used.

The *specific resistance* of a body is usually given according to an absolute unit by taking for this unit the specific resistance of a body whose absolute resistance with a length = 1 and a cross-section = 1 is equal to the fixed measure of resistance. But the determination of specific resistance according to this unit meets with a practical difficulty in the accurate measurement of the cross-section, especially in fine wires, and hence, to obviate this difficulty, Kirchhoff has indirectly ascertained the cross-section of the wire by determining its absolute and specific gravity.

Now the determination of specific resistance according to this unit, presupposes that the resistance of a wire whose length remains unchanged, but the thickness of which is increased or diminished, varies inversely as the cross-section. This has not, however, been proved, and,

with the small alterations of cross-section which are produced by pressure, can scarcely be proved. There is just as much reason for assuming that, if the mass and the length of the wire remain unchanged, the resistance does not alter even with a changing cross-section. On this assumption the absolute unit would have to be fixed in another way than as being the specific resistance of a body whose absolute resistance for the length $= 1$ and for a mass $= 1$ is equal to the fixed measure of resistance. According to this, the specific resistance of a body would be determined by multiplying the resistance of a wire formed of that substance expressed according to the fixed measure of resistance by its mass, and dividing by the square of its length.

The specific resistances of the wires used by Jacobi, Kirchhoff, and myself will be determined according to the unit thus fixed; for apart from the above considerations, this determination is in any case the most applicable and capable of execution.

The following Table exhibits the results of these determinations:

Quality of copper in	Length in millimetres	Mass in milligrammes	Resistance in absolute measure	Specific resistance	ε
Jacobi's wire	7 620	22 435	5 980 000 000	2 310 000	1/270
Kirchhoff's wire	355	4 278	58 500 000	1 916 000	1/224
Weber's wire	3 946 000	152 890 000	190 000 000 000	1 865 600	1/218

It will be seen that there is only a small difference between my copper and Kirchhoff's; while the difference in the case of Jacobi's is far more considerable, as the latter possesses a far smaller conductivity. In the supposition that Jacobi may have used galvanoplastic copper for his standard, I examined a wire of that material which I procured through the kindness of Professor Schellbach in Berlin,[362] and found the following result, which proves, contrary to the above supposition, that galvanoplastic copper is a somewhat better conductor.

Wire of galvanoplastic copper	Length in millimetres	Mass in milligrammes	Resistance in absolute measure	Specific resistance	ε
	12 780	221 295	1 243 000 000	1 684 000	1/196

In the last column here and in the upper Table are given the different values of ε which were obtained for the Neumann's induction-constant by adhering to the measure chosen by Kirchhoff, but using the different kinds of copper which have been mentioned. Adhering, however, to the absolute measure fixed as above, so is $C' = C$, $W' = W$, and ε has always the value 2.

11.7 On the Constants of the Electric Laws which Depend on the Choice of Measures

The law of induced currents propounded by Neumann represents the intensity of these currents as dependent on a constant the value of which must be determined from the measures according to which the magnitudes taken into consideration are to be determined. This constant Neumann has called the *induction-constant*. Such a constant occurs in the general

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[362][Note by AKTA:] Weber may be referring to Karl Heinrich Schellbach (1805-1892).

expression of any natural law which states how one magnitude is determined by another. I may here give a summary of these constants for all the fundamental laws which refer to *electromotive force*, *current intensity*, and *electric resistance*. Each of these laws represents the desired magnitude as an expression of other measurable magnitudes, which has a constant as a factor the value of which is to be determined from the measures chosen.

1. The fundamental law of the voltaic circuit represents the intensity of the current i as an expression of the electromotive force e, and of the resistance w; for, if the constant whose value is to be determined is called α,

$$i = \alpha \cdot \frac{e}{w} \ .$$

This constant α has the following meaning. If J, E, W are the absolute measures fixed as above for current intensities, electromotive forces, and resistance; and if J', E', W' are the measures actually used, we have

$$\alpha = \frac{JE'W}{J'EW'} \ .$$

Hence using the absolute measure itself,

$$\alpha = 1 \ .$$

2. The fundamental law of electro-magnetism represents the electromotive force F as an expression of the quantity of magnetic fluid μ, of the length ds, and of the intensity i of the current element, of their distance from one another r, and of a number which is given by the angle φ which r makes with ds; that is, if the constant whose value is to be determined from the measures chosen is β,

$$F = \beta \cdot \frac{\mu i ds}{r^2} \sin \varphi \ .$$

The constant β has the following signification: — If P is the absolute unit of measure of the momentum of rotation (the product of a millimetre into that force which in one second imparts to the mass of one milligramme, the absolute unit of measure of velocity), if M is the absolute unit of measure of the magnetic fluid, and J is the absolute measure for current intensities; if, further, P' M', and J' are the measures actually used,

$$\beta = \frac{PM'J'}{P'MJ} \ ,$$

consequently, using the absolute measure,

$$\beta = 1 \ .$$

3. Ampère's fundamental law of electrodynamics[363] represents the electrodynamic force of attraction F as an expression of the current intensities of two elements i and i', and of a number which is fixed by the relations of the lengths of the two elements to their distance ds/r and ds'/r; and by the three angles ε, φ and φ', which ds and ds' form with one another and with r; that is, if the constant whose value is to be determined from the given measures is designated by γ,

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[363][Note by AKTA:] See footnote 10 on page 13.

$$F = \gamma \cdot ii' \cdot \frac{dsds'}{r^2} \left(\cos \varepsilon - \frac{3}{2} \cos \vartheta \cos \vartheta' \right) .$$

The constant γ has the following signification: — If F is the absolute measure of force (that force which in a second imparts to the mass of a milligramme a velocity of a millimetre in a second), if J is the absolute measure for current intensities, and F', J' the measures actually used, we get

$$\gamma = \frac{FJ'J'}{F'JJ} ,$$

hence using the absolute measure,

$$\gamma = 2 .$$

4. The fundamental law of magneto-induction represents the electromotive force e as an expression of the mass of magnetic fluid μ, of the velocity of induced motion c, of the length of the induced element ds, and of its distance r from μ, and of a number given by the two angles φ and ψ which ds makes with r and c with the normal to the plane rds; that is, if the constant whose value is to be determined from the measures chosen is called δ,

$$e = \delta \cdot \frac{\mu cds}{r^2} \sin \varphi \cos \psi .$$

The constant δ has the following signification: — If E is the absolute unit of measure of electromotive force, M the absolute unit of measure of magnetic fluid, S the seconds of time, and E', M', S' the measures actually used, we get

$$\delta = \frac{EM'S}{E'MS'} ,$$

hence using the absolute measure,

$$\delta = 1 .$$

5. The fundamental law of voltaic induction represents the electromotive force e as an expression of the current intensity i and of its change di/dt, of the velocity of the inducing motion c, and of the distance r of the induced from the inducing element, and of several numbers which are given by the relations of the lengths of the two elements to their distance ds/r and ds'/r, and by the four angles ε, ϑ, ϑ' and φ which ds and c form with each other and with r, and which ds' forms with r; that is to say, if the constant whose value is to be determined from the measures chosen is called ζ,

$$e = \zeta \cdot \left[ci \cdot \frac{dsds'}{r^2} \left(\cos \varepsilon - \frac{3}{2} \cos \vartheta \cos \vartheta' \right) \cos \varphi + \frac{1}{2} \frac{di}{dt} \frac{dsds'}{r} \cos \vartheta \cos \vartheta' \right] .$$

The constant ζ has the following significance: — If E and J are the absolute units for electromotive forces and for current intensities, and C the absolute measure of velocity (a millimetre in a second), and E', J' and C' the measures actually used, we have

$$\zeta = 2 \cdot \frac{EJ'C'}{E'JC} ,$$

hence using the absolute measure itself,

$$\zeta = 2 \ .$$

6. The general fundamental law of electric action represents the electric force F as an expression of the electric masses v and v', of their distance r, their relative velocity dr/dt, and their change ddr/dt^2, that is, if the constant whose value is to be determined from the given measures is called η, we have

$$F = \eta \cdot \frac{vv'}{r^2} \left[1 - \frac{1}{aa} \left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2} \right) \right] \ .$$

(a stands for the number indicating the relation of that velocity with which two electric masses must be moved against each other in order that they exert no force on each other, to the velocity of a millimetre in a second.)

The constant η has the following signification: — If F is the absolute measure of force, N the absolute unit of electric fluid (that mass of electric fluid which at a distance of a millimetre exerts upon a similar mass the absolute unit of force), if R is a millimetre, and F', N' and R' the measures actually used, we have

$$\eta = \frac{FN'^2R^2}{F'N^2R'^2} \ ,$$

hence using the absolute measure itself,

$$\eta = 1 \ .$$

Every electric force can act, however, as electromotive force; and this latter e is represented, according to the general fundamental law of electric action, as an expression of the electric mass v, of the length of the element ds in which is contained the quantity of electricity acted upon; further, of the distance r of both from each other, of their relative velocity dr/dt, and their change ddr/dt^2, and of the angle φ which ds forms with r; that is, if the constant whose value is to be determined from the measures chosen is called k, we have

$$e = k \cdot \frac{vds}{r^2} \left[a - \frac{1}{a} \left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2} \right) \right] \cos \varphi \ .$$

The constant k has the following meaning: — If E is the absolute unit of measure of electromotive forces, N the absolute unit of measure of the electric fluid, C the absolute unit of velocity (a millimetre in a second), R a millimetre, and E', N', C' and R' the measures actually used, we have

$$k = \frac{1}{2\sqrt{2}} \frac{EN'C'R}{E'NCR'} \ ,$$

hence using the absolute measure,

$$k = \frac{1}{2\sqrt{2}} \ .$$

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Chapter 12

Editor's Introduction to Weber's Second Memoir on Electrodynamic Measurements

A. K. T. Assis[364]

I present here some important points related to Weber's Second major Memoir on Electrodynamic Measurements.[365] It has been translated by David Delphenich.[366]

One of the simplest and most basic phenomena of electromagnetism is the situation of a resistive circuit connected to a battery and carrying a steady current. Most scientists and textbook authors dealing with this configuration in the last 150 years have assumed implicitly or explicitly the following:

- The resistive wire is neutral not only along its interior, but also along its surface.

- Therefore, an external test charge located at rest relative to the wire experiences no force exerted by the stationary resistive circuit.

- In the language of field theory, these scientists assume that the wire carrying a constant current generates only a magnetic field, but no electric field.

These are wrong assumptions. Wilhelm Eduard Weber (1804-1891) studied this configuration in the present work. He first analyzed a long and thin straight conductor of circular cross-section, Figure 12.1 (a). It might be considered as the region close to the center of the upper portion BC of the rectangular closed circuit of Figure 12.1 (b).

Weber showed in the present work that in order to have a steady current flowing along this resistive wire, it was necessary to have a distribution of charges spread along the surface of the conductor. This distribution of surface charges would generate a constant electric force acting on any charge of the conductor, like a stationary ion at rest in the lattice or a mobile charge of the wire (as the modern conduction electron). This distribution of surface charges was maintained by the battery. It was constant in time for steady currents, but

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[365][Web52a] with English translation in [Web21c].
[366]feedback@neo-classical-physics.info and http://www.neo-classical-physics.info/index.html.

(a)   (b)
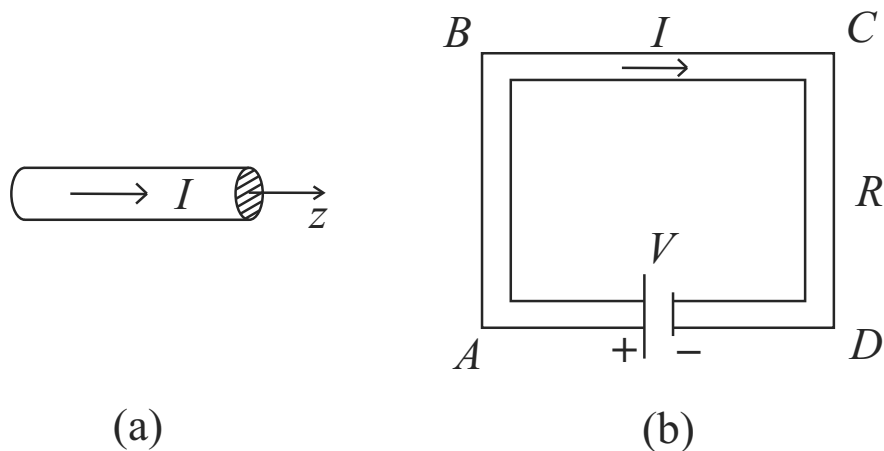
Figure 12.1: (a) Cylindrical resistive wire carrying a steady current I. (b) Rectangular circuit of circular cross-section. A battery generating a voltage V and producing a steady current I in the closed circuit of resistance R.

its magnitude varied along the length of the wire. In this case of a straight wire, Weber showed that the surface charge density σ increased linearly as a function of the longitudinal coordinate z, Figure 12.2 (a) and (b). That is, it became increasingly positive towards the positive terminal of the battery and increasingly negative towards the negative terminal of the battery.

(a)   (b)
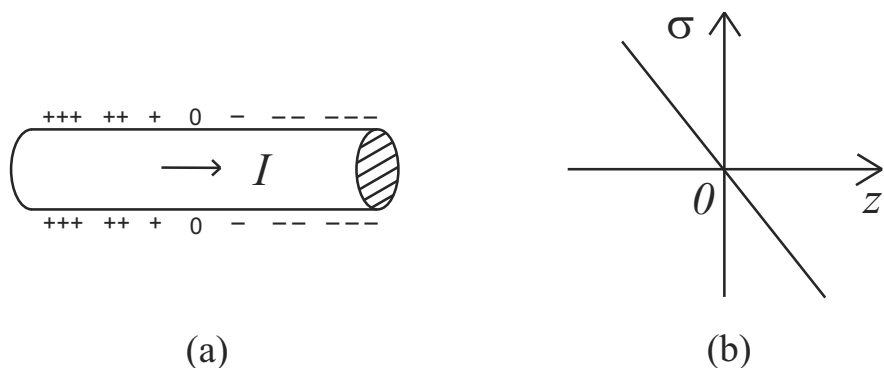
Figure 12.2: (a) Distribution of charges along the surface of a cylindrical wire. (b) Surface charge density σ as a function of the longitudinal coordinate z.

Weber calculated the force per unit charge acting on a test charge showing that it was constant anywhere inside the wire. For a steady current this force acting on the conduction electrons would be balanced by the resistive force exerted by the lattice, yielding Ohm's law. By extending his calculation to a stationary test charge located outside the wire, we obtain the lines of electric field shown in Figure 12.3.

Figure 12.3 shows that a test charge outside the wire, at rest relative to the conductor, should experience a force exerted by the resistive wire carrying a steady current.

Weber then considered a toroidal conductor in the form of a resistive ring connected to a battery and carrying a constant current I along the azimuthal direction, Figure 12.4 (a).

In this case he performed a brilliant approximate calculation of the density of surface charges. Once more he showed that in order to have a constant electromotive force anywhere inside the ring pointing along the azimuthal θ direction, it was necessary to have a
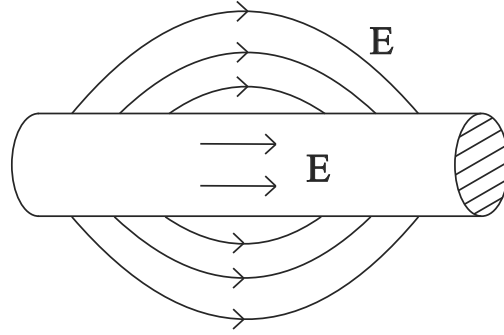
Figure 12.3: Force per unit charge, $\vec{F}/q = \vec{E}$, acting on an internal or external test charge at rest relative to the resistive cylindrical conductor carrying a steady current. This force is exerted by the charges spread on the surface of the wire.

(a)　　　　　　　　　　(b)　　　　　　　　　　(c)
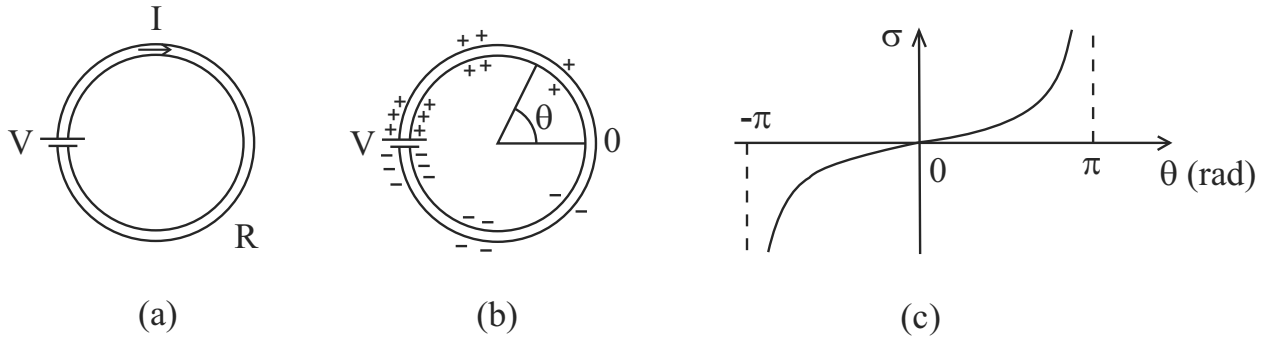
Figure 12.4: (a) Resistive ring carrying a steady current I. (b) Charges distributed along the surface of the ring. (c) Surface charge density σ as a function of the azimuthal angle θ.

distribution of charges along the surface of the wire. For a steady current this distribution was constant in time, but its magnitude varied along the azimuthal direction. It became increasingly positive towards the positive terminal of the battery and increasingly negative towards the negative terminal of the battery, Figure 12.4 (b).

However, in this case he was able to show that the surface charge density σ varied linearly with the azimuthal angle θ only far away from the battery, that is, close to $\theta = 0$ rad, as in Figure 12.4 (c). He also showed that when we are closer to both terminals of the battery, that is, close to $\theta = \pm\pi$ rad, the increase in the magnitude of σ as a function of θ was faster than linearly, as shown qualitatively in Figure 12.4 (c).

Weber calculated the force per unit charge acting on a test charge showing that it had a constant magnitude anywhere inside the ring, pointing along the azimuthal direction. By extending his calculation to a stationary test charge located outside the ring, we obtain the lines of electric field shown in Figure 12.5.

In 2007 we published the book "*The Electric Force of a Current: Weber and the Surface Charges of Resistive Conductors Carrying Steady Currents*". It is available in English, Portuguese and German.[367] In this book we discussed Weber's 1852 work and presented the analytical expressions for the distributions of surface charges, equipotential lines and also the lines of electric field. These calculations were made not only for the configurations of Figures 12.1 up to 12.5, but also in many other geometries (coaxial cable, solenoid, twin-lead etc.) We calculated the force per unit charge acting on internal and external test charges

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[367][AH07], [AH09] and [AH13].
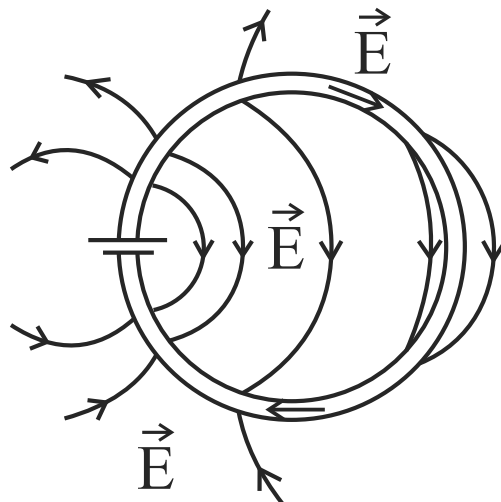
Figure 12.5: Force per unit charge, $\vec{F}/q = \vec{E}$, acting on an internal or external test charge at rest relative to the resistive ring carrying a steady current. This force is exerted by the charges spread on the surface of the ring.

at rest relative to different resistive circuits carrying steady currents. We also cited many references and quotations of different scientists and textbook authors presenting the wrong assumptions discussed in the beginning of this Introduction, including James Clerk Maxwell (1831-1879), Rudolf Clausius (1822-1888), Edward Mills Purcell (1912-1997), Richard Feynman (1918-1988) etc.

Moreover, we presented many experiments performed by several scientists related to this subject:

- Charge collectors have been utilized to study the sign and magnitude of the charge density spread along the surface of resistive conductors carrying steady currents.

- Grass seeds, semolina or flour have been spread around these conductors in order to map the electric field lines outside them. These seeds or particles are not affected by a magnetic field. However, in the presence of an electric field, they get electrically polarized and align themselves along these lines. These experiments utilizing grass seeds or semolina to map the lines of electric field are analogous to the usual experiments utilizing iron fillings in order to map the lines of magnetic field.

- Electronic electrometers connected to a radioactive alpha source have been utilized to measure the potential in different points inside and outside hollow conductors carrying steady currents. The measured equipotential lines are shown to be orthogonal to the electric field lines mapped with grass seeds or semolina.

- A torsion balance has been utilized in order to measure directly the force between a stationary test charge and a stationary resistive closed conductor carrying a steady current.

These experiments give strong support to Weber's pioneering calculations. Weber's fundamental work has been essentially forgotten during the XXth century. I hope this English translation will help to bring his brilliant measurements, calculations and ideas to the attention of a larger audience.

Chapter 13

[Weber, 1852a, EM2] Electrodynamic Measurements, Second Memoir, relating specially to Measures of Resistance

Wilhelm Weber[368,369,370]

I - Resistance Measurements with a Given Basic Unit

13.1   Tools

Measuring resistance, like any other measurement, assumes three things: *First of all*, a definition of the type of quantity that is to be measured, *secondly*, a certain unit,[371] and *thirdly*, a method for comparing quantities of that type with each other.

First of all, the *definition* of resistance that will be treated here can be expressed in the following way: From the laws that Ohm exhibited,[372] the quotient of the measured *electromotive force* and the measured *current intensity* always has the same value for a galvanic circuit with *unchanging closed conductors*, and that value depends upon only the size and nature of the conductor. Having assumed that, the property that lies in the size and nature of the conductor and upon which the value of that quotient depends will be referred to by the name of the *resistance* of the conductor, and it will be considered to be a *quantity that is proportional to that quotient*. In that way, there is the possibility of measuring resistance by determining that quotient.

As far as the *second* issue — viz., the *unit* of resistance — is concerned, the *basic unit*

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[368] [Web52a] with English translation in [Web21c].

[369] Translated by D. H. Delphenich, feedback@neo-classical-physics.info and http://www.neo-classical-physics.info/index.html. Edited by A. K. T. Assis.

[370] The Notes by Wilhelm Weber are represented by [Note by WW:]; the Notes by H. Weber, the Editor of Volume 3 of Weber's *Werke*, are represented by [Note by HW:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[371] [Note by AKTA:] In German: *Maass*.

[372] Note by AKTA:] See footnote 99 on page 53.

that Jacobi presented in St. Petersburg and sent to Professor Poggendorff in Berlin on 30 August 1846 with the following remarks shall be assumed. Mr. Jacobi wrote:[373]

> "On a previous occasion, I have commented on how interesting and important it would be if the physicists were to give their current measurements in electrolytic, hence absolute, units in their galvanic investigations. In order to do that, it would be necessary to refer the galvanometers that they work with to electrolytic actions in order to give the publication of the experiment performed the degree of precision that is true of the chosen instrument or the chosen method. Meanwhile, I have reserved a discussion of that for another occasion. It is no less important than the absoluteness of the current measurements that the physicists should express the conductive resistance that they measure in a common unit. However, no absolute determination can exist here, since it seems that differences exist between resistances that cannot be explained by a difference in dimension alone, even for the chemically-purest metals. If one assumes that one's resistance meter and multiplier are referred to copper wire of one meter in length and one millimeter in thickness, then one will still not be convinced that your copper wire and ours possess the same coefficient of resistivity. Now, all of those complications will be eliminated when one lets an arbitrarily-chosen copper (or other type of) wire circulate amongst the physicists, and that will beseech one to henceforth refer one's resistance-measuring instrument to it and give one's measurement in terms of that unit. Professor Magnus[374] then proposed a small black box provided with two screws, in which a copper wire that was wound around a board is cemented in a mastic that consists of wax and resin so it is protected from moisture and humidity. I would ask you to compare your resistance meters to that resistance standard, but also to what Professor Weber and other physicists addressed with galvanometric measurements ... The copper wire that is found in the box is precisely $25'$ (Russian-English) between the screws, weighs 22.5495 g, and its thickness, which was measured by a good Munich microscope that was provided with a micrometer, amounted to $0.0265''$ (English) at one end and $0.0260''$ at the other, and therefore a mean of $0.02625''$ (English). The measurements themselves are the means of three very-closely-corresponding observations. I would like to further point out that the length of the weighed wire amounted to $25\frac{1}{8}''$ (so $25\frac{1}{8}'' = 22.5495$ g) and that $\frac{3}{4}''$ of it at either end was soldered onto the screws. Expressed in French units, the length of the wire was $25' = 7.61975$ m, and its thickness was $0.02625'' = 0.000667$ m."

Finally, as far as the *third* issue is concerned, namely, comparing the resistances of two conductors or determining their resistance properties (for example, comparing a copy with the given basic unit), two instruments are required for that, along with several conductors:

1. An *electromotor*[375] with which galvanic currents can be produced.

2. A *galvanometer* with which the intensity of the produced currents can be measured.

In the first instrument, the conductors in which the currents are produced define essential components, while in the second instrument, it is the conductors through which the current

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[373][Note by AKTA:] M. H. v. Jacobi (1801-1874) and J. C. Poggendorff (1796-1877). This letter in French can be found at [Jac51].

[374][Note by AKTA:] Heinrich Gustav Magnus (1802-1870).

[375][Note by AKTA:] In German: *Elektromotor.*

must flow in order to be measured that define essential components. If one adds the conductors whose resistance properties are to be determined to those two conductors that are included already in the two instruments, then one will have a complete overview of all tools that are necessary for making a resistance comparison. With that overview:

1. The *electromotor*,
2. The *galvanometer*, and
3. The *conductor* and *its* combinations

that were used in the following experiment will be considered in particular.

13.2   The Electromotor

The choice of electromotor mainly came down to the decision as to whether one would be working with *continuous* or *instantaneous* currents. In the *first* case, one sees the advantages of the so-called constant cells, as Daniell, Grove, and Bunsen had employed for the purpose of such measurements.[376] By contrast, in the *second* case, there is greater advantage in working with *induction permanent magnets*, because when one utilizes instantaneous currents, it is neither the intensity of those currents nor their duration that one deals with, but the value of the product of the two, which one can call the *integral value of the current intensity*. However, only by the method of induction by permanent magnets that integral value can be represented by quantities that are always equal.

In the following experiments, preference is given to the instantaneous currents, and as a result, magnetic induction, on the following two grounds: *First of all*, for fine measurements, the utilization of metallic conductors (e.g., the use of nothing but copper wire, without needing to include a wet conductor, such as water, acids, or a salt solution, in the circuit) allows one greater certainty. It is known that polarization phenomena can perturb the measurements at the surface of a metal that is submerged in a wet conductor. One can avoid such perturbations by utilizing closed wire circuits in which one induces currents by moving them with respect to permanent magnets. Each repetition of such a motion produces a current with the same integral value no matter how short its duration might be. *Secondly*, with the utilization of continuous currents, which might be obtained with constant cells, the temperature of the conductor whose resistance properties are to be determined will increase, and that increase will be different in the various conductors. However, the resistance of the conductor will grow with temperature, and that variability of the resistance would make the determination of the resistance properties uncertain, which could be avoided by utilizing instantaneous currents that are of such short duration that no noticeable change of temperature can occur.

The appropriate equipment and the use of *magnetic inductors* for measurement, in general, was discussed already on another occasion. For that, see "*Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1838*," p. 86.[377,378] One finds the special equipment that was employed with the inductor that was used in the following experiments described more precisely at the end of this treatise in *Supplement* 13.39 and illustrated in Figure 1.[379]

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[376][Note by AKTA:] See footnotes 98, 108 and 179 on pages 53, 61 and 124.

[377][Note by HW:] Wilhelm Weber's *Werke*, Vol. II, p. 105.

[378][Note by AKTA:] [Web39b].

[379][Note by AKTA:] Weber is referring here to the first Figure of Supplement 13.39, see page 399. When Weber's paper was reprinted in his collected works, this Figure was renumbered as Figure 10.

13.3 The Galvanometer

To measure the intensity of a *continuous* current, one can work with the so-called sine galvanometer, as well as the tangent galvanometer.[380] However, in order to measure the intensity of an induced *instantaneous* current (i.e., the strength of a so-called induction pulse), one can use only the tangent galvanometer, because the use of the sine galvanometer assumes the persistence of the needle in its deflected position, which is not the case for an induction pulse because the needle will be merely set into oscillation by an induction pulse that it experiences in its rest position and will not take on a residual deflection in that way. It is most precise and convenient to observe the elongations of the needle oscillations that are caused by the induction pulse with a *magnetometer* that is provided with a multiplier, as Gauss instructed one to do in his "*Resultaten aus den Beobachtungen des magnetischen Vereins im Jahre 1837.*"[381] One should only observe that a large multiplier with a *large conduction resistance*, which is what larger magnetometers are ordinarily provided with, would be disadvantageous for the present measurements. Therefore, a magnetometer of very small dimensions was used for the following experiments, whose needle was only 100 millimeters long, and which was provided with a small multiplier of moderate resistance.

Performing the observations, especially since they should be repeated frequently and in rapid succession, was eased considerably by equipping the magnetometer with a strong damper in addition to the multiplier, and that damper returned the needle to rest after a small number of oscillations after it had been excited. Since the effectiveness of the damper was based mainly upon the magnetic force on the oscillating needle, one would need a magnetometer that is equipped with a very strongly-magnetized needle in order to do that. However, at the same time, it is necessary that the period of oscillation of the needle does not amount to less than 10 to 12 seconds if one is to perform the observations with any precision. That goal can also be achieved with a strong magnetization of the needle in such a way that one gives the needle a large thickness relative to its short length, e.g., 15 millimeters for a length of 100 millimeters. One finds a more precise description of the galvanometer that was used here at the end of the treatise in *Supplement* 13.39, and an illustration is also given in Figures 2, 3, 4.[382]

13.4 Combinations of Four Conductors

The four conductors are the *inductor wire*, the *multiplier wire*, the wire of the *original resistance unit*,[383] and the wire of the *copy*. Of those four conductors, the first two are necessary for all experiments and define the circuit, either alone or together with one or two of the other wires, which can be found in the following *combinations*:

1. The ends of the inductor and multiplier wires are connected to each other directly, and those two wires alone define the circuit.

2. The previous circuit is broken at one place and the wire of the original resistance unit is inserted there.

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[380][Note by AKTA:] See footnote 92 on page 48.

[381][Note by AKTA:] C. F. Gauss (1777-1855). See [Gau38b] with English translation in [Gau41c]. See also [Web38a] and [Web39a] with English translations in [Web41c], [Web66a], [Web41d] and [Web66b].

[382][Note by AKTA:] Weber is referring here to Figures 2, 3 and 4 of Supplement 13.39. When Weber's paper was reprinted in his collected works, these Figures were renumbered as Figures 11, 13 and 12, respectively.

[383][Note by AKTA:] In German: *des Original-Widerstandsmaasses.*

3. The wire of the copy is inserted instead of the wire of the original resistance unit.

4. The wires of the original resistance unit and the copy are put *together* and inserted in the circuit one after the other.[384]

5. The wire of the original resistance unit and the copy are set *next to each other* and inserted into the circuit while connected to each other at their beginnings and ends.[385]

6. The ends of the inductor and multiplier wires are connected to each other directly, but they do not define the circuit by themselves, as in 1., but the wire of the original resistance unit is inserted between their connection points, such that the current that is provided by the inductor wire is split between the latter and the multiplier wire.

7. The wire of the copy is inserted between the two connection points of the inductor and the multiplier, instead of the wire of the original resistance unit.

8. The wires of the original resistance unit and the copy are connected *in succession* and inserted between the two connection points of the inductor and multiplier.

9. The wires of the original resistance unit and the copy are put *next to each other*, connected to each other at their beginnings and ends, and inserted between the two connection points of the inductor and multiplier.

Only the last four of those nine different combinations were employed in the following experiments, because with the first five, the effect was too strong for one to measure the elongation of the needle with the same scale. However, the calculations with the observations will later show that three of those combinations already suffice to determine the resistance properties of the original and the copy, and the fourth one merely serves as a control on the accuracy of the measurement.

13.5   Methods of Observation

The instruments described can be applied to the task of observation by various methods that differ from each other by their precision in some cases, their convenience in others, and by the rules by which the observations are calculated. Instead of simply observing the elongation of the needle once it has been put into motion from a state of rest by an induction pulse, there is greater advantage to performing a system of elongation observations while the needle is given repeated induction pulses at prescribed moments. For those repetitions, one can generally propose the rule that all induction pulses shall take place at only those moments when the oscillating needle passes the position in which it would remain at rest. That is, in fact, the necessary condition for the calculation of the observations to be performed by simple rules.

For the purpose of all finer measurements, in the context of continuous, as well as instantaneous currents, it is important to gain a clear picture of the different methods for arranging the observations and experiments, as well as their calculation, and in particular, when the galvanometer is provided with a damper, as in our case, to understand the rules by which the observations must be calculated when one considers the influence of damping. However, in order to not linger here on a summary of the various methods of observation and their corresponding types of calculation, that will be given at the end of the treatise in *Supplement* 13.39, where the difference between the *multiplication method* and the *throw-back method*[386] will be discussed in more detail, since both of them are permissible when

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[384][Note by AKTA:] That is, they are connected in series.

[385][Note by AKTA:] That is, they are connected in parallel.

[386][Note by AKTA:] In German: *Zurückwerfungsmethode.*

instantaneous currents are applied. The first series of observations that are cited here were performed using the multiplication method.

13.6 Observations

The position of the galvanometer needle when initially at rest was observed on the scale. The *first* positive induction pulse imparted a positive velocity to the needle, and the greatest elongation or the highest position on the scale that the needle attained was observed. The *second* negative induction pulse was produced at the moment when the needle passed its rest position during its return swing, and the lowest position that the needle attained on the scale was observed. The *third*, once more positive, induction pulse was produced at the moment when the needle, which once more swung forward, passed its rest position, and the highest position on the scale was observed again. The observations were continued in that way up to twelve induction pulses, as a rule, and finally when the needle once more came to rest, its position on the scale was noted one more time. Similar series of observations were made several times in succession with different combination of the wires. The various series shall be denoted by A, B, C, D, such that A refers to 6., B, to 7., C, to 9., and D, to 8. in the combinations of wires that were cited above. The following Table gives an overview of the observations that were associated with those series.

	D	C	B	A	B	A	B	C	D
Position	494.8	492.9	493.2	493.7	493.7	493.0	494.3	494.1	494.3
1.	821.8	587.8	672.7	676.3	673.2	675.3	673.1	588.8	821.2
2.	88.3	376.1	271.8	268.8	272.2	268.1	273.1	377.8	88.1
3.	918.0	614.6	723.3	726.7	724.1	726.6	724.4	614.7	916.2
4.	64.5	370.5	260.4	257.0	260.9	257.5	261.9	372.7	64.5
5.	922.9	616.3	725.7	731.1	726.7	729.5	726.7	616.9	923.2
6.	64.0	369.8	259.5	256.1	259.9	257.5	261.7	371.9	62.9
7.	923.3	616.4	726.0	730.7	726.3	730.2	727.2	617.2	922.7
8.	63.4	369.8	259.6	256.2	259.9	257.3	261.6	371.6	62.6
9.	922.5	616.6	726.2	730.8	726.5	730.5	727.2	617.8	923.7
10.	62.9	370.2	259.2	255.7	260.0	257.5	261.9	371.5	61.7
11.	922.9	616.5	724.2	731.1	725.9	730.9	726.9	617.7	923.3
12.	61.9	370.2	262.7	255.7	260.3	257.2	261.7	371.6	62.9
Position	492.7	493.2	493.6	493.7	492.9	494.2	494.1	494.3	493.7

The observations are arranged in that Table in the sequence that they were made in immediate succession during a time interval that did not amount to a whole hour. The repetitions of those series of observations are spaced symmetrically so that the small, time-dependent influences (e.g., the influence of the variation in the directive force[387] of geomagnetism) can be eliminated almost completely by combinations of them.

A direct inspection of the Table above yields the following Table when one:

1. Subtracts the mean value of the observed rest position at the beginning and the end of the series from each number that is read on the scale,

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[387][Note by AKTA:] See footnote 109 on page 70. Weber is referring here to the variation in the magnetic torque exerted by the Earth on the galvanometer needle.

2. Finds the mean value of all corresponding observations in the series that is denoted by A, B, C, or D, and

3. Reduces the deflections that were observed on the scale, which are proportional to the tangent of twice the deflection angle, from the theory of the magnetometer, by that mean value, in such a way that they will be proportional to the deflection angles themselves.

It should be noted that the horizontal distance from the mirror to the scale amounted to 2150 scale divisions, so if x denotes the observed value, then the reduced value will be obtained when one reduces the observed one by $x^3/(13\,867\,500)$.

No.	D	C	B	A
1.	+325.05	+94.64	+178.96	+181.72
2.	−400.87	−116.53	−220.49	−224.38
3.	+417.74	+120.92	+229.42	+232.09
4.	−423.70	−121.87	−231.67	−235.45
5.	+423.45	+122.87	+231.82	+235.69
6.	−424.22	−122.62	−232.36	−235.89
7.	+423.40	+123.07	+231.96	+235.84
8.	−425.13	−122.77	−232.36	−235.94
9.	+423.50	+122.47	+232.09	+236.04
10.	−425.81	−122.62	−232.36	−236.09
11.	+423.50	+123.37	+231.13	+236.39
12.	−425.72	−122.57	−231.17	−236.24

One sees from this Table that the observed elongations of the magnetic needle in the galvanometer were indeed rapidly-increasing to begin with, but soon approached a limiting value as a result of the influence of the *damping* that the galvanometer was equipped with, which increased with the magnitude of the needle's oscillation. In order to reduce all individual measurements to that limiting value, the logarithmic decrement of the decrease in the oscillation arc was determined, for which special experiments were performed immediately before and after the series of observations above. The logarithmic decrement yields a mean of

$$0.633\,95$$

in those experiments, or the ratio of two successive elongations of the needle was:

$$1 : 0.2323 \ .$$

Since the deviations between those mean values was not large for the individual series, it sufficed to use those mean values in the calculations, instead of the true values. Thus, the first observation was reduced to the limiting value by increasing it by the ratio:

$$0.7677 : 1 \ ,$$

and the nth observation was increased by the ratio:

$$(1 - 0.2323^n) : 1 \ .$$

The following Table gives an overview of those reduced values and the means that were obtained for A, B, C, D.

	D	C	B	A
1.	+423.41	+123.28	+233.11	+236.71
2.	−423.73	−123.18	−233.07	−237.18
3.	+423.05	+122.46	+232.33	+235.04
4.	−424.93	−122.22	−232.34	−236.13
5.	+423.73	+122.95	+231.98	+235.85
6.	−424.29	−122.64	−232.40	−235.93
7.	+423.42	+123.07	+231.97	+235.85
8.	−425.13	−122.77	−232.36	−235.94
9.	+423.50	+122.47	+232.09	+236.04
10.	−425.81	−122.62	−232.36	−236.09
11.	+424.50	+123.37	+231.13	+236.39
12.	−425.72	−122.57	−231.17	−236.24
Mean	±424.19	±122.80	±232.19	±236.13

The same series of experiments was performed in the same way three times on three successive days, and the following Table gives an overview of the values of A, B, C, D from all three series of experiments.

	D	C	B	A
I.	424.19	122.80	232.19	236.13
II.	424.80	123.27	232.25	235.93
III.	423.00	122.59	231.38	235.53
Mean	424.00	122.89	321.94	235.86

13.7   Calculating the Observations

The four values that were denoted by A, B, C, D were determined precisely by the observations that were described above, and that further raised the question of how the desired resistance ratio of the original resistance unit a to the copy b could be derived from those four values. For the sake of simplicity, it will be first assumed that the part of the damping that originates in the circuit itself is so small in comparison to the part that is independent of the circuit that it can be neglected, and as a result, the *damping can be assumed to be equal for all observations A, B, C, D*. In that case, one easily convinces oneself that the reduced elongation observation is proportional to the *velocity* that the galvanometer needle possesses at the moment when it passes the rest position as a result of a current from an induction pulse that flows through the multiplier in the galvanometer, and that the velocity itself is proportional to the *integral value* of that current. Thus, the observed elongations can be used as a unit of that current.

However, the current that flows through the *multiplier* of the galvanometer and is *measured* by it was not the total current that was produced by an induction pulse in the *inductor* in the experiments above, but only a fraction of it, and *from the law of current distribution*, it will be expressed by the ratio of the resistance of the inserted wire to the sum of the resistances of the inserted wire and the multiplier wire. If m denotes the resistance of the *multiplier wire*, a, the resistance of the *basic unit*, and b, the resistance of the *copy*, then the resistance of the inserted wires will be equal to:

a for the observation A,

$$b \text{ for the observation } B,$$
$$ab/(a+b) \text{ for the observation } C,$$
$$a+b \text{ for the observation } D,$$

and as a result, the corresponding ratios were:

$$\frac{a}{a+m} \quad \text{for} \quad A \,,$$

$$\frac{b}{b+m} \quad \text{for} \quad B \,,$$

$$\frac{ab}{ab+am+bm} \quad \text{for} \quad C \,,$$

$$\frac{a+b}{a+b+m} \quad \text{for} \quad D \,.$$

However, from Ohm's law, the *total* current will be represented by a fraction whose numerator K is the same for all experiments and depends upon the electromotive force that corresponds to an induction pulse, while the denominator is given by the resistance of the circuit through which the current goes. If one denotes the resistance of the *inductor wire* by r then that will yield the resistance of the entire circuit as equal to:

$$r + \frac{am}{a+m} \quad \text{for the observation} \quad A \,,$$

$$r + \frac{bm}{b+m} \quad \text{for the observation} \quad B \,,$$

$$r + \frac{abm}{ab+am+bm} \quad \text{for the observation} \quad C \,,$$

$$r + \frac{(a+b)m}{a+b+m} \quad \text{for the observation} \quad D \,.$$

One then gets the following equations for the current intensities that are observed with the galvanometers, which shall be denoted by A, B, C, D:

$$A = \frac{a}{a+m} \cdot \frac{K}{r + \frac{am}{a+m}} = \frac{aK}{am + ar + mr} \,,$$

$$B = \frac{b}{b+m} \cdot \frac{K}{r + \frac{bm}{b+m}} = \frac{bK}{bm + br + mr} \,,$$

$$C = \frac{ab}{ab+am+bm} \cdot \frac{K}{r + \frac{abm}{ab+am+bm}} = \frac{abK}{ab(m+r) + (a+b)mr} \,,$$

$$D = \frac{a+b}{a+b+m} \cdot \frac{K}{r + \frac{(a+b)m}{a+b+m}} = \frac{(a+b)K}{(a+b)(m+r) + mr} \,.$$

If one sets:

$$\frac{1}{mr} \cdot K = \alpha \ , \qquad\qquad \frac{1}{m} + \frac{1}{r} = \beta \ ,$$

in them, for the sake of brevity, then that will yield:

$$A\left(\beta + \frac{1}{a}\right) = B\left(\beta + \frac{1}{b}\right) = C\left(\beta + \frac{1}{a} + \frac{1}{b}\right) = D\left(\beta + \frac{1}{a+b}\right) = \alpha \ ,$$

and therefore:

$$\frac{\frac{1}{b}B - \frac{1}{a}A}{A - B} = \frac{\left(\frac{1}{a} + \frac{1}{b}\right)C - \frac{1}{a}A}{A - C} = \frac{\frac{1}{a+b}D - \frac{1}{a}A}{A - D} = \beta \ ,$$

from which one will get the following two equations for determining the desired resistance ratio of the copy to the basic unit $b : a$:

$$
\begin{aligned}
(a - b)AB - aAC + bBC &= 0 \ , \\
(a^2 - b^2)AB + b^2 AD - a^2 BD &= 0 \ ,
\end{aligned}
$$

or:

$$
\begin{aligned}
\frac{b}{a} &= \frac{AB - AC}{AB - BC} \ , \\
\frac{b^2}{a^2} &= \frac{AB - BD}{AB - AD} \ .
\end{aligned}
$$

According to Ohm's law, the following relation will exist between the four observations A, B, C, D:

$$\frac{A^3}{B^3} = \left(\frac{A - C}{B - C}\right)^2 \cdot \frac{A - D}{B - D} \ ,$$

which is obtained by eliminating a and b from the foregoing equations.

With the given development, the formulas that were presented here are initially valid for only the cases in which the observations A, B, C, D give the induced currents and the currents that flow through the multiplier in terms of the same unit, i.e., where the damping of the galvanometer needle is not noticeably different for the various observations. However, those formulas require a special test in order to be also be able to apply them to the remaining cases in which *the damping varies*, because, as one easily sees, the observed elongations A, B, C, D are then, in fact, likewise proportional to the current strengths, but the strength of the damping is inversely proportional to them, in addition.

Now, the damping consists of a part that is *constant* for all observations that originates in the unvarying annular damper that the galvanometer is equipped with, and might be set equal to 1, and a *variable* part that depends upon the way that the multiplier is closed, which is inversely proportional to the resistance of the circuit that starts from the multiplier and returns to it. However, the resistance of that circuit is equal to:

$$m + \frac{ar}{a+r} \qquad \text{for} \qquad A \text{ ,}$$

$$m + \frac{br}{b+r} \qquad \text{for} \qquad B \text{ ,}$$

$$m + \frac{abr}{ab + ar + br} \qquad \text{for} \qquad C \text{ ,}$$

$$m + \frac{(a+b)r}{a+b+r} \qquad \text{for} \qquad D \text{ .}$$

As a result, if one sets $1/m + 1/r = \beta$ and γ denotes a constant factor, then the variable part of the damping can be represented by

$$\gamma \cdot \frac{\frac{1}{r} + \frac{1}{a}}{\beta + \frac{1}{a}} \qquad \text{for} \qquad A \text{ ,}$$

$$\gamma \cdot \frac{\frac{1}{r} + \frac{1}{b}}{\beta + \frac{1}{b}} \qquad \text{for} \qquad B \text{ ,}$$

$$\gamma \cdot \frac{\frac{1}{r} + \frac{1}{a} + \frac{1}{b}}{\beta + \frac{1}{a} + \frac{1}{b}} \qquad \text{for} \qquad C \text{ ,}$$

$$\gamma \cdot \frac{\frac{1}{r} + \frac{1}{a+b}}{\beta + \frac{1}{a+b}} \qquad \text{for} \qquad D \text{ .}$$

Now, for the cases in which that variable part of the damping can not be neglected in comparison to the constant equals to 1, then A, B, C, D must be replaced with their products with the associated value of the damping in the formulas that were developed above, i.e.:

$$A \quad \text{is replaced with} \quad A\left(1 + \gamma \cdot \frac{\frac{1}{r} + \frac{1}{a}}{\beta + \frac{1}{a}}\right) \text{ ,}$$

$$B \quad \text{is replaced with} \quad B\left(1 + \gamma \cdot \frac{\frac{1}{r} + \frac{1}{b}}{\beta + \frac{1}{b}}\right) \text{ ,}$$

$$C \quad \text{is replaced with} \quad C\left(1 + \gamma \cdot \frac{\frac{1}{r} + \frac{1}{a} + \frac{1}{b}}{\beta + \frac{1}{a} + \frac{1}{b}}\right) \text{ ,}$$

$$D \quad \text{is replaced with} \quad D\left(1 + \gamma \cdot \frac{\frac{1}{r} + \frac{1}{a+b}}{\beta + \frac{1}{a+b}}\right) \text{ .}$$

However, with those substitutions, one gets:

$$A\left[\beta + \frac{1}{a} + \gamma\left(\frac{1}{r} + \frac{1}{a}\right)\right] = B\left[\beta + \frac{1}{b} + \gamma\left(\frac{1}{r} + \frac{1}{b}\right)\right]$$

$$= C\left[\beta + \frac{1}{a} + \frac{1}{b} + \gamma\left(\frac{1}{r} + \frac{1}{a} + \frac{1}{b}\right)\right] = D\left[\beta + \frac{1}{a+b} + \gamma\left(\frac{1}{r} + \frac{1}{a+b}\right)\right] = \alpha \text{ ,}$$

and from this:

$$\frac{A\left(\beta+\frac{1}{a}\right)-B\left(\beta+\frac{1}{b}\right)}{B\left(\frac{1}{r}+\frac{1}{b}\right)-A\left(\frac{1}{r}+\frac{1}{a}\right)}=\frac{A\left(\beta+\frac{1}{a}\right)-C\left(\beta+\frac{1}{a}+\frac{1}{b}\right)}{C\left(\frac{1}{r}+\frac{1}{a}+\frac{1}{b}\right)-A\left(\frac{1}{r}+\frac{1}{a}\right)}$$

$$=\frac{A\left(\beta+\frac{1}{a}\right)-D\left(\beta+\frac{1}{a+b}\right)}{D\left(\frac{1}{r}+\frac{1}{a+b}\right)-A\left(\frac{1}{r}+\frac{1}{a}\right)}=\gamma\ ,$$

from which it follows that:

$$AB\left(\beta-\frac{1}{r}\right)\left(\frac{1}{a}-\frac{1}{b}\right)+AC\left(\beta-\frac{1}{r}\right)\frac{1}{b}-BC\left(\beta-\frac{1}{r}\right)\frac{1}{a}=0\ ,$$

$$AB\left(\beta-\frac{1}{r}\right)\left(\frac{1}{a}-\frac{1}{b}\right)+AD\left(\beta-\frac{1}{r}\right)\left(\frac{1}{a+b}-\frac{1}{a}\right)$$

$$-BD\left(\beta-\frac{1}{r}\right)\left(\frac{1}{a+b}-\frac{1}{b}\right)=0\ ,$$

or when one drops the common factor of $(\beta-1/r)$, one will get the same equations as before, namely:

$$\frac{b}{a}=\frac{AB-AC}{AB-BC}\ ,$$

$$\frac{b^2}{a^2}=\frac{AB-BD}{AB-AD}\ .$$

Finally, when one applies the rules that were found to the values of A, B, C, D that were given by the experiments that were described above, namely:

$$A=235.86\ ,$$
$$B=231.94\ ,$$
$$C=122.89\ ,$$
$$D=424.00\ ,$$

that will give, first of all:

$$\frac{A^3}{B^3}=1.051\,56\ ,$$

$$\left(\frac{A-C}{B-C}\right)^2\cdot\frac{A-D}{B-D}=1.051\,28\ .$$

The close agreement between those two values, which should be equal according to the rules above, can serve as a confirmation of Ohm's law, from which those rules were derived.

Furthermore, that gives the ratio of the resistance of the copy b to the basic unit a, and indeed, from the observed values of A, B, C:

$$\frac{b}{a} = \frac{AB - AC}{AB - BC} = 0.981\,616 \;,$$

and from the observed values of A, B, D:

$$\frac{b}{a} = \sqrt{\frac{AB - BD}{AB - AD}} = 0.981\,485 \;;$$

so the mean of those numbers is the resistance of the copy, expressed in units of the resistance of the given basic unit, which equals:

$$0.981\,55 \;.$$

One can also find the ratio of the resistance of other conductors to the basic unit in the same way that the ratio of the resistance of the copy to that of the basic unit was determined, and in that way the resistances of all of those conductors can be measured in terms of the given basic unit.

The arrangement of the observations in the example that was given here was consistent with the multiplication method. However, it was mentioned before that this arrangement could also be consistent in yet another way, namely, the throwback method, and the latter way of organizing things even possesses an advantage over the former. Therefore, the second method deserves to be discussed in more detail, which shall be done in *Supplement* 13.39 at the end of this treatise, where an example of measuring by that method will be added.

II - Converting the Resistance Measurements to Absolute Units

13.8   On the Meaning of an Absolute Resistance Unit

From what was shown in the first Section, namely, how the resistance of a conductor can be determined with the required sharpness with a given *basic unit*, in this second Section, those measurements will be converted to an *absolute unit*.

One can believe that such a conversion can be implemented in the simplest way by reverting to the spatial dimensions (length and cross-section) of the conductor, and in that way employ the metal that is most suitable to conductors and most frequently used, namely, copper. In fact, in that way, one would succeed in determining the resistance of the conductors that can be referred to with the name of absolute, but that would not, in fact, correspond to the true objective, namely, reducing the number of arbitrarily-assumed basic units. In that way, a basic unit for a specific resistance (namely, that of copper) would replace a basic unit of absolute resistance. However, for the stated objective, it is irrelevant whether one uses a unit of absolute resistance as a basis and derives the unit of specific resistance from it or conversely uses a unit of specific resistance as a basis and derives the unit of absolute resistance from it. Absolute resistance measurements have an intrinsic meaning only when they are performed in such a way that absolutely no new units, such as space and time, are used as a basis other than the ones that are present and have already been used and are indispensable for other purposes.

Thus, one can easily judge what Jacobi said on p. 199 and the following[388] in the reference that was cited above on the occasion of his proposal in regard to a fixed unit of resistance: In order to express the conductive resistance that the physicists measure in a common unit, no absolute method of determination can exist, because it seems that differences in resistance can exist between even the most chemically pure metals that cannot be explained by a difference in dimension alone, and therefore when one physicist refers his resistance meter and multipliers to copper wire of 1 meter in length and 1 millimeter in thickness, another physicist still cannot convince himself that his copper wire and the other one possess the same coefficient of resistance (i.e., whether the copper has the same specific resistance for the two wires). One sees that here Jacobi had in mind only a determination for which the unit of absolute resistance can be derived from a basic unit that is assumed for the specific resistance, which he had every right to reject. However, Jacobi did not even touch upon the question of whether a new basic unit was even necessary or whether resistance determinations might be possible without assuming anything about the two basic units. Nonetheless, that question is precisely the one whose resolution we would prefer to address. Moreover, while that answer implies that no new basic unit is, in fact, even necessary for the purpose of resistance measurements, it does not by any means follow that establishing such a basic unit of the kind that Jacobi proposed, and as it was used in the first part of this treatise, is entirely superfluous. Rather, it will be shown that assuming Jacobi's proposal will also remain the most desirable on practical grounds, because a direct absolute determination of resistance can be performed precisely only in isolated cases and under especially favorable conditions by assuming Jacobi's proposal, but that will build a bridge by which one will

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[388][Note by AKTA:] Page 199 of the original article, which corresponds to page 303 of Volume 3 of Weber's *Werke* and page 292 of this translation.

succeed in converting all other resistance measurement to an absolute unit with the help of a single absolute resistance determination that one can actually perform. Now, the fact that an absolute resistance determination is possible in a way that is completely different from the one that Jacobi spoke of and which is completely independent of the specific resistance or the resistance coefficient of any body, such as copper, namely, by a particular combination of magneto-electric and electromagnetic observations, was already expressed by Gauss soon after Faraday's discovery of magneto-electricity was made known.[389]

The essence of that method can be expressed briefly in words in the following way: If one considers the intensity of any galvanic current, then that will explain the fact that it can be determined in two essentially-different ways, in general: *First of all*, from the causes upon which it depends. *Secondly*, from the effects that it produces. However, as one can easily show, the current intensity that is defined by its effects can now be converted into *absolute units*, and since that explains the fact that the value of current intensity in absolute units must be the same, it might be defined by either its effects or its causes, so the result that must be obtained in the latter way will already be known in advance by way of the one that was known by the former way. Nevertheless, one knows that the current intensity depends upon only two causes, namely, the electromotive force and the resistance of the circuit, and that of the two, the electromotive force can be converted to an absolute unit. Now, just as the absolute value of the current intensity will be given immediately when the resistance is given in absolute units, along with the electromotive force, the converse is also true, since when the electromotive force, as well as the current intensity is given in absolute units, the value of the resistance in absolute units will be given, and one then sees that the resistance measurement can be performed without one having to use any new arbitrary basic unit for it, which was to be proved.

In general, that also explains the possibility that there can be an absolute unit of resistance in the given, narrower, sense of the word, but it will still be necessary to give a precise definition of that unit if an actual measurement is to be performed with that unit. However, one finds a complication in such a definition in the fact that it assumes that other absolute units are known, namely, the absolute unit for electromotive force and the absolute unit for current intensity (as determined from its effects). One would then be dealing with the foundation of an absolute unit of resistance on the basis of establishing a complete system of absolute units for all electrodynamics. If one goes even further back, then one will find that the latter units also assume yet other ones that lie beyond the scope of electrodynamics, and that the intended foundation of the resistance unit would necessitate a more detailed discussion of the absolute units of several different types of quantities that must precede the performance of our measurement.

13.9   On the Absolute Unit of Several Different Types of Quantities

It is known that physical research can be greatly simplified when one introduces no more independent specialized basic units for the different types of quantities than are unavoidably necessary and derives all other units from that minimal set of basic units. On those grounds, merely *distances*, *time intervals*, and *masses* are presented as basic units in mechanics, and the units of all other types of quantities that are considered in mechanics are derived from

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[389][Note by AKTA:] See footnote 24 on page 20.

the minimal set of basic units, which are then called *absolute units*. For example, no basic unit of velocity and density are defined, but the absolute units that are utilized for them can be reduced to those three basic units. Similarly, the units for moving and absolute forces, moment of rotation, moment of inertia, efficiency, etc., can be reduced to those three basic units using known laws. On the same basis, no further independent specialized basic unit is introduced for magnetism either, but one refers it to the absolute unit that Gauss had derived for magnetism in terms of the three basic units of mechanics in his treatise: *Intensitas vis magneticae terresris ad mensuram absolutam revocata. Gottingae 1833.*[390,391]

The unit for bar magnetism[392] is then, in fact, the magnetism of a bar that exerts a moment of rotation that has a ratio of $1 : R^3$ with the absolute unit of moment of rotation (when it acts at great distance R upon another equally-strong magnetic bar whose magnetic axis is parallel to the line that connects the midpoints of the two magnets, while its own magnetic axis is, by contrast, perpendicular to it).

The unit for the strength of geomagnetism (viz., the strength of the geomagnetic force) at any location is, accordingly, just the moment of rotation, expressed in absolute units, that geomagnetism exerts upon a bar magnet that is found at that location when the latter contains one absolute unit of magnetism and its magnetic axis makes a right angle with the direction of geomagnetism at that location.

13.10   Definition of the Absolute Units in Electrodynamics

The absolute units for the types of quantities that are considered in electrodynamics can now be defined completely and concisely in the following way by reducing them to the magnetic unit.

13.10.1   The Unit of Current Intensity

The unit of current intensity is the intensity of the current that exerts the same action at a distance as a bar magnet that contains the previously-defined unit of magnetism when it flows around a plane of unit area.

That definition of the unit of current intensity is the same as the one that was given in "*Resultaten aus den Beobachtungen des magnetischen Vereins im Jahre 1840*," p. 86.[393,394]

13.10.2   The Unit of Electromotive Force

The unit of electromotive force is the electromotive force that the previously-defined unit of geomagnetism would exert upon a closed circuit when the latter is rotated in such a way that the area that is bounded by its projection onto the plane that is perpendicular to the direction of geomagnetism would increase or decrease by a unit area during a unit time.

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[390][Note by HW:] Carl Friedrich Gauss's *Werke*, Vol. V, p. 79.

[391][Note by AKTA:] See footnote 97 on page 51.

[392][Note by AKTA:] That is, a magnetized bar with a magnetic moment = 1.

[393][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 9.

[394][Note by AKTA:] [Web41b, p. 86 of the *Resultaten* and p. 9 of Weber's *Werke*] with English translation in [Web20b].

13.10.3 The Unit of Resistance

The unit of resistance is the resistance of a closed circuit in which the previously-defined unit of electromotive force would produce the previously-defined unit of current intensity.

If one denotes the unit of current intensity that was defined above by I and denotes any current intensity that is measured with it by iI, in which i denotes a real number, and one further denotes the unit of electromotive force that was defined above by E, and any electromotive force that is measured with it by eE, in which e denotes a real number, then wW will be the resistance of a circuit upon which the electromotive force eE acts and in which a current of intensity iI is produced when W denotes the unit of resistance that was defined above and $w = e/i$ is a pure number. The resistance of that circuit will then be equal to the unit of resistance when one finds that $e = i$. One sees from this that a conductor that possesses the unit of resistance that was defined above can actually be realized.

13.11 Schema for Determining the Absolute Resistance of a Conductor

The following example will serve to explain how the electrodynamic unit that was defined before can be applied to the determination of the absolute resistance of a conductor:
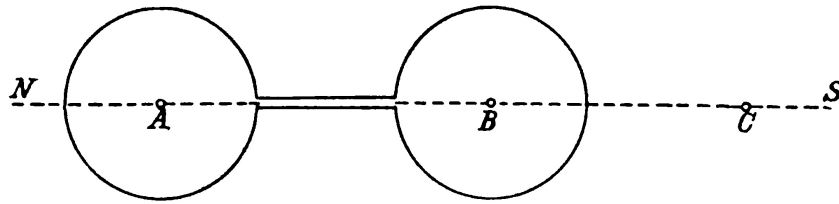
Fig. 1.

The line NS [in Figure 1] denotes the direction of geomagnetism, whose strength at the two locations A and B shall be equal to T with the unit defined above. The value of T is known to be found from magnetometric observations according to the guidance that Gauss gave in *Intensitas vis magneticae terrestris ad mensuram absolutam revocata*.[395] Now, a closed circuit consists of two circles whose centers are A and B. The line NS lies in the plane of those circles. However, two more wires belong to that circuit that lie close to each other and exhibit a double connection between the two circles. Finally, let each circle be cut between the two points where the two wires connect with it such that all parts together will define a curve that closes back on itself, as the Figure shows. r denotes the radius of both circles, which shall be assumed to be equal for the sake of simplicity. If one projects the circle A in the direction NS onto a plane that is perpendicular to NS, then the area that is bounded by the projection will be equal to zero. However, the flexibility of the wires that connect the two circles might allow the circle A to rotate and be located perpendicular to NS, at which point, the area of the surface that is bounded by the same projection will be equal to πr^2. That rotation takes place in a short time interval τ in such a way that the area that is bounded by the projection of the circle will grow uniformly from 0 to πr^2 during that time. From laws of *magneto-electrics*, that will then yield the electromotive force eE that

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[395][Note by AKTA:] See footnote 97 on page 51.

geomagnetism T exerts upon the circular conductor A during the time interval τ, which is determined in terms of the previously-defined unit E by the number:

$$e = \frac{\pi r^2}{\tau} \cdot T \ .$$

A current that flows through the entire closed circuit will be produced by that electromotive force during the time interval τ whose intensity shall be denoted by iI. That current also goes through the circle B and acts from that circle on a distant magnetic needle at C whose rotational axis is perpendicular to NS and lies in the plane of the circle B. Now, if I is the previously-defined unit of current intensity then, from the laws of *electromagnetism*, that will imply that the moment of rotation that the current that flows through the circle B exerts upon the needle is equal to the moment of rotation exerted by a bar magnet that is placed at the center of the circle B in such a way that its magnetic axis would be perpendicular to the plane of the circle, when the magnetism M of that bar is:

$$M = \pi r^2 i \ ,$$

as measured in the unit defined above.[396] Now, when the magnetism of the needle C, as measured in the same unit, is further equal to m, and $BC = R$, and φ denotes the angle that the magnetic axis of the needle C makes with the direction NS of geomagnetism, then from the known laws of magnetism, the moment of rotation that the bar magnet M exerts upon the needle m will be expressed by:

$$\frac{Mm}{R^3} \cos \varphi = \frac{\pi r^2}{R^3} \cdot im \cdot \cos \varphi \ .$$

That implies that if K denotes the moment of inertia of the needle, then the acceleration of the rotation will be equal to:

$$\frac{\pi r^2}{R^3} \cdot \frac{im}{K} \cdot \cos \varphi \ ,$$

and as a result, if the needle were previously at rest when $\varphi = 0$, then the angular velocity $d\varphi/dt$ at the end of the short time interval τ would be:

$$\frac{d\varphi}{dt} = \frac{\pi r^2}{R^3} \cdot \frac{im}{K} \cdot \tau \ .$$

Finally, one finds the greatest elongation α of the needle that is set into oscillation in that way from that velocity using the known laws of oscillation by multiplying is by the period of oscillation t[397] and dividing by the number π, namely:

$$\alpha = \frac{r^2}{R^3} \cdot \frac{im}{K} \cdot \tau t \ .$$

As is known, one has the following equation for the period of oscillation t:

$$mT = \frac{\pi^2 K}{t^2} \ ,$$

from which:

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[396][Note by AKTA:] The magnetism M of the bar is then its magnetic moment.
[397][Note by AKTA:] In German: *Schwingungsdauer*. See footnote 96 on page 51.

$$\frac{mt}{K} = \frac{\pi^2}{tT} \ ,$$

and as a result:

$$\alpha = \frac{\pi^2 r^2}{R^3} \cdot \frac{i\tau}{tT} \ ,$$

or

$$i = \frac{\alpha R^3}{\pi^2 r^2} \cdot \frac{t}{\tau} \cdot T \ .$$

Now, one can further calculate the action of the circular current A on the needle when one observes that the same current that flows through the circle B also flows through the circle A. Meanwhile, for the sake of simplicity, it might be assumed here that the distance AC is large enough that this action will vanish in comparison to the action of the circular current B. The observation of the actual width of the elongation will then give the value of α directly.

That will then imply that the electromotive force eE that was given above and determined in the previously-defined unit, for which it was found that:

$$e = \frac{\pi r^2}{\tau} \cdot T \ ,$$

will produce a current in the total circuit whose intensity in the previously-defined unit will be determined by iI, when:

$$i = \frac{\alpha R^3}{\pi^2 r^2} \cdot \frac{t}{\tau} \cdot T \ .$$

Finally, the resistance of the total circuit in the previously-defined unit will be determined from wW, when:

$$w = \frac{e}{i} = \frac{\pi^3 r^4}{\alpha R^3 t} \ .$$

Measuring the absolute resistance of the total circuit is then reduced to measuring the quantities:

$$r, \qquad R, \qquad \alpha, \qquad t \ ,$$

or in other words, the resistance of the total circuit can then be expressed in the previously-defined unit when one has found from the observations *first of all* the number α, which gives the width of elongation of the needle in parts of the radius, then *secondly* the number r/R, which gives the radius of the two circles in parts of the distance BC, then *thirdly* the velocity r/t with which the radius of that circle would move during an oscillation of the needle. It will then follow from this that a *unit of velocity* is the only dimensional unit upon which the absolute measurement of resistance rests.

With that overview of all of the observations that are required for determining the absolute resistance, we shall now go on to discuss how one performs those observations.

13.12   On Performing the Observations

Most of the observations that must be made for determining the absolute resistance of the total circuit according to the foregoing presentation can now be actually performed with no difficulty and with great precision, because the observations that are required to determine the period of oscillation of the needle admit a sharpness that is known to leave nothing to be desired. The same thing is true of measuring the radius of the circle and the distance $BC = R$. All that remains then is to observe the width of the elongation α of the oscillating needle. As is known, that can also be determined precisely up to an arc-second with the equipment that pertains to the magnetometer and will then likewise leave nothing to be desired when, e.g., the value of α is not less than $1°$. However, when this schema is followed exactly, that value will generally be much smaller and would not be realized, even with the best means of observations. The main problem for the practical measurement of the resistance of a circuit in *absolute* units then consists of modifying the equipment that was described in such a way that the observed elongation α will become as big as possible.

Such a modification consists, *first of all*, of relocating the magnetic needle C from a large distance to the center of the circle B, where the elongation will be increased by a well-defined ratio according to the laws of electromagnetism. In so doing, one must only be careful that the length of the needle is much smaller than the radius of the circle in order to not need to include the particular distribution of magnetism in the needle in the calculation, because a more detailed study of that distribution would bring some complications with it.

A *second* modification by which an increase in the elongation α can be achieved, consists of multiplying the winding number of both circles, which will convert them into rings that possess a significant cross-section. However, the influence of each individual winding must then be included in the calculation because they have different radii, and not all of them lie in the plane of the needle.

With those two essential modifications, one will arrive at an enlargement of the elongation that will also make it possible to perform that observation with the sharpness that was exhibited in the experiment to be described.

Before we move on to describe the experiment itself, a remark might be made in regard to another modification of the equipment to which one will arrive when one applies the exchange of an action at a distance with an action at the center that was already given for the circle B to the circle A, as well. In that way, the electromotive force that geomagnetism exerts upon the circle A at a distance would be replaced by the electromotive force of a magnet that is placed at the center of the circle A. From the laws of magneto-electricity, that will then make the effect when the magnet is at rest and the circle rotates forwards the same as when the circle is at rest and the magnet rotates backwards. One can then suspend a magnetic needle at the center of the circle at rest and let it oscillate, and that oscillating needle will then exert an electromotive force on the circle, and in that way, the circle and the magnetic needle can keep the same location for A that the circle and the magnetic needle do for B.

Finally, when both circles and their needles take the same form and arrangement, nothing at all stands in the way of completely combining them, because, in fact, *from the damping principle*, the magneto-electric action of the needle on the circle and the electromagnetic action of the circle on the needle can coexist without mutual perturbations of those needles and circles. One then needs a single needle that is set into oscillation, and in that way it will exert an electromotive force on a closed circle in whose center the needle is found

according to the laws of *magneto-electricity*, and that electromotive force will produce a galvanic current in that circle that acts back upon the needle that excited the electromotive force according to *electromagnetic* laws and which will then produce a *damping* or decrease in the oscillation arc of the oscillating needle. With that simplification, it is enough to observe the *arc of oscillation*, from whose *magnitude* the magnitude of the *electromotive force* can be determined and from whose *decrease* the strength of the *induced current* can be determined. The second and third series of experiments will give examples of how the resistance of a circuit can also be measured in *absolute* units with this method.

We shall now move on to describe the experiments that were performed according the methods that were explained, and we will first summarize the experiment that was carried out using the *first* method.

13.13   First Method

The following instruments were devised for performing the experiments using the *first* method:

1. The *Earth inductor*,[398] or a wire loop in which a galvanic current was generated by rotating it in the magnetism of the Earth.

2. A *multiplier*, whose wire-ends were connected to those of the Earth inductor.

3. A small *magnetometer*, whose needle was suspended at the center of the multiplier.

The following remarks should be made about those instruments:

13.13.1   The Earth Inductor

The copper wire that was employed in the Earth inductor, including the wool that was wound around it, had a weight of:

16 533 grams,

500 grams of which came from the wool. That wire was wound around a wooden frame that had the approximate form of a regular hexagon. All of the wire windings collectively defined a loop with a rectangular cross-section whose one side, which was perpendicular to the plane of the loop, was 64 millimeters long, while the other was about 16 millimeters long. The length of a tape that was laid around the wooden frame before the wire was wound gave a circumference of 3067 millimeters. The length of a tape that was laid around the wire windings gave a circumference of 3170 millimeters. The wire consisted of seven layers, each of which had 22 to 23 windings, although the seventh, or outer, layer was not complete and had only 10 windings, which gave:

145 windings,

in total. The lengths of the two protruding wire-ends, taken together, amounted to 550 millimeters. When one considers the slight deviation in form from a regular hexagon, that made the sum of the areas of the surface that bounded the projections of those 145 windings onto the plane of the loop equal to:

104 924 000 square millimeters.

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[398][Note by AKTA:] In German: *Erdinduktor*.

After the wire was wound, two strong wooden clamps were fixed at two opposite corners of the hexagon that enclosed the copper loop, each of which was equipped with a strong, round, outward-facing peg[399] around which the loop could be turned when it was inserted along with that peg in the bed of a large wooden frame that was joined to a very solid beam. The rotational axis that was defined by that peg was parallel to the plane of the loop and was vertical. One of the two pegs was hollow, and the two wire-ends were led through it and fixed at the end. Those two wire-ends that were fastened to the rotatable peg were connected by two brass coil springs that terminated on the fixed wooden frame where the connecting wires that connected the inductor to the multiplier were clamped. In that way, any loose connection was avoided that could result in an indeterminacy of the resistance in the circuit, and it likewise allowed for a rotation of the inductor in a semi-circle forwards or backwards while the remaining parts of the circuit remained immobile. A long crank for rotation was attached to the other peg, which was arrested at the end of each rotation by a fixed tooth that was attached to the wooden frame. The placement of that ratchet tooth was regulated in such a way that the rotation of the inductor amounted to exactly two right angles and the vertical plane of the loop was perpendicular to the magnetic meridian at the beginning and end of each rotation.

13.13.2   The Multiplier

The copper wire that was employed for the multiplier, including the wool that was wound around it, had a weight of:

$$157\,430 \text{ grams,}$$

of which, 4540 grams came from the wool. That wire was wound around a wooden spool that was bounded externally by a cylindrical surface whose radius amounted to:

$$303.51 \text{ millimeters.}$$

The wire that was wound lay between two parallel wooden protecting walls that were separated by a distance of 202.05 millimeters. The mean radius of the surface that bounded the outermost layer of wire windings was 374.41 millimeters, so the rectangular cross-section of the loop that was defined by all windings was 202.05 millimeters long and 70.9 millimeters wide. The wire defined 28 successive layers, each of which consisted of 66 to 68 windings. The 28th, or outermost, layer was not complete and had only 44 windings, which gave:

$$1854 \text{ windings,}$$

in total. The last winding was missing 155 millimeters. The length of the two protruding ends amounted to:

$$1340 \text{ millimeters,}$$

in total. The multiplier was placed so that its plane coincided with the magnetic meridian.

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[399][Note by AKTA:] In German: *Zapfen.*

13.13.3   The Small Magnetometer

The needle of the small magnetometer was a tempered and magnetized steel cylinder that was 60 millimeters long and 6.2 millimeters thick, and at its center it was equipped with a brass clip from which it was suspended and which carried a round plane mirror of diameter 30 millimeters whose normal defined a right angle with the magnetic axis. With the given length of the needle, which still did not amount to ten parts of the diameter of the multiplier, the influence of the particular distribution of magnetism no longer came under consideration and therefore did not need to be included in the calculation. The needle was lengthened at both ends by two 31-millimeter-long brass pins that carried two brass balls of diameter 11.7 millimeters. The weight served to increase the moment of inertia of the needle, which gave the period of oscillation a magnitude that was convenient for observation. That needle was suspended by four cocoon threads that were combined into one thread that was fastened to the inner wall of the multiplier in such a way that the center of the needle could lie at the midpoint of the multiplier. Finally, the space that was enclosed by the multiplier, in whose center the magnetic needle floated, was converted into a closed case by two wooden lids that were attached to both sides. In one of those lids, there was a small opening for the mirror on the needle that was sealed with a plane-parallel piece of glass. The reading telescope for the magnetometer was placed in the vertical plane of the normal to the mirror at a distance of about 4 meters, and a scale was fastened to it perpendicular to the normal to the mirror. Its horizontal distance from the mirror amounted to:

$$4087.5 \text{ millimeters,}$$

and the image of the scale could be observed by the telescope that was pointed towards the mirror.

13.14   Observations

Now, the following observations should be made about those instruments: The inductor was placed so that its plane coincided with the magnetic meridian, and the magnetic needle was brought to rest. The inductor was then turned suddenly through 90°. In that way, the needle would be set into oscillation, and the state of the needle in its greatest positive elongation on the scale, which it attained after one-half an oscillation period,[400] would be observed by the telescope. The needle attained its greatest negative elongation after $1\frac{1}{2}$ oscillation periods, which was likewise observed on the scale. The inductor was then turned 180° backwards at the moment when the forward-oscillating needle once more passed through its original rest position, i.e., two oscillation periods after the beginning of the experiment. The oscillating needle was then arrested in the middle of its motion and pushed backwards, at which point its greatest negative, and then its greatest positive, elongation on the scale was observed. After the course of four oscillation periods, at the moment when the needle returned from its last elongation to pass through its original rest position, the inductor was again moved 180° backwards, at which point the same elongation observations were made as the first time, and the experiment was continued in that way until a satisfactory series of observations was obtained. The following Table encompasses four such series of observations. For each series, the elongations that were observed on the scale are recorded successively in the first column.

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[400][Note by AKTA:] See footnote 96 on page 51.

In the second column, the mean value of each two successive positive or negative elongations was added. Finally, in the third column, the difference between the greatest positive and negative elongations or the magnitude of the total oscillation arc is given, and under each series, the mean value is recorded.

First Series			Second Series			Third Series			Fourth Series		
467.1			467.1			463.0			462.0		
540.7			540.5			536.7			534.7		
	543.70			543.65			539.65			538.20	
546.7			546.8			542.6			541.7		
		80.10			79.65			80.40			80.00
461.4			461.3			456.6			455.3		
	463.60			464.00			459.25			458.20	
465.8			466.7			461.9			461.1		
		79.75			79.55			80.35			79.75
540.6			540.8			537.6			535.1		
	543.35			543.55			539.60			537.95	
546.1			546.3			541.6			540.8		
		79.25			79.90			79.55			79.50
462.3			461.8			458.3			456.0		
	464.10			463.65			460.05			458.45	
465.9			465.5			461.8			460.9		
		79.45			80.00			79.70			79.50
541.4			542.1			537.7			535.3		
	543.55			543.65			539.75			537.95	
545.7			545.2			541.8			540.6		
		79.75			79.70			79.95			80.05
462.3			462.8			457.9			456.0		
	463.80			463.95			459.80			457.90	
465.3			465.1			461.7			459.8		
		79.70			79.85			79.85			79.85
542.0			542.3			537.6			536.1		
	543.50			543.80			539.65			537.75	
545.0			545.3			541.7			539.4		
		79.45			80.10			79.70			79.55
462.8			462.7			458.2			456.8		
	464.05			463.70			459.95			458.20	
465.3			464.7			461.7			459.6		
		79.45			79.80			80.10			79.65
542.0			542.3			537.6			536.0		
	543.50			543.50			540.05			537.85	
545.0			544.7			542.5			539.7		
		79.65			79.75			80.05			79.70
462.9			462.8			457.3			456.5		
	463.85			463.75			460.00			458.15	
464.8			464.7			462.7			459.8		
		79.85			79.60			79.50			79.60
542.7			541.9			536.6			535.8		
	543.70			543.35			539.50			537.75	
544.7			544.8			542.4			539.7		
		79.45			79.75			79.75			79.55
463.4			462.3			457.2			456.4		
	464.25			463.60			459.75			458.20	
465.1			464.9			462.3			460.0		
		79.70			79.85						79.55
542.6			541.3						535.7		
	543.95			543.45						537.75	
545.3			545.6						539.8		
		79.75									
462.8											
	464.20										
465.6											
Mean 79.64			Mean 79.79			Mean 79.90			Mean 79.69		

As a result, the mean value of the total oscillation arc over all observations was 79.755 scale divisions, which gave the measurement:

$$79.4 \text{ millimeters.}$$

However, that result must be made 1/2 millimeter larger if it is to be independent of the influence of *the duration of the rotation of the inductor* on it. One then gets:

$$79.9 \text{ millimeters.}[401]$$

To complete the measurement, the oscillation period of the needle was observed, and a period of oscillation equal to:

$$10.2818 \text{ seconds}$$

was found from 300 periods of oscillation.

Furthermore, the ratio of the magnetic directive force[402] to that of the thread was found to be:

$$1770 : 1 \ .$$

Finally, since other magnets that could not be removed were found in the hall in which the instruments were located, as well as in the adjacent rooms, one could not assume that the strength of the horizontal part of geomagnetism that generated the current in the *inductor* would be precisely equal to the strength of the horizontal part of geomagnetism that acted upon the needle at the midpoint of the *multiplier*. Therefore, both of them were compared to each other in such a way that the period of oscillation of one and the same needle at both locations would be observed directly in succession, and that gave these periods of oscillation at the midpoint:

$$2.9095 \text{ seconds for the damper,}$$
$$2.9126 \text{ seconds for the inductor.}$$

The squares of those periods of oscillation are inversely proportional to the strength of geomagnetism at both locations, i.e.:

$$100\,000 : 99\,787 \ .$$

Those were all of the experiments that were necessary for determining the resistance of the total circuit that consisted of the wire of the inductor, the damper, and the two connecting

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[401][Note by WW:] The rotation of the inductor should not be accomplished so rapidly that its duration cannot be neglected in comparison to the period of oscillation of the needle. It must then be performed in two seconds with the greatest possible uniformity. The intensity of the inducted current can then be determined for each moment of the rotation, and will be represented by $i \sin \pi\vartheta/2$, if i denotes the intensity in the middle of the rotation and the time ϑ is measured from the beginning of the rotation. That varying, two-second-long induction can be replaced with a uniform induction that generates a current of intensity i over $4/\pi$ seconds with almost the same effect. That current begins to act upon the needle $2/\pi$ seconds before the needle arrives at the magnetic meridian and then reverses, and after that, it again flows for $2/\pi$ seconds before the current ceases. If α denotes the greatest elongation of the needle and t its period of oscillation, then the deflection of the needle at the moment when the current begins or stops will be expressed approximately by α/t, and the mean deflection over the entire duration of the induction, by $\alpha/3t$. The acceleration of the needle due to its directive force that corresponds to such a deflection is equal to $\alpha/3t \cdot \pi^2/t^2$, and the velocity that it creates during the induction is equal to $4\pi \cdot \pi^2\alpha/3t^2$. One-half of that must be added to the velocity $\pi\alpha/t$ that the needle acquires due to its elongation α when it goes through the meridian in order to get the velocity that the needle would possess at the moment after its reversal when the induction happened *instantaneously*. Now, just as the velocities have the ratio $\pi\alpha/t : (1 + 2/3t^2)\pi\alpha/t$, so do the observed elongation of the needle α and the elongation that would have been found to exist for *instantaneous induction*. The latter then yields $(+2/3t^2)\alpha$. Now, since the total oscillation arc $2\alpha = 79.4$ millimeters, and one had $t = 10.2818$, the value of 79.9 millimeters that was cited above would follow from that.

[402][Note by AKTA:] See footnote 109 on page 70.

wires in *absolute* units using the first method. Before we move on to calculate the magnitude of the resistance of the circuit from those experiments, the experiments that were made using the *second* method shall be summarized and stated in advance.

13.15   Second Method

13.15.1 A

The second method includes the simplification that, of the instruments that were used in the first, the Earth inductor is made completely superfluous. Therefore, in the following experiments, the wire of the multiplier that was described above will define the entire circuit when its ends are connected to each other directly. The placement of the multiplier, which was converted into a *damper* in that way, remained unchanged. By contrast, the needle in the magnetometer was replaced with a larger and stronger one whose oscillations could exert a greater electromotive force on the closed circuit. That needle consisted of nine bar magnets that took the form of parallelepipeds, each of which was 90 millimeters long and 9 millimeters wide and thick, had parallel axes, and were spaced 5 millimeters apart from each other. They were coupled into a rigid system, while being equipped with a mirror for observing the oscillations.

The following experiment was then made with that simplified instrument: When it commenced, the wire-ends of the damper were *separate* from each other. The needle was then set into oscillation, and the period of oscillation of the needle and the decrease in its oscillation arc, or its logarithmic decrement, were determined according to the instructions that Gauss gave in "*Resultaten aus den Beobachtungen des magnetischen Vereins im Jahre 1837.*"[403],[404] The wire-ends of the circuit were then connected, or the *damper was closed*, and the same observations were repeated. The damper would then be opened again, and one would alternate in that way several more times. The results of those experiments are summarized in the following Table, in which the first column under A gives the logarithmic decrement of the decrease in the oscillation arc for a *closed damper*. In the second column, the same thing is given for an *open damper*, and in the third column under t, one finds the associated *period of oscillation*. At the bottoms of the columns, the mean values over the repeated determinations are added.

A	B	t
0.028 645	0.000 460	9.1128
0.027 955	0.000 360	9.1148
0.028 565	0.000 380	9.1107
0.028 388	0.000 400	9.1128

Finally, to complete the measurement, the following experiment was performed in order to determine the magnetism of the needle[405] and to also gain some knowledge about its distribution to the extent that it seemed necessary. Namely, a small galvanometer was placed as close as possible to the location where the oscillating needle was found, and its deflection v_1 was observed when that needle approached it. Similarly, the deflection v_2 was

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[403][Note by HW:] Carl Friedrich Gauss's *Werke*, Vol. V, p. 374.

[404][Note by AKTA:] [Gau38a].

[405][Note by AKTA:] That is, to determine its magnetic moment.

observed once the needle was turned around its midpoint by 180°. Finally, the corresponding deflections v_3 and v_4 were observed when the needle was displaced parallel to itself to a great distance from the galvanometer on its opposite side, and the value of:

$$v = \frac{1}{4}(v_1 - v_2 + v_3 - v_4)$$

was calculated from that. That experiment was then performed for different distances from the galvanometer and for different directions of the line that went through the middle of the needle and the galvanometer, namely, at distances of 400, 500, and 600 millimeters when the line was perpendicular to the magnetic meridian and a distance of 400 millimeters when it was parallel to the magnetic meridian. The magnetic axis of the deflecting needle was always perpendicular to the magnetic meridian. The results of those experiments are summarized in the following Table. Numbers 1, 2, 3 refer to the cases in which the line was perpendicular to the magnetic meridian, while number 4 refers to the case in which that line was parallel to the magnetic meridian. The distance between the midpoints of both needles is given in the second column under R, while the values that were found for v are given in the third column.

No.	R	v
1.	400 mm	32° 37′ 52.5″
2.	500 mm	18° 1′ 52.5″
3.	600 mm	19° 37′ 7.5″
4.	400 mm	17° 24′ 45.0″

It should be pointed out that this series of experiments was performed some time later than the observations above of the oscillating needle in the damper, and that it therefore cannot be assumed that the ratio of the needle magnetism to geomagnetism remained completely unchanged during that time. For that reason, one of those deflection experiments was performed in the time interval between the individual sets of oscillation observations above, which could then be used to reduce the ratio of the needle magnetism to geomagnetism that was obtained from the last complete series of deflection observations to the time when the oscillation observations above were performed. Namely, a comparison of the corresponding deflections gave the ratio:

$$10\,293 : 10\,000 \, ,$$

from which it emerged that the needle magnetism had decreased noticeably in the meantime. The ratio of the magnetic directive force to that of the thread for the oscillation observations was:

$$68 : 1 \, .$$

Those are all of the experiments that were necessary using the second method to determine the resistance of the circuit or the wire that defined the damper in absolute units.

13.15.2 B

It emerges from the experiments that were summarized in (A) that a needle whose length amounts to almost only the 7th part of the diameter of the damper and makes very small oscillations will still exert an electromotive force on the damper that is sufficient to generate a current whose reaction on the needle is not only perceptible, but can be measured precisely.

Now, should that experiment be based upon a calculation of the resistance of the circuit in absolute units, then some complications would arise in such a way that the distribution of magnetism in the needle cannot be ignored completely, due to the moderate dimensions of the needle in comparison to the diameter of the damper. That complication can be avoided completely when a smaller needle is suspended in the damper and the smaller needle possesses just as much magnetism as the larger one.

In the Physics Institute in Leipzig, one finds a natural magnet that has small size and great strength in comparison to its size, which weighs 40 grams, along with its frame, and is 24 millimeters long. Due to its smallness and strength, it was very suitable as a magnetometer needle for this experiment, and the diameter of the damper could be reduced considerably without it being necessary to make a more precise study of the distribution of the magnetism in it. However, the limited time during which the large wire mass of the damper was available for these experiments that was allotted did not permit any modification of the damper, and the natural magnet was then suspended in the unchanged damper and a second series of experiments performed with it that will be likewise summarized here, because it gives an interesting proof of the fineness that the observations of the decrease in the oscillation arc with the damper afforded one in order to recognize the effects of very weak electromotive forces and measure them with tolerable precision. To that end, the natural magnet was equipped with a frame for attaching the mirror and suspending it by a thread from the center of the damper. The instrument remained otherwise unchanged, and the experiment was then performed in exactly the same way as the foregoing one. The following Table gives an overview of the results that were obtained from it, namely, under A, one finds the logarithmic decrement of the decrease in the oscillation arc for a closed damper, under B, one finds the logarithmic decrement of the decrease in the oscillation arc for an open damper, and under t, one finds the associated period of oscillation.

A	B	t
0.006 01	0.002 54	3.955
0.006 13	0.002 67	3.954
0.006 15	0.002 67	3.953
0.006 05	0.002 66	3.949
0.006 085	0.002 635	3.9527

To complete this experiment, the magnetism of the small needle was determined by a special experiment in a way that was similar to the one in the foregoing series. However, since it was merely necessary to determine the moment of that small magnet, that experiment was limited to two different distances from the midpoint of the small auxiliary galvanometer in the direction perpendicular to the magnetic meridian east and west of the galvanometer. The following Table gives an overview of the results that were obtained in that way. Under R, one finds the distance from the center of the natural magnet to the center of the galvanometer, and under v, one finds the deflection of the galvanometer, which was calculated in the same way that it was in the previous series.

R	v
180.08 mm	20° 42′ 0″
240.18 mm	9° 4′ 52″

The results of those observations are valid for a temperature of $20°R$ for the copper wire, which was the mean temperature over the observations in this and the previous Section.

The data for determining the resistance of the circuit in absolute units are given completely by those experiments.

13.16 Rules for Calculating the Resistance from the Foregoing Observations

When the conditions under which the foregoing observations were performed correspond precisely with the conditions that were assumed in the schema for determining the absolute resistance of a conductor that was given in Section 13.11, the rules for calculating the resistance from the observational results that were announced will be included in the formula that is found at the conclusion of that schema:

$$w = \frac{\pi^3 r^4}{\alpha R^3 t} \ ;$$

because the value of the number α, which gives the elongation width from rest for the needle that is set into oscillation in parts of the radius, as well as the value of the number r/R, which gives the ratio of the radii of the circular conductors A and B to the distance BC, and finally, the velocity r/t with which the radius of the circular conductor will move during an oscillation of the needle, will be given directly from the results of observation. However, since the foregoing observations, with the given description, were not performed under precisely the conditions that were assumed in the aforementioned schema, those simple rules will require some alterations in order to make them applicable to the present observations.

Some of those alterations are easily obtained when one assumes that the radii of the two circular conductors are unequal in the derivation that was given for the equation $w = \pi^3 r^4 / \alpha R^3 t$ and distinguishes them by r' and r'', and one includes the number of their windings m and n in the calculation, and in addition, one considers the elasticity of the thread that suspends the needle, which will yield a directive force for the needle that has a ratio with its magnetic directive force of $\vartheta : 1$, and finally one observes the unequal strength of geomagnetism at the two locations A and B, whose ratio will be represented by T'/T''. One then finds that in the formula above, the square r^2 must be replaced with the product $r'r''$, and the total value for w must be multiplied by $mn/(1 + \vartheta) \cdot T'/T''$, and as a result:

$$w = \frac{mn}{(1 + \vartheta)} \cdot \frac{T'}{T''} \cdot \frac{\pi^3 r'^2 r''^2}{\alpha R^3 t} \ .$$

In addition, the following essential modifications of the observations that were carried out using the first method now come under consideration, namely, that *first of all*, the needle is moved to a distance of $BC = R$ from the midpoint of the circle B, which will increase the observed elongation width by a ratio of:

$$r''^3 : 2R^3 \ .$$

In that way, one must consider the fact that the circle A will be rotated through two quadrants each time, instead of one quadrant, which will likewise increase the elongation width by a ratio of:

$$1 : 2 \ .$$

If one then currently denotes that increased elongation width by α, then one must set:

$$w = \frac{mn}{1+\vartheta} \frac{T'}{T''} \cdot \frac{4\pi^3 r'^2}{\alpha r'' t}$$

accordingly. *Secondly*, the multiplicity of windings in both circles, which will convert them into loops with significant cross-sections, will come under consideration. For the loop A, if one recalls that it is not precisely circular in form, it suffices to replace $m\pi r'^2$ with the sum of the areas that are bounded by the projections of all of its windings onto the plane of the loop, and as a result, if that sum is denoted by S then:

$$w = \frac{n}{1+\vartheta} \cdot \frac{T'}{T''} \cdot \frac{4\pi^2 S}{\alpha r'' t} \ .$$

By contrast, the outer radius of the ring B is a'', the inner radius is a', the height of the loop is $2b'$, and in addition, in regard to the distribution of magnetism M in the needle, when one sets:

$$M = 2e'\mu \ ,$$

in which $\pm\mu$ denotes the amount of north or south magnetic fluid, which can be thought of as spread across the surface of the needle, according to Gauss's known theorem on the ideal distribution of magnetism,[406] the length e' can be calculated by replacing $1/r''$ with the following expression:

$$\frac{1}{a''-a'} \left\{ \log \frac{a'' + \sqrt{a''^2+b'^2}}{a' + \sqrt{a'^2+b'^2}} + \frac{1}{4} \left(\frac{a''^3}{(a''^2+b'^2)^{3/2}} - \frac{a'^3}{(a'^2+b'^2)^{3/2}} \right) \frac{e'^2}{b'^2} \right\} \ .$$

The changes to the formula that was found in Section 13.11 that were cited here, which are necessary if the resistance of the circuit is to be calculated from the experiments that were described in Section 13.14, are so numerous that, rather than go into a detailed discussion of them and how to establish them, I would prefer to derive the two equations that will be used in the following Section 13.17 to calculate the resistance from the experiments that were described in Section 13.14, namely:

$$
\begin{aligned}
w &= \frac{n}{1+\vartheta} \cdot \frac{T'}{T''} \cdot \frac{4\pi^2 S}{\alpha r'' t} \ , \\
\frac{1}{r''} &= \frac{1}{a''-a'} \left\{ \log \frac{a'' + \sqrt{a''^2+b'^2}}{a' + \sqrt{a'^2+b'^2}} \right. \\
&\quad \left. + \frac{1}{4} \left(\frac{a''^3}{(a''^2+b'^2)^{3/2}} - \frac{a'^3}{(a'^2+b'^2)^{3/2}} \right) \cdot \frac{e'^2}{b'^2} \right\} \ ,
\end{aligned}
$$

directly from the basic laws of electromagnetism and magneto-electricity. One finds that derivation in *Supplement* 13.39 at the end of the treatise.

Furthermore, the following equations will be used in Section 13.18 to calculate the resistance from the experiments that were described in Section 13.15:

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[406][Note by AKTA:] [Gau39] with English translations in [Gau41a] and [GT14].

$$w = \frac{n^2\pi^2}{1+\vartheta} \cdot \frac{\pi^2 + \lambda^2}{\lambda} \cdot \tan v_o \cdot \frac{r''}{t} ,$$

$$\frac{1}{r''} = \frac{1}{a'' - a'} \left\{ \log \frac{a'' + \sqrt{a''^2 + b'^2}}{a' + \sqrt{a'^2 + b'^2}} \right.$$

$$\left. + \frac{1}{4}\left(\frac{a''^3}{(a''^2 + b'^2)^{3/2}} - \frac{a'^3}{(a'^2 + b'^2)^{3/2}}\right) \cdot \frac{e'^2}{b'^2} \right\} ,$$

in which λ denotes the natural logarithm of the observed ratio of two successive oscillation arcs of the magnetometer needle as a result of damping for a closed circuit, and $\tan v_0$ was written instead of $2M/T''r''^3$. Those last two equations are also derived in *Supplement* 13.39 directly from the basic laws of electromagnetism and magneto-electricity.

We can now move on to calculate the resistance itself from the experiments that were described in Sections 13.14 and 13.15.

13.17 Calculating the Resistance from the First Series of Experiments

In the series of experiments in Section 13.14, which were performed using the first method, the circuit consisted of the wires of the inductor and multiplier and the two connecting wires, and the resistance to be calculated was the sum of the resistances of those four wires.

The immediate result of the experiment that was described in Section 13.14 was *first of all* the magnitude of the oscillation arc that was measured with the magnetometer, namely:

<center>79.9 millimeters,</center>

for a radius of 8175 millimeters long (= twice the horizontal distance from the mirror to the scale). That gave:

$$\alpha = \frac{79.9}{8175} .$$

(See *Supplement* 13.39 on that subject, where the throwback method that was used here is discussed in more detail.)

Secondly, the magnitude of the period of oscillation of the magnetometer needle is:

$$t = 10.2818 \text{ seconds.}$$

Thirdly, the part of the directive force on the needle that originates in the elasticity of the suspension thread is:

$$\vartheta = \frac{1}{1770} .$$

when expressed in units of its magnetic directive force.

Fourthly, the ratio of the strength of the horizontal part of geomagnetism at the location of the inductor T' to its strength at the location of the multiplier T'' is:

$$\frac{T'}{T''} = 0.997\,87 \ .$$

The results of the measurements of the inductor and multiplier must be added to these immediate results of observation. For the *inductor*, it suffices to know the result that the sum of the areas that are bounded by the projections of its 145 windings onto the plane of the loop amounts to:

$$S = 104\,924\,000 \text{ square millimeters.}$$

For the *multiplier*, the following results of measurement must be added:

$$
\begin{aligned}
\text{Inner radius } a' &= 303.51 \text{ millimeters,} \\
\text{Outer radius } a'' &= 374.41 \text{ millimeters,} \\
\text{Width } 2b' &= 202.05 \text{ millimeters,} \\
\text{Number of windings } n &= 1854.
\end{aligned}
$$

With those values of a', a'', b', one gets:

$$
\begin{aligned}
\frac{1}{r''} &= \frac{1}{a'' - a'} \left\{ \log \frac{a'' + \sqrt{a''^2 + b'^2}}{a' + \sqrt{a'^2 + b'^2}} \right. \\
&\qquad \left. + \frac{1}{4} \left(\frac{a''^3}{(a''^2 + b'^2)^{3/2}} - \frac{a'^3}{(a'^2 + b'^2)^{3/2}} \right) \frac{e'^2}{b'^2} \right\} \ , \\
\frac{1}{r''} &= 0.002\,835\,2 + 0.000\,000\,015\,875 \cdot e'^2 \ ,
\end{aligned}
$$

in which an approximate value for e' (say, 20 millimeters) would suffice, due to the smallness of the needle, so $1/r'' = 0.002\,841\,55$. That will then yield:

$$
\begin{aligned}
w &= \frac{n}{1 + \vartheta} \cdot \frac{T'}{T''} \cdot \frac{4\pi^2 S}{\alpha r'' t} \\
&= \frac{1770}{1771} \cdot 1854 \cdot 0.997\,87 \cdot \frac{4\pi^2 \cdot 104\,924\,000}{79.9 \cdot 10.2818} 8175 \cdot 0.002\,841\,55 \ ,
\end{aligned}
$$

or

$$w = 2166 \cdot 10^8 \ .$$

The resistance of the circuit that consists of the inductor and the multiplier wires, along with the two connecting wires, is then determined completely by the resistance unit W that was defined and the number w, in which it must only be pointed out that this absolute unit determination is based upon the millimeter as the linear unit and the second as the time unit, which can be expressed by the following notation:

$$2166 \cdot 10^8 \; \frac{\text{millimeters}}{\text{second}} .$$

If another linear unit has a ratio of $1 : r$ with the millimeter and another time unit has a ratio of $1 : t$ with the second, then the same resistance will be:

$$2166 \cdot 10^8 \cdot \frac{r}{t} \; ,$$

when the new units are used as a basis; e.g., when the basic linear unit is the mile, which has a ratio of $1 : 0.000\,000\,135$ with the millimeter, one will have:

$$29\,241 \; \frac{\text{miles}}{\text{second}} .$$

13.18 Calculating the Resistance from the Second Series of Experiments

In the second series of experiments, which were performed using the second method, the circuit consisted of merely the wire of the damper, i.e., the wire that defined the multiplier in the foregoing series of experiments.

The direct results of the experiments were:

First of all, the magnitude of the logarithmic decrement of the decrease in the oscillation arc, which was found to be equal to:

$$0.027\,988$$

in common logarithms, after subtracting the part that was independent of any electromagnetic influence. As a result:

$$\lambda = 0.064\,445 \; .$$

in common logarithms.

Secondly, the magnitude of the period of oscillation of the magnetometer needle was:

$$t' = 9.1128 \; ,$$

in which it should be pointed out that the magnetic directive force was increased by roughly 68 parts by the elasticity of the thread, so:

$$\vartheta = \frac{1}{68} \; .$$

Thirdly, the magnitude of the ratio of the needle magnetism to geomagnetism can be inferred from the results of the deflection experiments that are included in the following Table:

No.	R	v
1.	400 mm	32° 37′ 52.5″
2.	500 mm	18° 1′ 52.5″
3.	600 mm	10° 37′ 7.5″
4.	400 mm	17° 24′ 45″

The first three numbers refer to the experiments for which the midpoint of the magnetometer and its magnetic axis coincided with the perpendicular to the magnetic meridian, which was laid through the center of the auxiliary galvanometer. The fourth number refers to an experiment in which the magnetometer needle was likewise positioned with its axis perpendicular to the magnetic meridian, but its midpoint was found on the line that was laid through the center of the auxiliary galvanometer parallel to the magnetic meridian. It was also ascertained that the ratio of the needle magnetism to geomagnetism that emerged would have to be increased by the ratio:

$$10\,000 : 10\,293 \; ,$$

if it were to be valid at the time when the decrease in the oscillation arc and the period of oscillation of the magnetometer needle were observed.

The derivation of the values of e' and v_0 in the formulas that are presented for calculating the resistance:

$$w = \frac{n^2\pi^2}{1+\vartheta} \tan v_0 \cdot \frac{\pi^2 + \lambda^2}{\lambda} \cdot \frac{r''}{t} \; ,$$

$$\frac{1}{r''} = \frac{1}{a''-a'} \left\{ \log \frac{a'' + \sqrt{a''^2 + b'^2}}{a' + \sqrt{a'^2 + b'^2}} \right.$$
$$\left. + \frac{1}{4} \left(\frac{a''^3}{(a''^2 + b'^2)^{3/2}} - \frac{a'^3}{(a'^2 + b'^2)^{3/2}} \right) \frac{e'^2}{b'^2} \right\} \; ,$$

from the data that was cited above is the following one: One imagines that the aforementioned (page 321) ideal distribution of a magnetic fluid that is spread over the surface of the needle is concentrated into its midpoints (i.e., centers of mass), i.e., two points that lie at a distance e' from the center of the needle on a line that is parallel to the direction of the magnetic axis and whose separation distance is equal to $2e'$. The position of the midpoint on the needle and its magnetic axis relative to the midpoint of the deflecting galvanometer and the magnetic meridian is determined precisely for each experiment in the Table above. Now, if f' has the same meaning for the galvanometer that e' has for the magnetometer needle, then that will explain the fact that for any given deflection of the galvanometer v, the position of the four points at which the magnetic fluids of both needles are thought to be concentrated are determined completely with respect to each other and the magnetic meridian by e' and f', and the fact that the ratio of the rotational moment that the magnetometer needle exerts on the galvanometer to the one that geomagnetism exerts can then be determined from the ratio of the magnetism M of the magnetometer needle to geomagnetism T using the law by which two elements of the magnetic fluid act upon each other. The deflection v for which those two rotational moments prove to be equal and opposite is the one that is observed independently of e', f', and M/T. The equation that expresses the dependency of those quantities then gives:[407]

$$\frac{T}{M} \tan v = \frac{2}{R^3} + \frac{4e'^2 - 6(1 - 5\sin v^2) f'^2}{R^5} + \dots$$

[407][Note by AKTA:] The expression $\sin v^2$ should be understood as $\sin^2 v$.

for the case in which the line R that connects the midpoints of the two needles is perpendicular to the magnetic meridian and:

$$\frac{T}{M}\tan v = \frac{1}{R^3} - \frac{3}{2}\frac{e'^2 - (4 - 15\sin v^2)f'^2}{R^5} + \ldots$$

for the case in which R is parallel to the magnetic meridian. The assumption that the magnetic fluid is concentrated is admissible only when the terms that are divided by R^7 or higher powers can be neglected. Now, since the values of R and v are given for each experiment in the Table above, each experiment will give an equation between e', f', M/T, and as result, the four experiments that are included in the Table above will give four equations between those three quantities, three of which serve to determine those quantities and the fourth of which serves as a control to insure that the value of R is actually so large that the terms of higher order can be neglected. The values of the three quantities above that harmonize best with the observations are:

$$\begin{aligned}
e' &= 33.715 , \\
f' &= 14.856 , \\
\frac{M}{T} &= 20\,143\,000 .
\end{aligned}$$

The last value of M/T is true for the time when the deflection experiment was made and, as on page 325, it must be multiplied by 1.0293 if it is to be valid at the time when the period of oscillation and the decrease in the oscillation arc were observed. For the latter time, one then gets:

$$\frac{2M}{T} = 41\,466\,000 .$$

If one further substitutes the value that was found for e' in the equation:

$$\frac{1}{r''} = 0.002\,835\,2 + 0.000\,000\,015\,875 e'^2 ,$$

which is also true for the second series of experiments, because the measurements of the damper here are the same as the measurements of the multiplier in the first series of experiments, then one will get:

$$\frac{1}{r''} = 0.002\,853\,2 ,$$

or

$$r'' = 350.48 ,$$

and with that value:

$$\frac{2M}{Tr''^3} = \tan v_0 = 0.963\,14 .$$

In addition, as in the first series of experiments, one has:

$$n = 1854.$$

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That will then give:

$$w = \frac{n^2\pi^2}{1+\vartheta} \tan v_0 \cdot \frac{\pi^2 + \lambda^2}{\lambda} \cdot \frac{r''}{t}$$
$$= \frac{68}{69} \cdot 1854^2\pi^2 \cdot 0.963\,14 \cdot \frac{\pi^2 + 0.064445^2}{0.064\,445} \cdot \frac{350.48}{9.1128} \; ,$$

or

$$w = 1898 \cdot 10^8 \; .$$

The resistance of the circuit that consists of merely the damper wire is then determined completely by the defined unit of resistance W and the number w.

13.19 Calculating the Resistance from the Third Series of Experiments

In the third series of experiments, as well, the circuit whose resistance was to be determined consisted of merely the damper wire, and the experiment was performed using the second method. The essential difference in the second method then consisted of merely the fact that the natural magnet that was used for the magnetometer needle had much smaller dimensions, which meant that on the one hand, the calculations were indeed simplified, since with such small dimensions in comparison to the diameter of the damper, the type of distribution of free magnetism did not come under consideration. On the other hand, the measurement lost precision in that way, because the magnetism, despite being strong in comparison to the size of the magnet, amounted to almost the 19th part of the magnetism of the larger needle, which made the damping so weak that the observations did not allow a fine determination of the logarithmic decrement of the decrease in the oscillation arc.

The direct results of the experiments were: *First of all*, the magnitude of the logarithmic decrement for the decrease in the oscillation arc, which was found to be equal to:

$$0.003\,45$$

in common logarithms, after subtracting the part that was independent of electromagnetic influences, and as a result:

$$\lambda = 0.007\,944$$

in natural logarithms.

Secondly, the period of oscillation of the needle:

$$t' = 3.9527 \; .$$

The elasticity of the suspension thread cannot be neglected since it further increased the directive force by $1/2000$.

Thirdly, the magnitude of the needle magnetism in comparison to geomagnetism was inferred from the deflection experiments that are summarized in the following Table:

R	v
180.08 mm	20° 42.0′
240.18 mm	9° 4.52′

The line R, which connects the midpoints of the two deflecting and deflected needles, was there perpendicular to the magnetic meridian.

From the rule that Gauss gave in *Intensitas vis magneticae terrestris etc.*,[408] it then follows that:

$$\tan 20° \; 42' \;\; = \;\; \frac{2M}{T} \cdot 180.08^{-3} + a \cdot 180.08^{-5} \;,$$
$$\tan 9° \; 4' \; 52'' \;\; = \;\; \frac{2M}{T} \cdot 240.18^{-3} + a \cdot 240.18^{-5} \;,$$

so

$$\frac{2M}{T} = 2\,224\,660 \;.$$

However, it should be pointed out that with the low degree of precision that the deflection experiments possess when they are performed with such a small needle, the elimination of the second term, which depends upon the fifth power of the distance, is very uncertain, such that a similarly precise, or even more precise, result would be obtained if one did not consider that second term at all. One then finds that:

$$\tan 20° \; 42' \;\; = \;\; \frac{2M}{T} \cdot 180.08^{-3} \;,$$
$$\tan 9° \; 4' \; 52'' \;\; = \;\; \frac{2M}{T} \cdot 240.18^{-3} \;,$$

and that gives the two values for $2M/T$:

$$2\,206\,600 \;,$$
$$2\,214\,500 \;,$$

or the mean value:

$$2\,210\,550 \;.$$

From the doubt that exists about whether to prefer the first or second calculation in the present case, and since the results that were obtained in both ways differ only slightly anyway, the mean shall be taken from the results of both calculations, namely:

$$\frac{2M}{T} = 2\,217\,600 \;.$$

---

[408][Note by AKTA:] See footnote 97 on page 51.

Now, since the same measurements are true for the damper that were true in the foregoing series of experiments, in addition, but the term in the value of $1/r''$ that depends upon e' is negligible, due to the smallness of the needle, that will give:

$$
\begin{aligned}
\frac{1}{r''} &= 0.002\,835\,2 \ , \\
r'' &= 352.71 \ ,
\end{aligned}
$$

and as a result:

$$
\frac{2M}{Tr''^3} = \tan v_0 = 0.050\,54 \ .
$$

In addition, as in the foregoing series, one has:

$$
n = 1854.
$$

When ϑ is neglected, due to its smallness, that will then yield:

$$
\begin{aligned}
w &= n^2\pi^2 \cdot \tan v_0 \cdot \frac{\pi^2 + \lambda^2}{\lambda} \cdot \frac{r''}{t} \\
&= 1854^2 \cdot \pi^2 \cdot 0.05054 \cdot \frac{\pi^2 + 0,007944^2}{0,007\,944} \cdot \frac{352.71}{3.9527} \ ,
\end{aligned}
$$

or

$$
w = 1900 \cdot 10^8 \ .
$$

The difference between those values that were derived from the second series of experiments is smaller than the difference that can be attributed to the unavoidable observation error in the last series.

13.20 Comparing the Resistance in the Circuit in the First Series of Experiments with the Resistance in the Circuit for the Second and Third Series

The resistances in two circuits were measured in absolute units in the series of experiments above, the first of which was composed of:

1. A wire A, that served as the multiplier,

2. A wire B that served as an Earth inductor, and

3. Two short, thick connecting wires C.

By contrast, the last circuit consists of merely the wire A that was used as a damper. A comparison of the resistances in both circuits was based mainly upon a comparison of the resistance A with the resistance B, since the resistance C was so minor that its influence in proportion to its length and its cross-section could be easily included as a correction to the calculation.

Since the direct comparison of the resistances A and B led to a less certain result, due to the large difference between them, three auxiliary wires a, b, c were called upon in order to make it possible to base the ratio $A : B$ upon only those measurements in which only resistances that were very close to each other could be compared.

Those resistance comparisons were all performed using the method that is described in *Supplement* 13.39 and explained in an example, and it therefore suffices to summarize the results in the following Table without going into details about the observations. In the first column, the resistance comparisons that were made using the given method are distinguished by numbers. The second column under X gives the relationship to the desired resistance ratio, and the third column under q gives the numerical value that was found. Finally, the logarithms of q and $q + 1$ are appended in the last two columns.

No.	X	q	$\log q$	$\log(q+1)$
1.	B/c	1.043 54	0.018 51	0.310 38
2.	$b/(B+c)$	1.034 98		0.308 56
3.	$a/(B+b+c)$	1.007 52		0.302 66
4.	$A/(B+a+b+c)$	0.915 29	9.961 56	
	A/B	7.322 4	9.943 05	0.921 60

Now, one has:

$$\log\left(\frac{A}{B}\right) = \log\left(\frac{A}{B+a+b+c}\right) - \log\frac{B}{c}$$
$$+ \log\left(\frac{B}{c}+1\right)\left(\frac{b}{B+c}+1\right)\left(\frac{a}{B+b+c}+1\right) ,$$

that is, the difference between the two logarithms in the fourth column (which is given beneath it) is added to the sum of the three logarithms in the last column (which is likewise given beneath it) in order to obtain the logarithm of the desired ratio A/B, which is then recorded in the third column.

For C, it suffices to remark that the cross-section was three times larger than that of B, while the length was thirty times smaller. As a result, since both wires were copper, the resistance ratio was:

$$\frac{B}{C} = 90 ,$$

from which it would ultimately follow that a comparison of the resistance $A + B + C$ of the circuit that was used in the first series of experiments with the resistance of the circuit A in the other two series of experiments would yield:

$$\frac{A+B+C}{A} = 1 + \frac{1+90}{7.3224 \cdot 90} = 1.138 .$$

Now, $A + B + C$ was determined in absolute units from the first series of experiments to be:

$$A + B + C = 2166 \cdot 10^8 \ \frac{\text{millimeters}}{\text{second}}.$$

If one divides that value by the quotient above, then one will get the value of A that is derived from the first series of experiments:

$$A = 1903 \cdot 10^8 \ \frac{\text{millimeters}}{\text{second}}.$$

13.21 Overview of the Various Measurements of the Resistance of the Multiplier or Damper Wire A

I. From the first series of experiments:

$$A = 1903 \cdot 10^8 \ \frac{\text{millimeters}}{\text{second}}.$$

II. From the second series of experiments:

$$A = 1898 \cdot 10^8 \ \frac{\text{millimeters}}{\text{second}}.$$

III. From the third series of experiments:

$$A = 1900 \cdot 10^8 \ \frac{\text{millimeters}}{\text{second}} .$$

Of those three measurements of the same resistance, less weight should be given to the third one than the first two, as was remarked already. However, since it agrees with the other two very closely, there is no reason to exclude it, and that will yield the following mean value of all of them:

$$A = 19\,003 \cdot 10^7 \ \frac{\text{millimeters}}{\text{second}} .$$

The agreement that then exists between the two measurements of the resistance of the wire A, which were obtained by totally-different methods, namely, between the first and last two series of experiments, then presents a special interest, because it proved that the 1854 windings that the wire defined in the multiplier or damper were sufficiently insulated by the wool that was wound around them. That is because if conduction had taken place from one winding to the other through the wool, then the effect of the multiplier on the magnetometer would have been weakened by it in the first series of experiments, and the calculation would have given a resistance that was too large, as if the current that flowed through the entire circuit had been weakened by a larger resistance. By contrast, from the result that was calculated from the second series of experiments, the conduction through the wool from one winding to another had no influence whatsoever, because it is known that the damping force of a damper will not be changed by it in such a way that its wire windings will be conductively connected to each other. At the very least, the damping force cannot be reduced in that way. However, an increase in it (if it were perceptible at all) would have the effect that the calculation of the resistance would prove to be too small.

13.22   Standards for Resistance Measurements in Absolute Units

If the wire A whose resistance was known in absolute units from the measurements above were reserved for use as a standard resistance, then it would itself have to be able to serve to reduce all resistance measurements to absolute units without it being necessary to repeat the original measurement, as long as one could build upon the invariability of the standard. However, that wire was not determined for that purpose, and its use in the present investigation was permitted for only a short time. Therefore, if the utility that the results obtained might have had for the duration of the future measurements of resistance were not to be lost, then copies of the wire A would have to be made that were guaranteed to have equal resistance, or standards whose resistance was precisely comparable to the resistance of A. The three copper wires a, b, c that were cited above can initially serve as such standards that are compared to the resistances A and B as auxiliary wires, and their resistance ratios to A can be derived from the observations above. That is because from the observations above, one has:

$$\log \frac{B}{c} = 0.018\,51 \ ,$$
$$\log \frac{b}{B+c} = 0.014\,93 \ ,$$
$$\log \frac{a}{B+b+c} = 0.003\,25 \ .$$

If one adds:

$$\log \frac{A}{B} = 0.864\,65 \ ,$$
$$\log A = 11.278\,82 \ ,$$

from the above, then that will yield the resistances of the three copper wires a, b, c in absolute units, namely:

$$a = 10\,420 \cdot 10^7 \ \frac{\text{millimeters}}{\text{second}},$$
$$b = 5260 \cdot 10^7 \ \frac{\text{millimeters}}{\text{second}},$$
$$c = 2487 \cdot 10^7 \ \frac{\text{millimeters}}{\text{second}}.$$

Those three standard resistances that were thus determined, with the given notation and the attached resistance values, were recorded in the instrument collection of the Physics Institute at the University of Leipzig.

However, since many resistance measurements have been performed already with the standard resistance that Jacobi created and copies of it were distributed, it would seem most

convenient for practical applications to determine the value of that standard in *absolute* units, which can be easily achieved by comparing the resistance of that standard with the resistance of the copper wire that was denoted by c above. That comparison cannot be performed directly either, but only by means of a fourth copper wire d.

On page 303, the resistance of a copy of Jacobi's standard J was compared with the resistance of the original. One finds a comparison of that copy with another one in *Supplement* 13.39. That yielded the resistance:

$$
\begin{aligned}
\text{of the } \textit{first one} &= 0.9815 \cdot J \ , \\
\text{of the } \textit{second one} &= 0.9839 \cdot J \ , \\
\text{in total} &= 1.9654 \cdot J \ .
\end{aligned}
$$

A comparison of that resistance with that of the wire d, using the method that is described in *Supplement* 13.39, yields the following value for d:

$$ d = 1.1295 \cdot 1.9654 \cdot J = 2.220 \cdot J \ . $$

However, a comparison of the latter resistance, along with those of the two copies, with the wire c gave the following value for c:

$$ c = 0.993 \cdot (2.220 + 1.9654) \cdot J = 4.156 \cdot J \ , $$

and as a result, since c is $2487 \cdot 10^7$ (millimeters/second) in absolute units:

$$ J = 598 \cdot 10^7 \ \frac{\text{millimeters}}{\text{second}} = 807 \ \frac{\text{miles}}{\text{second}}. $$

Inspector Leyser[409] in Leipzig has produced a number of copies of Jacobi's standard whose resistance was given even more precisely by the test that Dr. Quintus Icilius[410] carried out, in Jacobi units, as well as in absolute units.

13.23 On Neumann's Induction Constant and Kirchhoff's Determination of It

A treatise by Dr. G. Kirchhoff appeared recently in Poggendorff's *Annalen*, Vol. 76, pp. 412 *et seq.* (1849), with the title "Bestimmung der Konstanten, von welcher die Intensität inducierter elektrischer Ströme abhängt."[411]

Kirchhoff said:

> "The mathematical law of induced currents was exhibited by Neumann[412] and Weber. In the expression that the two have found for the intensity of an induced current, in addition to quantities that must be measured in each given case, a constant appears that must be ascertained once and for all by experiment and which Neumann denoted by ε. I have attempted to determine it."

---

[409][Note by AKTA:] See footnote 105 on page 58.
[410][Note by AKTA:] Ernst Wilhelm Gustav von Quintus Icilius (1824-1885).
[411][Note by AKTA:] G. R. Kirchhoff (1824-1887), see [Kir49a].
[412][Note by AKTA:] F. E. Neumann (1798-1895).

Now, the constant ε that Kirchhoff determined has a simple relationship to the one that he used for a resistance unit and the absolute unit of resistance that was defined above that can be expressed in the following way:

With the units that were established above for current intensity, electromotive force, and resistance, one has the following equation for the current intensity i that is produced in a closed conductor, whose resistance is w, by the electromotive force e:

$$i = \frac{e}{w} \ .$$

If one now introduces some other units that relate to the absolute one like:

$$a : 1 \ ,$$
$$b : 1 \ ,$$
$$c : 1 \ ,$$

and one denotes the three quantities above when they are expressed in the new units by i', e', w', then one will get:

$$ai' = i \ , \qquad be' = e \ , \qquad cw' = w \ ,$$

and as a result:

$$ai' = \frac{be'}{cw'} \ .$$

A more precise test and comparison of those units, which are based upon Neumann's expression for the intensity of an induced current and Kirchhoff's calculation, with the units above will yield:

$$a = \sqrt{\frac{1}{2}} \qquad \text{and} \qquad b = \sqrt{2} \ ,$$

when the space and time units in the velocity measurement are based upon millimeters and seconds, respectively, and one will then have:

$$i' = \frac{2e'}{cw'} \ ,$$

for which one can also write:

$$i' = \frac{\frac{2}{c}e'}{w'} \ .$$

Now, the constant coefficient $2/c$ with which the electromotive force e' is multiplied in that expression for the induced current is the constant that Neumann and Kirchhoff denoted by ε. At the same time, one sees from the given presentation that $c = 2/\varepsilon$ is the number that gives how many times bigger the chosen basic unit of resistance is than the absolute unit of resistance that was defined in Section 13.10. If one chooses, e.g., a basic unit for which the induction constant $\varepsilon = 1$, then the ratio of that basic unit to the one defined in Section 13.10 will be $2 : 1$. Now, Kirchhoff found from his own observations that:

"The constant $\varepsilon = 1$ when one assumes that the unit of velocity is a velocity of 1000 feet per second and the unit of resistance is the resistance of a copper wire with a cross-section of one square line[413] and a length of 0.434 inches."

That data was based upon the Prussian unit of length. In metric units, it translates into: The constant $\varepsilon = 1$ when one assumes that the unit of velocity is a velocity of $313\,853$ millimeters per second and the unit of resistance is the resistance of a copper wire with a cross-section of 4.75 square millimeters and a length of 11.35 millimeters.

Now, it can be easily shown that $\varepsilon = 1$ will still be true as long as the ratio of the two units — viz., the velocity unit and the resistance unit — remains unchanged. Therefore, the constant ε will also be equal to 1 when one chooses the unit of velocity to be the velocity of 1 millimeter per second and the unit of resistance to be the resistance of a copper wire of cross-section $4.75 \cdot 313\,853$ square millimeters and a length of 11.35 millimeters.

Now since $c = 2$ for $\varepsilon = 1$, that will imply that this unit of resistance is twice as big as the unit of resistance that was defined in Section 13.10.

Kirchhoff's observations, after the given reduction to the unit that was defined in Section 13.10, will then imply that the absolute unit of resistance is equal to the resistance of a copper wire of cross-section $4.75 \cdot 313\,853$ square millimeters and length 11.35 millimeters or of cross-section $262\,752$ square millimeters and length 1 millimeter.

By contrast, from the observations that were communicated in this treatise, what was said in Section 13.22 implies that Jacobi's copper wire with a cross-section of $0.3335^2 \cdot \pi$ square millimeters and a length of 7619.75 millimeters possessed a resistance that was $598 \cdot 10^7$ times greater than the absolute resistance unit that was defined in Section 13.10, and as a result, with that type of copper, the resistance of a copper wire of cross-section $0.3335^2 \cdot 598 \cdot 10^7 \cdot \pi$ square millimeters and length 7619.75 millimeters or the resistance of a copper wire of cross-section $274\,250$ square millimeters and length 1 millimeters should be equal resistance units.

The agreement between those two pieces of data, which were obtained in entirely different ways, will become all the more unexpected when one observes that Jacobi's and Kirchhoff's wires were made of different types of copper, and that even greater differences often occur in the conductivity or resistance coefficients of copper. If one were to attribute the difference in the two pieces of data to merely the inclusion of the difference between the types of copper, then that would imply that the copper that Jacobi used possessed a somewhat lower conductivity, or a somewhat higher resistance coefficient, than the copper that Kirchhoff used. I have also found the resistance coefficient for the copper that I used to be smaller than it was for what Jacobi used, and the difference was even considerably larger than it was for the copper that Kirchhoff used. A direct comparison of the resistance of Kirchhoff's wire with Jacobi's basic unit would then be especially interesting for the sake of a more precise comparison of the results of both measurements.

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[413][Note by AKTA:] In German: *Quadratlinie.*

III - Examples of Applications of the Absolute Resistance Unit

13.24 Applying the Resistance Unit to the Measurement of Galvanic Currents in Their Engineering Uses

For the engineering applications of galvanism, e.g., for chemical purposes and galvano-optics, a simple and generally-understandable rule is often lacking. Every engineer is then required to test the relationships that give favorable results with their own experiments. The expenditure of time and cost that is created in that way complicates those applications of galvanism appreciably with massive undertakings. However, such rules are not lacking as much because satisfactory experiments have not been carried out yet as because the results of the experiments that were made cannot be expressed simply and unambiguously, because mere descriptions of the processes would not suffice. It is only by galvanic measurements that it is possible to present the results of the experiments that were performed with fewer words and numbers in a generally-understandable way and to give well-defined and precise rules for future uses, and galvanic measurements are likewise necessary in the applications if one is to guarantee that the prescribed rules have been fulfilled.

One then deals with the effectiveness of galvanic currents, which are however measured in very *different* way, due to the variety of the situations. Frequently, it is merely the *current intensity* that one deals with, e.g., for galvano-plastic precipitation. However, the current intensity is often only one factor that affects the effectiveness in question, while the other factor is the *length of the conductor* through which that current flows, e.g., when the conductor goes around an iron rod that is to be converted into an electromagnet. Finally, there is also the case in which each part of the length of the conductor through which the current flows is associated with a *special value* for the effectiveness in question, e.g., for a multiplier whose various windings each have a different ideal position with respect to the magnetic needle.

The simplest case for engineering applications, which is also the most important one, is the first one, in which the effectiveness in question depends upon merely the *current intensity*. The construction of galvanic workshops and the many types of projects that are carried out in them can be facilitated and promoted considerably when the current intensity that is ideal for each purpose is ascertained precisely and one has a convenient means at one's disposal for testing whether that current intensity is present when one performs that test.

As far as the study and precise determination of ideal current intensities is concerned, the *voltameter* that Faraday[414] specified for that purpose, in which the volume of gas that is produced by the decomposition of water in a certain time interval indicates that intensity, offers a simple means for achieving that goal, and therefore its use cannot be emphasized enough. It is only with *weak* currents, for which the decomposition of the water takes place very slowly, that it is not applicable. In addition, the voltameter is not always convenient in ordinary practice when it is to be used continually to test the prescribed current intensity because the time interval must be measured as an essential element. Finally, the voltameter

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[414][Note by AKTA:] See footnote 163 on page 111.

must continually remain in the circuit because when one removes it, the current intensity will no longer be the measured one, but something much stronger. However, the weakening of the current that is coupled with its insertion can be very unfavorable in many cases. In all cases where the use of the voltameter is not practical for the cited reasons, it can be replaced with a *tangent galvanometer*, which was described in "*Resultaten aus den Beobachtungen des magnetischen Vereins im Jahre 1840,*" pp. 85 *et seq.*[415],[416] and whose use for intensity measurements was explained there by examples. Every unit of current intensity that was measured with the tangent galvanometer by that rule decomposed in 1 second 0.009 376 or in 1 minute $46\frac{2}{3}$ seconds 1 milligram of water in the voltameter (about 1 grain every one-half hour). No clock is necessary for the use of the tangent galvanometer, and the insertion in or removal of the instrument from the circuit has no appreciable influence on the current intensity.

Finally, the *resistance measurement* offers a third practical tool for determining the *current intensity*. The current intensity depends upon two things: the *electromotive force* and the *resistance* of the circuit, of which only the variability of the latter comes under consideration in engineering use, as a rule. That is because, as a rule, in engineering establishments, one always uses the same type of cell, whose electromotive force can be determined once and for all with a precision that is sufficient for practical purposes. The current that is produced by those cells will sometimes flow through more vessels and sometimes less of them and through various fluids, which will vary the resistance considerably.

If one assumes that the electromotive force E is known and is therefore reduced to measuring only the resistance, then any *arbitrary galvanometer* can be used to determine the *current intensity* when one appeals to a *standard resistance w* as an absolute unit. That is because if a denotes the reading on the galvanometer when that standard is excluded from the circuit and b denotes the reading when it is included, then the resistance of the circuit in absolute units W will be determined by that, namely:

$$W = \frac{bw}{a - b} \ ,$$

and the current intensity will then prove to be simply:

$$\frac{E}{W} = \frac{a - b}{bw} \cdot E \ .$$

13.25 Applying the Resistance Measurement to the Measurement of Electromotive Forces in Absolute Units

The final remark in the previous Section leads to a further application that one can make of a known *standard resistance* in absolute units. That is because it follows from the statement that when one appeals to a tangent galvanometer or voltameter, or any other instrument with which the current intensity can be measured in *absolute* units, instead of an arbitrary galvanometer, the electromotive force E can itself be found in *absolute* units in the given way when it is still unknown. That is because if one denotes the current intensity, measured

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[415][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 8.
[416][Note by AKTA:] [Web41b] with English translation in [Web20b].

in absolute units, by α when the standard resistance is excluded from the circuit and by β when it is included, then just as before, that will yield the resistance W of the circuit in absolute units:

$$W = \frac{\beta w}{\alpha - \beta} \; ,$$

and one gets the electromotive force E in absolute units from that:

$$E = \alpha W = \frac{\alpha \beta w}{\alpha - \beta} \; .$$

One sees from this, for example, how the electromotive forces in *galvanic cells* can be determined in that way in the same absolute units as the electromotive forces that *geomagnetism* will exert upon closed circuits while they are moving. However, it is important to measure electromotive forces that are created by different sources, such as hydroelectric and magneto-electric forces, in the same units, because the road to a comparative study of those sources will be paved in that way. That will be easy and simple when it is applied to a conductor of known absolute resistance, without such a conductor being linked with great difficulties, such as, for example, when one wishes to make the comparison in the following way:

Let the galvanic cell whose electromotive force is to be compared with a magneto-electric force, without being applied to a conductor of known absolute resistance, be connected by a conductor of arbitrary length and form, and then let everything be rotatable. A second current will then arise in the circuit due to the rotation, namely, in addition to the current that arises from the electromotive force of the cell itself, there is another current that originates in the electromotive force of geomagnetism. It is in one's power to arrange that the direction of both currents in the circuit are opposite by choosing the *direction* of rotation. On the other hand, one can make the intensities of both currents equal, at least for a small time interval during which both currents will cancel, by choosing the *velocity* of rotation. However, if the intensities of both currents are equal, then it will follow that the electromotive forces are equal in that case, i.e., the equality of the electromotive force in the *cell* with the electromotive force of *geomagnetism*. That latter is given immediately in absolute units by the known value of geomagnetism and the form and rotation of the closed circuit. As a result, the electromotive force of the *cell* will also be found in the same units in that way. However, that in itself explains the fact that the comparison of those forces will be achieved much simpler and more easily in that aforementioned way with the help of absolute resistance units.

IV - On the Principles of Various Absolute Systems of Units in Electrodynamics

13.26 Self-sufficient Basis for Absolute Units in Electrodynamics, Without Reference to Magnetic Units

Just as no intrinsic *basic unit* needs to be defined for the magnitudes of velocities when such units for space and time are given already, as we have seen, no intrinsic *basic unit* needs to be defined for the magnitudes of galvanic resistances when units for the magnitudes of electromotive forces and current intensities are given already. However, no intrinsic *basic units* need to be assumed for the last two types of quantities either, since one can also give *absolute* units for them, which was done with the definitions that were given in Section 13.10 *by reducing the magnetic* unit to *three basic mechanical units*.

For most electrodynamic measurements, it is indeed sufficient and convenient to reduce the units of the electrodynamic quantities to the established *magnetic unit*, as was done in Section 13.10. However, it is in the nature of things that the dependency by which the electrodynamic units were converted to the magnetic units is by no means well-established, which is explained by the fact that the basic electrodynamic laws are independent of the magnetic ones. Rather, the electrodynamic unit can be established in yet another way that makes it completely independent of how the magnetic unit was defined. In order to do that, it is merely necessary to revert to the basic laws of *electrodynamics* and *voltaic induction*,[417] instead of starting from the basic laws of *electromagnetism* and *magneto-electricity*, as was done in Section 13.10.

The fundamental law of *electrodynamics* gives the following formula for the magnitude of the *force of repulsion* between two current elements α, α' with current intensities i, i' at a distance of r apart, and which makes angles ϑ, ϑ' with the two current directions, while the angle between the two current directions is equal to ε, namely:

$$-\frac{\alpha\alpha'}{r^2}ii'\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right) \ .$$

The fundamental law of *voltaic induction*, as it was given in Section 30 of the first treatise on "Elektrodynamische Maassbestimmungen,"[418] gives the following formula for the electromotive force that a current element α with current intensity i exerts upon another element α' at a distance of r, when r makes the angles ϑ and ϑ' with the current direction and the direction in which α' is displaced with a velocity of v, respectively, and the last two directions subtend an angle of ε:

$$-\frac{\alpha\alpha'}{r^2}vi\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right) - \frac{1}{2}\frac{\alpha\alpha'}{r}\cos\vartheta\cdot\frac{di}{dt} \ .$$

That force, which acts in the direction r, must be decomposed along the direction of α', because the component that is perpendicular to α' will be cancelled. If η denotes the angle that α' makes with r, then the formula above must be multiplied by $\cos\eta$.

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[417][Note by AKTA:] See footnote 139 on page 96.

[418][Note by AKTA:] See Section 5.30.

Now, *first of all*, an absolute unit of *current intensity* can be defined by that fundamental law independently of the magnetic unit in the following way:

When a current circulates around a unit area and acts upon a current that circulates around a similar area at a great distance of R, and both areas are perpendicular in such a way that the extension of the first area bisects the second, the unit of current intensity will be the current intensity that the former current will possess when it exerts a rotational moment on the latter current that has a ratio to the unit of rotational moment of $1 : 2R^3$.

That new *absolute* unit of current intensity can be defined even more simply when one is permitted to reduce the interaction of closed current to the interaction of individual current elements that cannot be observed directly, because such current elements are present only as components of closed currents, namely:

The unit of current intensity is the current intensity that a current element will possess when it exerts a force of attraction on an equal and parallel current element that is perpendicular to the connecting line at a distance that is equal to one unit of length, and the ratio of that force to the unit of force will be that of the square of the length of that current element to the unit of area.

This second *absolute* unit of current intensity is not equal to the first one, which depends

upon magnetic units, but has a ratio with it of $1 : \sqrt{2}$.[419],[420]

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[419][Note by WW:] The derivation of the definitions that were exhibited above from the fundamental laws of electrodynamics is as follows:

First definition. — In Section 9 of the earlier treatise "Elektrodynamische Maassbestimmungen," Leipzig 1846 [Wilhelm Weber's *Werke*, Vol. III, p. 86], the following expression for the rotational moment that a planar current exerts upon another one at a distance was derived already from the fundamental laws of electrodynamics, namely, [note by AKTA: the expression $\cos\psi^2$ should be understood as $\cos^2\psi$]:

$$\frac{1}{2}\frac{ii'\lambda\lambda'}{r^3}\sin\delta \cdot \sqrt{1 + 3\cos\psi^2} \; ,$$

in which i, i' denote the current intensities, λ, λ' are the areas that are circulated around, r is the distance between their midpoints, ψ is the angle between the normal to the first planar current and r, and δ is the angle that the second planar current subtends with the *directive force*. However, the directive force is included in the plane that is laid through the normal to the first planar current A and the midpoint of the second planar current C, and in the triangle ACB, rectangular at C, whose hypotenuse AB is the normal to the planar current A, it is parallel to the line CD that cuts the side AB of the triangle at D in such a way that $AD : DB = 1 : 2$. — Now, under the circumstances that were indicated in the first definition, one will have $i = i'$, $\lambda = \lambda' = 1$, $\delta = \psi = \pi/2$, $r = R$, which will make the *rotational moment* take the value:

$$\frac{i^2}{2R^3} \; ,$$

which will have a ratio with the *unit* of rotational moment of $1 : 2R^3$ when $i = 1$.

Second definition. — In the expression for the *force of attraction* between two current elements that is given directly by the fundamental law of electrodynamics:

$$\frac{\alpha\alpha'}{r^2}ii'\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right)$$

under the circumstances that were indicated in the definition, one will have $i = i'$, $\alpha = \alpha'$, $\vartheta = \vartheta' = \pi/2$, $\varepsilon = 0$, $r = 1$, which will make the *force of attraction* take the value:

$$\alpha^2 i^2 \; ,$$

which will have a ratio to the *force unit* of $\alpha^2 : 1$ when $i = 1$.

It still remains to be proved that the second *absolute* unit of current intensity that was presented here has a ratio to the first one, which depends upon magnetic units, of $1 : \sqrt{2}$.

The expression for the *rotational moment* that a magnet m exerts upon another one m' at a distance r was given before in Section 9, *loc. cit.*, according to known laws, namely:

$$\frac{mm'}{r^3}\sin\delta \cdot \sqrt{1 + 3\cos\psi^2} \; ,$$

where ψ and δ have the cited meanings when one exchanges the normals to the two planar currents with the axes of the two magnets in it. Now, if one denotes the first unit of current intensity by K, which depends upon magnetic units, and the second one by J, in order to distinguish the two units of current intensity, then kK and $k'K$ will be two well-defined current intensities that can be expressed in terms of the first unit, and iJ and $i'J$ will be the same current intensities, which can be expressed in terms of the second unit as follows:

$$iJ = kK \qquad \text{and} \qquad i'J = k'K \; .$$

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Secondly, the unit of *electromotive force* will be defined in the following way using the cited fundamental law of voltaic induction, independently of the magnetic unit:

> The unit of electromotive force is the electromotive force that a current that circulates around a unit area exerts upon a conductor that is bounded by a surface that has the same area as the first one, is perpendicular to it, and is bisected by it at a distance of R when its intensity has a ratio with the *absolute* unit that was presented of $2R^3 : 1$, while the conductor is rotated with a unit angular velocity around the line of intersection of the two areas.

If it were permissible to revert to the electromotive force on an isolated current element, then that definition could be made simpler in the following way:

> The unit of electromotive force is the electromotive force that a current element exerts upon an equally-long conductor element that is perpendicular to it, parallel to the connecting line, and at a distance of one unit of length, when its intensity has the same ratio to the absolute unit that was presented, as the ratio of the *unit area* to the *square of the length* of that element, while the conductor element is displaced with unit velocity in the direction that is parallel, but opposite to the direction of current.[421]

According to the fundamental law of electromagnetism, the *rotational moment* will remain unchanged when one sets the magnet m equal to the current kK that circulates around an area $\lambda = m/k$. If one sets, in the same way, the magnet m' equal to the current $k'K$ that circulates around an area of $\lambda' = m'/k'$, then one will get the *rotational moment* that the first planar current exerts upon the second one:

$$\frac{kk'\lambda\lambda'}{r^3}\sin\delta \cdot \sqrt{1 + 3\cos\psi^2} \ .$$

However, the following value for that rotational moment was found above:

$$\frac{1}{2}\frac{ii'\lambda\lambda'}{r^3}\sin\delta \cdot \sqrt{1 + 3\cos\psi^2} \ ,$$

from which it would follow that $ii'/2 = kk'$, i.e., when $k = k'$, $i = i'$:

$$i = k\sqrt{2} \ .$$

Thus, the equation $iJ = kK$ will give:

$$J : K = 1 : \sqrt{2} \ .$$

[420][Note by AKTA:] See Section 5.9. The expression $\cos\psi^2$ should be understood as $\cos^2\psi$.

[421][Note by WW:] In order to derive the first of those two new definitions of an *absolute* unit for electromotive force from the general law of voltaic induction, one first observes that the inducing current i in the definition is taken to be constant, so $di/dt = 0$, which means that the general expression for the electromotive force that is exerted in the direction r will be reduced to:

$$-\frac{\alpha\alpha'}{r^2}vi\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right) \ .$$

However, the similar expression for the force of attraction between two current elements:

$$-\frac{\alpha\alpha'}{r^2}ii'\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right)$$

yields the direction and magnitude of the force that a current i that circulates around an area λ will exert

The second *absolute* unit for electromotive force is not equal to the first, which depends upon magnetic units, but has a ratio to it of $\sqrt{2} : 1$.

Thirdly, it is self-explanatory that the definition of the third electrodynamic unit, namely, *resistance*, can also be made independent of the magnetic unit when the *absolute* units of current intensity and electromotive force in the definitions that were given in Section 13.10, which depend upon the magnetic unit, are switched with the new units that are independent of the magnetic unit, so the definition will remain completely unchanged. The ratio of this new unit to the old one that was given will then imply that the new *absolute* unit of resistance

upon the current element α', namely, that *first of all*, the direction is perpendicular to the plane that is laid through the current direction i' and the *directive force*. (The directive force is contained in the plane that is laid through the normal to the inducing planar current A and the midpoint of the induced element C, and is parallel to the line CD in the triangle ACB, rectangular at C, whose hypotenuse AB is the normal to the planar current A that cuts the side AB of the triangle at D in such a way that $AD : DB = 1 : 2$.). *Secondly*, the magnitude of the force is equal to:

$$\frac{1}{2}\frac{\lambda \alpha'}{r^3} ii' \sin \delta \cdot \sqrt{1 + 3\cos \psi^2} \; ,$$

where ψ denotes the angle between the normal to the planar current and r, and δ denotes the angle that the current direction in α' makes with the directive force. — Similarly, from the foregoing expression of the electromotive force that a current element exerts upon a conductor element in the direction of the line that connects them, results the direction and magnitude of the electromotive force that the entire current i that circulates around the area λ exerts upon the conductor element α', namely, *first of all*, the direction is perpendicular to the plane that is defined by the path along which α' is displaced and the direction of the *directive force*, and *secondly*, the magnitude is equal to:

$$\frac{1}{2}\frac{\lambda \alpha'}{r^3} vi \sin \delta \cdot \sqrt{1 + 3\cos \psi^2} \; ,$$

where ψ denotes the angle that the normal to the planar current makes with r, and δ is the angle that the direction in which α' is displaced makes with the directive force. (See Section 9, *loc. cit.*, Note on page 264 [Wilhelm Weber's *Werke*, Vol. III, p. 86], about which it should be remarked that ε has the same meaning there that δ has here, but the factor $\sin \varepsilon$ was mistakenly omitted from the formula for the force that a planar current exerts upon the moving element, which depends upon the direction of motion of that element.)

Now, if the conductor element α' also belongs to the boundary of an area λ' whose normal is parallel to the direction in which the conductor element was displaced (as a result of a rotation of the conductor around its plane bisector), and therefore makes an angle of δ with the *directive force*, then one must decompose each element α' of the boundary line into two elements ds and $d\sigma$ such that one of them is parallel to the line in which a plane that is normal to the directive force CD cuts the plane of the conductor, while the other one is perpendicular to that line of intersection. One can arrange the former elements to have equal length $ds = ds'$ pair-wise and be connected by the perpendicular x to that line of intersection. If one lets a, b, c denote the distances between the elements ds and ds' and the point of intersection of the perpendicular x with the rotational axis of that line of intersection, and further lets γ denote the angular velocity, while δ' is the angle that the rotational axis that bisects the plane of the conductor makes with that line of intersection, then if v and v' denote the velocities with which the elements ds and ds', respectively, are displaced, one will have:

$$\begin{aligned} a - b &= x \; , \\ (a - c)\gamma \cos \delta' &= v \; , \\ (b - c)\gamma \cos \delta' &= v' \; . \end{aligned}$$

If one further observes that the direction of the electromotive force that was given above is directly parallel to the one element ds and parallel, but opposite, to the other one ds', then one will get the electromotive force, when it is decomposed in the directions of both elements:

has a ratio of $2 : 1$ with the one that was defined in Section 13.10.

$$+\frac{1}{2}\frac{\lambda i}{r^3}\sin\delta\cdot\sqrt{1+3\cos\psi^2}\cdot\gamma\cos\delta'\cdot(a-c)ds\ ,$$

$$-\frac{1}{2}\frac{\lambda i}{r^3}\sin\delta\cdot\sqrt{1+3\cos\psi^2}\cdot\gamma\cos\delta'\cdot(b-c)ds\ .$$

As a result, their sum will be:

$$+\frac{1}{2}\frac{\lambda i}{r^3}\cdot\gamma\cos\delta'\sin\delta\cdot\sqrt{1+3\cos\psi^2}\cdot xds\ .$$

It ultimately follows from this that the sum of all of the electromotive forces that are exerted by the elements of the closed conductor that are parallel to the line of intersection above, when decomposed along the direction of the conductor, will be:

$$+\frac{1}{2}\frac{\lambda i}{r^3}\cdot\gamma\cos\delta'\sin\delta\cdot\sqrt{1+3\cos\psi^2}\cdot\int xds\ ,$$

that is, since the integral $\int xds$ denotes the magnitude λ' of the area that is bounded:

$$+\frac{1}{2}\frac{\lambda\lambda'}{r^3}i\gamma\cos\delta'\sin\delta\cdot\sqrt{1+3\cos\psi^2}\ .$$

If one similarly considers the electromotive forces that act upon all elements $d\sigma$ that are perpendicular to the line of intersection above and decomposes them along their directions, then one will find that their sum is equal to zero. As a result, the formula above will express the total electromotive force that the planar current exerts upon the closed conductor.

If one applies that expression to the ratio that was indicated in the first definition, in which one has $i = i'$, $\lambda = 1$, $r = R$, $\gamma = 1$, $\delta = 0$, $\varepsilon = \psi = \pi/2$, then that will yield the value of the electromotive force:

$$\frac{i}{2R^3}\ ,$$

i.e., it will be equal to 1 when $i = 2R^3$.

Second definition. — The general expression for the electromotive force of a current element on a conductor element that was cited above:

$$-\frac{\alpha\alpha'}{r^2}vi\left(\cos\varepsilon-\frac{3}{2}\cos\vartheta\cos\vartheta'\right)\cos\eta-\frac{1}{2}\frac{\alpha\alpha'}{r}\cos\vartheta\cos\eta\cdot\frac{di}{dt}$$

will reduce to the value:

$$\alpha^2 i\ ,$$

when one applies it to the situation that was indicated in the second definition of the *absolute* unit of electromotive force, where $\alpha = \alpha'$, $\varepsilon = \eta = 0$, $\vartheta = \vartheta' = \pi/2$, $r = 1$, $v = -1$, $di/dt = 0$, i.e., to *unity*, when the intensity of the inducing current i has a ratio of $1 : \alpha^2$ to the established unit of intensity.

Finally, the ratio of this second absolute *unit* of electromotive force that was presented here to the first one, which depends upon the magnetic unit, is obtained as follows: In the expression for the *rotational moment* of the previous footnote that a magnet m exerts upon another one m' at a distance of r, namely:

$$\frac{mm'}{r^3}\sin\delta\cdot\sqrt{1+3\cos\psi^2}\ ,$$

13.27 On the Relationship Between the Absolute Units in Electrodynamics and the Ones in Mechanics

An *electromotive* force is any force that seeks to move the two electric fluids at a location in opposite directions. However, such forces are all forces that are determined by the basic law of *electrostatics*, because all of those forces are forces of attraction and repulsion, and indeed the same force that is a force of attraction for the one electric fluid will necessarily be a force of repulsion for the other one. Now, since all types of electromotive forces are comparable to each other, and therefore all of them can be expressed in the units by which any one of them was measured, that will explain the fact that all types of electromotive

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if one replaces the magnet m' by the current $k'K$ that circulates around the area $\lambda' = m'/k'$, then one will obtain the rotational moment that the magnet m exerts upon that current:

$$\frac{m\lambda'}{r^3}k'\sin\delta \cdot \sqrt{1 + 3\cos\psi^2}\ ,$$

and from this, using known relations that exist between the electromagnetic laws and the magneto-electric ones, and which one will find discussed in detail in *Supplement* D at the end of this treatise, one will obtain the electromotive force that the magnet m exerts upon the closed current conductor when the latter is rotated with unit angular velocity in the direction that is opposite to that rotational moment, when one sets $k' = 1$, namely:

$$\frac{m\lambda'}{r^3}\sin\delta \cdot \sqrt{1 + 3\cos\psi^2}\ .$$

Finally, if one also sets the magnet m in this equal to a current kK that circulates around the area $\lambda = m/k$, then one will obtain the electromotive force that this current will exert upon that closed current conductor under the rotation of it that was described as:

$$\frac{\lambda\lambda'}{r^3}k\sin\delta \cdot \sqrt{1 + 3\cos\psi^2}$$

when expressed in terms of the *first unit*, which was expressed in terms of the *second unit* as:

$$\frac{1}{2}\frac{\lambda\lambda'}{r^3}i\gamma\cos\delta'\sin\delta \cdot \sqrt{1 + 3\cos\psi^2}\ ,$$

i.e.:

$$\frac{1}{2}\frac{\lambda\lambda'}{r^3}i\sin\delta \cdot \sqrt{1 + 3\cos\psi^2}\ ,$$

when one observes that $\gamma = 1$ and $\cos\delta' = 1$.

If one now denotes the first unit by E and the second one by E', in order to distinguish the two units, and denotes the same electromotive force in both units by eE and $e'E'$, respectively, when one observes that one had $i = k\sqrt{2}$, that will imply that:

$$e = \sqrt{\frac{1}{2} \cdot \frac{\lambda\lambda'}{r^3}i\sin\delta \cdot \sqrt{1 + 3\cos\psi^2}}\ ,$$

$$e' = \frac{1}{2} \cdot \frac{\lambda\lambda'}{r^3}i\sin\delta \cdot \sqrt{1 + 3\cos\psi^2}\ ,$$

and as a result, since $eE = e'E'$, one will have:

$$E' : E = e : e' = \sqrt{2} : 1\ .$$

forces must be capable of being expressed in the unit that is established for electric forces in *electrostatics*, and the fact that one will then require no other unit for electromotive forces than the one that is used for electrostatic forces. However, in electrostatics, the electric forces are measured, not with a special unit, but with the same unit as all forces in *mechanics*, since that force unit will be taken to be the force that imparts one unit of acceleration upon a unit of ponderable mass when that force acts upon the mass. The electric force that is exerted upon an electric particle is then equal to 1 when the unit of ponderable mass that the electric particle is endowed receives a unit of acceleration as a result of it. One sees from this that establishing a special unit for the *electromotive* forces is not at all necessary, but that the unit that is established for all forces in mechanics is sufficient.

A similar argument can be applied to the *intensity* of electric currents when one takes the unit in mechanics to be those current strengths or current intensities for which a unit mass of any fluid will flow through the cross-section of the channel in a unit time interval. Now, since the mass-unit of electric fluids in electrostatics has been determined already, namely, the mass[422] that will exert a force on an equal mass at a distance R that has a ratio to the unit of force of $1 : R^2$, that will explain the fact that one does not require a special unit for the intensity of electric currents.

Now, if the use of all special units for electromotive forces and current intensities is to be avoided completely, then a rule must be found for reducing the measurements that were quoted in terms of special units up to now in order to represent them independently of those special units.

In order to find that rule, it does not suffice to revert to the basic laws of electrostatics, electrodynamics, and induction, but it is necessary to revert to the fundamental general law of the theory of electricity, which likewise subsumes and connects electrostatics, electrodynamics, and induction, and which was presented in the previous treatise "Elektrodynamische Maassbestimmungen," Leipzig, 1846.[423],[424] According to the latter law, the force that the electric mass e exerts upon the electric mass e' at a distance r with a relative velocity dr/dt and a relative acceleration d^2r/dt^2 is represented by:[425]

$$\frac{ee'}{r^2}\left(1 - \frac{1}{c^2}\cdot\frac{dr^2}{dt^2} + \frac{2r}{c^2}\frac{d^2r}{dt^2}\right)\ ,$$

in which $1/c^2$ is the same constant factor that was denoted by $a^2/16$ in that treatise.

For a constant value of the relative velocity dr/dt, one will have $d^2r/dt^2 = 0$, and as a result, the force will equal:

$$\frac{ee'}{r^2}\left(1 - \frac{1}{c^2}\cdot\frac{dr^2}{dt^2}\right)\ ,$$

which implies that *c means the constant value of the relative velocity dr/dt for which two electric masses have no effect at all on each other.*

Now, it was further verified in Section 21 of the cited treatise that the number i, which is attached to the unit J that was defined in the foregoing Section, will determine a *current intensity*:[426]

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[422][Note by AKTA:] That is, the electric mass or the electric charge.
[423][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 157.
[424][Note by AKTA:] See footnote 198 on page 149.
[425][Note by AKTA:] The expression dr^2/dt^2 should be understood as $(dr/dt)^2$.
[426][Note by AKTA:] See Section 5.21.

$$i = aeu = \frac{4}{c} \cdot eu \ ,$$

where eu denotes the amount of electricity that goes through the cross-section of the conductor during a unit time interval with that current intensity. Now, if that current intensity were expressed in the general current unit K that is established in mechanics by:

$$kK = iJ \ ,$$

then one would have:

$$k = eu = \frac{c}{4} i \ .$$

That implies the rule by which the measurements that were carried out in the previous Section and defined in special units can be reduced in order to make them independent of those special units, namely: One multiplies the values obtained by $c/4$. In that way, one will get the value of the electric current strength, as expressed in the general current unit of mechanics.

It was likewise found in Section 24 of the cited treatise that an *electromotive force*, that is determined by a number e and special unit E that was defined in the previous Section, will be determined in the general unit F for all forces in mechanics by the number f such that $fF = eE$ when one makes:[427]

$$f = \frac{4}{c} e \ ,$$

because the following expression for the electromotive force that a constant current exerts upon a moving conductor, in the general unit of force in mechanics, was given in Section 24 of the cited reference by:

$$f = -\frac{\alpha \alpha'}{r^2} i \left(\cos \varepsilon - \frac{3}{2} \cos \vartheta \cos \vartheta' \right) \cdot au' \cos \varphi \ .$$

However, under certain circumstances for which the electromotive force that is determined in that way is equal to the special unit that was defined in the previous Section, one will have:

$$\frac{\alpha \alpha'}{r^2} i = 1, \qquad \varepsilon = 0, \qquad \vartheta = \frac{1}{2}\pi, \qquad \varphi = \pi, \qquad u' = 1 \ ,$$

and as a result, for $e = 1$, one will have $f = a = 4/c$, or more generally:

$$f = \frac{4}{c} e \ .$$

That implies the rule by which the measurements of electromotive forces that were performed in the previous Section in terms of the special units are reduced in order to make them independent of that special unit, namely: One multiplies the values obtained by $4/c$. In that way, one will get the value of the electromotive force, as expressed in terms of the general force unit of mechanics.

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[427][Note by AKTA:] See Section 5.24.

Finally, should an absolute *unit of resistance* be derived from those general force and current units of mechanics when one uses them for electromotive forces and electric current in the same way as in the previous Section from the special units that were defined there, namely, such that the resistance that was taken to be a unit was the resistance that a circuit would have to possess in order for a unit of electromotive force to produce a unit of current, then if w denotes that resistance with the unit that was defined in the previous Section, which is v in the new unit, that will yield the following equation:

$$v = \frac{16}{c^2} w \ .$$

The *velocity c* with which two electric masses must move with respect to each other if they are to not act upon each other at all has still not been ascertained up to now,[428] and that is the reason why the special unit, as it was defined in the previous Section, is still indispensable for practical use in electrodynamics at the moment, because without knowing the velocity c, the reduction of measured *current intensities*, *electromotive forces*, and *resistances* to the known units of mechanics cannot be performed.

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[428][Note by AKTA:] Weber and Rudolf Kohlrausch (1809-1858) determined this constant experimentally a few years later, in 1855-6, [KW57] with English translation in [KW21].

V - On the Connection Between the Theory of Galvanic Circuits and the Basic Laws of Electricity

13.28   On the Adjustment of the Electromotive Forces in the Circuit through the Distribution of Free Electricity

The theory of galvanic circuits, by itself, defines a part of electrodynamics, and the connection between the laws of galvanic circuits and the basic laws of electricity must then be developed.[429] That has not happened up to now. Rather, the theory of galvanic circuits was considered by itself, and the laws of galvanic circuits are partly inferred directly from experience and partly derived from assumptions that are made completely independently of the basic laws of electricity. Namely, that is true of the laws of galvanic circuits, as Ohm gave them,[430] whose validity and practical significance are generally acknowledged, moreover. The reason why such a development of the theory of galvanic circuits from the basic laws of electricity has still not been given up to now might lie mainly in the mathematical complexity that one would find in such a development if it were to be complete and rigorous. Nonetheless, a special discussion regarding the connection between the theory of galvanic circuits and the basic laws of electricity that is more closely linked with the topics that are considered in this treatise might be appropriate here.

In the course of this treatise, it was frequently necessary to refer to Ohm's law of galvanic circuits, which was necessary since all resistance measurements are essentially based upon that law, and even the definition of the resistance unit must be based upon it. That is because the resistance is basically defined only by the constant that is given by Ohm's law for any closed conductor as the ratio of the electromotive force to the current intensity.

Ohm's law assumes that the intensity of the electric current is the same in all parts of the closed circuit since that must actually be the case when it is steady. By that assumption, the domain in which Ohm's law is valid is restricted and does not subsume all motions of electricity in the circuit. That is because it excludes, e.g., all motions that the electricity must make in the circuit before it reaches a steady state. That also explains why that law is based empirically only to the extent that it concerns the dependency of the current intensities in all parts of the circuit where it becomes equal upon the sum of all electromotive forces in the circuit and the sum of all resistances in all of its parts, while an actual basic law of current intensity must make the current intensity at any location in the circuit depend upon only the electromotive force that acts at that location and the resistance that exists at that location. Now, Ohm had indeed directed his attention to the difference between the electric charges in the various parts of the circuit in order to arrive at an actual basic law and had sought to base the law upon the assumption that for equal current intensities, the difference between the charges at two locations between which there is no electromotive force (e.g., there is no point of contact between different metals) will be proportional to the resistance of the part of the circuit that lies between the two locations and that, by contrast,

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[429][Note by AKTA:] Sections 13.28 up to 13.36 have been discussed in detail in Appendix A of the book *The Electric Force of a Current: Weber and the Surface Charges of Resistive Conductors Carrying Steady Currents.* It is available in English, [AH07], Portuguese, [AH09], and German, [AH13].

[430][Note by AKTA:] See footnote 99 on page 53.

at any such location where an electromotive force exists (e.g., where two different metals are in contact), the charge will make a sudden jump from one side to the other and that the difference in charge between both sides will be proportional to the electromotive force that exists at that location; finally, that for different current intensities, the difference between the electric charges at two well-defined locations of the same conductor will be proportional to the current intensity. Guided by that, Ohm then exhibited a basic law for the dependency of the electric current in each part of the circuit upon the distribution of electric charge that was analogous to the one that Fourier[431] had exhibited for the dependency of the heat current in any part of a heat conductor on the distribution of temperature and had proved that the consequences that could be inferred from that analogy agreed with experiments to the extent that those results could be authenticated.

Ohm had really found in the distribution of electric charge the true key for opening the door to the path from the empirically-based law that subsumed all closed circuits to the true basic law that would have to be exhibited for each part of the circuit in general. However, as far as the effect of that electric distribution on the motion of the electricity was concerned, which he had considered merely by analogy with the effect of the temperature distribution on the motion of heat, it was based upon assumptions that were neither necessary nor did they even seem admissible. That is because the effect of free electricity is given already by the fundamental general law of the theory of electricity, or when one abstracts from relative motions, by the basic law of electrostatics, and can be calculated from it for any distribution in the conductor, from which one can easily verify the inadmissibility of arbitrary assumptions about mere analogies with the effect of the temperature distribution on the motion of heat. Even as far as the distribution itself is concerned, it seems inadmissible from that to assume a distribution of free electricity that is not one on the *surface* of the conductor. Furthermore, that will also explain an essential difference, namely, that a relation must necessarily exist between the propagation of heat and the temperature decrease that exists in its direction, such that the former is not even possible without the latter. Such a dependency of the electric current on the distribution of free electricity does not exist in galvanic circuits because the forces that produce the electric current act, not merely upon the immediate neighborhood, but also at great distances, and can therefore exist completely outside of the conductor, which is not possible for a heat conductor.

Take, e.g., a circular copper ring whose cross-section is the same everywhere to be the conductor and move a magnet along the line that is drawn through the center of the ring perpendicular to its plane. It is known that the magnet exerts the same electromotive force on all elements of the ring as a result of that motion, and since all elements are also endowed with the same resistance, an equal electric current will be generated simultaneously in all of the elements by that motion, from which it will follow that a greater or smaller accumulation of positive or negative electricity cannot arise at any location on the ring. We will then have the case here of a current in a closed circuit with no distribution of free electricity in the circuit. The law for the dependency of the current intensity on the distribution of free electricity in the conductor will not be applicable then in all of the cases for which the given electromotive forces extend over the entire closed circuit and act proportionally in all parts of the resistances. It is only for a nonuniform action of the given electromotive forces in the various parts of the circuit that a distribution of free electricity will occur, and the fact that a steady, uniform current exists in all parts of the circuit will then prove that this distribution of free electricity in the conductor has the effect of balancing out all of the inequalities

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[431][Note by AKTA:] J.-B. J. Fourier (1768-1830). See [Fou22] with English translation in [Fou52].

in the way that the electromotive forces acted originally. However, if that balancing were considered to be proof of the fact that a steady current exists, then it would still remain: *first of all*, to prove that such a distribution is possible from the basic law of electricity and how it must be addressed, and *secondly*, to show how it could be created and maintained.

13.29 Verifying the Possibility of a Distribution of Free Electricity in a Conductor for which the Inequalities in the Effectiveness of Given Electromotive Forces in Various Parts of the Circuit can be Compensated in Proportion to Their Resistances

Any particle of free (positive or negative) electricity that is found on the surface of a conductor exerts electromotive forces on all parts of the conductor that weaken the given electromotive forces in the circuit at some locations and strengthen them at others, and one then asks whether such a distribution of free electricity on the entire surface of the conductor is possible that would make the electromotive force stronger wherever it is weak and weaker wherever it is strong, and in that way bring about an equilibration of the electromotive forces in all parts of the circuit in proportion to their resistances, which is the condition for a steady, uniform current. When the influence of the relative motions of the electric particles with respect to each other has been removed beforehand, that question must be decided by the basic law of electrostatics that says that the forces that are exerted upon all points in the interior of the conductor is determined by the electricity in any arbitrary distribution on the surface.

Poisson[432] is known to have proved the following theorem from the basic law of electrostatics:

> When arbitrary electric forces act upon a conductor of arbitrary form from the outside, a distribution of free electricity on the surface of the conductor is always possible — but only one of them — for which the electric forces that originate from that distribution of free electricity will likewise be in equilibrium with the electric forces at all points of the interior of the conductor that act from the outside.

One next imagines a conductor of cylindrical form and a concentrated mass of free (positive or negative) electric fluid in the direction of its axis at a great distance that exerts forces on all parts of the cylinder that are equal and parallel to its axis. It will then follow from the proposition above that it is possible to have a distribution of free electricity on the surface of the cylinder for which, in the absence of that distant mass, electromotive forces would result on all parts of the cylinder that are equal and parallel to its axis, namely, the forces that had maintained the equilibrium of the forces that were exerted by the distant mass before it was removed.

By contrast, if one images a bent rod and a concentrated mass of free (positive or negative) fluid in the direction of the tangent to one of its elements and at a great distance, then it

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[432][Note by AKTA:] S. D. Poisson (1781-1840).

will likewise follow that it is possible to have a distribution of free electricity on the surface of that element such that in the absence of that distant mass, electromotive forces will result in all parts of the element that are equal and parallel to its tangent, and that possibility will also remain when the electric charges in all other elements of the bent rod act upon the element in question, no matter how those charges might have been obtained, except that the way that the free electricity is distributed on the surface of the element considered will then depend upon the charge in the rest of the rod.

That argument can now be applied to all elements of the bent rod such that electromotive forces will result in all elements that are equal and parallel to their tangents. The charges in all individual elements will be made dependent upon the charge in the entire rod in that way, and the charge in the entire rod, in turn, must ultimately be set equal to the sum of the charges on all elements.

Now that the charge in the entire bent rod has been determined in that way, the rod might define only a smaller or larger part of a circle. The charges in the contact surfaces between any two elements that bound each other must neutralize in such a way that the distribution of free electricity is still restricted to the *surface of the rod*, of which essentially the parts on the *initial surface* and the *final surface* will be calculated, which cannot coincide then.

The necessity of keeping the initial and final surfaces of the rod separate when the free electricity that is distributed on the surface is supposed to exert equal electromotive forces on all elements of the rod in tangential directions follows from the fact that the charges on the initial and final surface of the rod cannot approach a well-defined limiting value when they approach each other, but must grow to infinity. One can convince oneself of that by the following argument:
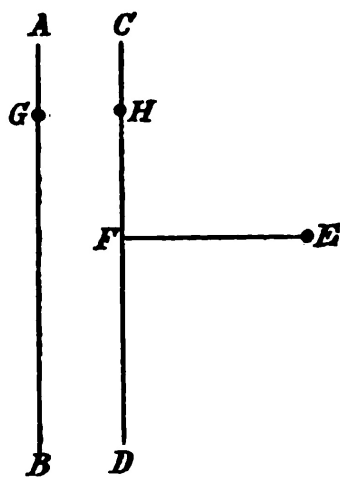
Fig. 2.

Let AB [in Figure 2] represent the initial surface of the rod and let CD represent its final surface; call the very small distance between both surfaces δ. It can be assumed that the distribution of free electricity on the entire surface of the rod, with the exception of AB and CD, remains almost unchanged when δ gets smaller, from which it will follow that the electromotive force that results at a point E of the rod can be regarded as unchanged as long as the electromotive force that results at E from the charges on the two surfaces AB and CD stays the same. Let G and H be two equal mutually-opposed elements of the

surfaces AB and CD. The charge on the element G is denoted by $-e$, while the charge on the element H is denoted by $+e$. The distance FH, which is perpendicular to the direction of the electromotive force that results at E, is called β; the distance FE is called α. From the basic law of electrostatics, the force that acts from H to F in the tangential direction EF then proves to be equal to:

$$\frac{+\alpha e}{(\alpha^2 + \beta^2)^{3/2}} \, ,$$

while the force that acts from G to F in the same direction is equal to:

$$\frac{-(\alpha + \delta)e}{[(\alpha + \delta)^2 + \beta^2]^{3/2}} \, ,$$

and as a result, the sum of the two forces will be:

$$= \frac{2\alpha^2 - \beta^2}{(\alpha^2 + \beta^2)^{5/2}} \cdot \delta e \, ,$$

when δ is very small compared to α. It then follows from this that the electromotive force that results at E will remain unchanged as δ gets smaller when the product δe keeps the same value. For vanishing values of δ, the charge e must then grow to infinity, which was to be proved.

That likewise explains the fact that when the electromotive force, which is the same in the entire rod, is supposed to increase or decrease, the value of the product δe must change proportionally.

Finally, if K denotes a point that lies between the surfaces AB and CD, then that will explain the fact that the charges on the surfaces AB and CD exert an electromotive force on K that has the opposite direction to the one on E. Therefore, should a closed circuit be defined in which equal electromotive forces act everywhere in the same sense (which would be necessary for a steady, uniform current to exist), then K would have to be the site of an electromotive force that is independent of the distribution of free electricity on the surface of the rod, which would be the case, e.g., when copper and zinc contact each other at the point K. It can also be proved that, under otherwise-equal conditions, the given electromotive force at all points K of the line δ that connects the two oppositely-charged surfaces, must be proportional to the product δe, and that this product can be considered to be a unit for the given electromotive force.[433]

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[433][Note by WW:] AB and CD [in Figure 3] represent the two oppositely-charged surfaces whose separation distance is equal to δ.
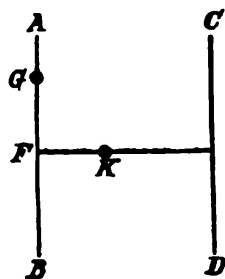

Fig. 3.

Let G be the element of the surface AB whose charge will be denoted by e. The distance FG, which is

The following results can be inferred from those general considerations, which can be compared with the known laws of galvanic circuits:

1. It follows from the argument above that it is not possible to produce current in a closed ring by merely distributing free electricity on its surface, but electromotive forces must be given, at least in *some cross-section* of the ring, e.g., by copper contacting zinc, if a steady, uniform current is supposed to come about in the entire ring by means of a certain distribution of free electricity on the surface of the ring.

2. If the current is to be doubled in a certain circuit, then the amount of free electricity on the entire surface must be doubled. As a result, a doubling of the factor e in the product δe must also take place, i.e., a doubling of the electromotive force that is proportional to it. A doubling of the electromotive force then corresponds to a doubling of the current intensity in that circuit.

3. If *all* dimensions of a circuit are doubled, while the electromotive forces at all points remain just as large as before, then the density of the layer of electricity at corresponding locations on the surface will have to remain unchanged, while the location on the surface that covers it will have to be four times larger. At the same time, the proportional expansion of all dimensions implies that the distance δ in the product δe must be thought of as being doubled, so since e is to remain unchanged, the product δe and the electromotive force that is proportional to it must be doubled. It follows from this that a doubled electromotive force will be required in order to produce an electric motion in a circuit of twice the length and four times the cross-section that is just as strong as it is in a circuit of one-fold length and one-fold cross-section. However, such an electric motion that is equally-strong at all points would give four times the current intensity for four times the cross-section. The doubled electromotive force would then produce four times the current intensity in a circuit with twice the length and four times the cross-section, which is also actually the case from the known laws of galvanic circuits.

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perpendicular to the direction of the electromotive force that results at K, will be denoted by β, and the distance FK, by α. The force that acts from G to F in the direction FK is then obtained from the basic law of electrostatics, and is equal to:

$$\frac{e\alpha}{(\alpha^2 + \beta^2)^{3/2}} \, ,$$

and as a result, for all points K from $\alpha = 0$ to $\alpha = \delta$, it will equal:

$$e\left(\frac{1}{\beta} - \frac{1}{\sqrt{\beta^2 + \delta^2}}\right) \, .$$

For all surface elements that lie at an equal distance β from F, one will then get:

$$2\pi e\left(1 - \frac{\beta}{\sqrt{\beta^2 + \delta^2}}\right) \, ;$$

upon multiplying by $2\pi\beta$. Finally, for all surface elements from $\beta = 0$ to $\beta = b$, one will get:

$$2\pi e\left(\delta + b - \sqrt{b^2 + \delta^2}\right) \, ,$$

or, since δ is very small compared to b:

$$2\pi e\delta \, .$$

The same result will be obtained for the force that is exerted upon the surface CD, and as a result, the sum of both forces will be equal to $4\pi e\delta$, i.e., something proportional to the product δe.

A complete development of the laws of galvanic circuits requires a closer determination of the distribution of the free electricity on the surface of the circuit.

13.30 On the Law of Distribution of Free Electricity on the Surface of the Conductor of a Steady, Uniform Current

For a *linear* conductor, it is permissible to replace the distribution of free electricity on the surface with a distribution of it along the line that defines the axis of the conductor. This is self-evident in relation to all parts of the conductor that lie at a great distance from the points for which the electromotive force that is exerted by that free electricity is to be determined, and all that remains then it to prove that the statement is true for those parts of the conductor that lie next to those points.

Let A [in Figure 4] be the point for which the electromotive force that is exerted by the free electricity in the element of the conductor $BCDE$ is to be determined.
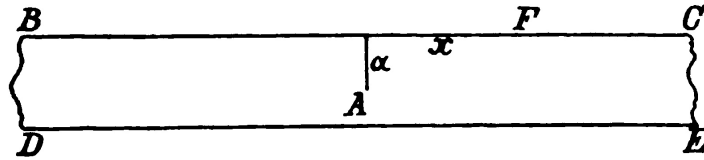
Fig. 4.

Let α denote the infinitely-small radius of the conducting wire. The thickness of the layer of free electricity at the point F, whose small distance from the cross-section of the conductor that goes through A will be denoted by x, can be represented by:

$$a + bx \ ,$$

and the electromotive force that the free electricity of the surface element $2\pi\alpha dx$ at F exerts upon the point A can be represented by:

$$\frac{2\pi\alpha(a + bx)dx}{\alpha^2 + x^2} \ ,$$

from which the component of that force in the direction of the axis will follow, namely, it is equal to:

$$\frac{2\pi\alpha(a + bx)xdx}{(\alpha^2 + x^2)^{3/2}} \ .$$

The value of the integral of that between the limits $x = -\lambda$ and $x = +\lambda$ is then:[434]

$$2\pi\alpha b \int_{-\lambda}^{+\lambda} \frac{x^2 dx}{(\alpha^2 + x^2)^{3/2}} = 2\pi\alpha b \left(\log \frac{\sqrt{\lambda^2 + \alpha^2} + \lambda}{\sqrt{\lambda^2 + \alpha^2} - \lambda} - \frac{2\lambda}{\sqrt{\lambda^2 + \alpha^2}} \right) \ ,$$

or, since α is very small compared to λ:

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[434][Note by AKTA:] In the next equation "log" should understood as the natural logarithm to the base $e = 2.718$, that is, ln.

$$= 4\pi\alpha b \cdot \log \frac{2\lambda}{e\alpha} = 4\pi\alpha b \left(\log \lambda - \log \frac{e}{2}\alpha \right) \; ,$$

in which e denotes the base of the system of natural logarithms.

Now, if that free electricity were concentrated on the axis of the conductor, instead of its surface, then an electromotive force would be exerted from the axis element where the free electricity $2\pi\alpha(a + bx)dx$[435] is concentrated upon A in the direction of the axis, and it would be represented by:

$$\pm \frac{2\pi\alpha(a + bx)dx}{x^2} \; ,$$

according to whether x had a positive or negative value. The value of the integral of that between the limits $x = -\lambda$ and $x = -e\alpha/2$ would then be equal to:

$$2\pi\alpha b \left(\log \lambda - \log \frac{e}{2}\alpha \right) + 2\pi\alpha a \left(\frac{1}{\lambda} - \frac{2}{e\alpha} \right) \; ,$$

and between the limits $x = +e\alpha/2$ and $x = +\lambda$, it would equal:

$$2\pi\alpha b \left(\log \lambda - \log \frac{e}{2}\alpha \right) - 2\pi\alpha a \left(\frac{1}{\lambda} - \frac{2}{e\alpha} \right) \; ;$$

and as a result, the value of the integral between the limits $x = -\lambda$ and $x = +\lambda$, excluding the part that falls between the limits $x = -e\alpha/2$ and $x = +e\alpha/2$, would equal:

$$4\pi\alpha b \left(\log \lambda - \log \frac{e}{2}\alpha \right) \; ,$$

from which it would emerge that it would be permissible to substitute a distribution of free electricity on the axis of the conductor for its distribution on the surface when one excludes the part of the integral of the electromotive force that lies between the limits $x = -e\alpha/2$ and $x = +e\alpha/2$.

If the conductor has the form of, e.g., a circle whose radius is equal to r, and if A denotes the starting point of an arc $AB = r\varphi$ that is the location of the given electromotive force in the circuit, then let:

$$f\varphi \cdot d\varphi$$

be the free electricity in the arc-length element $rd\varphi$ at the end of the arc $r\varphi$.[436] The value of the *potential* of that electric mass at the point C at the end of the arc $AC = r\psi$ is then equal to:

$$\frac{f\varphi \cdot d\varphi}{2r \sin \frac{1}{2}(\varphi - \psi)} \; ,$$

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[435][Note by AKTA:] Due to a misprint in the original, this expression appeared as $2\pi\alpha(a + x)dx$.

[436][Note by AKTA:] $f\varphi$ should be understood as an angular density of charge, that is, the amount of charge per unit angle. It is a function of the angle φ along the ring, that is, $f(\varphi)$. Weber will try to determine approximately the function $f(\varphi)$ that will produce an electromotive force which has the same value no matter the value of the angle φ.

and as a result, the value of the *potential* of the electric mass of the entire conductor at the point C will be:[437]

$$\frac{1}{2r} \int \frac{f\varphi \cdot d\varphi}{\sin \frac{1}{2}(\varphi - \psi)} = F\psi \ ,$$

where the integration extends from $\varphi = \psi + e\alpha/2r$ to $\varphi = 2\pi + \psi - e\alpha/2r$. That will imply that the electromotive force that is exerted upon the point C, as expressed in terms of the differential quotient of the potential with respect to the arc-length $r\psi$, is equal to:

$$\frac{d \cdot F\psi}{rd\psi} \ .$$

Now, should that electromotive force be equal in all parts of the conductor, i.e., should $d \cdot d(F\psi)/rd\psi$ have a constant value c, then one would get:

$$F\psi = c\psi + \text{constant} \ ,$$

or for a symmetric distribution of the free positive and negative electricity in the conductor, where $F\pi = c\pi + \text{constant} = 0$,

$$F\psi = c(\psi - \pi) \ .$$

Now, should one encounter difficulties in discovering the general form of the function $f\varphi$, then it would nonetheless not be difficult to subject the hypotheses that Ohm proposed in that regard to a test and decide whether, and to what extent, it is admissible.

Ohm's hypothesis consists essentially of saying that the value $f\varphi$ grows in proportion to φ from $\varphi = 0$ to $\varphi = 2\pi$, so for the case of the symmetric distribution of positive and negative electricity in the conductor, where $f(0) = -f(2\pi)$:

$$f\varphi = a(\varphi - \pi) \ .$$

Once that is assumed, one can determine the value of the *potential* of the free electricity in the entire conductor at the points for which $\varphi = \psi$ in the following way:

Let A be the starting point of the arc $r\varphi$, so $AB = BD = r\psi$. All elements of the arc $r\varphi$ from A to D can be arranged pair-wise by their distance from B. Namely, if one element belongs to $\varphi = \psi - \chi$, and its distance from B is equal to $2r \sin \frac{1}{2}\chi$, then the element that belongs to $\varphi = \psi + \chi$ will have the same distance from B. The electric masses that belong to those two elements are:

$$a(\psi - \chi - \pi)d\chi \qquad \text{and} \qquad a(\psi + \chi - \pi)d\chi \ ,$$

and the values of the *potentials* of those masses at the point B will be:

$$\frac{a(\psi - \chi - \pi)d\chi}{2r \sin \frac{1}{2}\chi} \qquad \text{and} \qquad \frac{a(\psi + \chi - \pi)d\chi}{2r \sin \frac{1}{2}\chi} \ ,$$

and as a result, their sum will be equal to:

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[437][Note by AKTA:] In the next equation $F\psi$ should be understood as a function of the angle ψ, that is, $F(\psi)$.

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$$\frac{a(\psi - \pi)d\chi}{r \sin \frac{1}{2}\chi} \ .$$

The value of the potential of the free electricity of the entire arc AD at the point B then proves to be:[438]

$$\frac{a(\psi - \pi)}{r} \int_{e\alpha/(2r)}^{\psi} \frac{d\chi}{\sin \frac{1}{2}\chi} = \frac{2a(\psi - \pi)}{r} \cdot \left(\log \tan \frac{1}{4}\psi - \log \tan \frac{e\alpha}{8r} \right) \ .$$

The point C on the circle lies diametrically opposite to the point B, and as a result the arc-length $ABC = r(\psi + \pi)$. All elements of the arc $r\varphi$ from D over C to A can likewise be arranged pair-wise according to their distance from C. Namely, if one of the elements belongs to $\varphi = \psi + \pi - \chi$, and its distance from C is equal to $2r \sin \frac{1}{2}\chi$, then the element that belongs to $\varphi = \psi + \pi + \chi$ will have the same distance from C. The electric masses that belong to those two elements are:

$$a(\psi - \chi)d\chi \qquad \text{and} \qquad a(\psi + \chi)d\chi \ ,$$

and the values of the *potentials* of those masses at the point B will be:

$$\frac{a(\psi - \chi)d\chi}{2r \sin \frac{1}{2}(\pi - \chi)} \qquad \text{and} \qquad \frac{a(\psi + \chi)d\chi}{2r \sin \frac{1}{2}(\pi - \chi)} \ ,$$

and as a result, there sum will be equal to:

$$\frac{a\psi}{r} \cdot \frac{d\chi}{\cos \frac{1}{2}\chi} \ .$$

The value of the potential of the free electricity of the entire arc DCA at the point B then proves to be:

$$\frac{a\psi}{r} \int_0^{\pi - \psi} \frac{d\chi}{\cos \frac{1}{2}\chi} = -\frac{2a\psi}{r} \log \tan \frac{1}{4}\psi \ ,$$

so the value of the potential of the free electricity of the entire circle will then be equal to:

$$-\frac{2a\psi}{r} \log \tan \frac{e\alpha}{8r} - \frac{2a\pi}{r} \left(\log \tan \frac{1}{4}\psi - \log \tan \frac{e\alpha}{8r} \right) \ .$$

That yields the electromotive force that is exerted upon the point B, as expressed in terms of the differential quotient of the potential with respect to the arc-length $r\psi$, which equals:

$$-\frac{2a}{r^2} \log \tan \frac{e\alpha}{8r} - \frac{a\pi}{r^2 \sin \frac{1}{2}\psi}$$

or

$$= \frac{2a}{r^2} \log \cot \frac{e\alpha}{8r} - \frac{a\pi}{r^2 \sin \frac{1}{2}\psi} \ .$$

For values of ψ that differ only slightly from π, the electromotive forces prove to be almost equal. However, the more the value of ψ approaches the value of 0 or 2π, the lower the

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[438][Note by AKTA:] The expression $\log \tan \frac{1}{4}\psi - \log \tan \frac{e\alpha}{8r}$ should be understood as $\ln \left(\tan \frac{\psi}{4} \right) - \ln \left(\tan \frac{e\alpha}{8r} \right)$.

electromotive force will drop below that limiting value, from which it will follow that Ohm's hypothesis about the distribution of free electricity is approximately admissible only for the middle part of the circuit.

Now, just as the value of the electromotive force is smaller in all parts of the circuit than the limiting value that is valid for the middle of the circuit, according to that hypothesis, one can also easily propose a hypothesis by which it would be larger. Namely, Ohm's hypothesis necessarily requires an extension if it is to not contradict the law that an electromotive force that is everywhere equal in the interior of the conductor can result from the distribution of free electricity on the surface only when two cross-sectional surfaces of the conductor belong to that surface (see, page 352). That is because that would imply that in our linear representation, all of the free electricity that is found in those two cross-sectional surfaces would have to be thought of as concentrated at two points, while only the electricity that is found on the boundary line of a cross-section would be thought of as concentrated at a point in the rest of the circuit. That would imply that a concentration of free electricity that Ohm did not consider would have to exist, at least on those two endpoints that represent the two cross-sections. If one denotes them by $\pm\varepsilon$, where the upper sign is true for the one point, and the lower sign is true for the other one, and δ denotes the small distance between the two points, then the electromotive force that must be added for each point of the circuit in that way can be determined by the same laws that Gauss gave for the action of a magnet at a distance. See "*Resultaten aus den Beobachtungen des magnetischen Vereins im Jahre 1840*," pages 33, 34.[439],[440] Namely, if ACA' is the circular conductor and the contact location is at A, and if one has to determine the electromotive force that is added by the free electricity $\pm\varepsilon$ on both sides of A at the point C of the conductor, then one must draw the tangent to A and extend it until it cuts the extended line $A'C$ at B, where A' denotes the point of the circle that lies diametrically opposite to the point A. If one further makes $AD = \frac{1}{3}AB$ and draws CD, then CD will be the direction of the electromotive force that $\pm\varepsilon$ exerts upon C, and the magnitude of that force will be represented by:

$$\frac{CD}{AD} \cdot \frac{\delta\varepsilon}{AC^3} \,.$$

Finally, if one draws the tangent to the circle at C and drops the perpendicular DE from it, then that will yield the component in the direction of the tangent to the circle at C, i.e., the desired electromotive force, which equals:

$$\frac{CE}{CD} \cdot \frac{CD}{AD} \cdot \frac{\delta\varepsilon}{AC^3} = \frac{CE}{AD} \cdot \frac{\delta\varepsilon}{AC^3} \,.$$

If one denotes the radius of the circle by r and the circular arc AC by ψ, then one will find the following expression for it:[441]

$$\frac{1 + \cos\frac{1}{2}\psi^2}{\sin\frac{1}{2}\psi^3} \cdot \frac{\delta\varepsilon}{8r^3} \,.$$

If one now adds that electromotive force to the one that was found under Ohm's hypothesis, then one will get:

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[439][Note by HW:] Gauss's *Werke*, Vol. V, pp. 434 and 435.

[440][Note by AKTA:] [Gau41d].

[441][Note by AKTA:] The expressions $\cos\frac{1}{2}\psi^2$ and $\sin\frac{1}{2}\psi^3$ should be understood as $\cos^2(\psi/2)$ and $\sin^3(\psi/2)$, respectively.

$$\frac{2a}{r^2} \log \cot \frac{e\alpha}{8r} - \frac{a\pi}{r^2 \sin \frac{1}{2}\psi} + \frac{1 + \cos \frac{1}{2}\psi^2}{\sin \frac{1}{2}\psi^3} \cdot \frac{\delta\varepsilon}{8r^3} .$$

That value is also almost constant for values of ψ that differ only slightly from π, as one will see when one develops the differential quotients, namely:

$$\frac{\cos \frac{1}{2}\psi}{2r^2 \sin \frac{1}{2}\psi^2} \left(a\pi - \frac{\delta\varepsilon}{4r} \left(1 + \frac{3}{2} \frac{1 + \cos \frac{1}{2}\psi^2}{\sin \frac{1}{2}\psi^2} \right) \right) ,$$

which is equal to zero when $\psi = \pi$. However, in addition, the value of $\delta\varepsilon$ can be determined in such a way that the second and third differential quotients are also zero for $\psi = \pi$, which will be the case when:

$$\delta\varepsilon = \frac{8}{5} a\pi r .$$

If one substitutes that value of $\delta\varepsilon$ in the expression for the electromotive force, then one will get:

$$\frac{2a}{r^2} \log \cot \frac{e\alpha}{8r} + \frac{2a\pi}{5r^2 \sin \frac{1}{2}\psi^3} \left(3 \cos \frac{1}{2}\psi^2 - 2 \right) ,$$

whose differential quotient is equal to:[442]

$$-\frac{3}{5} \frac{a\pi \cos \frac{1}{2}\psi^3}{r^2 \sin \frac{1}{2}\psi^4} ,$$

and it will be equal to zero for $\psi = \pi$, because it has $\cos \frac{1}{2}\pi = 0$ as a factor. One also sees that the two following differential quotients will be zero for $\psi = \pi$, because they likewise have the factor $\cos \frac{1}{2}\pi = 0$.

One sees from this that according to the present hypothesis, the value of the electromotive force in all parts of the circuit is greater than the limiting value that is true for the middle of the circuit, whereas according to Ohm's hypothesis, it would be smaller. The correct hypothesis about the distribution of free electricity that should yield an everywhere equal electromotive force is then included between the limits that are given by the two hypotheses above, which means the same thing as: The electric charge in the circuit does not grow uniformly from the indifference point to the contact point, but accelerates gradually. The everywhere-equal electromotive force that emerges from that will then presumably lie between the limiting values that are given by the two hypotheses above, namely:

$$\frac{2a}{r^2} \left(\log \cot \frac{e\alpha}{8r} - \frac{1}{2}\pi \right)$$

and

$$\frac{2a}{r^2} \left(\log \cot \frac{e\alpha}{8r} - \frac{2}{5}\pi \right) .$$

The factor a then refers to the *gradient* of the electric charge in the middle of the circuit when one, with Ohm, understands the gradient to mean the differential quotient of the charge $f\varphi$ with respect to the arc-length φ.

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[442][Note by AKTA:] The expressions $\cos \frac{1}{2}\psi^3$ and $\sin \frac{1}{2}\psi^4$ should be understood as $\cos^3(\psi/2)$ and $\sin^4(\psi/2)$, respectively.

13.31

The distribution of free electricity in a *linear* conductor through which a constant current flows and the electromotive force that depends upon that magnitude of that distribution can be determined approximately in each individual case in the following way: For the sake of simplicity, the conductor shall also be assumed to have form of a circle here, and an electromotive force equal to a shall be given at each of its individual points.

If one divides the circle into four equal parts by means of the points A, (A^1), B, (A_1), and if B is the point at which the electromotive force equal to a is given, then a distribution of free electricity can be easily given at the two points (A^1) and (A_1), by means of which the electromotive forces at the two points A and B can be compared. If $+e$ then denotes the free electricity at (A^1) and $-e$ the free electricity at (A_1), and r is the radius of the circle, then the distances from the points A and B to (A^1) or (A_1) will be $2r\sin\frac{1}{4}\pi = r\sqrt{2}$. From the basic law of electrostatics, that will then give the electromotive force in the direction of the tangent to the circle:

At B:[443]

$$= a - \frac{2e}{4r^2 \sin\frac{1}{4}\pi^2} \cdot \cos\frac{1}{4}\pi = a - \frac{e}{r^2}\sqrt{\frac{1}{2}} \ ,$$

at A:

$$= +\frac{2e}{4r^2 \sin\frac{1}{4}\pi^2} \cdot \cos\frac{1}{4}\pi = +\frac{e}{r^2}\sqrt{\frac{1}{2}} \ ,$$

and as a result the desired comparison will be:

$$a = \frac{e}{r^2} \cdot \sqrt{2} \ ,$$

or

$$+e = +ar^2 \cdot \sqrt{\frac{1}{2}} \ ,$$

respectively.

In the same way, when the circle through the points A, (A^1), A^1, (A^2), etc., is divided into $4n$ equal parts and the electromotive force equal to a is given at the point B that is diametrically opposite to A, one will get a distribution of free electricity at $2n$ points (A^1), (A^2), etc., by means of which the electromotive forces at the $2n$ points A, A^1, etc. can be compared. That is because if $\pm e_1$ denotes the free electricity at (A^1), (A_1), and $\pm e_2$ denotes the free electricity at (A^2), (A_2), and r is the radius of the circle, and one sets:

$$\frac{\cos\frac{(2m-1)\pi}{4n}}{4r^2\left[\sin\frac{(2m-1)\pi}{4n}\right]^2} = p_m \ ,$$

then one will find the electromotive force in the direction of the tangent to the circle:
At B

$$= a - 2p_n \cdot e_1 - 2p_{n-1} \cdot e_2 - \dots - 2p_1 \cdot e_n \ ,$$

---

[443][Note by AKTA:] The expression $\sin\frac{1}{4}\pi^2$ should be understood as $\sin^2(\pi/4)$.

at A:

$$= 2p_1 e_1 + 2p_2 e_2 + \ldots + 2p_n e_n ,$$

at A^m or A_m:

$$\begin{aligned}= \quad & - p_m e_1 - p_{m-1} e_2 - \ldots - p_1 e_m + p_1 e_{m+1} + \ldots + p_{n-m} e_n \\ & + p_{m+1} e_1 + p_{m+2} e_2 + \ldots + p_n e_{n-m} - p_n e_{n-m+1} - \ldots - p_{n-m+1} e_n \end{aligned}$$

in which m can be set to all whole numbers from 1 to $n-1$. By setting all of those $(n+1)$ values equal to each other, one will get n equations for determining the n unknown quantities e_1, e_2, ... , e_n.

Moreover, that yields the mean value of the first two of the $(n+1)$ electromotive forces k that were set equal above:

$$k = \frac{1}{2}a + (p_1 - p_n)e_1 + (p_2 - p_{n-1})e_2 + \ldots ,$$

and the sum of all of them will be:

$$(n+1)k = a + (p_1 - p_n)e_1 + (p_2 - p_{n-1})e_2 + \ldots ,$$

and as a result:

$$(n+1)k - a = k - \frac{1}{2}a ,$$

or

$$a = 2nk .$$

For example, for $n = 2$, that will give:

$$\begin{aligned} e_1 &= 0.015\,67 \cdot 4r^2 a , \\ e_2 &= 0.058\,33 \cdot 4r^2 a , \\ k &= \frac{1}{4}a ; \end{aligned}$$

for $n = 4$:

$$\begin{aligned} e_1 &= 0.001\,537 \cdot 4r^2 a , \\ e_2 &= 0.004\,744 \cdot 4r^2 a , \\ e_3 &= 0.008\,570 \cdot 4r^2 a , \\ e_4 &= 0.015\,922 \cdot 4r^2 a , \\ k &= \frac{1}{8}a ; \end{aligned}$$

for $n = 8$:

$$
\begin{aligned}
e_1 &= 0.000\,158\,2 \cdot 4r^2 a \ , \\
e_2 &= 0.000\,477\,1 \cdot 4r^2 a \ , \\
e_3 &= 0.000\,804\,7 \cdot 4r^2 a \ , \\
e_4 &= 0.001\,149\,5 \cdot 4r^2 a \ , \\
e_5 &= 0.001\,527\,1 \cdot 4r^2 a \ , \\
e_6 &= 0.001\,972\,6 \cdot 4r^2 a \ , \\
e_7 &= 0.002\,595\,1 \cdot 4r^2 a \ , \\
e_8 &= 0.004\,118\,7 \cdot 4r^2 a \ , \\
k &= \frac{1}{16} a \ .
\end{aligned}
$$

The larger the number n is, the closer that the value of e_1 will approach the values:

$$
\frac{1}{3} e_2 \ , \qquad \frac{1}{5} e_3 \qquad \dots \ .
$$

If one distributes the masses e_1, e_2, ... , e_m, whose deviations from the masses e_1, $3e_1$, ... , $(2m-1)e_1$ can be neglected as imperceptible, along the m circular arcs $\pi r/n$ at whose midpoints they lie, and whose distances x from the point A are proportional, then if b denotes a constant factor, one will have:

$$
b \int_0^{m\pi r/n} x \, dx = \frac{1}{2} b \cdot \frac{m^2 \pi^2 r^2}{n^2} = e_1 + e_2 + \dots + e_m = m^2 e_1 \ ,
$$

and as a result:

$$
b = \frac{2n^2}{\pi^2} \cdot \frac{e_1}{r^2} \ .
$$

Now, if the circular arc $m/n \cdot \pi r$ is small enough that its deviation from a straight line can be considered to be imperceptible, then the electromotive force at the point A that is due to the masses e_1, e_2, ... , e_m that are concentrated at the midpoints of the m circular arcs $\pi r/n$ will be equal to:

Due to e_1:

$$
= \frac{4n^2}{\pi^2 r^2} \cdot e_1 \ ,
$$

due to e_2:

$$
= \frac{4n^2}{\pi^2 r^2} \cdot \frac{1}{9} e_2 = \frac{4n^2}{\pi^2 r^2} \cdot \frac{1}{3} e_1 \ ,
$$

due to e_m:

$$
= \frac{4n^2}{\pi^2 r^2} \cdot \frac{1}{(2m-1)^2} e_m = \frac{4n^2}{\pi^2 r^2} \cdot \frac{1}{2m-1} \cdot e_1 \ ,
$$

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so the total electromotive force that is exerted by those m masses at the point A will be equal to:

$$\frac{4n^2}{\pi^2 r^2}\left(1 + \frac{1}{3} + ... + \frac{1}{2m-1}\right)e_1 = 2\left(1 + \frac{1}{3} + ... + \frac{1}{2m-1}\right)b \ .$$

By contrast, the electromotive force that is exerted at point A by the same mass that is distributed continuously along the entire arc $m\pi r/n$ according to the given law, when that linear distribution takes the place of the actual distribution on the surface of a thin wire of radius α, as in Section 13.30, will be found to be:

$$b\int_{e\alpha/2}^{m\pi r/n}\frac{dx}{x} = b\log\operatorname{nat}\frac{2m\pi r}{ne\alpha} \ .$$

The two expressions for the electromotive force that are exerted by the m masses at A will be equal when α takes a value such that:

$$2\left(1 + \frac{1}{3} + ... + \frac{1}{2m-1}\right) = \log\operatorname{nat}\frac{2m\pi r}{ne\alpha} \ ,$$

i.e.:

$$e\alpha = \frac{2m}{n}\pi r e^{-2(1+\frac{1}{3}+...+\frac{1}{2m-1})} \ ,$$

in which e denotes the base of the system of natural logarithms. The greater the number n (and as a result, the number m, as well), the less influence that it will have on the value of α whether one takes the number m to be larger or smaller by a few units. That is because if m denotes a larger number and α' denotes the value that α takes when m is increased by 1, then α' can be represented by $(2m^2 + 3m + 1)\alpha/(2m^2 + 3m)$, which will differ from α only slightly for large values of m. For that value of α, the masses of free electricity that are concentrated at the midpoints of the m circular arcs $\pi r/n$ can be set equal to an equally-large mass that is distributed continuously on the surface of the conductor. That is because for the parts of the circuit that are closest to the points considered, that fact would follow from the equality of the electromotive forces that was just proved, but for the distant parts of the circuit, it would be self-explanatory, just as in Section 13.30.

For the case that was considered above, when $n = 8$, one will easily see that m cannot be taken to be greater than 2; as a result:

$$e\alpha = \frac{1}{2}\pi r e^{-8/3} = 0.109\,15 \cdot r \ .$$

Now, because it is based upon such small values of n and m, that value of α must not be considered to be precise, and in addition it proves to be too large for the rules that were developed in Section 13.30 to be applicable with sufficient precision, since they were true for only small values of α. A more precise application of those rules would require that n could not be smaller than 32, and one would get:

$$e\alpha = \frac{1}{4}\pi r e^{-352/105} = 0.027\,49 \cdot r \ ,$$

when one assumes that $m = 4$. The foregoing case might then serve to explain only the distribution of free electricity in the conductor, and the electromotive force that results from

it can be determined somewhat approximately in the given way, despite the inaccuracy and magnitude of α. In addition, one can get the value of $e\alpha$:

$$e\alpha = 0.109\,15r \ ,$$

namely, for that case, one will have:

$$b = \frac{32}{\pi^2} \cdot \frac{e_1 + e_2}{r^2} = 0.008\,239 \cdot a \ ,$$

and the electromotive force, which is equal in the entire circuit, will be:

$$k = \frac{1}{16}a \ .$$

That result can only be compared with the formulas that were given on page 360, from which that electromotive force can be represented approximately by the following expressions, namely:[444]

$$\frac{2a}{r^2} \left(\log \cot \frac{ea}{8r} - \frac{1}{2}\pi \right)$$

or

$$\frac{2a}{r^2} \left(\log \cot \frac{ea}{8r} - \frac{2}{5}\pi \right) \ ,$$

in which one must observe that the mass-element of free electricity in the arc-length element $rd\varphi$, which was found at a small distance $r\varphi$ from the indifference point A, was expressed by $a\varphi d\varphi$ there, while the same mass-element was denoted by $bxdx$ here, where $x = r\varphi$ and $dx = rd\varphi$; one must then set $a = br^2$ in those two formulas. One will then get either:

$$k = 2b \left(\log \cot \frac{ea}{8r} - \frac{1}{2}\pi \right) = 0.004\,88 \cdot a$$

or

$$k = 2b \left(\log \cot \frac{ea}{8r} - \frac{2}{5}\pi \right) = 0.050\,06 \cdot a \ ,$$

approximately, instead of $k = \frac{1}{16}a = 0.0625 \cdot a$, which was found above. One then sees from this that when the value of k that was calculated above does not agree precisely with the latter two approximate values either (which is impossible, due to the imprecision and magnitudes of the value of α), that path itself will at least lead to a value of k with the same order of magnitude under these unfavorable circumstances. A better agreement might be expected when the calculation is performed, e.g., for $n = 32$ or even greater numbers. The distribution of free electricity in the linear conductor could be determined approximately, but more sharply, along with the electromotive force that depends upon it, when the numbers n and m are increased appreciably.

Furthermore, it is hardly necessary to point out, in particular, that the circular form of the conductor in the presentation above was chosen only as an example to simplify the calculation, but that the method would still be applicable to any other linear form for the

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[444][Note by AKTA:] In the next four equations the expression $\log \cot \frac{ea}{8r}$ should be replaced with $\log \cot \frac{e\alpha}{8r}$.

conductor. The same thing is also true when several electromotive forces at different places on the conductor are given, instead of one of them, or when the conductor is decomposed into pieces with different specific resistances, so a non-uniform distribution of electromotive force that is proportional to those resistances must exist. Above all, the application of this method is restricted only by the assumption of a *linear* conductor, besides the complexity of the calculations.

13.32 Verifying How the Distribution of Free Electricity on the Surface of a Closed Conductor that is Necessary for a Steady, Uniform Current Is Produced

That explains the fact that when electromotive forces are given only at the individual points in a closed circuit, an electric current can begin directly at only those points and not in the entire circuit. That is because the electric fluid will not be moving in any of the parts of the circuit upon which no electromotive forces act either. However, if the electric fluids begin to move at places where electromotive forces are given, and indeed the positive fluid moves in one direction and the negative one moves in the opposite direction, while the fluid between them persists in a state of rest, then that displacement of the positive fluid from one side to the other will produce an accumulation of free positive electricity that exerts an electromotive force forwards, as well as backwards. The effect of the given electromotive force behind it is weakened or cancelled out, while in the forward direction, it exerts an electromotive force with the same sense as the given one, except at a different location in the circuit. The same thing is also true for the negative fluid that is displaced in the opposite direction, as long as the electric fluid in the parts of the circuit that lie before it persists in a state of rest. At the same time, the accumulation of free negative electricity that will also act forwards and backwards, namely, the effect of the electromotive force will be weaker behind and it will exert an electromotive force in the forward direction with the same sense as the given one, except at a different location in the circuit. If one considers that argument, then one will see that in general this accumulation of free electricity can stop growing and become stationary only for a uniform current in all parts of the circuit, and that any deviation of the current from uniformity would lead to a change in that accumulation that would increase until the nonuniformity in the current had vanished again.

Now, the distribution of free electricity on the surface of the conductor that was discussed in Sections 13.29, 13.30 is indeed of the type for which no equilibrium can exist in the distributed free electricity for it. That is because in order for it to exist, it would be necessary for the resultant of all forces that a particle of free electricity on the surface experiences as a result of all of the remaining ones to point perpendicularly to the surface and outwards, which is not the case. That is because the presentation that was given in Section 13.29, in its own right, explained how a tangential force on each particle of free electricity on the surface would result, in addition to an outward-pointing force that is perpendicular to the surface, from which it would follow that this free electricity on the surface could not persist in a state of rest, but must take part in the current that exists in its interior. However, that participation of the free electricity on the surface in the interior current can probably exist with an unchanging distribution of free electricity on the surface of the conductor. That is

because if one represents the distribution of all positive electricity that flows on the boundary and in the interior of a conductor that is thought to extend along a straight line AA' by the ordinates of another straight line BC and likewise represents the distribution of all negative electricity that flows on it by the ordinates of a third straight line $B'C'$ that cuts the line BC at O, then an equal amount of both fluids will be present in the cross-section OP, [Figure 5].
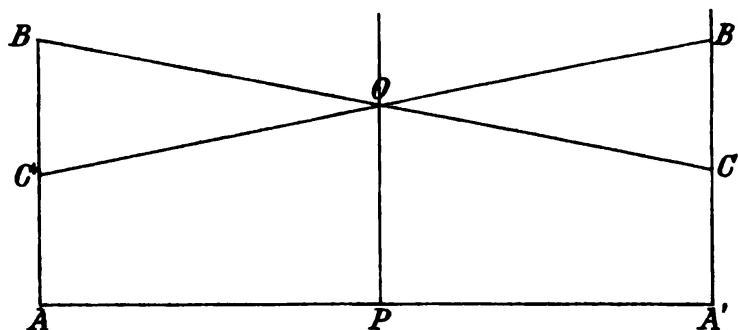

Fig. 5.

However, the excess of positive electricity increases from P to A in proportion to the distance from P. From P to A', the excess of negative electricity likewise increases in proportion to the distance from P. The general flow of current will then be represented by an equally-fast advance of the lines BC and $B'C'$ in opposite directions parallel to AA', from which it would follow easily that the ordinate of the point of intersection of the two lines PO — i.e., the indifference point of the circuit — would remain unshifted and that the growth of that excess at either of the two sources of electricity at a distance of P would remain unchanged under that advance, as long as one can assume that the shifted electricity at the contact points A, A' is always replaced with the newly-separated electricity in such a way that the shifted lines BC and $B'C'$ are always lengthened until they extend to the ordinates of the points A, A'. From the illustration, it might seem as though the amount of electricity that flows between A and A' will always become larger. That is due to the fact that in that way the always-newly-separated electricity at A and A' that moves in opposite direction is included in the calculation, while no consideration is given to the electricity that comes between A and A' upon reuniting to a state of rest. However, that gradual reunion of the two electric fluids between A and A' can also be easily illustrated by an advance of the abscissa line upwards, which can take place with a velocity such that the ordinate PO would always keep the same length, which expresses the idea that the amount of positive and negative electricity that is found there will remain unchanged.

Ohm's law of proportionality is assumed for the charge in the circuit in that representation. Should one give some consideration to the deviation from that law that was discussed in the foregoing Sections, then the calculation must also include the difference in the velocity with which the two types of electricity must flow if equal quantities of the two are to go through the cross-section for a given excess. From a more precise discussion, one would also find that the electrostatic principle that was used as a basis here for the sake of simplicity would no longer be satisfied, so a return to the fundamental general law of electric action would be deemed necessary.

13.33   On Kirchhoff's Deduction of Ohm's law, in Connexion with the Theory of Electrostatics

During the printing of this treatise, a publication by Dr. Kirchhoff of the Berlin Physical Society entitled "Ueber eine Ableitung der Ohm'sche Gesetze, welches sich an die Theorie der Elektrostatik anschliesst" appeared in Poggendorff's *Annalen*, Vol. 79, p. 506, in which the principles upon which the foregoing discussion was based were subjected to a more precise test.[445] In particular, it was shown that Ohm's law of galvanic circuits did not necessarily have any connection with the assumption that Ohm made in his derivation, in contradiction to the basic laws of electrostatics, namely, that the electricity in a conductor can be found in a state of rest when it fills up its volume with a uniform density. Rather, the derivation of that law remains unchanged when one replaces the assumption that contradicts the basic laws of electrostatics with another one that agrees with it and necessarily results from it, namely, that the neutral electric fluid in a conductor can be found in a state of rest when the *potential* of free electricity that is distributed on its surface has the same value everywhere in the interior of the conductor and when, in the course of the derivation, one sets the value of the potential in the interior of the conductor that is due to the free electricity that is found on the surface equal to the density of electricity that should be found in the interior according to Ohm. The proof that Kirchhoff gave of that is so brief that it admits no summary, and for that reason, one must refer to the original itself. One might only quote the concluding remark that Kirchhoff added in regard to it, in which he sought to justify the reduction of the laws of galvanic circuits to the basic laws of *electrostatics*, since the laws of galvanic circuits were concerned with *electrodynamic* phenomena, so the basic law of electrostatics would not generally suffice to explain them. He said, *loc. cit.*, p. 512:

> "The considerations that were developed were based upon the electrostatic law of action for electric particles. Ampère's electrodynamic phenomena[446] and the induction phenomena cannot be explained by that law. Weber has found a more general law with which he could succeed in explaining those phenomena, which is a law whose expression includes the relative velocity of the particles whose mutual action is being considered and which goes back to the electrostatic law when that velocity vanishes. In order to unify the various branches of the theory of electricity under a common viewpoint, one must then pose the problem of deriving the laws of currents in closed circuits from Weber's law. That derivation seems difficult, but it is easy to prove *a posteriori* that the representation of the currents to which the assumption of the electrostatic law would lead will also be consistent with Weber's law when one appeals to a certain hypothesis, namely, the hypothesis that when one calculates the force that produces a separation of the two types of electricity in the volume element v of a conductor, the types of electricity in v must be regarded as being at rest. There will be nothing objectionable about that assumption when one imagines that the motion of the electricity in a conductor only goes from molecule to molecule such that each electric particle finds a point of rest at the molecule to which it arrives. With that picture, one can easily concede that the amount of electricity that goes from one molecule to a neighboring one is determined by only the forces that were exerted upon the electric particle while it was found to be at rest at that molecule, but not

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[445][Note by AKTA:] See Chapter 10.
[446][Note by AKTA:] See footnote 10 on page 13.

by the forces that acted upon it while it was already on the way to the following molecule. In the context of the theory of induction that Weber gave, it is irrelevant whether one does or does not make that assumption. If one makes it and imagines, moreover, that the currents in the circuit are the ones that assuming the electrostatic law would yield, then it is also irrelevant whether one starts from the electrostatic law or Weber's in order to find the magnitude and direction of the force that tries to separate the types of electricity in the element v (so the electromotive force, as Weber called it). The possible difference between them must, in fact, originate in the forces that the types of electricity that flow in the other parts of the system exert, and from what Weber has proved, those forces contribute nothing to that electromotive force, since the currents are constant, and equal amounts of the two types of electricity move in opposite directions with equal velocities."

13.34   Determining the Relative Velocity of Two Electric Masses at which Neither Attraction Nor Repulsion will Take Place by Comparing Electromotive and Galvanometric Observations of the Galvanic Circuit

If the law of distribution of free electricity on the surface of a conductor for a steady, uniform current is given, then that can be the basis for an application that is generally important in the study of electricity. Namely, it explains the fact that the electromotive force can be determined in two ways, namely, *first of all*, from its effect, i.e., from the intensity of the current that it produces for a known resistance in the circuit. In that way, the determination of the electromotive force will be made to depend upon the measurements of the current intensity and resistance of the circuit, both of which can be accomplished in absolute units, as was shown in this treatise. *Secondly*, it can be determined from its source, i.e., from the free electricity that is distributed on the surface of the conductor. If the current intensity i and the resistance of the circuit w are found in the units that were defined in Section 13.26, then the electromotive force in the entire circuit will be determined in the units that are given for it by the product:

$$iw \ ,$$

and according to Section 13.27, that value can be reduced to the general force unit in mechanics by multiplying it by $4/c$, where c denotes the relative velocity with which two electric masses must move relative to each other if they are to neither attract nor repel. The electromotive force in the entire circuit is then calculated to be equal to:

$$\frac{4}{c}iw$$

in the general force units of mechanics. In order to determine the electromotive force in that circuit from its cause, one might now base it upon the expression that is found in Section 13.30:

$$\frac{2a}{r^2}\left(\log\cot\frac{e\alpha}{8r}-\beta\pi\right)$$

in which β has a value that is smaller than $1/2$ and larger than $2/5$. From page 357, the a in that equation denotes the factor that will give the mass of free electricity that is distributed along the element of length $rd\varphi$ in the circuit at the end of the arc $r\varphi$ when it is multiplied by $(\varphi-\pi)d\varphi$. Now, if the mass of free electricity in two elements of the circuit of length dx, one of which it at the end of the arc $\pi-\chi$, while the other is at the end of the arc $\pi+\chi$, is actually measured, and the former is found to be equal to Edx, while the latter is equal to $E'dx$, then one must set:

$$\begin{aligned} Edx &= -a\chi d\chi\ , \\ E'dx &= +a\chi d\chi\ , \end{aligned}$$

and $rd\chi = dx$; as a result:

$$a = \frac{r}{2\chi}(E'-E)\ .$$

If one now substitutes that value of a in the expression above, then one will get:

$$\frac{E'-E}{r\chi}\left(\log\cot\frac{e\alpha}{8r}-\beta\pi\right)\ .$$

However, that expression does not give the electromotive force for the entire length of the circuit, but only for a piece of the circuit that has unit length, and must be multiplied by the length of the circuit ($=2\pi r$) if one is to obtain the electromotive force in the entire circuit, namely:

$$\frac{2\pi}{\chi}(E'-E)\left(\log\cot\frac{e\alpha}{8r}-\beta\pi\right)\ .$$

Thus, upon setting the electromotive forces in the entire circuit that are determined in both ways equal to each other, that will ultimately yield the following equation:

$$\frac{4}{c}iw = \frac{2\pi}{\chi}(E'-E)\left(\log\cot\frac{e\alpha}{8r}-\beta\pi\right)$$

or

$$c = \frac{2\chi}{\pi}\frac{iw}{E'-E}\cdot\frac{1}{\log\cot\frac{e\alpha}{8r}-\beta\pi}\ .$$

One has then determined the velocity c with which two electric masses must move relative to each other in order for them to neither repel not attract. The importance of the determination of that velocity c itself is explained by the fundamental law of interaction for two electric masses that was expressed in the first treatise on "Elektrodynamische Maassbestimmungen,"[447] as well as by Section 13.27 of this treatise, where it was shown that when that velocity c is known, all electromotive forces can be expressed in the force units that are established in mechanics. However, from that significance of c itself, it is already interesting

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[447][Note by AKTA:] See Chapter 5 on page 33.

to verify the possibility of such a determination even when the actual performance of it encounters obstacles that cannot be overcome because instruments that would be suitable for that determination are lacking. In fact, such obstacles would now prevent one from performing the delicate electrometric measurement by which the quantities E' and E would be found. None of our modern electroscopes and electrometers seem suitable for performing those measurements. It would only be possible then to determine the ratio of the quantities E and E', but not their absolute values; at least no attempt of that kind has been made up to now. However, the construction of new electroscopes and electrometers that would be suitable for that purpose defines a problem in its own right, which we shall not address here because we shall restrict ourselves to only electrodynamic measurements in this treatise.

13.35   On the Relationship between the Drift Velocity and the Velocity of Propagation of a Current

There is no data whatsoever about the velocity with which the electric fluid itself moves in conductors. One knows only that the velocity with which many electric *phenomena*, such as lightning, propagate must be very large, since their propagation through even the largest spaces does not require even the smallest measurable time intervals. One likewise knows only that the propagation of a galvanic current through a long circuit happens with an extraordinary velocity because the time that is required for a current that is generated at a certain location in the circuit to achieve the same intensity in all parts of the circuit is so small that there has been no way of measuring it up to now. Wheatstone's experiments[448] regarding the non-simultaneity of sparks that are produced at different places in an interrupted conducting wire when the accumulated positive and negative electricity in two conductors combine with each other through the conducting wire likewise give no information about the velocity with which the electric fluids are moving, but only about the propagation of the motion through the electrically-neutral medium in the conducting wire. That is because the appearance of sparks assumes that the electrically-neutral medium that is found at the location in question has been set into motion. However, it by no means assumes that the positive or negative electricity that had accumulated in the two conductors beforehand was itself forced to go to that location. The non-simultaneity of sparks that Wheatstone observed at various points of interruption in the conducting wire can therefore give information about only the velocity of the propagation of motion through the electrically-neutral medium in the parts of the conducting wire that lie between them. In a closed and nowhere-interrupted circuit in which the equilibrium of the electric fluid is perturbed continually by electromotive forces, one must also distinguish two types of velocities, namely, the velocity of the motion that propagates from particle to particle and the velocity of the motion that is peculiar to each particle: The former is called the *velocity of current distribution*,[449] while the latter is called the *drift velocity*.[450] The drift velocity is everywhere equal for a steady current in a

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[448][Note by AKTA:] See footnote 169 on page 119.

[449][Note by AKTA:] In German: *Geschwindigkeit der Stromverbreitung*. This expression can also be translated as *velocity of current propagation*. Weber is referring here to the velocity of propagation along the circuit of a perturbation in the current, or the velocity of propagation along the circuit of the distribution of free electricity.

[450][Note by AKTA:] In German: *Stromgeschwindigkeit*. This expression can also be translated as *current velocity*. Weber is referring here to the velocity of each electrified particle relative to the matter of the conductor. See also footnotes 41 and 42 on pages 24 and 24.

homogeneous circuit. Such a current is called uniform, because it has *propagated* uniformly throughout the entire circuit, and as long as it persists unchanged, one can no longer speak of any further *current distribution*. If one is to once more speak of current distribution, then one must introduce a *change* in the current: The current must become stronger or weaker. That raises the question then of whether any change in the current strength, i.e., any change in the *drift velocity*, takes place simultaneously in all parts of the circuit or gradually from one part to the others. In the *former* case, one would say that the current propagates with infinite speed through the circuit or that the speed of current distribution is immeasurable. In the *other* case, one would say that the current propagates with finite velocity through the circuit or that the speed of current distribution is measurable. It emerges from this that the measurement of the speed of current distribution assumes a change or modification in the current strength in the circuit, since without such a thing one could not speak of such a measurement at all.

Now, it was already explained in an example on page 350 that changes in the current strength or *drift velocity* that take place in all parts of the circuit simultaneously are, in fact, possible, namely, when the given electromotive forces that cause the change act upon all parts of the circuit directly in proportion to its resistance. However, such a special case still does not prove the immeasurability of the speed of current distribution in general. If the speed of current distribution is to be called immeasurable in general, then that simultaneity of the change in current must take place in all parts of the circuit in all cases, and in particular, even when the given electromotive force that causes the change acts directly upon only part of the circuit. However, in that case, the connection between the laws of galvanic circuits and the basic laws of electricity that was discussed in the foregoing Section implies that the changed drift velocity must persist for some time in all of the parts of the circuit where it was produced directly by the given electromotive force before it can enter into the other parts of the circuit, namely, because in order for that current change to enter into other parts of the circuit, it must have necessarily been preceded by a new accumulation of the free electricity that would be necessary for producing the current change in those parts. However, that new accumulation of free electricity can be produced only by a current change in a part of the circuit during the time in which that current changes still did not happen in the remaining parts of the circuit. That therefore implies that it is impossible for the change in current that is produced directly at one location in the circuit by a given electromotive force to take place in all other parts of the circuit exactly simultaneously, since it can only arise gradually in one part after the other, after the accumulation of free electricity that is necessary to produce it in each part has been formed previously.

If, for the sake of simplicity, one focusses on, e.g., the approximately-allowable Ohm hypothesis about the distribution of free electricity in the conductor, by which the free electricity in the element of length $rd\varphi$ of the circuit at the end of the arc φ is represented by $a(\varphi - \pi)d\varphi$, then that will give the integral value:

$$= a \int_0^\pi (\varphi - \pi)d\varphi = -\frac{\pi^2}{2}a \ ,$$

for the free negative electricity in the one half of the circular conductor and the integral value:

$$= a \int_\pi^{2\pi} (\varphi - \pi)d\varphi = +\frac{\pi^2}{2}a \ ,$$

for the free negative electricity in the other half, in which, from page 370:

$$a = \frac{r}{2\chi}(E' - E) ,$$

if Edx denotes the mass of free electricity in the element of length dx at the end of the arc $r(\pi - \chi)$ and $E'dx$ denotes the mass of free electricity in an equally-long element dx at the end of the arc $r(\pi + \chi)$. The length of the piece of the circuit that lies between those two elements is then equal to $2r\chi$. If one now denotes the mass of free electricity in two such elements dx, but between which only a piece of the circuit of unit length lies, by εdx and $\varepsilon'dx$, then one will get:

$$\varepsilon' - \varepsilon = \frac{E' - E}{2r\chi} ,$$

and as a result:

$$a = r^2(\varepsilon' - \varepsilon) ,$$

and when one substitutes that value of a in the expression above for the integral value of the free negative and positive electricity, one will get:

$$-\frac{\pi^2 r^2}{2}(\varepsilon' - \varepsilon) \qquad \text{and} \qquad +\frac{\pi^2 r^2}{2}(\varepsilon' - \varepsilon) .$$

From page 370, the electromotive force that results from that is:

$$\frac{2\pi}{\chi}(E' - E)\left(\log \cot \frac{e\alpha}{8r} - \beta\pi\right) = 4\pi r(\varepsilon' - \varepsilon)\left(\log \cot \frac{e\alpha}{8r} - \beta\pi\right) .$$

If one lets k denote the resistance of a unit length and cross-section of the conductor in the units that were established in Section 13.27 (so the resistance of the entire circuit whose length equals $2\pi r$ and whose cross-section in equal to $\pi\alpha^2$ will be denoted by $2rk/\alpha^2$), then the quotient of that electromotive force and that resistance will represent the current intensity eu, where e is the mass of the positive or negative that is included in a piece of the circuit of unit length and u denotes the drift velocity, so:

$$4\pi r(\varepsilon' - \varepsilon)\left(\log \cot \frac{e\alpha}{8r} - \beta\pi\right) = \frac{2r}{\alpha^2}k \cdot eu .$$

Now, should the current intensity in that circuit change by a ratio of $1 : n$, then neu would have to enter in place of eu, and as a result $n(\varepsilon' - \varepsilon)$ would also have to enter in place of $(\varepsilon' - \varepsilon)$, and in that way, the free negative and positive electricity would take on the following expression:

$$-\frac{\pi^2 r^2}{2} \cdot n(\varepsilon' - \varepsilon) \qquad \text{and} \qquad +\frac{\pi^2 r^2}{2} \cdot n(\varepsilon' - \varepsilon) .$$

That would then yield a change in that integral value that is equal to:

$$-\frac{\pi^2 r^2}{2}(n - 1)(\varepsilon' - \varepsilon) \qquad \text{and} \qquad +\frac{\pi^2 r^2}{2}(n - 1)(\varepsilon' - \varepsilon) .$$

However, the possibility of that change assumes that the increase in drift velocity that is equal to $(n-1)u$ at the beginning of the circuit where the strengthening of the electromotive

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force takes place by which the change in current intensity is accomplished, happened earlier than in the middle of the circuit, which is most distant from that location, and indeed during a time interval T in which a mass of negative or positive electricity that is equal to $(n-1)eu \cdot T$ flowed through the cross-section of the circuit as a result of the change in velocity $(n-1)u$, which is equal to the change in the integral value above, which will yield the following equation:

$$\frac{\pi^2 r^2}{2}(n-1)(\varepsilon' - \varepsilon) = (n-1)eu \cdot T .$$

It follows from that, in conjunction with the equation that was found before:

$$4\pi r(\varepsilon' - \varepsilon)\left(\log \cot \frac{e\alpha}{8r} - \beta\pi\right) = \frac{2r}{\alpha^2}k \cdot eu ,$$

that the time interval T is:

$$T = \frac{\pi^2 r^2}{2} \cdot \frac{\varepsilon' - \varepsilon}{eu} = \frac{\pi r^2}{4\alpha^2}\frac{k}{\log \cot \frac{e\alpha}{8r} - \beta\pi} .$$

It is assumed here that n goes to nu at the same time as the first moment of the change in drift velocity in the first element of the circuit and that this new drift velocity nu will persist unchanged in that element from that point onward. With the assumption that a similar sudden transition in the drift velocity from u to nu takes place in all parts of the circuit, one can ultimately determine the velocity of the current distribution in each part of the circuit. Namely, under that assumption, the time t that it takes for the current to propagate through a piece of the circuit corresponding to an arc $r\psi$ is determined by the following equation:

$$t = \frac{\psi^2 r^2}{4\pi\alpha^2} \cdot \frac{k}{\log \cot \frac{e\alpha}{8r} - \beta\pi} .$$

One obtains the speed of propagation $rd\psi/dt$ by differentiating that equation with respect to t and ψ:

$$\frac{rd\psi}{dt} = \frac{2\pi\alpha^2}{kr\psi}\left(\log \cot \frac{e\alpha}{8r} - \beta\pi\right) ,$$

which says that this speed will get smaller as the piece $r\psi$ of the circuit through which the change in the current has already propagated gets larger.

In that expression for the speed of propagation, k denotes the resistance of the conductor per unit length and cross-section and indeed in the units that were defined in Section 13.27. If one lets q denote the resistance of a conductor with the same length and cross-section in the units that were defined in Section 13.26 (which is measurable by known methods), then from Section 13.27 one will have:

$$k = \frac{16}{c^2}q ,$$

and if one substitutes that value of k in the equation above, then one will get the speed of propagation $rd\psi/dt$:

$$\frac{rd\psi}{dt} = \frac{\pi c^2 \alpha^2}{8qr\psi}\left(\log \cot \frac{e\alpha}{8r} - \beta\pi\right) ,$$

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from which it will follow that when one knows the velocity c with which two electric masses must move relative to each other when they are neither attracted nor repelled, the speed of propagation $rd\psi/dt$ can be calculated from that, and conversely, when the speed of propagation $rd\psi/dt$ is measured, the speed c can be calculated from it. However, if both speeds c and $rd\psi/dt$ can be determined by independent observations, then one would have obtained the means to test the validity of the equation above experimentally. That equation implies that the speed of propagation $rd\psi/dt$ differs, not merely between different circuits, but also at different places in the same circuit. That is because the numerical coefficient $\frac{1}{8}(\log \cot e\alpha/8r - \beta\pi)$ has different values for different circuits, and in one and the same circuit for which the numerical coefficient is given as

$$\frac{1}{8}\left(\log \cot \frac{e\alpha}{8r} - \beta\pi\right) = n \ ,$$

the speed of propagation at a well-defined location in the circuit will be inversely-proportional to the resistance in that piece $r\psi$ from which the propagation of the current took place from its source to there. If one denotes that resistance in the unit that was defined in Section 13.26 by $w = q \cdot r\psi/\pi\alpha^2$, then $rd\psi/dt = n \cdot c^2/w$. The speed of propagation will then decrease as the propagation gets more distant from its source and will therefore be much easier to measure in very long circuits that it is in shorter ones.

However, as far as the drift velocity u is concerned, one easily sees that ultimately, except for the obstacles that one finds in performing the measurement of the speed c in the way that was given in Section 13.34 or in this Section, its determination primarily breaks down due to one's complete ignorance of the masses of positive or negative electricity $\pm e$ that are contained in a piece of the conductor of unit length. That is because in order to determine the product eu, from Section 13.27, one has the equation:

$$eu = \frac{c}{4}i \ ,$$

where i can be measured in the known way. It would seem that the possibility of getting detailed information about the values of e and u would be based upon the fact that the resistance of a conductor, which is defined only by its effects up to now, namely, by the dependency of the current intensity in it upon a given electromotive force, could also be defined more closely by its causes. Namely, if one succeeds in ascertaining the sources of the resistance and if that implies, for example, that the resistance in a conductor depends upon the value e that pertains to the conductor, and indeed the fact that it will be larger or smaller according to whether the value of e is smaller or larger, and it can be represented by d/e, where d is determined independently of e from the other properties of the conductor, then that would explain the fact that, from Section 13.27, i can be set equal to the quotient of the electromotive force $\varepsilon \cdot 4/c$ (in which ε is measurable, as in Section 13.26) and the resistance d/e, and as a result:

$$eu = \frac{c}{4}i = \frac{\varepsilon}{d}e \ ,$$

so

$$u = \frac{\varepsilon}{d} \ .$$

The value of e would, at the same time, be obtained from that determination of u, as well. That shows the importance of a closer investigation of the causes of resistance in the study

of electricity, which have still not been discussed up to now.

13.36   On the Causes of Resistance in a Conductor

For a complete understanding of resistance, it does not suffice to define the magnitude of the resistance by its effects, i.e., by the strength of a current that is produced by a given electromotive force, but it is also necessary to define the magnitude of the resistance from its causes. Without that essential extension, our knowledge of the essence of resistance would be lacking, and the magnitude that is ascertained would be merely a tool for electrodynamics whose true physical significance would still be unknown. Now, if resistance has merely been considered in terms of its effects up to now, then that is based upon the fact that nothing at all of a fundamental nature regarding its causes has been ascertained up to now. Merely the dependency of resistance on the external dimensions of the conductor (namely, its length and cross-section) has been ascertained, but that dependency is merely concerned with the absolute resistance of a conducting wire and has no relationship to the specific resistance of conducting metals, about whose causes nothing at all is known. The causes seem to lie hidden so deeply in the nature of the body that they are inaccessible by the paths of research up to now. Briefly, the question of the causes of galvanic resistance lead to a realm of science that is still entirely unconstructed. I shall therefore restrict myself to only a specialized discussion, namely, a discussion of the relationship of resistance to the nature of the electric fluid itself, how it is to be defined, and what relationship it has to the double electric currents, and how they are to be adapted to and established in the usual representation.

The question of the causes of resistance first leads one to focus upon the extent to which those causes lie in the ponderable carrier of the current and the extent to which they lie in the electric fluid that it contains. The fact that there exist ponderable parts of channels through which electric fluids flow that restrict it more or less and can therefore influence the electric current is self-explanatory. However, that raises the question of whether that cause alone is already sufficient to explain resistance. That cause would merely restrict the mass of electric fluid that can take part in the current. However, it is in the nature of resistance, as we understand it by its effects, that the magnitude of the resistance will not merely restrict the mass of electric fluid that takes part in the fluid motion, but that it will also restrict the motion itself. However, that restriction of motion itself cannot be based upon merely the presence of ponderable parts, but must necessarily assume that there are forces that preserve the equilibrium of the continuing electromotive forces, because without those forces the electric fluids would have to keep accelerating their motion, which is not the case for a steady, uniform current.

One then further asks, from whence originate the forces that preserve the equilibrium of the continuing electromotive forces for a steady, uniform current, and in that way prevent further acceleration of the electric fluid in its motion? Are those forces purely-electric forces or are they forces that the ponderable parts exert upon the electric fluids that flow through them? If we assume, as we have always done, that there are two electric fluids in the galvanic current that simultaneously flow through that conductor in opposite directions, then we are very close to looking for a cause of the resistance to the motion of that fluid in the fluid that opposes it. Namely, the positive and negative fluid will combine into a neutral mixture at the moment that they meet, and no matter how easy that it might be to also separate that neutral combination, such a new separation could result only from a new electromotive force, and not as a consequence of a persistence of the motions that the

two fluids possessed before their combination, because they must be considered as cancelling each other out as a result of their meeting and combination. That implies that whereas each fluid must be ascribed a persistent motion by itself, the motions of both fluids together will not persist in the double current. However, even if that reason for the motion of the electric fluids to not persist in the double current is also the correct one, nonetheless, one would not gain any clear insight into the phenomenon itself in that way as long as one does not know the forces that act upon the mixing and combination of the electric fluids when they meet and which must be overcome during their repeated separation. One then asks whether even more forces come into consideration in that way besides the ones that are already determined by the general fundamental law of electricity, e.g., whether special molecular forces act upon the electric fluid, as well. If that were not the case, then the evolution of the alternating combination and separation of the electric fluid in the double current would have to be determined more precisely according to the known fundamental law of electric action. Without such a more precise determination, one can generally assume with some plausibility only that the intensity of a double electric current must depend upon not just the mass of the electric fluid that takes part in the current, but also upon the number of separations that result in a certain time interval, and that the number of those separations must be proportional to the electromotive force that continues to act during that time interval. If that implies, e.g., the fact that every electric particle will always suffer an equal number of combinations and separations in an equal time interval for an equal electromotive force, and in that way would proceed along an equal path segment, then the drift velocity u would always be the same for an equal electromotive force, and the current intensity for equal electromotive force would vary merely with the amount of electricity e that is contained in such a path segment (e.g., in a unit length of the conductor), and would indeed be proportional to it, which implies that the so-called resistance would likewise vary with only e, and indeed it would be inversely proportional to e, which is the case that was cited at the end of the previous Section as an explanation.

Should the cause of resistance be actually contained in the alternating combination and separation of the electric fluids when they meet in the double current, then it would further follow that a *steady* double current without continual external electromotive forces would be impossible, and one would then have to ask how the assumption of *steady molecular currents* could be compatible with the explanation for magnetic and diamagnetic phenomena. The possibility of such molecular currents must then necessarily rest upon an action of ponderable molecules by which the paths of the electric fluids that move around that molecule in opposite directions would remain separate from each other when, e.g., the one fluid describes a narrower orbit, while the other fluid describes a wider one, such that the two fluids can never meet and combine during their motions.

The following argument will serve to explain the evolution of the alternating combination and separation of the electric fluids in the double current, as it would be derived from the fundamental law of electric action without resorting to special molecular forces in that fluid: Let positive electric masses be at A, B, C, ... , about which one might initially assume that they are fixed at the locations where they are found, [Figure 6].
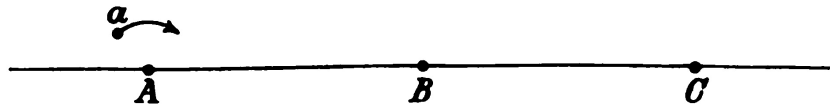
Fig. 6.

At a, one presently finds a mobile negative electric mass upon which the neighboring positive mass at A acts strongly enough that the action of the more distant masses B, C, ... can be neglected. The masses at A and a act upon each other with a force that depends upon their magnitudes, relative velocity, and the change in it. Meanwhile, for the sake of simplicity, one might assume that the correction to the electrostatic force (which depends upon the masses and the distance)[451] from the relative velocity and its change proves to be so small in comparison to the latter force that it can likewise be neglected. With those assumptions, it will follow that when no other force acts upon the mass at a, that mass must obey the laws of motion in central forces, which are inversely proportional to the square of the distance. As a result, the mass at a will describe, e.g., an elliptical orbit around A, according to Kepler's laws.[452] However, a perturbation of that motion of the mass in question will occur at A as long as an electromotive force parallel to the line AB with constant intensity acts upon the mass considered, in addition to the central force. The elements of the elliptical motion up to now would then be changed continually, and the orbit that is described by the mass considered would go over to a spiral curve in that way, in which the mass considered would ultimately be led so far from A that it would leave the sphere of action of A and arrive in the sphere of action of B, and so on, until it had described a number of spiral windings around B that also led it so far from B that it left the sphere of action of B and entered the sphere of action of C. In that way, an electromotive force can then cause the negative electricity to flow away in the direction ABC, which the positive masses at A, B, C take no part in. The essence of that argument consists of the fact that as soon as the electromotive force ceases to act, the mass in question will, at the same time, once more move in an elliptical orbit according to Kepler's laws around the positive mass in whose neighborhood it is found, because after the perturbing force disappears, no other change in the elements of its central motion will take place. One also easily sees that nothing would change in that fundamental relationship if the positive masses at A, B, C, ... were likewise assumed to be mobile, and that the negative masses in whose neighborhoods they are found would be subject to not only the central force, but also the perturbing effect of the same electromotive forces, although it would have a direction for the positive masses that is the opposite of the one that it had for the negative ones. That implies the following result: If the electromotive force c acts upon the negative mass considered alone, then it would impart a velocity ct upon that mass in the direction ABC during the time t, with which the mass must advance steadily in the direction ABC, also after the force c has ceased to act. However, when combined with the central forces of the positive masses at A, B, C, ..., as long as the electromotive force c acts, it will indeed likewise cause the mass considered to advance in the direction ABC, but as soon as the force c ceases to act, that advance will also cease, i.e., that advance of the mass considered cannot happen in the direction ABC with a velocity that persists after the force that produced the advance ceases to act. Therefore, the reason why the mass considered will not advance further in the direction ABC after the

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[451][Note by AKTA:] That is, which depends upon the amount of electric charge on the two interacting particles and upon their distance.

[452][Note by AKTA:] J. Kepler (1571-1630).

electromotive force has ceased to act lies in the central forces that the positive masses exert upon the negative mass in question. However, in the theory of galvanic currents, the word *resistance* refers to essentially nothing other than the fact that the advance of the electric fluid in a galvanic current is proportional to the electromotive force, i.e., it will cease as soon as the electromotive force ceases to act. It then follows from this that the reason for resistance might lie in the *central forces* that the positive and negative masses mutually exert upon each other when they meet in the electric double current. It would be important for the sake of further theoretical investigations to derive a precise and unambiguous definition for resistance on that basis and develop its relationship to the way that it is defined by its effect. In that way, one would mainly arrive at a determination of the time that a particle needs in order to pass from one winding around a central mass A to the corresponding winding around the central mass B that follows it in its spiral orbit. However, the theory of perturbation in astronomy shows that such a determination can also encounter great difficulties, even when all of the essential elements for the calculation are given.

VI - Comparing the General Principles of Neumann's Mathematical Theory of Induced Electric Currents with the Laws of Induction that are Derived from the Fundamental Law of Electric Action

13.37   On the Difference Taking Place at Sliding Contacts According to Neumann

The treatise that Neumann presented to the Berliner Akademie der Wissenschaften in 1845, namely, "Die mathematischen Gesetze der inducierten Ströme,"[453] was cited before in Section 26 of the first treatise on *Electrodynamic Measurements*.[454] That treatise, which has yet to be published, can be cited only in the form of the abstract that appeared in Poggendorff's *Annalen*. Since then, Neumann presented a more comprehensive work on the same topic to the Berliner Akademie der Wissenschaften: "Ueber ein allgemeines Princip der mathematischen Theorie inducierter Ströme," which appeared in the Schriften der Berliner Akademie der Wissenschaften in 1847 and was published by Reimer in Berlin in 1848.[455] In that treatise, Neumann established the following general theorem:

> "If a closed, unbranched, system of conducting arcs $A_|$ is carried over to another one $A_{||}$ with a new form and position by an arbitrary displacement of its elements that does not alter the connectivity of the conductors, and if that alteration of $A_|$ to $A_{||}$ takes place under the influence of a system of electric currents $B_|$ that simultaneously experiences a change in position, form, and intensity from $B_|$ to $B_{||}$ by an arbitrary displacement of its elements, then the sum of the electromotive forces that are induced by that change in the system of conducting arcs will be equal to the induction constant ε times the difference in potential values of the current $B_{||}$ relative to $A_{||}$ and the current $B_|$ relative to $A_|$, when the unit current is thought of as flowing through $A_{||}$ and $A_|$."

Once Neumann had developed that theorem, along with its consequences, in the first four paragraphs of his treatise, he continued in § 5:

> "In his treatise "Elektrodynamische Maassbestimmungen, etc.," W. Weber had paved the way that would bridge the chasm in our knowledge of the electrostatic and electrodynamic action of electricity. He showed how Ampère's laws for the action of two current elements[456] could be derived from the action of positive and negative electricity in the one element on the two electric currents in the other element. That analysis of Ampère's laws led to the fundamental law of two electric masses, by which the law depended upon not only their relative distance, but also on the relative velocity and its variation. At the same time, as Weber showed, that fundamental law explained the induction phenomena and their laws. The subject of this paragraph is to establish the extent to which the results that are contained in the foregoing agree

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[453][Note by AKTA:] See [Neu46]. See also [Neu47] with French translation in [Neu48].
[454][Note by AKTA:] See Section 5.26.
[455][Note by AKTA:] [Neu49].
[456][Note by AKTA:] See footnote 10 on page 13.

with the laws of induction that are deduced from Weber's fundamental law of electric action."

Neumann, *loc. cit.*, developed a general expression for the induction from that fundamental law of electric action, as it was presented in the first treatise on *Electrodynamic Measurements*,[457] in his own treatise, which he then applied to various types of induction, namely, to the cases in which:

1. Neither the current element nor the conducting element suffers a change in position, and the induction originates in merely a change in current intensity.

2. The induction is produced by merely a change in position of the conducting element that takes place under the influence of a constant and immovable current.

3. The induced conductor was at rest and the induction is excited by a motion of the entire carrier of a constant current.

All of those cases implied the result that the law of induction that was derived from that fundamental general law of electric action agreed completely with the results of the general principle of the mathematical theory of induced currents that Neumann had presented.

Neumann continued:

> "Things are different with the equation that expresses the electromotive force that is induced by a simple circulating current[458] under the assumption that it consists of a moving piece of conductor and one at rest. The sum of the electromotive forces that are excited during the traversal of the inducing element is the same in both formulas, but the direction of the induced current is the opposite one."

Now, in order to decide whether a discrepancy existed in those individual cases between the results of the law of induction that Neumann had derived from the fundamental general law of electric action and the results of his own general principle of the mathematical theory of induced currents, and which of them was actually correct, Neumann cited an experiment in his treatise that had proved that the formula that was derived from Neumann's general principle was also the correct one in that case. I have also repeated that experiment, which will be described below, and completely confirmed the result that Neumann obtained. Once the law that applied to that case was guaranteed to be factual by that experiment, Neumann subjected the derivation of that law of induction that he himself has given for that case from the fundamental general law of electric action to a more thorough proof. He said:

> "One must examine where the derivation of the formula from Weber's fundamental law breaks down. The fact that the contradiction in question occurs only for inducing currents with *sliding contacts*, immediately leads one to consider that fact. In that case, new elements will enter or leave the current path, in which the current strengths will change from i to 0 within a very short time, and by means of which their change in intensity will exert an inducing effect that is already contained in my formulas, but must still be considered in order to apply Weber's fundamental law."

[457][Note by AKTA:] See Chapter 5 on page 33.
[458][Note by AKTA:] In German: *von einem einfachen Stromumgang.*

That test led Neumann to the result that this second part of the induction that was not considered in the first derivation of the fundamental general law of electric action compensated for the one-half of the contradiction in question, since the sum of the electromotive forces then proved to be equal to zero in the first and second parts.

Finally, from that test of the calculation, which did not lead to a satisfactory result, Neumann went on to test the assumption that was at the basis for that calculation in regard to the physical circumstances that existed in that case under which the induction took place. That assumption consisted of assuming that the current strengths in the conducting elements that entered or exited that current path at the sliding contact changed from 0 to i or from i to 0 within a very short time interval. However, it is a condition for a steady current that the same current intensity must be found in all elements of the closed circuit, and therefore when the current intensities in the elements that enter or exit the sliding contact also vary, it would seem that the *mean* value of the current intensity over the short time interval in which it varies would also have to satisfy that condition, here as well, and if the current intensity in the entire circuit is to be the same $(= i)$, then that would assume that the current strengths vary from 0 to $2i$ or from $2i$ to 0 in the elements that enter or exit the sliding contacts. Now, with that assumption about the physical circumstances under which induction takes place, it can be easily proved that the initially-remarked contradiction will vanish completely, and the law of induction that is derived from the fundamental general law of electric action will also agree with Neumann's general principle of the mathematical theory of induced currents for this case.

However, as far as the assumption itself is concerned, upon which the elimination of the contradiction in question rests, Neumann said that it was "justified less by its evidence than by its success." Nonetheless, despite the concerns that might be raised in regard to that assumption itself, it seems to me that if the assumption were true, then it would necessarily lead to a consequence that would once more negate that success entirely. Namely, if it is given that the current strengths in the elements that enter or exit the sliding contact actually change from 0 to $2i$ or from $2i$ to 0 within a very short time interval, then it seems to me that this would necessarily have to lead to the consequence that immediately after the current strengths in the entering elements increased to $2i$, they will likewise once more drop down to i in the elements that remain in the chain, because i refers to the current intensities in all parts of the circuit, which are necessarily equal. In the same way, for the parts that exit the sliding contact, in which the current intensity was constantly equal to i, before that current intensity would decrease from $2i$ to 0, it would first have to rise from i to $2i$. If one introduces not only the change that was assumed above into the calculation, but also that one, which is necessarily linked with it, then that will give the same results as when one ignores that assumption and simply assumes, as was done before, that the current strengths in the elements that enter or exit the sliding contact change from 0 to i or from i to 0 within a very short time interval.

It then seems to me that the insoluble contradiction will solve itself with the assumption above when one makes a closer study of whether all of the given relative motions of the electric fluid and their variations are actually included in the calculations in the derivation of the law of induction from the fundamental general law of electric action that Neumann gave, and that solution will be given once I have presented my description of the aforementioned experiment that Neumann carried out in order to resolve that important question, along with my repetition of it.

13.38 Description of Neumann's Experiment and Its Repetition

Neumann said on p. 59 of the cited treatise [related to Figure 7 below]:
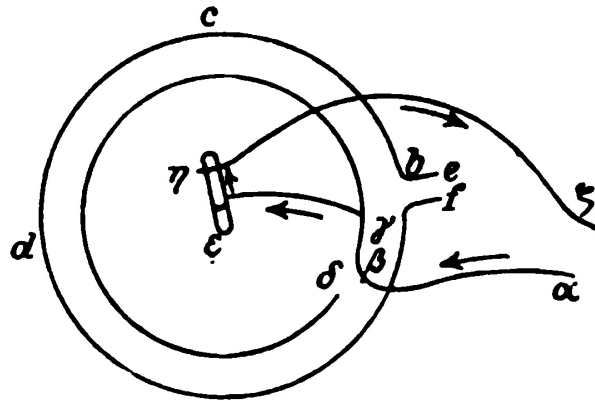

Fig. 7.

"Although I have excluded any description of experiments from this treatise, due to the importance of this case, I will give a brief outline of the device to which I appealed in order to test the formulas that were spoken of. One part of the closure wire of a galvanic circuit α is bent into a circle $\beta\gamma\delta$. The end δ of that circle comes very close to its starting point β without making a conductive connection with it. A rotating axle $\varepsilon\eta$ that is perpendicular to the plane of the circle at its center leads the moving part of the path $\varepsilon\gamma$ around in a circle, and indeed in such a way that its end γ will continually drag against the ring. Starting from α, the inducing current enters the ring at β and leaves it at γ to enter the moving part of the path, and then leaves that part and enters the conducting axle $\varepsilon\eta$, and at η, it returns to α through the conducting wire $\eta\zeta$, which is at rest. The direction of the current is suggested by arrows in the figure. Concentric to the ring, there is a circular conductor bcd, in which a current is induced by the motion of the path segment $\varepsilon\gamma$. Due to the short distance from δ to β, the path itself can be regarded as closed when the moving path segment β advances from γ to δ, which is why the given formulas can be applied to the determination of the electromotive force that is developed during a cycle... In order to observe the direction and magnitude of the induced current, the following arrangement was employed: The induced circular conductor was broken at b[459] and equipped with two continuations e and f there, one of which was connected directly to the end of the multiplier wire, while the other one was connected to a metal spring that made a sliding contact with a metal case that was isolated from the axle $\varepsilon\eta$. The induced current then went through that spring into the case, left it by way of a second metal spring that pressed against it, and then left it to go to the other end of the multiplier wire. The case had a cutout that was filled with wood, upon which the one spring lay at the moment when the moving path segment $\gamma\varepsilon$ left the ring $\beta\gamma\delta$ at δ, in order to once more become connected with it conductively at β. Namely, at that moment, the closure of the inducing current will be broken and then

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[459][Note by AKTA:] Due to a misprint in the original, we have β here instead of b.

reestablished, so that current will vanish and then reappear, but no induction will be excited in the conductor in so doing, because the arrangement that was given above will offer no closed, connecting path. Therefore, only the current that is induced by the motion of the path segment $\gamma\varepsilon$ will arrive at the multiplier, and its direction and intensity can be observed, since it always flows in the same direction under continued rotation of the axle $\varepsilon\eta$. Observation indicated an induced current, and as far as its direction was concerned, it was the same as the one that my formula required. In order to prove that not only the direction, but also the strength of the induced current was expressed correctly by that formula, I proceeded as follows: The spring that broke the conductive connection in the inducing current path was placed so high that it no longer met the wood-filled cutout in the case through which that break was effected. The induced currents are now always offered a closed path. With a rapid rotation of the axle $\varepsilon\eta$, three currents arrived at the multiplier within a very short time, namely, the one that was induced by the motion of the path segment $\gamma\varepsilon$, then the one that was induced by the vanishing of the inducing current at the moment when the moving path segment left the ring at δ, and finally, the one that was induced by its reappearance once the piece again reached the ring at β. The force that those three currents exerted during the brief duration of a traversal of the path segment $\gamma\varepsilon$ on the magnetic needle of the multiplier is proportional to the sum of their electromotive forces. The needle will take its almost-fixed position on one side or the other of the meridian according to whether the sign of that sum is positive or negative, or it will keep its position at the meridian when that sum is equal to zero... Observation showed that when the rotation happened rapidly, the needle remained at the meridian, which proves the validity of my formula in regard to both the direction and strength of the induced current."

In order to repeat that experiment, 1 kilogram of copper wire that was 2/3 millimeters thick and wound with silk was wound around a thin brass ring of diameter 120 millimeters. A wooden cylinder whose diameter was somewhat smaller than the brass ring was placed inside that brass ring, and that cylinder was provided with a metal axle by which it could be rotated rapidly by means of a gear. A copper strip was laid in the wooden cylinder that reached from the metal axle to the periphery. Three brass springs were connected to that copper strip on its periphery that contacted the brass ring at three internal points that lay along a line that was parallel to the axis of rotation. Those three springs served to establish a secure contact in order for the connection between the brass ring and the other two springs to be maintained when one of the three springs failed for a moment. One of the two conducting wires from a Grove cell[460] was fixed at the position of the rotational axis, while the other one was fixed to any point on the brass ring. The two ends of the silk-wound copper wire that was wound around the brass ring were connected to the multiplier of the galvanometer, whose needle possessed a period of oscillation of about 10 seconds.

The apparatus that was described differed from Neumann's in essentially just one respect, namely, due to the fact that the brass ring was not cut, which had the effect that the current in the rod that entered into the metal rotational axis could follow two paths from the place on the brass ring to which it was led by the brass spring to another place on the brass ring form which it returned to the rod. The current then split between those two paths, namely, between the two parts of the brass ring that connected the contact point of the

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[460][Note by AKTA:] See footnote 98 on page 53.

brass springs with the locations where the other conducting wire of the rod was attached to the brass ring. Essentially the same thing was achieved by that splitting of the current as the intention in Neumann's second experiment for maintaining the closure of the induced circuit at the moment when the sliding contact passed the cut in the brass ring, namely, that the sum of the electromotive forces that were exerted by the current-elements that entered and exited the sliding contact would be equal to zero for an entire rotation of the axle, and therefore under rapid rotation, the observed effect on the galvanometer would depend upon just the sum of those electromotive forces that originated in the motion of the path segment $\gamma\varepsilon$. The splitting of the current that was just described likewise made it possible for the sum of the electromotive forces that were exerted from elements that entered and exited the sliding contact to be equal to zero, and indeed, for not only the entire duration of a rotation, but for each individual instant, which gave one an advantage in performing the experiment that its success was no longer coupled with the condition of rapid rotation, as was the case in Neumann's experiment.[461] Another apparatus that was devised in order to repeat Neumann's first experiment, as well as the second one, completely unchanged shall

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[461][Note by WW:] The fact that the splitting of the current that was described actually has the given effect can be shown in the following way: One denotes the constant intensity of the undivided current by i and splits that current into two parts when it enters the brass ring, one of which has an intensity $i_|$ and goes through the circular arc ψ to the exit point, while the other one has an intensity $i_{||}$ and goes through the arc $2\pi - \psi$ to the exit point. Ohm's law then gives the following equations for the splitting:

$$\begin{aligned} i_| + i_{||} &= i \ , \\ i_| : i_{||} &= (2\pi - \psi) : \psi \ . \end{aligned}$$

If one now increases ψ by $d\psi$, then the current intensity $i_{||}$ in the arc element $d\psi$ will vanish, and in its place, the current intensity $-i_|$ (where the negative sign means that the direction of the newly-created current has the opposite direction to the increasing arc ψ) will arise in the same element. The vanishing of a positive current $i_{||}$ in the element $d\psi$ creates an electromotive force that is proportional to $i_{||}d\psi$, and the creation of a negative current $-i_|$ in the element $d\psi$ will produce an electromotive force that is proportional to $-(-i_|d\psi) = i_|d\psi$, whose sum is then equal to:

$$a(i_| + i_{||})d\psi = aid\psi \ ,$$

if a denotes a constant factor. However, when ψ grows by $d\psi$, the ratio of $i_| : i_{||} = (2\pi - \psi) : \psi$ will likewise change, while the sum $i_| + i_{||} = i$ will remain unchanged, which will imply the two differential equations:

$$\begin{aligned} di_| + di_{||} &= 0 \ , \\ \psi di_| - (2\pi - \psi)di_{||} &= -id\psi \ , \end{aligned}$$

and as a result, $di_| = -id\psi/2\pi$ and $di_{||} = +id\psi/2\pi$. The change in intensity $di_|$ of the current $i_|$ in the arc ψ in the direction of decreasing values of ψ will generate an electromotive force $+a\psi di_| = -a\psi id\psi/2\pi$ that is proportional to $\psi di_|$. The change in intensity $di_{||}$ in the arc $(2\pi - \psi)$ in the direction of increasing values of ψ will generate an electromotive force $= -a(2\pi - \psi)di_{||} = -a(2\pi - \psi)id\psi/2\pi$ that is proportional to $-(2\pi - \psi)di_{||}$. That makes the sum of all electromotive forces that result from the increase $d\psi$ in the arc ψ equal to:

$$aid\psi - a\psi\frac{id\psi}{2\pi} - a(2\pi - \psi)\frac{id\psi}{2\pi} = 0 \ ,$$

which was to be proved.

be described later on.

The following two experiments were made: *First of all*, the wooden cylinder was rotated around its axis 10 times per second by way of a gear, while the inducing current flowed through the rotational axis and the brass ring, and it was observed that no current was induced in that way. The unperturbed state of the galvanometer needle could be authenticated to within 1/2 a scale division. That result agreed completely with that of Neumann's second experiment. *Secondly*, yet another auxiliary wire was wound around the brass ring, and its ends were connected with the rod, such that the current would have to go through that wire, instead of the rotational axis and the brass ring. At the moment when that circuit was closed, an induced current was observed with the galvanometer whose direction was opposite to that of the inducing current. Opening the circuit produced an induced current of the same strength, but the same direction as the inducing one. In both cases, the galvanometer needle obtained a deflection of about 22 scale divisions. The second experiment served to prove that in the first experiment, for 100 rotations of the moving current segment during one oscillation, the galvanometer needle would have to take on a deflection of 1000 scale divisions if each rotation were to generate an electromotive that would be equal to the one that was determined in the second experiment. Such a force was not present.

That experiment deserved especial interest with the current splitting that was described, since it corresponded precisely to the known electrodynamic rotation experiment in which a moving current segment was found inside of a fixed circular current that pointed to the center of that circle. For that electrodynamic rotation experiment, see Poggendorff in the *Annalen*, Vol. 77 (1849), pp. 22 *et seq.*[462] It is known that the circular current made the moving radial current rotate in the direction of the circular current itself or the opposite one according to whether the direction of the current in the moving current segment pointed towards the center or away from it. From the otherwise-valid rule that says that electromagnetic or electrodynamic experiments will invert into magneto-electric or voltaic induction experiments, it would seem that when that moving radial current is rotated (as was the case in our experiment), a current must be induced in the fixed circular conductor that is parallel or opposite to the direction of rotation according to whether the current in the moving conductor points towards the center or away from it, respectively. That also explains the fact that replacing the mercury trough in which one cares to immerse the moving current segment in the aforementioned rotation experiment with a brass ring that contacts the moving current segment is inessential and can have no influence on the result. However, the experiment has taught us that the induction current that we would expect from the cited rule did not exist in this case. That rule of inversion is not true in general then, but an exception to it will exist when the closed *inducing* current consists of a moving current segment and an immobile one that are connected by a *sliding contact*. As is known, the induction current will exist when the *induced* conductor consists of two parts that are connected by a *sliding contact*.

Furthermore, I have also repeated Neumann's experiment unaltered, in which the brass ring was cut through next to the place where the conducting wire that came from the rod was attached to it. The one connection between the wire that was wound around the brass ring and the multiplier of the galvanometer was established by a spring and could be broken by compressing that spring. That compression was effected by a wooden rod that was attached to the wooden cylinder and placed in such a way that releasing the spring would take place at the moment when the brass springs that were attached to the wooden cylinder came to be

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[462][Note by AKTA:] [Pog49].

at the cut location on the brass ring. It should be further pointed out that the wire that was wound around the brass ring had a smaller number of windings than before. The following experiment was then made: *First of all*, the wooden cylinder was rotated around its axis by means of the gear 10 times per second, and an induced current was observed with the galvanometer that was so strong that the deflection of the needle amounted to over 500 scale divisions and could no longer be measured with that scale. *Secondly*: After removing the wooden rod, the wooden cylinder was fixed in the position where the brass springs that were attached to it contact the end of the sliced brass ring that was not connected to the pile, such that the current would have to flow through the entire brass ring. Now, at the moment when the pile was closed, an induced current was observed in the galvanometer that deflected the needle by 13.5 scale divisions in the same direction as in the first experiment, assuming that the direction of the inducing current was the same, and that the first attempt was turned in that direction, in which brass springs would advance from their aforementioned position to the cut in the brass ring. *Thirdly*: An auxiliary wire was wound around the brass ring one time, and the pile was then closed in such a way that the current flowed through that wire winding in the same direction as it did before in the brass ring. At the moment when the pile was closed, an induced current was then observed with the galvanometer that deflected the needle by 13.8 scale divisions in the same direction as before. *Fourth*: the multiplier was weakened, and the first experiment was repeated again. The induced current then produced a residual deflection of the magnetometer needle of 377 scale divisions, although a finer measurement of that deflection could not be performed, due to significant fluctuations that were probably based in the incompleteness of the engineering implementation of the rotation device. *Fifth*: the second experiment was also repeated once more, and it yielded only a deflection of 8 scale divisions with the weakened multiplier, instead of the previously-observed deflection of 13.5 scale divisions. *Sixth*: and finally, the second of Neumann's experiments was also repeated, which differed from the fourth experiment only in that the wooden rod was removed from the wooden cylinder, which had the effect that the induced circuit would always remain closed under the rotation of the wooden cylinder. No deflection of the galvanometer at all was observed for a rotation that was just as rapid as in the first and fourth experiment, and that state of rest could be guaranteed up to a pair of scale divisions inside of which the needle fluctuated.

The results of the measurements that were made in the fourth and fifth experiments admit a comparison that deserves to be pointed out, even when those measurements did not possess any great precision. Namely, the result of the measurement that was made in the fourth experiment allowed one to calculate the greatest elongation from rest that the magnetometer needle would achieve as a result of the motion that would be imparted upon it by a single instantaneous rotation of the wooden cylinder. To that end, it should only be added that the logarithmic decrement of the decrease in the arc of oscillation of the needle was equal to 0.471 60, or that when one denoted that number, divided by the modulus of the system of logarithms, by λ, one would have $\lambda = 1.088$. If, in addition, one lets y denote the deflection that was observed in the fourth experiment for n rotations during the period of oscillation of the needle, then that would yield the following expression for the greatest elongation that the needle would have attained as a result of motion that was imparted upon it by one rotation:[463]

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[463][Note by WW:] See *Supplement* 13.39, where the angular velocity that is imparted upon the needle at rest, when one includes damping, is expressed by

$$x = \frac{y}{n} \cdot \sqrt{\pi^2 + \lambda^2} \cdot e^{-\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} \ .$$

Now, it was found that $y = 377$ in scale divisions and that $\lambda = 1.088$, moreover $n = 100$ (because 10 rotations took place in 1 second, and the period of oscillation was $\tau = 10$ seconds). As a result, the greatest elongation from rest that the magnetometer needle attained as a result of the motion that was imparted upon it by one rotation was:

$$x = 8.164 \ ,$$

in scale divisions, while an elongation of 8 scale divisions was actually observed in the fifth experiment when the wooden cylinder was not rotated, but left fixed in the position in which the current that arose from closing the pile would have to flow through the entire brass ring. It follows from the agreement that emerges from that comparison that the current that was induced in the fourth experiment was only a direct consequence of the rotation that was induced by the current that was generated in the entire brass ring by each rotation (whose re-vanishing could have no influence, since the multiplier circuit was opened at the moment when it vanished). The rotation of the moving current segment itself then had no component in the induced current. One then finds that the determinations that Neumann gave are confirmed completely by that experiment.

13.39   The law of Induction for Inducing Currents with a Sliding Contact

The general principle of the mathematical theory of induced electric currents that Neumann proposed is a theorem that refers to the currents and conductors as a whole, and indeed, it merely refers to their strengths and positions at the beginning and end of the induction considered and represents the desired sum of the electromotive forces independently of any consideration of all interacting elements individually and the consideration of the gradual transition of the currents and conductors from their states at the beginning of the induction to the end of it. The simplification that a theorem of such simplicity and generality contributes whenever it can be applied to the actual determination of the desired sum of electromotive forces is obvious. Things are quite different with the fundamental general law of electric action, because it should give a rule that is valid only for all elementary actions from which the desired sum of electromotive forces will not be obtained directly, but they can be found only indirectly from a summation of all completely-combined elementary actions. When

$$\frac{x}{\tau} \sqrt{\pi^2 + \lambda^2} \cdot e^{\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} \ ,$$

if x denotes the desired elongation and τ denotes the period of oscillation under the influence of damping. However, that angular velocity is given by the angular momentum F that corresponds to the deflection y, divided by the moment of inertia of the needle K, and multiplied by the time of one rotation τ/n in the fourth experiment, so it equals $\tau/n \cdot F/K$. Finally, the angular momentum that corresponds to the deflection y is $F = \frac{\pi^2 + \lambda^2}{\tau^2} Ky$, and as a result:

$$\frac{x}{\tau} \sqrt{\pi^2 + \lambda^2} \cdot e^{\frac{\lambda}{n} \arctan \frac{\pi}{\lambda}} = \frac{\tau}{n} \cdot \frac{\pi^2 + \lambda^2}{\tau^2} y \ ,$$

which will yield the expression for x that was cited above.

one is deriving a law of induction for a particular case, one mainly arrives at a complete combination of all of the elementary actions that the given case assumes. The derivation of the law of induction from the fundamental general law of electric action then demands a very special attention to all of the relationships that should be determined in each given case. That is what happens in the case of the inductions that current elements exert upon other current elements or other conducting elements in the first treatise on *Electrodynamic Measurements*,[464] as well as in the derivation that Neumann gave in § 5 of the cited treatise, which is why one can exhibit essentially two different types of elementary actions for that case, namely, the ones that a current element exerts due to its motion relative to the induced element and the ones that a current element exerts due to changes in its current intensity.

Now, Neumann had also applied that classification of elementary actions to the case of an inducing current with *sliding contact.* That current splits into a moving and an immobile current segment that have conducting connections at two places, at least one of which is a sliding contact. That easily implies that the elementary actions of the moving current segment belong to the first kind, namely, the ones that the current elements exert due to their motion relative to the induced elements. That likewise implies that the elementary actions of the immobile current segment belong to the second kind, namely, the ones that the current elements exert due to changes in their current intensities. Neumann has calculated only the part of the electromotive force that originates in the former sources, but then added the following test of the part of the electromotive force that originates in the other sources.

A further test can be directed to only the question of whether the combination of the elementary actions of the two given types is actually exhaustive in the case of an inducing current with *sliding contact.* In fact, it would be truly exhaustive only if the induced *current elements* were given in that case, because they must belong to either the moving or the immobile current segments, which implies that their elementary actions must be of either the former or latter type, respectively. However, if one examines more closely in the present case whether all given motions of the electric fluid and its variations can truly be traced back to motions of the electricity *in the current elements* and their variations, then that will easily imply that this reduction is possible everywhere, *with the exception of sliding contact.* Namely, a sudden change in the motion of all electric particles occurs at the sliding contact, since the ones that go from the moving current segment to the immobile one cease to take part in the motion of the former component, and the ones that go from the immobile current segment to the moving one begin to take part in the motion of the latter. That sudden change in the motion of all electric particles at the sliding contact cannot be addressed along with the changes that take place *in the current elements* themselves, because that change does not occur in either the current elements of the moving current segment (since all electric particles must also take part in the motion of those current elements, as long they belong to those components) or in the current elements of the immobile current segment. No sudden change in the motion of all electric particles at the sliding contact can be attributed to changes in the motions in the current elements themselves then, and it is therefore the source of a third type of elementary action that must be distinguished from the two types of elementary action that pertain to inducing current elements. One then asks only whether the electromotive forces actually do or do not arise from the given sudden change in the motion of all electric particles at the sliding contact according to the fundamental general law of electric action. In the former case, one sees that those electromotive forces must still be added to the sum of electromotive forces that Neumann calculated, since Neumann did

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[464][Note by AKTA:] See Chapter 5 on page 33.

not include them in his calculations.

The derivation of the electromotive forces that arise in an electric fluid from the sudden change in the motion of the electric fluid at a *sliding contact* is also not included in the derivation of the law of voltaic induction from the basic general of electric action in Section 30 of the first treatise on *Electrodynamic Measurements*,[465] because that law was restricted expressly to the induction in *current elements*, so only the changes in motion of the electric fluid that occurred in the current elements needed to be considered. However, when there are changes in the motion of the electric fluid that do not occur in any current element, but only at the *boundary* between two current elements or at the moment when the electric fluid goes from one current element to the other (and such a case actually occurs at a *sliding contact*), the law of induction above will require some extension if it is to subsume that case. That extension can be given easily, because in order to do that, it is only necessary that the electric masses that suffer such sudden changes in the velocity of their motion and the magnitudes of those changes must be determined precisely in order to apply the basic general of electric action to them, as well. In that way, for ease of understanding, the same relationships shall be used as in the derivation that was given in Section 30 of the first treatise on *Electrodynamic Measurements*. Moreover, for the sake of brevity, none of the arguments that were valid in the same way shall be developed once more but shall only be borrowed from that reference.

As far as the change in *mass*[466] that the electric fluid suffers due to a sudden change in its motion at the sliding contact is concerned, it cannot be expressed by the product $\pm\alpha e$, as it is for a current element, where α denotes the length of the current element,[467] instead, α must be replaced by the length of the path-element udt which the electricity would pass through with the velocity u with which it crosses the sliding contact in the time-element dt. By contrast, just as in Section 30 of the cited treatise, the induced masses can be represented by $+\alpha'e'$ and $-\alpha'e'$, where α' denotes the length of the induced element, and $\pm e'$ denotes the positive or negative electricity that is contained in a unit length of the induced conductor.

The motion of those inducing masses $+eudt$ and $-eudt$, and the paths that they flow through can be depicted in the following way, [Figure 8]:

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[465][Note by AKTA:] See Section 5.30.

[466][Note by AKTA:] That is, the change in the amount of electric charge of an infinitesimal length of the conductor.

[467][Note by AKTA:] And $\pm e$ denotes the positive or negative charge per unit length for this element.
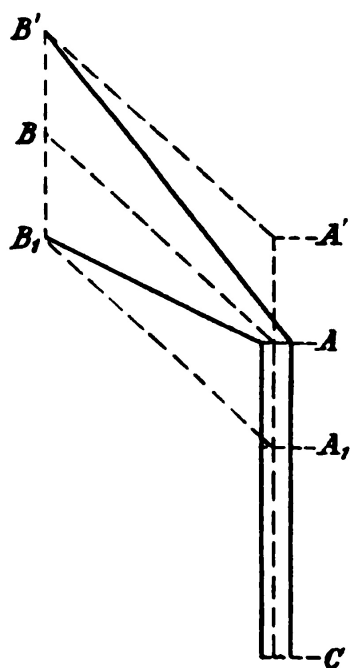
Fig. 8.

Let A be the sliding contact, let AB be the bounded part of the moving part of the current segment, and let AC be the bounded part of the immobile one. The electric fluid will flow through the path $CA = AB$ with a velocity of u during the same time interval in which the moving current segment advances from $A_1 B_1$ to AB or from AB to $A'B'$. The composition of the two motions yields the path $B_1 AC$ for the negative mass (when it goes from the moving current segment to the immobile one), and the path segment $B_1 A$ will be traversed in the same amount of time as AC. For the positive mass, one likewise gets the path CAB', and the segments CA and AB' will be traversed in the same amount of time. In that representation, it is assumed, for the sake of clarity, that the current makes a sudden turn at the sliding contact A and goes from the direction CA to the direction AB. In reality, such a sudden turn cannot happen, but one can assume that the two elements of the true current paths CA and AB define something close to a straight line, [as in Figure 9].

Fig. 9.

If one then lets v denote the velocity of the moving current segment, then $A_1 A = AA' = B_1 B = BB' = vdt$, while the length of the current element is $CA = AB = udt$. That will then imply that in two equal successive time elements dt, the positive mass will traverse the paths $CA = udt$ and $AB' = (u + v)dt$, while the negative mass will traverse the paths $B_1 A = -(u - v)dt$ and $AC = -udt$ in the same time elements. The velocity of the positive electricity then goes suddenly from $+u$ to $+(u + v)$ at A. By contrast, the velocity of the negative electricity goes suddenly from $-(u - v)$ to $-u$ at A. Should that change in velocity happen according to the law of continuity, then one can denote the time during which that transition takes place (although it is also quite small) by τ. One denotes the velocity of the positive electricity at any moment $d\sigma$ at the end of the time period σ within the time interval τ by $+(u + v \cdot \sigma/\tau)$, and likewise denotes the velocity of the negative electricity by $-(u + v \cdot \sigma/\tau - v)$. In addition, as in Section 30 of the cited treatise, one lets ϑ denote the angle that the direction of $+u$ (i.e., AB) makes with $A\alpha' = r$, while ϑ' denotes the angle that the direction in which the positive electricity moves with the velocity $+u'$ in the immobile induced element α' makes with lengthened line $A\alpha'$, and ω denotes the angle between the two planes that are laid through $A\alpha'$ parallel to the directions of $+u$ and $+u'$. Finally, r_1 denotes the distance from the mass $+eudt$ to the mass $+\alpha'e'$, r_2 denotes the distance from the mass $-eudt$ to the mass $-\alpha'e'$, r_3 denotes the distance from the mass $+eudt$ to the mass $-\alpha'e'$, and r_4 denotes the distance from the mass $-eudt$ to the mass $+\alpha'e'$, which are all equal to r at the moment considered, but they do not remain the same under the differing motions of those masses. The fundamental general law of electric action then implies that the *difference* in the forces that act upon the *positive* and *negative* electricity in the element α', which is what the *induction* depends upon, will be:

$$-\frac{a^2}{16} \cdot \frac{eudt \cdot \alpha'e'}{r^2} \left\{ \left(\frac{dr_1^2}{dt^2} - \frac{dr_2^2}{dt^2} + \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) \right.$$

$$\left. - 2r \left(\frac{d^2r_1}{dt^2} - \frac{d^2r_2}{dt^2} + \frac{d^2r_3}{dt^2} - \frac{d^2r_4}{dt^2} \right) \right\} .$$

That equation differs from the expression for the difference in an inducing current element that was derived in Section 30 of the cited treatise (p. 197) by only the fact that *eudt* appears in place of αe. Furthermore, one finds, in the way that was given there, that for our case:

$$\frac{dr_1}{dt} = -\left(u + \frac{\sigma}{\tau}v \right) \cos \vartheta + u' \cos \vartheta' ,$$

$$\frac{dr_2}{dt} = +\left(u + \frac{\sigma}{\tau}v - v \right) \cos \vartheta - u' \cos \vartheta' ,$$

$$\frac{dr_3}{dt} = -\left(u + \frac{\sigma}{\tau}v \right) \cos \vartheta - u' \cos \vartheta' ,$$

$$\frac{dr_4}{dt} = +\left(u + \frac{\sigma}{\tau}v - v \right) \cos \vartheta + u' \cos \vartheta' ,$$

which differ from the equations that were found in *loc. cit.* only by the fact that the velocity of the inducing positive electricity was set to $+(u + v \cdot \sigma/\tau)$, instead of $+u$, the velocity of the inducing negative electricity was set to $-(u + v \cdot \sigma/\tau - v)$, instead of $-u$, and the term that depended upon the motion of the induced element α' was dropped. One then has:[468]

$$\left(\frac{dr_1^2}{dt^2} - \frac{dr_2^2}{dt^2} + \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) = +4 \left(u + \frac{\sigma}{\tau}v - \frac{1}{2}v \right) v \cos \vartheta^2 .$$

One gets the second differential quotient from this in the manner that was given in the cited reference when one considers the fact that u, u', and v have given constant values here, namely:

$$\frac{d^2r_1}{dt^2} = +\left(u + \frac{\sigma}{\tau}v \right) \sin \vartheta \frac{d\vartheta_1}{dt} - u' \sin \vartheta' \frac{d\vartheta_1'}{dt} - \frac{v}{\tau} \cos \vartheta ,$$

$$\frac{d^2r_2}{dt^2} = -\left(u + \frac{\sigma}{\tau}v - v \right) \sin \vartheta \frac{d\vartheta_2}{dt} + u' \sin \vartheta' \frac{d\vartheta_2'}{dt} + \frac{v}{\tau} \cos \vartheta ,$$

$$\frac{d^2r_3}{dt^2} = +\left(u + \frac{\sigma}{\tau}v \right) \sin \vartheta \frac{d\vartheta_3}{dt} + u' \sin \vartheta' \frac{d\vartheta_3'}{dt} - \frac{v}{\tau} \cos \vartheta ,$$

$$\frac{d^2r_4}{dt^2} = -\left(u + \frac{\sigma}{\tau}v - v \right) \sin \vartheta \frac{d\vartheta_4}{dt} - u' \sin \vartheta' \frac{d\vartheta_4'}{dt} + \frac{v}{\tau} \cos \vartheta .$$

One then has:

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[468][Note by AKTA:] The expression $\cos \vartheta^2$ should be understood as $\cos^2 \vartheta$.

$$\left(\frac{d^2 r_1}{dt^2} - \frac{d^2 r_2}{dt^2} + \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2}\right)$$

$$= + \left(u + \frac{\sigma}{\tau}v\right)\sin\vartheta \left(\frac{d\vartheta_1}{dt} + \frac{d\vartheta_2}{dt} + \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt}\right)$$

$$- u'\sin\vartheta' \left(\frac{d\vartheta'_1}{dt} + \frac{d\vartheta'_2}{dt} - \frac{d\vartheta'_3}{dt} - \frac{d\vartheta'_4}{dt}\right)$$

$$- v\sin\vartheta \left(\frac{d\vartheta_2}{dt} + \frac{d\vartheta_4}{dt}\right) - 4\frac{v}{\tau}\cos\vartheta$$

and one finds, in the manner that was given in the cited reference, that:

$$
\begin{aligned}
r\frac{d\vartheta_1}{dt} &= +\left(u + \frac{\sigma}{\tau}v\right)\sin\vartheta - u'\sin\vartheta'\cos\omega \ , \\
r\frac{d\vartheta_2}{dt} &= -\left(u + \frac{\sigma}{\tau}v - v\right)\sin\vartheta + u'\sin\vartheta'\cos\omega \ , \\
r\frac{d\vartheta_3}{dt} &= +\left(u + \frac{\sigma}{\tau}v\right)\sin\vartheta + u'\sin\vartheta'\cos\omega \ , \\
r\frac{d\vartheta_4}{dt} &= -\left(u + \frac{\sigma}{\tau}v - v\right)\sin\vartheta - u'\sin\vartheta'\cos\omega \ , \\
r\frac{d\vartheta'_1}{dt} &= -u'\sin\vartheta' + \left(u + \frac{\sigma}{\tau}v\right)\sin\vartheta\cos\omega \ , \\
r\frac{d\vartheta'_2}{dt} &= +u'\sin\vartheta' - \left(u + \frac{\sigma}{\tau}v - v\right)\sin\vartheta\cos\omega \ , \\
r\frac{d\vartheta'_3}{dt} &= +u'\sin\vartheta' + \left(u + \frac{\sigma}{\tau}v\right)\sin\vartheta\cos\omega \ , \\
r\frac{d\vartheta'_4}{dt} &= -u'\sin\vartheta' - \left(u + \frac{\sigma}{\tau}v - v\right)\sin\vartheta\cos\omega \ .
\end{aligned}
$$

If one substitutes those values, then one will get:

$$r\left(\frac{d\vartheta_1}{dt} + \frac{d\vartheta_2}{dt} + \frac{d\vartheta_3}{dt} + \frac{d\vartheta_4}{dt}\right) = +2v\sin\vartheta \ ,$$

$$r\left(\frac{d\vartheta'_1}{dt} + \frac{d\vartheta'_2}{dt} - \frac{d\vartheta'_3}{dt} - \frac{d\vartheta'_4}{dt}\right) = 0 \ ,$$

$$r\left(\frac{d\vartheta_2}{dt} + \frac{d\vartheta_4}{dt}\right) = -2\left(u + \frac{\sigma}{\tau}v - v\right)\sin\vartheta \ ,$$

and as a result:[469]

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[469][Note by AKTA:] The expression $\sin\vartheta^2$ should be understood as $\sin^2\vartheta$.

$$r \left(\frac{d^2 r_1}{dt^2} - \frac{d^2 r_2}{dt^2} + \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right)$$

$$= +4 \left(u + \frac{\sigma}{\tau} v - \frac{1}{2} v \right) v \sin \vartheta^2 - 4 \frac{rv}{\tau} \cos \vartheta \ ,$$

which finally yields the *difference* between the forces that act upon the *positive* and *negative* electricity in the element α', and upon which the *induction* depends, namely:

$$-\frac{a^2}{16} \cdot \frac{eudt \cdot \alpha' e'}{r^2} \left\{ \left(\frac{dr_1^2}{dt^2} - \frac{dr_2^2}{dt^2} + \frac{dr_3^2}{dt^2} - \frac{dr_4^2}{dt^2} \right) \right.$$

$$\left. - 2r \left(\frac{d^2 r_1}{dt^2} - \frac{d^2 r_2}{dt^2} + \frac{d^2 r_3}{dt^2} - \frac{d^2 r_4}{dt^2} \right) \right\}$$

$$= -\frac{a^2}{4} \cdot \frac{eudt \cdot \alpha' e'}{r^2} \left(u + \frac{\sigma}{\tau} v - \frac{1}{2} v \right) v \left(\cos \vartheta^2 - 2 \sin \vartheta^2 \right)$$

$$- \frac{a^2}{2} \frac{eudt \cdot \alpha' e'}{r} \frac{v}{\tau} \cos \vartheta \ .$$

If one multiplies that expressions by the time element $d\sigma$ and integrates from $\sigma = 0$ to $\sigma = \tau$, then one will get the value of the integral of that difference over the duration of the transition τ, which is equal to:

$$-\frac{a^2}{4} \frac{eudt \cdot \alpha' e'}{r^2} \cdot uv\tau \cdot \left(\cos \vartheta^2 - 2 \sin \vartheta^2 \right) - \frac{a^2}{2} \cdot \frac{eudt \cdot \alpha' e'}{r} \cdot v \cos \vartheta \ ,$$

or when τ is vanishingly small (i.e., when the change in velocity in the electric fluid at the sliding contact happens very rapidly), it is equal to:

$$-\frac{a^2}{2} \cdot \frac{eudt \cdot \alpha' e'}{r} \cdot v \cos \vartheta \ .$$

If one now sets $aeu = i$ in this (as was done in *loc. cit.*, p. 202) and multiplies by $\cos \vartheta'/e'$, then that will give the *electromotive* force that is exerted upon the electricity that flows through the sliding contact onto the induced element α' during the time element dt, which is equal to:

$$-\frac{1}{2} \frac{\alpha' v dt}{r} \cdot ai \cos \vartheta \cos \vartheta' \ .$$

However:

$$vdt = \alpha$$

is the length of the conducting element that newly enters into the circuit at the sliding contact during the time dt, in which the current strength then grows from 0 to i. However, the *electromotive* force that is exerted on α' by the element α due to the growth in current intensity di/dt (*loc. cit.*, p. 202) was found to be equal to:

$$-\frac{1}{2} \cdot \frac{\alpha \alpha'}{r} a \cos \vartheta \cos \vartheta' \frac{di}{dt} \ ,$$

and as a result, the electromotive force when the current intensity grows from 0 to i will be equal to:

$$-\frac{1}{2} \frac{\alpha \alpha'}{r} ai \cos \vartheta \cos \vartheta' \ .$$

Finally, if one sets α equal to its value vdt in that, then one will see that the electromotive force that is exerted upon the induced element during the time element dt due to the electricity that goes through the sliding contact is equal in magnitude, as well as in direction, to the electromotive force that is exerted upon the induced element α' by the recently-entering current element ai at the sliding contact during the same time element dt, and that in order to include the former force in the calculation, one must merely double the latter force. However, as Neumann proved, that doubling is the condition for the law of induction that is derived from the fundamental general law of electric action to agree with the results of Neumann's general principle in the mathematical theory of induced currents, as well with experiments, in the case of a sliding contact, as well. That agreement is verified by that fact. The sudden change in the motion of all electric particles that occurs at the sliding contact is then the source of electromotive forces that Neumann had not included in his calculations for the derivation of the law of induction from the fundamental general law of electric action that he gave, and if one adds the sum of the electromotive forces that arise from that source to the sum that Neumann found, then one will find that the contradiction that seems to exist between the results of the fundamental general law of electric action and Neumann's general principle in the mathematical theory of induced currents disappears completely, which was to be proved.

Finally, the results of all of the experiments that were described in Section can be predicted with the laws that were developed here. Namely, if one lets R denote the radius of the circle that is described by the sliding contact (which differs from the radius $R_|$ of the induced circle only slightly), and if m is the number of the latter, while n is the number of rotations of the moving current segment per unit time, and if i is the strength of the inducing current, while one ultimately sets:

$$R_0 = \frac{R^2 R'^2}{(R^2 + R'^2)^{3/2}} \cdot \left\{ 1 + \frac{15}{8} \left(\frac{RR'}{R^2 + R'^2} \right)^2 + \frac{315}{64} \left(\frac{RR'}{R^2 + R'^2} \right)^4 + ... \right\} \ ,$$

for the sake of brevity, then the laws above will imply that:

1. The sum of the electromotive forces in the moving current segment is equal to:

$$+mn\pi^2 \cdot aiR_0 \ .$$

2. The sum of the electromotive forces on the current segment that enters gradually at the sliding contact (when the action of its sudden vanishing is cancelled by an instantaneous opening of the induced circuit with each rotation, as was the case in Neumann's first experiment) is equal to:

$$-mn\pi^2 \cdot aiR_0 \ .$$

3. The sum of the electromotive forces on the electricity that goes through the sliding contact due to the sudden change in its velocity at the sliding contact is equal to:

$$-mn\pi^2 \cdot aiR_0 \ .$$

For the derivation of that value, see *Supplement* 13.39. The total electromotive force for all of the experiments that were described in Section 13.38 can be easily combined from those partial sums. Namely, one gets:

a) The electromotive force for the first of Neumann's experiments by adding all three partial sums, which is equal to:

$$-mn\pi^2 \cdot aiR_0 \ .$$

The same thing is true for both repetitions of that experiment, as long as one sets m equal to its appropriate value in each experiment. The negative sign means that the current in the induced circle has the opposite direction to the current in the circular inducing current segment when the latter increases through the newly-entering element at the sliding contact.

b) For Neumann's second experiment, where the electromotive force on the newly-entering current element at the sliding contact was cancelled by its sudden vanishing at the end of each rotation, the partial sum in (2) drops out, and one merely adds the two partial sums in (1) and (3), which gives an electromotive force that is:

$$= 0 \ .$$

The same thing is true for the repetition of that experiment, as well as for the modification of it where the same effect that was produced by the sudden vanishing at the end of the rotation of all new current elements that enter gradually during a rotation would be achieved by a current division.

c) All that remain then are the experiments in which the inducing current that goes through a circular conductor suffers a change in intensity from either 0 to i or from i to 0, and in which that current either did not flow through the moving segment of the conductor at all or that segment was not moving while the current was flowing. For that experiment, the partial sums in (1) and (3) drop out completely, and what remains as the electromotive force is just the partial sum in (2), in which one sets n equal to the value 1, so that sum equals:

$$-m\pi^2 \cdot aiR_0 \ .$$

The negative sign means that when the circuit is closed, the current in the induced circle has the opposite direction to the current in the inducing circle.

All of those electromotive forces are expressed in the general force units of mechanics, and from Section 13.27, they can be converted into the absolute unit that was defined in Section 13.26 upon multiplying them by $c/4 = 1/a$. The unknown factor a in the expression for those forces will drop out by that reduction, and the reduced value can be determined by measurement. Moreover, the expressions above give the mean strengths of the electromotive force or its integral value per unit time in all of the experiments in which the action persisted uniformly. By contrast, for the experiments in which the action was only instantaneous, the expressions above give the integral value of the electromotive force over the total duration of the action. If T_0 generally denotes the time for which the integral value of the electromotive force is valid (so one sets $T_0 = 1$ in all of the experiments for which the action persisted

uniformly), then one will get the mean strength of the electromotive force by dividing the integral value that was found by T_0, and it can then be represented by:

$$\pm mn\pi^2 \cdot \frac{R_0}{T_0} \cdot i \ ,$$

for (1), (2), and (3), in which m denoted the number of the induced circle, and n denotes the number of rotations. If one divides that mean strength of the electromotive force by the resistance in the induced circuit, as it is found according to the unit that was defined in Section 13.26, then one will get the mean intensity of the induced current. However, it now appears that the resistance can be represented by:

$$p\frac{R'}{T'} \ ,$$

in the given units, where p is a pure number, but R', just like R_0, refers to the chosen length unit, and T', just like T_0, refers to the chosen time unit. As a result, that will give the following expression for the mean intensity of the induced current:

$$\pm\frac{mn\pi^2}{p} \cdot \frac{R_0}{R'} \cdot \frac{T'}{T_0} \cdot i \ ,$$

in which $(mn\pi^2/p) \cdot (R_0/R') \cdot (T'/T_0)$ is a pure number that is calculated from the measurements and gives the ratio of the strength of the induced current to that of the inducing one. In that way, it is also possible to predict the strength of the induced current in the given units.

Supplements

A - Description of a Magnetic Inductor to be Used for Resistance Measurements

In the attempts to compare the resistances of two conductors, a magnetic inductor was used as an electromotor that was designed in the following way: Two cylindrical magnetic rods of length 300 millimeters and thickness 15 millimeters were fixed in a wooden tube in such a way that they both pointed towards the same pole (viz., the North pole), but in order that they did not weaken each other in that position, they were separated from each other by a gap of 150 millimeters. The wooden tube AB (Figure 10),[470] together with the magnets sn, sn that were included in it, could be raised perpendicularly by a level apparatus $CDEF$ that was set in motion with one's foot and once more lowered, as well as displaced back and forth through the cavity in an inductor coil GG, which was immovably attached to the upper side of the frame $HHHH$, which was screwed to the floor.
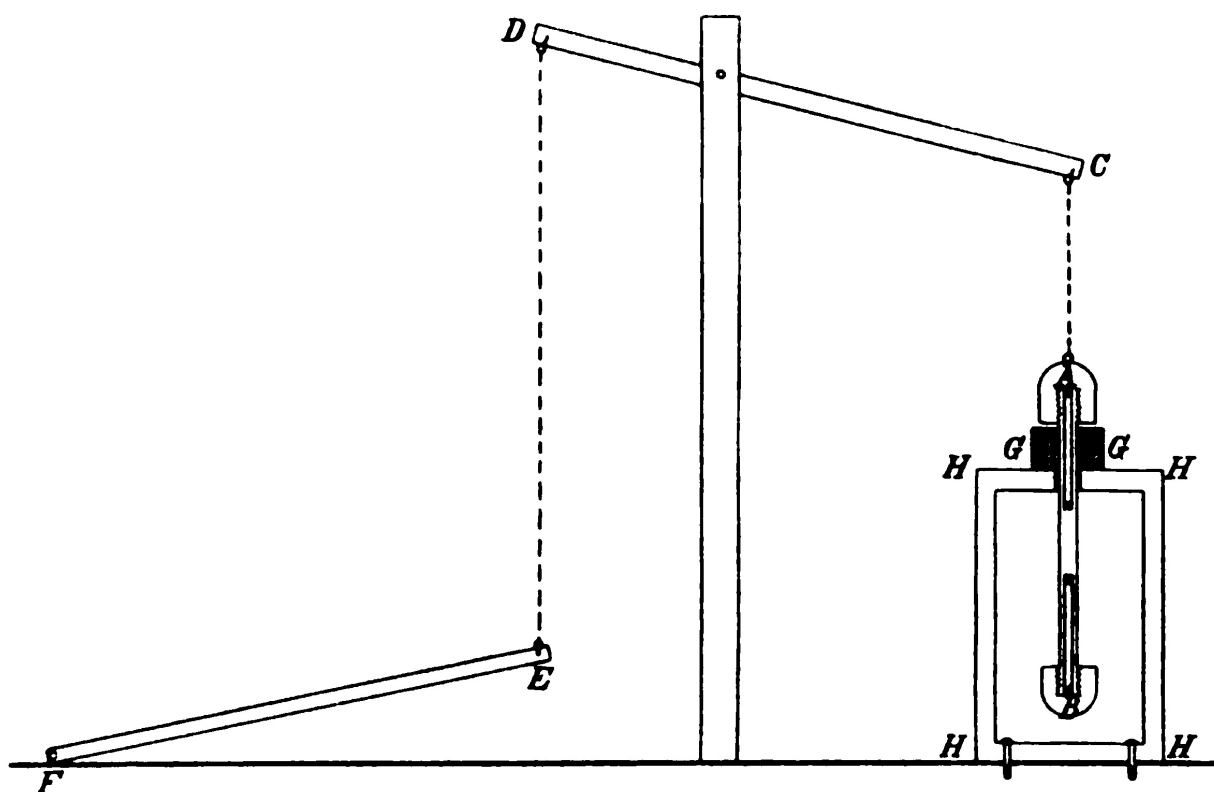
Fig. 10.

Figure 10 represents a vertical Section of that apparatus. The South poles of the two magnetic rods are denoted by s and the North poles, by n. The wooden tube in which the magnetic rods are fixed is sealed at both ends with caps that could be unscrewed. If the tube in the inductor coil were displaced so far downwards (as in Figure 10) that the upper cap contacted the inductor coil GG, then one would find the midpoint of the upper magnetic rod at the center of the inductor coil. By contrast, if the tube were displaced so far upwards

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[470][Note by AKTA:] See footnote 379 on page 293.

that the lower cap contacted the frame HH to which the inductor coil GG was attached, then the midpoint of the lower magnetic rod would be found at the center of the inductor coil. The induction would be zero in those two extreme positions since the electromotive forces of the poles that lay symmetrically on the two sides of the inductor coil when both of them moved simultaneously upwards or downwards would cancel. The induction took place in the same sense during the entire displacement of the tube from below to above and was strongest when the two North ends of the magnetic rod went through the induction coil. The induction took place in the opposite sense during the entire displacement in the opposite direction from above to below. Each such motion is called an induction thrust, and indeed it will be positive or negative according to whether the displacement happened upwards or downwards, respectively. The fact that each induction thrust began and ended in a position for which the induction was zero had the objective of making the total value of the induction that corresponded to an induction thrust be a maximum and remain unchanged, even when those extreme positions were not attained exactly. The simple displacement by which such a total induction thrust was accomplished could be performed very rapidly, and was therefore especially suited to measurements for which the induction thrusts had to take place at the moments when the galvanometer needle went through its equilibrium position. In order to be able to adhere to those moments exactly, the convention was established that the positive induction thrust (i.e., the displacement down to up) was accomplished by pushing the foot down on the lever EF, while the negative induction thrust (i.e., the displacement from up to down) resulted from the weight of the inductor itself when the foot was raised again. In that way, the observer that followed the course of the galvanometer needle with the telescope could see the induction thrust at precisely the moment when the needle passed through its equilibrium position without abandoning the telescope.

B - Description of the Galvanometer

The following description was given by the mechanic Leyser in Leipzig,[471] who had already constructed several such instruments and gave them the price that is noted below. The galvanometer was also designed so that the strengths of the currents that were observed with it could be determined in the absolute units that were established in Section 13.10, for which two multipliers that could displace along yardsticks at various distance from the needle could be used. Since that device was not necessarily coupled with the instrument and was not used here, it shall not be mentioned any further in the following description.[472]

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[471][Note by AKTA:] See footnote 105 on page 58.
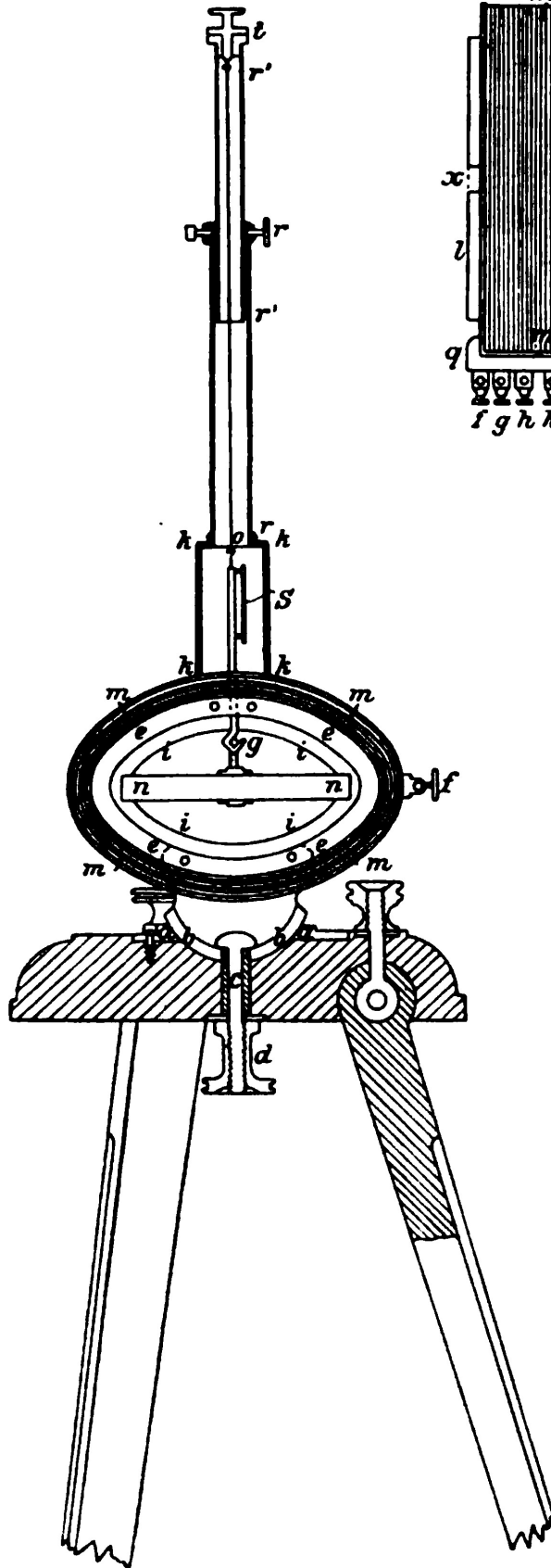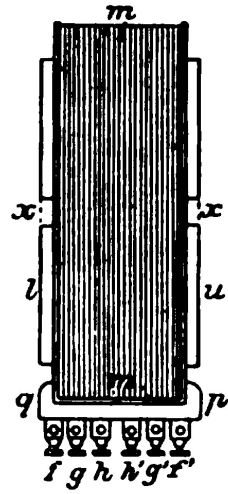[472]Another reproduction of Figures 11, 12 and 13 appear on page 402.
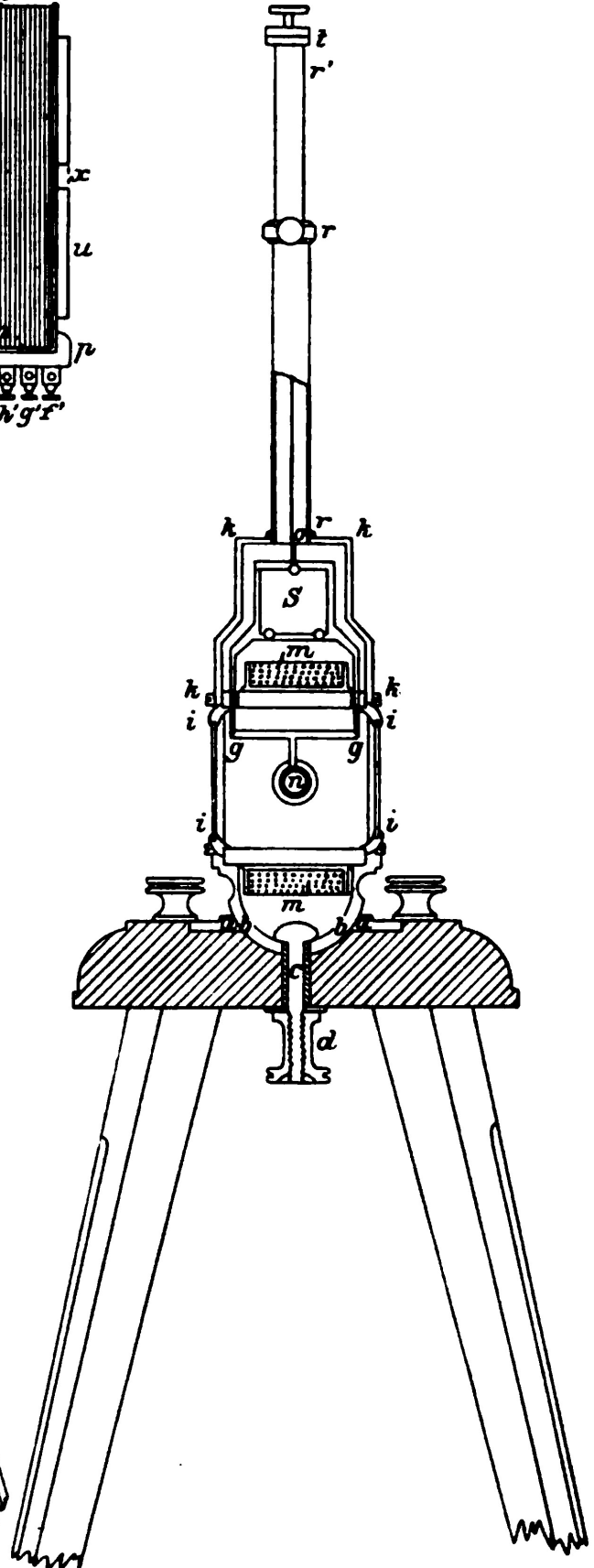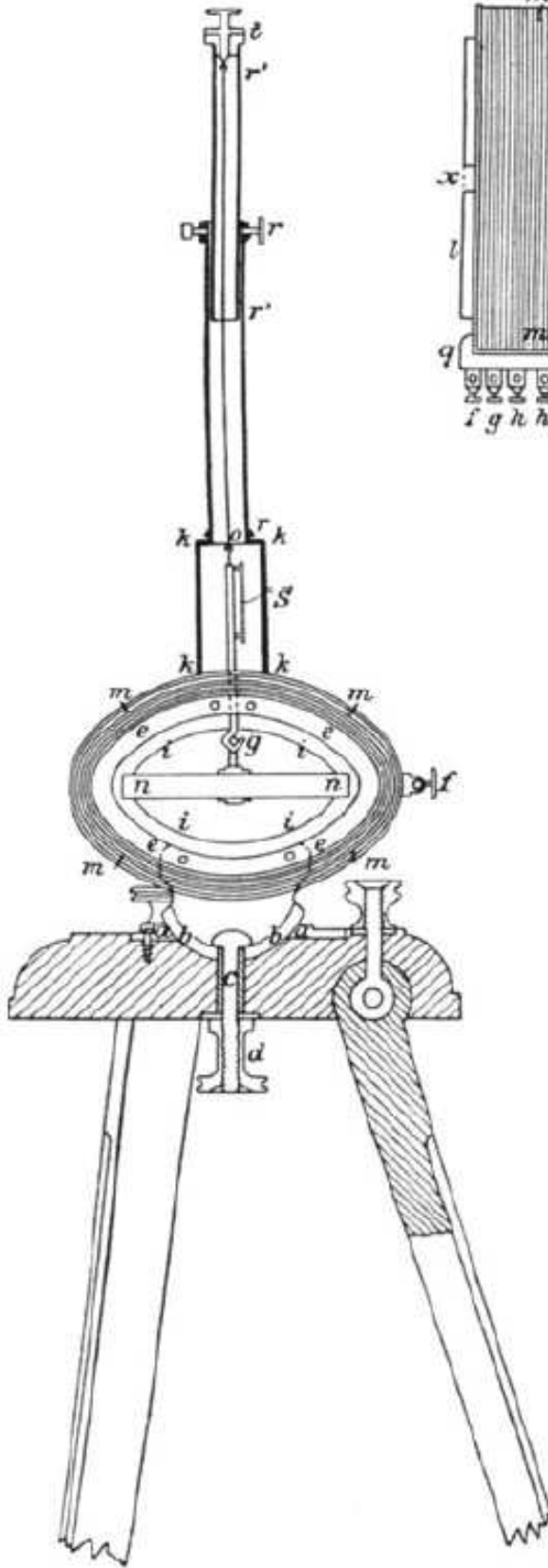
Fig. 11.

Fig. 12.

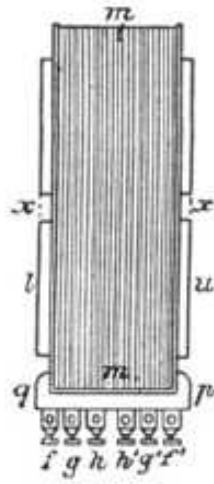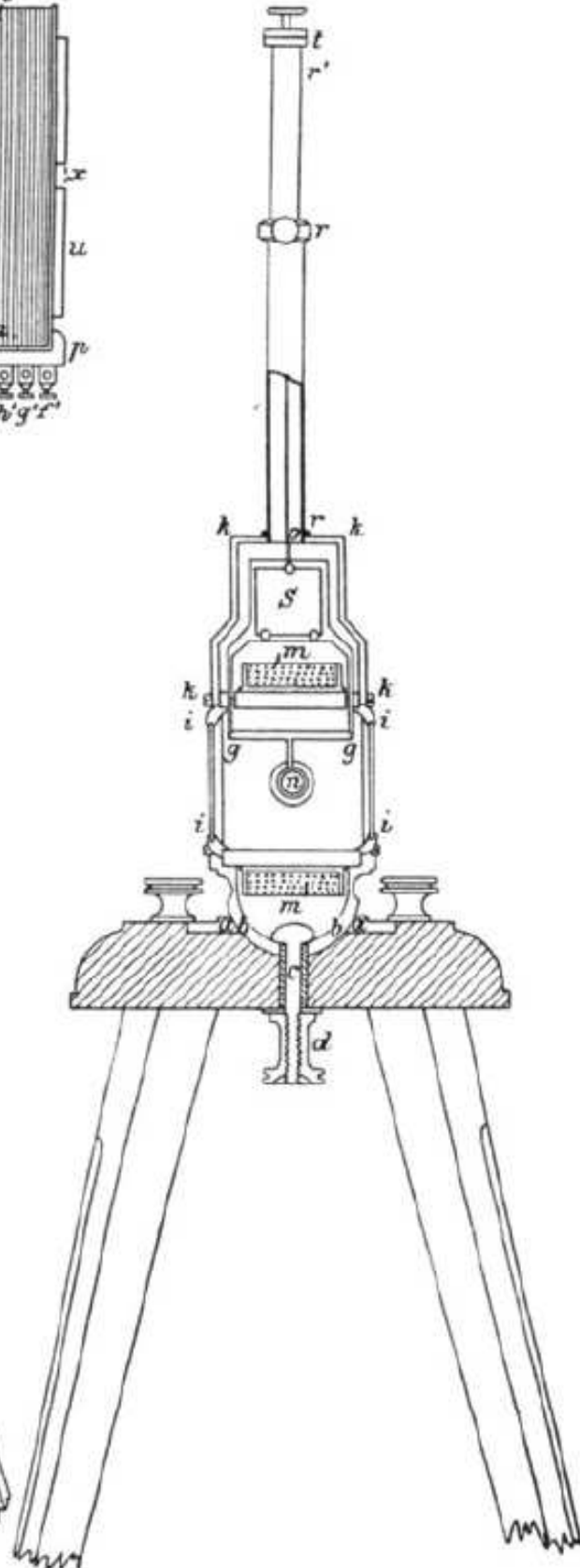Fig. 13.

Fig. 11.

Fig. 12.

Fig. 13.

"The associated markings represent the fifth parts of the galvanometer's actual linear dimension, and indeed Figure 11[473] shows a longitudinal section of the galvanometer in the direction of the magnetic meridian. Figure 13 shows a section of it that is perpendicular to the direction of the magnetic meridian. Except for the magnetic needle, iron and steel were carefully avoided in the construction of the instrument, such that some of its parts consist of copper, and some of them consist of iron-free brass. The frame itself upon which the instrument stands is a circular wooden disc that has three movable feet attached to it whose ends converge to metal points, in the manner of the tripods that are used for measuring instruments. Next, a circular metal plate aa is embedded in that circular wooden disc whose center has been removed and machined to be spherical, such that a suitable spherical surface bb can be installed in all possible directions and attached to the circular plate aa by means of a bolt c and a nut d. Now, the actual galvanometer is attached to the aforementioned spherical surface bb. To that end, that spherical surface bb is continued upwards to two parallel side plates, which are suggested by dotted lines in Figure 11, while they are seen clearly in Figure 13. Between those side plates, there is the copper damper $eeee$, which is bent into the form of an oval, self-returning ring of width 80 millimeters and thickness 8 millimeters that is attached to each side with two screws. The cross-section of that damper is an ellipse along whose major axis the needle floats. A frame that is prepared from thin brass sheets is slid sideways over that damper. It is equipped with vertical walls and incorporates a quantity of spun copper wire that is wound around it, so it then represents a multiplier $mmmm$ above the damper. The wire windings of that multiplier go around the damper elliptically and consist of nine concentric layers that are one on top of another, each layer of which consists of 80 windings that are laid next to each other; the wire had a thickness of about 2/3 millimeter. In that way, the device is constructed so that the multiplier can sometimes be used completely and sometimes partially in three sections with three locations. Those three sections can also be connected in such a way that the same current will flow through them simultaneously that gets split between them. To that end, the device that gives the multiplier is clearly illustrated in Figure 12, which is seen from above. qp is a crossbar of boxwood that is attached to the foregoing walls of the frame for the multiplier mm. The first section of the multiplier begins with the first — or lowest — layer of the windings, which starts at the button f. It goes around the copper damper towards its right-hand side u and thus defines the first layer. It then turns to the left-hand side l, and in that way defines the second layer. From there, it once more goes to the right-hand side u, which thus creates the third layer, whose end is found at the button f'. Now, in complete analogy with the way that the first section of the multiplier defined three layers that had their start at f and their end at f', the second and third sections each have three layers. However, the start of the second section is at g, while its end is at g', and finally the start of the third section is at h, while its end is at h'. (Those six copper buttons are drilled through transversely and firmly inserted in the boxwood crossbar qp with screws, but isolated from each other. With that arrangement for the multiplier, one easily sees that the three sections that the multiplier consists of can be combined in different ways according to how the buttons are coupled by wires.) One finds a little frame $kkkk$ over the damper $eeee$ that is attached to it with screws, one of

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[473][Note by AKTA:] See footnote 382 on page 294.

whose open sides is sealed with a soft metal plate, while the other is sealed with a glass plate with parallel outer surfaces, which then represents a closed, rectangular space. A tube rr is mounted above it that can be lengthened or shortened by an adjustable graduated extension tube $r'r'$. That extension tube $r'r'$ concludes with a torsion circle t whose construction is elucidated directly in the figures. That torsion circle has a small eyelet in which is fixed the cocoon thread that hangs down inside the combined tube and carries a light rectangular metal plate on a hook o, to which a planar mirror s is attached by three screws. That rectangular plate goes underneath, and in fact through two cutouts that are installed in the sides of the damper $eeee$ (which are suggested by xx in Figure 12), beyond two soft rods whose ends appear as the hook gg in the figures. The magnetic needle nn was inserted into that hook, and its end was enclosed in a narrow sleeve and equipped with a small crossbar that was attached to that sleeve, whose ends extended cylindrically and were inserted in that hook gg. The position of the needle relative to its tube was regulated by the extension tube $r'r'$. The torsion in the filament can be brought to zero by rotating the torsion circle t. However, the entire system of the instrument can be positioned vertically by means of the spherical motion that the spherical surface bb with the plate aa allows, and that positioning is easiest when one fastens the extension tube in the vicinity of the torsion circle and effects the positioning by very lightly tightening the screw d, and the correct position of the instrument is then established by tightening the screw d. The still-open viewing ports on both sides of the damper $eeee$ are closed off by inserting glazed wooden frames whose cross-sections are illustrated by $iiii$. The price of the instrument with the device for absolute measurements is 80 Thaler, and 60 Thaler without it."

C - Overview of the Method of Observation for Galvanic Measurements when One Includes the Influence of Damping

Ordinarily with galvanic measurements, only those galvanic currents are included in the calculations that are excited outside of the multiplier and flow through it in order to be measured by the deflection that is imparted to the needle. If that current is *constant* and the deflection of the needle had not been measured before it came to rest, then the deflection of the needle would truly depend upon just that current. However, if the current is not constant or lasts for only a very short time, and one observes the deflection before the needle comes to rest (e.g., one observes the first elongation of the needle), then other currents besides the current to be measured will be present that frequently have a great influence on the observations that cannot remain unobserved. Those currents originate in the motion of the magnetic needle, which *induces* galvanic currents in all of the surrounding conductors whose intensities will be proportional to the magnetism of the needle and to the velocity with which it moves, and whose directions will always be such that their reactions on the needle will slow or *damp* the motion of it that exists.

Such a current will be induced by the moving magnetic needle, *first of all*, in the multiplier itself, and will be stronger when the metallic cross-section of the entire multiplier and the fraction that the resistance of the multiplier represents in the resistance of the total circuit

are larger. It will be strongest when the multiplier is closed and weakest when the circuit of the multiplier is open.

Secondly, such a current will also be excited by the moving magnetic needle in all metallic parts of the instrument, and the reaction on the magnetic needle will be especially strong when vertical plates are found in the direction of the magnetic meridian close to the needle or when the needle is surrounded by a vertical metal ring, which is why such a ring, when it is installed for that purpose intentionally, is called a *damper*. For most measurements, the application of such a damper not only affords much more ease of observation, but also often contributes to greater precision in them. See the "*Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1837,*" p. 18.[474],[475]

One then comes to the problem of making the results of the observations independent of the damping influence of those two currents or of determining the *correction* that must be made to the observations due to the *damping*. That correction will become especially important and meaningful when one makes one's galvanic measurements with either a *magnetometer* that is equipped with a damper or an *astatic double needle*,[476] which is composed of two strong magnetic needles and is either likewise equipped with a damper or is tightly enclosed in a strong multiplier. Such a correction can be especially necessary in the latter case, where the damping arises from the multiplier and can be very different for different closings of the multiplier, if one is to make the experiments comparable to each other.

In order to determine that correction, *first of all*, the damping force of the instrument must be determined more closely, which can be done easily by observing the *decrease in the arc of oscillation. Secondly*, one must then show how the influence of that damping force on the various methods of observation can be determined in or eliminated from the calculation of the results.

1. Determining the Damping Force of a Galvanometer.

The damping force of a galvanometer is split into two parts that must be separated from each other, namely, a *constant* part that is independent of the circuit to which the multiplier belongs and a *variable* one.

One obtains the *constant* part of the damping force by observing the decrease in the arc of oscillation when the circuit to which the multiplier belongs is closed. Namely, when the arc of oscillation is not too large, it defines a decreasing geometric series that can be represented by Ae^0, $Ae^{-\lambda'}$, $Ae^{-2\lambda'}$, ... , $Ae^{-n\lambda'}$, where n denotes the number of oscillations that the needle has made, as counted from the moment when the arc was equal to A. After one period of oscillation,[477] the arc will then have decreased by the ratio:

$$e^{\lambda'} : 1 \ ,$$

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[474][Note by HW:] Gauss's *Werke*, Vol. V, p. 372.

[475][Note by AKTA:] [Gau38b] with English translation in [Gau41c].

[476][Note by AKTA:] The adjective "astatic" is used in physics with the meaning of something having no tendency to take a definite position or direction. An astatic needle can be a combination of two parallel magnetized needles having equal magnetic moments, but with their poles turned opposite ways, that is, in antiparallel position. The arrangement protects the system from the influence of terrestrial magnetism. It was invented by Ampère, [Amp21] and [LA98]. An earlier system composed of a single magnetized needle had also been created by Ampère, [Amp20b, p. 198] with Portuguese translation in [CA09, p. 133], [Amp20a, p. 239] and [Amp, p. 2], see also [AC15, p. 57].

[477][Note by AKTA:] See footnote 96 on page 51.

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after two periods of oscillation, it will have decreased by the ratio:

$$e^{2\lambda'} : 1 \; ,$$

and after n periods of oscillation, it will have decreased by the ratio:

$$e^{n\lambda'} : 1 \; .$$

If one then takes the exponent λ' to be the unit of damping during the period of one oscillation, or during τ' seconds, if τ' expresses the period of oscillation of the needle in seconds, then $2\lambda'$ will be unit of damping for $2\tau'$ seconds, and $n\lambda'$ will be the unit for $n\tau'$ seconds. The ratio of the damping force thus-determined to the time interval to which it refers, then ultimately gives the constant $\lambda'/\tau' = 2\lambda'/2\tau' = n\lambda'/n\tau'$, which expresses the *unit of damping, when reduced to the unit of time.* However, λ' is nothing but the natural logarithm of the ratio of two successive oscillation arcs, and τ' is the period of oscillation of the needle under the influence of damping. One then obtains the unit of damping, when reduced to the unit of time, when one divides that logarithm by that period of oscillation, both of which can be easily determined from the observations.

In order to determine the *variable* part of the damping force, the decrease in the oscillation arc is observed when the multiplier is closed. If $e^{\lambda''} : 1$ is the ratio of two successive oscillation arcs that is found by observation and τ'' is the period of oscillation, then the *unit of damping per unit time* will be equal to:

$$\frac{\lambda''}{\tau''} \; .$$

That will then yield the damping force in the closed multiplier as being equal to:

$$\frac{\lambda''}{\tau''} - \frac{\lambda'}{\tau'} \; .$$

In most cases, the difference between the periods of oscillations τ'' and τ' is negligible, and the unit of damping for the closed multiplier will then be equal to:

$$\frac{1}{\tau'}(\lambda'' - \lambda') \; .$$

Now, one can determine the *variable* part of the damping from that, when one knows the fraction of the total resistance of the circuit that the resistance of the multiplier defines. If a denotes the resistance of the multiplier, while $a + b$ is resistance of the entire circuit, then the desired *value of the variable part of the damping* will be equal to:

$$\frac{a}{a+b}\left(\frac{\lambda''}{\tau''} - \frac{\lambda'}{\tau'}\right) \; ,$$

in which only b is variable and must be determined for each individual case in particular. If one combines that with the unit of constant part of the damping (which equals λ'/τ'), then that sum will give the *value of the actual damping*, which equals λ/τ:

$$\frac{\lambda}{\tau} = \frac{a}{a+b}\cdot\frac{\lambda''}{\tau''} + \frac{b}{a+b}\cdot\frac{\lambda'}{\tau'} \; ,$$

in which λ denotes the natural logarithm of the ratio of two successive oscillation arcs for the case considered, and τ denotes the period of oscillation.

2. Calculating the Galvanic Measurements when one Includes Damping.

If one has determined the damping force in the instrument in that way, then that determination can be employed to eliminate the influence of damping from the calculation of the results of the various methods of observations, in which one follows the advice of Gauss in "*Resultaten aus den Beobachtungen des magnetischen Vereins im Jahre 1837*," pp. 58 *et seq.*[478],[479] for determining the period of oscillation of a magnetic needle, in which one will find the development of the laws by which the damping acts upon the state and period of oscillation of the needle. The various methods of observation shall be considered here, but only for small oscillations of the needle in all cases.

Observation of the First Elongation.

1. If one observes only the *first elongation* that the magnetic needle makes after a *constant* current enters it in galvanic measurements, then it is known that when *no damping* is present, that elongation will be twice the deflection of the needle for which it would persist in equilibrium under the effect of that current. By contrast, when there is *damping*, the deflection E of the needle that corresponds to equilibrium can be determined from the

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[478][Note by HW:] Gauss's *Werke*, Vol. V, p. 374.
[479][Note by AKTA:] [Gau38a].

observed first elongation x of the needle in the following way:[480]

$$E = \frac{x}{1 + e^{-\lambda}} \ ,$$

which one can set to:

$$E = \frac{1}{2}x + \frac{1}{4}\lambda x \ .$$

for small values of λ.

2. In galvanic measurements, if one observes the *first elongation* after the resting needle is set into motion by an *instantaneous* current (e.g., by an induction thrust), then one will essentially have to derive the *velocity* that the instantaneous current has imparted upon the

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[480][Note by WW:] *When there is no damping*, one has the expression:

$$x = p + A \sin \frac{\pi}{T}(t - B) \ ,$$

for the state of the oscillating needle that equals x at the end of a time equal to t, where T denotes the period of oscillation. By contrast, *when there is damping*, one has:

$$x = p + A e^{-\lambda t/\tau} \cdot \sin \frac{\pi}{\tau}(t - B) \ ,$$

where τ expresses the period of oscillation of the needle under the influence of damping, and is determined from the following equation:

$$\frac{\pi^2}{\tau^2} = \frac{\pi^2}{T^2} - \frac{\lambda^2}{\tau^2} \ .$$

See "*Resultate*," 1837, pp. 74, 75 [Note by HW: Gauss's *Werke*, Vol. V, p. 389; Note by AKTA: [Gau38a]], in which ε denotes the same thing that λ/τ denotes here, and T' denotes the same thing that τ denotes here. Now, if the starting point of the time t is chosen to be the moment when the constant current begins to move the needle, when the velocity will then be $dx/dt = 0$, which will make $\tan(-B\pi/\tau) = \pi/\lambda$, and as a result

$$-B = \frac{\tau}{\pi} \cdot \arctan \frac{\pi}{\lambda} = \frac{1}{2}\tau - \frac{\tau}{\pi} \cdot \arctan \frac{\lambda}{\pi} \ ;$$

and if the state of the needle up to that starting point were taken to be the elongation x, so $x = 0$ for $t = 0$, then the equation above would take the following form:

$$x = -\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}} + A e^{-\lambda t/\tau} \cos\left(\frac{\pi}{\tau}t - \arctan\frac{\lambda}{\pi}\right) \ ,$$

where $-\pi A/\sqrt{\pi^2 + \lambda^2} = E$ denotes the rest state of the needle under the influence of the constant current. As a result, at the moment of the *first elongation*, $t = \tau$, when one considers that

$$\cos\left(\pi - \arctan\frac{\lambda}{\pi}\right) = -\frac{1}{\sqrt{1 + \lambda^2/\pi^2}} \ ,$$

one will have:

$$x = -\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}} \cdot \left(1 + e^{-\lambda}\right) = E\left(1 + e^{-\lambda}\right) \ ,$$

and as a result:

$$E = \frac{x}{1 + e^{-\lambda}} \ .$$

needle from the observed elongation of the needle $(= x)$. The velocity C is obtained from the following equation:[481]

$$C = x \cdot \frac{\pi}{T} \cdot e^{\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} \,,$$

in which T denotes the period of oscillation of the needle when there is no damping. For small values of λ, one can set:

$$C = \frac{\pi}{T} x + \frac{1}{2} \frac{\pi}{T} \lambda x \,.$$

Method of Multiplication.

1. Due to the weakness of the *constant* current that one measures, if one does not just observe the first elongation, but lets the needle swing back and forth while one inverts the direction of the current in the multiplier at the end of each oscillation and then observes the increasing magnitude of the successive oscillation arcs, which will be denoted by x_1, x_2, x_3, ..., then the following equations will yield the deflection E of the needle that corresponds to equilibrium:[482]

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[481][Note by WW:] Upon differentiating the equation that was given in the foregoing footnote, namely,

$$x = p + Ae^{-\lambda t/\tau} \sin \frac{\pi}{\tau}(t - B) \,,$$

one will get:

$$\frac{dx}{dt} = -\frac{\lambda}{\tau} Ae^{-\lambda t/\tau} \sin \frac{\pi}{\tau}(t - B) + \frac{\pi}{\tau} Ae^{-\lambda t/\tau} \cos \frac{\pi}{\tau}(t - B) \,.$$

If one then calculates the time t of the moment when the instantaneous current acts upon the needle and imparts the velocity C upon it, then one will have $B = 0$ and $dx/dt = C$ for $t = 0$; as a result, $A\pi/\tau = C$ or $A = C\tau/\pi$. If one then sets the original state of the needle to $p = 0$, to simplify, then one will get:

$$x = \frac{\tau}{\pi} C e^{-\frac{\lambda}{\tau} t} \sin \frac{\pi}{\tau} t \,,$$

and as a result, at the end of the first elongation, for which $dx/dt = 0$, so

$$\tan \frac{\pi t}{\tau} = \frac{\pi}{\lambda} \,, \qquad t = \frac{\tau}{\pi} \cdot \arctan \frac{\pi}{\lambda} \,, \qquad \sin \frac{\pi t}{\tau} = \frac{1}{\sqrt{1 + \lambda^2/\pi^2}} \,,$$

one will have:

$$x = C \cdot \frac{\tau}{\sqrt{\pi^2 + \lambda^2}} \cdot e^{-\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} \,.$$

However, one has $\tau/\sqrt{\pi^2 + \lambda^2} = T/\pi$, which one would get from the equation that was cited above $\pi^2/\tau^2 = \pi^2/T^2 - \lambda^2/\tau^2$, and as a result:

$$x = C \cdot \frac{T}{\pi} e^{-\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} \qquad \text{or} \qquad C = x \frac{\pi}{T} e^{\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} \,.$$

[482][Note by WW:] The same equation is true here that was true in the footnote [480] on page 408 up to the end of the first elongation, namely:

$$x = -\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}} + Ae^{-\lambda t/\tau} \cos\left(\frac{\pi}{\tau} t - \arctan \frac{\lambda}{\pi}\right) \,,$$

so at the moment of the first elongation, for which $t = \tau$, one will have

$$-\frac{x_1}{E} = 1 + e^{-\lambda} ,$$

$$+\frac{x_2}{E} = 2 + 3e^{-\lambda} + e^{-2\lambda} ,$$

$$-\frac{x_3}{E} = 2 + 4e^{-\lambda} + 3e^{-2\lambda} + e^{-3\lambda} ,$$

$$+\frac{x_4}{E} = 2 + 4e^{-\lambda} + 4e^{-2\lambda} + 3e^{-3\lambda} + e^{-4\lambda} .$$

$$x = -\left(\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}}\right) \cdot \left(1 + e^{-\lambda}\right) .$$

At that moment, when the first oscillation ends and the second one begins, the current in the multiplier will reverse, which will convert the previous rest state of the needle $-\pi A/\sqrt{\pi^2 + \lambda^2}$ into $+\pi A/\sqrt{\pi^2 + \lambda^2}$. The deflection of the needle from its rest state, which was

$$x + \frac{\pi A}{\sqrt{\pi^2 + \lambda^2}} = -\left(\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}}\right) \cdot e^{-\lambda}$$

at the end of the first oscillation, will then be converted into

$$-\left(\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}}\right) \cdot \left(2 + e^{-\lambda}\right) ,$$

which will yield the amplitude of the second oscillation from $t = \tau$ to $t = 2\tau$:

$$x = +\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}} + A\left(2 + e^{-\lambda}\right) e^{-\frac{\lambda}{\tau}(t - \tau)} \cos\left(\frac{\pi}{\tau}t - \arctan\frac{\lambda}{\pi}\right) ,$$

so at the end of the second elongation, at $t = 2\tau$, one will have:

$$x = +\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}} \left(1 + 2 \cdot e^{-\lambda} + e^{-2\lambda}\right) .$$

One likewise obtains the amplitude of the third oscillation from $t = 2\tau$ to $t = 3\tau$:

$$x = -\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}} + A\left(2 + 2e^{-\lambda} + e^{-2\lambda}\right) e^{-\frac{\lambda}{\tau}(t - 2\tau)} \cos\left(\frac{\pi}{\tau}t - \arctan\frac{\lambda}{\pi}\right) ,$$

so at the end of the third elongation, at $t = 3\tau$, one will have:

$$x = -\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}} \left(1 + 2e^{-\lambda} + 2e^{-2\lambda} + e^{-3\lambda}\right) ,$$

and so on. If one writes the obtained values of x for $t = 0$, $t = \tau$, $t = 2\tau$, $t = 3\tau$ and so on under one another

$$0 ,$$

$$-\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}} \left(1 + e^{-\lambda}\right) ,$$

$$+\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}} \left(1 + 2e^{-\lambda} + e^{-2\lambda}\right) ,$$

$$-\frac{\pi A}{\sqrt{\pi^2 + \lambda^2}} \left(1 + 2e^{-\lambda} + 2e^{-2\lambda} + e^{-3\lambda}\right) ,$$

then the differences of two successive values of x, one after the other, give the desired oscillation arcs x_1, x_2, x_3, where $\pi A/\sqrt{\pi^2 + \lambda^2} = E$.

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The larger that λ is, the faster that x/E will approach a limiting value, for which one will get the following expression:

$$\pm \frac{x}{E} = \frac{4}{1 - e^{-\lambda}} - 2 \ .$$

As a result, when one continues the experiment until the arc of oscillation stops increasing, one will find the deflection E that corresponds to equilibrium of the needle from the coincident values x of the last oscillation arcs in the following way:

$$E = \frac{x}{2} \cdot \frac{1 - e^{-\lambda}}{1 + e^{-\lambda}} \ .$$

2. Due to the weakness of the *instantaneous* current to be measured, if one does not just observe the first elongation after the resting needle is set into motion, but lets the needle swing back and forth while one lets the same instantaneous current for accelerating the needle flow through the multiplier in the opposite direction each time at the next moment when the needle once more passes its original position and then observes the increasing magnitudes of the oscillation arc, which will be denoted by x_1, x_2, x_3, ..., then that will give the *velocity C* that the *instantaneous* current imparts upon the needle each time in the following way. If one sets:

$$B = C \cdot \frac{T}{\pi} \cdot e^{-\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} \ ,$$

then:[483]

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[483][Note by WW:] For the *first* oscillation period from $t = 0$ to $t = \tau$, the same equation will be true that was true in the footnote [481] on page 409, namely:

$$x = \frac{\tau}{\pi} \cdot C e^{-\frac{\lambda}{\tau} t} \sin \frac{\pi}{\tau} t \ ,$$

and as a result, at the moment of the first elongation, for which one had

$$t = \frac{\tau}{\pi} \cdot \arctan \frac{\pi}{\lambda} \ , \qquad \sin \frac{\pi}{\tau} t = \frac{1}{\sqrt{1 + \lambda^2/\pi^2}} = \frac{T}{\tau} \ ,$$

then

$$x = \frac{T}{\pi} \cdot C e^{-\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} \ .$$

At the end of the oscillation period, when $t = \tau$, that will yield $dx/dt = -C e^{-\lambda}$. Now, at that moment, the velocity of the needle will be changed by $-C$ due to the renewed instantaneous current, i.e., it will be converted into $-C(1 + e^{-\lambda})$, so for the *second* period of oscillation from $t = \tau$ to $t = 2\tau$, that will imply that:

$$x = \frac{\tau}{\pi} \cdot C \left(1 + e^{-\lambda}\right) e^{-\frac{\lambda}{\tau}(t - \tau)} \cdot \sin \frac{\pi}{\tau} t \ ,$$

and as a result, at the moment of the second elongation, for which:

$$t = \tau + \frac{\tau}{\pi} \arctan \frac{\pi}{\lambda} \ , \qquad \sin \frac{\pi}{\tau} t = -\frac{T}{\tau} \ ,$$

one will have:

$$x = -\frac{T}{\pi} C \left(1 + e^{-\lambda}\right) e^{-\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} \ .$$

For the *third* period of oscillation from $t = 2\tau$ to $t = 3\tau$, one likewise has that:

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$$+\frac{x_1}{B} = 1 \ ,$$

$$-\frac{x_2}{B} = 2 + e^{-\lambda} \ ,$$

$$+\frac{x_3}{B} = 2 + 2e^{-\lambda} + e^{-2\lambda} \ .$$

Here as well, x/B will approach a limiting value more rapidly as λ increases, and that will yield a limiting value of:

$$\frac{x}{B} = \frac{2}{1 - e^{-\lambda}} \ .$$

As a result, when one continues the experiment until the oscillation arc ceases to increase, one will find the *velocity C* that the instantaneous current to be measured imparts upon the needle each time from the coincident values x of the last-observed oscillation arc in the following way:

$$C = \frac{x}{2} \cdot \frac{\pi}{T} \left(1 - e^{-\lambda}\right) e^{\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} \ .$$

Throw-back Method.

Finally, the application of the method of observation that Gauss described in "*Resultaten*

$$x = \frac{\tau}{\pi} \cdot C \left(1 + e^{-\lambda} + e^{-2\lambda}\right) e^{-\frac{\lambda}{\tau}(t - 2\tau)} \cdot \sin \frac{\pi}{\tau} t \ ,$$

and from this, at the moment of the third elongation, for which one has

$$t = 2\tau + \frac{\tau}{\pi} \cdot \arctan \frac{\pi}{\lambda} \ , \qquad \sin \frac{\pi t}{\tau} = +\frac{T}{\pi} \ ,$$

one will get

$$x = +\frac{T}{\pi} \cdot C \left(1 + e^{-\lambda} + e^{-2\lambda}\right) e^{-\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} \ ,$$

and so on. If one writes the values of x that are found for $t = 0$, $t = \tau/\pi \cdot \arctan \pi/\tau$, $t = \tau + \tau/\pi \cdot \arctan \pi/\lambda$, $t = 2\tau + \tau/\pi \cdot \arctan \pi/\lambda$ and so on one after the other, in which one replaces $\frac{T}{\pi} C e^{-\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}}$ with B, for brevity, namely:

$$0 \ ,$$
$$+B \ ,$$
$$-B \left(1 + e^{-\lambda}\right) \ ,$$
$$+B \left(1 + e^{-\lambda} + e^{-2\lambda}\right) \ ,$$
$$\cdots \ ,$$

then the differences between each successive pair of values will give the desired sequence of oscillation arcs x_1, x_2, x_3, etc.

aus den Beobachtungen des magnetischen Vereins im Jahre 1838," pp. 98 *et seq.*[484],[485] belongs here, which is also especially important because it gives one a precise and convenient method of measuring the *damping* when it is strong, whereas the method that was cited above, which was based upon observing the decrease in the oscillation arc, is recommended for only weak damping. The aforementioned method of measurement is then especially suited when one employs a galvanometer whose magnetic needle has a large period of oscillation and never deviates more than a few degrees from its normal state, which is the case for a magnetometer that is provided with a multiplier. If the instrument is not equipped with a damper, then the influence of the weak damping that is still present and originates in the multiplier, along with other influences, will be eliminated from the results by the combination of observations that are peculiar to that method. On the other hand, for stronger damping, the method of observation remains essentially the same, but the calculation of the results from the observations must be modified if those results are to agree completely with the ones that were obtained with no damping.

Now, that method essentially consists of suddenly setting the needle in motion with an *instantaneous current* and observing its *first elongation*, and then when the needle again passes its original state for the first time, an instantaneous current will again act upon it that is, however, twice as strong as the first, and the same thing will be true for all of the following ones. That second current shall have the same direction as the first one. The motion of the needle will not only be suddenly inhibited by that current, but it will even take on a velocity that is in the same direction as the one that it came from. One then once more observes the *first elongation* that the needle makes thereafter, which will be almost equal to the previous one with no damping, and lets the needle swing over to the other side of its rest position, where one also observes the *second elongation*. It is only when the needle again passes its rest position from the other side that one can make an instantaneous current act in the opposite direction to the second time that will push the needle back to the same side as the one from which it came and then observe the first and second elongations that follow, whereupon one makes the instantaneous current act in the opposite direction to the previous time as soon as the needle again passes its rest position, etc. The elongations that are thus observed are then arranged in pairs of alternating position and negative elongations, whose mean is taken when they differ from each other only slightly, which is the case for weak damping. The differences between those successive positive and negative mean values are found to be almost equal, and they give a unit for the intensity of the *instantaneous* current that is to be measured.

It was assumed in this that only weak damping was present. However, the same method can also be applied for *strong* damping, and it can even achieve greater precision then. However, the derivation of the results from the observations will suffer an essential modification then.

Let it next be pointed out that for strong damping, the first instantaneous current should not be exactly more than one-half of the following one, but when m denotes the ratio of two successive oscillation arcs, the first current should be the $(m + 1/m)$th part of the following one. However, when that ratio is not adhered to exactly, the observations will not suffer essentially because of that, since one needs only to exclude the first observation

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[484][Note by HW:] Wilhelm Weber's *Werke*, Vol. II, p. 115.

[485][Note by AKTA:] [Web39b, p. 98 of the *Resultate* and p. 115 of Weber's *Werke*]. See also [Gau38b] with English translation in [Gau41c], [Gau38a], [Gau39] with English translations in [Gau41a] and [GT14], and [Web38b].

from the calculation of the results, since the influence of that initial nonuniformity on the following observations will vanish very rapidly due to the damping. One then sees that the corresponding observations (namely, 1st, 5th, 9th, etc., or 2nd, 6th, 10th, etc., or 3rd, 7th, 11th, etc., or 4th, 8th, 12th, etc.) will approach four limiting values very rapidly. If one then denotes the difference between the first and third limiting values by b and the difference between the second and fourth limiting values by a, then the ratio $a : b$ will be equal to the ratio of two successive oscillation arcs, and it will follow that:

$$\lambda = \log \mathrm{nat}\, \frac{a}{b} \ .$$

Moreover, the velocity c that each instantaneous current (except for the first one) imparts upon the needle will be:

$$c = \frac{\pi}{2T} \frac{a^2 + b^2}{\sqrt{ab}} \cdot e^{-\frac{\lambda}{\pi} \arctan \frac{\lambda}{\pi}} \ ,$$

so when a and b differ only slightly (i.e., for weak damping):

$$c = \frac{\pi}{2T} \cdot \frac{a^2 + b^2}{\sqrt{ab}} \ ,$$

and for even weaker damping, one can set:

$$c = \frac{\pi}{2T}(a + b) \ .$$

The proof of that is similar to the ones for the previous rules. Namely, if one calculates the time t from the moment when the instantaneous current pushed the needle towards the side of positive elongation, then x will be:

$$x = A e^{-\lambda t/\tau} \sin \frac{\pi}{\tau} t$$

for the duration of the following two unperturbed oscillations. For the two observed elongations x' and x'', one will have $dx/dt = 0$, or:

$$0 = -\frac{\lambda}{\tau} A e^{-\lambda t/\tau} \sin \frac{\pi}{\tau} t + \frac{\pi}{\tau} A e^{-\lambda t/\tau} \cos \frac{\pi}{\tau} t \ ,$$

and as a result, one will have:

$$t = \frac{\tau}{\pi} \arctan \frac{\pi}{\lambda}$$

for the first moment of observation and:

$$t = \tau + \frac{\tau}{\pi} \arctan \frac{\pi}{\lambda}$$

for the second one. If one substitutes those values for t in the equation for x, then one will get:

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$$x' = + \frac{Ae^{-\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}}}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} ,$$

$$x'' = - \frac{Ae^{-\frac{\lambda}{\pi} \arctan\left(\frac{\pi}{\lambda}\right) - \lambda}}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} .$$

After a time $t = 2\tau$ has passed, the oscillation of the needle will again be changed by the action of the current, namely, the velocity $-c$ will be added to the velocity:

$$\frac{dx}{dt} = \frac{\pi}{\tau} Ae^{-2\lambda} ,$$

that it has at the end of time $t = 2\tau$, which will yield the amplitude of the following two oscillations:

$$x = \left(Ae^{-2\lambda} - \frac{\tau}{\pi}c \right) e^{-\lambda(t-2\tau)/\tau} \sin \frac{\pi}{\tau} t .$$

For the two elongations x''' and x'''' that are observed during that time interval from $t = 2\tau$ to $t = 4\tau$, one will have $dx/dt = 0$ or:

$$\begin{aligned} 0 = & -\frac{\lambda}{\tau} \left(Ae^{-2\lambda} - \frac{\tau}{\pi}c \right) e^{-\lambda(t-2\tau)/\tau} \sin \frac{\pi}{\tau} t \\ & + \frac{\pi}{\tau} \left(Ae^{-2\lambda} - \frac{\tau}{\pi}c \right) e^{-\lambda(t-2\tau)/\tau} \cos \frac{\pi}{\tau} t , \end{aligned}$$

and as a result:

$$t = 2\tau + \frac{\tau}{\pi} \arctan \frac{\pi}{\lambda}$$

for the first moment of observation, while:

$$t = 3\tau + \frac{\tau}{\pi} \arctan \frac{\pi}{\lambda}$$

for the second. If one substitutes those values for t in the new equation for x, then one will get:

$$x''' = + \left(Ae^{-2\lambda} - \frac{\tau}{\pi}c \right) \frac{e^{-\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}}}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} ,$$

$$x'''' = - \left(Ae^{-2\lambda} - \frac{\tau}{\pi}c \right) \frac{e^{-\frac{\lambda}{\pi} \arctan\left(\frac{\pi}{\lambda}\right) - \lambda}}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} .$$

After a time $t = 4\tau$ has passed, the oscillation of the needle will again be changed by the renewed effect of the instantaneous current, namely, the velocity $+c$ will be added to the velocity:

$$\frac{dx}{dt} = \frac{\pi}{\tau}\left(Ae^{-2\lambda} - \frac{\tau}{\pi}c\right)e^{-2\lambda} \ ,$$

that it has at the end of the time $t = 4\tau$, and in that way cause the needle to have from now on the same motion again that it had at the start for $t = 0$. However, the velocity for $t = 0$ was equal to:

$$\frac{\pi}{\tau}A \ ,$$

so:

$$\frac{\pi}{\tau}A = c + \frac{\pi}{\tau}\left(Ae^{-2\lambda} - \frac{\tau}{\pi}c\right)e^{-2\lambda} \ ,$$

from which it will follow that:

$$c = \frac{\pi}{\tau}A\left(1 + e^{-2\lambda}\right) \ .$$

If one substitutes that value in the expression above for x''' and x'''', then one will find that $x''' = -x'$, $x'''' = -x''$, and as a result:

$$\begin{aligned}
a &= x' - x''' = \frac{2Ae^{-\frac{\lambda}{\pi}\arctan\frac{\pi}{\lambda}}}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} \ , \\[2ex]
b &= x'''' - x'' = \frac{2Ae^{-\frac{\lambda}{\pi}\arctan\left(\frac{\pi}{\lambda}\right) - \lambda}}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} \ ,
\end{aligned}$$

so:

$$\frac{a^2 + b^2}{\sqrt{ab}} = \frac{2Ae^{-\frac{\lambda}{\pi}\arctan\frac{\pi}{\lambda}}}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} \cdot \frac{1 + e^{-2\lambda}}{e^{-\lambda/2}} = \frac{2Ae^{\frac{\lambda}{\pi}\arctan\frac{\lambda}{\pi}}}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} \cdot \left(1 + e^{-2\lambda}\right) \ ,$$

which implies that:

$$c = \frac{\pi}{2\tau}\sqrt{1 + \frac{\lambda^2}{\pi^2}}\frac{a^2 + b^2}{\sqrt{ab}}e^{-\frac{\lambda}{\pi}\arctan\frac{\lambda}{\pi}} \ .$$

Now, the equation:

$$\frac{\pi^2}{\tau^2} = \frac{\pi^2}{T^2} - \frac{\lambda^2}{\tau^2} \ ,$$

was quoted in the footnote [480] on page 408 above, from which it will follow that:

$$\frac{1}{T} = \frac{1}{\tau}\sqrt{1 + \frac{\lambda^2}{\pi^2}}$$

and

$$c = \frac{1}{2}\frac{\pi}{T} \cdot \frac{a^2 + b^2}{\sqrt{ab}} \cdot e^{-\frac{\lambda}{\pi}\arctan\frac{\lambda}{\pi}} \ .$$

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At the same time, one sees that:

$$\frac{a}{b} = e^{\lambda} \qquad \text{or} \qquad \lambda = \log \text{nat} \frac{a}{b} \, ,$$

from which it will emerge that one can get a precise determination of the unit of damping that is reduced to the period of oscillation by measuring a and b, and it will be especially useful when no precise determination can be obtained due to the rapid decrease in oscillation arc in the observations of the latter.

In order to explain the method of observation that was just developed, one might appeal to those observations that were made in order to compare the resistance of two copies of Jacobi's basic unit using that method and were mentioned before on page 333. The *first* copy consisted of an unvarnished wire that was wound around an unvarnished serpentine cylinder, while the *second* copy consisted of an unvarnished wire that was wound around an unvarnished glass tube. The experiments split into five sets. The ends of the inductor wire were connected with the ends of the multiplier wire in the same way in all of them. In the *first* set, the wires of the two copies and the inductor and multiplier wire were combined in the manner that was described in item (8.) on page 295 and denoted by D. In the *second* set, they were combined in the manner that was described in item (7.) and denoted by B. In the *third* set, they were combined in the manner that was described in item (6.) and denoted by A, in which the first copy enters in place of the basic unit. The *fourth* set was a repetition of the second one, and finally, in the *fifth* set, the wires were combined in the way that was described in item (9.) and denoted by C. The experiment began when the galvanometer needle was at rest. The first positive induction thrust set the needle into oscillation. The first positive elongation was not observed, and likewise the second negative elongation was not observed, either. The second negative induction thrust took place at the moment when the needle arrived at the location that corresponded to the rest state from that second elongation in the positive direction, which not only inhibited the needle in the middle of its positive motion, but it even pushed it back to the side from which it had come. The third elongation that followed it was then, in turn, a negative one, and it was not observed, as well as the fourth positive elongation. The third positive induction thrust took place at the moment when the needle arrived at the location that corresponded to its equilibrium from that fourth elongation in the negative direction, which not only inhibited the needle in the middle of its negative motion, but even pushed it back to the side from which it came. The experiment was continued for a long time in the same way, and the elongation of the needle was recorded as it was observed on the scale from then on. The first four recorded elongations are placed next to each other in the horizontal rows of the following Tables, but the 5th is below the 1st, the 6th is below the 2nd, etc. Finally, the mean value of each column of observations is given.

D.

775.8	436.6	199.6	538.2
775.7	436.3	199.5	537.9
775.0	435.9	198.9	537.3
774.6	435.4	198.5	537.2
774.6	435.4	198.8	537.2
774.2	435.3	198.5	537.1
774.0	435.1	198.2	536.8
773.8	434.7	197.9	536.6
773.5	434.4	197.6	536.6
774.0	434.0	197.5	536.0
774.52	435.31	198.50	537.09

B.

692.1	448.0	277.0	521.1
691.8	447.8	276.7	521.0
691.7	447.4	276.5	520.8
691.3	447.3	276.2	520.5
691.3	447.2	276.0	520.7
691.4	447.2	276.0	520.6
691.4	447.2	275.9	520.5
691.3	447.1	275.9	520.5
691.4	447.0	275.9	520.4
691.3	447.0	275.8	520.3
691.50	447.32	276.19	520.64

A.

691.9	447.7	276.9	521.0
691.6	447.7	276.8	520.9
691.7	447.7	276.8	521.0
691.8	447.6	276.7	521.1
691.8	447.8	276.8	521.0
691.8	447.8	277.0	521.1
691.8	447.8	276.9	521.1
691.6	447.6	276.8	520.8
691.3	447.3	276.4	520.7
691.0	446.9	275.9	520.1
691.63	447.59	276.60	520.88

B.

691.6	447.0	275.7	520.5
691.4	446.9	275.6	520.3
691.2	446.7	275.3	520.0
690.9	446.3	275.2	520.0
690.7	446.2	275.0	519.7
690.5	446.1	274.9	519.8
690.5	446.5	274.8	519.7
690.3	445.9	274.6	519.4
690.1	445.8	274.6	519.2
690.1	445.6	274.3	519.2
690.73	446.30	275.00	519.78

C.

615.8	459.3	350.2	506.2
615.6	459.2	350.1	506.1
615.2	459.0	349.8	505.8
615.1	458.8	349.4	505.6
614.8	458.4	349.2	505.3
614.4	458.1	349.1	505.2
614.2	458.1	348.8	505.0
614.1	458.0	348.8	504.9
613.9	457.8	348.7	504.8
613.8	457.6	348.2	504.3
614.69	458.43	349.23	505.32

It should be remarked in regard to those Tables that the horizontal distance from the mirror to the scale amounted to 2218 scale divisions. Moreover, one should observe that the inductor coil that was used here was different from the one that was used before, page 296 [on Section 13.6]. The new inductor coil has a much smaller number of windings, but much stronger wire, such that its resistance was much smaller than the resistance of the first inductor coil. That fact had a significant effect on the relationship of the observations A, B, C, D to each other.

The mean values of the observations above are clearly summarized in the following Table, and the differences between the first and third values, as well as the second and fourth values, for each set of experiments is added, and those two differences are denoted by a and b, just as on page 414.

D.	774.52	
	435.31	$a = 576.02$
	198.50	$b = 101.78$
	537.09	
B.	691.50	
	447.32	$a = 415.31$
	276.19	$b = 73.32$
	520.64	
A.	691.63	
	447.59	$a = 414.93$
	276.70	$b = 73.29$
	520.88	
B.	690.73	
	446.30	$a = 415.73$
	275.00	$b = 73.48$
	519.78	
C.	614.69	
	458.43	$a = 265.46$
	349.23	$b = 46.89$
	505.32	

The values of a and b that are summarized in that Table now first require a correction, because they are the tangents of twice the elongation angles, according to the laws of catoptrics. With the help of the given distance from the mirror to the scale, it is easy to reduce them to values that are proportional to the elongation angles themselves, and that reduction is sufficient because of the smallness of all of those elongations. To that end, if x denotes the value of a or b that is given in scale divisions, then one must reduce the number x by:

$$\frac{1}{3}\frac{x^3}{4436^2} = \frac{x^3}{59\,034\,288}\,.$$

After that reduction, one will get the following values for a and b:

	a	b
D.	572.78	101.76
B.	414.10	73.31
A.	413.72	73.28
B.	414.51	73.47
C.	265.14	46.89

If one now takes the mean of the two values a and b that are quoted in B, then one will get the following summary:

	a	b	$\log \operatorname{nat}\frac{a}{b} = \lambda$	$\frac{a^2+b^2}{\sqrt{ab}} \cdot e^{-\frac{\lambda}{\pi} \arctan \frac{\lambda}{\pi}}$
A.	413.72	73.28	1.730 902	768.22
B.	414.305	73.39	1.730 814	769.23
C.	265.14	46.89	1.732 454	492.44
D.	572.78	101.76	1.727 884	1063.11

Now, as was shown on page 414 *et seq.*, the values that are given in the last column can serve as a unit of current strength in the multiplier, i.e, they can be considered to be the values that were denoted by A, B, C, D on page 302. Finally, with those values, one will get:

$$\frac{AB - BC}{AB - AC} = 0.997\,65 ,$$

$$\sqrt{\frac{AB - AD}{AB - BD}} = 0.997\,62 ,$$

from the formulas that were presented there. With that, one will get the ratio of the resistance of the first copy to the second one as equal to:

$$0.997\,64 : 1 ,$$

in the mean.

D - Basis for the Rules for Calculating the Resistance of a Conductor from the Observations

In order to base the rules for calculating the resistance of a conductor from the observations, we shall start from the following two fundamental laws from the study of *electromagnetism* and the study of *magneto-electricity.*

First law. The linear element of a galvanic current ds exerts a driving force on an element of the magnetic fluid μ that is inversely proportional to the square of the distance r between them. However, at the same time, an entirely anomalous situation enters in that way, namely, that the direction of the force is not along the straight line that connects them, but perpendicular to the plane that is laid through μ and the direction of ds, and in addition, the strength of the force does not depend upon the distance alone, but likewise on the angle that r makes with the direction of ds. If one calls that angle ϑ then:

$$\frac{\sin \vartheta \cdot \mu ds}{r^2}$$

will be the unit of the driving force that ds exerts upon μ, and it is just as large as the force that μ exerts upon the current element ds or its ponderable carrier, whose direction is parallel, but opposite, to the former.

Remark. — One must understand the current element that is denoted by ds to mean the product of its length α with the intensity i of the current that flows through it, so $ds = \alpha i$. — That fundamental law of electromagnetism is repeated here verbatim just as Gauss expressed it in "*Resultaten aus den Beobachtungen des magnetischen Vereins im Jahre 1839,*" pp. 1, 2.[486,487]

Second law. If the element of magnetic fluid μ moves with a velocity u parallel to the force that acts upon the current element $ds = \alpha i$ according to the first law, then an electromotive

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[486][Note by HW:] Gauss's *Werke*, Vol. V, p. 198.
[487][Note by AKTA:] [Gau40] with English translation in [Gau43].

force that is parallel to the direction of the current i will be exerted upon the linear element of the conductor α, whose strength will be represented by the expression that was given in the first law, namely, $\sin \vartheta \cdot \mu ds/r^2$, when one exchanges $ds = \alpha i$ with αu, so by:

$$\frac{\sin \vartheta \cdot \mu \alpha u}{r^2} \ .$$

By contrast, if the element of the magnetic fluid μ were to move in a different direction that makes an angle ψ with the one that was referred to above, then the expression for the strength of the force must be multiplied by $\cos \psi$.

Remark. — If one introduces two other angles in place of the two angles ϑ and ψ, namely, the angle φ that the direction in which μ moves makes with r, and the angle ε that the direction of α makes with the normal to a plane that is laid parallel to the direction in which μ moves and goes through r, then the expression $\sin \vartheta \cdot \mu \alpha u \cdot \cos \psi / r^2$ will be converted into $\sin \varphi \cdot \mu \alpha u \cos \varepsilon / r^2$. — The latter expression agrees with the one that one obtains when one decomposes the elementary electromotive force that was given in the first treatise on "Elektrodynamische Maassbestimmungen," p. 345,[488],[489] in the direction of the induced element α. The expression thus-obtained will indeed contain a constant factor whose value, however, depends upon the choice of unit for the electromotive force and is equal to 1 for a certain unit.

The arguments that follow were derived from these two laws:

1. A relationship exists between the electromagnetic and magneto-electric forces such that when two arbitrarily-placed magnetic elements μ and μ' along a current element $ds = \alpha i$ exert equal and equally-directed electromagnetic forces, then their electromotive forces on the linear element of the conductor α (when it is moving) will be equal. The same thing will be true when μ and μ' are replaced with an ensemble of arbitrarily-distributed magnetic elements. It then follows from this that when the geomagnetism at a location is equal and equally-directed to the electromagnetic force that a distant bar magnet would exert, then the electromotive force of geomagnetism on an inductor that is itself moving will likewise be equal to that of the bar magnet, no matter how the magnetism of the Earth might be distributed.

2. When the current element ds belongs to a circular current, the component of the electromagnetic force that ds exerts upon μ that is perpendicular to the plane of the circle will be obtained when $\sin \vartheta \cdot \mu ds/r^2$ is multiplied by the cosine of the angle that the plane of the circle defines with the plane that goes through μ and the direction of ds. This component is called C.

The current element ds might be decomposed into its factors, namely, into its current intensity i and its length, which will be denoted by $ad\alpha$ as the length of a circular element, when a is the radius of the circle that it belongs to and α is the angle that the associated radius defines with the radius that lies in a plane that is perpendicular to the plane of the circle, along with μ. Furthermore, if one lets b denote the perpendicular that is dropped from μ to the plane of the circle, while x denotes the distance from the foot of that perpendicular to the midpoint, then:

$$r^2 = a^2 + b^2 + x^2 - 2ax \cos \alpha \ ,$$

and one will get the expression:

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[488][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 177.
[489][Note by AKTA:] See page 169 on Subsection 5.25.4.

$$\frac{\sin\vartheta \cdot i\mu \cdot a d\alpha}{a^2 + b^2 + x^2 - 2ax\cos\alpha},$$

for the total force that ds exerts upon μ. Moreover, the cosine of the angle that the plane of the circle defines with the plane that is laid through μ and the direction of ds is:

$$\frac{a - x\cos\alpha}{r\sin\vartheta} = \frac{a - x\cos\alpha}{\sin\vartheta \cdot \sqrt{a^2 + b^2 + x^2 - 2ax\cos\alpha}}.$$

The product of that cosine with the expression for the total force above gives the expression for the desired component C, namely:

$$C = i\mu \cdot a d\alpha \frac{a - x\cos\alpha}{(a^2 + b^2 + x^2 - 2ax\cos\alpha)^{3/2}}.$$

With those two stated laws, which are the basic laws of magneto-electricity, one gets the *electromotive force* that μ exerts upon ds, when μ moves with a velocity u parallel to the direction of the force C, upon multiplying by the value that C has, when $i = -1$, with u, namely:

$$-u\mu \cdot a d\alpha \frac{a - x\cos\alpha}{(a^2 + b^2 + x^2 - 2ax\cos\alpha)^{3/2}}.$$

By contrast, when ds moves with a velocity u in the same direction that is perpendicular to the plane of the circle, the electromotive force that μ exerts upon ds will be:

$$+u\mu \cdot a d\alpha \frac{a - x\cos\alpha}{(a^2 + b^2 + x^2 - 2ax\cos\alpha)^{3/2}}.$$

If one further develops the expression for C above in powers of $\cos\alpha$, then one will get:[490]

$$\begin{aligned}
C \;=\; & \frac{i\mu}{(a^2 + b^2 + x^2)^{3/2}} \left\{ a^2 d\alpha + (2a^2 - b^2 - x^2)\frac{ax}{a^2 + b^2 + x^2} \cdot \cos\alpha\, d\alpha \right. \\
& + \frac{3}{2}(3a^2 - 2b^2 - 2x^2)\frac{a^2 x^2}{(a^2 + b^2 + x^2)^2} \cdot \cos\alpha^2\, d\alpha \\
& + \left. \frac{5}{2}(4a^2 - 3b^2 - 3x^2)\frac{a^3 x^3}{(a^2 + b^2 + x^2)^3} \cdot \cos\alpha^3\, d\alpha + ... \right\}.
\end{aligned}$$

3. The expression for the *electromagnetic* force that the *total* circular current exerts upon μ, perpendicular to the plane of the circle, is obtained in the following way: Since the radius a and the current intensity i, like b and x, are equal for all circle elements, the desired force, or the sum of all *electromagnetic* forces that all current elements exert upon μ, which are perpendicular to the plane of the circle, will be:

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[490][Note by AKTA:] The expressions $\cos\alpha^2$, $\cos\alpha^3$ etc. in this equation and in the next ones should be understood as $\cos^2\alpha$, $\cos^3\alpha$ etc., respectively.

$$i\mu \cdot a \int_0^{2\pi} \frac{a - x\cos\alpha}{(a^2 + b^2 + x^2 - 2ax\cos\alpha)^{3/2}} d\alpha$$

$$= \frac{i\mu}{(a^2+b^2+x^2)^{3/2}} \cdot \left\{ a^2 \int_0^{2\pi} d\alpha + (2a^2 - b^2 - x^2)\frac{ax}{a^2+b^2+x^2} \int_0^{2\pi} \cos\alpha\, d\alpha \right.$$

$$+ \frac{3}{2}(3a^2 - 2b^2 - 2x^2)\frac{a^2 x^2}{(a^2+b^2+x^2)^2} \cdot \int_0^{2\pi} \cos\alpha^2\, d\alpha$$

$$\left. + \frac{5}{2}(4a^2 - 3b^2 - 3x^2)\frac{a^3 x^3}{(a^2+b^2+x^2)^3} \cdot \int_0^{2\pi} \cos\alpha^3\, d\alpha + ... \right\},$$

that is:

$$\frac{2\pi a^2 \cdot i\mu}{(a^2+b^2+x^2)^{3/2}} \left\{ 1 + \frac{3}{4}(3a^2 - 2b^2 - 2x^2)\frac{x^2}{(a^2+b^2+x^2)^2} + ... \right\} . \qquad (I)$$

Moreover, one can easily convince oneself that the current intensity in that must be determined in the absolute units that were established in Section 13.10, when one sets the area of the circle $\pi a^2 = 1$, from which one finds that one must have $i = 1$ in order for the force that is exerted upon μ at a distance (where a^2 vanishes in comparison to $b^2 + x^2$), perpendicular to the plane of the circle, to be equal to the force that is exerted upon a magnet in the same direction, when that magnet has one absolute unit of magnetism[491] and its axis is normal to the plane of the circle. Let A [in Figure 14] be the midpoint of the magnet, let AB be the direction of its axis, and let the element μ be found at C.
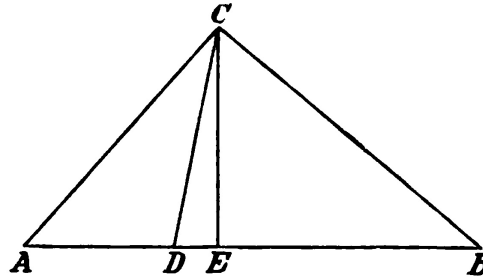
Fig. 14.

Let ABC be a rectangular triangle at C, and let $AD = AB$. From a known theorem ("*Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1840*," pp. 33, 34),[492,493] CD will be the direction of the force that acts upon μ, and its strength will be $\mu/AC^3 \cdot CD/AD$. If one drops CE perpendicular to AB, then the component that is parallel to the axis of the magnet will be $\mu/AC^3 \cdot CD/AD \cdot ED/CD = \mu/AC^3 \cdot ED/AD$. Now, if AE and CE are the lines that were denoted above by b and x, then that will make:

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[491][Note by AKTA:] See footnote 392 on page 306.
[492][Note by HW:] Gauss's *Werke*, Vol. V, p. 434.
[493][Note by AKTA:] [Gau41d].

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$$AC = \sqrt{b^2 + x^2} ,$$

$$AD = \frac{1}{3}AB = \frac{1}{3}\frac{AC^2}{AE} = \frac{b^2 + x^2}{3b}$$

and

$$ED = AE - AD = \frac{2b^2 - x^2}{3b} ;$$

and as a result, the desired force will be:

$$\mu \cdot \frac{2b^2 - x^2}{(b^2 + x^2)^{5/2}} .$$

By contrast, when one sets $\pi a^2 = 1$ and $\frac{a^2}{b^2+x^2} = 0$ in the expression above, one will get:

$$i\mu \cdot \frac{2b^2 - x^2}{(b^2 + x^2)^{5/2}} ,$$

which implies that one must have $i = 1$, if the current that flows around a unit area is supposed to have the same effect as the absolute unit of magnetism. However, that current intensity is the absolute unit that was established in Section 13.10, which explains the fact that the current intensities must be determined in the absolute unit that was established in the applications of the electromagnetic laws above.

4. The expression for the *electromotive force* that μ exerts upon the *entire* circle when it[494] moves with a velocity u in a direction that is perpendicular to the plane of the circle is obtained in the following way: The two laws that were stated previously imply that the electromotive force that μ exerts upon ds when it moves with a velocity u parallel to the direction of the force C is obtained by multiplying u by the value of C, in which one has set $i = -1$, namely:

$$-\frac{\mu u}{(a^2 + b^2 + x^2)^{3/2}} \cdot \left\{ a^2 d\alpha + (2a^2 - b^2 - x^2)\frac{ax}{a^2 + b^2 + x^2} \cdot \cos\alpha d\alpha \right.$$
$$+ \frac{3}{2}(3a^2 - 2b^2 - 2x^2)\frac{a^2 x^2}{(a^2 + b^2 + x^2)^2} \cdot \cos\alpha^2 d\alpha$$
$$\left. + \frac{5}{2}(4a^2 - 3b^2 - 3x^2)\frac{a^3 x^3}{(a^2 + b^2 + x^2)^3} \cdot \cos\alpha^3 d\alpha + ... \right\} .$$

As a result, the sum of the electromotive force that μ exerts upon all elements of the circle is as follows:

$$-\frac{\mu u}{(a^2 + b^2 + x^2)^{3/2}} \left\{ a^2 \int_0^{2\pi} d\alpha + (2a^2 - b^2 - x^2)\frac{ax}{a^2 + b^2 + x^2} \int_0^{2\pi} \cos\alpha d\alpha \right.$$
$$+ \frac{3}{2}(3a^2 - 2b^2 - 2x^2)\frac{a^2 x^2}{(a^2 + b^2 + x^2)^2} \int_0^{2\pi} \cos\alpha^2 d\alpha$$
$$\left. + \frac{5}{2}(4a^2 - 3b^2 - 3x^2)\frac{a^3 x^3}{(a^2 + b^2 + x^2)^3} \int_0^{2\pi} \cos\alpha^3 d\alpha + ... \right\} ,$$

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[494][Note by AKTA:] That is, when μ.

that is:

$$-\mu u \frac{\pi a^2}{(a^2 + b^2 + x^2)^{3/2}} \cdot \left\{ 2 + \frac{3}{2}(3a^2 - 2b^2 - 2x^2) \frac{x^2}{(a^2 + b^2 + x^2)^2} + \ldots \right\} . \quad (II)$$

Moreover, one can easily convince oneself that this value of the electromotive force is expressed in terms of the absolute unit that was established in Section 13.10. Namely, if one sets $b = 0$ in that value, then the linear velocity u of the element μ will be identical to an angular velocity $\gamma = u/x$ around diameter of the circle that is perpendicular to x, which can then be set equal to the opposite angular velocity of the circle $-\gamma$ around the same axis without changing the electromotive force. The expression for the electromotive force of μ on the circle that rotates with a velocity of $-\gamma$ will then be:

$$-\mu\gamma \frac{\pi a^2 x}{(a^2 + x^2)^{3/2}} \left\{ 2 + \frac{3}{2}(3a^2 - 2x^2) \frac{x^2}{(a^2 + x^2)^2} + \ldots \right\} ,$$

which is therefore $-\mu\gamma \cdot \pi a^2/x^2$ on the circle that rotates with a velocity of $+\gamma$ when μ acts at a distance, where a^2 vanishes in comparison to x^2. That implies that the sum of the two electromotive forces when one has, first of all, $\mu = +m$ and $x = R + e$ and, secondly, $\mu = -m$ and $x = R - e$, i.e., the electromotive force of a magnet $M = 2em$ that acts at a distance R will be:

$$-m\gamma \frac{\pi a^2}{(R + e)^2} + m\gamma \frac{\pi a^2}{(R - e)^2} = M\gamma \frac{2\pi a^2}{R^3} .$$

If one now makes the angular velocity γ of the circle large enough that its projection onto a plane that is normal to x changes by one unit of area in a unit time, that is, $\gamma\pi a^2 = 1$, then one will find that the value of the electromotive force above must be equal to 1 when M is the magnetism of the bar that has the same effect as the unit of geomagnetism whose direction is parallel to x. If, namely, in the direction of terrestrial magnetism T of a place at the distance R there is an identically directed magnet M, then the effect of M in magnetic units will be equal to the effect of T at that location when one has:

$$\frac{2M}{R^3} = T .$$

M will then denote the magnetism of the bar that acts the same as one unit of geomagnetism when one has:

$$\frac{2M}{R^3} = 1 ,$$

from which it emerges that when, at the same time, $\gamma\pi a^2 = 1$, the value of the electromotive force above will also be equal to 1. However, that electromotive force is itself the absolute unit that was established in Section 13.10, which explains the fact that the electromagnetic force is determined in the given absolute unit in the applications of the magneto-electric laws that were developed here.

   Up to now, we have considered the forces that an element of a magnetic fluid μ exerts or experiences. The application of that to experiments requires that all elements of both magnetic fluids that are contained in a magnetic needle should be included in the calculation. However, that explains the fact that, with Gauss,[495] one can then focus on the elements of

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[495][Note by AKTA:] See footnote 406 on page 321.

the ideal magnetic fluids that are distributed over the surface, that are completely separated from each other. If the sum of the positive elements is equal to $+m$, then the sum of the negative ones will be equal to $-m$, and if one denotes the distance from the midpoint of the one from the midpoint of the other by $2e$, then the [magnetic] moment of the needle will be equal to $2em$, and the length of the line e is a measurable quantity. That also explains the fact that when all of the positive elements lie close together, and similarly for all the negative ones, their effect will be almost the same as if they were concentrated at their respective midpoints. Only the effect of two points would be included in the calculation with the needle used then, namely, the one at which all north-magnetic fluid is thought to be concentrated and the one at which the south-magnetic fluid is concentrated. That will then imply:

5. The comparison of the angular moment that the multiplier exerts upon the needle that is found at its *midpoint* with the one that it would exert if the needle were *at a great distance* as follows: The meridian plane in which the needle lies divides the multiplier in such a way that an equal number of windings lie on both sides. If one draws a horizontal line in that plane through the midpoint of the multiplier, then the point at which all north-magnetic fluid $+m$ is concentrated will lie along that line, and the distance from it to the midpoint will be called $+e$. The distance from the point along the same line at which the south-magnetic fluid $-m$ is thought to be concentrated will be called $-e$. Let a' and a'' be the inner and outer diameters of the multiplier and let $2b'$ be the width of its cross-section, which will then be $2(a'' - a')b'$. The part of that cross-section of the entire ring that comes to a winding, whose diameter is a and whose plane lies at a distance b from the midpoint of the multiplier, will be denoted by $dadb$: From (I), the product of the cross-section of a winding with the *force* that it exerts upon $+m$ will then be:

$$+im\frac{2\pi a^2 dadb}{(a^2 + b^2 + e^2)^{3/2}}\left\{1 + \frac{3}{4}(3a^2 - 2b^2 - 2e^2)\frac{e^2}{(a^2 + b^2 + e^2)^2} + \dots\right\}.$$

If one multiplies that product with the perpendicular e that is dropped from the rotational axis to the direction of the force, then one will get the product of the cross-section of the winding with the *angular moment* that it exerts. Finally, if one substitutes $-m$ for $+m$ and $-e$ for $+e$ in that expression, then one will get an equal value for the product of that cross-section with the angular moment that is exerted upon the negative fluid by that winding. As a result, the product of the angular moment that is exerted by that winding on the entire needle will be:

$$iM\frac{2\pi a^2 dadb}{(a^2 + b^2 + e^2)^{3/2}}\left\{1 + \frac{3}{4}(3a^2 - 2b^2 - 2e^2)\frac{e^2}{(a^2 + b^2 + e^2)^2} + \dots\right\},$$

when one denotes the magnetism of the needle by $M = 2em$. Since e is a smaller fraction of a, all parts that have the fourth or higher powers of it as a factor can be dropped, and one will then get:

$$iM\frac{2\pi a^2 dadb}{(a^2 + b^2)^{3/2}}\left\{1 + \frac{3}{4}\frac{a^2 - 4b^2}{(a^2 + b^2)^2} \cdot e^2\right\}.$$

It then follows from this that the sum of the products of the cross-section of each winding with the rotational moment that it exerts will be:

$$iM \cdot 2\pi \int_{\alpha'}^{\alpha''} a^2 da \int_{-b'}^{+b'} \frac{db}{(a^2+b^2)^{3/2}} \left\{ 1 + \frac{3}{4} \frac{a^2-4b^2}{(a^2+b^2)^2} e^2 \right\}$$

$$= iM \left\{ 4\pi b' \cdot \log \frac{a''+\sqrt{a''^2+b'^2}}{a'+\sqrt{a'^2+b'^2}} + \frac{\pi}{b'} \left(\frac{a''^3}{(a''^2+b'^2)^{3/2}} - \frac{a'^3}{(a'^2+b'^2)^{3/2}} \right) \cdot e^2 \right\} .$$

If one divides that expression by the product of the cross-section of a winding with the number of windings — i.e., by the cross-section of the entire ring $2(a''-a')b'$, — then one will get the mean rotational moment that a winding exerts upon a needle, from which will emerge the rotational moment of the multiplier on the needle that is found at its midpoint when one multiplies by the number of windings n, namely:

$$iM \cdot 2n\pi \cdot \frac{1}{a''-a'} \left\{ \log \frac{a''+\sqrt{a''^2+b'^2}}{a'+\sqrt{a'^2+b'^2}} \right.$$

$$\left. + \frac{1}{4} \left(\frac{a''^3}{(a''^2+b'^2)^{3/2}} - \frac{a'^3}{(a'^2+b'^2)^{3/2}} \right) \frac{e^2}{b'^2} \right\} .$$

For the case in which b' vanishes compared to a', and in which a' differs only slightly from a'', that will be $iM \cdot 2n\pi/a'$, and a' will be the radius of the multiplier in that case. If one now generally understands that the radius of the multiplier of a given *central needle* has the expression:

$$\frac{a''-a'}{\log \frac{a''+\sqrt{a''^2+b'^2}}{a'+\sqrt{a'^2+b'^2}} + \frac{1}{4} \left(\frac{a''^3}{(a''^2+b'^2)^{3/2}} - \frac{a'^3}{(a'^2+b'^2)^{3/2}} \right) \frac{e^2}{b'^2}}$$

and one denotes it by r', then the rotational moment will be:

$$\frac{2n\pi}{r'} \cdot Mi .$$

By contrast, if the needle is at a great distance from the multiplier, but $+m$ and $-m$ remain on the same line at distances of $R+e$ and $R-e$, respectively, from the midpoint, then one must revert to the expression C on page 423 for the force that an element ds exerts upon μ perpendicular to the meridian plane, in which one substitutes $+m$ or $-m$ for μ, and $R+e$ or $R-e$ for x. One will then get:

$$im \cdot ad\alpha \frac{a-(R+e)\cos\alpha}{(a^2+b^2+(R+e)^2-2a(R+e)\cos\alpha)^{3/2}} ,$$

for $+m$, and:

$$-im \cdot ad\alpha \frac{a-(R-e)\cos\alpha}{(a^2+b^2+(R-e)^2-2a(R-e)\cos\alpha)^{3/2}} ,$$

for $-m$. The sum of the former value, multiplied by $+e$, and the latter one, multiplied by $-e$, will give the rotational moment that ds exerts upon the needle, namely, when one writes M for $2em$ and observes that a, b, and e vanish in comparison to R:

$$iM \cdot ad\alpha \cdot \frac{a - R\cos\alpha}{(R^2 - 2aR\cos\alpha)^{3/2}}$$

$$= -\frac{iM}{a}\left\{\frac{a^2}{R^2}\cos\alpha d\alpha + \frac{a^3}{R^3}(3\cos\alpha^2 - 1)d\alpha + ...\right\} ,$$

and as a result, the rotational moment that the entire circle to which ds belongs exerts upon on the needle will be:

$$-\frac{iM}{a}\left\{\frac{a^2}{R^2}\int_0^{2\pi}\cos\alpha d\alpha + \frac{a^3}{R^3}\int_0^{2\pi}(3\cos\alpha^2 - 1)d\alpha + ...\right\}$$

$$= -\frac{\pi a^2}{R^3} \cdot Mi ,$$

because the following terms, which include fourth or higher powers of a/R, can be dropped.

If one integrates that value, multiplied by $dadb$, between the limits from $a = a'$ to $a = a''$ and from $b = -b'$ to $b = +b'$, then the product of that integral with $n/[2(a'' - a')b']$ will be the rotational moment that the multiplier exerts upon the distant needle:

$$-\frac{1}{3}\frac{n\pi Mi}{R^3} \cdot \frac{a''^3 - a'^3}{a'' - a'} = -\frac{n\pi Mi}{R^3} \cdot \frac{a'^2 + a'a'' + a''^2}{3} .$$

For the case in which a' differs slightly from a'' that will give:

$$-\frac{n\pi a'^2}{R^3} \cdot Mi ,$$

and a' will be the radius of the multiplier in that case. If one now generally understands the radius of the multiplier of a *distant needle* to mean the expression:

$$\sqrt{\frac{1}{3}(a'^2 + a'a'' + a''^2)} ,$$

and denotes it by r'', then the rotational moment will be:

$$-\frac{n\pi r''^2}{R^3} \cdot Mi .$$

Finally, if one compares the expression that was found for the rotational moment that the multiplier exerts upon the needle that is found at its *midpoint* with the one that it would exert if the needle were *at a great distance*, then that would give the ratio:

$$\frac{2n\pi}{r'} \cdot Mi : -\frac{n\pi r''^2}{R^3} \cdot Mi = 1 : -\frac{r'r''^2}{2R^3} .$$

Rules for Calculating the Resistance from Observations
that are Performed by the First Method in Section 13.14.

Let the line NS [in Figure 15] denote the direction of the horizontal component of geomagnetism, whose strength equals T' at the location A, and T'' at the location B.
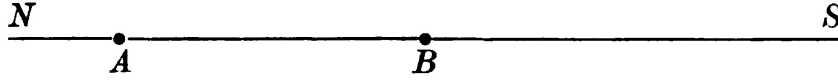
N ———•—•————————————•———————————————— S
        A   B

<center>**Fig. 15.**</center>

Let a closed circuit consist of two vertical rings whose centers are at A and B. Let the ring B, which defines the multiplier, be fixed, while the ring A, which defines the inductor, can rotate about its vertical diameter. Let the sum of the areas that are bounded by all windings of the ring A be S, and if ψ denotes the angle that the normal to the plane of the ring defined with the direction NS at the end of the time interval t, then the projection of S onto a plane that is perpendicular to NS at that moment will be equal to $S \cos \psi$, and the increase in that during the time element dt will be $-S \sin \psi \cdot d\psi$. That will give the absolute value of the *electromotive force* that geomagnetism T' exerts upon the ring A as in Section 13.10:

$$eE = -ST' \cdot \sin \psi \frac{d\psi}{dt} \cdot E \ .$$

The value of the integral of that from the moment when $\psi = \pi$ to the moment when $\psi = 0$ will be denoted by $e'E$, so:

$$e' = 2ST' \ .$$

The current that is produced by that electromotive force in the entire closed circuit, whose intensity at the end of the time interval t will be denoted by iJ, goes through the multiplier ring B, and when it flows, it will exert a rotational moment on the needle that is found at its midpoint whose magnetic axis coincides with NS and can be expressed in the following way:

$$\frac{2n\pi}{r'} \cdot Mi \ ,$$

as on page 428, in which:

$$\frac{1}{r'} = \frac{1}{a'' - a'} \left\{ \log \frac{a'' + \sqrt{a''^2 + b'^2}}{a' + \sqrt{a'^2 + b'^2}} + \frac{1}{4} \left(\frac{a''^3}{(a''^2 + b'^2)^{3/2}} - \frac{a'^3}{(a'^2 + b'^2)^{3/2}} \right) \frac{e^2}{b'^2} \right\} \ .$$

Here, n denotes the number of windings of the multiplier ring B, a' and a'' denote the smallest and largest radii, respectively, and $2b'$ denotes their height, while M denotes the magnetism of the needle in absolute units, and $2e$ is the quotient M/m, when m expresses the amount of north-magnetic fluid that is spread over the surface of the needle in the ideal distribution.

If K denotes the moment of inertia of the needle, then that will yield the acceleration of the rotation of the needle as equal to:

$$\frac{2n\pi}{r'} \cdot \frac{Mi}{K} \ .$$

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Furthermore, if one denotes the value of the integral of the current intensity iJ over the time interval from the moment when $\psi = \pi$ to the moment when $\psi = 0$ by $i'J$, then the value of the integral of the acceleration over the same time interval — i.e., the angular velocity that is imparted to the needle by the induction thrust — will be:

$$\frac{d\varphi}{dt} = \frac{2n\pi}{r'} \cdot \frac{Mi'}{K} \; ,$$

from which, upon multiplying by t'/π, the width of the elongation α will be:

$$\alpha = \frac{2nt'}{r'} \cdot \frac{i'M}{K} \; ,$$

and as a result, one will get:

$$i' = \frac{\alpha K r'}{2nMt'} \; ,$$

in which t' denotes the period of oscillation of the needle.

If $1 : (1 + \vartheta)$ is the increased ratio of the magnetic directive force to the elasticity of the thread from which the needle hangs, and T'' is the strength of the horizontal component of geomagnetism at the location of the multiplier, then:

$$t'^2 = \frac{\pi^2 K}{(1 + \vartheta)MT''} \; ,$$

or

$$\frac{K}{Mt'} = \frac{(1 + \vartheta)T''t'}{\pi^2} \; ,$$

and as a result:

$$i' = \frac{(1 + \vartheta)T''r't'}{2n\pi^2} \cdot \alpha \; .$$

Finally, if wW denotes the resistance of the entire closed circuit, then that will give the following rule for calculating the coefficient w:

$$w = \frac{e'}{i'} = \frac{n}{1 + \vartheta} \cdot \frac{T'}{T''} \cdot \frac{4\pi^2 S}{\alpha r' t'} \; ,$$

which was to be proved.

<p style="text-align:center">Rule for Calculating the Resistance from Observations
that are Performed According to the Second Method in Section .</p>

If the fixed ring B that is parallel to the magnetic meridian is closed and the needle that hangs from its center is made to oscillate, then an *electromotive force* will be exerted by that needle on the ring B that can be determined in the following way:

Let $+m$ denote the north-magnetic fluid, which is thought of as spread over the surface of the needle in the ideal distribution, and let $+e$ denote the distance at which the midpoint of that magnetic mass lies from the center of the ring B. Furthermore, let $-m$ denote the south-magnetic fluid, which is thought of as spread over the surface of the needle in the ideal

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distribution, and let $-e$ denote the distance at which the midpoint of that magnetic mass lies from the center of the ring B. If:

$$M = 2em ,$$

as a result of the needle magnetism, then if γ is the angular velocity of the needle, for small elongation widths of the needle, the electromotive force that the needle exerts upon a winding of the ring B whose radius is equal to a and whose plane lies at a distance b from the center of the ring B, from page 426, can be determined by the following expression:

$$-\gamma M \frac{\pi a^2}{(a^2 + b^2 + e^2)^{3/2}} \left\{ 2 + \frac{3}{2}(3a^2 - 2b^2 - 2e^2) \frac{e^2}{(a^2 + b^2 + e^2)^2} \right\} ,$$

in which one *first* sets $\mu = +m$ and $u = +e\gamma$ in it and *secondly* sets $\mu = -m$ and $u = -e\gamma$ and takes the sum of both values. The electromotive force that the needle exerts upon the entire ring, whose inner and outer radii are a' and a'', respectively, whose height in $2b'$, and which possesses n windings, follows from that to equal eE, where:

$$e = -\frac{n}{2(a'' - a')b'} \cdot \gamma M \pi \int_{a'}^{a''} a^2 da \int_{-b'}^{+b'} \frac{db}{(a^2 + b^2 + e^2)^{3/2}} \left(2 \right.$$
$$\left. + \frac{3}{2}(3a^2 - 2b^2 - 2e^2) \frac{e^2}{(a^2 + b^2 + e^2)^2} \right) ,$$

or

$$e = -\gamma M \cdot 2n\pi \frac{1}{a'' - a'} \left\{ \log \frac{a'' + \sqrt{a''^2 + b'^2}}{a' + \sqrt{a'^2 + b'^2}} \right.$$
$$\left. + \frac{1}{4} \left(\frac{a''^3}{(a''^2 + b'^2)^{3/2}} - \frac{a'^3}{(a'^2 + b'^2)^{3/2}} \right) \frac{e^2}{b'^2} \right\} ,$$

or

$$e = -\frac{2n\pi}{r'} \cdot M\gamma ,$$

when one sets:

$$\frac{1}{r'} = \frac{1}{a'' - a'} \left\{ \log \frac{a'' + \sqrt{a''^2 + b'^2}}{a' + \sqrt{a'^2 + b'^2}} \right.$$
$$\left. + \frac{1}{4} \left(\frac{a''^3}{(a''^2 + b'^2)^{3/2}} - \frac{a'^3}{(a'^2 + b'^2)^{3/2}} \right) \frac{e'^2}{b'^2} \right\}$$

as above.

The current that is induced in the multiplier ring, whose intensity will be denoted by $-\gamma i J$, where $i J$ is the current that would be produced by the electromotive force $2n\pi/r'$,

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once more exerts a rotational moment backwards on the oscillating needle that is expressed in the following way, according to page 428:

$$-\frac{2n\pi}{r'} \cdot M\gamma i \ .$$

If K denotes the moment of inertia of the needle, then that will imply a retardation of the angular velocity γ that is equal to:

$$-\frac{2n\pi}{r'} \cdot \frac{M\gamma i}{K} \ .$$

Finally, if one lets φ denote the small angle that the oscillating needle makes with the magnetic meridian at any moment, so $\gamma = d\varphi/dt$, then the rotational moment that geomagnetism T exerts upon the needle will be equal to:

$$-MT\varphi \ ,$$

and that will cause a retardation of the velocity γ that equals:

$$-\frac{MT}{K}\varphi \ ,$$

to which one adds the part of the retardation that originates in the elasticity of the suspension thread, and which is equal to:

$$-\frac{\vartheta MT}{K}\varphi$$

when ϑ expresses the directive force on the needle that arises from it, in units of its magnetic directive force. The total retardation of the velocity $\gamma = d\varphi/dt$ then amounts to:

$$-\frac{d^2\varphi}{dt^2} = (1+\vartheta)\frac{MT}{K}\varphi + \frac{2n\pi}{r'} \cdot \frac{Mi}{K} \cdot \frac{d\varphi}{dt} \ ,$$

from which it follows that:

$$\varphi = Ae^{-\frac{n\pi Mi}{Kr'}t} \sin t \sqrt{(1+\vartheta)\frac{MT}{K} - \left(\frac{n\pi Mi}{Kr'}\right)^2} \ ,$$

where e is the base for natural logarithms, t is the time that is measured for the needle to go through the meridian once, and:

$$\frac{\pi}{\sqrt{(1+\vartheta)\frac{MT}{K} - \left(\frac{n\pi Mi}{Kr'}\right)^2}}$$

is the period of oscillation of the needle t', and finally:

$$1 : e - \frac{n\pi Mit'}{Kr'}$$

is the ratio of two successive oscillation arcs. The natural logarithm of that ratio or the logarithmic decrement of the decrease in the oscillation arc is:

$$\lambda = \frac{n\pi Mit'}{Kr'} \ .$$

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It follows from that equation that one can determine the intensity iJ of the current in the closed ring B that is produced by the electromagnetic force $e = 2n\pi M/r'$:

$$i = \frac{Kr'}{n\pi Mt'} \cdot \lambda \; ,$$

and it finally follows from this that one can determined the resistance wW of the ring B:

$$w = \frac{e}{i} = \left(\frac{n\pi M}{r'}\right)^2 \cdot \frac{2t'}{K\lambda} \; .$$

That expression for w can be put into another form when one observes that the period of oscillation of the needle is:

$$t' = \frac{\pi}{\sqrt{(1+\vartheta)\frac{MT}{K} - \left(\frac{n\pi Mi}{Kr'}\right)^2}}$$

and

$$\lambda = \frac{n\pi Mi}{Kr'} \cdot t' \; ,$$

from which, one gets that:

$$\frac{MT}{K} = \frac{\pi^2 + \lambda^2}{(1+\vartheta)t'^2} \; ,$$

and when one observes that, in addition, when one sets:

$$\frac{2M}{Tr'^3} = \tan v_0 \; ,$$

v_0 can be determined from the magnetometric deflection experiments by known methods. Upon multiplying those two equations together, one will get:

$$\frac{2M^2}{Kr'^3} = \frac{\pi^2 + \lambda^2}{(1+\vartheta)t'^2} \cdot \tan v_0 \; ,$$

and as a result:

$$w = \frac{n^2\pi^2}{1+\vartheta} \cdot \tan v_0 \cdot \frac{\pi^2 + \lambda^2}{\lambda} \cdot \frac{r'}{t'} \; ,$$

which was to be proved.

E - Rules for calculating the Current that is Induced by a Current with a Sliding Contact

Suppose that one is given a current of constant intensity i that enters into the circular arc ab at a and flows through it to the location b, at which point, it flows through the radius bc to the center c and is conducted back to a from c, [as in Figure 16].[496]

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[496][Note by AKTA:] The letter α in Figure 16 should be replaced by letter a.
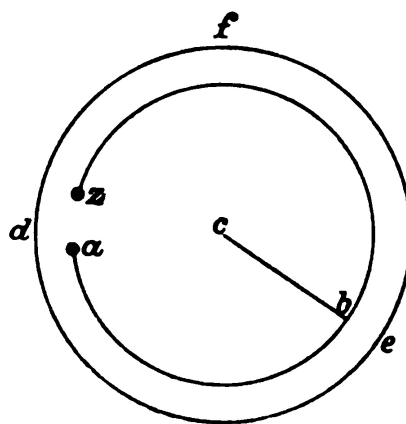
Fig. 16.

One needs to calculate the electromotive force that this current exerts upon one or more concentric circles def while the moving current segment bc describes a circle about c, or more precisely, while the end b of the moving current segment bc traverses the arc abz, which is smaller than the total periphery by the arbitrarily small gap za. One must then distinguish three types of electromotive forces, namely:

1. The electromotive forces that are exerted upon the elements of the moving current segment bc.

2. The electromotive forces that newly-entering current elements exert upon the ends of the arc ab as a consequence of the advance of the moving current segment bc.

3. The electromotive forces that the electricity at the sliding contact b of the arc ab exerts upon the radius bc, or that bc exerts upon ba, as a result of the change in velocity that it experiences.

As far as the *first* type of electromotive force is concerned, from Section 30, p. 367,[497],[498] of the first treatise on *Electrodynamic Measurements*, the electromotive force of an element α of the moving current segment bc on an element α' of the induced conductor def can be determined from the following expression:

$$-\frac{\alpha\alpha'}{r^2}i\left(\sin\vartheta\sin\eta\cos\varepsilon - \frac{1}{2}\cos\vartheta\cos\eta\right)\cdot av\cos\vartheta' \ .$$

The explanation for the symbols that was given in *loc. cit.* is clarified by applying this to the present case. Namely, let C [in Figure 17] be the center of the circular arc $A'A$, through which the inducing current i flows from A' to A, and the moving radius $CA = R$ defines the moving current segment through that current i flows from A to C.

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[497][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 202.
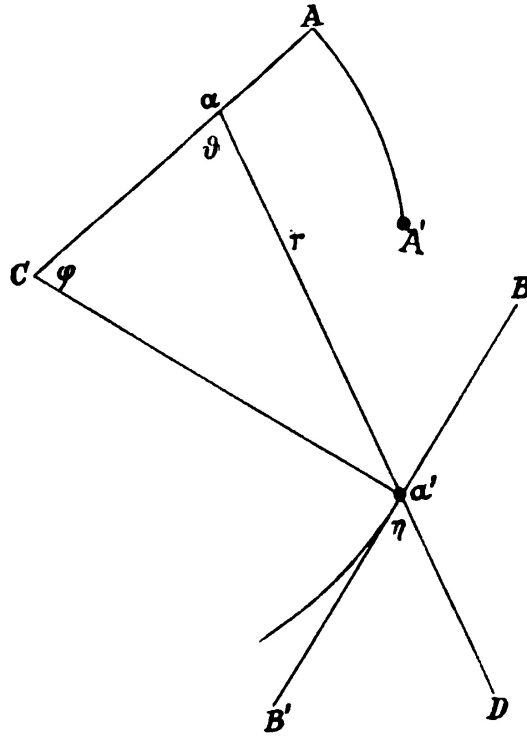[498][Note by AKTA:] See page 192 on Section 5.30.

Fig. 17.

The inducing element α lies along that radius at a distance of ρ from the center C. Let the induced element α' be an element of a concentric circle whose radius is equal to R', and the moving radius CA that defines an angle of $\varphi = AC\alpha'$. Let r be the line that is drawn from α to α', that makes an angle of $\alpha'\alpha C = \vartheta$ with the direction of the current at α, namely, αC. Since the induction depends upon only the relative motion of the two elements α and α', that explains the fact that one can replace the rotation of α around the center C, while α' remains unmoved, with the rotation of α' around that center C with the same value of the arc and the opposite direction, while α remains unmoved. One then assumes that the element α' moves with a velocity v in the direction of the negative tangent $\alpha'B'$. That direction defines an angle of $D\alpha'B' = \eta$ with the lengthened r, i.e., with $\alpha'D$. Moreover, since α' itself is an element of the circle whose direction coincides with the positive tangent to the circle $\alpha'B$, the angle that its direction makes with the lengthened r will be $\vartheta' = \eta + \pi$. Finally, the angle ε between the two planes that are laid through r parallel to the direction of the current at α and parallel to the direction in which α' is displaced, will be $\varepsilon = 0$ when ϑ and η are either both smaller or both larger than π, or $\varepsilon = \pi$ when one of the two angles ϑ, η is smaller, while the other is larger than π. If one sets the extended angle ϑ or η equal to the 2π (as long as they are greater than π), then one will always have $\cos \varepsilon = +1$. One then gets:

$$+\frac{\alpha\alpha'}{r^2}i\left(\sin\vartheta\sin\vartheta' - \frac{1}{2}\cos\vartheta\cos\vartheta'\right) \cdot av\cos\vartheta'$$

for the expression above, in which the value of ϑ is always smaller than π and that of ϑ' is always larger, and furthermore:

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$$r^2 = R'^2 + \rho^2 - 2R'\rho\cos\varphi \ ,$$
$$r\sin\vartheta = R'\sin\varphi \ ,$$
$$r\cos\vartheta = \rho - R'\cos\varphi \ ,$$
$$r\sin\vartheta' = \rho\cos\varphi - R' \ ,$$
$$r\cos\vartheta' = -\rho\sin\varphi \ .$$

If one sets $\alpha = -d\rho$ and $\alpha' = R'd\varphi$, in addition, then one will get the following expression:[499]

$$+\frac{1}{2}avi \cdot R'\sin\varphi^2 d\varphi \cdot \left(1 - \frac{3R'}{r^2}(R' - \rho\cos\varphi)\right)\frac{\rho d\rho}{r^3} \ .$$

If one sets $r^2 = R'^2 + \rho^2 - 2R'\rho\cos\varphi$, then one will find that:

$$\int\left(1 - \frac{3R'}{r^2}(R' - \rho\cos\varphi)\right)\frac{\rho d\rho}{r^3} = -\frac{\rho^2}{r^3} + \text{constant}.$$

The sum of all electromotive forces that all of the elements of the moving current segment from $\rho = R$ to $\rho = 0$ exert upon the induced element α' when one sets $R'^2 + R^2 - 2R'R\cos\varphi = r'^2$, is:

$$+\frac{1}{2}avi \cdot R'R^2 \cdot \frac{\sin\varphi^2 d\varphi}{r'^3} \ .$$

Finally, the sum of the electromagnetic forces over all induced elements of the circle def, i.e., for all elements from $\varphi = 0$ to $\varphi = 2\pi$, is:

$$+\frac{1}{2}avi \cdot R'R^2 \cdot \int_0^{2\pi}\frac{\sin\varphi^2 d\varphi}{r'^3} \ .$$

The product of that expression with the time t is the value of the integral of the sum of the electromagnetic forces for the time interval t or for the path vt that is traversed by the inducing current during that time interval. As a result, if one sets $vt = 2n\pi R'$, i.e., equal to n times the length of the orbit, then one will get the value of the integral of the electromagnetic force for n revolutions of the inducing current, namely:

$$+ai \cdot n\pi R^2 R'^2 \int_0^{2\pi}\frac{\sin\varphi^2 d\varphi}{r'^3} \ .$$

If the induced conductor consists of not just one winding, but m of them whose radii are not appreciably different, then one will get the sum of the electromotive forces that are exerted upon all m windings of the induced conductor by the n windings of the inducing one, namely:

$$+ai \cdot mn\pi R^2 R'^2 \int_0^{2\pi}\frac{\sin\varphi^2 d\varphi}{r'^3} \ ,$$

in which one sets $r'^2 = R'^2 R^2 - 2R'R\cos\varphi$. If one sets $R = kR'$, in which $k < 1$, then one will get:[500]

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[499][Note by AKTA:] The expression $\sin\varphi^2$ should be understood as $\sin^2\varphi$.

[500][Note by AKTA:] The expressions $\cos\varphi^2$, $\cos\varphi^3$ and $\cos\varphi^4$ should be understood as $\cos^2\varphi$, $\cos^3\varphi$ and $\cos^4\varphi$, respectively.

$$\frac{1}{r'^3} = \frac{1}{R'^3}\left\{\frac{1}{(1+k^2)^{3/2}} + \frac{3k\cos\varphi}{(1+k^2)^{5/2}} + \frac{15}{2}\frac{k^2\cos\varphi^2}{(1+k^2)^{7/2}} + \frac{35}{2}\frac{k^3\cos\varphi^3}{(1+k^2)^{9/2}}\right.$$
$$\left. + \frac{315}{8}\frac{k^4\cos\varphi^4}{(1+k^2)^{11/2}} + \ldots\right\},$$

and as a result:

$$\int_0^{2\pi}\frac{\sin\varphi^2 d\varphi}{r'^3} = \frac{\pi}{(1+k^2)^{3/2}R'^3}\left\{1 + \frac{15}{8}\frac{k^2}{(1+k^2)^2} + \frac{315}{64}\frac{k^4}{(1+k^2)^4} + \ldots\right\}.$$

If one again sets k equal to its value R/R' in this and:

$$R_0 = \frac{R^2 R'^2}{(R^2+R'^2)^{3/2}}\cdot\left\{1 + \frac{15}{8}\left(\frac{RR'}{R^2+R'^2}\right)^2 + \frac{315}{64}\left(\frac{RR'}{R^2+R'^2}\right)^4 + \ldots\right\},$$

then one will get the expression:

$$+ai\cdot mn\pi^2 R_0 ,$$

which was to be proved.

As far as the *second* type of electromotive forces are concerned, from Section 30, p. 367,[501],[502] of the first treatise on *Electrodynamic Measurements*, the electromotive force that an element of the immobile current segment α in which the current strength increases by di during the time element dt exerts upon an induced element α' will be determined by the following expression:

$$-\frac{1}{2}\frac{\alpha\alpha'}{r}a\cos\vartheta\cos\vartheta'\cdot\frac{di}{dt} .$$

Now, let α and α' be elements of two circular arcs $A'\alpha$ and $B'\alpha'$, that have a common center C and radii R and R', respectively, so $A'\alpha$ is the immobile segment of the inducing current and αC is the moving segment [in Figure 18].[503]

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[501] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 202.
[502] [Note by AKTA:] See page 192 on Section 5.30.
[503] [Note by AKTA:] I replaced B by B' at the bottom left of this Figure.

Fig. 18.

The angle $\alpha C\alpha' = \varphi$ is the angle that the moving segment defines with the radius of the induced element α'; α is the conducting element that enters the current circuit while the end of the moving current segment advances by $Rd\varphi = \alpha$. The current direction αA in the element α makes and angle of $D'\alpha A + \pi = \vartheta$ with the direction $\alpha\alpha' = r$. The direction $\alpha'B$ of the induced element α' makes an angle of $\alpha\alpha'B + \pi = \vartheta'$ with the direction of the lengthened r, i.e., with $\alpha'D$. If one drops the perpendicular αE from α to $C\alpha'$, and the perpendicular $\alpha'F$ from α' to the lengthened $C\alpha$, then one will have $\alpha\alpha'F = D'\alpha A = \vartheta - \pi$ and $\alpha'\alpha E = \alpha\alpha'B = \vartheta' - \pi$. As a result, the perpendiculars are:

$$\begin{aligned}
\alpha'F &= R'\sin\varphi = r\cos\alpha\alpha'F = -r\cos\vartheta\ , \\
\alpha E &= R\sin\varphi = r\cos\alpha'\alpha E = -r\cos\vartheta'\ .
\end{aligned}$$

That implies that:

$$\alpha\cos\vartheta\cos\vartheta' = \frac{R^2 R'}{r^2}\sin\varphi^2 d\varphi\ .$$

If one substitutes that value in the expression above for the electromotive force, then one will get:

$$-\frac{\alpha'}{2}\frac{R^2 R'}{r^3}\sin\varphi^2 d\varphi \cdot a\frac{di}{dt}\ .$$

If one sets $di/dt = i/t$, in which t denotes time during which the current strength increases from 0 to i in the inducing element $\alpha = Rd\varphi$, then the electromotive force of the current element α that newly enters with current strength i will be the product of the latter expression with the time t, viz.:

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$$-\frac{\alpha'}{2}ai \cdot \frac{R^2 R'}{r^3} \sin\varphi^2 d\varphi \; ,$$

and the sum of the electromotive forces over all newly-entering current elements during one rotation of the moving current segment will be:

$$-\frac{\alpha'}{2}ai \cdot R^2 R' \int_0^{2\pi} \frac{\sin\varphi^2 d\varphi}{r^3} = -\frac{\alpha'}{2}ai \cdot \pi\frac{R_0}{R'} \; ,$$

when one sets:

$$R_0 = \frac{R^2 R'^2}{(R^2 + R'^2)^{3/2}} \cdot \left\{ 1 + \frac{15}{8}\left(\frac{RR'}{R^2 + R'^2}\right)^2 + \frac{315}{64}\left(\frac{RR'}{R^2 + R'^2}\right)^4 + \ldots \right\} \; .$$

That sum will be proportional to the length of any induced element of the circle to which α' belongs. If the induced conductor then forms m windings whose radii are roughly equal to R', so its length will be equal to $2m\pi R'$, then when one substitutes that length for α' in the expression above, one will get the sum of the electromagnetic forces that the newly-entering current elements exert upon the entire induced conductor during one rotation of the moving current segment, or when one multiplies by n, the sum over n rotations of the moving current segment, assuming that the effect of the sudden vanishing of all current elements entering at the end of that rotation is cancelled by closing the induced circuit at that moment. The desired electromotive force will then be:

$$-ai \cdot mn\pi^2 R_0 \; ,$$

which was to be proved.

The *third* type of electromotive forces still remains to be considered, namely, the ones that are exerted by the electricity that goes from immobile current segment to the moving one at the sliding contact as a result of the change of velocity that they experience during that transition. However, as was shown in Section 13.39, all of the elementary effects that are produced in that way prove to be equal to the *second* type of electromotive forces. As a result, that equality is also found to be true for the sum, so it is likewise valid for the expression that was just found.

The total electromotive force over the duration of n rotations of the inducing current is the sum of the three expressions that were found, the first of which is equal and opposite to the last two, and it is then:

$$-ai \cdot mn\pi^2 R_0 \; .$$

Finally, if one lets T denote the duration of n rotations of the inducing current and lets w denote the resistance of the induced conductor, then one will get the following equation for calculating the strength i' of the induced current, in comparison to the strength i of the inducing current that has a sliding contact:

$$\frac{i'}{i} = -\frac{a}{w} \cdot mn\pi^2\frac{R_0}{T} \; ,$$

where the negative sign in the second term means that the direction of the induced current is opposite to the direction of the inducing one, assuming that new elements are always added to the immobile current segment by the rotation of the inducing current. On the other hand,

it is obvious that the second term in the equation would take the opposite value under the inverse rotation of the inducing current, by which elements would be withdrawn from the immobile current segment.

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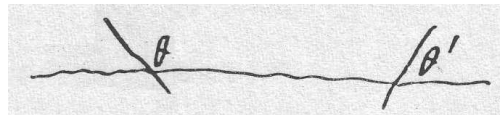Available in PDF format at www.ifi.unicamp.br/~assis

- Page 14, the 1st line should be replaced by:
2.3 Gauss to Weber, 19 March 1845

- Page 15, in the paragraph below the equation, two Figures from Gauss were missing: and that I do not know, whether Ampère (whose Memoire, as I said, I do not have at hand) used the first or the second notation. Both of them signify the same thing, and one uses the first form, when one measures the angle θ, θ' with the same delimited straight line; thus, this line determines the side of the second angle in the opposite way,
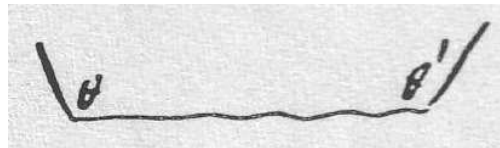
but determines the other form, when one is considering a straight line of indeterminate length, and, for the measurement of angle θ, θ', one resorts to that line twice, in one sense or another.

And, likewise, one can place a $+$ sign in front of the whole formula instead of the $-$ sign, if one is considering as a positive effect, not repulsion, but attraction.

- Page 39, the last paragraph of footnote 83 should be replaced by:
According to this, the voltaic circuit is formed: (1) by the circular arc BB' together with the conductors MN and $M'N'$; (2) by a circuit which consists of the parts RJP and $P'J'R'$ of the apparatus, of the curvilinear conductor which goes from R' to S, and of the battery itself. The latter circuit acts as a closed one, because it is only interrupted by the thickness of the glass plate which isolates the two cups P and P': it is therefore sufficient to observe its action on the circular arc BB' in order to empirically establish the action of a closed current on a circular arc in the various positions which one can give them in relation to each other.

- Page 45, the 10th line of the 2nd paragraph should be replaced by:
2). The two suspension wires are made of copper, 1 meter long, and 1/6 millimeter thick; their distance

- Page 48, the words "derivation", "drainage" or "diversion" should be utilized instead of "deduction" as the translation of the German word "Ableitung" in the context of the 5th line of Section 5.3, footnote 93.

- Page 55, the 10th line should be replaced by:

values of the deflection for the observations in the Table from 11 to 11 are derived from the 11

- Page 60, the 1st line should be replaced by:
which are held in a fixed position 30 mm distant from each other by a 5.5 mm thick brass

- Page 61, the 1st sentence of the 2nd paragraph should be replaced by:
In order to make a system of electrodynamic measurements carried out with this instrument comparable with each other, it was necessary to measure independently the intensity of the current which was passed through the two coils during each measurement.

- The last line of page 74 and the first two lines of page 75 should be replaced by:
this action for both current elements coincides with their connecting-line, and is opposite for the two current elements, it is repulsive, when the above expression has a positive value, attractive in the opposite case.

- Page 77, the 12th line should be replaced by:
which the perpendicular to the plane of the circle n makes with the coordinate axes, then the

- Page 78, the 2nd line should be replaced by:
planes are perpendicular to one another, and the perpendiculars erected on them at their centers intersect

- Page 84, the last line of the 1st paragraph below the Table should be replaced by:
(Section 5.5) in the determination of the dynamometer deflection at a 300-mm distance.

- Page 84, the two last lines of the 3rd paragraph below the Table should be replaced by:
The signs of the calculated values in the foregoing Table thus teach us, that if the fixed coil acts on the bifilar coil at a distance from north or south, a rotation of the bifilar coil results, which, if it amounts to 90 degrees, will cause the currents to go in *opposite* direction around axes aligned in the same way; if, on the other hand, the fixed coil acts at a distance from east or west, there will be a rotation of the bifilar coil, which, if it amounts to 90 degrees, would cause the currents to go about co-directed axes in the *same* direction. The latter also takes place, according to calculation, if the centers of both coils coincide.

- Page 85, the first two lines of the 2nd paragraph should be replaced by:
For the purpose of our measurements, we have, however, based the unit of current intensity on the *electromagnetic principle*, according to which the fundamental unit of current

- Page 87, the 7th line of the footnote should be replaced by:
simple association of these different classes of phenomena can easily be overlooked, which does not immediately

- Page 89, the 2nd line should be replaced by:
and the comparison of the factor thusly determined with that utilized above, namely, with

- Page 91, the 1st sentence of the 3rd paragraph should be replaced by:
The observed dynamometer deflections are measured in *scale divisions* and are therefore to be divided by twice the horizontal distance of the mirror from the scale (= 6612.6 scale divisions) in order to bring them to absolute *angular units*, given the smallness of the angles.

- Page 92, the 8th line from bottom to top should be replaced by:
in which the value of ψ for our case $= 0$, and δ is the complement of the observed displacement

- Page 97, the 4th line of item (c) should be replaced by:
against the bounded conductor; however, if the bounded conductor moves against

- Page 98, the last sentence of the 2nd paragraph should be replaced by:
After these experiments, he ascribes to these induced currents in parallel wires alternately opposite directions; to the first, however, the same direction as the vanishing current of Volta's battery due to the *breaking* of the circuit.

- Page 98, the 2nd line of the last paragraph should be replaced by:
coil is *swinging*, with a voltaic battery, then, in order to *induce* a current in the *other* coil,

- Page 102, the following sentence should be included at the end of the 1st paragraph:
It is also easy to see that the position in the last half of the observations was somewhat greater than in the former.

- Page 116, the 3rd sentence of the 2nd paragraph should be replaced by:
The use of a wet string is, however, not absolutely necessary to this fundamental experiment, but seems to be advantageous only when one wants to apply the electricity *accumulated* in Leyden jars or batteries, and is dispensable if one directly connects the wire ends of the multiplier of a sensitive galvanometer with the positive and negative conductors of an electrical machine. One then also observes the deflection of the needle in the direction predetermined by the electromagnetic laws during the rotation of the electrical machine.

- Page 118, the 3rd line from bottom to top should be replaced by:
relevant, in which a battery of 8 such jars as before was used, and a hemp string of

- Page 126, the 6th line of the 1st paragraph should be replaced by:
the effect, as with an electromagnetic galvanometer, is proportional

- Page 131, the 7th line of the 3rd paragraph should be replaced by:
acting on the *ponderable mass of the current conductor*. Ampère did not deal with the *electrical*

- Page 138, due to a misprint in the original German text, there should be a minus sign in front of Ampère's force on the 3rd line, so that this equation becomes compatible with the way Weber presented it in Sections 8 and 21, namely:

$$-\frac{ii'}{r^2}\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right)dsds' \ ,$$

- Page 148, the 3rd line from bottom to top should be replaced by:
with Ampère's formula, then a third will be derived from them for the same force, likewise

- Page 157, the 7th line from bottom to top should be replaced by:
If you subtract the square of this from the found value of $r_1\frac{d^2 r_1}{dt^2} + \frac{dr_1^2}{dt^2}$, you get:

- Page 163, the 26th line should be replaced by:

obtained as

- Page 166, the 15th line should be replaced by:
the plane of the magnetic axis and of line r, from whose angle with straight line r the sine is to $\sin\psi$

- Page 175, the 4th line from bottom to top should be replaced by:
has an equal value according to both coordinate systems, then

- Page 175, the 2nd line from bottom to top should be replaced by:
which was to be proven. That $zdx - xdz$ has the same value for all right-angled

- Page 176, the 1st line should be replaced by:
evident from the fact that $\frac{1}{2}(zdx - xdz)$ represents the area of the triangle projected on a plane normal to

- Page 179, the 8th line should be replaced by:
or, since according to page[240] 144, $aeu = i$, consequently, because u is variable, $ae{\cdot}du = di$,

- Page 180, the 6th and 7th lines of the 2nd paragraph of Section 5.29 should be replaced by:
which a current element of *constant* intensity i elicits, while the induced element is infinitely removed from a given position parallel to itself in the direction of the straight line r, or, from an

- Pages 183 and 184, "conical surface" should be utilized instead of "cone".

- Page 183, the 4th line of the 1st paragraph should be replaced by:
given element. On this theorem, see Section 5.31 below.

- Page 183, the 11th line from bottom to top should be replaced by:
have to be delimited by the same conical surfaces. And specifically, a current, which goes into α' *from the outer conical surface to*

- Page 184, the 4th line should be replaced by:
through them, it can be easily recognized, that if in A, $\sin(\gamma' - \gamma) = \mp\cos\vartheta'$, then in A',

- Page 184, the 7th line should be replaced by:
It can occur, that in addition to α' and $\mathfrak{a}'$, yet a *third* element of the conductor is bounded

- Page 185, the 4th line from bottom to top should be replaced by:
positions through which it passes and of the speed, with which it runs through it, and merely depends upon the difference in the *potential*

- Page 205, the 3rd line of the 1st paragraph should be replaced by:
translation has been published in 1852, 1966 and 2007.[269] I am maintaining the original title

- Page 210, the 3rd line should be replaced by:
hole d, and projects from it. When the wire is placed upon the reel and the end fixed by

- Page 212, the 3rd line from bottom to top should be replaced by:

wires, since these wires act at b and b' as it were at the ends of a lever, the centre of motion

- Page 219, the 2nd line of the 3rd paragraph should be replaced by:
directed towards any side of a hall or room as required. All the figures are drawn one-fourth of

- Page 220, the following paragraph should be inserted just below the 1st Table:
These observations are reduced so that the former furnish a measure of the electro-dynamic force with which the two conductors of the dynamometer act upon each other, when currents of equal intensity are transmitted through them, whilst the latter furnish a measure of this intensity itself.

- Page 228, the 4th line of the 2nd paragraph should be replaced by:
of the momentary current can be determined from these two observations. This mutual complementarity

- Page 230, the 7th line of the 1st paragraph should be replaced by:
occurrence of the deflection of the dynamometer to one side or the other does not, as in the

- Page 233, the 5th line should be replaced by:
Finally, if, *fourthly*, in the original expression we consider $-\alpha e \alpha' e'$ as the product of the

- Page 235, the last line should be replaced by:
summation of the last four equations, bearing this in mind, we have

- Page 238, the 6th line of the 2nd paragraph should be replaced by:
of application in those cases where the latter is inapplicable; hence its greater utility.

- Page 244, the last line should be replaced by:
element α, upon the induced element α', in the ordinary meaning of the word,

- Page 246, the 3rd line of the 4th paragraph should be replaced by:
element of a conductor that is not moving, or the value of the electromotive force

- Page 246, the 4th line of the 5th paragraph should be replaced by:
consequently the induction for any period of time in which the intensity of the inducing

- Page 257, the 5th line from bottom to top should be replaced by:
currents, whose direction however is not invariable, exist in iron independently of any external

- Page 264, the 1st line should be replaced by:

$$R = -\frac{du}{dN} \ ,$$

- Page 265, the 13th line from bottom to top should be replaced by:
to a neighbouring one is only conditioned by the forces which are exerted upont the particles

- Page 269, the 5th and 6th lines of the 3rd paragraph should be replaced by:

from which it is clear that if the numbers e and i are determined by measurement, the number w is also indirectly obtained without needing a special measurement.

- Page 269, the 10th and 11th lines of the 4th paragraph should be replaced by:

and hence it is clear that if the three numbers f, t, s are determined by measurement, the number e is also thereby indirectly given without necessitating a special measurement.

- Page 270, the 7th and 6th lines from bottom to top should be replaced by:

velocity. But if, by an alteration of the given measures, the standard of velocity is diminished or increased n times, an n times larger or smaller value is obtained for the factor r'/s, and

- Page 272, the 3rd line should be replaced by:

$BC = R$, and φ the angle which the magnetic axis of the needle in C makes with the direction

- Page 273, the 1st line of the 3rd paragraph should be replaced by:

Hence the execution of the measurement of an electric resistance in absolute terms depends on the mea-

- Page 273, the last line of the 3rd paragraph should be replaced by:

is to be determined by measurement according to an absolute standard.

- Page 274, the 7th line should be replaced by:

into rings of large cros-section. In that case, however, the influence of all the windings must

- Page 274, the 5th line of the 2nd paragraph should be replaced by:

a semicircle around its vertical diameter, so that the perpendicular upon the plane of the ring at the commencement and

- Page 274, the 2nd line of the 3rd paragraph should be replaced by:

its plane coincided with the magnetic meridian, and the needle in the middle of the ring B was

- Page 274, the 4th line of the 3rd paragraph should be replaced by:

needle in the middle of the ring B was set in oscillation,[348] and by means of the telescope the

- Page 276, the last line of the 2nd pargraph of Section 11.4 should be replaced by:

to form a single ring which serves at once for inductor and multiplier.

- Page 276, the 4th line of Section 11.4 should be replaced by:

in the *centre* of the ring formed by the induced conductor. The magnet may then either be

- Page 277, the 3rd line should be replaced by:

powerful to cause a strong damping; and also that the length of the needle be very small as compared with the diameter

- Page 285, the 2nd line of item 4. should be replaced by:

expression of the mass of magnetic fluid μ, of the velocity of inducing motion c, of the length

- Page 292, the 34th and 35th lines should be replaced by:

like to further point out that the length of the weighed wire amounted to $25\frac{1}{8}'$ (so $25\frac{1}{8}' = 22.5495\ g$) and that $\frac{3}{4}''$ of it at either end was soldered onto the screws.

- Page 294, the 7th line of Section 13.3 should be replaced by:

that it experiences in its rest position and will not take on a permanent deflection in that way.

- Page 300, the 7th and 8th lines from bottom to top should be replaced by:

B, C, D are then, in fact, likewise proportional to the current strengths, but are also inversely proportional to the strength of the damping.

- Page 304, the last line of the 2nd paragraph should be replaced by:

Absolute resistance measurements have an intrinsic meaning only when they are performed in such a way that absolutely no new units are used as a basis, other than the ones that are present and have already been used and are indispensable for other purposes, such as, for example, those of space and time.

- Page 304 , the last line should be replaced by:

Rather, it will be shown that assuming Jacobi's proposal will also remain the most desirable on practical grounds, because a direct absolute determination of resistance can be performed precisely only in isolated cases and under especially favorable conditions, but accepting Jacobi's proposal creates a bridge by which one will

- Page 307, the 5th, 6th and 7th lines should be replaced by:

any current intensity that is measured with it by iI, in which i denotes a pure number, and one further denotes the unit of electromotive force that was defined above by E, and any electromotive force that is measured with it by eE, in which e denotes a pure number, then

- Page 307, the 11th and 12th lines from bottom to top should be replaced by:

plane of those circles. However, two more wires belong to that circuit that lie close to each other, are insulated from one another, and exhibit a double connection between the two circles. Finally, let each circle be cut

- Page 308, the 6th line from bottom to top should be replaced by:

way from that velocity using the known laws of oscillation by multiplying it by the period

- Page 313, the 5th line of Subsection 13.13.3 should be replaced by:

length of the needle, which was not even a tenth of the diameter of the multiplier,

- Page 313, the 1st line of Section 13.14 should be replaced by:

The following observations were made using these instruments. The inductor was

- Page 313, the 5th line of Section 13.14 should be replaced by:

the scale, which it attained after half an oscillation period,[400] would be observed by the

- Page 316, the 1st line of footnote 401 should be replaced by:

The rotation of the inductor, given its size, could not be accomplished so quickly that its duration would be negligible against the period of oscillation of the needle.

- Page 316, the penultimate line of footnote 401 should be replaced by:
$(1 + 2/3t^2)\alpha$. Now, since the total oscillation arc $2\alpha = 79.4$ millimeters, and one had $t = 10.2818$, the value of

- Page 317, the 3rd line from bottom to top should be replaced by:
distribution to the extent that it seemed necessary. Namely, a small compass was

- Page 318, the 2nd and 3rd lines should be replaced by:
deflections v_3 and v_4 were observed when the needle was displaced parallel to itself at an equal distance on the opposite side from the compass, and the value of:

- Page 318, the 6th and 7th lines should be replaced by:
the compass and for different directions of the line that went through the middle of the needle and the compass, namely, at distances of 400, 500, and 600 millimeters when the

- Page 319, the 1st sentence should be replaced by:
If these experiments are to be used as a basis for calculating the resistance of the circuit in absolute units, some complication arises from the fact that even with the moderate dimensions of the needle compared to the diameter of the damper, the distribution of the magnetism in the needle must not be completely disregarded.

- Page 319, the 13th line of the 2nd paragraph should be replaced by:
equipped with a frame for attaching the mirror and for hanging it on a thread in the

- Page 319, in the 3rd paragraph, the word "compass" should be used instead of the word "galvanometer".

- Page 320, the 7th line from bottom to top should be replaced by:
moved from the distance $BC = R$ into the center of the circle B itself, which will increase the

- Page 321, the 15th line should be replaced by:
ideal distribution of magnetism,[406] the length e' must be taken into account, which is done by replacing $1/r''$ with

- Page 322, in the 4th line from bottom to top we should have a comma "," instead of the full point ".", namely:

$$\vartheta = \frac{1}{1770} \; ,$$

- Page 324, the 10th and 11th lines of Section 13.18 should be replaced by:

$$\lambda = 0.064\,445 \; ,$$

in natural logarithms.

- Page 325, in the 1st paragraph, the word "compass" should be used instead of the word "galvanometer".

- Page 325, in the last paragraph, the word "compass" should be used instead of the word "galvanometer".

- Page 327, the 3rd line of Section 13.19 should be replaced by:
method. The essential difference from the second series of experiments was therefore only that

- Page 327, the 4th line from bottom to top should be replaced by:
The elasticity of the suspension thread could be neglected, since it did not increase the

- Page 329, the 12th line should be replaced by:

$$= 1854^2 \cdot \pi^2 \cdot 0.05054 \cdot \frac{\pi^2 + 0.007944^2}{0.007944} \cdot \frac{352.71}{3.9527} \, ,$$

- Page 329, the 15th line should be replaced by:
The difference between this value and that derived from the second series of experiments

- Page 342, the 5th line should be replaced by:
same area as the first one, is perpendicular to it, and is bisected by it at a great distance

- Page 343, the 11th line from botto to top of the footnote should be replaced by:
denote the distances of the elements ds and ds' and the point of intersection of the perpendicular x

- Page 350, the 4th and 5th lines from bottom to top should be replaced by:
electromotive forces extend over the entire closed circuit and act proportionally to the resistances in all parts. It is only for a nonuniform action of the given electromotive forces in the

- Page 353, the 4th line should be replaced by:
the basic law of electrostatics, the force that acts from H to E in the tangential direction

- Page 353, the 7th line should be replaced by:
while the force that acts from G to E in the same direction is equal to:

- Page 361, the 5th line of the 1st paragraph should be replaced by:
electromotive force equal to a shall be given for a single point of it.

- Page 361, the 4th line of the 2nd paragraph should be replaced by:
electromotive forces at the two points A and B are balanced. If $+e$ then denotes the

- Page 361, the 5th line of the 3rd paragraph should be replaced by:
consequently for the requested balancing:

- Page 361, the 5th and 6th lines of the 4th paragraph should be replaced by:
balanced. That is because if $\pm e_1$ denotes the free electricity at (A^1), (A_1), and $\pm e_2$ denotes the free electricity at (A^2), (A_2), etc., and r is the radius of the circle, and one sets:

- Page 366, the 1st line of Section 13.32 should be replaced by:
It is clear that when electromotive forces are given only at the individual points

- Page 366, the 7th line of Section 13.32 should be replaced by:

in front of them persists in a state of rest, then that displacement of the positive fluid from

- Page 373, the 1st line should be replaced by:
for the free positive electricity in the other half, in which, from page 370:

- Page 373, the 21st line should be replace by:
intensity eu, where e is the mass of the positive or negative electricity that is included in a piece of the

- Page 374, the 2nd line of the last paragraph should be replaced by:
for the unit of its length and cross-section and indeed in the units that were defined in Section 13.27.

- Page 378, the 4th line should be replaced by:
upon their magnitudes, distance, relative velocity, and the change in it. Meanwhile, for the sake of

- Page 378, the 12th line should be replaced by:
mass in question around A will occur as soon as an electromotive force parallel to the line AB

- Page 378, the 27th and 28th lines should be replaced by:
likewise assumed to be mobile and, in addition to the central force exerted by the negative masses in whose neighborhoods they are found, were subjected to the perturbing influence of the

- Page 378, the 31st line should be replaced by:
If only the electromotive force c acts upon the negative mass considered, then it would

- Page 380, the 3rd and 4th lines of Section 13.37 should be replaced by:
tion 26 of the firste treatise on *Electrodynamic Measurements*.[454] That treatise, which was not yet printed at that time, could be cited there only in the form of the abstract that appeared in Poggendorff's

- Page 380, the 5th line of the 4th paragraph of Section 13.37 should be replaced by:
electricity in one element on the two electricities in the other element. That

- Page 384, the 3rd line from the bottom to the top should be replaced by:
ring from which it returned to the rod. The current then split between those two paths,

- Page 386, the 16th line should be replaced by:
divisions if each rotation were to generate an electromotive force that would be equal to the one

- Page 387, the 20th line should be replaced by:
produced a permanent deflection of the magnetometer needle of 377 scale divisions, although a

- Page 390, the 4th line should be replaced by:
derivation of the law of voltaic induction from the basic general law of electric action in Section

- Page 395, the 9th and 10th lines from bottom to top should be replaced by:

then that will give the *electromotive* force that is exerted upon the induced element α' during the time element dt by the electricity that flows through the sliding contact, which

- Page 398, the 4th line should be replaced by:

for (1), (2), and (3), in which m denotes the number of induced circles, and n denotes

- Page 404, the 4th line of Section C should be replaced by:

the deflection of the needle had not been measured until it has come to rest, then the deflection

- Page 405, the 6th line from bottom to top should be replaced by:

arc of oscillation when the circuit to which the multiplier belongs is open. Namely, when

- Page 406, the 10th and 11th lines from bottom to top should be replaced by:

From this, the *variable* part of the damping is determined if one knows the fraction that the resistance of the multiplier forms from that of the whole circuit. If

- Page 408, the 3rd line of footnote 480 should be replaced by:

for the position of the oscillating needle that equals x at the end of a time equal to t, where T denotes the

- Page 408, the 15th line of footnote 480 should be replaced by:

and if the position of the needle up to that starting point were taken to be the elongation x, so $x = 0$ for $t = 0$,

- Page 409, the 5th line of footnote 481 should be replaced by:

If one then calculates the time t from the moment when the instantaneous current acts upon the needle and

- Page 409, the 7th line of footnote 481 should be replaced by:

or $A = C\tau/\pi$. If one then sets the original position of the needle $p = 0$, to simplify, then one will get:

- Page 413, the 3rd line of the 2nd paragraph should be replaced by:

passes its original position for the first time, an instantaneous current will again act upon it

- Page 413, the 16th line of the 2nd paragraph should be replaced by:

are thus observed are then arranged in pairs of alternating positive and negative elongations,

- Page 417, the 5th and 6th lines of the 2nd paragraph should be replaced by:

cylinder, while the *second* copy consisted of a varnished wire that was wound around a varnished glass tube. The experiments split into five sets. The ends of the inductor wire

- Page 422, the 2nd and 3rd lines of item 1. (that is, the 2nd and 3rd lines of the 4th paragraph) should be replaced by:

when two arbitrarily-placed magnetic elements μ and μ' exert equal and equally directed electromagnetic forces on a current element $ds = \alpha i$, then their electromotive forces on

- Page 422, the last line of item 1. (that is, the last line of the 4th paragraph) should be replaced by:

From this it follows that if the Earth's magnetism in one place exerts the same electromagnetic force in the same direction as a distant magnetic bar, the electromotive force of the Earth's magnetism on an inductor moving there is also equal to that of the magnetic bar, no matter how the magnetism of the Earth might be distributed.

- Page 424, the 5th line below Equation (I) should be replaced by:

to the plane of the circle, to be equal to the force that is exerted by a magnet in the same

- Page 427, the 1st line of item (5) should be replaced by:

5. The comparison of the torque that the multiplier exerts upon the needle that

- Page 431, the first 3 lines should be replaced by:

Furthermore, if one denotes the integral value of the current intensity iJ over the time interval from the moment when $\psi = \pi$ to the moment when $\psi = 0$ by $i'J$, then the integral value of the acceleration over the same time interval — i.e., the angular velocity that

- Page 432, the 2nd, 3rd and 4th lines should be replaced by:

lies from the center of the ring B. Consequently the needle magnetism is

$$M = 2em ,$$

then if γ is the angular velocity of the needle, for

- Page 433, the 21st and 22nd lines should be replaced by:

where e is the base for natural logarithms, t is the time counted from one passage of the needle through the meridian, and:

- Page 434, the 2nd line should be replaced by:

closed ring B that is produced by the electromotive force $e = 2n\pi M/r'$:

- Page 435, the 2nd paragraph should be replaced by:

1. The electromotive forces that are exerted by the elements of the moving current segment bc.

- Page 436, the 2nd and 3rd lines should be replaced by:

the induced element α' be an element of a concentric circle whose radius is equal to R', and let the movable radius CA form an angle $\varphi = AC\alpha'$ with the radius through α'. Let r be the line that is

- Page 437, the 10th line should be replaced by:

The sum of all electromotive forces that all of the elements of the moving current segment

- Page 437, the 14th line should be replaced by:

the electromotive forces for the time interval t or the path vt that is traversed by

- Page 437, the 17th line should be replaced by:

electromotive force for n revolutions of the inducing current, namely:

- Page 440, the 10th line should be replaced by:

expression above, one will get the sum of the electromotive forces that the newly-entering

- Page 440, the 3rd, 4th and 5th lines of the 2nd paragraph should be replaced by:

one at the sliding contact as a result of the change of velocity that it experiences during that transition. However, as was shown in Section 13.39, all of the elementary actions resulting from this are equal to those resulting from the *second* type of electromotive forces. As

This is the second of 4 volumes of the book "Wilhelm Weber's Main Works on Electrodynamics Translated into English".

This second Volume begins with the text of the Gauss-Weber correspondence of 1845. It is related to Ampère's force between current elements and Weber's ideas about the unification of the electrostatic and electrodynamic laws. It is followed by a paper by Fechner published in 1845 in which he presented some qualitative ideas in the same direction. That is, to unify Ampère's force and Faraday's law of induction with electrostatics. To this end he suggested a force depending not only on the distance between the interacting electrified particles, but also on their velocities. At the end of his paper, Fechner mentioned that his work might be seen as a forerunner of Weber's investigations.

Then comes Weber's First major Memoir on Electrodynamic Measurements, published in 1846. This work is probably Weber's most important publication. He introduced his bifilar electrodynamometer with which he could measure currents with high precision. Initially he utilized this instrument in order to prove Ampère's force. He then utilized Ampère's force between current elements in order to deduce his own force law between electrified particles. Weber's force between two electrified particles depends not only on their distance, but also on their relative velocity and relative acceleration. He showed that it was possible to unify the laws of Coulomb, Ampère and Faraday with his force law.

This Volume contains also Weber's 1848 paper in which he introduced his velocity dependent potential energy. It is followed by Kirchhoff's 1849 paper on a deduction of Ohm's law in connection with the theory of electrostatics.

This Volume finishes with Weber's Second major Memoir on Electrodynamic Measurements (1852). The main focus of this work was the absolute measure of resistance. Weber's Memoir contains also his pioneering calculation of the distribution of charges along the surfaces of resistive conductors carrying steady currents. In particular, he considered a cylindrical straight conductor and a resistive ring.

About the Editor: Prof. Andre Koch Torres Assis has been working on Weber's law applied to electromagnetism and gravitation for more than 30 years: https://www.ifi.unicamp.br/~assis