Weber's Electrodynamics

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Maxwell's equations (1861-64)

Gauss's law

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

There are no magnetic monopoles

$$\nabla \cdot \vec{B} = 0$$

Faraday's law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

"Ampère's" law with displacement current

$$\nabla \times \vec{B} = \mu \, \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

Lorentz's force (1895)

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Wilhelm Weber (1804 – 1891)

J. C. Maxwell (1831 – 1879)

1831-1843 in Göttingen with Gauss

1843-1849 in Leipzig

1849-1891 in Göttingen

Weber in 1846:

Coulomb (1785)

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2}$$

Ampère (1822)

$$\vec{F} = -\frac{\mu_0}{4\pi} I_1 I_2 \frac{\hat{r}}{r^2} f(\alpha, \beta, \gamma)$$

Faraday (1831)

$$emf = -M \frac{dI}{dt}$$

Idea: $Id\vec{\ell} \Leftrightarrow q\vec{v}$

Weber's force

$$\vec{F} \approx \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} \left(1 + k_1 v_1 v_2 + k_2 a_{12}\right)$$

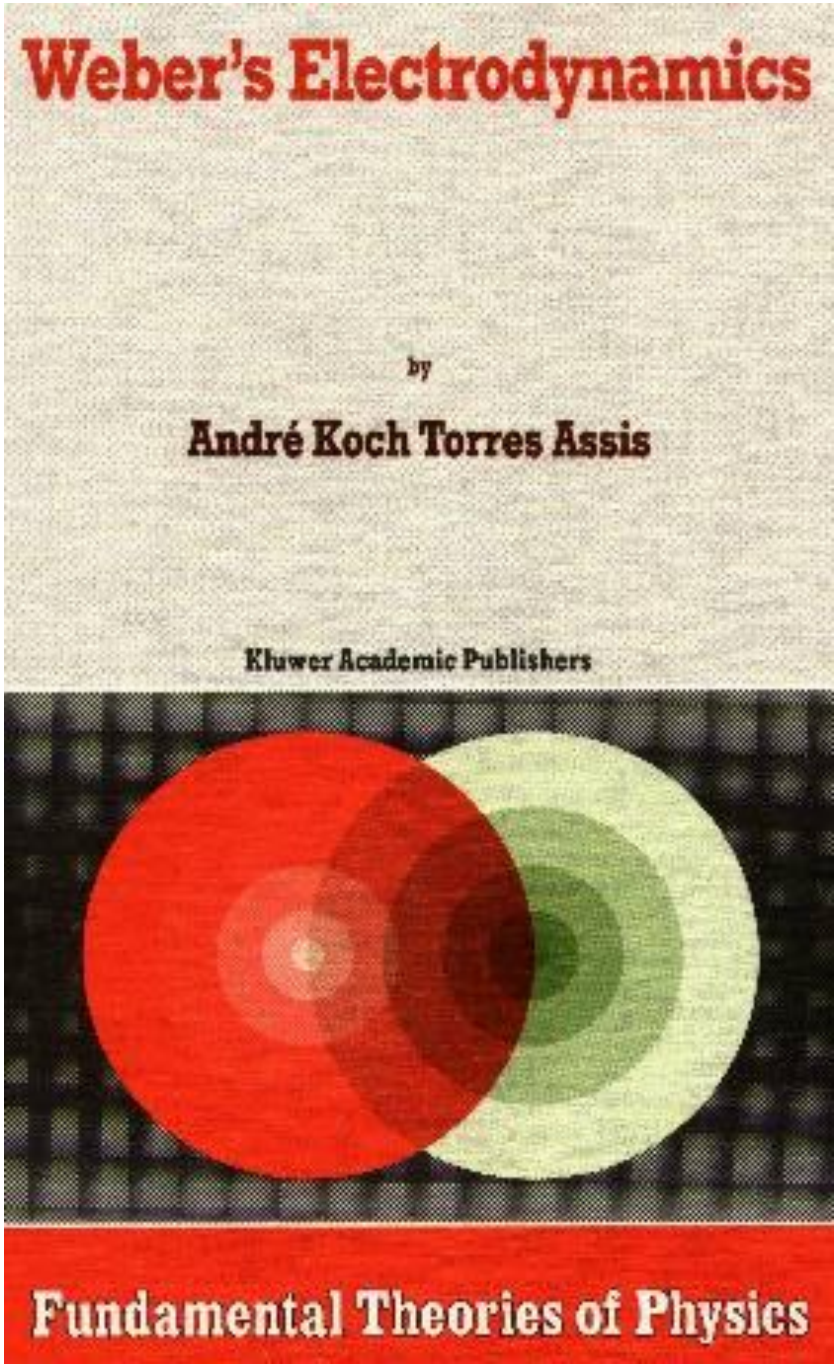
Weber's force

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2}\right)$$

$$\dot{r} = \frac{dr}{dt}$$

$$\ddot{r} = \frac{d^2 r}{dt^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s}$$


Weber's Electrodynamics
by
André Koch Torres Assis
Kluwer Academic Publishers
Fundamental Theories of Physics

Kluwer
1994

Properties of Weber's force

- In the static case ($dr/dt = 0$ and $d^2r/dt^2 = 0$) we return to the laws of Coulomb and Gauss.
- Action and reaction. Conservation of linear momentum.
- Force along the straight line connecting the particles. Conservation of angular momentum.
- It can be derived from a velocity dependent potential energy:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}\left(1 - \frac{\dot{r}^2}{2c^2}\right)$$

- Conservation of energy: $$\frac{d(K+U)}{dt} = 0$$

- Faraday's law of induction can be derived from Weber's force (see Maxwell, *Treatise*, Vol. 2, Chap. 23).
- Ampère's circuital law can be derived from Weber's force.
- It is completely **relational**. It depends only on r, dr/dt and d^2r/dt^2. It has the same value to all observers and to all systems of reference. It depends only on magnitudes intrinsic to the the system of interacting charges. It depends only on the relation between the bodies.

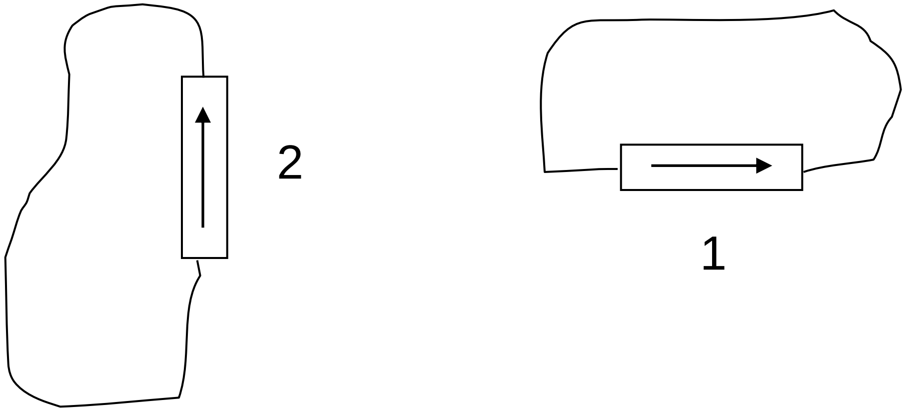
Weber $\rightarrow$ Ampère's force (1822)

$$\vec{F}^{A} = -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}\left[2(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - 3(\hat{r}\cdot d\vec{\ell}_1)(\hat{r}\cdot d\vec{\ell}_2)\hat{r}\right]$$

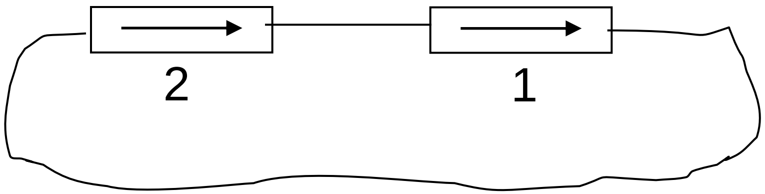
Lorentz $\rightarrow$ Grassmann's force (1845)

$$\vec{F}^{G} = I_1 d\vec{\ell}_1 \times d\vec{B}_2 = I_1 d\vec{\ell}_1 \times \left(\frac{\mu_0}{4\pi}\frac{I_2 d\vec{\ell}_2 \times \hat{r}}{r^2}\right)$$

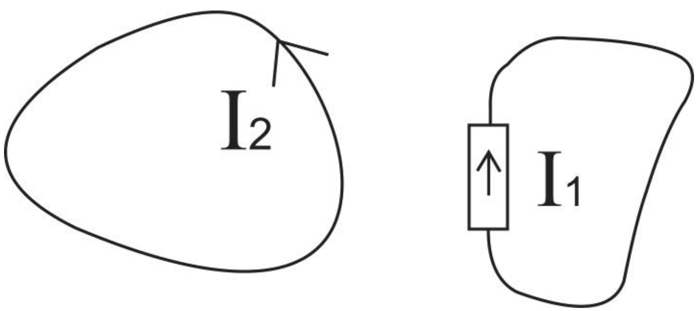
$$= -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}\left[(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - (d\vec{\ell}_1 \cdot \hat{r})d\vec{\ell}_2\right]$$



	$F_{2\text{ in }1}$	$F_{1\text{ in }2}$
A	0	0
G	↑	0

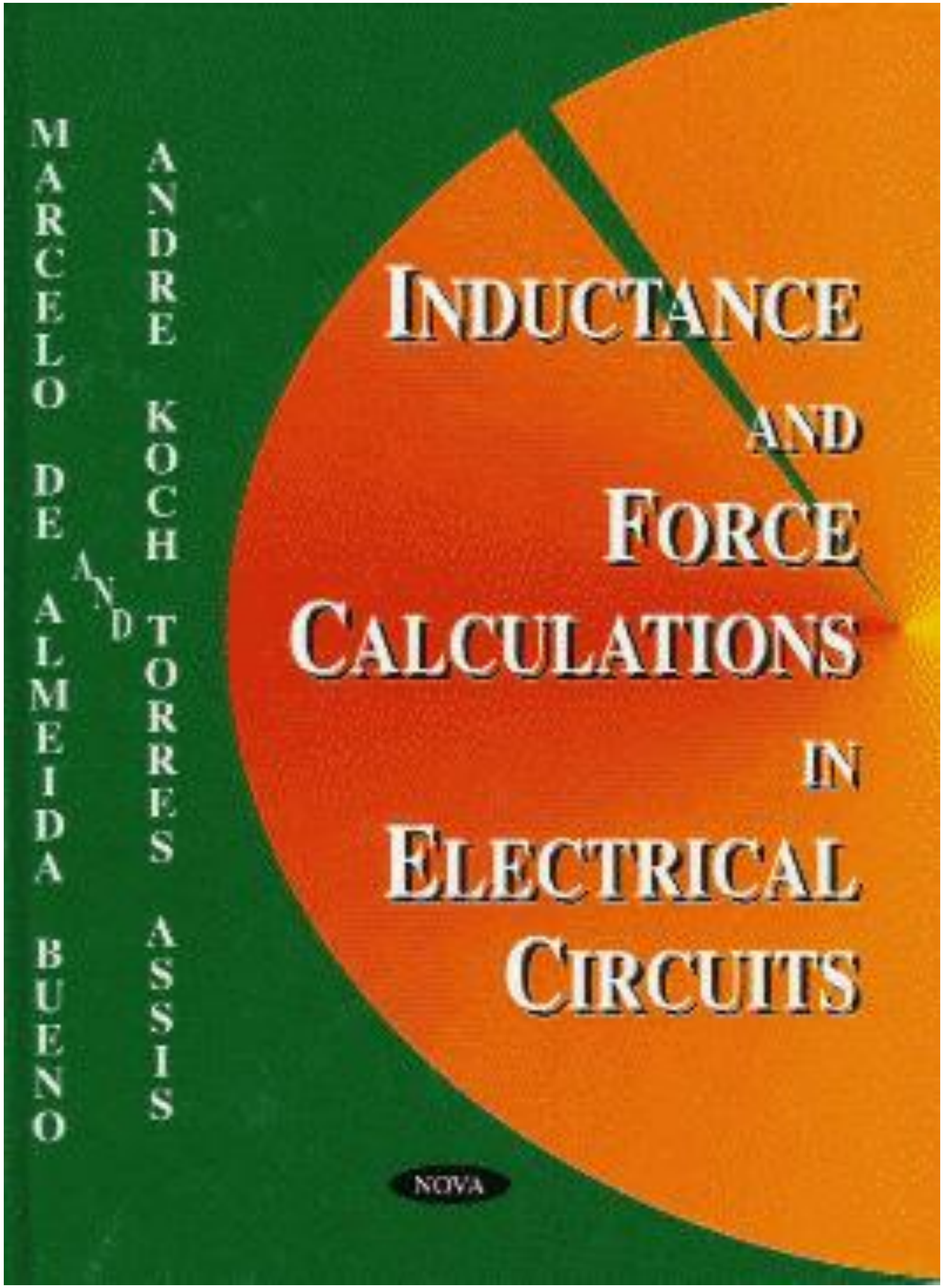


	$F_{2\text{ in }1}$	$F_{1\text{ in }2}$
A	→	←
G	0	0



$$F^{A}_{2\,\mathrm{in}\,1} = F^{G}_{2\,\mathrm{in}\,1} =$$

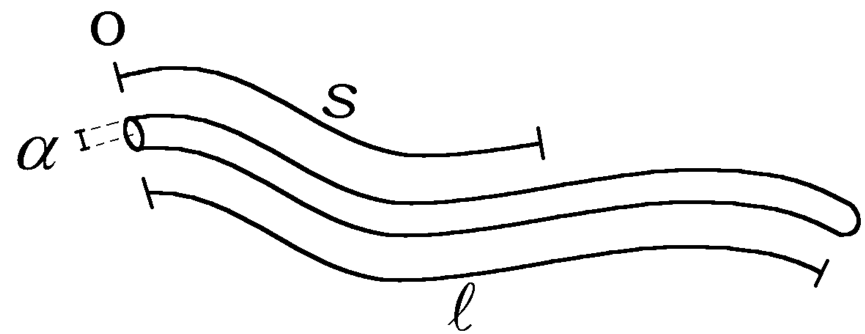
$$I_1 d\vec{\ell}_1 \times \left(\frac{\mu_0}{4\pi} \oint \frac{I_2 d\vec{\ell}_2 \times \hat{r}}{r^2} \right)$$



Nova Publishers
2001

Propagation of electromagnetic signals (Weber and Kirchhoff, 1857)

$$\vec{J} = g\vec{E} = -g\left(\nabla\phi + \frac{\partial \vec{A}}{\partial t}\right)$$



$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\frac{\partial^2 \xi}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} = \frac{2\pi\varepsilon_0 R}{\ell \ln \frac{\ell}{\alpha}}\frac{\partial \xi}{\partial t}$$

with $\xi = I, \sigma, \phi, A$

Weber versus Lorentz

Weber's force
$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2}\right)$$

Lorentz's force
$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

Weber versus Lorentz

$$\vec{F}^{\,Weber}_{2\text{ in }1} = \vec{F}(v_1, v_2, a_1, a_2) =$$

$$\frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left\{1 + \frac{(\vec{v}_1 - \vec{v}_2)\cdot(\vec{v}_1 - \vec{v}_2)}{c^2} - \frac{3\left[\hat{r}\cdot(\vec{v}_1 - \vec{v}_2)\right]^2}{2c^2} + \frac{\vec{r}\cdot(\vec{a}_1 - \vec{a}_2)}{c^2}\right\}$$

$$\vec{F}^{\,\text{Lorentz}}_{2\text{ in }1} = q_1\vec{E} + q_1\vec{v}_1 \times \vec{B} = \vec{F}(v_1, v_2, a_2) =$$

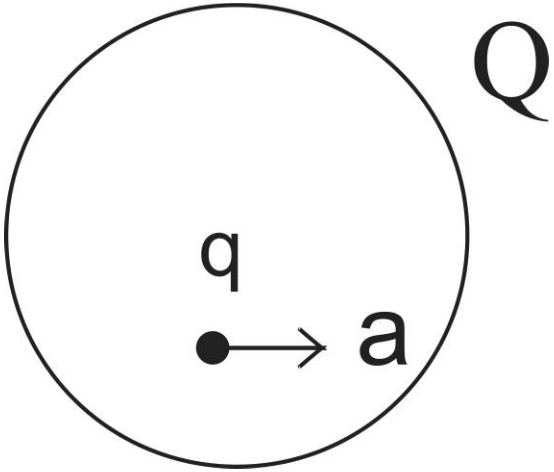
$$q_1\left\{\frac{q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\left[\left(1 + \frac{v_2^2}{2c^2} - \frac{3(\hat{r}\cdot\vec{v}_2)^2}{2c^2} - \frac{\vec{r}\cdot\vec{a}_2}{2c^2}\right)\hat{r} - \frac{r\,\vec{a}_2}{2c^2}\right]\right\}$$

$$+\ q_1\vec{v}_1 \times \left\{\frac{q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\frac{\vec{v}_2 \times \hat{r}}{c^2}\right\}$$

Force of a uniformly charged insulating spherical shell upon an internal accelerated test body

$$\vec{F}^{\,Lorentz} = q\vec{E} + q\vec{v} \times \vec{B} = 0$$

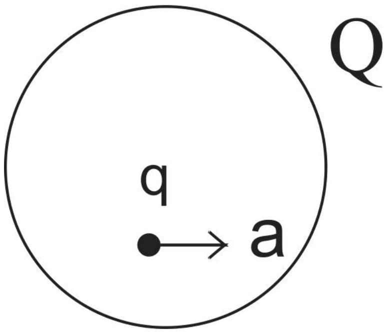
$$\vec{F}^{\,Weber} = \frac{\mu_0 q Q}{12\pi R}\vec{a}$$



Assis, J. Phys. Soc. Japan, Vol. 62, p. 1418 (1993), Changing the inertial mass of a charged particle.

Equation of motion for an electron accelerated inside a uniformly charged spherical shell

Lorentz $$\vec{F} = m\vec{a}$$



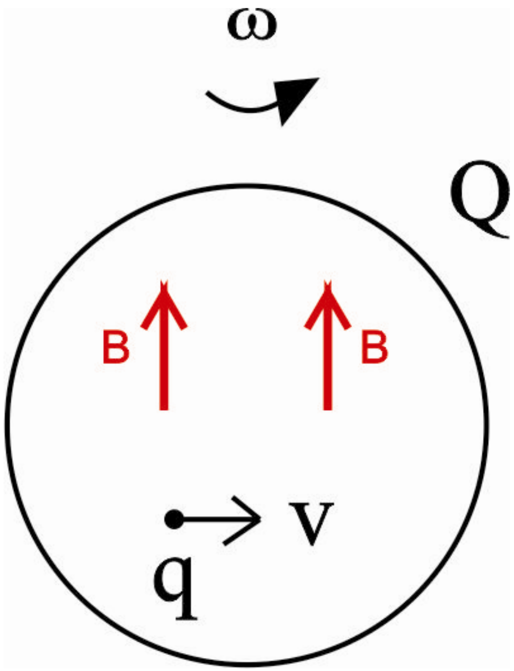
Weber $$\vec{F} + \frac{\mu_0 q Q}{12\pi R}\vec{a} = m\vec{a}$$

According to Weber's electrodynamics, the electron should behave as if it had an effective inertial mass depending upon the surrounding charges:

$$m_{effective} = m - \frac{qV}{3c^2}$$

This means that we can double the effective mass of an electron with a potential of 1.5 MV.

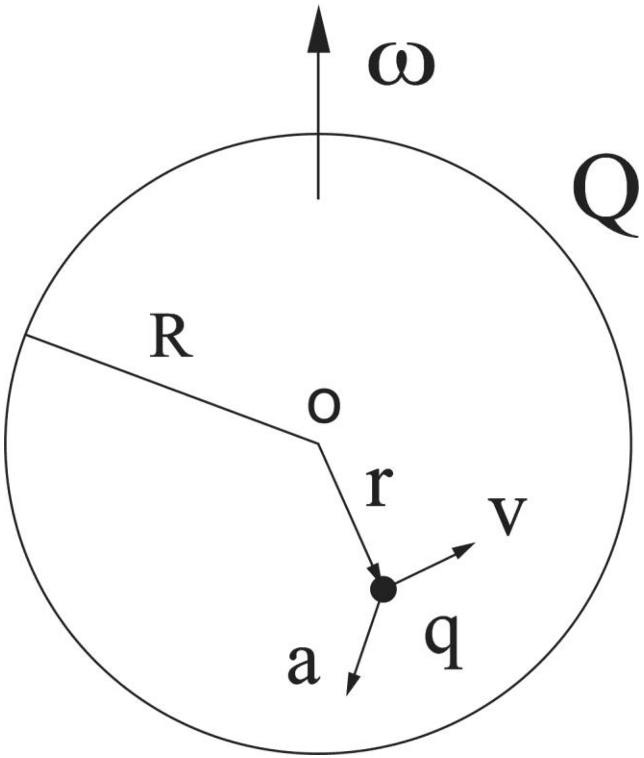
Force of a uniformly charged and spinning spherical shell upon an internal test body



$$\vec{F}^{\,Lorentz} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$= q\vec{v} \times \frac{\mu_0 Q \vec{\omega}}{6\pi R}$$

Force of a uniformly charged and spinning spherical shell upon an internal test body



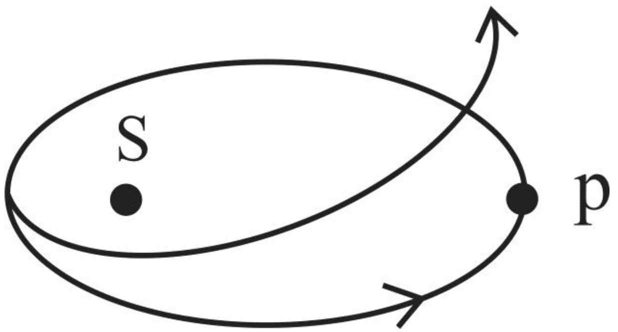
$$\vec{F}^{\,Lorentz} = q\vec{v} \times \frac{\mu_0 Q \vec{\omega}}{6\pi R}$$

$$\vec{F}^{\,Weber} = \frac{\mu_0 q Q}{12\pi R}\left[\vec{a} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + 2\vec{v} \times \vec{\omega}\right]$$

Evidences for a component of the force depending upon the acceleration of the test body

Schrödinger derived the precession of the perihelion of the planets utilizing Weber's potential energy for gravitation, Annalen der Physik, V. 77, p. 325 (1925):

"The presence of the Sun has, in addition to the gravitational attraction, also the effect that the planet has a somewhat greater inertial mass radially than tangentially."

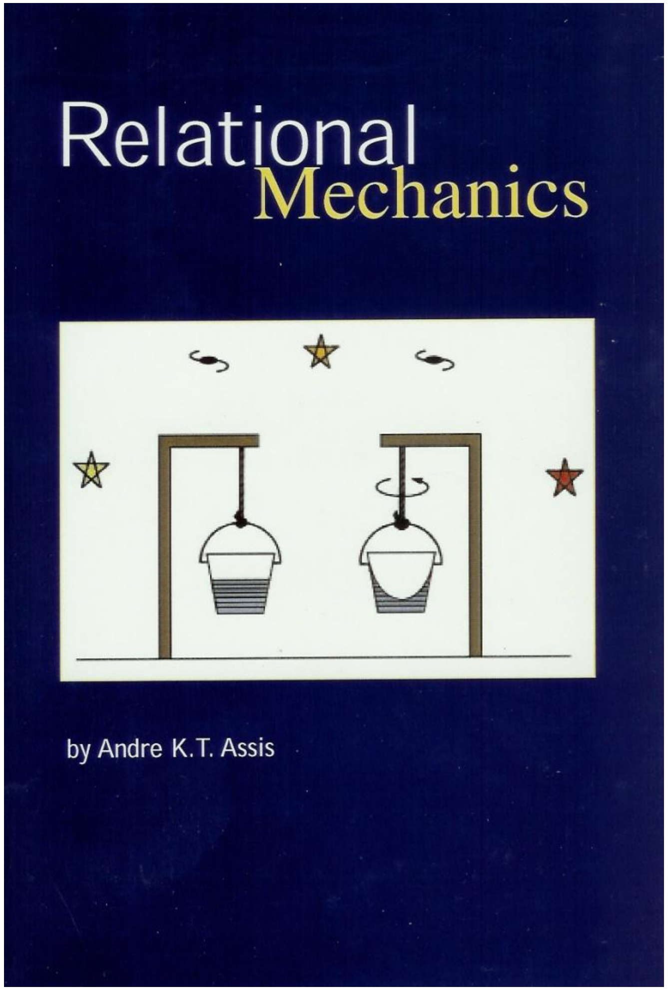


Relational Mechanics

Based upon a Weber's force law for gravitation.
It leads to a mathematical implementation of Mach's principle:

- The inertial mass is due to a gravitational interaction between the test body and the distant galaxies. It is derived the proportionality between inertial and gravitational masses.

- All inertial forces (– ma, centrifugal, Coriolis) are real interactions, due to a relative acceleration between the test body and the distant galaxies.



Relational Mechanics, A. K. T. Assis
(Apeiron, Montreal, 1999)

V. F. Mikhailov published an experiment showing an effective inertial mass of test electrons depending upon the surrounding charges:

- The action of an electrostatic potential on the electron mass:
Ann. Fond. Louis de Broglie, Vol. 24, p. 161 (1999).
A neon glow lamp RC-oscillator placed inside a glass sphere of radius 5 cm having In-Ga plating charged up to 2 kV. The oscillation frequency of the lamp depended upon the potential of the shell according to Weber's law.

Junginger and Popovic did not confirm these results:

- An experimental investigation of the influence of an electrostatic potential on electron mass as predicted by Weber's force law:
Can. J. Phys., Vol. 82, p. 731 (2004).

However, instead of a coated glass shell, which may have worked as a charged insulator in Mikhailov's experiment, they utilized a conductive enclosure foil 40 X 40 X 40 cm^3 .

Mikhailov published two other experiments confirming his earlier results:

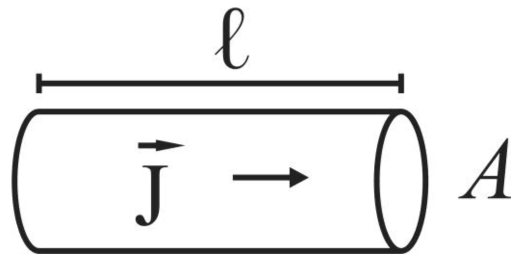
– Ann. Fond. Louis de Broglie, Vol. 26, p. 33 (2001). Frequency of a Barkhausen-Kurz generator (flux of mobile electrons) depending upon the voltage of the surrounding glass sphere.

– Ann. Fond. Louis de Broglie, Vol. 28, p. 231 (2003). Oscillation frequency of a neon glow lamp inside two spherical concentric shells. The internal shell is connected to the circuit of the generator and may be connected either to the earth or to a source of high voltage by a switch. The external shell is connected to a source of high voltage which may be changed at will. It is observed that the oscillation frequency of the lamp depends upon the voltage of the shell.

The self-inductance of a circuit may be interpreted as being due to an effective inertial mass of the conduction electrons due to their acceleration in relation to the positive lattice of the metal.

Assis, Circuit theory in Weber electrodynamics, Eur. J. Phys., Vol. 18, p. 241 (1997)

RL circuit:



$$V = L\frac{dI}{dt} + RI$$

Newton's second law

$$F = ma = qE - bv$$

with $a = 0$

$$E\ell = \frac{b\ell}{q}v = \frac{b\ell}{q\rho\, A}\cdot \rho\, Av$$

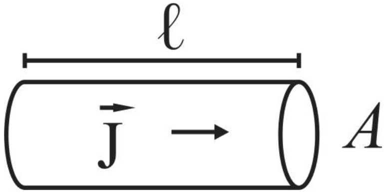
$V = RI$ with $R = \frac{b}{q\rho}\frac{\ell}{A} = r\frac{\ell}{A}$

with $a \neq 0$

$$V = RI + \frac{m\ell}{q\rho\, A}\frac{dI}{dt}$$

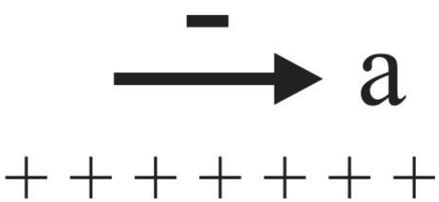
but $\frac{m\ell}{q\rho\, A} \approx 10^{-16}H$

while $L = \frac{\mu_0 \ell}{2\pi}\ln\frac{2\ell}{d} \approx 10^{-6}H$



Newton’s second law with Weber’s force

$$F = ma = F_W - bv \qquad \vec{F}_W = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2}\right)$$

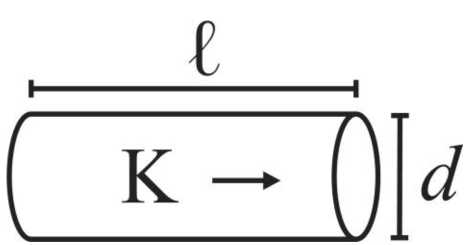


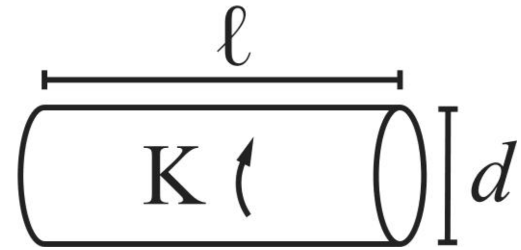
$$F = ma = qE - bv - \left(\frac{\mu_0 q\rho\, d^2}{8}\ln\frac{2\ell}{d}\right)a$$

$$E\ell = \frac{b\ell}{q}v + \left(m_W + m\right)\frac{\ell}{q}a$$

where $m = 9\times 10^{-31} kg$ and $m_W \approx 10^{-20} kg >> m$

$$V = RI + L\frac{dI}{dt}$$ with $$m_W = \frac{q\,\rho\,A}{\ell}L$$

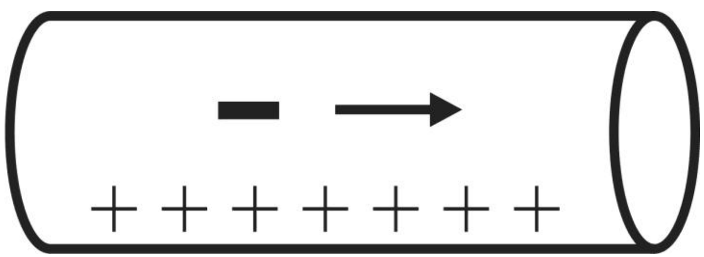


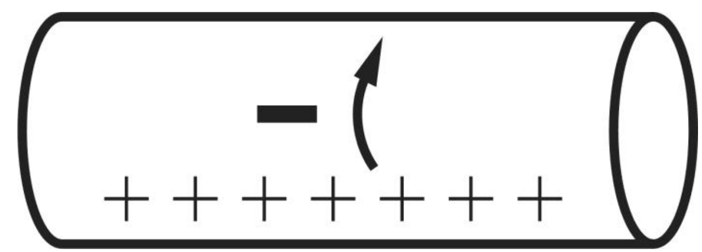


$$V = RI + L\frac{dI}{dt}$$

$$L = \frac{\mu_0 \ell}{2\pi} \ln \frac{2\ell}{d}$$

$$L = \frac{\mu_0 \pi d^{\,2}}{4\ell}$$

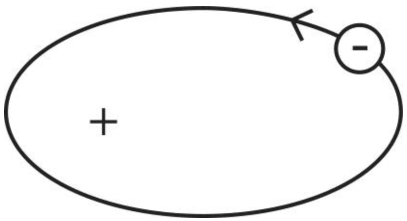




$$F = ma = F_w - bv$$

$$m_w = \frac{q\,\rho\,\pi\,d^{\,2}}{4\ell} L$$

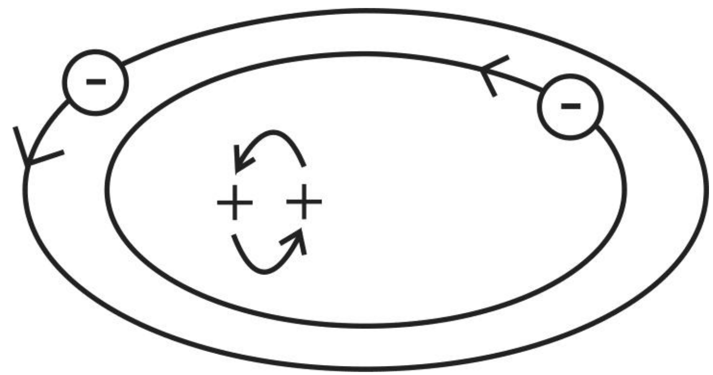
Weber's planetary model of the atom (1871 to 1880s)



$$F = ma$$

$$\frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\left(1-\frac{\dot{r}^2}{2c^2}+\frac{r\,\ddot{r}}{c^2}\right)\approx qE + m_W a = ma$$

$$qE = (m - m_W)\,a$$



where $$m_W = \frac{\mu_0}{4\pi}\frac{q_1 q_2}{r}$$

two positrons with $m = 9\times10^{-31}kg$ and $q = 1.6\times10^{-19}C$

attract one another for distances smaller than

$$r_C = 10^{-15}m$$

Conclusion

- Weber’s force is completely <u>relational</u>, depending upon the relative velocities and relative accelerations between the interacting bodies.
- Weber’s law conserves energy, linear and angular momentum.
- It is compatible with the laws of Gauss, Ampère and Faraday.
- It leads to the propagation of electromagnetic signals at light velocity.

There are many indications of a component in the force law depending upon the acceleration of the test body:

- In gravitation: precession of the perihelion, Mach's principle, proportionality between inertial and gravitational masses.

- Relational Mechanics: Derivation of Newton's 2nd law
  $F = ma$, derivation of $m_i = m_g$

- In electromagnetism: nuclear forces (Weber's planetary model of the atom), Mikhailov's experiments, self-inductance.

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