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COMPLIANCE OF A WEBER'S FORCE LAW FOR GRAVITATION WITH MACH'S PRINCIPLE

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Abstract-We discuss the different aspects and meanings of Mach's principle and present the consequences a dynamic model should lead in order to incorporate this principle. Then we analyse a Weber's force law for gravitation from this point of view and conclude that it seems to be in full compliance with Mach's principle.

Recently we applied a Weber's force law for gravitation in order to implement quantitatively Mach's principle, [1]. In this work we will discuss if this model is in agreement with the ideas of Mach.

To begin with we must discuss the meaning of «Mach's principle». What does it mean? Let us quote Mach in order to have some idea of the meanings of his principle. All quotations are from his main work related to this subject, »The Science of Mechanics», [2]:

«Try to fix Newton's bucket and rotate the heaven of fixed stars and then prove the absence of centrifugal forces.» (p. 279)

«The principles of mechanics can, indeed, be so conceived, that even for relative rotations centrifugal forces arise.» (p. 284)

«Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces but that such forces are produced by its relative rotation with respect to the mass of the earth and the other celestial bodies.» (p. 284)

«I have remained to the present day the only one who insists upon referring the law of inertia to the earth, and in the case of motions of great spatial and temporal extent, to the fixed stars.» (p. 336)

This last quotation indicates that to Mach it is meaningless to speak of absolute space, as in practice we are always referring motion to the earth or (in the case of planetary motion) to the frame of fixed stars (nowadays we could speak of the frame in which the most distant galaxies are seem without rotation, or the frame in which the cosmic background radiation is isotropic). In this respect Pais is correct when afirming that Mach's innovation was the abolition of absolute space and the introduction of the fixed stars idealized as a rigid system in its place, [3]. Instead of Newton's three laws of notion Mach propoced a set of alternative propositions of his own, [2, pp. 264-71 and p. 303]. Although in his key definition of inertial mass («The mass-ratio of any two bodies is the negative inverse ratio of the mutually induced acceleration of those bodies») he did not specify clearly the frame of reference with respect to which the accelerations in this definition should be measured, it is evident from his writings that he had in mind the frame of fixed stars. This has been shown conclusively in an important paper by Yourgrau and van der Merwe, [4].

But if Mach's principle were restricted to this aspect we could suspect that it was only a question of language. That is, intead of Newton's absolute space we could speak of Mach's frame of fixed stars and then all would be settled. But the first three quotations of Mach which we presented earlier indicate a stronger meaning. In fact they show a dynamic origin, for Mach, of the centrifugal force. That is, the centrifugal force is a real force which appears in a frame of reference in which the sky of stars is rotating. This aspect cannot be derived from Newton's laws of motion nor even from his universal law of gravitation. As a matter of fact a spherical shell (or any isotropic distribution of mass) exerts no force in a body anywhere inside this shell. This happens not only if the spherical shell is at rest (as Newton showed in the Principia) but also if this spherical shell is spinning or has any other motion (this follows from the fact that Newton's law of universal gravitation does not depend on the velocity or acceleration of the interacting masses). So the fixed stars (idealized here as an isotropic distribution of masses) cannot exert anything similar to a centrifugal force, according to Newton's laws. Although Mach did not present a dynamical theory of his own showing how the rotating «fixed stars» could exert the centrifugal force, he believed such a causal law could be discovered, as is apparent from his second statement presented above.

Let us now give a look of what is «Mach's principle» according to some authors:

«Inertial frames are those which are unaccelerated relative to the 'fixed stars', that is, relative to a suitably defined mean of all the matter in the universe.» (Sciama, [5]).

«Inertia is not due to movement with respect to 'absolute space', but due to surrounding matter.» (Brown, [6]).

«The motion and consequently the mass of every single body is determined (caused, produced) by the remaining bodies in the universe.» (Bunge, [7]).

«The inertial properties of matter on the local scene derive in some way from the existence of the distant masses of the universe and their distribution in space.» (Schiff, [8]).

«The inertial mass of a body is caused by its interactions with the other bodies in the universe.» (Reinhardt, [9]).

«Inertial forces should be generated entirely by the motion of a body relative to the bulk of matter in the universe.» (Raine, [10]).

«Mach suggested that inertial motion here on the earth and in the solar system is causally determined in accordance with some quite definite but as yet unknown law by the totality of the matter in the universe.» (Barbour, [11]).

It seems that Einstein was the first to coin the term «Mach's principle» [12], for the conjecture that the inertial properties of local matter are determined by the overall matter distribution in the universe.

From these views we perceive that in order to satisfy Mach's principle a dynamical model must satisfy some properties: The inertial mass of any body must be completely derived from its interaction with the remaining universe; this causal model should be able to show that Newton's first law of motion (or an equivalent to that) can be valid only in a reference frame which is not accelerated relative to the frame of the «fixed stars»; all inertial forces, including the «fictitious» ones (centrifugal, Coriolis, etc) must appear as real forces due to an interaction with the remaining universe (as regards centrifugal and Coriolis forces, they should appear only in a reference frame relative to which the sky as a whole is rotating).

As is well known, [9, 10], Mach's principle is not contained in Einstein's general theory of relativity, as Einstein himself emphasized, [5]. One of the reasons is that in general relativity, inertial mass is an intrinsic local and invariant property of bodies [10]. In particular Einstein showed that his field equations imply that a test-particle in an otherwise empty universe has inertial properties, [5]. Moreover, according to general relativity there are no observable effects (increase in the inertial mass of a body, for instance) in a laboratory from a spherically symmetric agglomeration of matter surrounding it, [9].

Let us then analyse to what extent Mach's principle is incorporated in a Weber's force law for gravitation. First one remark. Mach did not specify what kind of interaction (gravitational, electric, magnetic, nuclear, ...) was the responsible for inertia. The first to state clearly that such an interaction had a gravitational origin was Einstein, [13, 14]. This is a natural assumption (which we also used) because one aspect of Mach's principle is to derive the inertial mass from an interaction with the remaining universe. Now it is a well established fact known since Galileo and Newton that the inertial mass is proportional to the gravitational mass of the body and not, for instance, to its electrical charge. This remarkable fact strongly suggests that the interaction responsible for inertia has a gravitational origin.

The main aspects of our formulation can be stated in two assumptions, [1]:

I) The sum of all forces (gravitational, electric, nuclear, etc) on any material body is always zero in all coordinate frames.

II) The gravitational force exerted by a material point j on a material point i is given by a Weber's law, namely

$$\vec{F}_{ji} = -H_g m_{gi} m_{gj} \frac{\hat{r}_{ij}}{r_{ij}^2} \left[1 - \frac{\xi}{c^2} \left(\frac{\dot{r}_{ij}^2}{2} - r_{ij} \ddot{r}_{ij} \right) \right] \qquad (1)$$

In this expression H_g is a constant, m_{gi} and m_{gj} are the gravitational masses of bodies i and j, $\xi = 6$ and c is a constant with the same value of the velocity of light in vacuum.

Moreover $\vec{r}_i$ ($\vec{r}_j$) is the position vector of body $i(j)$ relative to an arbitrary frame of reference and $\vec{r}_{ij} \equiv \vec{r}_i - \vec{r}_j$, $r_{ij} \equiv |\vec{r}_{ij}|$, $\dot{r}_{ij} \equiv dr_{ij}/dt$, $\ddot{r}_{ij} \equiv d^2 r_{ij}/dt^2$.

There are Machian aspects in these two assumptions. The advantage of the first assumption as compared with Newton's second law of motion $\left(\sum_{all\, j} \vec{F}_{ji} = d(m_i \vec{v}_i)/dt\right)$ stems from the fact that we didn't need to introduce a priori the key concepts of inertial mass and absolute space
(or inertial frame). As Newton began with this equation he was obliged to introduce previous to that the meaning of inertial mass (the mass which appears in the right hand side of this equation) and of absolute space (the frame in which is valid his equation of motion). As regards our second assumption the Machian aspects are the utilization of only relative distances, velocities and accelerations between the interacting bodies (r_{ij}, $\dot{r}_{ji}$ and $\ddot{r}_{ij}$), and the utilization of Newton's action and reaction law in the strongest form (there is no other meaningful direction for two bodies interacting with one another than the straight line joining them).

Some authors have worked previous to us with models like this [5, 6, 15-18]. Sciama, [5], in particular, seems to have been the first to state a particular form of our first assumption. The first limitation of his hypotesis $\left(\sum_{all\, j} \vec{F}_{ji} = 0\right)$ was that he supposed it valid only for gravitational interactions, while we applied it to all kinds of interaction. But more serious to that was the fact that he restricted the validity of his postulate only to the rest-frame of the test body wich feels the interaction. We, on the other hand, supposed it valid in all coordinate systems. The reason for that is very simple. He utilized as his force law a similar to Lorentz's force as applied to gravitation. As is well known Lorentz's force depends on the position and velocity of the test body, but not on its acceleration. So Sciama was able to derive $\Sigma \vec{F} = -m\vec{a}$ (where $\vec{a}$ here is the acceleration of the universe relative to the test body) only in the rest frame of the particle. He could not derive such a law in the rest frame of the universe (i. e., in the frame of the fixed stars). As we will see in the sequence, we suceeded in deriving a similar to Newton's second law in this last frame.

Returning to our work, we divided all bodies interacting with an arbitrary body 1 in two parts. The first one is the isotropic (relative to the arbitrary coordinate system which specified the radius vectors) distribution of matter surrounding this body 1. The second part is the anisotropic distribution of bodies surrounding it (as, for instance, objects in its proximity as a spring and a magnet, the Earth, the Sun, the Milk Way, etc). This part is what gives rise to the usual Newtonian forces, and which we represent by $\sum_{j=2}^{N} \vec{F}_{j1}$. According to Newton's law of universal gravitation (but not according to Weber's law) the first part will exert no net force on the internal body 1. On the other hand our main result was to show that the resultant force of this part on m_{g1} (the force of the distant galaxies on m_{g1}) is usually different from zero. In a frame of reference in which the universe is rotating with $\vec{\omega}(t)$ this force is given by, [1]:

$$-\Phi m_{g1}\left[\vec{a}_1+\vec{r}_1\times\frac{d\vec{\omega}}{dt}+2\vec{v}_1\times\vec{\omega}+\vec{\omega}\times(\vec{\omega}\times\vec{r}_1)\right],$$

where $\Phi=\frac{2\pi}{3}\xi H_g\frac{\rho_0}{H_0^2}$ (H_0 is Hubble's constant and ρ_0 is the mean estimated gravitational mass density of the universe). In this expression $\vec{r}_1$, $\vec{v}_1$ and $\vec{a}_1$ are, respectively, the radius vector, velocity and acceleration of body 1 relative to this frame of reference in which the isotropic universe rotates with $\vec{\omega}(t)$.

Combining these two expressions with our first assumption yields an equivalent to Newton's second law of motion, namely:

$$\frac{\sum_{j=2}^{N}\vec{F}_{j1}}{\Phi}=m_{g1}\left[\vec{a}_1+\vec{r}_1\times\frac{d\vec{\omega}}{dt}+2\vec{v}_1\times\vec{\omega}+\vec{\omega}\times(\vec{\omega}\times\vec{r}_1)\right] \quad (2)$$

Now we can discuss more precisely to what extent this model implements Mach's principle. As was first pointed out by Einstein, [19], a model of interaction satisfying Mach's principle should lead to some consequences, namely, [9]:

1) The inertial mass of a body should increase with the agglomeration of masses in its neighborhood.

2) A body in an otherwise empty universe should have no inertia.

3) A body should experience an acceleration if nearby bodies are accelerated. The accelerating force should be in the same direction as the acceleration of the latter.

4) A rotating body should generale inside it a Coriolis force.

All these consequences follow from Weber's force law. If we surround body 1 by an extra spherical shell there will be an extra force proportional to m_{g1} and $\vec{a}_1$, which can be interpreted either as an increase in the inertial mass of 1 or as a change in the coupling «constant» $G = H_g/\Phi$. This answers to consequence (1). Consequence (2) follows from our equation (2) simply making $\rho_0 = 0$, so that $\Phi = 0$ and there will be no inertia. It is easier to observe consequence (3) from Eq. (1). Considering bodies i and j moving along the X axis, with $x_i < x_j$, Eq. (1) can be written as

$$\vec{F}_{ji} = + H_g m_{gi} m_{gj} \frac{\hat{x}}{|x_i - x_j|^2} \left[1 - \frac{\xi}{c^2} \left(\frac{(\dot{x}_i - \dot{x}_j)^2}{2} - - (x_i - x_j)(\ddot{x}_i - \ddot{x}_j) \right) \right] \quad (3)$$

This means that if j is accelerated to the right ($\ddot{x}_j > 0$) there will be a component of the force acting on m_i proportional to $\ddot{x}_j$ and pointing also to the right, in agreement with consequence (3). The last consequence is evident from Eq. (2), where $\vec{\omega}$ is the angular rotation of the surrounding body (a spherical distribution of mass).

Just a few further remarks. The proportionality between inertial and gravitational masses follows at once from this formulation. This is due to the fact that we only began with gravitational masses so that in the right hand side of Eq. (2) the mass which appears in these inertial forces is already a gravitational mass. We can recover Newton's second law without the «fictitious» forces simply by moving to a coordinate system in which the universe is not rotating, so that $\vec{\omega} = 0$. This is what should be expected from Mach's ideas. In our work we derived also the kinetic energy (another place where the inertial mass appears in classical mechanics) as an energy arising from a gravitational interaction with the remaining universe, [1]. All these aspects seem to be worthy of consideration.

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