Applications of the Principle of Physical Proportions to Gravitation

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We propose the principle of physical proportions, according to which all laws of physics may depend only on the ratio of quantities of the same type. We present examples of laws that satisfy this principle, and others that do not. These examples suggest that the theories leading to these laws must be incomplete.

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The Principle of Physical Proportions

Newton, in his *Principia* (1687), introduced the concepts of absolute space, absolute time and absolute motion. Leibniz, Berkeley and Mach were against these concepts and proposed that only relative space, relative time and relative motion could be conceived and perceived by the senses. We agree with these authors and propose a generalization of their ideas through the principle of physical proportions, which can be stated as follows: "All laws of physics can depend only on the ratio of quantities of the same type." The meaning of this principle is illustrated by the examples below.

The law of the lever satisfies this principle. According to Archimedes two weights P_1 and P_2 at distances d_1 and d_2 from a fulcrum remain in horizontal static equilibrium only when $P_1/P_2 = d_2/d_1$. Only ratios of local weights and local distances are relevant here.

On the other hand, classical mechanics does not satisfy this principle. For instance, the acceleration of free fall near the surface of the earth is given by

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$$a = \frac{GM_e}{R_e^2} = \frac{4\pi}{3} G R_e \rho_e \,.$$

Here $G = 6.67 \times 10^{-11} Nm^2/kg^2$ is the constant of gravitation, $M_e = 5.98 \times 10^{24} kg$ is the earth's mass, $R_e = 6.37 \times 10^6 m$ is its average radius and $\rho_e = 5.52 \times 10^3 kg/m^3$ its average mass density. This acceleration of free fall depends only on the mass (or density) of the earth, and not on the ratio of this mass (or density) to other masses (or densities) in the universe. Although the constant *G* has the dimensions of acceleration divided by (distance times density), it is not dependent on other bodies in the universe, since it is a universal constant. This situation is in conflict with the principle of physical proportions.

Relational mechanics (Assis 1999, Section 8.1) has resolved this problem, as it is completely compatible with the principle of physical proportions. It is based on Weber's law for gravitation and electromagnetism, and on the principle of dynamical equilibrium: The sum of all forces of any nature (gravitational, electric, magnetic, elastic, nuclear, *etc*.) acting on any body is always zero in all frames of reference. As the sum of all forces is zero, only ratios of forces will be detectable or measurable. The system of units (MKSA, cgs, *etc*.) to be employed is not relevant. Moreover, the unit or dimension of the forces can be whatever we wish.

According to relational mechanics the acceleration of free fall is given by (Assis, 1999, Sections 8.5 and 9.2)

$$a = \frac{2}{\alpha} R_e H_o^2 \frac{\rho_e}{\rho_o} \,.$$

Here H_o is Hubble's constant and ρ_o is the average matter density of the distant universe. Moreover, α is a dimensionless number with value 6 if we work with a finite universe and integrate Weber's law for gravitation up to Hubble's radius c/H_o. If we work with Weber's law and an exponential decay in gravitation we can integrate up to infinity, and in this case $\alpha = 12$.

The important aspect of this result is that only a ratio of densities is important here. Doubling the earth's density while keeping the mass density of the distant universe unaltered is equivalent to keeping the earth's density unaltered while halving the mass density of the distant universe. In both cases the acceleration of free fall doubles compared to its present value of $9.8 m/s^2$.

Next we consider the figure of the earth.

The Flattening of the Earth

Due to its diurnal rotation around the North-South axis the earth takes essentially the form of an ellipsoid of revolution. Its equatorial radius $R_>$ is greater than the polar radius $R_<$. According to classical mechanics the fractional change f is given by (Assis, 1999, Section 3.3.2):

$$f \equiv \frac{R_> - R_<}{R_<} \approx \frac{15\omega^2}{16\pi \, G\rho_e} \approx 0.004 \, .$$

Here $\omega = 7.29 \times 10^{-5} \, rad/s$ is the angular rotation of the earth relative to an inertial frame of reference with a period of one day.

Several observations may be made considering this result, which is based on classical mechanics. In the first place the fractional change depends on the angular rotation of the earth relative to absolute space or to an inertial frame of reference. In principle, the distant universe composed of stars and galaxies can disappear without affecting f. If the earth remained stationary in an inertial frame of reference and the distant universe rotated around its North-South direction in the opposite direction compared with the previous situation, the earth would not be flattened. This is against Mach's point of view. Moreover, the fractional change depends only on the density of the earth, but not on the density of distant matter. If it were possible to double the average matter density of the distant universe without affecting the matter density of the earth, the previous result would not be affected. This shows that not only space and time, but also mass and matter density are absolute quantities in classical mechanics. All of these aspects are against the principle of physical proportions.

The flattening of the earth according to relational mechanics is given by (Assis, 1999, Sections 8.5 and 9.5.1)

$$f \equiv \frac{R_> - R_<}{R_<} \approx \frac{5\alpha}{8} \frac{\omega_{eU}^2}{H_o^2} \frac{\rho_o}{\rho_e} \, .$$

As there are many uncertainties concerning the precise value of Hubble's constant and the average matter density of the universe, it is not possible to give a precise value for the above ratio. But the order of magnitude is compatible with the observed value of 0.004. We can also utilize the fact that this is the observed value of *f*, and in this way (together with the known value of the angular rotation of the earth and its matter density) derive the value of $\alpha\ \rho_o / H_o^2$.

But what we want to emphasize here are the Machian aspects of this result. The first is that the angular rotation ω_{eU}, which appears in relational mechanics, is the angular rotation of the earth relative to the distant universe (distant galaxies). It is no longer the angular rotation of the earth relative to free space. According to relational mechanics, there will be the same flattening of the earth no matter whether the earth rotates relative to an arbitrary reference frame while the distant universe remains stationary in this frame, or if the distant universe rotates while the earth remains stationary in this frame, provided the relative rotation between the earth and the distant universe is the same in both cases. The flattening of the earth can no longer be considered as a proof of the real or absolute rotation of the earth. The second aspect is that this flattening depends on the ratio of densities of the distant universe and of the earth. We can increase the flattening by decreasing the density of the earth or increasing the density of the distant universe. Only ratios of quantities are important here. Mass and matter density are not absolute in relational mechanics. The last aspect to be considered here is the ratio of the angular rotation of the earth and Hubble's constant. If we double the rotation of the earth relative to the distant universe, the flattening increases four times, as it is proportional to the square of the angular rotation of the earth. To say that the rotation of the earth has increased we must compare it with something else (for instance, with a clock). The same result should appear if the earth did not change its rate of rotation, but all other motions in the universe became slower by half. This means that Hubble's constant must somehow be like an average frequency of oscillation and/or rotation of the matter in the universe. If we decrease by a factor of two all of these frequencies (except the frequency of rotation of the earth relative to the distant universe), the present value of Hubble's constant must then be divided by 2, and the flattening increases by a factor of four, as in the previous situation. This happens in relational mechanics but not in classical mechanics.

Applications to Other Situations

We now propose applications of this principle to other situations involving different physical concepts.

We first analyze electrostatics. Consider two charges q_1 and q_2 of the same sign repelling one another. We can keep them separated at a constant distance d by applying an external force, for instance, placing a dielectric spring of elastic constant k and relaxed length ℓ_o between them. By equating the coulombian force with the elastic force $k(d-\ell_o)$, we find that the fractional displacement f of the spring is given by

$$f \equiv \frac{d-\ell_o}{\ell_o} = \frac{q_1 q_2}{4\pi\ \varepsilon_o\ d^2 \ell_o\ k}\ .$$

Here $\varepsilon_o = 8.85\times 10^{-12} C^2 s^2 / kgm^3$ is called the vacuum permittivity. Doubling the value of the two charges increases f four times. The fractional displacement should also increase four times according to the principle of dynamical equilibrium if q_1 and q_2 are kept unaltered but all other charges in the universe are halved (*i.e.*, the charges of all atoms and molecules of the spring, the earth and of all other bodies of the universe, excepting q_1 and q_2). However, this consequence is not implemented in present theories, indicating that they must be incomplete. The influence may be completely local (halving all the charges of the spring and distance galaxies changes only the elastic constant to k/4, without affecting ε_o), completely cosmological (halving all the charges of the spring and of all astronomical bodies does not change k, but does change the vacuum permittivity to $\varepsilon_o/4$), or a mixture of both effects (halving all the charges of the spring and of all astronomical bodies affects the elastic constant and the vacuum permittivity, their new values becoming k/2 and $\varepsilon_o/2$).

Suppose now we remove the spring, releasing the charges. They will then be accelerated in opposite directions. The value of the acceleration of q_1 relative to an inertial frame or to the universal frame of distant galaxies is given by

$$a_1 = \frac{q_1 q_2}{4\pi\ \varepsilon_o\ \mathrm{d}^2 m_1}\ .$$

This acceleration is increased four times when q_1 and q_2 are doubled. The same must happen when q_1 and q_2 are kept unaltered but all other charges in the universe are halved (that is, the charges of all atoms and molecules of distant galaxies, and the microscopic charges composing bodies 1 and 2 are all halved). Again the effect may be totally cosmological (affecting only the vacuum permittivity), totally local (affecting only the masses m_1 and m_2) or a mixture of both effects (affecting the vacuum permittivity and both masses).

One example of how the mass of a body may depend on its microscopic constituent charges has already been given (Assis, 1992). The Newtonian gravitational force between two bodies of masses m_1 and m_2 was derived as a residual electromagnetic force arising from the interaction between the neutral oscillating dipoles composing body 1 and the neutral oscillating dipoles belonging to body 2, where each dipole consisted of a negative charge oscillating around a positive one. The mass of each body was then found proportional to the number of oscillating dipoles composing it, and to q^2 / ε_o, where q represents the positive (or negative) charge of each neutral dipole.

Another situation is Ampère's force between electrical circuits carrying currents I_1 and I_2, proportional to $I_1 I_2$. As the currents are proportional to the drift velocities of the electrons, we can increase the force four times, doubling these drift velocities. The consequences of this effect can be seen statically (an increase in the tension on a spring holding the two circuits at a constant distance) or dynamically (an increase in the acceleration of the two circuits when the spring is released). The same consequences must be found if we keep I_1 and I_2 unaltered, but make all other bodies in the universe move with half their present velocities. As modern theories do not implement this property, they must be incomplete.

Consider now the equation of state of an ideal gas, $PV = k_B NT$ (P being the pressure, V the volume, $k_B = 1.38 \times 10^{-23}$ J/K Boltzmann's constant, N the number of atoms and T the temperature). This equation is not compatible with the principle of physical proportions. The equation of an ideal gas compatible with this principle should take the form $(p/p_o)(V/V_o) = a\,(N/N_o)(T/T_o)$, where "$a$" is a dimensionless number and p_o, N_o and T_o are local and/or cosmological pressures, the number of particles and temperature. When the theory leading to this new equation is found, it will be possible to relate Boltzmann's constant k_B to the properties (such as pressure, density and temperature) of the local or cosmological environment. For instance, relational mechanics showed that the universal constant of gravitation G is proportional to H_o^2 / ρ_o. This shows that it is no longer a constant, but a function of the properties of the distant universe. An analogous situation must hold for Boltzmann's constant.

The same can be said of almost all relations in physics. The universal constants, such as the light velocity *in vacuo*, Planck's constant, *etc*. must all be functions of properties of the distant universe (macrocosm, holistic relations) or of local particles (microcosm, microscopic relations).

We hope this paper will motivate others to search for these relations in all branches of physics. Many new things will be learned in this process, and certainly many novel developments and deeper theories will come out of this endeavour.

References

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