32. Find an expression analogous to Eq. (4.116) for the angle of recoil of the target particle (ϑ_2 in Fig. 4.7) in terms of the scattering angle Θ in the equivalent one-body problem. Show that, for an elastic collision,

$$\vartheta_2 = \tfrac{1}{2}(\pi - \Theta).$$

33. Assume that $m_2 \gg m_1$, and that $\Theta = \vartheta_1 + \delta$, in Eq. (4.117). Find a formula for δ in terms of ϑ_1. Show that the first-order correction to the Rutherford scattering cross section [Eq. (3.276)], due to the finite mass of m_2, vanishes.

34. An elastic sphere of radius a collides with an identical elastic sphere at rest. Assume that in the center of mass coordinate system, each sphere rebounds from the other so that the relative velocities before and after impact make equal angles with the normal to the spheres at the point of contact. Find the cross section for scattering the incident sphere through an angle ϑ_1.

35. A pair of masses m_1, m_2, connected by a spring of force constant k, slide without friction along the x-axis. Show that the center of mass moves with uniform velocity and that the masses oscillate with frequency $(k(m_1+m_2)/m_1m_2)^{1/2}$.

36. Set up the equations of motion for Fig. 4.10, assuming that the relaxed length of each spring is l, and that the distance between the walls is $3l+a$, so that the springs are stretched, even in the equilibrium position. Show that the equations can be put in the same form as Eqs. (4.135) and (4.136).

37. For the normal mode of vibration given by Eqs. (4.162) and (4.163), find the force exerted on m_1 through the coupling spring, and show that the motion of x_1 satisfies the equation for a simple harmonic oscillator subject to this driving force.

38. In Fig. 4.10, $m_1 = m_2 = m$, $k_1 = k$, $k_2 = 0.9k$, $k_3 = 0.1k$. Initially mass m_2 is held fixed at its equilibrium position and mass m_1 is pulled a distance A from its equilibrium position; then both masses are released. Find $x_1(t)$ and $x_2(t)$ and show that your result agrees qualitatively with Fig. 4.13.

39. Find the two normal modes of vibration for a pair of identical damped coupled harmonic oscillators [Eqs. (4.180), (4.181)]. That is, $m_1 = m_2$, $b_1 = b_2$, $k_1 = k_2$. [*Hint*: If $k_3 = 0$, you can certainly find the solution. You will find this point helpful in factoring the secular equation.]

40. Set up the equations of motion for the system shown in Fig. 4.16. The relaxed lengths of the two springs are l_1, l_2. Separate the problem into two problems, one involving the motion of the center of mass, and the other involving the "internal motion" described by the two coordinates x_1, x_2. Find the normal modes of vibration.

41. The system of coupled oscillators shown in Fig. 4.10 is subject to an applied force

$$F = F_0 \cos \omega t,$$

applied to mass m_1. Set up the equations of motion and find the steady-state solution. Sketch the amplitude and phase of the oscillations of each oscillator as functions of ω.