F-313 C Mecânica Geral
Lista de Exercícios #7

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Os problemas sugeridos estão destacados em vermelho

1. **Centro de massa.**

   No cálculo do Centro de Massa (CM) de corpos rígidos que apresentam algum tipo de simetria, são úteis alguns teoremas simplificadores. Demonstre então os teoremas enunciados abaixo:

   (a) **Teorema 1.** Se um corpo for simétrico em relação a um plano, o seu CM estará nesse plano.

   (b) **Teorema 2.** Se um corpo for simétrico em relação a dois planos, o seu CM estará na linha de interseção dos planos.

   (c) **Teorema 3.** Se um corpo for simétrico em relação a um eixo, o seu CM estará neste eixo.

   (d) **Teorema 4.** Se um corpo for simétrico em relação a três planos que tem um ponto em comum, esse ponto será seu CM.

   (e) **Teorema 5.** Se um corpo tiver simetria esférica em relação aum ponto, este ponto será seu CM. □

2. **Centro de massa do corpos compostos**

   Mostre que, se um corpo for composto de duas ou mais partes cujos CM's são conhecidos, então o CM do corpo composto poderá ser calculado, considerando-se suas partes componentes como partículas localizadas nos cntros de massa respectivos. □

3. **Teoremas de Pappus**

   Quando a densidade ρ de um corpo é uniforme, a posiçaõ do CM só depende da geometria, pois a densidade pode ser eliminada:

$$\vec{\mathbf{R}} = \frac{\rho}{M}\iiint\limits_{Corpo} dV\ \vec{\mathbf{r}} = \frac{1}{V}\iiint\limits_{Corpo} dV\ \vec{\mathbf{r}}\ , \tag{1}$$

com $M = \rho V$. O ponto cuja coordenada $\vec{\mathbf{R}}$ é dada por (1) chama-se *centróide* do volume V. O conceito pode ser generalizado para dimensão arbitrária, como:

$$\vec{\mathbf{R}} = \frac{1}{\Omega_n} \int ... \int dx^n \; \vec{\mathbf{r}} \; ,$$

onde Ω_n é o 'volume' do corpo em $n-$ dim. Para $n = 1$, trata-se de uma curva e para $n = 2$, de uma área. Em geral, Ω_n é um hipervolume. Mostre que temos os seguintes teoremas:

(a) **Teorema de Pappus 1.** Se uma curva plana girar em torno de um eixo situado em seu próprio plano que não a intercepta, a área da superfície de revolução gerada será igual ao comprimento da curva multiplicado pelo comprimento da trajetória percorrida por seu centróide.

(b) **Teorema de Pappus 2.** Se uma área planar rodar em torno de um eixo situado no seu próprio plano que não a intercepte, o volume gerado será igual a área multiplicada pelo comprimento da trajetória percorrida pelo seu centróide.

(c) Use um dos teoremas de Pappus para calcular o CM de uma área semicircular de raio a, de densidade uniforme. $\square$

eliminated from Eq. (5.190) by means of Eq. (5.187). If we eliminate the density, we have

$$\frac{dp}{dz} = -\frac{Mg}{RT}p. \tag{5.192}$$

As an example, if we assume that the atmosphere is uniform in temperature and composition, we can solve Eq. (5.192) for the atmospheric pressure as a function of altitude:

$$p = p_0 \exp\left(-\frac{Mg}{RT}z\right). \tag{5.193}$$

PROBLEMS

1. (a) Prove that the total kinetic energy of the system of particles making up a rigid body, as defined by Eq. (4.37), is correctly given by Eq. (5.16) when the body rotates about a fixed axis.
b) Prove that the potential energy given by Eq. (5.14) is the total work done against the external forces when the body is rotated from θ_s to θ, if N_z is the sum of the torques about the axis of rotation due to the external forces.

2. Using the scheme of analogy in Section 5.2, formulate a theorem analogous to that given by Eq. (2.8) and prove it, starting from Eq. (5.13).

3. Prove, starting with the equation of motion (5.13) for rotation, that if N_z is a function of θ alone, then $T + V$ is constant.

4. The balance wheel of a watch consists of a ring of mass M, radius a, with spokes of negligible mass. The hairspring exerts a restoring torque $N_z = -k\theta$. Find the motion if the balance wheel is rotated through an angle θ_0 and released.

5. A wheel of mass M, radius of gyration k, spins smoothly on a *fixed* horizontal axle of radius a which passes through a hole of slightly larger radius at the hub of the wheel. The coefficient of friction between the bearing surfaces is μ. If the wheel is initially spinning with angular velocity ω_0, find the time and the number of turns that it takes to stop.

6. A wheel of mass M, radius of gyration k is mounted on a horizontal axle. A coiled spring attached to the axle exerts a torque $N = -K\theta$ tending to restore the wheel to its equilibrium position $\theta = 0$. A mass m is located on the rim of the wheel at distance $2k$ from the axle at a point vertically above the axle when $\theta = 0$. Describe the kinds of motion which can occur, locate the positions of stable or unstable equilibrium of the wheel if any, and find the frequencies of small oscillations about the equilibrium points. Consider two cases: (a) $K > 2mgk$. (b) $K = 4mgk/\pi$. What if $K < 4mgk/5\pi$? [*Hint*: Solve the trigonometric equation graphically.]

7. An airplane propeller of moment of inertia I is subject to a driving torque

$$N = N_0(1 + \alpha \cos \omega_0 t),$$

and to a frictional torque due to air resistance

$$N_f = -b\dot{\theta}.$$

Find its steady-state motion.

8. A motor armature weighing 2 kg has a radius of gyration of 5 cm. Its no-load speed is 1500 rpm. It is wound so that its torque is independent of its speed. At full speed, it draws a current of 2 amperes at 110 volts. Assume that the electrical efficiency is 80%, and that the friction is proportional to the square of the angular velocity. Find the time required for it to come up to a speed of 1200 rpm after being switched on without load.

9. Derive Eqs. (5.35) and (5.36).

10. Assume that a simple pendulum suffers a frictional torque $-mb_1\dot{\theta}$ due to friction at the point of support, and a frictional force $-b_2 v$ on the bob due to air resistance, where v is the velocity of the bob. The bob has a mass m, and is suspended by a string of length l. Find the time required for the amplitude to damp to $1/e$ of its initial (small) value. How should m, l be chosen if it is desired that the pendulum swing as long a time as possible? How should m, l be chosen if it is desired that the pendulum swing through as many cycles as possible?

11. A child of mass m sits in a swing of negligible mass suspended by a rope of length l. Assume that the dimensions of the child are negligible compared with l. His father pulls the child back until the rope makes an angle of one radian with the vertical, then pushes with a force $F = mg$ along the arc of a circle until the rope is vertical and releases the swing. (a) How high up will the swing go? (b) For what length of time did the father push on the swing? (Assume that it is permissible to write $\sin\theta \doteq \theta$ for $\theta < 1$.) Compare with the time required for the swing to reach the vertical if he simply releases the swing without pushing on it.

12. A baseball bat held horizontally at rest is struck at a point O' by a ball which delivers a horizontal impulse J' perpendicular to the bat. Let the bat be initially parallel to the x-axis, and let the baseball be traveling in the negative direction parallel to the y-axis. The center of mass G of the bat is initially at the origin, and the point O' is at a distance h' from G. Assuming that the bat is let go just as the ball strikes it, and neglecting the effect of gravity, calculate and sketch the motion $x(t)$, $y(t)$ of the center of mass, and also of the center of percussion, during the first few moments after the blow, say until the bat has rotated a quarter turn. Comment on the difference between the initial motion of the center of mass and that of the center of percussion.

13. A compound pendulum is arranged to swing about either of two parallel axes through two points O, O' located on a line through the center of mass. The distances h, h' from O, O' to the center of mass, and the periods τ, τ' of small amplitude vibrations about the axes through O and O' are measured. O and O' are arranged so that each is approximately the center of oscillation relative to the other. Given $\tau = \tau'$, find a formula for g in terms of measured quantities. Given that $\tau' = \tau(1+\delta)$, where $\delta \ll 1$, find a correction to be added to your previous formula so that it will be correct to terms of order δ.

14. Prove that if a body is composed of two or more parts whose centers of mass are known, then the center of mass of the composite body can be computed by regarding its component parts as single particles located at their respective centers of mass. Assume that each component part k is described by a density $\rho_k(\mathbf{r})$ of mass continuously distributed over the region occupied by part k.

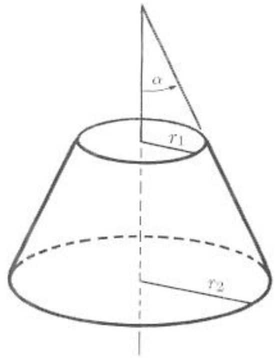

Fig. 5.28 Frustum of a cone.

15. A circular disk of radius a lies in the xy-plane with its center at the origin. The half of the disk above the x-axis has a density σ per unit area, and the half below the x-axis has a density 2σ. Find the center of mass G, and the moments of inertia about the x-, y-, and z-axes, and about parallel axes through G. Make as much use of labor-saving theorems as possible.

16. (a) Work out a formula for the moments of inertia of a cone of mass m, height h, and generating angle α, about its axis of symmetry, and about an axis through the apex perpendicular to the axis of symmetry. Find the center of mass of the cone.

b) Use these results to determine the center of mass of the frustum of a cone, shown in Fig. 5.28, and to calculate the moments of inertia about horizontal axes through each base and through the center of mass. The mass of the frustum is M.

17. Find the moments of inertia of the block shown in Fig. 5.8, about axes through its center of mass parallel to each of the three edges of the block.

18. Through a sphere of mass M, radius R, a plane saw cut is made at a distance $\frac{1}{2}R$ from the center. The smaller piece of the sphere is discarded. Find the center of mass of the remaining piece, and the moments of inertia about its axis of symmetry, and about a perpendicular axis through the center of mass.

19. How many yards of thread 0.03 inch in diameter can be wound on the spool shown in Fig. 5.29?

20. Given that the volume of a cone is one-third the area of the base times the height, locate by Pappus' theorem the centroid of a right triangle whose legs are of lengths a and b.

21. Prove that Pappus' second theorem holds even if the axis of revolution intersects the surface, provided that we take as volume the difference in the volumes generated by the two parts into which the surface is divided by the axis. What is the corresponding generalization of the first theorem?

22. Find the center of mass of a wire bent into a semicircle of radius a. Find the three radii of

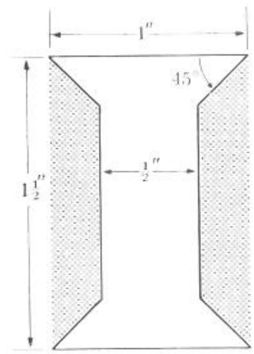

Fig. 5.29 How much thread can be wound on this spool?

gyration about x-, y-, and z-axes through the center of mass, where z is perpendicular to the plane of the semicircle and x bisects the semicircle. Use your ingenuity to reduce the number of calculations required to a minimum.

23. (a) Find a formula for the radius of gyration of a uniform rod of length l about an axis through one end making an angle α with the rod.

b) Using this result, find the moment of inertia of an equilateral triangular pyramid, constructed out of six uniform rods, about an axis through its centroid and one of its vertices.

24. Find the radii of gyration of a plane lamina in the shape of an ellipse of semimajor axis a, eccentricity ε, about its major and minor axes, and about a third axis through one focus perpendicular to the plane.

25. Forces 1 kg-wt, 2 kg-wt, 3 kg-wt, and 4 kg-wt act in sequence clockwise along the four sides of a square $0.5 \times 0.5\ \text{m}^2$. The forces are directed in a clockwise sense around the square. Find the equilibrant.

26. Forces 2 lb, 3 lb, and 5 lb act in sequence in a clockwise sense along the three sides of an equilateral triangle. The sides of the triangle have length 4 ft. Find the resultant.

27. (a) Reduce the system of forces acting on the cube shown in Fig. 5.30 to an equivalent single force acting at the center of the cube, plus a couple composed of two forces acting at two adjacent corners.

b) Reduce this system to a system of two forces, and state where these forces act.

c) Reduce this system to a single force plus a torque parallel to it.

28. A sphere weighing 500 g is held between thumb and forefinger at the opposite ends of a horizontal diameter. A string is attached to a point on the surface of the sphere at the end of a perpendicular horizontal diameter. The string is pulled with a force of 300 g in a direction parallel to the line from forefinger to thumb. Find the forces which must be exerted by forefinger and thumb to hold the sphere stationary. Is the answer unique? Does it correspond to your physical intuition about the problem?