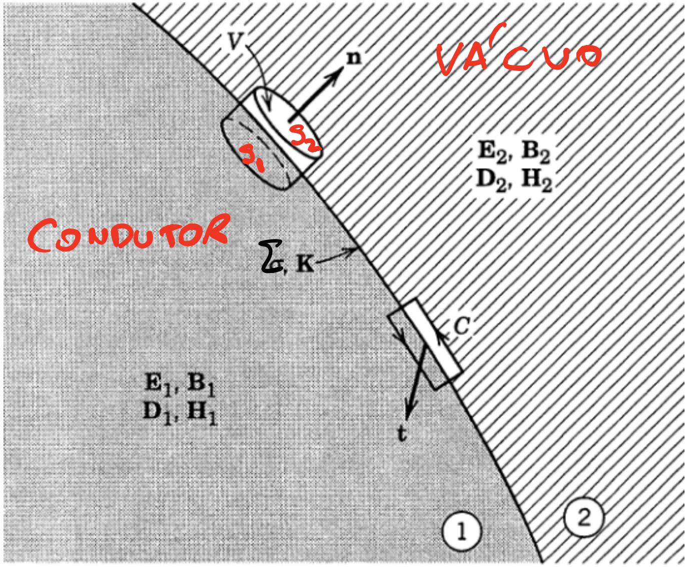
FI 008 – Eletrodinâmica I

1º Semestre de 2020
16/04/2020
Aula 11

Guias de ondas e cavidades ressonantes

Condições de contorno



$\bar{\nabla}\cdot\bar{B} = 0 \rightarrow$ APLICADA AO CILINDRO

$$\oint_{S(V)} \vec{B}\cdot\hat{n}\, dS = 0$$

$$\int_{S_2} \vec{B}\cdot\hat{n}\, dS - \int_{S_1} \vec{B}\cdot\hat{n}\, dS + \int_{S_{lat}} \vec{B}\cdot\hat{n}'\, dS = 0$$

CILINDRO TEM BASE CIRCULAR DE RAIO $\underline{a}$ E ALTURA h

$h \ll a \qquad h \to 0,\ a \to 0$

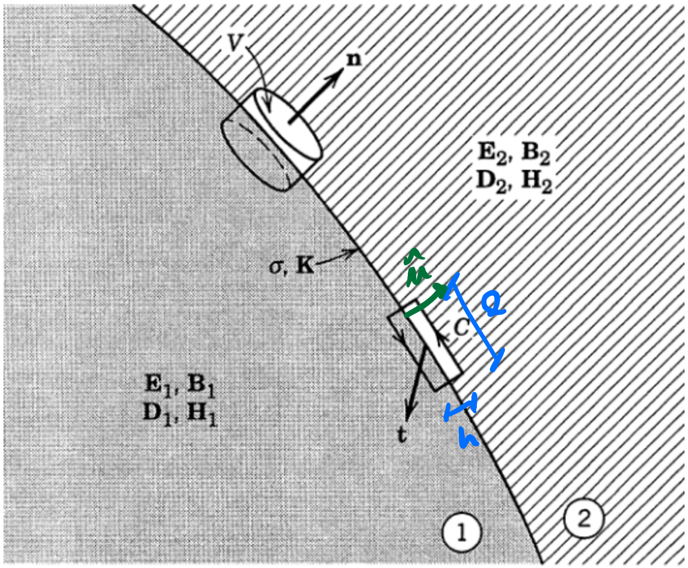
$$\int_{S_{lat}} \vec{B}\cdot\hat{n}'\, dS \sim (\vec{B}\cdot\hat{n}')\, 2\pi a h \to 0$$

$$\int_{S_1, S_2} \vec{B}\cdot\hat{n}\, dS \sim (\vec{B}\cdot\hat{n})\, \pi a^2$$

$$\int_{S_2} \vec{B}\cdot\hat{n}\, dS = \int_{S_1} \vec{B}\cdot\hat{n}\, dS$$

$$\vec{B}_2\cdot\hat{n} = \vec{B}_1\cdot\hat{n} \Rightarrow \boxed{B_{1n} = B_{2n}}$$

Condições de contorno



$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$ AO CIRCUITO $\underline{C}$

$$\oint_C \bar{E} \cdot d\bar{l} = -\frac{d}{dt}\left[\int_{S(C)} \bar{B} \cdot d\vec{S}\right]$$

$h << l \qquad h \to 0 \;, \; l \to 0$

$$\int_{S(C)} \vec{B} \cdot d\vec{S} \sim \cancelto{0}{B h l}$$

$$\oint_C \bar{E} \cdot d\bar{l} = \underbrace{\cancelto{0}{\int_{C_h} \vec{E} \cdot d\bar{l}}}_{\propto h} + \underbrace{\int_{C_l} \bar{E}_2 \cdot d\bar{l} - \int_{C'_l} \bar{E}_1 \cdot d\bar{l}}_{l}$$

$$\Rightarrow \int_{C_l} \bar{E}_2 \cdot d\bar{l} = \int_{C'_l} \bar{E}_1 \cdot d\bar{l}$$

$d\bar{l} \propto \pm \hat{t} \times \hat{n}$

$$\vec{E}_2 \cdot (\hat{t} \times \hat{n}) \cancel{l} = \bar{E}_1 \cdot (\hat{t} \times \hat{n}) \cancel{l} \Rightarrow \hat{t} \cdot (\hat{n} \times \bar{E}_2) = \hat{t} \cdot (\hat{n} \times \bar{E}_1)$$

$$\Rightarrow \hat{n} \times \bar{E}_1 = \hat{n} \times \vec{E}_2 \Rightarrow \boxed{E_{1t} = E_{2t}}$$

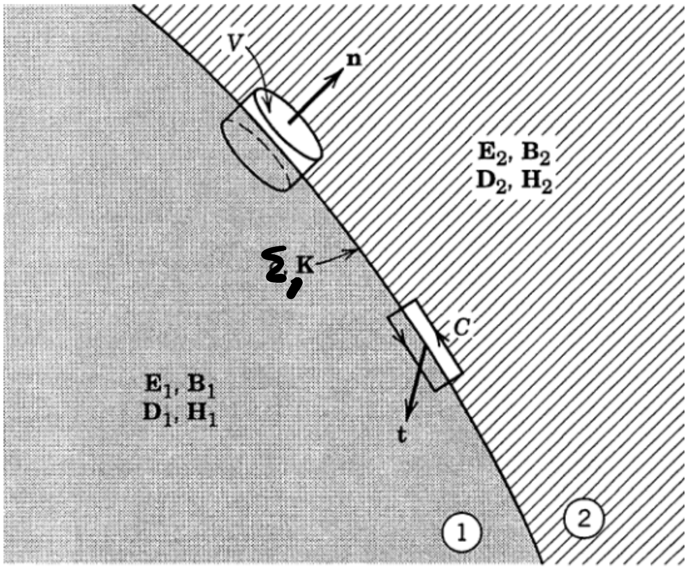
SE O CONDUTOR FOR PERFEITO

$(\sigma = \infty)$ $\vec{E}_c = 0$ $\vec{B}_c = 0$

$\Rightarrow \boxed{E_{tv} = 0}$ $V \rightarrow$ VÁCUO

$\Rightarrow \boxed{B_{nv} = 0}$

Condições de contorno



$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \longrightarrow \text{VOLUME } V$$

$$\int_{S_2}\vec{E}_2\cdot\hat{n}\,ds - \int_{S_1}\vec{E}_1\cdot\hat{n}\,ds + \cancelto{0}{\int_{S_{lat}}\vec{E}\cdot\hat{n}'\,ds}$$

$$\frac{1}{\epsilon_0}Q(V) \xrightarrow[\substack{h\to 0\\ a\to 0}]{} \frac{1}{\epsilon_0}\Sigma(\pi a^2)$$

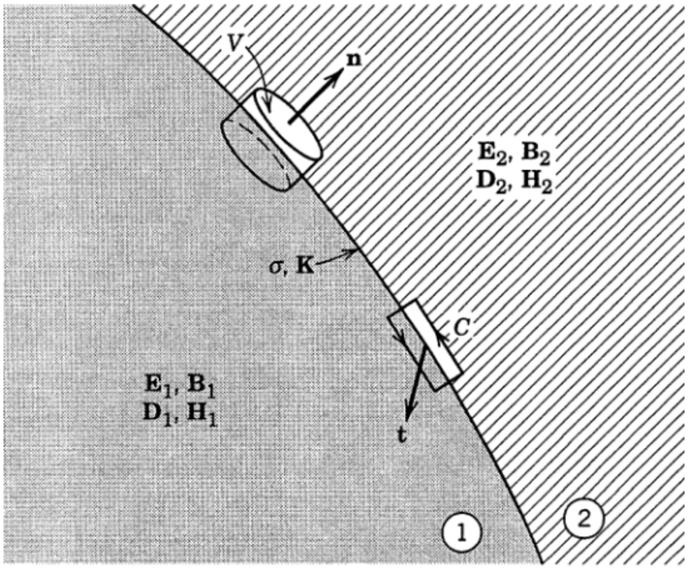
$$(\vec{E}_2\cdot\hat{n})\cancel{(\pi a^2)} - (\vec{E}_1\cdot\hat{n})\cancel{(\pi a^2)} = \frac{\Sigma}{\epsilon_0}\cancel{(\pi a^2)}$$

$$E_{2n} - E_{1n} = \frac{\Sigma}{\epsilon_0}$$

CONDUTOR PERFEITO: $\vec{E}_1 = 0$

$$\Rightarrow \boxed{E_{nV} = \frac{\Sigma}{\epsilon_0}}$$

Condições de contorno



$$\vec\nabla\times\bar B = \mu_0 \bar J + \mu_0\epsilon_0 \frac{\partial \bar E}{\partial t} \longrightarrow \text{CIRCUITO } C$$

$$\int \bar B_2\cdot d\bar l - \int \bar B_1\cdot d\bar l = \mu_0\epsilon_0 \cancelto{0}{\frac{d}{dt}} \underbrace{\left[\int_{S(C)} \bar E\cdot d\bar l\right]}_{hl} + \mu_0 I(S)$$

$$I(S) = (\vec K\cdot\hat t)\,l$$

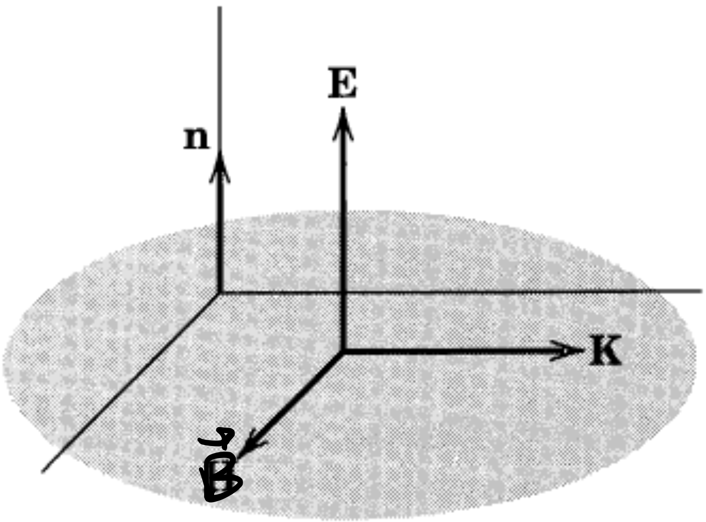
$$\int \bar B_2\cdot d\bar l - \int \bar B_1\cdot d\bar l = \vec B_2\cdot(\hat t\times\hat n)\,l - \bar B_1\cdot(\hat t\times\hat n)\,l$$

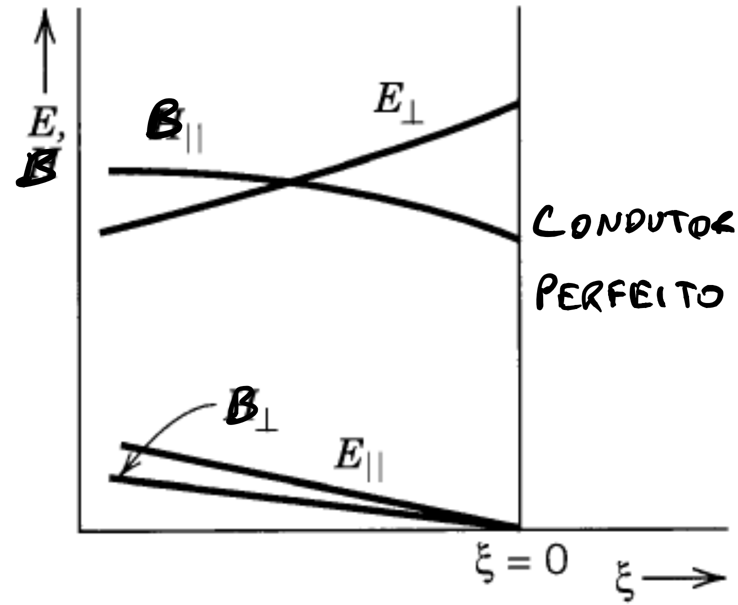
$$\underbrace{(\bar B_2-\bar B_1)\cdot(\hat t\times\hat n)}= \vec K\cdot\hat t\,\mu_0$$

$$\hat t\cdot\left[\hat n\times(\bar B_2-\bar B_1)\right] = \vec K\cdot\hat t\,\mu_0 \Rightarrow \hat n\times(\bar B_2-\bar B_1) = \vec K\mu_0$$

CONDUTOR PERFEITO: $\bar B_1 = 0 \Rightarrow \boxed{\hat n\times\vec B_V = \mu_0\vec K}$

Interface vácuo/condutor perfeito

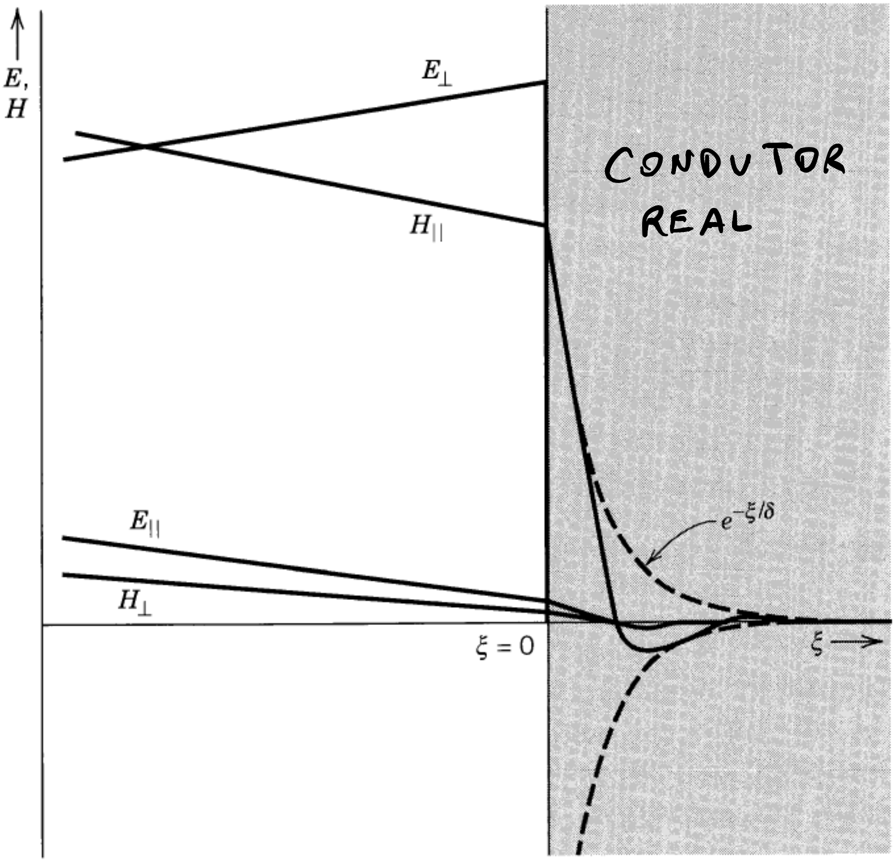




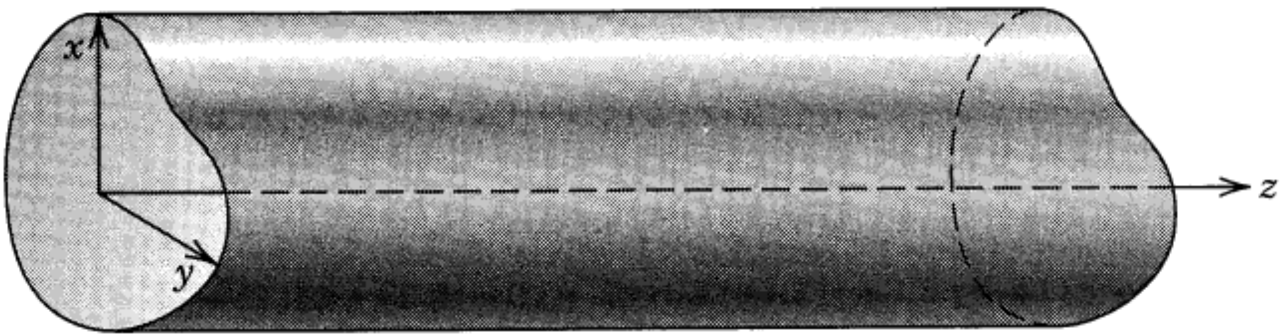
$\vec{E}_{tv} = 0$ $\bar{E}$ SÓ TEM COMPONENTE NORMAL

$B_{nv} = 0$ $\bar{B}$ " " " TANGENCIAL

Interface vácuo/condutor real



Guias de onda



$$\left.\begin{aligned}\bar{E}(\bar{x},t) &= \bar{E}(\bar{x})\, e^{-i\omega t}\\ \bar{B}(\bar{x},t) &= \bar{B}(\bar{x})\, e^{-i\omega t}\end{aligned}\right\} \Rightarrow \quad \begin{aligned}\bar{\nabla}\cdot\bar{E} &= 0 & \bar{\nabla}\times\bar{E} &= i\omega\bar{B} \quad (*)\\ \bar{\nabla}\cdot\bar{B} &= 0 & \bar{\nabla}\times\bar{B} &= -\frac{i\omega}{c^2}\bar{E}\end{aligned}$$

$$\left\{\begin{aligned}\bar{E}(\bar{x}) &= \bar{E}(x,y)\, e^{\pm ikz}\\ \bar{B}(\bar{x}) &= \bar{B}(x,y)\, e^{\pm ikz}\end{aligned}\right.$$

ONDAS PROPAGANTES NA DIREÇÃO $\pm z$

$$\underbrace{\bar{\nabla}\times(\bar{\nabla}\times\bar{E})}_{+\bar{\nabla}(\bar{\nabla}\cdot\bar{E})-\nabla^2\bar{E}} = i\omega\,\bar{\nabla}\times\bar{B} = \frac{\omega^2}{c^2}\bar{E} \quad \left\{\begin{aligned}\nabla^2\bar{E} + \frac{\omega^2}{c^2}\bar{E} &= 0\\ \nabla^2\bar{B} + \frac{\omega^2}{c^2}\bar{B} &= 0\end{aligned}\right.$$

$\bar\nabla = \bar\nabla_t + \bar\nabla_z$ ONDE $\bar\nabla_t = \hat x \dfrac{\partial}{\partial x} + \hat y \dfrac{\partial}{\partial y}$ $\quad \bar\nabla_z = \hat z \dfrac{\partial}{\partial z} = \pm i k \hat z$

$$\nabla^2 = \nabla_t^2 + \nabla_z^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \nabla_t^2 - k^2$$

$$\left.\begin{aligned} &\nabla_t^2 \vec E + \left(\frac{\omega^2}{c^2} - k^2\right)\bar E = 0 \\ &\nabla_t^2 \bar B + \left(\frac{\omega^2}{c^2} - k^2\right)\bar B = 0 \end{aligned}\right\} \quad \left(\nabla_t^2 + \gamma^2\right)\begin{Bmatrix}\vec E \\ \vec B\end{Bmatrix} = 0$$

$\bar E = \bar E_t + \bar E_z$ ONDE $\vec E_t = E_x \hat x + E_y \hat y$ $\quad \vec E_z = E_z \hat z$

$\bar B = \bar B_t + \bar B_z$ $\qquad \bar B_t = B_x \hat x + B_y \hat y$ $\quad \bar B_z = B_z \hat z$

DE MANEIRA GERAL, PARA UM CAMPO $\vec V$ QUALQUER:

$$\bar\nabla \times \bar V = (\bar\nabla_t + \bar\nabla_z)\times(\vec V_t + \bar V_z) = \underbrace{\bar\nabla_t \times \bar V_t}_{\parallel \hat z} + \underbrace{\bar\nabla_t \times \bar V_z + \bar\nabla_z \times \bar V_t}_{\perp \hat z \text{ (TRANSVERSAL)}} + \cancelto{0}{\bar\nabla_z \times \bar V_z}$$

$$\bar\nabla \cdot \bar V = \bar\nabla_t \cdot \bar V + \bar\nabla_z \cdot \bar V = \bar\nabla_t \cdot \bar V_t + \bar\nabla_z \cdot \bar V_z$$

$$\bar{\nabla}\times\bar{E} = i\omega\bar{B} : \qquad \bar{\nabla}_t\times\bar{E}_t = i\omega B_z \quad (1)$$

$$\bar{\nabla}_t\times\bar{E}_z + \bar{\nabla}_z\times\bar{E}_t = i\omega\bar{B}_t \quad (2)$$

$$\bar{\nabla}\times\bar{B} = -i\frac{\omega}{c^2}\bar{E} : \qquad \bar{\nabla}_t\times\bar{B}_t = -i\frac{\omega}{c^2}\bar{E}_z \quad (3)$$

$$\bar{\nabla}_t\times\bar{B}_z + \bar{\nabla}_z\times\vec{B}_t = -i\frac{\omega}{c^2}\bar{E}_t \quad (4)$$

$$\bar{\nabla}\cdot\bar{E} = 0 \qquad \bar{\nabla}_t\cdot\bar{E}_t = -\nabla_z E_z = \mp ik E_z \quad (5)$$

$$\bar{\nabla}\cdot\bar{B} = 0 \qquad \bar{\nabla}_t\cdot\bar{B}_t = -\nabla_z B_z = \mp ik B_z \quad (6)$$

DE (2) E (4):

$$\gamma^2 \bar{B}_t = i\left[\pm k\, \bar{\nabla}_t B_z + \frac{\omega}{c^2}\hat{z}\times\bar{\nabla}_t E_z\right] \quad (7)$$

$$\gamma^2 \bar{E}_t = i\left[\pm k\, \bar{\nabla}_t E_z - \omega\, \hat{z}\times\bar{\nabla}_t B_z\right] \quad (8)$$

ONDE $\gamma^2 \equiv \frac{\omega^2}{c^2} - k^2$

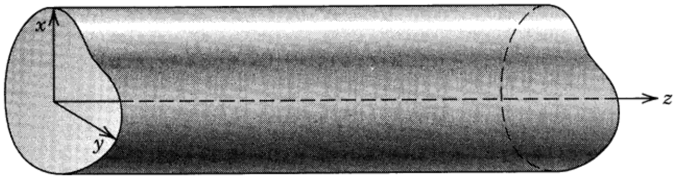
AS COMPONENTES TRANSVERSAIS $\bar{E}_t$ E $\bar{B}_t$ SÃO COMPLETAMENTE DETERMINADAS SE EU SOUBER E_z E B_z

MAS E_z E B_z SATISFAZEM:

$$(\nabla_t^2 + \gamma^2) E_z = 0 \Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \gamma^2\right)\begin{Bmatrix} E_z \\ B_z \end{Bmatrix} = 0$$

$$(\nabla_t^2 + \gamma^2) B_z = 0 \Rightarrow$$

Resumo



(2) $$\frac{\partial \mathbf{E}_t}{\partial z} + i\omega\hat{\mathbf{z}} \times \mathbf{B}_t = \nabla_t E_z,$$ $$\hat{\mathbf{z}} \cdot (\nabla_t \times \mathbf{E}_t) = i\omega B_z$$ (1)

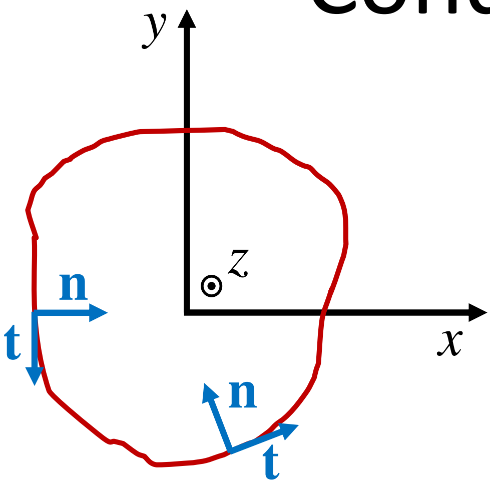
(4) $$\frac{\partial \mathbf{B}_t}{\partial z} - i\mu\epsilon\omega\hat{\mathbf{z}} \times \mathbf{E}_t = \nabla_t B_z,$$ $$\hat{\mathbf{z}} \cdot (\nabla_t \times \mathbf{B}_t) = -i\mu\epsilon\omega E_z$$ (3)

(5) $$\nabla_t \cdot \mathbf{E}_t = -\frac{\partial E_z}{\partial z},$$ $$\nabla_t \cdot \mathbf{B}_t = -\frac{\partial B_z}{\partial z}$$ (6)

$$\mathbf{B}_t = \frac{i}{(\mu\epsilon\omega^2 - k^2)}[k\nabla_t B_z + \mu\epsilon\omega\,\hat{\mathbf{z}} \times \nabla_t E_z]$$ (7)

$$\mathbf{E}_t = \frac{i}{(\mu\epsilon\omega^2 - k^2)}[k\nabla_t E_z - \omega\hat{\mathbf{z}} \times \nabla_t B_z]$$ (8)

Condições de contorno



$\hat{n}$ APONTA PARA DENTRO DA SEÇÃO RETA

$\hat{t} = \hat{n} \times \hat{z}$

CONDIÇÕES DE CONTORNO:

$E_{\parallel}\big|_S \to 0 \Rightarrow \boxed{E_z\big|_S = 0}$ E $\hat{t}\cdot\vec{E}\big|_S = 0$

$B_n\big|_S = 0 \Rightarrow \vec{B}\cdot\hat{n}\big|_S = 0$

DIRICHLET

DE (4), PODE-SE PROVAR QUE:

$$\frac{\partial B_z}{\partial n}\bigg|_S = (\hat{n}\cdot\vec{\nabla}_t)B_z\Big|_S = 0 \qquad \text{(NEUMANN)}$$

$$(\nabla_t^2 + \gamma^2) E_z = 0 \qquad E_z\big|_S = 0$$

PROBLEMA ANÁLOGO AO POÇO INFINITO 2D DA MECÂNICA QUÂNTICA

$$(\nabla_t^2 + \gamma^2) B_z = 0 \qquad \frac{\partial B_z}{\partial n}\bigg|_S = 0$$

AMBAS SÃO EQUAÇÕES DE AUTO-VALORES E AUTO-VETORES:

$$\gamma \to \gamma_\lambda \to \text{VALORES DISCRETOS}$$

$$\gamma_\lambda^2 = \frac{\omega^2}{c^2} - k^2 \Rightarrow \omega = \sqrt{c^2\gamma_\lambda^2 + k^2c^2} \neq \omega = ck$$

$$\omega = c\sqrt{\gamma_\lambda^2 + k^2}$$

Resumo

$$(\nabla_t^2 + \gamma^2)\psi = 0 \qquad \gamma^2 = \frac{\omega^2}{c^2} - k^2$$

TRANSVERSE MAGNETIC (TM) WAVES

$$B_z = 0 \text{ everywhere; boundary condition, } E_z|_S = 0$$

TRANSVERSE ELECTRIC (TE) WAVES

$$E_z = 0 \text{ everywhere; boundary condition, } \left.\frac{\partial B_z}{\partial n}\right|_S = 0$$

Modos TEM

MODOS TEM : $E_z = B_z = 0$

DE (1) : $\vec{\nabla}_t \times \vec{E}_t = 0$
DE (5) : $\vec{\nabla}_t \cdot \vec{E}_t = 0$
COMO NA ELETROSTÁTICA!

$\vec{E}_t = -\vec{\nabla}_t \Phi_t \Rightarrow \nabla_t^2 \Phi_t = 0$

$E_t|_S = 0 \Rightarrow \underline{S}$ É UMA EQUIPOTENCIAL, $\Phi_t|_S = \text{CONST.}$

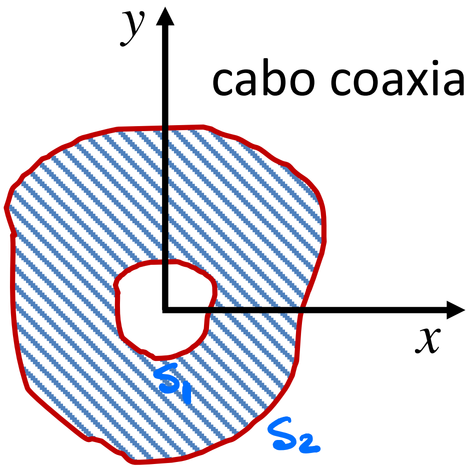
DENTRO DA SEÇÃO RETA: $\Phi_t = \text{CONST}$ É SOLUÇÃO

$\Rightarrow \vec{E}_t = 0$

DE (2) $\Rightarrow \vec{B}_t = 0$

NÃO HÁ MODOS TEM EM GUIAS DE ONDAS CILÍNDRICOS!

ENTRETANTO, PARA CABOS COAXIAIS



$\Phi_t = \Phi_1$ NA SUPERFÍCIE S_1

$\Phi_t = \Phi_2 \neq \Phi_1$ " " S_2

$\Rightarrow$ SOLUÇÃO NÃO TRIVIAL

$\Rightarrow$ PODE HAVER MODOS TEM !

NOTE QUE OS MODOS TEM:

$$\nabla_t^2 \Phi_t = 0$$

TOMANDO $-\bar{\nabla}_t \Rightarrow \nabla_t^2 (\underbrace{-\bar{\nabla}_t \Phi_t}_{\bar{E}_t}) = 0$

$$\Rightarrow \boxed{\nabla_t^2 \bar{E}_t = 0}$$

MAS TÍNHAMOS $(\nabla_t^2 + \gamma^2)\bar{E} = 0 \Rightarrow \boxed{\gamma = 0}$

$$\Rightarrow \gamma^2 = \frac{\omega^2}{c^2} - \vec{k} \Rightarrow \boxed{\omega = ck}$$

AS MESMAS CARACTERÍSTICAS DA PROPAGAÇÃO NO VÁCUO.

DA (2): $\bar{B}_t = \pm \frac{1}{c} \hat{z} \times \bar{E}_t$