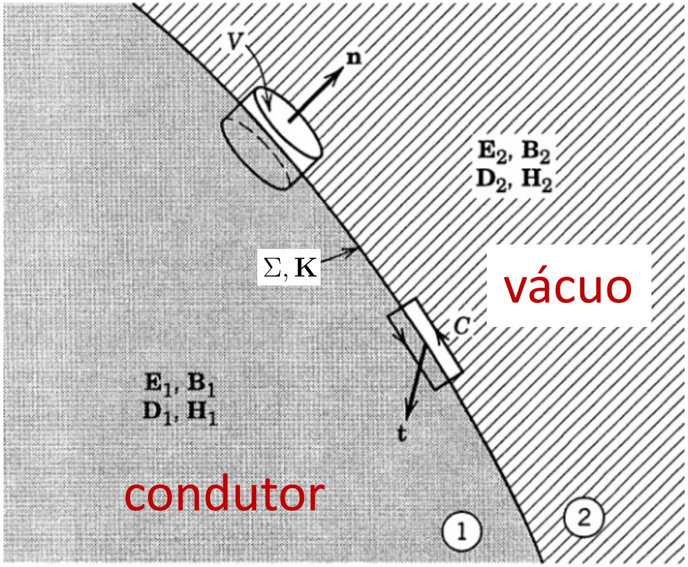
FI 008 – Eletrodinâmica I

1º Semestre de 2021
22/04/2021
Aula 11

Guias de ondas e cavidades ressonantes

Condições de contorno



$$
\begin{aligned}
\hat{\mathbf{n}} \cdot \mathbf{B}_2 &= \hat{\mathbf{n}} \cdot \mathbf{B}_1 \\
\hat{\mathbf{n}} \times \mathbf{E}_2 &= \hat{\mathbf{n}} \times \mathbf{E}_1 \\
\hat{\mathbf{n}} \cdot \mathbf{E}_2 - \hat{\mathbf{n}} \cdot \mathbf{E}_1 &= \frac{\Sigma}{\epsilon_0} \\
\hat{\mathbf{n}} \times \mathbf{B}_2 - \hat{\mathbf{n}} \times \mathbf{B}_1 &= \mu_0 \mathbf{K}
\end{aligned}
$$

Se o meio 1 é um condutor perfeito:

$$\mathbf{E}_1 = \mathbf{E}_c = 0; \;\; \mathbf{B}_1 = \mathbf{B}_c = 0$$

$$
\boxed{
\begin{aligned}
\hat{\mathbf{n}} \cdot \mathbf{B}_V &= 0 \\
\hat{\mathbf{n}} \times \mathbf{E}_V &= 0 \\
\hat{\mathbf{n}} \cdot \mathbf{E}_V &= \frac{\Sigma}{\epsilon_0} \\
\hat{\mathbf{n}} \times \mathbf{B}_V &= \mu_0 \mathbf{K}
\end{aligned}
}
$$

Condições de contorno

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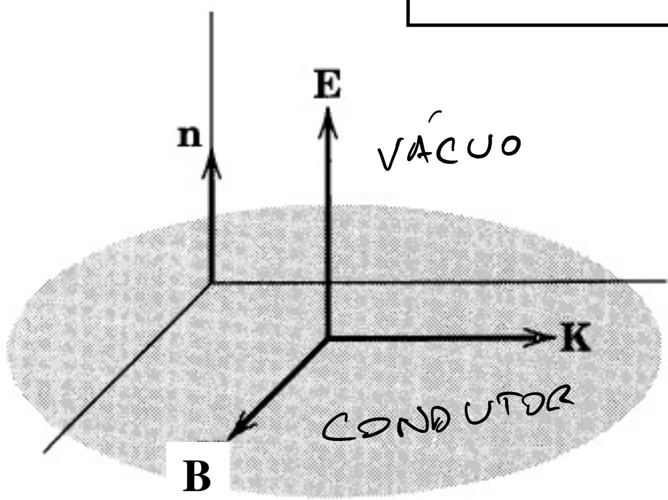
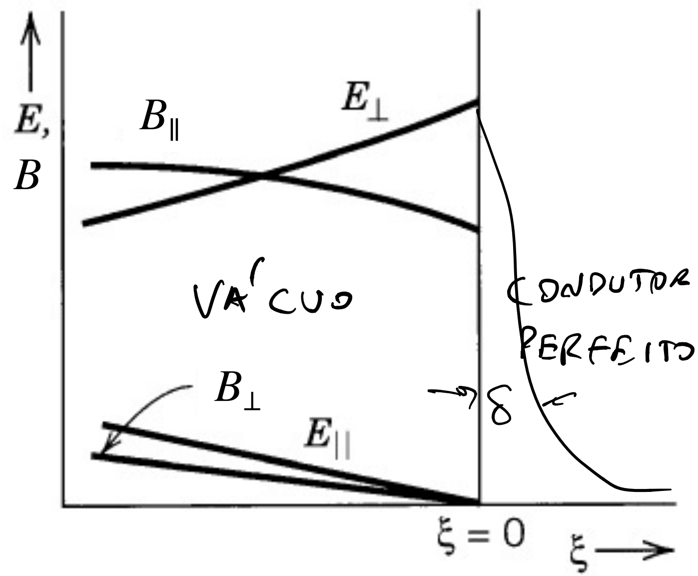
\end{aligned}$$

$\delta \sim \frac{1}{\sqrt{\sigma\omega}}$

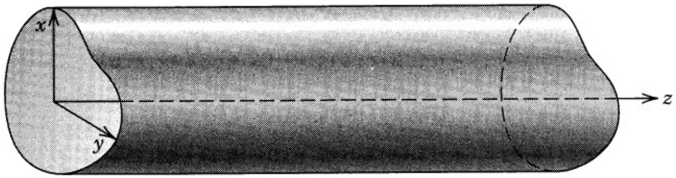




Guias de ondas

$z \pm ct$

$k(z \pm ct) = kz \pm \omega t$



$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}(x,y)\,e^{i(\pm kz-\omega t)}$$
$$\mathbf{B}(\mathbf{x},t) = \mathbf{B}(x,y)\,e^{i(\pm kz-\omega t)}$$

$$\boldsymbol{\nabla}_t = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y}$$

Das equações de Maxwell:

$$\boxed{\begin{aligned}\left(\nabla_t^2+\gamma^2\right)\mathbf{E}(x,y) &= 0\\ \left(\nabla_t^2+\gamma^2\right)\mathbf{B}(x,y) &= 0\end{aligned}}$$

$$\boxed{\gamma^2 = \frac{\omega^2}{c^2} - k^2}$$

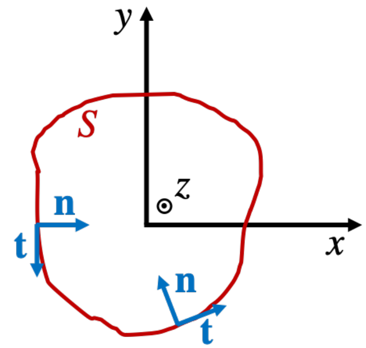
$$\mathbf{E} = \mathbf{E}_t + E_z\hat{\mathbf{z}}$$
$$\mathbf{B} = \mathbf{B}_t + B_z\hat{\mathbf{z}}$$

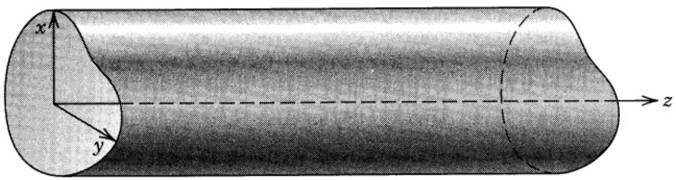
$$\boxed{\begin{aligned}\gamma^2\mathbf{B}_t &= i\left[\pm k\boldsymbol{\nabla}_t B_z + \frac{\omega}{c^2}\hat{\mathbf{z}}\times\boldsymbol{\nabla}_t E_z\right]\\ \gamma^2\mathbf{E}_t &= i\left[\pm k\boldsymbol{\nabla}_t E_z - \omega\hat{\mathbf{z}}\times\boldsymbol{\nabla}_t B_z\right]\end{aligned}}$$

As componentes transversais são completamente determinadas pelas componentes z: basta achar E_z e B_z

Seção transversal

Guias de ondas





$$\nabla_t^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

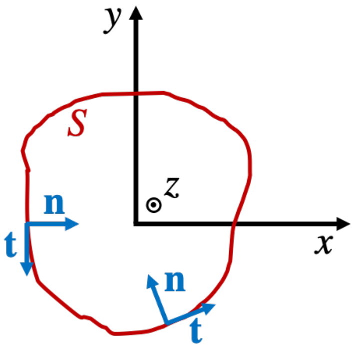
E_z e B_z satisfazem a Equação de Helmholtz na seção transversal do guia:

$$\boxed{\begin{aligned} \left(\nabla_t^2 + \gamma^2\right) E_z &= 0 \\ \left(\nabla_t^2 + \gamma^2\right) B_z &= 0 \end{aligned}}$$

E_z e B_z satisfazem as seguintes condições de contorno no perímetro da seção transversal:

$$\boxed{\begin{aligned} E_z|_S &= 0 \\ \hat{\mathbf{n}} \cdot \boldsymbol{\nabla}_t B_z|_S &= 0 \end{aligned}}$$

Seção transversal



Guias de ondas

Os modos TM (B_z=0) e TE (E_z=0) são independentes e devem ser resolvidos separadamente: são problemas de auto-valores.

Modo TM

$$\left(\nabla_t^2 + \gamma^2\right) E_z \;=\; 0$$

$$E_z\big|_S \;=\; 0$$

$$\gamma^2 \mathbf{E}_t \;=\; \pm ik \boldsymbol{\nabla}_t E_z$$

$$\mathbf{B}_t \;=\; \pm \frac{\omega}{c^2 k} \hat{\mathbf{z}} \times \mathbf{E}_t$$

$\gamma \to \gamma_\lambda$ QUANTIZADO

Modo TE

$$\left(\nabla_t^2 + \gamma^2\right) B_z \;=\; 0$$

$$\hat{\mathbf{n}} \cdot \boldsymbol{\nabla}_t B_z\big|_S \;=\; 0$$

$$\gamma^2 \mathbf{B}_t \;=\; \pm ik \boldsymbol{\nabla}_t B_z$$

$$\mathbf{E}_t \;=\; \mp \frac{\omega}{k} \hat{\mathbf{z}} \times \mathbf{B}_t$$

$\gamma \to \gamma_\lambda$ QUANTIZADOS

Modos TEM

$E_z = B_z = 0$?

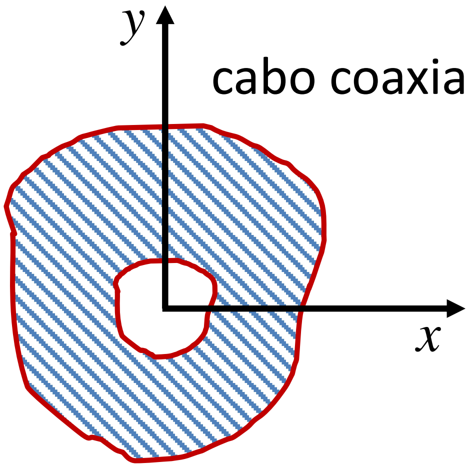
- Não existem modos TEM em guias cilíndricos ocos (E_z=B_z=0).
- Modos TEM podem ser propagados em guias como cabos coaxiais.
- Nesse caso:

$$\nabla_t \cdot \mathbf{E}_{\mathrm{TEM}} = 0$$

$$\nabla_t \times \mathbf{E}_{\mathrm{TEM}} = 0$$

$$\mathbf{B}_{\mathrm{TEM}} = \pm \frac{1}{c} \hat{\mathbf{z}} \times \mathbf{E}_{\mathrm{TEM}}$$

$$\boxed{\omega = ck}$$



EM 1D: EQ. DE ONDA $\frac{\partial^2 f}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0$

SOLUÇÃO GERAL:

$$f_G(z,t) = F(z-ct) + G(z+ct)$$

$$f(z,t) = e^{i(\pm kz - \omega t)} = e^{ik(\pm z - ct)} \qquad \omega = ck$$

$$f_G(z,t) = \int dk\, A(k)\, e^{ik(z-ct)}$$

$$+ \int dk\, B(k)\, e^{-ik(z+ct)}$$

$$= F(z-ct) + G(z+ct)$$

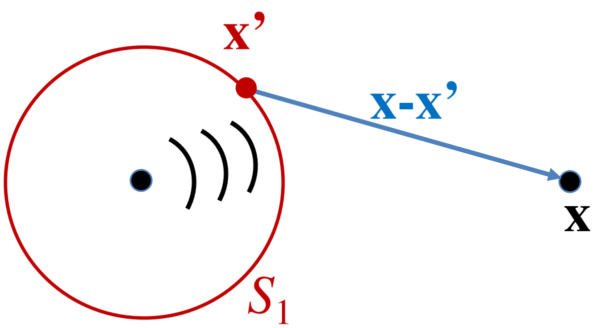
Fórmula integral de Kirchhoff e princípio de Huygens

Para cada componente de campo com frequência ω : $\sim e^{-i\omega t}$

$$\left[\nabla^2 + \left(\frac{\omega}{c}\right)^2\right]\psi = 0$$

vale:

$$\psi(\mathbf{x}) = -\frac{1}{4\pi}\oint_{S_1} \frac{e^{i\omega|\mathbf{x}-\mathbf{x}'|/c}}{|\mathbf{x}-\mathbf{x}'|}\left[\boldsymbol{\nabla}'\psi(\mathbf{x}') + \left(\frac{i\omega}{c} - \frac{1}{|\mathbf{x}-\mathbf{x}'|}\right)\left(\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|}\right)\psi(\mathbf{x}')\right]\cdot d\mathbf{S}'$$



Mas note que, diferentemente do princípio de Huygens original, o campo em $\mathbf{x}$ depende do campo e de sua derivada espacial na superfície S_1.