

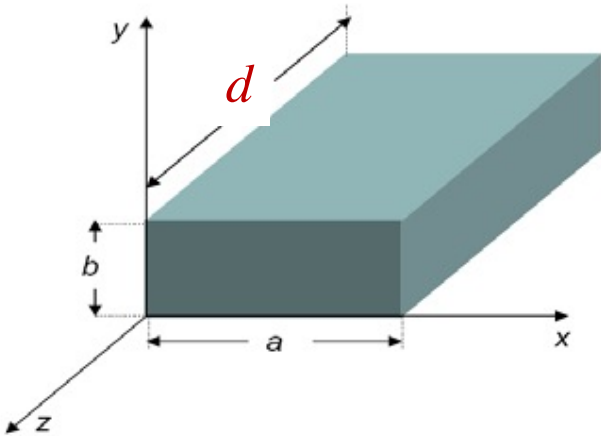
FI 008 – Eletrodinâmica I

1º Semestre de 2021

06/05/2021

Aula 14

Aula passada



Cavidades ressonantes como guias de ondas com “tampas” em $z=0$ e $z=d$:
novas condições de contorno.

$$\begin{aligned} \mathbf{E}_t|_{z=0,d} &= 0 \\ B_z|_{z=0,d} &= 0 \end{aligned}$$

O confinamento na direção z transforma a onda propagante do guia numa **onda estacionária na cavidade**: $e^{\pm ikz} \rightarrow A \cos(kz) + B \sin(kz)$

As novas condições de contorno “quantizam” o **vetor de onda k** e impõem **valores discretos de frequências**:

$$k = \frac{p\pi}{d}, \text{ onde } p = \begin{cases} 0, 1, 2, \dots \\ 1, 2, 3, \dots \end{cases} \quad \omega_{\lambda,p} = c \left[\gamma_{\lambda}^2 + \left(\frac{p\pi}{d} \right)^2 \right]^{1/2}$$

Aula passada

Modos TM:

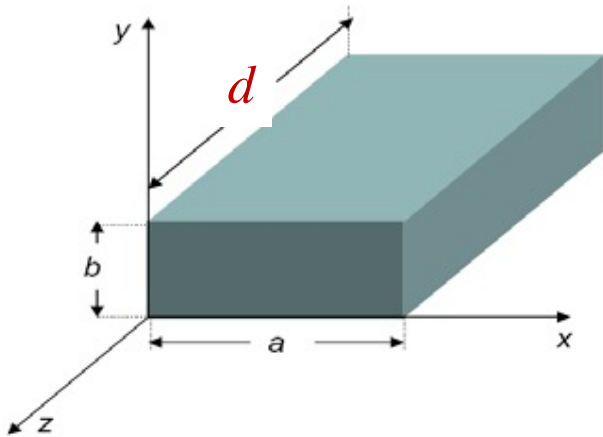
$$E_z(\mathbf{x}, t) = E_z(x, y) \cos\left(\frac{p\pi z}{d}\right) e^{-i\omega t} \quad (p = 0, 1, 2, 3, \dots)$$
$$\mathbf{E}_t(\mathbf{x}, t) = -\frac{p\pi}{d\gamma^2} \sin\left(\frac{p\pi z}{d}\right) \nabla_t E_z(x, y) e^{-i\omega t}$$
$$\mathbf{B}_t(\mathbf{x}, t) = \frac{i\omega}{\gamma^2 c^2} \cos\left(\frac{p\pi z}{d}\right) \hat{\mathbf{z}} \times \nabla_t E_z(x, y) e^{-i\omega t}$$

Modos TE:

$$B_z(\mathbf{x}, t) = B_z(x, y) \sin\left(\frac{p\pi z}{d}\right) e^{-i\omega t} \quad (p = 1, 2, 3, \dots)$$
$$\mathbf{B}_t(\mathbf{x}, t) = \frac{p\pi}{d\gamma^2} \cos\left(\frac{p\pi z}{d}\right) \nabla_t B_z(x, y) e^{-i\omega t}$$
$$\mathbf{E}_t(\mathbf{x}, t) = -\frac{i\omega}{\gamma^2} \sin\left(\frac{p\pi z}{d}\right) \hat{\mathbf{z}} \times \nabla_t B_z(x, y) e^{-i\omega t}$$

$$\omega_{\lambda,p} = c \left[\gamma_\lambda^2 + \left(\frac{p\pi}{d}\right)^2 \right]^{1/2}$$

Aula passada



Seção reta retangular:

$$\omega_{m,n,p} = \pi c \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 + \left(\frac{p}{d} \right)^2 \right]^{1/2}$$

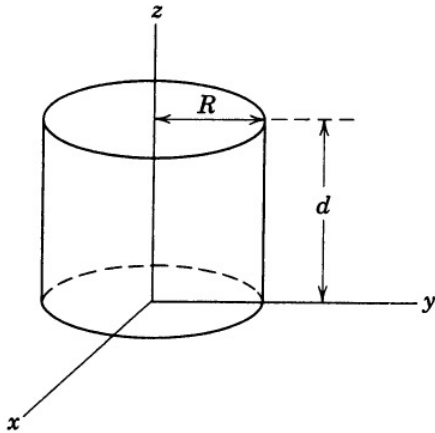
$$\text{TM} : \begin{cases} m = 1, 2, 3, \dots \\ n = 1, 2, 3, \dots \\ p = 0, 1, 2, \dots \end{cases}$$

$$\text{TE} : \begin{cases} m = 0, 2, 3, \dots \\ n = 0, 2, 3, \dots \\ m + n \neq 0 \\ p = 1, 2, 3, \dots \end{cases}$$

$$\omega_{1,1,0}^{\text{TM}} = \pi c \left[\frac{1}{a^2} + \frac{1}{b^2} \right]^{1/2}$$

$$\omega_{1,0,1}^{\text{TE}} = \pi c \left[\frac{1}{a^2} + \frac{1}{d^2} \right]^{1/2} \quad (a > b)$$

Cavidade cilíndrica



Modos TM:

$$E_z = E_0 J_m \left(\frac{x_{mn} \rho}{R} \right) e^{\pm im\phi} e^{-i\omega t}, \quad (m = 0, 1, 2, \dots \quad n = 1, 2, 3, \dots)$$

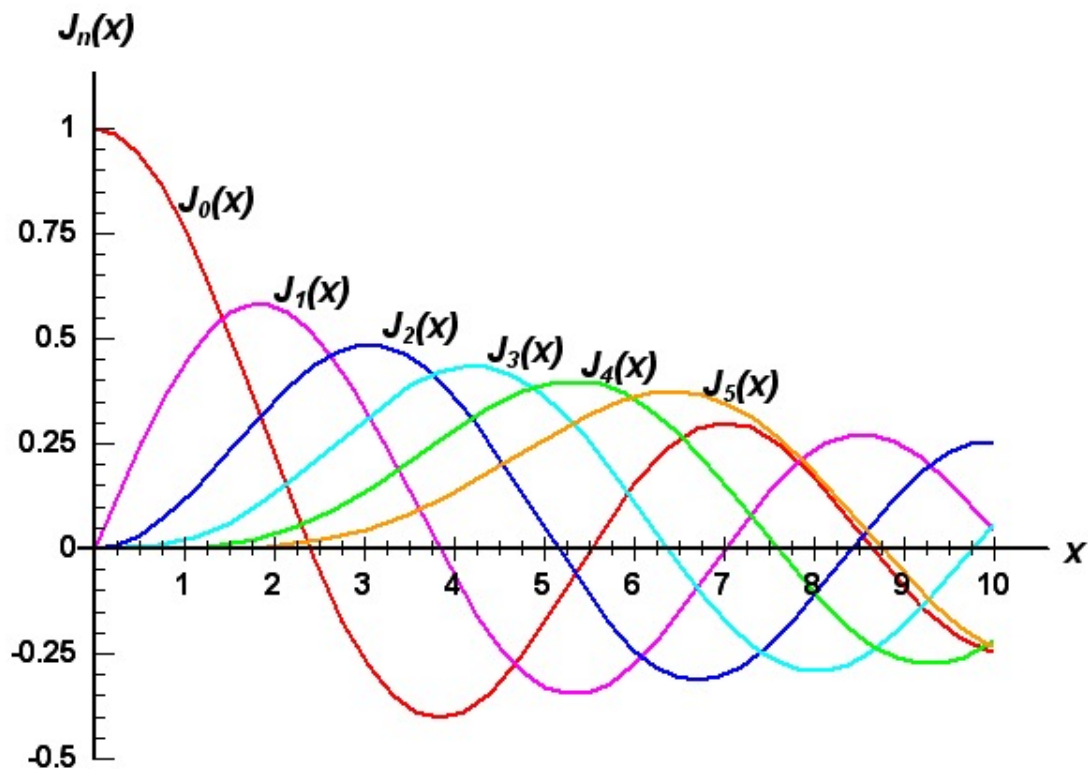
$$\omega_{mnp}^{\text{TM}} = c \left[\frac{x_{mn}^2}{R^2} + \left(\frac{p\pi}{d} \right)^2 \right]^{1/2}$$

$$\text{TM} : \begin{cases} m = 0, 1, 2, \dots \\ n = 1, 2, 3, \dots \\ p = 0, 1, 2, \dots \end{cases}$$

Modos TE: $B_z = B_0 J_m \left(\frac{x'_{mn} \rho}{R} \right) e^{\pm im\phi} e^{-i\omega t}, \quad (m = 0, 1, 2, \dots \quad n = 1, 2, 3, \dots)$

$$\omega_{mnp}^{\text{TE}} = c \left[\frac{x'_{mn}{}^2}{R^2} + \left(\frac{p\pi}{d} \right)^2 \right]^{1/2} \quad \text{TE} : \begin{cases} m = 0, 1, 2, \dots \\ n = 1, 2, 3, \dots \\ p = 1, 2, 3, \dots \end{cases}$$

Funções de Bessel do 1o. tipo



Raíces de $J_\nu(x) = 0$

$$\nu = 0, \quad x_{0n} = 2.405, 5.520, 8.654, \dots$$

$$\nu = 1, \quad x_{1n} = 3.832, 7.016, 10.173, \dots$$

$$\nu = 2, \quad x_{2n} = 5.136, 8.417, 11.620, \dots$$

Roots of $J'_m(x) = 0$

$$m = 0: \quad x'_{0n} = 3.832, 7.016, 10.173, \dots$$

$$m = 1: \quad x'_{1n} = 1.841, 5.331, 8.536, \dots$$

$$m = 2: \quad x'_{2n} = 3.054, 6.706, 9.970, \dots$$

$$m = 3: \quad x'_{3n} = 4.201, 8.015, 11.336, \dots$$

Modo TM mais baixo

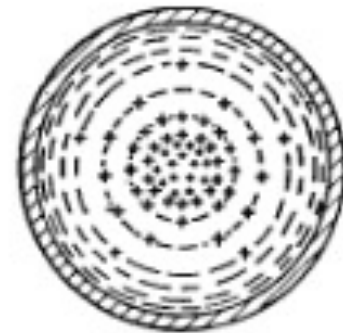
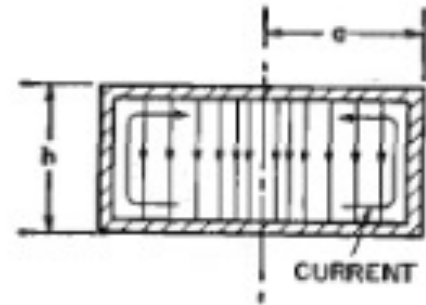
TM₀₁₀

$$E_z = E_0 J_0 \left(\frac{2.405\rho}{R} \right) e^{-i\omega t}$$

$$B_\phi = -\frac{i}{c} E_0 J_1 \left(\frac{2.405\rho}{R} \right) e^{-i\omega t}$$

$$\omega_{010}^{\text{TM}} = \frac{cx_{01}}{R}$$

TM₀₁₀ CYLINDER $R=a, h=d$



Campo elétrico **E**: linhas contínuas
Campo magnético **B**: linhas tracejadas

Moto TE mais baixo

$$\omega_{111}^{\text{TE}} = c \left[\frac{x_{11}^{\prime 2}}{R^2} + \left(\frac{\pi}{d} \right)^2 \right]^{1/2}$$

TE_{111}

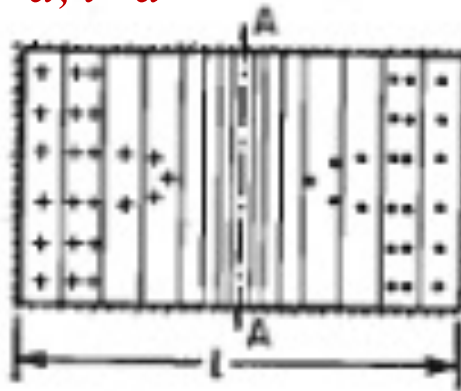
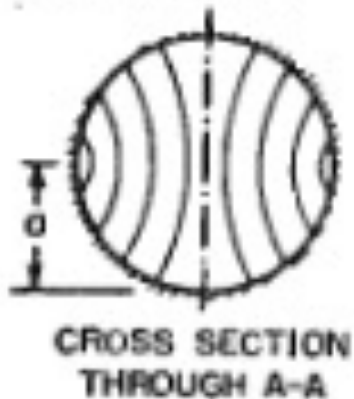
$$B_z = B_0 J_1 \left(\frac{1.841\rho}{R} \right) \cos \phi \sin \left(\frac{\pi z}{d} \right) e^{-i\omega t}$$

$$\mathbf{E}_t = -\frac{i\omega}{\gamma_{11}^2} \hat{\mathbf{z}} \times \nabla_t B_z$$

$$\mathbf{B}_t = \frac{1}{\gamma_{11}^2} \nabla_t \frac{\partial B_z}{\partial z}$$

TE_{111} , CYLINDER

$$R=a, l=d$$



Campo elétrico **E**: linhas contínuas

Campo magnético **B**: linhas tracejadas

Atenuação em cavidades ressonantes

Decaimento dos campos:

$$\begin{aligned}\omega_0 &\rightarrow (\omega_0 + \Delta\omega_0) - i\frac{\Gamma}{2} \\ e^{-i\omega_0 t} &\rightarrow e^{-i(\omega_0 + \Delta\omega_0)t} e^{-\Gamma t/2}\end{aligned}$$

Decaimento da energia armazenada:

$$U(t) = U_0 e^{-\Gamma t} \Rightarrow \Gamma = -\frac{1}{U} \frac{dU}{dt}$$

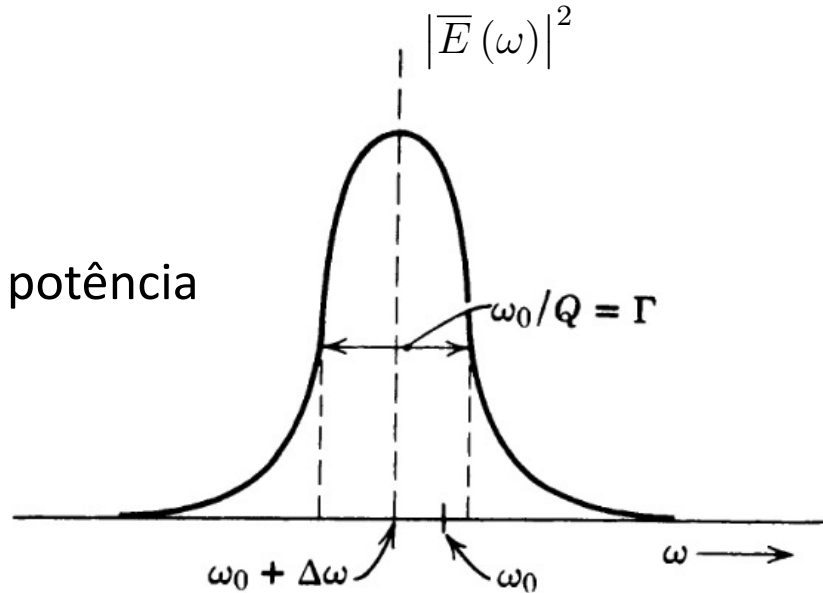
Fator de qualidade da cavidade: número de oscilações antes de decair

$$Q = \frac{\omega_0}{\Gamma}$$

$$E(t) = E_0 e^{-i(\omega_0 + \Delta\omega_0)t} e^{-\Gamma t/2}$$

$$|\bar{E}(\omega)|^2 \sim \frac{1}{(\omega - \omega_0 - \Delta\omega_0)^2 + (\Gamma/2)^2}$$

Espectro de potência



Energia armazenada em cavidades

$$U_{TM} = \frac{d\varepsilon_0}{4} \left[1 + \left(\frac{p\pi}{\gamma_\lambda d} \right)^2 \right] \int_{S.R.} |E_z|^2 da \quad (p \neq 0),$$

$$U_{TM} = \frac{d\varepsilon_0}{2} \int_{S.R.} |E_z|^2 da \quad (p = 0),$$

$$U_{TE} = \frac{d}{4\mu_0} \left[1 + \left(\frac{p\pi}{\gamma_\lambda d} \right)^2 \right] \int_{S.R.} |B_z|^2 da.$$

Perdas ôhmicas nas paredes:

$$\frac{dP_{loss}}{da} = \frac{1}{2\mu_0^2\sigma\delta} |B_{\parallel V}|^2 \quad \delta = \sqrt{\frac{2}{\mu_c\sigma\omega}}$$

Modos TM:

$$-\frac{dU}{dt} = \frac{1}{2\mu_0^2\sigma\delta_\lambda} \frac{1}{c^2\gamma_\lambda^2} \left[1 + \left(\frac{p\pi}{\gamma_\lambda d} \right)^2 \right] \left\{ \frac{d}{2} (1 + \delta_{p,0}) \oint \left| \frac{\partial E_z}{\partial n} \right|^2 dl + 2\gamma_\lambda^2 \int_{S.R.} |E_z|^2 da \right\}$$

Modos TE:

$$-\frac{dU}{dt} = \frac{1}{2\mu_0^2\sigma\delta_\lambda} \left\{ \frac{d}{2} \oint \left[|B_z|^2 + \left(\frac{p\pi}{\gamma_\lambda^2 d} \right)^2 |\hat{\mathbf{n}} \times \nabla_t B_z|^2 \right] dl + 2 \left(\frac{p\pi}{\gamma_\lambda^2 d} \right)^2 \int_{S.R.} |\nabla_t B_z|^2 da \right\}$$

Intuição física sobre Q

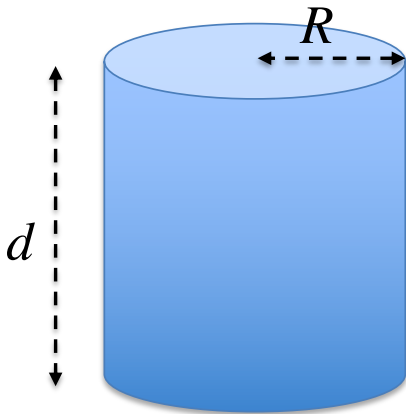
$$Q \propto \frac{V}{S\delta}$$

V = volume da cavidade

S = área total da superfície da cavidade

δ = penetração pelicular (“skin depth”)

$S\delta$ = volume “penetrado” pelos campos.



$$V = \pi R^2 d$$

$$S = 2\pi R^2 + 2\pi R d = 2\pi R (R + d)$$

$$\frac{V}{S\delta} = \frac{Rd}{2\delta (R + d)} \sim \frac{l}{\delta} \sim 10^4 \text{ (Cu, 1 GHz)}$$