FI 008 – Eletrodinâmica I

1º Semestre de 2021

11/05/2021

Aula 15

Radiação eletromagnética de distribuições localizadas de correntes

Radiação

Fontes harmônicas:

$$\rho(\mathbf{x},t) = \rho(\mathbf{x})\, e^{-i\omega t}$$

$$\mathbf{J}(\mathbf{x},t) = \mathbf{J}(\mathbf{x})\, e^{-i\omega t}$$

$$\boldsymbol{\nabla}\cdot\ \mathbf{J} = i\omega\rho$$

Notação complexa!

Soluções harmônicas:

$$\mathbf{A}(\mathbf{x},t) = \mathbf{A}(\mathbf{x})\, e^{-i\omega t}$$

$$\mathbf{B}(\mathbf{x},t) = \mathbf{B}(\mathbf{x})\, e^{-i\omega t}$$

$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}(\mathbf{x})\, e^{-i\omega t}$$

Solução geral:

$$\mathbf{A}(\mathbf{x},t) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{x}',t-R/c)}{R} d^3x'$$

$$\boxed{R = |\mathbf{x}-\mathbf{x}'|}$$

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{x}')\, e^{ikR}}{R} d^3x'$$

$$k = \frac{\omega}{c}$$

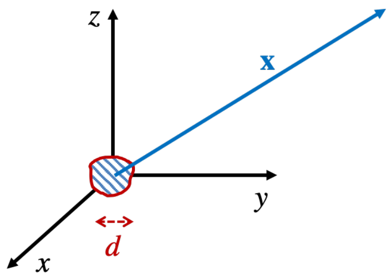
Região próxima

$R = |\vec{x} - \vec{x}'|$

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}') e^{ikR}}{R} d^3x'$$

Hipóteses:

- Fontes pequenas: $d \ll r$
- Região próxima ou estática: $r \ll \lambda$



$$\begin{cases} d \ll r = |\mathbf{x}| \\ d \ll \lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega} \\ r \ll \lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega} \end{cases}$$

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

Como na magnetostática mas não é estática!

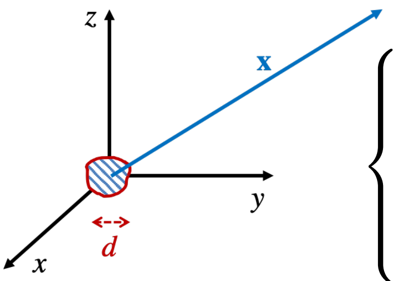
Região de radiação

$R = |\vec{x} - \vec{x}'|$

Hipóteses:

- Fontes pequenas: $d \ll r$
- Região de radiação: $r \gg \lambda$

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{x}')\, e^{ikR}}{R} d^3x'$$



$$\begin{cases} d \ll r = |\mathbf{x}| \\ d \ll \lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega} \\ r \gg \lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega} \end{cases}$$

$$\boxed{\hat{\mathbf{n}} = \frac{\mathbf{x}}{|\mathbf{x}|} = \hat{\mathbf{r}}}$$

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\mathbf{F}(k\hat{\mathbf{n}})$$

$$\mathbf{F}(k\hat{\mathbf{n}}) = \int \mathbf{J}(\mathbf{x}')\, e^{-ik\hat{\mathbf{n}}\cdot\mathbf{x}'} d^3x'$$

Onda esférica: $\sim \dfrac{e^{i(kr-\omega t)}}{r}$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} ik \frac{e^{ikr}}{r}\hat{\mathbf{n}} \times \mathbf{F}(k\hat{\mathbf{n}})$$

$$\mathbf{E}(\mathbf{x}) = -\frac{\mu_0}{4\pi} i\omega \frac{e^{ikr}}{r}\hat{\mathbf{n}} \times [\hat{\mathbf{n}} \times \mathbf{F}(k\hat{\mathbf{n}})]$$

$$c\mathbf{B}(\mathbf{x}) = \hat{\mathbf{n}} \times \mathbf{E}(\mathbf{x})$$

Distribuição angular da potência irradiada:

$$\frac{dP}{d\Omega} = \frac{\mu_0 \omega^2}{32\pi^2 c}\left|\hat{\mathbf{n}} \times \mathbf{F}(k\hat{\mathbf{n}})\right|^2$$

Expansão multipolar da radiação

$$\mathbf{F}(k\hat{\mathbf{n}}) = \int \mathbf{J}(\mathbf{x}')\, e^{-ik\hat{\mathbf{n}}\cdot\mathbf{x}'} d^3x'$$

$d \ll \lambda$

$$\boxed{\mathbf{F}(k\hat{\mathbf{n}}) = \sum_{n=0}^{\infty} \frac{(-ik)^n}{n!} \int \mathbf{J}(\mathbf{x}')\,(\hat{\mathbf{n}}\cdot\mathbf{x}')^n\, d^3x'}$$

$n=0$: radiação de dipolo elétrico

$$\mathbf{p} = \int \mathbf{x}\rho(\mathbf{x})\, d^3x \in \mathbb{C}$$

$$\mathbf{A}_D(\mathbf{x}) = -i\frac{\mu_0\omega}{4\pi}\frac{e^{ikr}}{r}\mathbf{p}$$

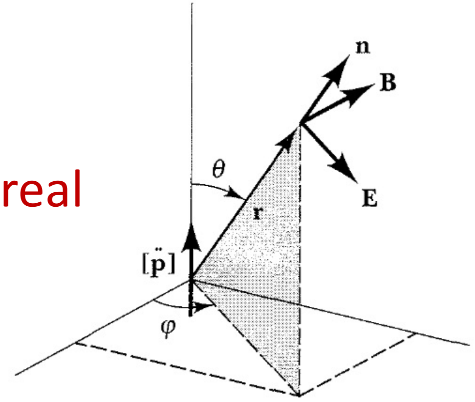
$$\mathbf{B}_D(\mathbf{x}) = \frac{\mu_0\omega^2}{4\pi c}\frac{e^{ikr}}{r}\hat{\mathbf{n}}\times\mathbf{p}$$

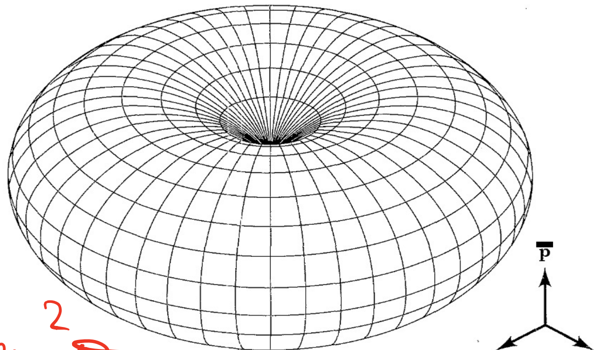
$$\mathbf{E}_D(\mathbf{x}) = -\frac{\mu_0\omega^2}{4\pi}\frac{e^{ikr}}{r}\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{p})$$

$$\frac{dP}{d\Omega} = \frac{\mu_0\omega^4}{32\pi^2 c}|\hat{\mathbf{n}}\times\mathbf{p}|^2$$

$$P = \frac{\mu_0\omega^4}{12\pi c}|\mathbf{p}|^2$$

se **p** for real





$\frac{dP}{d\Omega} \propto \sin^2\theta$

A sequência temporal da radiação de dipolo

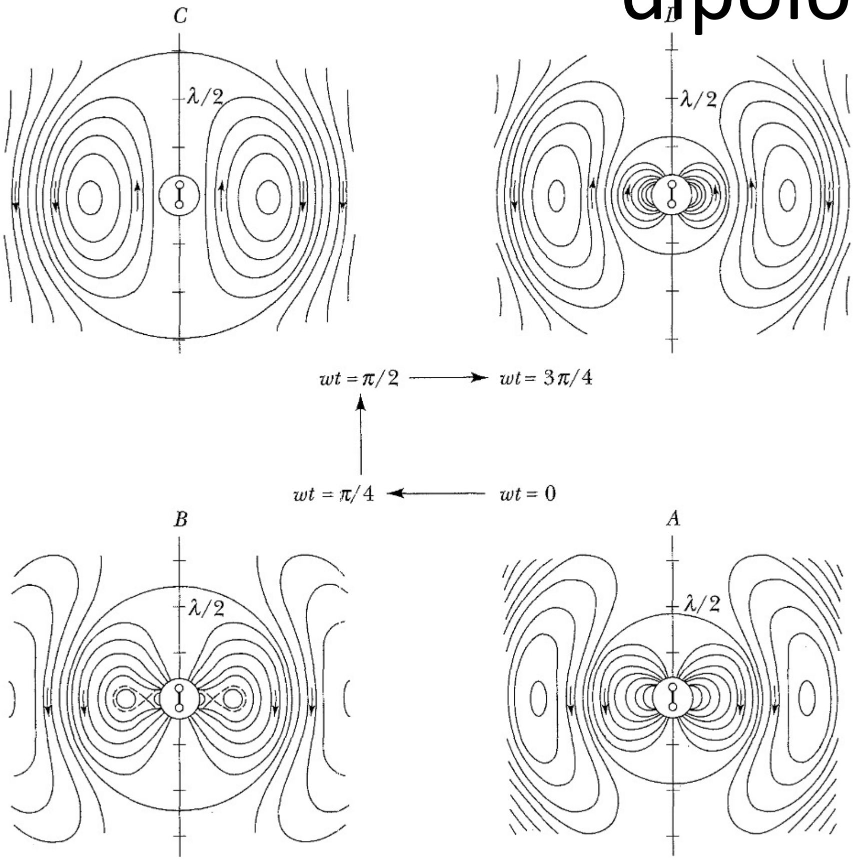


FIGURE 9-6. Snapshots of oscillating dipole. [From Hertz, *Wiedemann's Ann.* **36,** 1 (1889); reprinted in (He62).]