

FI 008 – Eletrodinâmica I

1º Semestre de 2020

07/05/2020

Aula 16

Aula passada

Fontes harmônicas:

$$\rho(\mathbf{x}, t) = \rho(\mathbf{x}) e^{-i\omega t}$$

$$\mathbf{J}(\mathbf{x}, t) = \mathbf{J}(\mathbf{x}) e^{-i\omega t}$$

$$\nabla \cdot \mathbf{J} = i\omega \rho$$



Soluções harmônicas:

$$\mathbf{A}(\mathbf{x}, t) = \mathbf{A}(\mathbf{x}) e^{-i\omega t}$$

$$\mathbf{B}(\mathbf{x}, t) = \mathbf{B}(\mathbf{x}) e^{-i\omega t}$$

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{E}(\mathbf{x}) e^{-i\omega t}$$

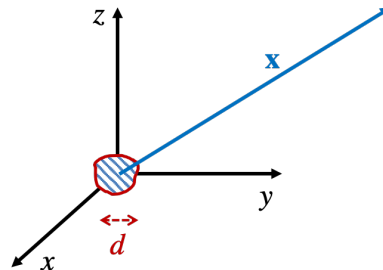
Solução geral:

$$\mathbf{A}(\mathbf{x}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}', t - R/c)}{R} d^3x'$$

$$R = |\mathbf{x} - \mathbf{x}'|$$

Hipóteses:

- Fontes pequenas
- Região de radiação



$$\left\{ \begin{array}{l} d \ll r = |\mathbf{x}| \\ d \ll \lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega} \\ \lambda \ll r \end{array} \right.$$

Aula passada

Onda esférica:

$$\hat{m} = \hat{n} = \frac{\mathbf{x}}{r}$$

$$k = \frac{\omega}{c}$$

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \mathbf{F}(k\hat{n})$$

$$\mathbf{F}(k\hat{n}) = \int \mathbf{J}(\mathbf{x}') e^{-ik\hat{n}\cdot\mathbf{x}'} d^3x'$$

$\sim k a \sim \frac{d}{\lambda} \ll 1$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} ik \frac{e^{ikr}}{r} \hat{n} \times \mathbf{F}(k\hat{n})$$

$$\mathbf{E}(\mathbf{x}) = -\frac{\mu_0}{4\pi} i\omega \frac{e^{ikr}}{r} \hat{n} \times [\hat{n} \times \mathbf{F}(k\hat{n})]$$

$$c\mathbf{B}(\mathbf{x}) = \hat{n} \times \mathbf{E}(\mathbf{x})$$

$$\frac{dP}{d\Omega} = \frac{\mu_0 \omega^2}{32\pi^2 c} |\hat{n} \times \mathbf{F}(k\hat{n})|^2$$

$$\mathbf{F}(k\hat{n}) = \sum_{n=0}^{\infty} \frac{(-ik)^n}{n!} \int \mathbf{J}(\mathbf{x}') (\hat{n} \cdot \mathbf{x}')^n d^3x'$$

Aula passada

$n=0$: radiação de dipolo elétrico

se \mathbf{p} for real

$$\mathbf{p} = \int \mathbf{x} \rho(\mathbf{x}) d^3x \in \mathbb{C}$$

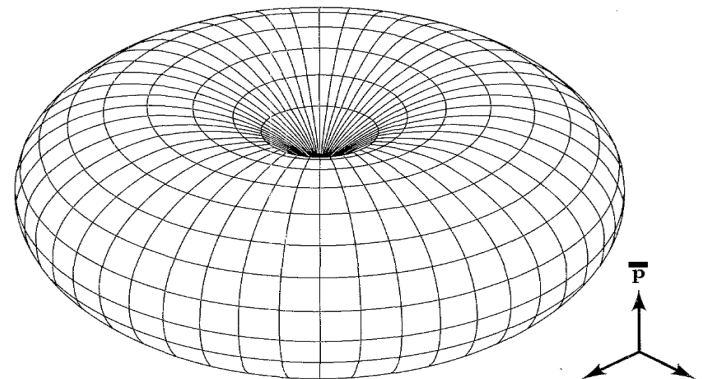
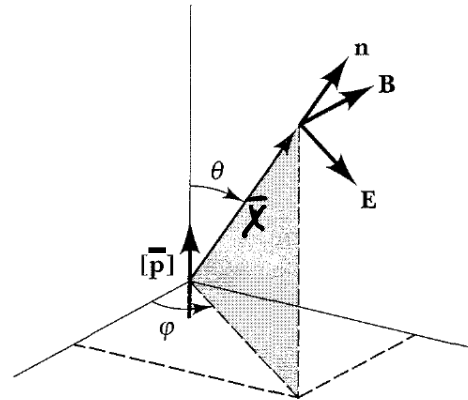
$$\mathbf{A}_D(\mathbf{x}) = -i \frac{\mu_0 \omega}{4\pi} \frac{e^{ikr}}{r} \mathbf{p}$$

$$\mathbf{B}_D(\mathbf{x}) = \frac{\mu_0 \omega^2}{4\pi c} \frac{e^{ikr}}{r} \hat{\mathbf{n}} \times \mathbf{p}$$

$$\mathbf{E}_D(\mathbf{x}) = -\frac{\mu_0 \omega^2}{4\pi} \frac{e^{ikr}}{r} \hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \mathbf{p})$$

$$\frac{dP}{d\Omega} = \frac{\mu_0 \omega^4}{32\pi^2 c} |\hat{\mathbf{n}} \times \mathbf{p}|^2$$

$$P = \frac{\mu_0 \omega^4}{12\pi c} |\mathbf{p}|^2$$

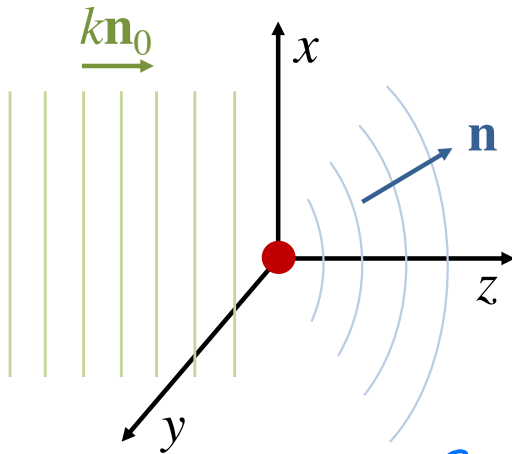


Espalhamento por uma esfera dielétrica

Seção 10.1.B do Jackson

ESFERA DIELETRICA ($\epsilon_r = \frac{\epsilon}{\epsilon_0}$)

RAIO $a \ll \lambda$



ONDA INCIDENTE:

$$\vec{E}_{INC} = E_0 \hat{e}_0 e^{i k \hat{m}_0 \cdot \vec{r}} \quad \hat{m}_0 = \hat{z}$$

$$\vec{B}_{INC} = \frac{1}{c} \hat{m}_0 \times \vec{E}_{INC} \quad \lambda = \frac{2\pi}{k}$$

O CAMPO ELÉTRICO DA ONDA INCIDENTE VAI POLARIZAR A ESFERA QUE TERÁ UM DIPOLO ELÉTRICO INDUZIDO OSCILANTE \vec{P} :

$$\vec{P} = 4\pi\epsilon_0 \left(\frac{\epsilon_r - 1}{\epsilon_r + 2} \right) a^3 E_0 \hat{e}_0$$

Distribuição angular em termos do campo elétrico:

$$\frac{dP}{d\Omega} = \frac{\mu_0 \omega^4}{32\pi^2 c} \left| [\hat{n} \times (\hat{n} \times \mathbf{p})] \right|^2$$

A DISTRIBUIÇÃO ANGULAR PARA UMA DADA POLARIZAÇÃO
É DO CAMPO IRRADIADO É:

$$\frac{dP(\hat{e})}{d\Omega} = \frac{\mu_0 \omega^4}{32\pi^2 c} \left| \hat{e}^* \cdot [\hat{u} \times (\hat{u} \times \vec{p})] \right|^2$$

$$\hat{e}^* \cdot [(\hat{u} \cdot \vec{p})\hat{u} - \vec{p}] = (\hat{u} \cdot \vec{p}) \hat{e}^* \cdot \hat{u} - \hat{e}^* \cdot \vec{p}$$

$$\frac{dP(\hat{e})}{d\Omega} = \frac{\mu_0 \omega^4}{32\pi^2 c} \left| \hat{e}^* \cdot \vec{p} \right|^2$$

Seção de choque diferencial

INTENSIDADE DA ONDA INCIDENTE:

$$\frac{P}{A} = I = \langle \vec{S}_{inc} \rangle \cdot \hat{m}_0 = \frac{|E_0|^2}{2\mu_0 c}$$

SEÇÃO DE CHOQUE DIFERENCIAL DE ESPALHAMENTO:

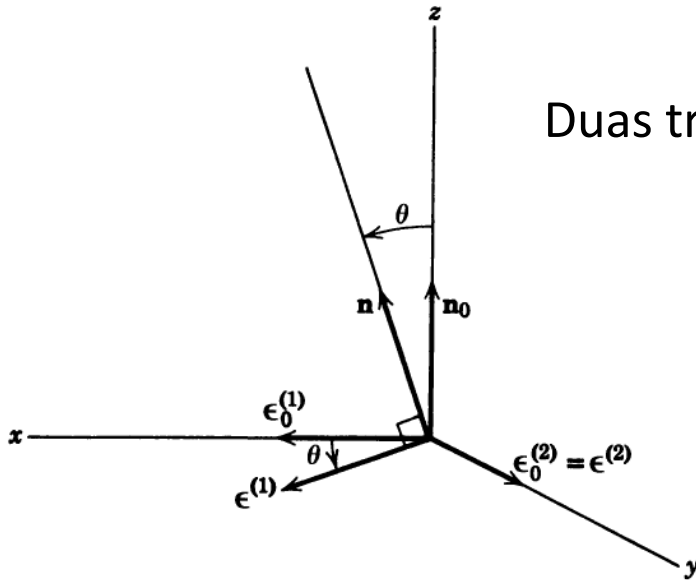
$$\frac{d\sigma(\hat{e})}{d\Omega} = \frac{dP(\hat{e})}{d\Omega} \frac{1}{I} = \frac{\mu_0^2 \omega^4}{16\pi^2} \frac{|\hat{e}^* \cdot \vec{P}|^2}{|E_0|^2}$$

$$\Rightarrow \frac{d\sigma(\hat{e})}{d\Omega} = \frac{\omega^4 a^6}{c^4} \left| \frac{\epsilon_{\lambda-1}}{\epsilon_{\lambda+2}} \right|^2 |\hat{e}^* \cdot \hat{e}_0|^2$$

$$\left[\frac{d\sigma}{d\Omega} \right] = A = L^2$$

COMO VAMOS APLICAR ESSE RESULTADO PARA A LUZ SOLAR, QUE É NÃO POLARIZADA, PRECISAMOS PROMEDIAR SOBRE \hat{e}_0

Média sobre a polarização incidente



Duas tríades ortonormais:

$$\left[\hat{\epsilon}_0^{(1)}, \hat{\epsilon}_0^{(2)}, \hat{n}_0 \right]$$

$$\left[\hat{\epsilon}^{(1)}, \hat{\epsilon}^{(2)}, \hat{n} \right]$$

POLARIZAÇÃO INCIDENTE:

$$\hat{\epsilon}_0 = \cos \phi \hat{\epsilon}_0^{(1)} + \sin \phi \hat{\epsilon}_0^{(2)}$$

POLARIZAÇÃO DE OBSERVAÇÃO

$$\hat{\epsilon}^* = \epsilon_1^* \hat{\epsilon}_0^{(1)} + \epsilon_2^* \hat{\epsilon}_0^{(2)} + \epsilon_3^* \hat{n}_0$$

$$\langle |\hat{\epsilon}^* \cdot \hat{\epsilon}_0|^2 \rangle = \int_0^{2\pi} \frac{d\phi}{2\pi} |\hat{\epsilon}^* \cdot \hat{\epsilon}_0|^2 = \int_0^{2\pi} \frac{d\phi}{2\pi} |\epsilon_1^* \cos \phi + \epsilon_2^* \sin \phi|^2$$

$$= \frac{1}{2} (|\epsilon_1|^2 + |\epsilon_2|^2) \quad (\text{VER NOTAS})$$

$$\text{SE } \hat{\epsilon} = \hat{\epsilon}^{(1)} = \cos \theta \hat{\epsilon}_0^{(1)} - \sin \theta \hat{m}_0 \Rightarrow \epsilon_1 = \cos \theta, \epsilon_2 = 0$$

$$\Rightarrow \langle |\hat{\epsilon}^* \cdot \hat{\epsilon}_0|^2 \rangle = \frac{1}{2} \cos^2 \theta$$

$$\text{SE } \hat{\epsilon} = \hat{\epsilon}^{(2)} = \hat{\epsilon}_0^{(2)} \Rightarrow \epsilon_2 = 1, \epsilon_1 = 0$$

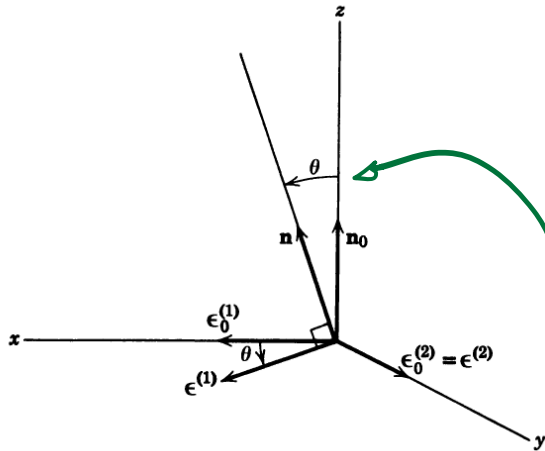
$$\Rightarrow \langle |\hat{\epsilon}^* \cdot \hat{\epsilon}_0|^2 \rangle = \frac{1}{2}$$

$$\frac{d\sigma(\hat{\epsilon}^{(1)})}{d\Omega} = \frac{k^4 a^6}{2} \left| \frac{\epsilon_n - 1}{\epsilon_n + 2} \right|^2 \cos^2 \theta$$

$$\frac{d\sigma(\hat{\epsilon}^{(2)})}{d\Omega} = \frac{k^4 a^6}{2} \left| \frac{\epsilon_n - 1}{\epsilon_n + 2} \right|^2$$

$\hat{E}^{(1)} \parallel$ AO PLANO DE ESPALHAMENTO

$\hat{E}^{(2)} \perp$ " " " "

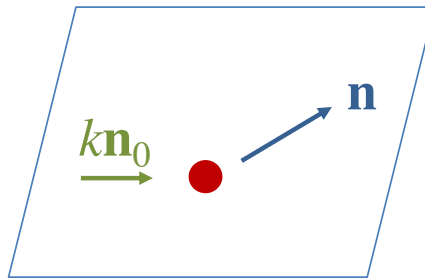
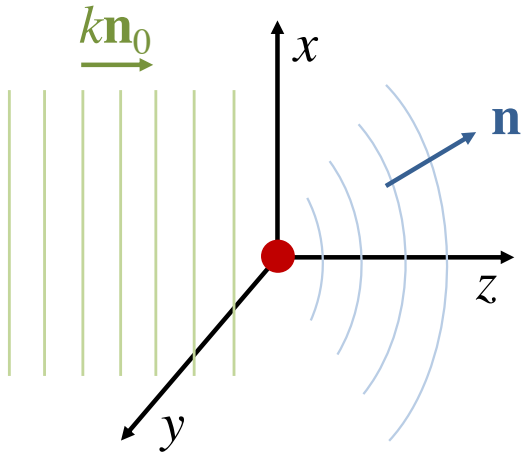


$$\frac{d\sigma_{\parallel}}{d\Omega} = \frac{k^4 a^6}{2} \left| \frac{\epsilon_r - 1}{\epsilon_r + 2} \right|^2 \cos^2 \theta$$

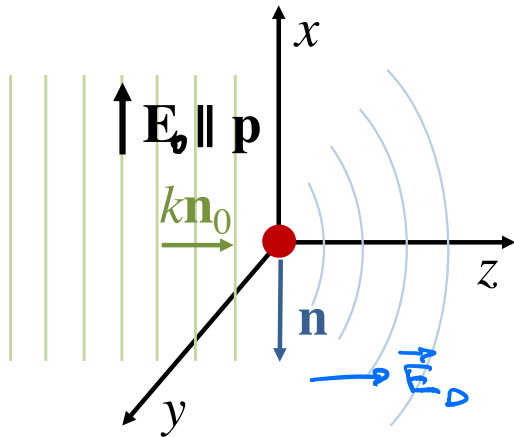
$$\frac{d\sigma_{\perp}}{d\Omega} = \frac{k^4 a^6}{2} \left| \frac{\epsilon_r - 1}{\epsilon_r + 2} \right|^2$$

SE $\theta = \frac{\pi}{2} \Rightarrow \frac{d\sigma_{\parallel}}{d\Omega} = 0$

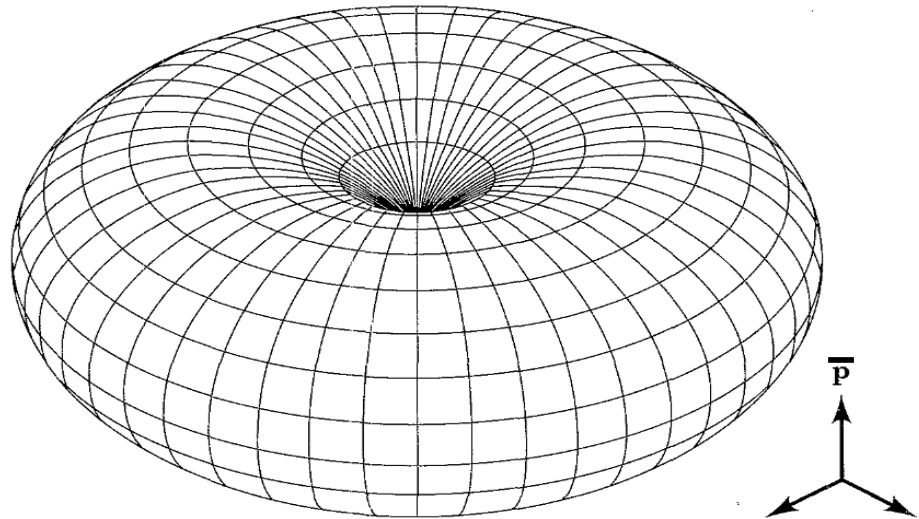
Plano de espalhamento (xz)



Porque a radiação polarizada no plano de espalhamento é 0 a 90° ?



A radiação é nula para $\theta=0,\pi$



Polarização da radiação espalhada

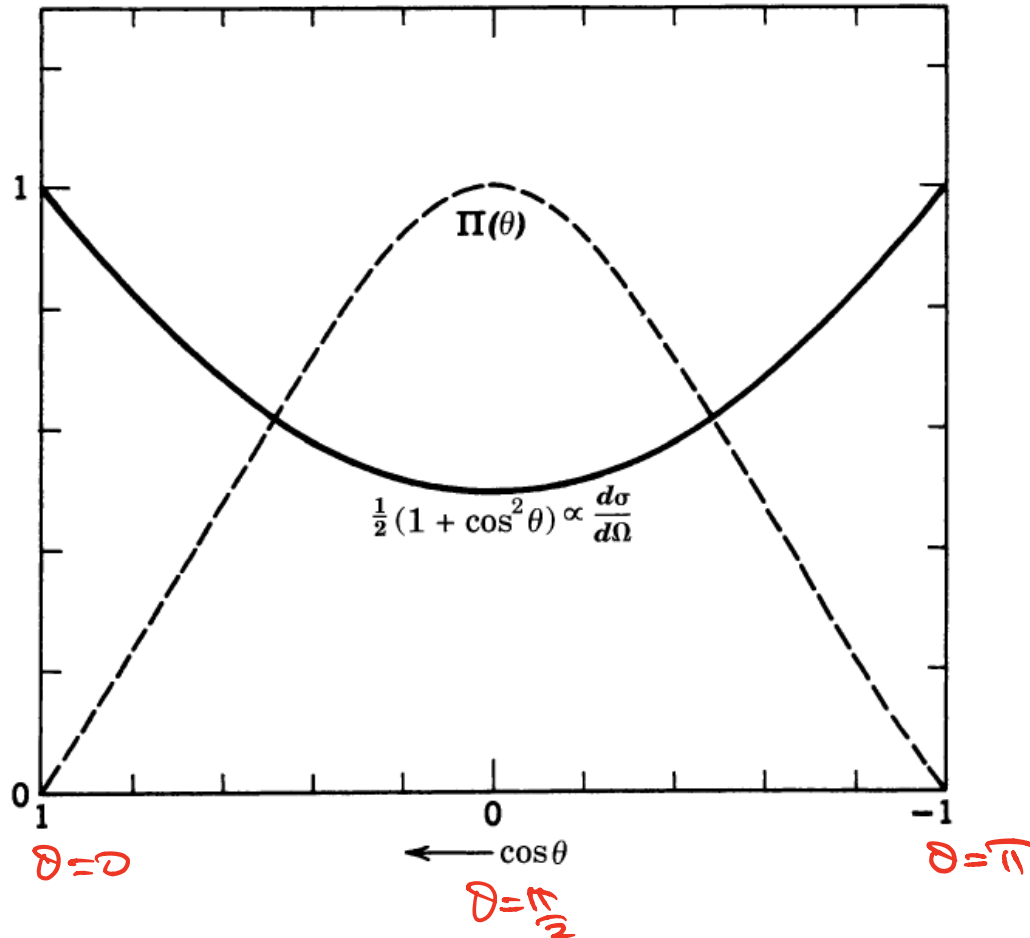
$$\Pi(\theta) = \frac{\frac{d\sigma_{\perp}}{d\Omega} - \frac{d\sigma_{\parallel}}{d\Omega}}{\frac{d\sigma_{\perp}}{d\Omega} + \frac{d\sigma_{\parallel}}{d\Omega}} = \frac{1 - \cos^2\theta}{1 + \cos^2\theta} = \frac{\sin^2\theta}{1 + \cos^2\theta}$$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{\parallel}}{d\Omega} + \frac{d\sigma_{\perp}}{d\Omega} = \frac{k^4 a^6}{2} \left| \frac{\epsilon_r - 1}{\epsilon_r + 2} \right|^2 (1 + \cos^2\theta)$$

$$\sigma = \int \frac{d\sigma}{d\Omega} \sin\theta d\theta d\phi = \frac{8\pi}{3} (k^4 a^6) \left| \frac{\epsilon_r - 1}{\epsilon_r + 2} \right|^2$$

$$\sigma \propto \omega^4$$

Polarização da radiação espalhada



Espalhamento da luz do Sol pela atmosfera

Intensidade no **solo relativa** à intensidade acima da atmosfera

<i>Color</i>	<i>Zenith</i>	<i>Sunrise-Sunset</i>
Red (6500 Å)	0.96	0.21
Green (5200 Å)	0.90	0.024
Violet (4100 Å)	0.76	0.000065

A intensidade no solo é menor que 1 porque parte foi espalhada.

Espalhamento da luz do Sol pela atmosfera

A: Espectro acima da atmosfera

B: Espectro no solo (Sol no zênite)

Curvas tracejadas: previsões de Rayleigh (zênite e pôr/nascer do Sol)

Desvios são devido a vapor d'água, ozônio,...

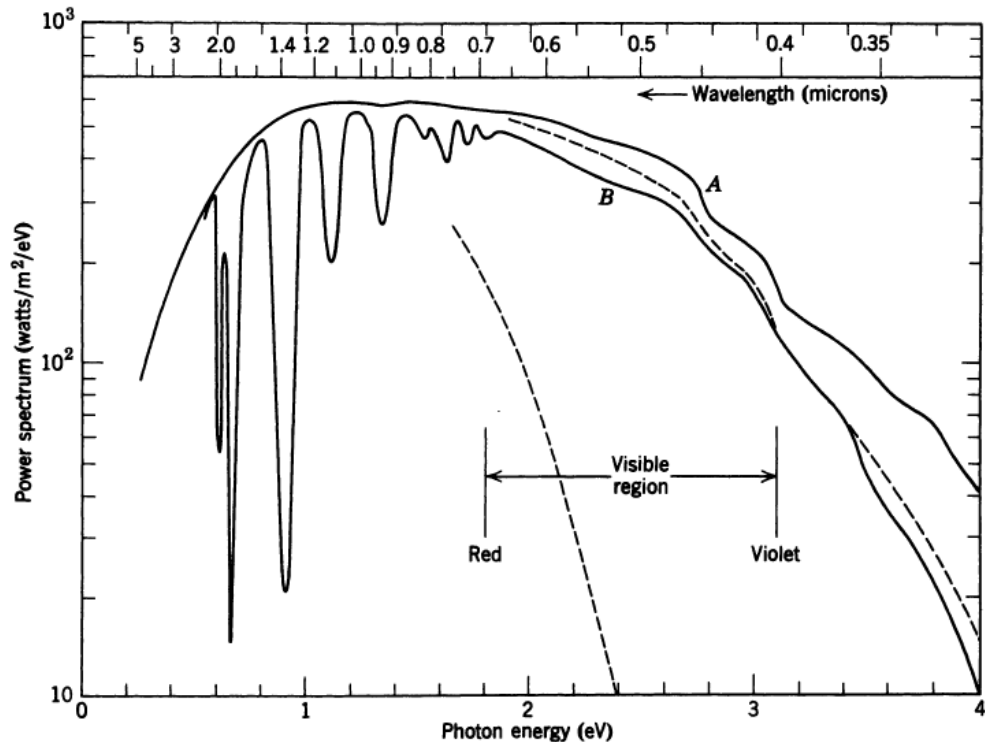


Figure 10.4 Power spectrum of solar radiation (in watts per square meter per electron volt) as a function of photon energy (in electron volts). Curve *A* is the incident spectrum above the atmosphere. Curve *B* is a typical sea-level spectrum with the sun at the zenith. The absorption bands below 2 eV are chiefly from water vapor and vary from site to site and day to day. The dashed curves give the expected sea-level spectrum at zenith and at sunrise-sunset if the only attenuation is from Rayleigh scattering by a dry, clean atmosphere.

Espalhamento da luz do Sol pela atmosfera

$$\theta = \frac{\pi}{2}$$

Polarização a 90° tem um máximo de aproximadamente **75%** em **550 nm (verde)**.

Desvio de 100% devido a:

- espalhamento múltiplo (6%)
- anisotropia molecular (6%)
- reflexão no solo (5%)
- aerossóis (8%)

Radiação de dipolo magnético e quadrupolo elétrico

TERMO $m=1$.

$$\vec{F}(k\hat{n}) = -ik \int \vec{J}(\vec{x}') (\hat{n} \cdot \vec{x}') d^3x'$$

$$F_i(k\hat{n}) = -ik \int J_i(\vec{x}') (\hat{n} \cdot \vec{x}') d^3x' = -ik \sum_j M_j \underbrace{\int J_i(\vec{x}') x'_j d^3x'}_{R_{ij}}$$

$$F_i(k\hat{n}) = -ik \sum_j R_{ij} M_j$$

R_{ij} É UM TENSOR DE ORDEM 2. PODE SER ESCRITO COMO A SOMA DE UM TENSOR SIMÉTRICO E UM ANTI-SIMÉTRICO:

$$R_{ij} = \underbrace{\frac{1}{2} (R_{ij} + R_{ji})}_{\text{SIMÉTRICO } S_{ij}} + \underbrace{\frac{1}{2} (R_{ij} - R_{ji})}_{\text{ANTI-SIMÉTRICO } A_{ij}}$$

$$A_{ij} = \frac{1}{2} \int [\mathcal{J}_i(\vec{x}') x'_j - \mathcal{J}_j(\vec{x}') x'_i] d^3x'$$

3 COMPONENTES NÃO NULAS

PODE SER ESCRITO EM TERMOS DE UM (PSEUDO-) VETOR!

$$m_k = -\frac{1}{2} \epsilon^{kij} A_{ij} \quad \text{E} \quad A_{ij} = -\epsilon^{ijk} m_k \quad (\text{VER NOTAS})$$

$$m_k = -\frac{1}{4} \epsilon^{kij} \int [\mathcal{J}_i x'_j - \mathcal{J}_j x'_i] d^3x'$$

$$= -\frac{1}{2} \epsilon^{kij} \int [\mathcal{J}_i x'_j] d^3x' = \frac{1}{2} \int [\vec{x}' \times \vec{\mathcal{J}}(\vec{x}')]_k d^3x'$$

$$\vec{m} = \frac{1}{2} \int [\vec{x}' \times \vec{\mathcal{J}}(\vec{x}')] d^3x'$$

MOMENTO DE DIPLO

MAGNÉTICO DA DIST. $\vec{\mathcal{J}}(\vec{x})$

$$\vec{m} \in \mathbb{C}$$

$$F_i^A(k\hat{u}) = -ik \sum_j A_{ij} m_j = ik \sum_j \epsilon^{ijk} m_j m_k = ik (\hat{u} \times \vec{m})_i$$

$$\vec{A}(\vec{x}) = ik \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} (\hat{u} \times \vec{m})$$

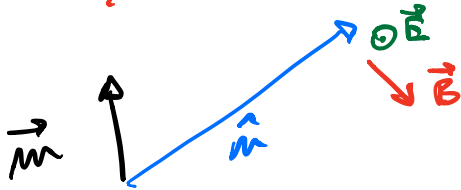
RADIAÇÃO DE
DIPLO MAGNÉTICO

$$S \in \vec{m} \in \mathbb{R}$$

$$\frac{dP}{dr} \propto |\hat{m} \times \vec{m}|^2 \propto m^2 \sin^2 \theta$$

POLARIZAÇÃO DA RADIAÇÃO

$$\vec{E} \parallel \hat{m} \times \vec{m}$$



Radiação de dipolo magnético

$$\mathbf{A} = ik \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} (\hat{\mathbf{n}} \times \mathbf{m})$$

$$\mathbf{B} = -k^2 \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \mathbf{m})$$

$$\mathbf{E} = -ck^2 \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} (\hat{\mathbf{n}} \times \mathbf{m})$$

$$\frac{dP}{d\Omega} = \frac{\mu_0 c k^4}{32\pi^2} |\hat{\mathbf{n}} \times \mathbf{m}|^2$$

$$P = \frac{\mu_0 \omega^4}{12\pi c^3} |\mathbf{m}|^2$$

Quadrupolo elétrico

$$S_{ij} = \frac{1}{2} \int [\mathcal{J}_i x_j' + \mathcal{J}_j x_i'] d^3x'$$

$$\vec{\nabla}' \cdot (x_i' x_j' \vec{\mathcal{J}}) = (x_i' x_j') \underbrace{\vec{\nabla}' \cdot \vec{\mathcal{J}}}_{i\omega\rho} + \mathcal{J}_i x_j' + \mathcal{J}_j x_i'$$

$$S_{ij} = -\frac{i\omega}{2} \int x_i' x_j' \rho(\vec{x}') d^3x' \in \mathbb{C}$$

COMO MOSTRADO NAS NOTAS :

$$S_{ij} = -\frac{i\omega}{6} [\Theta_{ij} + \Sigma \delta_{ij}]$$

$$\text{ONDE } \Theta_{ij} = \int (3x_i' x_j' - \delta_{ij} r'^2) \rho(\vec{x}') d^3x'$$

$$\Sigma = \int r'^2 \rho(\vec{x}') d^3x'$$

TENSOR QUADRUPOLO
ELÉTRICO

$$F_i^s(k\hat{m}) = (-ik) \sum_j S_{ij} u_j = -\frac{\omega k}{6} \sum_j u_j \left[\underbrace{Q_{ij}}_{Q_{ji}} + \sum \delta_{ij} \right]$$

$$\vec{F}^s(k\hat{m}) = -\frac{\omega k}{6} \left[\underbrace{\hat{m} \cdot \vec{Q}}_{\text{VECTOR}} + \sum \hat{m} \right]$$

$$\vec{A}(\vec{r}) = -\frac{\omega k}{6} \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \left[\hat{m} \cdot \vec{Q} + \sum \hat{m} \right]$$

$\vec{B} \propto \hat{m} \times \vec{A}$ \square \circ TERMO EM \sum NÃO CONTRIBUI

Radiação de quadrupolo elétrico

$$\mathbf{A} = -\frac{\omega k}{6} \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \left(\overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}} \right)$$

$$\mathbf{B} = -i \frac{\omega k^2}{6} \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \hat{\mathbf{n}} \times \left(\overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}} \right)$$

$$\mathbf{E} = i \frac{\omega^2 k}{6} \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \left[\left(\hat{\mathbf{n}} \cdot \overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}} \right) \hat{\mathbf{n}} - \overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}} \right]$$

$$\frac{dP}{d\Omega} = \frac{\mu_0 \omega^6}{1152 \pi^2 c^3} \left[\left| \overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}} \right|^2 - \left| \hat{\mathbf{n}} \cdot \overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}} \right|^2 \right]$$

$$P = \frac{\mu_0 \omega^6}{1440 \pi c^3} \sum_{ij} |Q_{ij}|^2$$

$$\begin{aligned} \hat{\mathbf{n}} &= \sin \theta \cos \phi \hat{\mathbf{x}} + \sin \theta \sin \phi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}} \\ &= \mu_1 \hat{\mathbf{x}} + \mu_2 \hat{\mathbf{y}} + \mu_3 \hat{\mathbf{z}} \end{aligned}$$

Exemplo

SUPONHA UMA DISTRIBUIÇÃO DE CARGAS E CORRENTES DANDO ORIGEM A UM TENSOR QUADRUPOLO ELÉTRICO

$$Q_{ij} = \begin{bmatrix} -Q_0/2 & 0 & 0 \\ 0 & -Q_0/2 & 0 \\ 0 & 0 & Q_0 \end{bmatrix} \quad Q_0 \in \mathbb{R}$$

$$\vec{Q} \cdot \hat{m} = Q_0 \begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = Q_0 \left[-\frac{1}{2}(m_1 \hat{x} + m_2 \hat{y}) + m_3 \hat{z} \right]$$

$$|\vec{Q} \cdot \hat{m}|^2 = |Q_0|^2 \left[\frac{1}{4}(m_1^2 + m_2^2) + m_3^2 \right] = |Q_0|^2 \left(\frac{\sin^2 \theta}{4} + \cos^2 \theta \right)$$

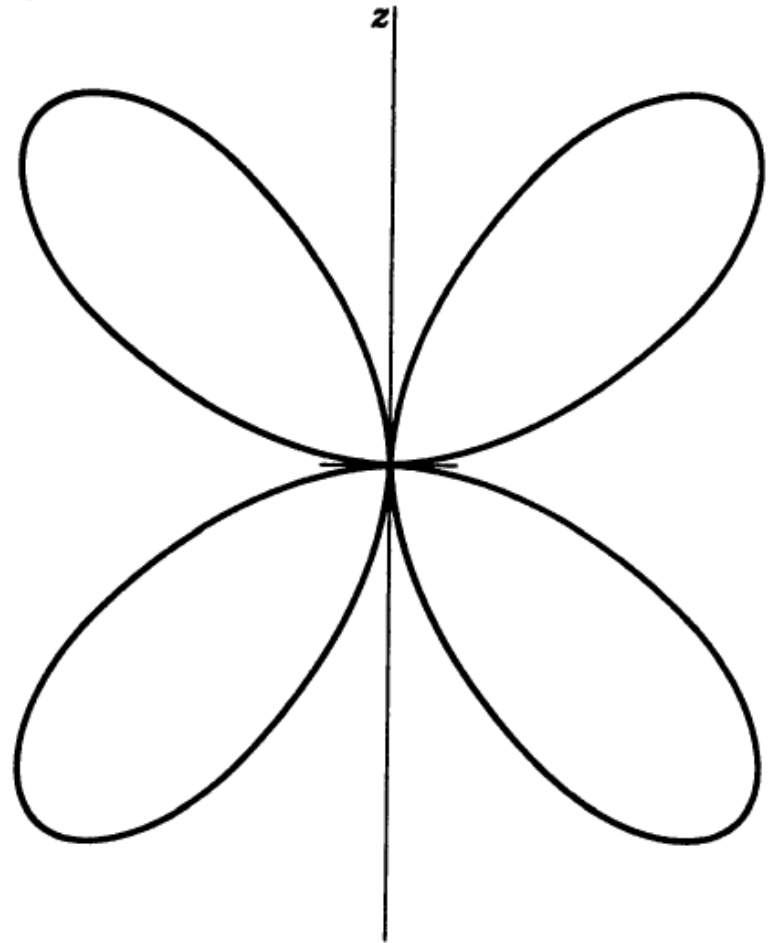
$$\begin{aligned} \hat{m} \cdot \vec{Q} \cdot \hat{m} &= \hat{m} (Q_0) \left[-\frac{1}{2}(m_1 \hat{x} + m_2 \hat{y}) + m_3 \hat{z} \right] \\ &= Q_0 \left[-\frac{1}{2}(m_1^2 + m_2^2) + m_3^2 \right] = Q_0 \left[\cos^2 \theta - \frac{1}{2} \sin^2 \theta \right] \end{aligned}$$

SOMANDO TODO (VER NOTAS)

$$\frac{dP_{\theta E}}{d\Omega} = \frac{\mu_0}{512\pi^2} \frac{\omega^6}{c^3} |a_0|^2 \sin^2\theta \cos^2\theta$$

$$P_{\theta E} = \frac{\mu_0 \omega^6}{960\pi c^3} |a_0|^2$$

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0 k^6}{512 \pi^2} Q_0^2 \sin^2 \theta \cos^2 \theta$$



Exemplo de radiação de
quadrupolo elétrico

MÁXIMOS EM

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

ZEROS EM

$$\theta = 0, \frac{\pi}{2}, \pi$$