FI 008 – Eletrodinâmica I

1º Semestre de 2021
13/05/2021
Aula 16

Radiação

Fontes harmônicas: $(\omega > 0)$

$$\rho(\mathbf{x}, t) = \rho(\mathbf{x})\, e^{-i\omega t}$$

$$\mathbf{J}(\mathbf{x}, t) = \mathbf{J}(\mathbf{x})\, e^{-i\omega t}$$

$$\boldsymbol{\nabla}\cdot\ \mathbf{J} = i\omega\rho$$

Notação complexa!

Soluções harmônicas:

$$\mathbf{A}(\mathbf{x}, t) = \mathbf{A}(\mathbf{x})\, e^{-i\omega t}$$

$$\mathbf{B}(\mathbf{x}, t) = \mathbf{B}(\mathbf{x})\, e^{-i\omega t}$$

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{E}(\mathbf{x})\, e^{-i\omega t}$$

Solução geral:

$$\mathbf{A}(\mathbf{x}, t) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{x}', t - R/c)}{R} d^3x' \quad \Rightarrow \quad \mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{x}')\, e^{ikR}}{R} d^3x'$$

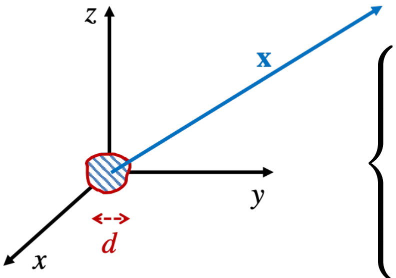
$$\boxed{R = |\mathbf{x} - \mathbf{x}'|}$$

$$k = \frac{\omega}{c}$$

Região de radiação

Hipóteses:

- Fontes pequenas: $d \ll r$
- Região de radiação: $r \gg \lambda$



$$\begin{cases} d \ll r = |\mathbf{x}| \\ d \ll \lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega} \\ r \gg \lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega} \end{cases}$$

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}') e^{ikR}}{R} d^3x'$$

$$\boxed{\hat{\mathbf{n}} = \frac{\mathbf{x}}{|\mathbf{x}|} = \hat{\mathbf{r}}}$$

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \mathbf{F}(k\hat{\mathbf{n}})$$

$$\mathbf{F}(k\hat{\mathbf{n}}) = \int \mathbf{J}(\mathbf{x}') e^{-ik\hat{\mathbf{n}}\cdot\mathbf{x}'} d^3x'$$

Onda esférica:

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} ik \frac{e^{ikr}}{r} \hat{\mathbf{n}} \times \mathbf{F}(k\hat{\mathbf{n}})$$

$$\mathbf{E}(\mathbf{x}) = -\frac{\mu_0}{4\pi} i\omega \frac{e^{ikr}}{r} \hat{\mathbf{n}} \times [\hat{\mathbf{n}} \times \mathbf{F}(k\hat{\mathbf{n}})]$$

$$c\mathbf{B}(\mathbf{x}) = \hat{\mathbf{n}} \times \mathbf{E}(\mathbf{x})$$

Distribuição angular da potência irradiada:

$$\frac{dP}{d\Omega} = \frac{\mu_0 \omega^2}{32\pi^2 c} |\hat{\mathbf{n}} \times \mathbf{F}(k\hat{\mathbf{n}})|^2$$

Expansão multipolar da radiação

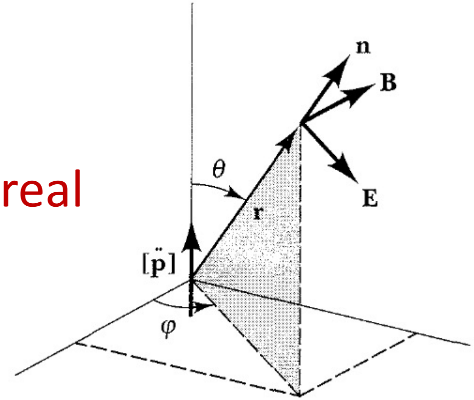
$$\mathbf{F}(k\hat{\mathbf{n}}) = \int \mathbf{J}(\mathbf{x}')\, e^{-ik\hat{\mathbf{n}}\cdot\mathbf{x}'} d^3x' \xrightarrow{d \ll \lambda} \boxed{\mathbf{F}(k\hat{\mathbf{n}}) = \sum_{n=0}^{\infty} \frac{(-ik)^n}{n!} \int \mathbf{J}(\mathbf{x}')\,(\hat{\mathbf{n}}\cdot\mathbf{x}')^n\, d^3x'}$$

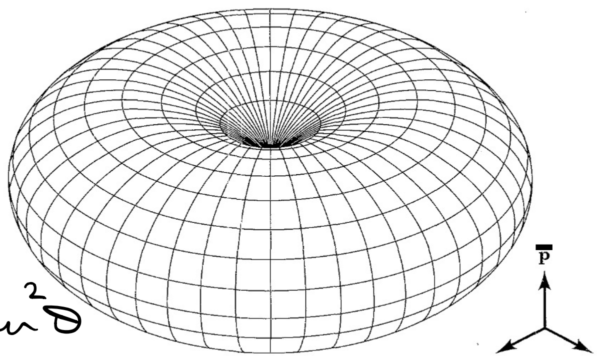
$n=0$: radiação de dipolo elétrico

$$\mathbf{p} = \int \mathbf{x}\rho(\mathbf{x})\, d^3x \in \mathbb{C}$$

$$\begin{aligned}
\mathbf{A}_D(\mathbf{x}) &= -i\frac{\mu_0\omega}{4\pi}\frac{e^{ikr}}{r}\mathbf{p} \\
\mathbf{B}_D(\mathbf{x}) &= \frac{\mu_0\omega^2}{4\pi c}\frac{e^{ikr}}{r}\hat{\mathbf{n}}\times\mathbf{p} \\
\mathbf{E}_D(\mathbf{x}) &= -\frac{\mu_0\omega^2}{4\pi}\frac{e^{ikr}}{r}\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{p}) \\
\frac{dP}{d\Omega} &= \frac{\mu_0\omega^4}{32\pi^2 c}\left|\hat{\mathbf{n}}\times\mathbf{p}\right|^2 \\
P &= \frac{\mu_0\omega^4}{12\pi c}\left|\mathbf{p}\right|^2
\end{aligned}$$

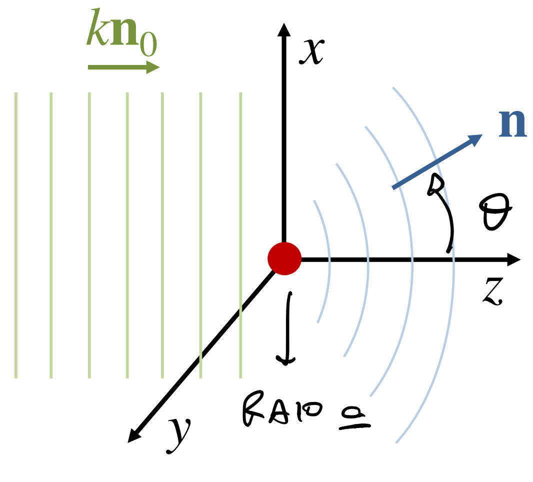
se $\mathbf{p}$ for real



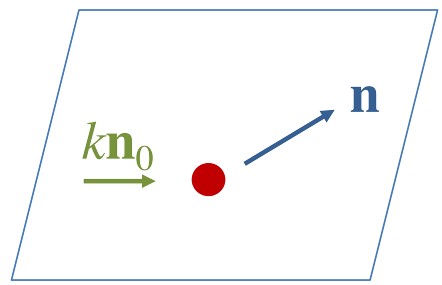


$$\frac{dP}{d\Omega} \propto \sin^2\theta$$

Espalhamento por esfera dielétrica



Plano de espalhamento (xz)



$$\lambda = \frac{2\pi}{k} >> a$$

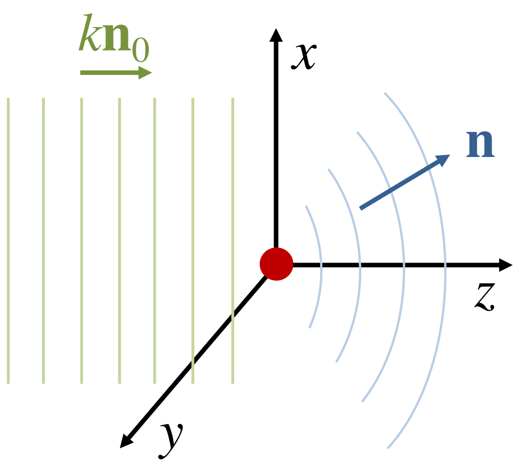
Espalhamento com polar. no plano de espalhamento: $$\frac{d\sigma_{\parallel}}{d\Omega}(\theta) = \frac{k^4 a^6}{2}\left|\frac{\epsilon_r - 1}{\epsilon_r + 2}\right|^2 \cos^2\theta$$

Espalhamento com polar. perp. ao plano de espalhamento: $$\frac{d\sigma_{\perp}}{d\Omega}(\theta) = \frac{k^4 a^6}{2}\left|\frac{\epsilon_r - 1}{\epsilon_r + 2}\right|^2$$

$$\underline{\vec{p} = \alpha \vec{E}} = \chi_e \epsilon_0 \vec{E}$$

$\hookrightarrow \epsilon_r$

Espalhamento por esfera dielétrica



$$\Pi(\theta) = \frac{\frac{d\sigma_\perp}{d\Omega} - \frac{d\sigma_\parallel}{d\Omega}}{\frac{d\sigma_\perp}{d\Omega} + \frac{d\sigma_\parallel}{d\Omega}} = \frac{\sin^2\theta}{1+\cos^2\theta}$$

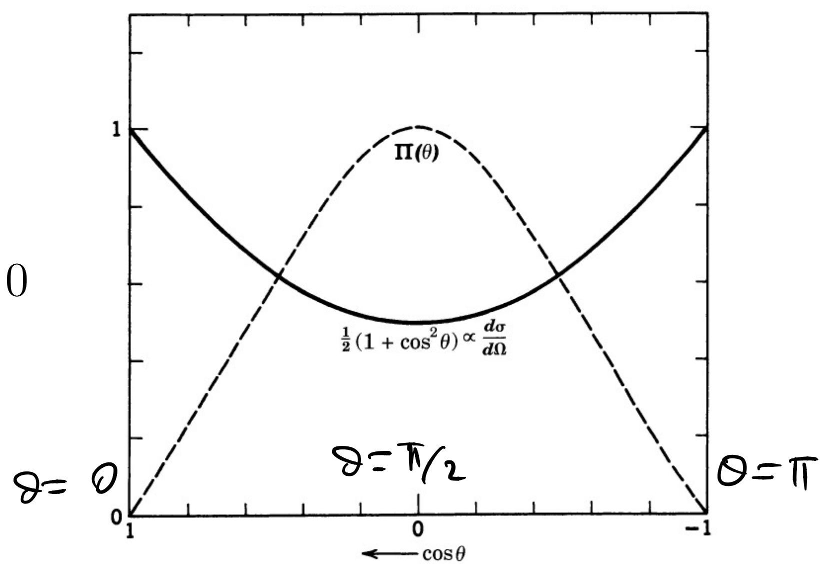
$$\frac{d\sigma}{d\Omega}(\theta) = \frac{d\sigma_\perp}{d\Omega} + \frac{d\sigma_\parallel}{d\Omega} = \frac{k^4 a^6}{2}\left|\frac{\epsilon_r - 1}{\epsilon_r + 2}\right|^2 \left(1+\cos^2\theta\right)$$

$$\sigma = \frac{8\pi a^2}{3}(ka)^4\left|\frac{\epsilon_r - 1}{\epsilon_r + 2}\right|^2$$

Polarização completa a 90º:

$$\frac{d\sigma_\parallel}{d\Omega}\left(\theta = \frac{\pi}{2}\right) \propto \left|\hat{\boldsymbol{\epsilon}}_\parallel \cdot \mathbf{E}_{\text{rad}}\right|^2_{\theta=\frac{\pi}{2}} = 0$$

$\hat{\boldsymbol{\epsilon}}_\parallel \parallel$ plano de espalhamento

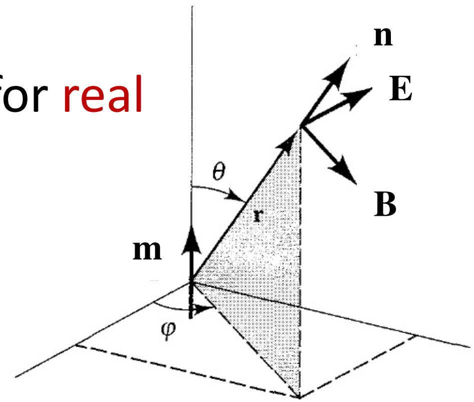


Dipolo magnético

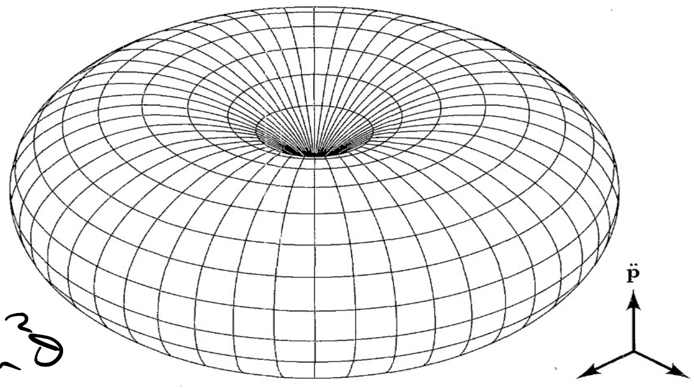
n=1: radiação de dipolo magnético

$$\mathbf{m} = \frac{1}{2}\int \left[\mathbf{x}\times\mathbf{J}\left(\mathbf{x}\right)\right] d^3x \in \mathbb{C}$$

se **m** for real



$$\begin{aligned}
\mathbf{A}_{DM} &= ik\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\left(\hat{\mathbf{n}}\times\mathbf{m}\right)\\
\mathbf{B}_{DM} &= -k^2\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\hat{\mathbf{n}}\times\left(\hat{\mathbf{n}}\times\mathbf{m}\right)\\
\mathbf{E}_{DM} &= -ck^2\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\left(\hat{\mathbf{n}}\times\mathbf{m}\right)\\
\frac{dP_{DM}}{d\Omega} &= \frac{\mu_0 ck^4}{32\pi^2}\left|\hat{\mathbf{n}}\times\mathbf{m}\right|^2\\
P_{DM} &= \frac{\mu_0\omega^4}{12\pi c^3}\left|\mathbf{m}\right|^2
\end{aligned}$$



$$\frac{dP_{DM}}{d\Omega} \propto \sin^2\theta$$

Quadrupolo ~~magnético~~ ELÉTRICO

n=1: radiação de quadrupolo elétrico

$$Q_{ij} = \int \left(3x_i x_j - \delta_{ij} r^2\right) \rho(\mathbf{x})\, d^3x \in \mathbb{C}$$

$$Q_{ij} = Q_0 \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left(\overline{\overline{Q}} \cdot \hat{n}\right)_i = \sum_j Q_{ij} n_j$$

$$\frac{dP}{d\Omega} = \frac{\mu_0 \omega^6 Q_0^2}{512 \pi^2 c^3} \sin^2\theta \cos^2\theta$$

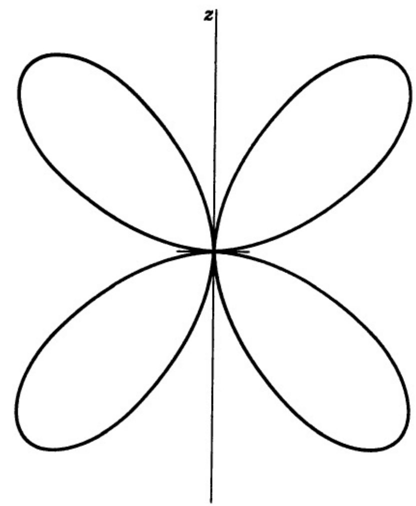
$$\mathbf{A}_{QE} = -\frac{\omega k}{6} \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \left(\overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right)$$

$$\mathbf{B}_{QE} = -i \frac{\omega k^2}{6} \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \hat{\mathbf{n}} \times \left(\overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right)$$

$$\mathbf{E}_{QE} = i \frac{\omega^2 k}{6} \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \left[\left(\hat{\mathbf{n}} \cdot \overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right) \hat{\mathbf{n}} - \overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right]$$

$$\frac{dP_{QE}}{d\Omega} = \frac{\mu_0 \omega^6}{1152 \pi^2 c^3} \left[\left|\overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right|^2 - \left|\hat{\mathbf{n}} \cdot \overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right|^2\right]$$

$$P_{QE} = \frac{\mu_0 \omega^6}{1440 \pi c^3} \sum_{ij} \left|Q_{ij}\right|^2$$



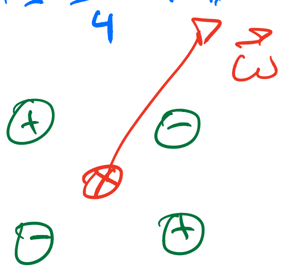
$$\hat{n} = \hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$

NA PÁGINA DO CURSO NA INTERNET, HÁ A DIST. ANGULAR DA POTÊNCIA IRRADIADA DE QUADRUPOLO ELÉTRICO PARA UM TENSOR DE QUADRUPOLO MAIS GERAL

$$Q_{ij} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & -a-b \end{bmatrix}$$

$$a = Q\cos\psi$$
$$b = Q\sin\psi$$
$$\psi \in [0, 2\pi]$$

$$\psi = \frac{\pi}{4} \Rightarrow a = b = \frac{Q}{\sqrt{2}}$$



$$\rho(\bar{x},t) \in \mathbb{R}$$

$$\bar{\rho}(\bar{x},\omega) = \int_{-\infty}^{+\infty} dt\, e^{i\omega t}\, \rho(\bar{x},t)$$

$$\left[\bar{\rho}(\bar{x},\omega)\right]^{*} = \left[\bar{\rho}(\bar{x},-\omega)\right]$$

$$\rho(\bar{x},t) = \dots + \bar{\rho}(\bar{x},\omega)\, e^{-i\omega t} + \underbrace{\bar{\rho}(\bar{x},-\omega)}_{\left[\bar{\rho}(\bar{x},\omega)\right]^{*}} e^{+i\omega t}$$

$(\hat{r}, \hat{\theta}, \hat{\phi}) \Rightarrow \hat{r} \times \hat{\theta} = \hat{\phi}$

2) $\dfrac{\partial [\rho E(\rho)]}{\partial \rho} = 0$ $\qquad \rho E(\rho) = \text{CONST.}$

$$E(\rho) = \frac{K}{\rho}$$

$$\langle \vec{S} \rangle = \frac{1}{2\mu_0} \mathrm{Re}\left[\vec{E}^* \times \vec{B}\right]$$

SEÇÃO RETA $\displaystyle \int_a^b \rho \, d\rho \int_0^{2\pi} d\phi$

$$\frac{d[\rho E]}{d\rho} = E + \rho\frac{dE}{d\rho} = 0$$

$$\frac{dE}{d\rho} = -\frac{E}{\rho}$$

$$\int\frac{dE}{E} = -\int\frac{d\rho}{\rho}$$

$$\ln(E) = -\ln(\rho) + \text{const.}$$

$$E = \frac{1}{\rho} \times e^{\text{const}} = \frac{K}{\rho}$$

$$\cancel{\boxed{E = K\rho}}$$

$$u = \frac{1}{8\pi}(E^2 + B^2)$$

$$\bar{S} = \frac{1}{4\pi}\,\bar{E} \times \bar{B}$$

$$T_{ij} = \frac{1}{2}\left[-(E^2 + B^2)\,\delta_{ij} + E_i E_j + B_i B_j\right]$$

(?)