FI 008 – Eletrodinâmica I

1º Semestre de 2020
12/05/2020
Aula 17

Aulas passadas

Fontes harmônicas:

$$\rho(\mathbf{x},t) = \rho(\mathbf{x})\,e^{-i\omega t}$$

$$\mathbf{J}(\mathbf{x},t) = \mathbf{J}(\mathbf{x})\,e^{-i\omega t}$$

$$\boldsymbol{\nabla}\cdot\mathbf{J} = i\omega\rho$$

Soluções harmônicas:

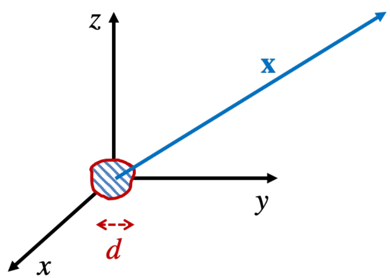
$$\mathbf{A}(\mathbf{x},t) = \mathbf{A}(\mathbf{x})\,e^{-i\omega t}$$

$$\mathbf{B}(\mathbf{x},t) = \mathbf{B}(\mathbf{x})\,e^{-i\omega t}$$

$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}(\mathbf{x})\,e^{-i\omega t}$$

Hipóteses:

- Fontes pequenas
- Região de radiação



$$\begin{cases} d \ll r = |\mathbf{x}| \\ d \ll \lambda \\ \lambda \ll r \end{cases}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega}$$

Aulas passadas

Onda esférica:

$$\hat{n} = \frac{|\vec{x}|}{r} = \hat{r}$$

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\mathbf{F}(k\hat{\mathbf{n}})$$

$$\mathbf{F}(k\hat{\mathbf{n}}) = \int \mathbf{J}(\mathbf{x}')\, e^{-ik\hat{\mathbf{n}}\cdot\mathbf{x}'} d^3x'$$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} ik \frac{e^{ikr}}{r}\hat{\mathbf{n}} \times \mathbf{F}(k\hat{\mathbf{n}})$$

$$\mathbf{E}(\mathbf{x}) = -\frac{\mu_0}{4\pi} i\omega \frac{e^{ikr}}{r}\hat{\mathbf{n}} \times \left[\hat{\mathbf{n}} \times \mathbf{F}(k\hat{\mathbf{n}})\right]$$

$$c\mathbf{B}(\mathbf{x}) = \hat{\mathbf{n}} \times \mathbf{E}(\mathbf{x})$$

$$\frac{dP}{d\Omega} = \frac{\mu_0\omega^2}{32\pi^2 c}\left|\hat{\mathbf{n}} \times \mathbf{F}(k\hat{\mathbf{n}})\right|^2$$

$$\boxed{\mathbf{F}(k\hat{\mathbf{n}}) = \sum_{n=0}^{\infty}\frac{(-ik)^n}{n!}\int \mathbf{J}(\mathbf{x}')\left(\hat{\mathbf{n}}\cdot\mathbf{x}'\right)^n d^3x'}$$

Aulas passadas

n=0: radiação de dipolo elétrico

se $\mathbf{p}$ for real

$$\mathbf{p} = \int \mathbf{x}\rho(\mathbf{x})\, d^3x$$

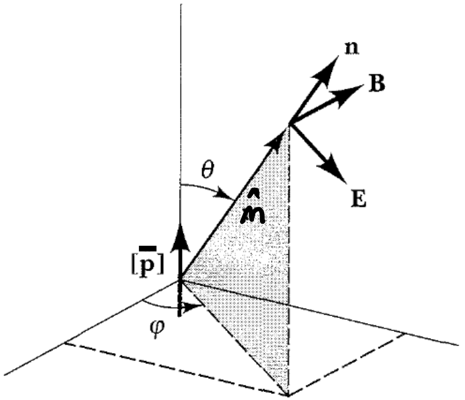
$$\mathbf{A}_{DE}(\mathbf{x}) = -i\frac{\mu_0\omega}{4\pi}\frac{e^{ikr}}{r}\mathbf{p}$$

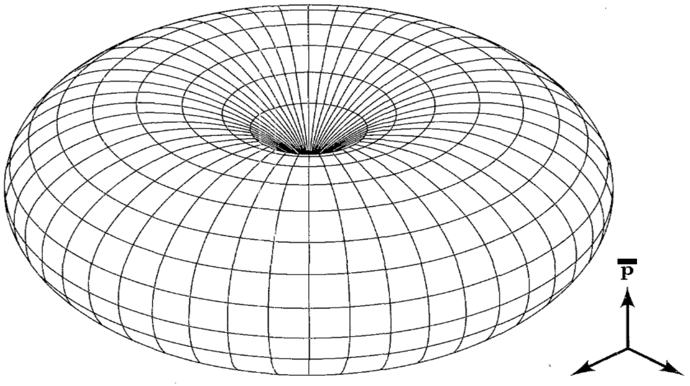
$$\mathbf{B}_{DE}(\mathbf{x}) = \frac{\mu_0\omega^2}{4\pi c}\frac{e^{ikr}}{r}\hat{\mathbf{n}}\times\mathbf{p}$$

$$\mathbf{E}_{DE}(\mathbf{x}) = -\frac{\mu_0\omega^2}{4\pi}\frac{e^{ikr}}{r}\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{p})$$

$$\frac{dP_{DE}}{d\Omega} = \frac{\mu_0\omega^4}{32\pi^2 c}\left|\hat{\mathbf{n}}\times\mathbf{p}\right|^2$$

$$P_{DE} = \frac{\mu_0\omega^4}{12\pi c}\left|\mathbf{p}\right|^2$$





Aulas passada

n=1: radiação de dipolo magnético

se **m** for real

$$\mathbf{m} = \frac{1}{2}\int\left[\mathbf{x}\times\mathbf{J}\left(\mathbf{x}\right)\right]d^3x$$

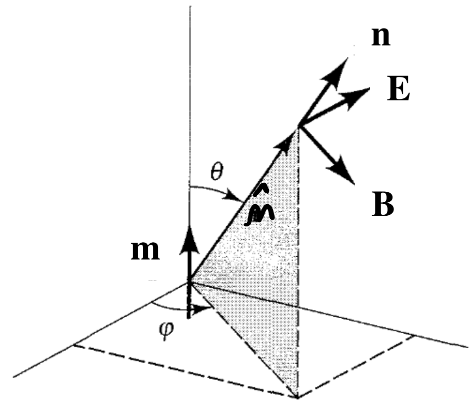
$$\mathbf{A}_{DM} = ik\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\left(\hat{\mathbf{n}}\times\mathbf{m}\right)$$

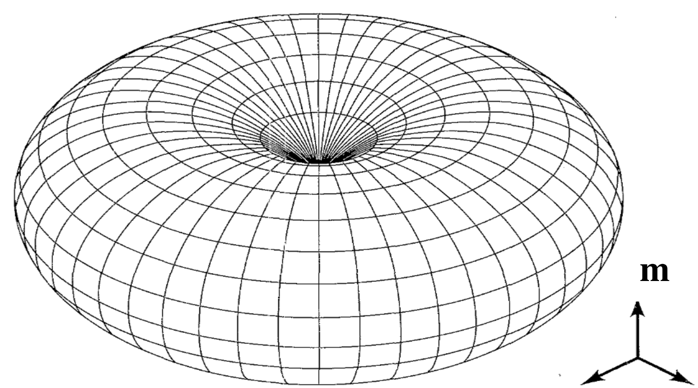
$$\mathbf{B}_{DM} = -k^2\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\hat{\mathbf{n}}\times\left(\hat{\mathbf{n}}\times\mathbf{m}\right)$$

$$\mathbf{E}_{DM} = -ck^2\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\left(\hat{\mathbf{n}}\times\mathbf{m}\right)$$

$$\frac{dP_{DM}}{d\Omega} = \frac{\mu_0 ck^4}{32\pi^2}\left|\hat{\mathbf{n}}\times\mathbf{m}\right|^2$$

$$P_{DM} = \frac{\mu_0\omega^4}{12\pi c^3}\left|\mathbf{m}\right|^2$$





Aula passada

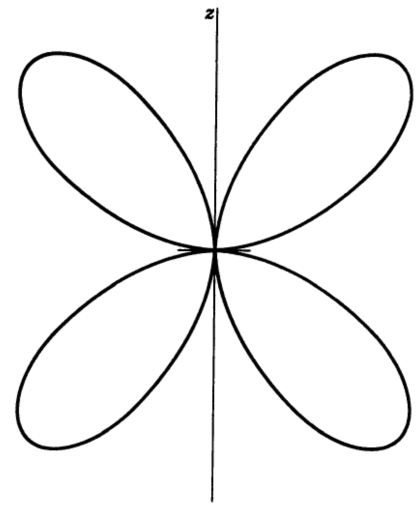
n=1: radiação de quadrupolo elétrico

$$Q_{ij} = \int \left(3x_i x_j - \delta_{ij} r^2\right) \rho\left(\mathbf{x}\right) d^3x$$

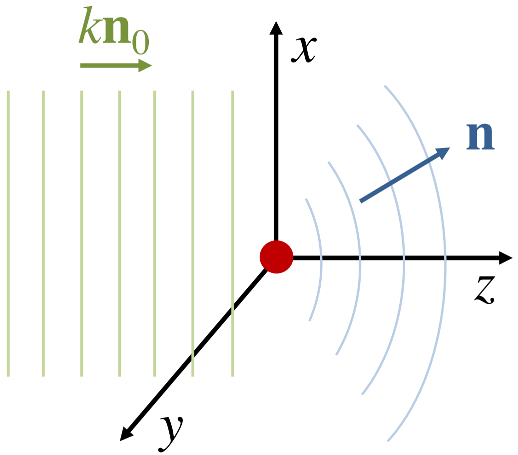
$$Q_{ij} = Q_0 \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\frac{dP}{d\Omega} = \frac{\mu_0 \omega^6 Q_0^2}{512\pi^2 c^3} \sin^2\theta \cos^2\theta$$

$$\begin{aligned} \mathbf{A}_{QE} &= -\frac{\omega k}{6} \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \left(\overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right) \\ \mathbf{B}_{QE} &= -i \frac{\omega k^2}{6} \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \hat{\mathbf{n}} \times \left(\overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right) \\ \mathbf{E}_{QE} &= i \frac{\omega^2 k}{6} \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \left[\left(\hat{\mathbf{n}} \cdot \overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right) \hat{\mathbf{n}} - \overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right] \\ \frac{dP_{QE}}{d\Omega} &= \frac{\mu_0 \omega^6}{1152\pi^2 c^3} \left[\left|\overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right|^2 - \left|\hat{\mathbf{n}} \cdot \overline{\overline{\mathbf{Q}}} \cdot \hat{\mathbf{n}}\right|^2\right] \\ P_{QE} &= \frac{\mu_0 \omega^6}{1440\pi c^3} \sum_{ij} \left|Q_{ij}\right|^2 \end{aligned}$$



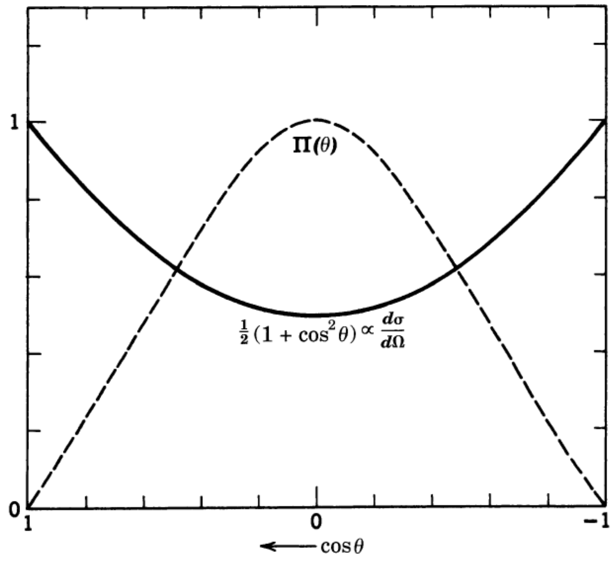
Aula passada



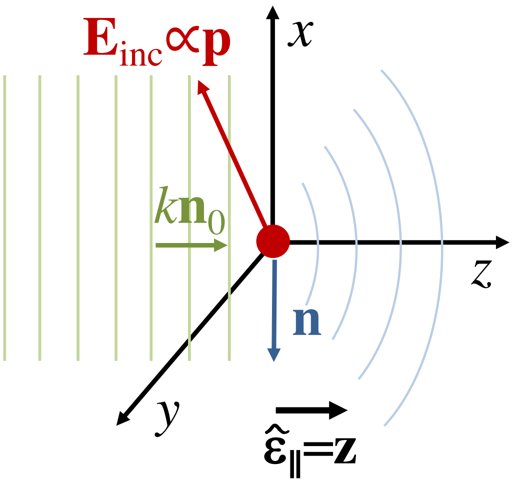
Polarização completa a 90º:

$$\frac{d\sigma_{\parallel}}{d\Omega}\left(\theta = \frac{\pi}{2}\right) \propto \left|\hat{\boldsymbol{\epsilon}}_{\parallel} \cdot \mathbf{E}_{\text{rad}}\right|^2_{\theta=\frac{\pi}{2}} = 0$$

$\hat{\boldsymbol{\epsilon}}_{\parallel} \parallel$ plano de espalhamento xz



Porque a radiação polarizada no plano de espalhamento é 0 a 90º?



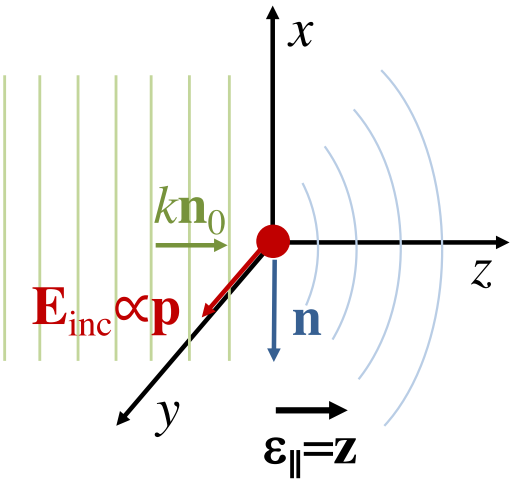
Polarização de observação:

$$\hat{\boldsymbol{\epsilon}}_{\|} = \hat{\mathbf{z}}$$

$$\mathbf{E}_{\text{inc}} = E_{0x}\hat{\mathbf{x}} + E_{0y}\hat{\mathbf{y}} \Rightarrow \mathbf{p} = p_x\hat{\mathbf{x}} + p_y\hat{\mathbf{y}}$$

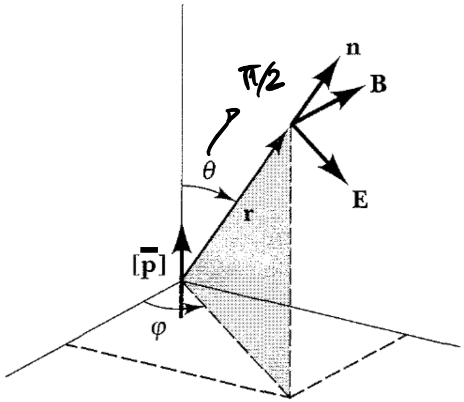
Vamos analisar separadamente as componentes de $\mathbf{E}$.

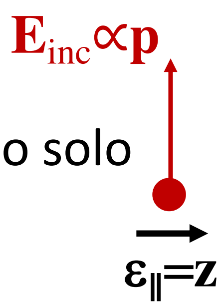
Onda incidente polarizada na direção **y**



$$\hat{\boldsymbol{\epsilon}}_{\parallel} = \hat{\mathbf{z}}$$

$$\mathbf{E}_{\rm inc} \parallel \hat{\mathbf{y}} \Rightarrow \mathbf{p} \parallel \hat{\mathbf{y}} \Rightarrow \boxed{\mathbf{E}_{\rm rad} \parallel \hat{\mathbf{y}}} \Rightarrow \hat{\boldsymbol{\epsilon}}_{\parallel} \cdot \mathbf{E}_{\rm rad} = 0$$

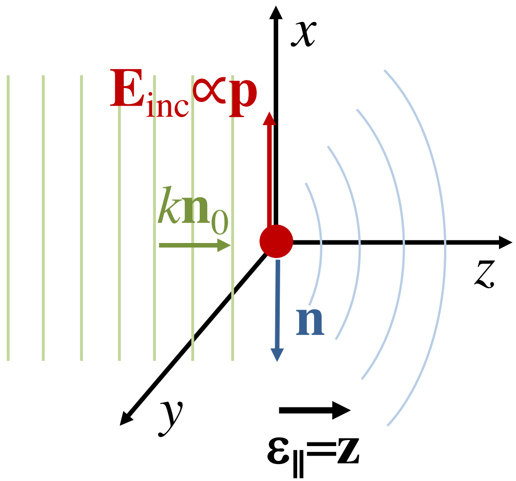




Vista do solo

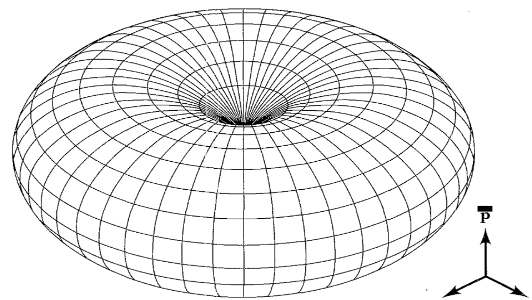
A polarização da onda irradiada é fora do plano.

Onda incidente polarizada na direção $\mathbf{x}$

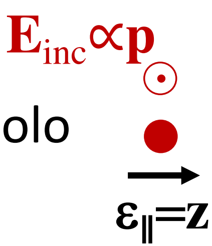


$$\hat{\boldsymbol{\epsilon}}_{\parallel} = \hat{\mathbf{z}}$$

$$\mathbf{E}_{\mathrm{inc}} \parallel \hat{\mathbf{x}} \Rightarrow \mathbf{p} \parallel \hat{\mathbf{x}} \Rightarrow \boxed{\mathbf{E}_{\mathrm{rad}} = 0} \Rightarrow \hat{\boldsymbol{\epsilon}}_{\parallel} \cdot \mathbf{E}_{\mathrm{rad}} = 0$$

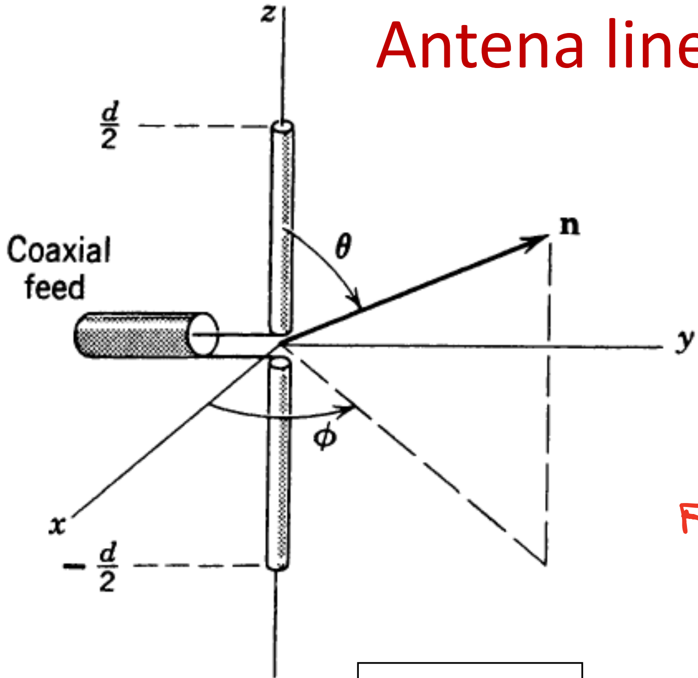


Vista do solo



Não há onda irradiada com essa polarização.

Antena linear com corrente senoidal



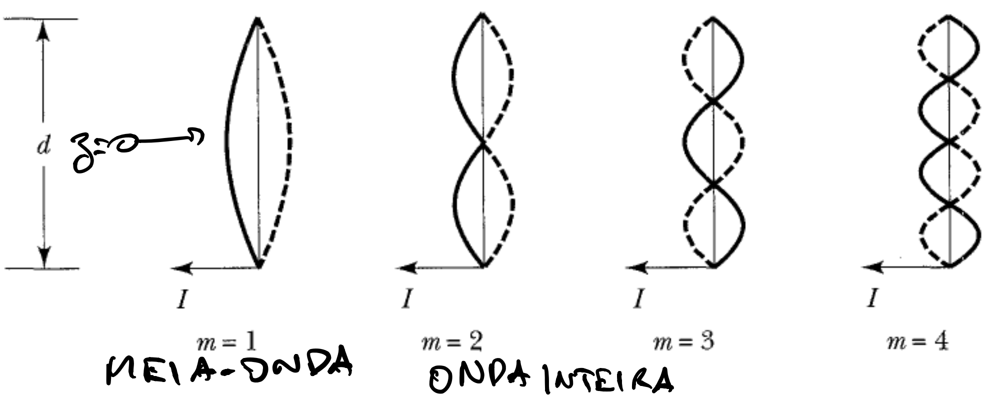
$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\mathbf{F}(k\hat{\mathbf{n}})$$

$$\mathbf{F}(k\hat{\mathbf{n}}) = \int \mathbf{J}(\mathbf{x}')\, e^{-ik\hat{\mathbf{n}}\cdot\mathbf{x}'} d^3x'$$

$$\vec{J}(\vec{x}) = I \sin\left(\frac{kd}{2} - k|z|\right)\delta(x)\delta(y)\,\hat{z}$$

$$F_z = I \int_{-d/2}^{d/2} \sin\left(\frac{kd}{2} - k|z'|\right) e^{-ikz'\cos\theta}\, dz'$$

$$\boxed{kd = m\pi}$$



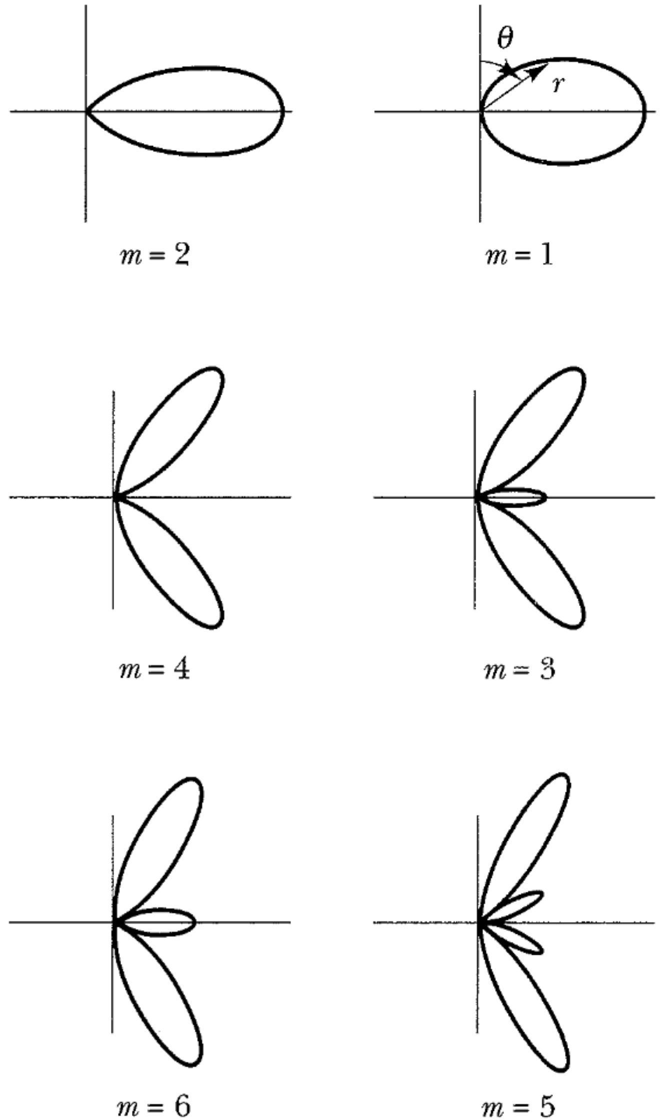
$$\vec{A}(\vec{x}) = A_z(\vec{x})\,\hat{z}$$

$$A_z(\bar{x}) = \frac{\mu_0 I}{2\pi}\,\frac{e^{ikr}}{kr}\left[\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin^2\theta}\right]$$

$$\vec{B} = ik\,\hat{n}\times\vec{A} \Rightarrow |\vec{B}| = k|A_z|\sin\theta$$

$$\Rightarrow \frac{dP}{d\Omega} = \frac{c\,r^2}{2\mu_0}|\vec{B}|^2 = \frac{\mu_0 c\, I^2}{8\pi^2}\left|\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta}\right|^2$$

$$\boxed{kd = m\pi}$$



9.1 A common textbook example of a radiating system (see Problem 9.2) is a configuration of charges fixed relative to each other but in rotation. The charge density is obviously a function of time, but it is not in the form of (9.1).

(a) Show that for rotating charges one alternative is to calculate *real* time-dependent multipole moments using $\rho(\mathbf{x}, t)$ directly and then compute the multipole moments for a given harmonic frequency with the convention of (9.1) by inspection or Fourier decomposition of the time-dependent moments. Note that care must be taken when calculating $q_{lm}(t)$ to form linear combinations that are real before making the connection.

(b) Consider a charge density $\rho(\mathbf{x}, t)$ that is periodic in time with period $T = 2\pi/\omega_0$. By making a Fourier *series* expansion, show that it can be written as

$$\rho(\mathbf{x}, t) = \rho_0(\mathbf{x}) + \sum_{n=1}^{\infty} \mathrm{Re}[2\rho_n(\mathbf{x})e^{-in\omega_0 t}]$$

where

$$\rho_n(\mathbf{x}) = \frac{1}{T}\int_0^T \rho(\mathbf{x}, t)e^{in\omega_0 t}\, dt$$

This shows explicitly how to establish connection with (9.1).

(c) For a single charge q rotating about the origin in the x-y plane in a circle of radius R at constant angular speed ω_0, calculate the $l = 0$ and $l = 1$ multipole moments by the methods of parts a and b and compare. In method b express the charge density $\rho_n(\mathbf{x})$ in cylindrical coordinates. Are there higher multipoles, for example, quadrupole? At what frequencies?

$\omega > 0$

$$\boxed{\begin{aligned} \rho(\mathbf{x}, t) &= \rho(\mathbf{x})e^{-i\omega t} \\ \mathbf{J}(\mathbf{x}, t) &= \mathbf{J}(\mathbf{x})e^{-i\omega t} \end{aligned} \qquad (9.1)}$$

(a) VOU DISCUTIR O CASO DO DIPOLO ELÉTRICO MAS OS OUTROS MOMENTOS DE MULTIPOLO SÃO ANÁLOGOS

DADO $\rho(\vec{x},t) \in \mathbb{R}$, CALCULO:

$$\vec{p}(t) = \int \vec{x}\, \rho(\vec{x},t)\, d^3x \in \mathbb{R}$$

FAÇO A ANÁLISE DE FOURIER DE $\vec{p}(t)$.

$\vec{p}(t)$ É PERIÓDICO COM PERÍODO $T = \dfrac{2\pi}{\omega_0}$

$$\Rightarrow \vec{p}(t) = \sum_{n=-\infty}^{+\infty} \vec{p}_n e^{-in\omega_0 t} \qquad \vec{p}_n = \frac{1}{T}\int_0^T \vec{p}(t) e^{in\omega_0 t} dt$$

$\vec{p}_n^{\,*} = \vec{p}_{-n}$ (PORQUE $\vec{p}(t) \in \mathbb{R}$)

$$\vec{p}(t) = \vec{p}_0 + \sum_{n=1}^{\infty} \Big[\vec{p}_n e^{-in\omega_0 t} + \underbrace{\vec{p}_{-n}}_{\vec{p}_n^{\,*}} e^{+in\omega_0 t} \Big]$$

$$\vec{p}(t) = \vec{p}_0 + \sum_{n=1}^{\infty} \mathrm{Re}\Big[2\vec{p}_n e^{-in\omega_0 t} \Big] \rightarrow \text{FORMA HARMÔNICA}$$

$\vec{p}_0$: NÃO IRRADIA!

CADA TERMO $n \geq 1$ TEM A FORMA HARMÔNICA COM DIPOLO $\underline{2\vec{p}_n}$ E FREQUÊNCIA $\underline{n\omega_0}$

$$q_{\ell m}(t) = \int \underbrace{r^\ell Y^*_{\ell m}(\theta,\phi)}_{\in \mathbb{C}} \rho(\vec{x},t)\, d^3x$$

$$q_{\ell m}(t) = (-1)^m q^*_{\ell,-m}(t)$$

$$\boxed{\text{Usando } q_{l,-m}(t) = (-1)^m q_{lm}^*(t)}$$

$$\begin{aligned}
p_x(t) &= \sqrt{\frac{2\pi}{3}}\left[q_{1-1}(t) - q_{11}(t)\right] = -2\sqrt{\frac{2\pi}{3}}\mathrm{Re}\left[q_{11}(t)\right] \\
p_y(t) &= -i\sqrt{\frac{2\pi}{3}}\left[q_{1-1}(t) + q_{11}(t)\right] = 2\sqrt{\frac{2\pi}{3}}\mathrm{Im}\left[q_{11}(t)\right] \\
p_z(t) &= \sqrt{\frac{4\pi}{3}}q_{10}(t)
\end{aligned}$$

$$\begin{aligned}
Q_{11}(t) &= \sqrt{\frac{6\pi}{5}}\left[q_{2-2}(t) + q_{22}(t)\right] - \sqrt{\frac{4\pi}{5}}q_{20}(t) = 2\sqrt{\frac{6\pi}{5}}\mathrm{Re}\left[q_{22}(t)\right] - \sqrt{\frac{4\pi}{5}}q_{20}(t) \\
Q_{22}(t) &= -\sqrt{\frac{6\pi}{5}}\left[q_{2-2}(t) + q_{22}(t)\right] - \sqrt{\frac{4\pi}{5}}q_{20}(t) = -2\sqrt{\frac{6\pi}{5}}\mathrm{Re}\left[q_{22}(t)\right] - \sqrt{\frac{4\pi}{5}}q_{20}(t) \\
Q_{33}(t) &= 2\sqrt{\frac{4\pi}{5}}q_{20}(t) \\
Q_{12}(t) &= i\sqrt{\frac{6\pi}{5}}\left[q_{22}(t) - q_{2-2}(t)\right] = -2\sqrt{\frac{6\pi}{5}}\mathrm{Im}\left[q_{22}(t)\right] \\
Q_{13}(t) &= \sqrt{\frac{6\pi}{5}}\left[q_{2-1}(t) - q_{21}(t)\right] = -2\sqrt{\frac{6\pi}{5}}\mathrm{Re}\left[q_{21}(t)\right] \\
Q_{23}(t) &= -i\sqrt{\frac{6\pi}{5}}\left[q_{2-1}(t) + q_{21}(t)\right] = 2\sqrt{\frac{6\pi}{5}}\mathrm{Im}\left[q_{21}(t)\right]
\end{aligned}$$

(b) $\rho(\bar{x},t) \in \mathbb{R}$ PERÍODO $T = \frac{2\pi}{\omega_0}$

$$\rho(\bar{x},t) = \sum_{n=-\infty}^{+\infty} e^{-in\omega_0 t}\,\rho_n(\bar{x}) \qquad \rho_n(\bar{x}) = \frac{1}{T}\int_0^T e^{+in\omega_0 t}\,\rho(\bar{x},t)\,dt$$

$$\rho(\bar{x},t) = \rho_0(\bar{x}) + \sum_{n=1}^{\infty} \mathrm{Re}\left[2\rho_n(\bar{x})\,e^{-in\omega_0 t}\right]$$

FREQUÊNCIAS: $n\omega_0$

DENSIDADES DE CARGA: $2\rho_n(\bar{x})$

c) $\rho(\vec{x},t) = \frac{q}{R^2}\,\delta(r-R)\,\delta(\theta-\frac{\pi}{2})\,\delta(\phi-\omega_0 t)$

MÉTODO DO ITEM (a)

$l=0$, NÃO HÁ RADIAÇÃO DE MONOPOLO ELÉTRICO

$l=1$, DIPOLO ELÉTRICO

$$\vec{p}(t) = \int \vec{x}\,\rho(\vec{x},t)\,d^3x = \int \overset{R}{r}(\sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}) \times \frac{q}{\cancel{R^2}} \times$$

$$\delta(r-R)\,\delta(\theta-\tfrac{\pi}{2})\,\delta(\phi-\omega_0 t)\,\cancel{r^2}\,dr\,\sin\theta\,d\theta\,d\phi$$

$$= qR\int(\cos\phi\,\hat{x} + \sin\phi\,\hat{y})\,\delta(\phi-\omega_0 t)\,d\phi$$

$$\vec{p}(t) = qR\,[\cos\omega_0 t\,\hat{x} + \sin\omega_0 t\,\hat{y}] = qR\,\mathrm{Re}\left[(\hat{x}+i\hat{y})e^{-i\omega_0 t}\right]$$

$2\vec{p}_1 = qR(\hat{x}+i\hat{y})$

$\Rightarrow$ USAR NAS FÓRMULAS DO CAPÍTULO

FREQUÊNCIA: ω_0

MÉTODO DO ITEM (b) $\quad T = \frac{2\pi}{\omega_0}$

$$\rho_m(\vec{x}) = \frac{1}{T}\int_0^T \frac{q}{R^2}\delta(r-R)\,\delta\left(\theta-\frac{\pi}{2}\right)\delta(\phi-\omega_0 t)\,\times$$

$$\times\, e^{im\omega_0 t}\,dt = \frac{\omega_0}{2\pi}\,\frac{q}{R^2}\,\delta(r-R)\,\delta\left(\theta-\frac{\pi}{2}\right)\times$$

$$\times \underbrace{\int_0^T dt\, e^{im\omega_0 t}\,\delta(\phi-\omega_0 t)}_{\frac{e^{im\phi}}{\omega_0}}$$

$$2\rho_m(x) = \frac{q}{\pi R^2}\,\delta(r-R)\,\delta\left(\theta-\frac{\pi}{2}\right)e^{im\phi}$$

FREQUÊNCIA: $m\omega_0$

DIPOLO ELÉTRICO:

$$\vec{p}_m = \int \vec{x}\,[2\rho_m(\vec{x})]\,d^3x = \int R(\cos\phi\,\hat{x} + \sin\phi\,\hat{y})$$

$$\frac{q}{\cancel{\pi R^2}}\,\delta(r-R)\,\delta\left(\theta-\frac{\pi}{2}\right)e^{im\phi}\,\cancel{r^2}\,dr \qquad \cancel{d\theta}\,d\phi$$

$$= qR\int_0^{2\pi}(\cos\phi\,\hat{x} + \sin\phi\,\hat{y})\,e^{im\phi}\,d\phi$$

$\longrightarrow$ SÓ DIFERENTE DE ZERO SE $m =$

$$2\vec{P}_1 = q\,R \int_0^{2\pi} d\phi \,(\cos\phi\,\hat{x} + \sin\phi\,\hat{y})\, e^{i\phi}$$

$$2\vec{P}_1 = q\,R\,(\hat{x} + i\,\hat{y})$$

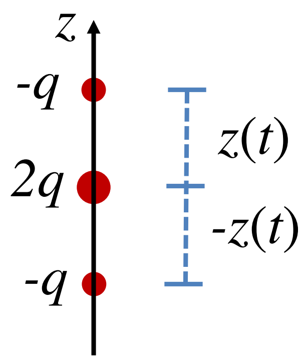
FREQUÊNCIA: ω_0

↳ FÓRMULAS DO CAPÍTULO

PARA $m > 1$ → QUADRUPOLO, OCTUPOLO, ETC.

9.11 Three charges are located along the z axis, a charge $+2q$ at the origin, and charges $-q$ at $z = \pm a \cos \omega t$. Determine the lowest nonvanishing multipole moments, the angular distribution of radiation, and ~~the total power~~ radiated. Assume that $ka \ll 1$. $\rightarrow d \ll \lambda$

$$\rho(\vec{x},t) = \Big[2q\,\delta(z) - q\,\delta(z - \underbrace{a\cos\omega t}_{z(t)}) - q\,\delta(z + a\cos\omega t)\Big] \times \delta(x)\,\delta(y)$$



$$\vec{J}(\vec{x},t) = \rho(\vec{x},t)\,\vec{v}$$

$$\vec{J}(\vec{x},t) = \hat{z}\, q a \omega \sin\omega t\, \big[\delta(z - a\cos\omega t) - \delta(z + a\cos\omega t)\big] \times \delta(x)\,\delta(y)$$

DIPOLO ELÉTRICO: $\vec{p}(t) = \int \underset{\substack{\downarrow \\ x\hat{x} + y\hat{y} + z\hat{z}}}{\vec{x}}\, \rho(\vec{x},t)\, d^3x = \hat{z}\, q\, [z(t) - z(t)] = 0$

DIPOLO MAGNÉTICO: $\vec{m}(t) = \frac{1}{2}\int (z\,\hat{z}) \times J_z \hat{z}\, dz = 0$

QUADRUPOLO ELÉTRICO:

$$Q_{ij}^{(t)} = \int (3x_i x_j - \delta_{ij} r^2)\, \rho(\bar{x},t)\, d^3x$$

$\delta(x)\delta(y) \longrightarrow Q_{ij} = 0$ SE $i \neq j$

$$Q_{11}^{(t)} = \int (2x^2 - y^2 - z^2)\, \rho(\bar{x},t)\, d^3x = -\int z^2 \rho(\bar{x},t)\, d^3x = +2q\, z^2(t)$$

$$Q_{22}(t) = \int (2y^2 - x^2 - z^2)\, \rho(\bar{x},t)\, d^3x = -\int z^2 \rho(\bar{x},t)\, d^3x = 2q\, z^2(t)$$

$$Q_{33}(t) = -4q^2 z^2(t) \quad [\ \mathrm{Tr}(Q_{ij}) = 0\ ]$$

NÃO IRRADIA

$$z^2(t) = a^2 \cos^2 \omega t = \frac{a^2}{2}(\cancel{1} + \cos 2\omega t) = \text{CONST} + \frac{a^2}{2} \mathrm{Re}[e^{-2i\omega t}]$$

$$Q_{ij}(t) = q a^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} e^{-2i\omega t} = \underbrace{-2qa^2}_{Q_0} \begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} e^{-2i\omega t}$$

FREQUÊNCIA $\rightarrow 2\omega$

$$\frac{dP}{d\Omega} = \frac{\mu_0 \omega^6 q^2 a^4}{2\pi^2 c^3} \sin^2\theta \cos^2\theta$$

$$P = \frac{4}{15} \frac{\mu_0 \omega^6 q^2 a^4}{\pi c^3}$$