FI 008 – Eletrodinâmica I

1º Semestre de 2020
28/05/2020
Aula 21

Aulas passadas

Quadrivetores: $V^\alpha \rightarrow \left(V^0, V^1, V^2, V^3\right) \equiv \left(V^0, \mathbf{V}\right)$

Quadritensores: $F^{\alpha\beta} \rightarrow \begin{pmatrix} F^{00} & F^{01} & F^{02} & F^{03} \\ F^{10} & F^{11} & F^{12} & F^{13} \\ F^{20} & F^{21} & F^{22} & F^{23} \\ F^{30} & F^{31} & F^{32} & F^{33} \end{pmatrix}$

Leis de transf. (Lorentz):

$$\begin{aligned} V'^{\alpha} &= A^{\alpha}_{\lambda} V^{\lambda} \\ F'^{\alpha\beta} &= A^{\alpha}_{\lambda} A^{\beta}_{\mu} F^{\lambda\mu} = A^{\alpha}_{\lambda} F^{\lambda\mu} \left(A^T\right)^{\mu}_{\beta} = \left(A F A^T\right)^{\alpha\beta} \end{aligned}$$

"Boost" de Lorentz:

$$A^{\alpha}_{\beta}\left(\boldsymbol{\beta} = \frac{v}{c}\hat{\mathbf{x}}\right) = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Aulas passadas

Leis de transf. (Lorentz):

$$V'^{\alpha} = A^{\alpha}_{\lambda} V^{\lambda}$$

$$F'^{\alpha\beta} = A^{\alpha}_{\lambda} A^{\beta}_{\mu} F^{\lambda\mu} = A^{\alpha}_{\lambda} F^{\lambda\mu} \left(A^T\right)^{\mu}_{\beta} = \left(AFA^T\right)^{\alpha\beta}$$

Matricialmente:

$$\begin{pmatrix} V^{0\prime} \\ V^{1\prime} \\ V^{2\prime} \\ V^{3\prime} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V^0 \\ V^1 \\ V^2 \\ V^3 \end{pmatrix}$$

$$\begin{pmatrix} F'^{00} & F'^{01} & F'^{02} & F'^{03} \\ F'^{10} & F'^{11} & F'^{12} & F'^{13} \\ F'^{20} & F'^{21} & F'^{22} & F'^{23} \\ F'^{30} & F'^{31} & F'^{32} & F'^{33} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} F^{00} & F^{01} & F^{02} & F^{03} \\ F^{10} & F^{11} & F^{12} & F^{13} \\ F^{20} & F^{21} & F^{22} & F^{23} \\ F^{30} & F^{31} & F^{32} & F^{33} \end{pmatrix} \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Aulas passadas

Contravariantes x covariantes:

$$
\begin{aligned}
A_\alpha &= g_{\alpha\beta} A^\beta \\
A^\alpha &= g^{\alpha\beta} A_\beta \\
F_{\alpha\beta} &= g_{\alpha\lambda} g_{\beta\mu} F^{\lambda\mu} \\
F^{\alpha\beta} &= g^{\alpha\lambda} g^{\beta\mu} F_{\lambda\mu}
\end{aligned}
$$

Tensor métrico de Minkowski (espaço-tempo plano):

$$
g_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad g^{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = (g_{\alpha\beta})^{-1}
$$

Aulas passadas

$$\begin{pmatrix} V^0 \\ V^1 \\ V^2 \\ V^3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} V_0 \\ -V_1 \\ -V_2 \\ -V_3 \end{pmatrix}$$

Matricialmente:

$$\begin{pmatrix} F^{00} & F^{01} & F^{02} & F^{03} \\ F^{10} & F^{11} & F^{12} & F^{13} \\ F^{20} & F^{21} & F^{22} & F^{23} \\ F^{30} & F^{31} & F^{32} & F^{33} \end{pmatrix} = \begin{pmatrix} F_{00} & -F_{01} & -F_{02} & -F_{03} \\ -F_{10} & F_{11} & F_{12} & F_{13} \\ -F_{20} & F_{21} & F_{22} & F_{23} \\ -F_{30} & F_{31} & F_{32} & F_{33} \end{pmatrix}$$

Aulas passadas

Quadri-nabla:

$$\partial_\alpha \equiv \frac{\partial}{\partial x^\alpha} \to \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right) = \left(\frac{1}{c}\frac{\partial}{\partial t}, \boldsymbol{\nabla}\right)$$

$$\partial^\alpha \equiv \frac{\partial}{\partial x_\alpha} \to \left(\frac{\partial}{\partial x_0}, \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right) = \left(\frac{\partial}{\partial x^0}, -\frac{\partial}{\partial x^1}, -\frac{\partial}{\partial x^2}, -\frac{\partial}{\partial x^3}\right) = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\boldsymbol{\nabla}\right)$$

Quadri-divergente: $$\partial_\alpha V^\alpha = \frac{1}{c}\frac{\partial V^0}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{V}$$

Quadri-Laplaciano ou d'Alembertiano ("caixa"):

$$\Box \equiv \partial_\alpha \partial^\alpha = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$

Covariância da Eletrodinâmica

ESCREVER TODAS AS EQS. DA ELETRODINÂMICA COM QUANTIDADES COM MESMAS PROPRIEDADES DE TRANSFORMAÇÃO (SOB LORENTZ) DOS DOIS LADOS DA EQUAÇÃO.

É ANÁLOGO A:

$$\vec{F} = m\vec{a}$$

ONDE AMBOS OS LADOS SÃO 3-VETORES.

Covariância da Eletrodinâmica

Unidades gaussianas:

$$\boxed{\begin{aligned} \nabla \cdot \mathbf{E} &= 4\pi\rho \\ \nabla \times \mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{B} &= \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \end{aligned}}$$

$$\mathbf{F} = q\left(\mathbf{E} + \frac{1}{c}\mathbf{u} \times \mathbf{B}\right)$$

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0 \quad (*)$$

$$\boxed{\begin{aligned} \mathbf{E} &= -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \\ \mathbf{B} &= \nabla \times \mathbf{A} \end{aligned}}$$

VAMOS USAR VERDE PARA AS "VERSÕES" NO S.I.

$$(*) \quad \frac{1}{c}\frac{\partial (c\rho)}{\partial t} + \bar{\nabla} \cdot \bar{J} = 0 \Rightarrow \boxed{J^\mu = (c\rho, \bar{J})}$$

$$\Rightarrow \boxed{\partial_\mu J^\mu = 0}$$

A quadri-corrente

$J^\mu = (c\rho, \vec{J})$ É UM QUADRI-VETOR SE A CARGA ELÉTRICA FOR UM ESCALAR.

COMO A CARGA ELÉTRICA É QUANTIZADA:

$$\pm e, \pm \frac{e}{3}, \pm \frac{2e}{3}$$

É RAZOÁVEL ASSUMIR QUE ELA É UM ESCALAR.

$$c\rho = c\frac{\delta q}{\delta V}$$

SE NO REFERENCIAL PRÓPRIO K' DA CARGA TIVERMOS

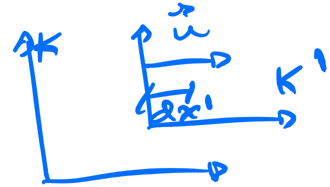
$$\delta V'$$

ENTÃO NO REFERENCIAL K ONDE ELA TEM VELOCIDADE

$$\vec{u} \Rightarrow \delta V = \delta x \, \delta y \, \delta z = \gamma^{-1} dx' \, dy' \, dz' = \gamma^{-1} \delta V'$$

$$\Rightarrow \rho' = \frac{\delta q}{\delta V'} = \frac{\rho}{\gamma} \Rightarrow \boxed{c\rho = c\gamma\rho'}$$

Em K':
$$dx' = \gamma\, dx$$
$$dy' = dy$$
$$dz' = dz$$



$$\delta V' = \gamma\, \delta V$$

$$\boxed{\vec{J} = \rho \vec{u} = \gamma \rho' \vec{u}}$$

$$\Rightarrow J^\mu = (c\rho, \vec{J}) = (c\gamma\rho', \gamma\rho'\vec{u}) = \rho'\gamma(c, \vec{u})$$

$$\boxed{J^\mu = \rho' U^\mu}$$ $\longrightarrow$ QUADRI-VETOR

JÁ QUE ρ' É A DENSIDADE DE CARGA PRÓPRIA QUE PODE SER TRATADA COMO ESCALAR SE A CARGA FOR UM ESCALAR

O quadri-potencial

No calibre de Lorenz:

$$\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} - \nabla^2\Phi = 4\pi\rho \qquad \left[= \frac{\rho}{\epsilon_0}\right]$$

$$\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \nabla^2\mathbf{A} = \frac{4\pi}{c}\mathbf{J} \qquad \left[= \mu_0 \vec{J}\right]$$

$$\boxed{\frac{1}{c}\frac{\partial\Phi}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{A} = 0}$$

SUGERE QUE $\boxed{(\Phi, \vec{A}) = A^\mu} \Longleftrightarrow \boxed{\partial_\mu A^\mu = 0}$

$\left[\left(\frac{\Phi}{c}, \vec{A}\right) = A^\mu\right]$

ALÉM DISSO:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\Phi = \frac{4\pi}{c}(c\rho)$$

$$\underbrace{\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)}_{\partial_\alpha\partial^\alpha \equiv \Box}\vec{A} = \frac{4\pi}{c}\vec{J}$$

$$\Rightarrow \boxed{\Box A^\mu = \frac{4\pi}{c} J^\mu}$$

$$\left[\Box A^\mu = \mu_0 J^\mu\right]$$

O quadri-tensor dos campos

SABEMOS QUE $\vec{E}$ E $\vec{B}$ SÃO DERIVADAS ESPAÇO-TEMPORAIS DE Φ E $\vec{A}$. ISSO SUGERE COMBINAR ∂_μ E A^μ

$\partial_\mu A^\mu = 0$ (CONDIÇÃO DE LORENTZ)

PODEMOS TENTAR $\partial^\mu A^\nu$

$$\partial^\mu A^\nu = \underbrace{\frac{1}{2}\left(\partial^\mu A^\nu + \partial^\nu A^\mu\right)}_{\text{SIMÉTRICO}} + \underbrace{\frac{1}{2}\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)}_{\text{ANTI-SIMÉTRICO}}$$

A COMBINAÇÃO SIMÉTRICA NÃO DÁ NADA DE NOVO.

MAS:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

COMO $F^{\mu\nu}$ É ANTI-SIMÉTRICO: $F^{00} = F^{11} = F^{22} = F^{33} = 0$

$$F^{0i} = \partial^0 A^i - \partial^i A^0 = \partial_0 A^i + \partial_i A^0 = \frac{1}{c}\frac{\partial \bar{A}_i}{\partial t} + \frac{\partial \Phi}{\partial x^i} = -\bar{E}_i$$

$$\bar{E} = -\bar{\nabla}\Phi - \frac{1}{c}\frac{\partial \bar{A}}{\partial t} \Rightarrow F^{i0} = E_i$$

$$F^{12} = \partial^1 A^2 - \partial^2 A^1 = \partial_2 A^1 - \partial_1 A^2 = \frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x} = -B_z$$

$F^{13} = B_y$ e $F^{23} = -B_x$

$F^{\mu\nu} \Rightarrow$ QUADRI-TENSOR DE CAMPOS

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

시간 → -시간
공간 → 공간

$$F_{\alpha\beta} = g_{\alpha\gamma}F^{\gamma\delta}g_{\delta\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

Transformações de Lorentz dos campos

$$F'^{\alpha\beta} = A^{\alpha}_{\lambda} A^{\beta}_{\mu} F^{\lambda\mu} = A^{\alpha}_{\lambda} F^{\lambda\mu} \left(A^T\right)^{\mu}_{\beta} = \left(AFA^T\right)^{\alpha\beta}$$

Para um "boost" na direção x:

$$\begin{pmatrix} 0 & -E_1' & -E_2' & -E_3' \\ E_1' & 0 & -B_3' & B_2' \\ E_2' & B_3' & 0 & -B_1' \\ E_3' & -B_2' & B_1' & 0 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix} \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{\begin{aligned} E_1' &= E_1 \\ E_2' &= \gamma\left(E_2 - \beta B_3\right) \\ E_3' &= \gamma\left(E_3 + \beta B_2\right) \end{aligned}} \qquad \boxed{\begin{aligned} B_1' &= B_1 \\ B_2' &= \gamma\left(B_2 + \beta E_3\right) \\ B_3' &= \gamma\left(B_3 - \beta E_2\right) \end{aligned}}$$

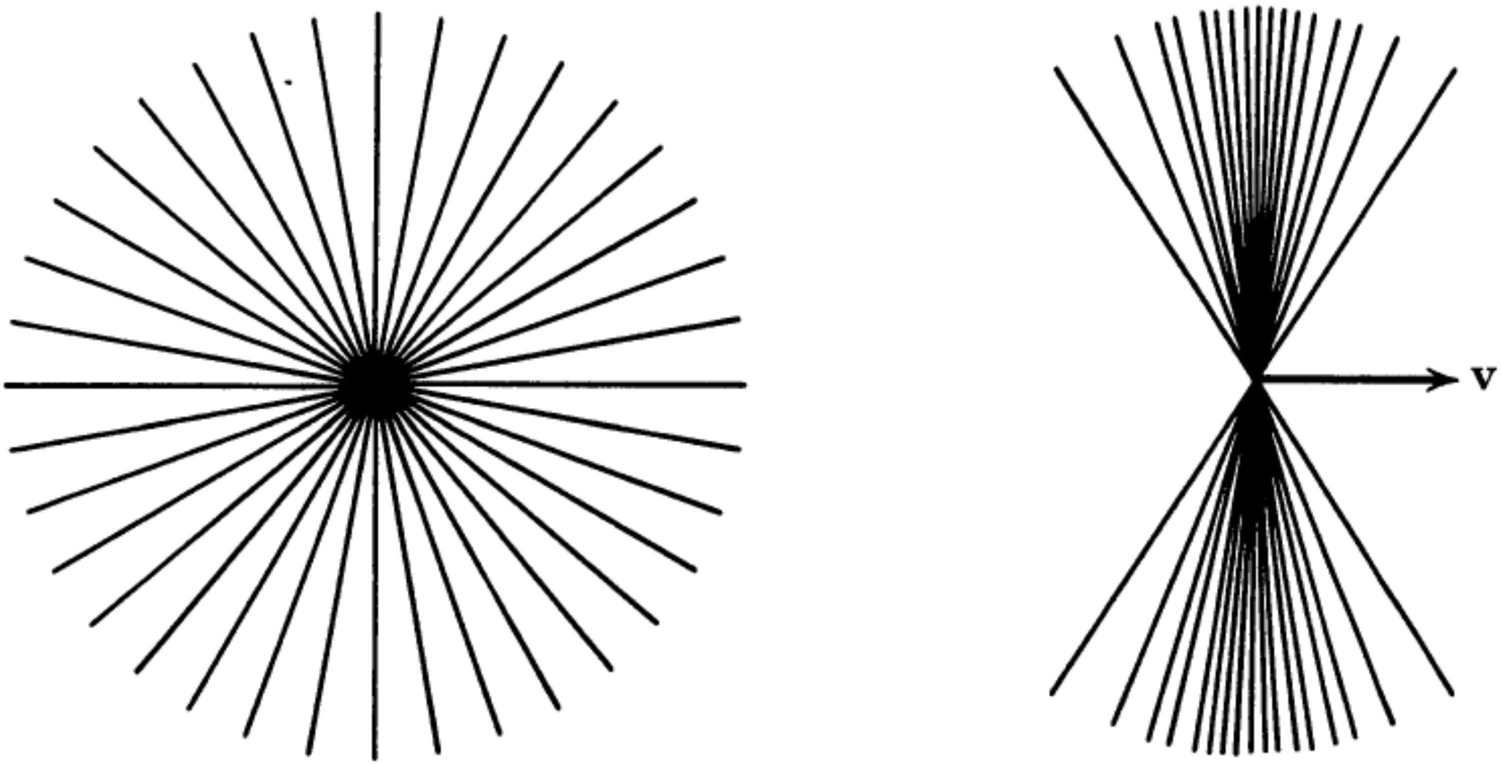
Para um "boost" qualquer ($\boldsymbol{\beta} = \mathbf{v}/c$):

$$\mathbf{E}' = \gamma\left(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B}\right) - \frac{\gamma^2}{\gamma + 1}\boldsymbol{\beta}\left(\boldsymbol{\beta} \cdot \mathbf{E}\right) \underset{\beta \ll 1}{\longrightarrow} \mathbf{E} + \boldsymbol{\beta} \times \mathbf{B}$$

$$\mathbf{B}' = \gamma\left(\mathbf{B} - \boldsymbol{\beta} \times \mathbf{E}\right) - \frac{\gamma^2}{\gamma + 1}\boldsymbol{\beta}\left(\boldsymbol{\beta} \cdot \mathbf{B}\right) \underset{\beta \ll 1}{\longrightarrow} \mathbf{B} - \boldsymbol{\beta} \times \mathbf{E}$$

Campo elétrico de uma carga em movimento

Ver discussão no Jackson (Seção 11.10)



Eqs. de Maxwell com fontes em forma covariante

QUEREMOS COMBINAR ∂_μ COM $F^{\mu\nu}$:

$\partial_\mu F^{\mu\nu}$ & J^ν ??

PROVA: $\nu = 0$:
$$\partial_\mu F^{\mu 0} = \partial_0 \cancelto{0}{F^{00}} + \partial_i F^{i0} = \frac{\partial E_i}{\partial x^i} = \bar{\nabla}\cdot\bar{E}$$

$\nu = i$:
$$\partial_\mu F^{\mu i} = \partial_0 F^{0i} + \partial_j F^{ji} = -\frac{1}{c}\frac{\partial E_i}{\partial t} + (\bar{\nabla}\times\bar{B})_i$$

$$\left.\begin{aligned}
\partial_\mu F^{\mu 0} &= \bar{\nabla}\cdot\bar{E} = 4\pi\rho = \frac{4\pi}{c}J^0 \\
\partial_\mu F^{\mu i} &= (\bar{\nabla}\times\bar{B})_i - \frac{1}{c}\frac{\partial E_i}{\partial t} = \frac{4\pi}{c}\bar{J}
\end{aligned}\right\}\quad \boxed{\partial_\mu F^{\mu\nu} = \frac{4\pi}{c}J^\mu}$$

$$\left[\partial_\mu F^{\mu\nu} = \mu_0 J^\mu\right]$$

O tensor de campos dual

4-TENSOR TOTALMENTE ANTI-SIMÉTRICO

$$e^{\alpha\beta\gamma\delta} = \begin{cases} +1 & \text{for } \alpha = 0,\ \beta = 1,\ \gamma = 2,\ \delta = 3, \text{ and} \\ & \text{any even permutation} \\ -1 & \text{for any odd permutation} \\ 0 & \text{if any two indices are equal} \end{cases}$$

GENERALIZAÇÃO DO TENSOR DE LEVI-CIVITA ϵ^{ijk} PARA 4 DIMENSÕES.

PERMUTAÇÃO PAR (ÍMPAR) DE (0,1,2,3) É UMA PERMUTAÇÃO OBTIDA POR UM NÚMERO PAR (ÍMPAR) DE TRANSPOSIÇÕES DE ÍNDICES (TROCAS DE 2 ÍNDICES

APENAS:

(0123) → (1023), (0213), (1302) → ÍMPARES

(0123) → (1032), (0231), (3102) → PARES

$$\epsilon^{\mu\nu\alpha\beta} = -\epsilon^{\nu\mu\alpha\beta} = \epsilon^{\nu\alpha\mu\beta} = \ldots$$

INVARIANTE POR LORENTZ

$$\epsilon_{\alpha\beta\mu\nu} = -\epsilon^{\alpha\beta\mu\nu}$$

DEFINIMOS O TENSOR DUAL

$$\tilde{F}^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\mu\nu} F_{\mu\nu}$$

$$= \frac{1}{2}\epsilon^{\alpha\beta\mu\nu}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)$$

MAS:

$$\epsilon^{\alpha\beta\mu\nu}\partial_\nu A_\mu = \epsilon^{\alpha\beta\nu\mu}\partial_\mu A_\nu$$

$$= -\epsilon^{\alpha\beta\mu\nu}\partial_\mu A_\nu$$

$$\Rightarrow \boxed{\tilde{F}^{\alpha\beta} = \epsilon^{\alpha\beta\mu\nu}\partial_\mu A_\nu}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$\vec{E} \to \vec{B}$
$\vec{B} \to -\vec{E}$

$$F_{\alpha\beta} = g_{\alpha\gamma} F^{\gamma\delta} g_{\delta\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\tilde{F}^{\alpha\beta} = \mathscr{F}^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta} F_{\gamma\delta} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z & -E_y \\ B_y & -E_z & 0 & E_x \\ B_z & E_y & -E_x & 0 \end{pmatrix}$$

$\left[\vec{E} \to c\vec{B} \, ; \, \vec{B} \to -\frac{\vec{E}}{c} \right]$

Eqs. de Maxwell sem fontes em forma covariante

$$\partial_\alpha \tilde{F}^{\alpha\beta} = \partial_\alpha\left[\epsilon^{\alpha\beta\mu\nu}\partial_\mu A_\nu\right] = \underbrace{\epsilon^{\alpha\beta\mu\nu}}_{\text{ANTI-SIMÉTRICO POR } \alpha \rightleftarrows \mu}\ \underbrace{\partial_\alpha\partial_\mu}_{\text{SIMÉTRICO POR } \alpha \rightleftarrows \mu} A_\nu$$

$$\epsilon^{\alpha\beta\mu\nu}\partial_\alpha\partial_\mu = \epsilon^{\alpha\beta\mu\nu}\partial_\mu\partial_\alpha \underset{\alpha \rightleftarrows \mu}{=} \epsilon^{\mu\beta\alpha\nu}\partial_\alpha\partial_\mu = -\epsilon^{\alpha\beta\mu\nu}\partial_\alpha\partial_\mu$$

$$\Rightarrow \epsilon^{\alpha\beta\mu\nu}\partial_\alpha\partial_\mu = 0$$

$$\Rightarrow \boxed{\partial_\alpha \tilde{F}^{\alpha\beta} = 0}$$

$$(\beta=0)\quad \partial_0 \cancelto{0}{\tilde{F}^{00}} + \partial_i \tilde{F}^{i0} = \bar{\nabla}\cdot\bar{B} = 0$$

$$(\beta=i)\quad \partial_0 \tilde{F}^{0i} + \partial_j \tilde{F}^{ji} = -\frac{1}{c}\frac{\partial \bar{B}}{\partial t} - (\bar{\nabla}\times\bar{E})_i = 0$$

$$
\begin{cases}
\partial_\mu F^{\mu\nu} = \dfrac{4\pi}{c} J^\nu \\
\partial_\mu \tilde{F}^{\mu\nu} = 0
\end{cases}
$$

Força de Lorentz

$$\mathbf{F} = q\left(\mathbf{E} + \frac{\mathbf{u}}{c} \times \mathbf{B}\right) = \frac{d\mathbf{p}}{dt} \qquad \mathbf{p} = \gamma_u m \mathbf{u}$$

$$\frac{d}{dt} = \frac{1}{\gamma_u}\frac{d}{d\tau} \Rightarrow \frac{d\vec{p}}{dt} = \frac{1}{\gamma_u}\frac{d\vec{p}}{d\tau} \Rightarrow q\,\gamma_u\left(\vec{E} + \frac{\vec{u}}{c}\times\vec{B}\right) = \frac{d\vec{p}}{d\tau} \quad (*)$$

$\vec{p}$ É A PARTE ESPACIAL DE $p^\mu = \left(\frac{E}{c}, \vec{p}\right)$

$(*)$ → PARTE ESPACIAL DE UMA EQUAÇÃO COVARIANTE

$\Rightarrow q\,\gamma_u\left(\vec{E} + \frac{\vec{u}}{c}\times\vec{B}\right)$ → PARTE ESPACIAL DE UM 4-VETOR

COMBINAR $F^{\mu\nu}$ COM U^μ (4-VELOCIDADE) $= \gamma_u(c, \vec{u})$

$q\,F^{\mu\nu}U_\nu = \,?$

$$\mu = i: \quad F^{i\nu} U_\nu = F^{i0} U_0 + F^{ij} U_j$$

$$= F^{i0} U^0 - F^{ij} U^j$$

$$= E_i c \gamma_u - \gamma_u \underbrace{F^{ij} u_j}_{(\vec{B} \times \vec{u})_i}$$

$$= \gamma_u c \left(\vec{E} + \frac{1}{c} \vec{u} \times \vec{B} \right)_i$$

$$\Rightarrow \boxed{\frac{q}{c} F^{\mu\nu} U_\nu = \frac{d p^\mu}{d\tau}}$$

PARTE ESPACIAL $\Rightarrow$ $q \gamma_u \left(\vec{E} + \frac{1}{c} \vec{u} \times \vec{B} \right) = \frac{d\vec{p}}{d\tau}$

A PARTE TEMPORAL É DE GRAÇA:

$$F^{0\nu} U_\nu = \cancel{F^{00}} U_0 - F^{0i} U^i = E_i \gamma_u u_i = \gamma_u \vec{u} \cdot \vec{E}$$

$$\frac{q}{\cancel{c}} \gamma_u \vec{u} \cdot \vec{E} = \frac{dp^0}{d\tau} = \frac{1}{\cancel{c}} \frac{\partial E}{d\tau} \Rightarrow \boxed{\frac{dE}{dt} = q \, \vec{u} \cdot \vec{E}}$$

$$\frac{dE}{dt} = q \, \vec{u} \cdot \vec{F} = q \, \vec{u} \cdot \vec{E}$$

TEOREMA TRABALHO-ENERGIA CINÉTICA RELATIVÍSTICO PARA A FORÇA LORENTZ

$$\frac{dp^\mu}{d\tau} = \frac{q}{c} F^{\mu\nu} U_\nu$$

$$\left[\frac{dp^\mu}{d\tau} = q F^{\mu\nu} U_\nu \right]$$

FICAM FALTANDO TEOREMA DE POYNTING E CONSERVAÇÃO DE MOMENTO LINEAR

$\longrightarrow$ PRÓXIMO CAPÍTULO.