FI 008 – Eletrodinâmica I

1º Semestre de 2021
24/06/2021
Aula 26

Aula passada

Potenciais Φ e $\mathbf{A}$ gerados por uma partícula de carga e e **trajetória dada** $\mathbf{r}(t)$: potenciais de Liénard-Wiechert

$$\Phi(\mathbf{x},t) = \frac{e}{|\mathbf{x}-\mathbf{r}(t_r)|}\frac{1}{1-\frac{\dot{\mathbf{r}}(t_r)\cdot[\mathbf{x}-\mathbf{r}(t_r)]}{c|\mathbf{x}-\mathbf{r}(t_r)|}} = \frac{e}{R}\frac{1}{1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}}}\Bigg|_{t=t_r}$$

$$\mathbf{A}(\mathbf{x},t) = \frac{e/c}{|\mathbf{x}-\mathbf{r}(t_r)|}\frac{\dot{\mathbf{r}}(t_r)}{1-\frac{\dot{\mathbf{r}}(t_r)\cdot[\mathbf{x}-\mathbf{r}(t_r)]}{c|\mathbf{x}-\mathbf{r}(t_r)|}} = \frac{e}{R}\frac{\boldsymbol{\beta}}{1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}}}\Bigg|_{t=t_r}$$

Notação:

$$\mathbf{R} = \mathbf{x}-\mathbf{r}(t_r)$$

$$R = |\mathbf{x}-\mathbf{r}(t_r)|$$

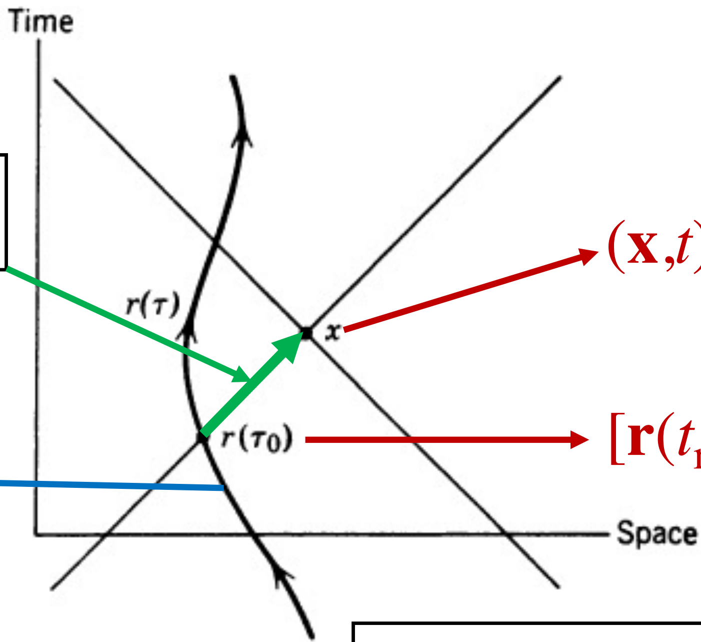
$$\hat{\mathbf{n}} = \frac{\mathbf{R}}{R} = \frac{\mathbf{x}-\mathbf{r}(t_r)}{|\mathbf{x}-\mathbf{r}(t_r)|}$$

$$\boldsymbol{\beta} = \frac{\dot{\mathbf{r}}(t_r)}{c}$$

Aula passada

$[\mathbf{r}(t_r), t_r]$ está no cone de luz do passado de $(\mathbf{x}, t)$: interseção da trajetória com o cone de luz de $(\mathbf{x}, t)$

$$c\left(t - t_r\right) = \left|\mathbf{x} - \mathbf{r}\left(t_r\right)\right|$$



$(\mathbf{x}, t)$

$[\mathbf{r}(t_r), t_r]$

Trajetória: $[\mathbf{r}(t), t]$

É fundamental para o entendimento posterior notar que t_r e $\mathbf{r}(t_r)$ são funções de $(\mathbf{x}, t)$:

$$\begin{aligned} t_r &= t_r\left(\mathbf{x}, t\right) \\ \mathbf{r}\left(t_r\right) &= \mathbf{r}\left[t_r\left(\mathbf{x}, t\right)\right] \end{aligned}$$

Campos de Liénard-Wiechert

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = \frac{e}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3}\left\{\frac{\hat{\mathbf{n}}-\boldsymbol{\beta}}{\gamma^2 R^2} + \frac{1}{c}\frac{\hat{\mathbf{n}}\times\left[(\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]}{R}\right\}\Bigg|_{t=t_r}$$

$$\mathbf{B} = (\hat{\mathbf{n}}\times\mathbf{E})\big|_{t=t_r}$$

Notação:

$$\mathbf{R} = \mathbf{x} - \mathbf{r}(t_r)$$

$$R = |\mathbf{x} - \mathbf{r}(t_r)|$$

$$\hat{\mathbf{n}} = \frac{\mathbf{R}}{R} = \frac{\mathbf{x}-\mathbf{r}(t_r)}{|\mathbf{x}-\mathbf{r}(t_r)|}$$

$$\boldsymbol{\beta} = \frac{\dot{\mathbf{r}}(t_r)}{c}$$

$$\dot{\boldsymbol{\beta}} = \frac{\ddot{\mathbf{r}}(t_r)}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Distribuição angular da potência total irradiada

$$\frac{dP(t)}{d\Omega} = R^2\hat{\mathbf{n}}\cdot\mathbf{S} = \frac{c}{4\pi}R^2\hat{\mathbf{n}}\cdot\left(\mathbf{E}_a\times\mathbf{B}_a\right)$$

$$= \frac{e^2}{4\pi c}\frac{1}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}}\right)^6}\left.\left|\hat{\mathbf{n}}\times\left[\left(\hat{\mathbf{n}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2\right|_{\mathrm{ret}}$$

$$\frac{dP(t_r)}{d\Omega} = \frac{dP(t)}{d\Omega}\frac{\partial t}{\partial t_r} = \frac{dP(t)}{d\Omega}\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}}\right)$$

$$\frac{dP(t_r)}{d\Omega} = \frac{e^2}{4\pi c}\frac{1}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}}\right)^5}\left.\left|\hat{\mathbf{n}}\times\left[\left(\hat{\mathbf{n}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2\right|_{\mathrm{ret}}$$

Distribuição angular da potência total irradiada

$$\frac{dP\left(t_r\right)}{d\Omega} = \frac{e^2}{4\pi c}\frac{1}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}}\right)^5}\left|\hat{\mathbf{n}}\times\left[\left(\hat{\mathbf{n}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2\Bigg|_{\mathrm{ret}}$$

$$\frac{dP_{\text{não-relat.}}}{d\Omega} = \frac{e^2}{4\pi c}\left|\hat{\mathbf{n}}\times\left[\hat{\mathbf{n}}\times\dot{\boldsymbol{\beta}}\right]\right|^2 = \frac{e^2a^2}{4\pi c^3}\sin^2\theta$$

θ = ÂNGULO ENTRE $\dot{\vec{\beta}} \sim \vec{a}$ E $\hat{n}$

Potência total irradiada

$$P = \frac{2}{3}\frac{e^2}{c}\gamma^6\left[\left|\dot{\boldsymbol{\beta}}\right|^2 - \left|\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}}\right|^2\right] \quad \text{(Fórmula de Liénard)}$$

$$P_{\text{não}-\text{relat.}} = \frac{2}{3}\frac{e^2}{c}\left|\dot{\boldsymbol{\beta}}\right|^2 = \frac{2}{3}\frac{e^2a^2}{c^3} \quad \text{(Fórmula de Larmor)}$$

$$\Delta \mathcal{L} = \partial^{\mu} G_{\mu} = \partial_{\mu} G^{\mu}$$

$$\Delta S = \int \Delta \mathcal{L} \, d^4x = \int \partial_{\mu} G^{\mu} d^4x$$

$$= \int_{S_\infty} G^{\mu} \, d\Sigma_{\mu}$$

$$\delta \Delta S = \int_{S_\infty} \delta G^{\mu} \, d\Sigma_{\mu}$$

POR HIPÓTESE DO PRINCÍPIO DE HAMILTON, $\delta G^{\mu} = 0$ NA BORDA S_∞ DO ESPAÇO-TEMPO

ANÁLOGO A:

$$\Delta L = \frac{df}{dt}[\{q_i\}, t]$$

$$\Rightarrow \Delta S = \int_{t_1}^{t_2} \Delta L \, dt = \int_{t_1}^{t_2} \frac{df}{dt} dt = f[\{q_i(t_2)\}, t_2] - f[\{q_i(t_1)\}, t_1]$$

$$\delta q_i(t_1) = 0 = \delta q_i(t_2)$$

$$\Rightarrow \delta \Delta S = 0$$