FI 008 – Eletrodinâmica I

1º Semestre de 2021

01/07/2021

Aula 27

Aulas passadas

Potenciais de Liénard-Wiechert:

$$\Phi(\mathbf{x},t) = \frac{e}{|\mathbf{x}-\mathbf{r}(t_r)|}\frac{1}{1-\frac{\dot{\mathbf{r}}(t_r)\cdot[\mathbf{x}-\mathbf{r}(t_r)]}{c|\mathbf{x}-\mathbf{r}(t_r)|}} = \left.\frac{e}{R}\frac{1}{1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}}}\right|_{t=t_r}$$

$$\mathbf{A}(\mathbf{x},t) = \frac{e/c}{|\mathbf{x}-\mathbf{r}(t_r)|}\frac{\dot{\mathbf{r}}(t_r)}{1-\frac{\dot{\mathbf{r}}(t_r)\cdot[\mathbf{x}-\mathbf{r}(t_r)]}{c|\mathbf{x}-\mathbf{r}(t_r)|}} = \left.\frac{e}{R}\frac{\boldsymbol{\beta}}{1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}}}\right|_{t=t_r}$$

Campos de Liénard-Wiechert:

$$\mathbf{E} = \frac{e}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3}\left.\left\{\frac{\hat{\mathbf{n}}-\boldsymbol{\beta}}{\gamma^2R^2}+\frac{1}{c}\frac{\hat{\mathbf{n}}\times\left[(\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]}{R}\right\}\right|_{t=t_r}$$

$$\mathbf{B} = \left.(\hat{\mathbf{n}}\times\mathbf{E})\right|_{t=t_r}$$

Notação:

$$\mathbf{R} = \mathbf{x}-\mathbf{r}(t_r)$$

$$R = |\mathbf{x}-\mathbf{r}(t_r)|$$

$$\hat{\mathbf{n}} = \frac{\mathbf{R}}{R} = \frac{\mathbf{x}-\mathbf{r}(t_r)}{|\mathbf{x}-\mathbf{r}(t_r)|}$$

$$\boldsymbol{\beta} = \frac{\dot{\mathbf{r}}(t_r)}{c}$$

$$\dot{\boldsymbol{\beta}} = \frac{\ddot{\mathbf{r}}(t_r)}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

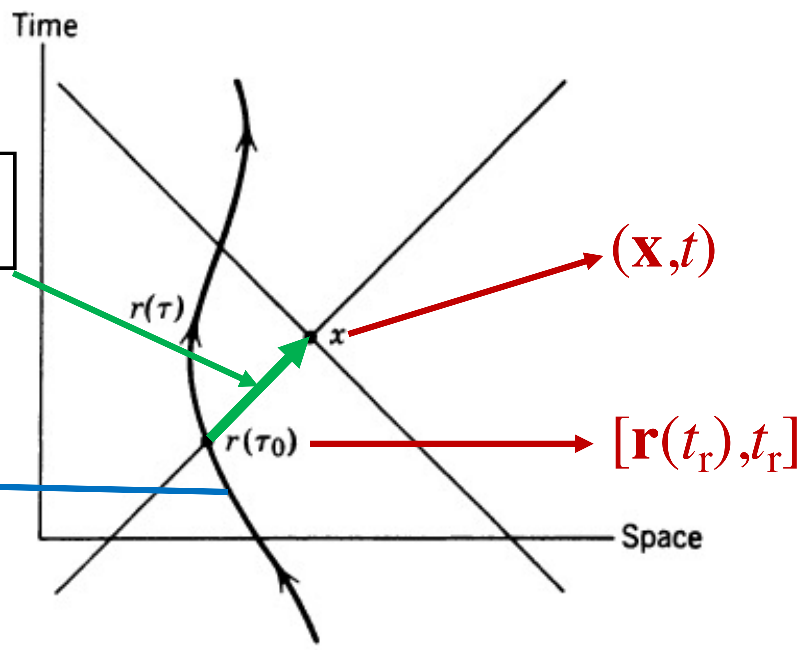
Aulas passadas

O tempo retardado t_r:

$(\mathbf{x},t)$ está no cone de luz do futuro de $[\mathbf{r}(t_r),t_r]$

$$c\left(t-t_r\right)=\left|\mathbf{x}-\mathbf{r}\left(t_r\right)\right|$$

Trajetória: $[\mathbf{r}(t),t]$



Aulas passadas

Distribuição angular da potência total irradiada:

$$\frac{dP\left(t_r\right)}{d\Omega} = \frac{e^2}{4\pi c}\frac{1}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}}\right)^5}\left|\hat{\mathbf{n}}\times\left[\left(\hat{\mathbf{n}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2\Bigg|_{\mathrm{ret}}$$

$$\frac{dP_{\text{não}-\text{relat.}}}{d\Omega} = \frac{e^2}{4\pi c}\left|\hat{\mathbf{n}}\times\left[\hat{\mathbf{n}}\times\dot{\boldsymbol{\beta}}\right]\right|^2 = \frac{e^2a^2}{4\pi c^3}\sin^2\theta$$

Potência total irradiada

$$P = \frac{2}{3}\frac{e^2}{c}\gamma^6\left[\left|\dot{\boldsymbol{\beta}}\right|^2-\left|\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}}\right|^2\right] \quad \text{(Fórmula de Liénard)}$$

$$P_{\text{não}-\text{relat.}} = \frac{2}{3}\frac{e^2}{c}\left|\dot{\boldsymbol{\beta}}\right|^2 = \frac{2}{3}\frac{e^2a^2}{c^3} \quad \text{(Fórmula de Larmor)}$$

Perdas em aceleradores lineares

Da fórmula de Liénard:

$$P_{lin} = \frac{2}{3}\frac{e^2}{c}\gamma^6\dot{\beta}^2 = \frac{2}{3}\frac{e^2}{c}\left(\frac{F}{mc}\right)^2 = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dE}{dx}\right)^2$$

$$\frac{P_{lin}}{P_{gain}} = \frac{2}{3}\frac{e^2}{m^2c^4}\left(\frac{dE}{dx}\right) \approx \frac{dE/dx}{10^{14}\ \mathrm{MeV/m}}$$

Usando valores típicos:

$$\frac{dE}{dx} = 50\frac{\mathrm{MeV}}{\mathrm{m}} \Rightarrow \frac{P_{lin}}{P_{gain}} \approx 10^{-13}$$

Perdas por radiação são <u>desprezíveis</u>.

Perdas em aceleradores circulares

Da fórmula de Liénard:

$$P_{circ} = \frac{2}{3}\frac{e^2}{c}\gamma^4\dot{\beta}^2 = \frac{2}{3}\frac{e^2}{c}\frac{\gamma^4}{\rho^2} = \frac{2}{3}\frac{e^4B^2}{m^2c^3}\gamma^2$$

$$\delta E_{volta} \approx \frac{4\pi}{3}\frac{e^3B}{mc^2}\gamma^3$$

Usando valores típicos:

$$\left.\begin{array}{c} 10\ \mathrm{GeV} \\ 100\ \mathrm{m} \\ 3.3\ \mathrm{kG} \end{array}\right\} \Rightarrow \delta E_{volta} \approx 10\ \mathrm{MeV}$$

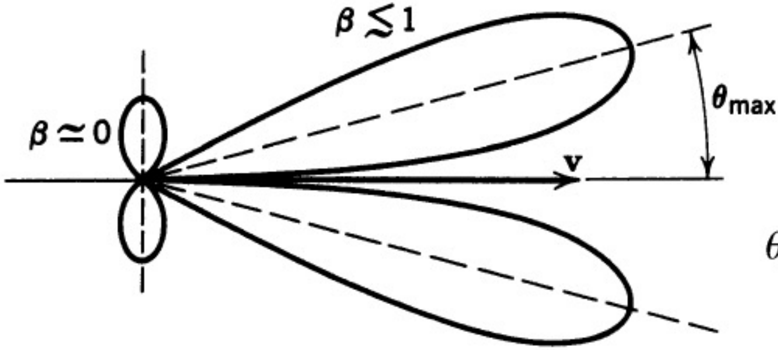
Perdas não são desprezíveis, mas da ordem do ganho de energia gerado pela radiofrequência.

Distribuição angular da radiação para velocidade e aceleração paralelas

$\vec{\beta} \parallel \dot{\vec{\beta}}$

$$\frac{dP(t')}{d\Omega} = \frac{e^2\dot{v}^2}{4\pi c^3}\frac{\sin^2\theta}{(1 - \beta\cos\theta)^5}$$

$$\gamma \gg 1 \quad \frac{dP(t')}{d\Omega} \simeq \frac{8}{\pi}\frac{e^2\dot{v}^2}{c^3}\gamma^8\frac{(\gamma\theta)^2}{(1 + \gamma^2\theta^2)^5}$$

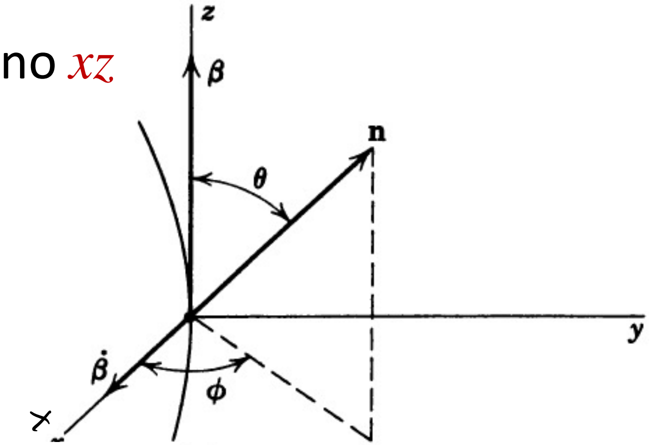


$$\theta_{\max} = \cos^{-1}\left[\frac{1}{3\beta}\left(\sqrt{1 + 15\beta^2} - 1\right)\right] \to \frac{1}{2\gamma}$$

$$\langle\theta^2\rangle^{1/2} = \frac{1}{\gamma} = \frac{mc^2}{E}$$

Distribuição angular da radiação: velocidade e aceleração perpendiculares

Tomando o movimento circular no plano *xz*



$$\boldsymbol{\beta} = \beta \hat{\boldsymbol{z}}$$

$$\dot{\boldsymbol{\beta}} = \dot{\beta} \hat{\boldsymbol{x}}$$

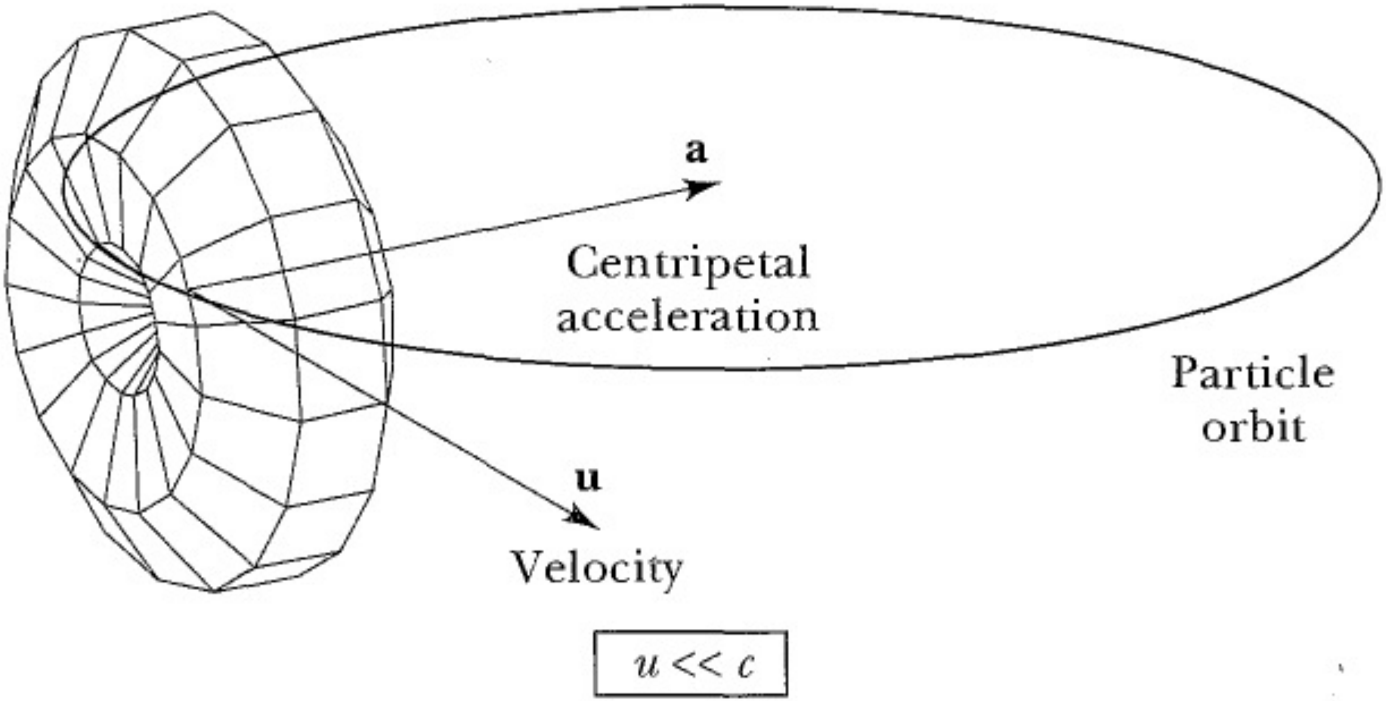
$$\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c^3} \frac{|\dot{\mathbf{v}}|^2}{(1 - \beta \cos\theta)^3} \left[1 - \frac{\sin^2\theta \cos^2\phi}{\gamma^2 (1 - \beta \cos\theta)^2} \right]$$

$$\gamma \gg 1 \quad \frac{dP(t')}{d\Omega} \simeq \frac{2}{\pi} \frac{e^2}{c^3} \gamma^6 \frac{|\dot{\mathbf{v}}|^2}{(1 + \gamma^2\theta^2)^3} \left[1 - \frac{4\gamma^2\theta^2 \cos^2\phi}{(1 + \gamma^2\theta^2)^2} \right]$$

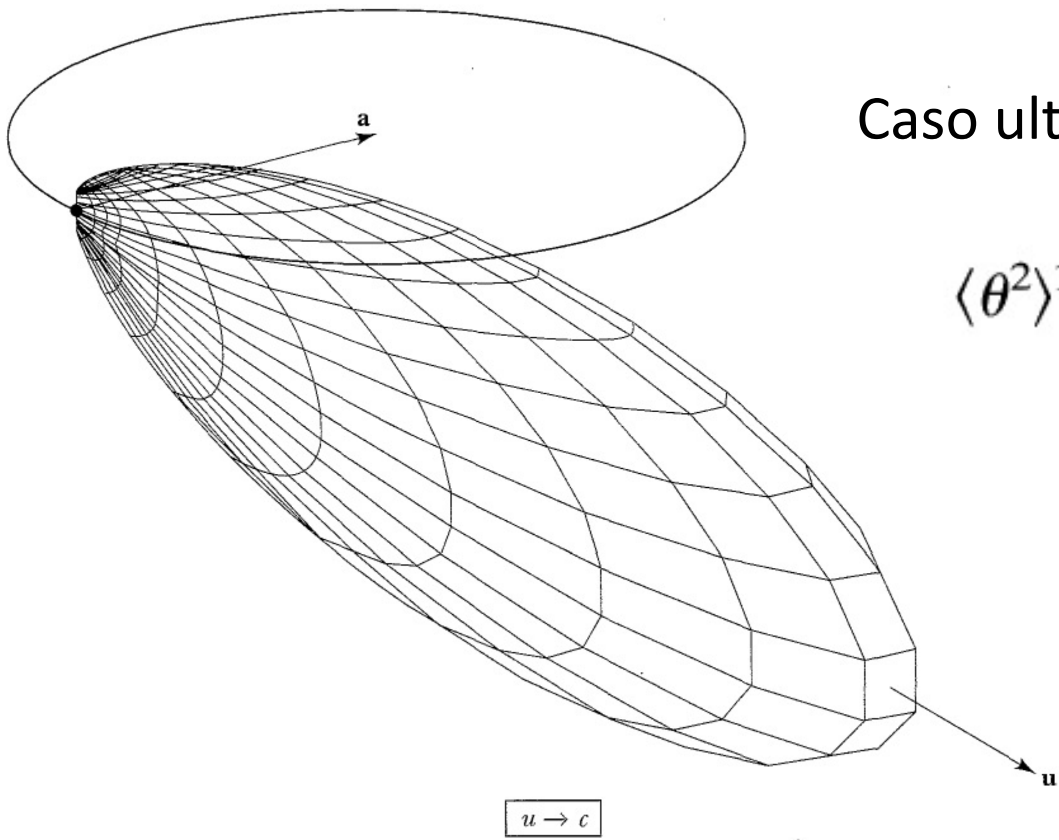
$$\langle \theta^2 \rangle^{1/2} = \frac{1}{\gamma} = \frac{mc^2}{E}$$

Distribuição angular da radiação: velocidade e aceleração perpendiculares

Caso não-relativístico:



Distribuição angular da radiação: velocidade e aceleração perpendiculares

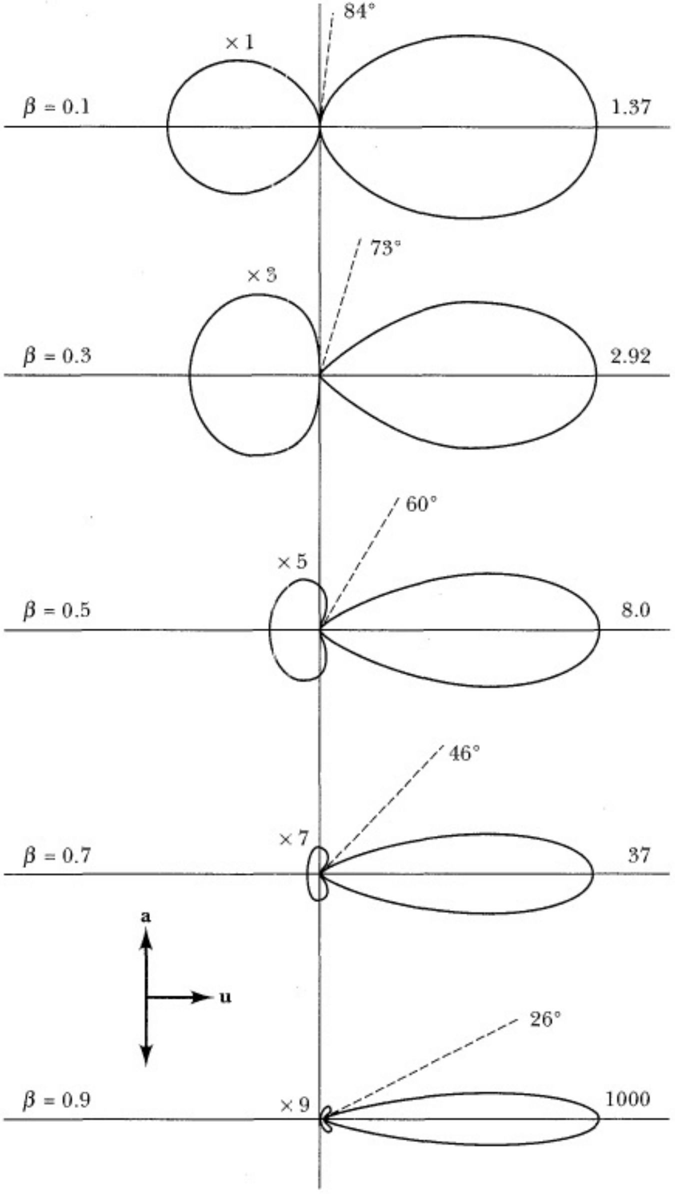


Caso ultra-relativístico:

$$\langle \theta^2 \rangle^{1/2} = \frac{1}{\gamma} = \frac{mc^2}{E}$$

Dependência com a velocidade:

$$\langle \theta^2 \rangle^{1/2} = \frac{1}{\gamma} = \frac{mc^2}{E}$$



Distribuição espectral da radiação

Energia total irradiada por ângulo sólido $d\Omega$ e no intervalo $[\omega,\omega+d\omega]$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta})\, e^{i\omega[t-\hat{\mathbf{n}}\cdot\mathbf{r}(t)/c]}dt\right|^2$$