

FI 008 – Eletrodinâmica I

1º Semestre de 2020

30/06/2020

Aula 28

Aulas passadas

Potenciais de Liénard-Wiechert:

$$\Phi(\mathbf{x}, t) = \frac{e}{|\mathbf{x} - \mathbf{r}(t_r)|} \frac{1}{1 - \frac{\dot{\mathbf{r}}(t_r) \cdot [\mathbf{x} - \mathbf{r}(t_r)]}{c|\mathbf{x} - \mathbf{r}(t_r)|}} = \frac{e}{R} \frac{1}{1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}}} \Big|_{t=t_r}$$
$$\mathbf{A}(\mathbf{x}, t) = \frac{e/c}{|\mathbf{x} - \mathbf{r}(t_r)|} \frac{\dot{\mathbf{r}}(t_r)}{1 - \frac{\dot{\mathbf{r}}(t_r) \cdot [\mathbf{x} - \mathbf{r}(t_r)]}{c|\mathbf{x} - \mathbf{r}(t_r)|}} = \frac{e}{R} \frac{\boldsymbol{\beta}}{1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}}} \Big|_{t=t_r}$$

Campos de Liénard-Wiechert:

$$\mathbf{E} = \frac{e}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}})^3} \left\{ \frac{\hat{\mathbf{n}} - \boldsymbol{\beta}}{\gamma^2 R^2} + \frac{1}{c} \frac{\hat{\mathbf{n}} \times [(\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{R} \right\} \Big|_{t=t_r}$$

$$\mathbf{B} = (\hat{\mathbf{n}} \times \mathbf{E}) \Big|_{t=t_r}$$

$$\mathbf{R} = \mathbf{x} - \mathbf{r}(t_r)$$

$$R = |\mathbf{x} - \mathbf{r}(t_r)|$$

$$\hat{\mathbf{n}} = \frac{\mathbf{R}}{R} = \frac{\mathbf{x} - \mathbf{r}(t_r)}{|\mathbf{x} - \mathbf{r}(t_r)|}$$

$$\boldsymbol{\beta} = \frac{\dot{\mathbf{r}}(t_r)}{c}$$

$$\dot{\boldsymbol{\beta}} = \frac{\ddot{\mathbf{r}}(t_r)}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$c(t - t_r) = |\mathbf{x} - \mathbf{r}(t_r)|$$

Aulas passadas

Distribuição angular da potência total irradiada:

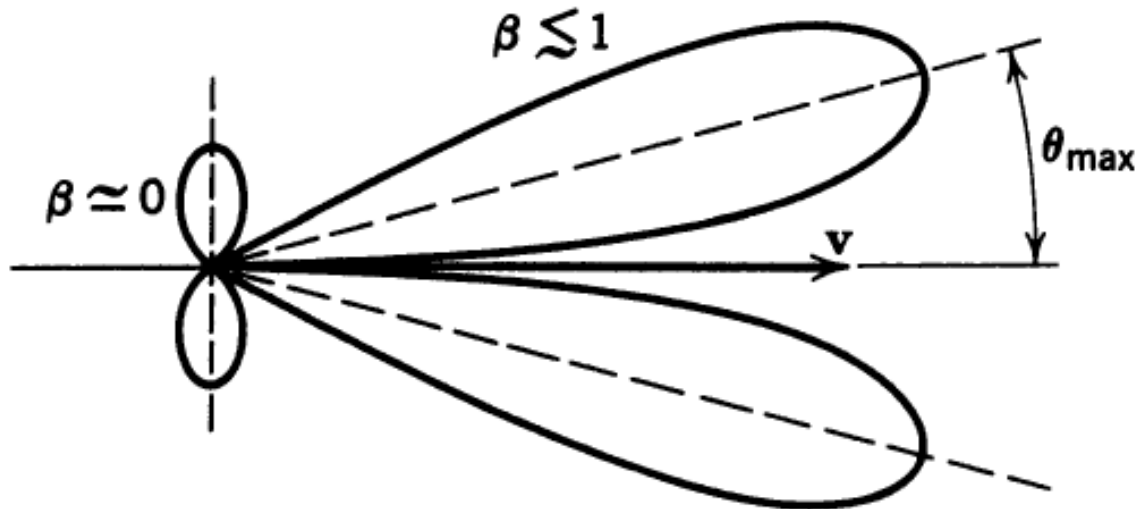
$$\frac{dP(t_r)}{d\Omega} = \frac{e^2}{4\pi c} \frac{1}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}})^5} \left| \hat{\mathbf{n}} \times [(\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}] \right|_{\text{ret}}^2$$
$$\frac{dP_{\text{n\~{a}o-relat.}}}{d\Omega} = \frac{e^2}{4\pi c} \left| \hat{\mathbf{n}} \times [\hat{\mathbf{n}} \times \dot{\boldsymbol{\beta}}] \right|^2 = \frac{e^2 a^2}{4\pi c^3} \sin^2 \theta$$

Potência total irradiada

$$P = \frac{2}{3} \frac{e^2}{c} \gamma^6 \left[\left| \dot{\boldsymbol{\beta}} \right|^2 - \left| \boldsymbol{\beta} \times \dot{\boldsymbol{\beta}} \right|^2 \right] \quad (\text{Fórmula de Liénard})$$
$$P_{\text{n\~{a}o-relat.}} = \frac{2}{3} \frac{e^2}{c} \left| \dot{\boldsymbol{\beta}} \right|^2 = \frac{2}{3} \frac{e^2 a^2}{c^3} \quad (\text{Fórmula de Larmor})$$

Aula passada

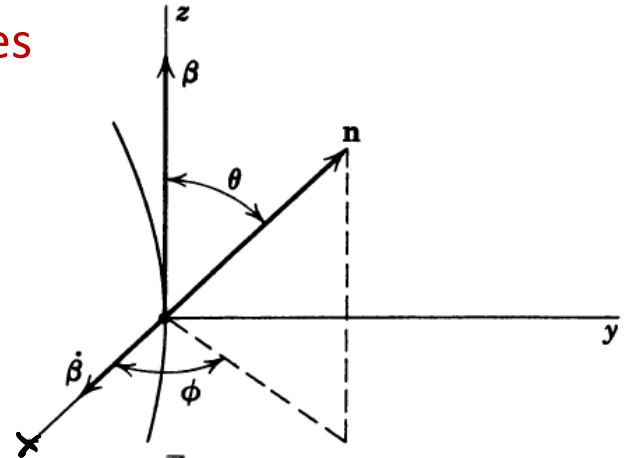
Velocidade e aceleração paralelas



$$\theta_{\max} = \cos^{-1} \left[\frac{1}{3\beta} (\sqrt{1 + 15\beta^2} - 1) \right] \rightarrow \frac{1}{2\gamma}$$

Aula passada

Velocidade e aceleração perpendiculares



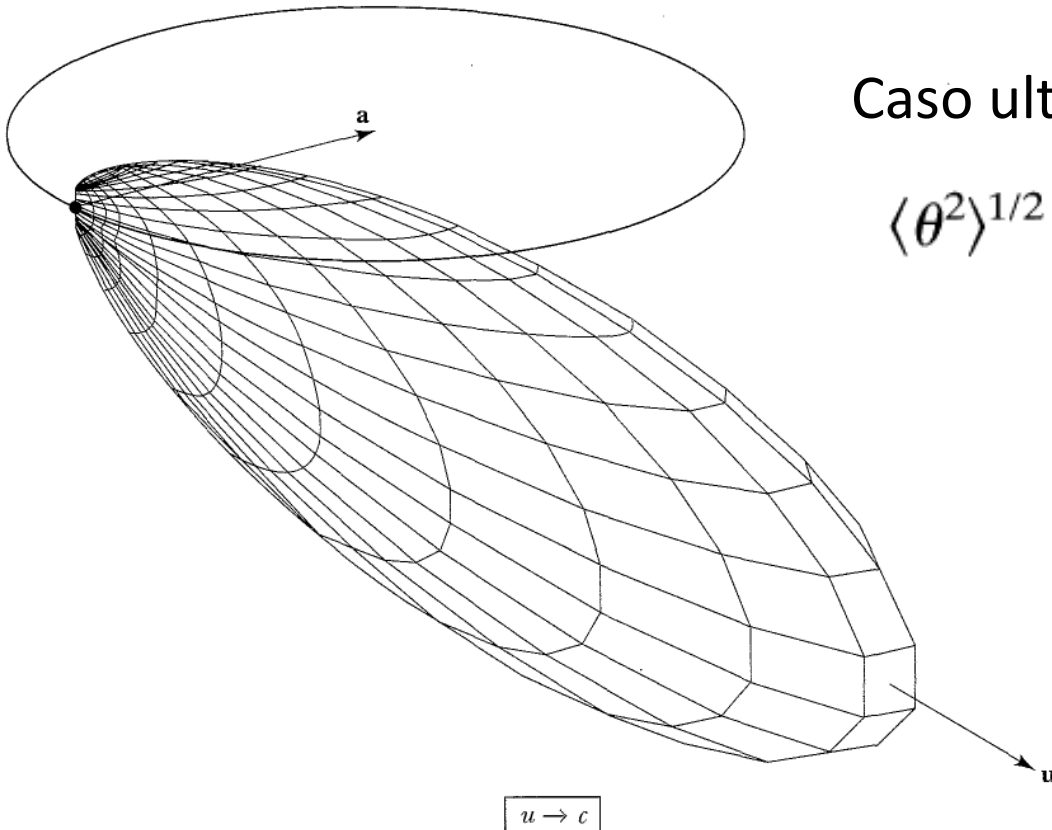
$$\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c^3} \frac{|\dot{\mathbf{v}}|^2}{(1 - \beta \cos \theta)^3} \left[1 - \frac{\sin^2 \theta \cos^2 \phi}{\gamma^2 (1 - \beta \cos \theta)^2} \right]$$

$$\gamma \gg 1 \quad \frac{dP(t')}{d\Omega} \simeq \frac{2}{\pi} \frac{e^2}{c^3} \gamma^6 \frac{|\dot{\mathbf{v}}|^2}{(1 + \gamma^2 \theta^2)^3} \left[1 - \frac{4\gamma^2 \theta^2 \cos^2 \phi}{(1 + \gamma^2 \theta^2)^2} \right]$$

$$\langle \theta^2 \rangle^{1/2} = \frac{1}{\gamma} = \frac{mc^2}{E}$$

Aula passada

Velocidade e aceleração perpendiculares



Caso ultra-relativístico:

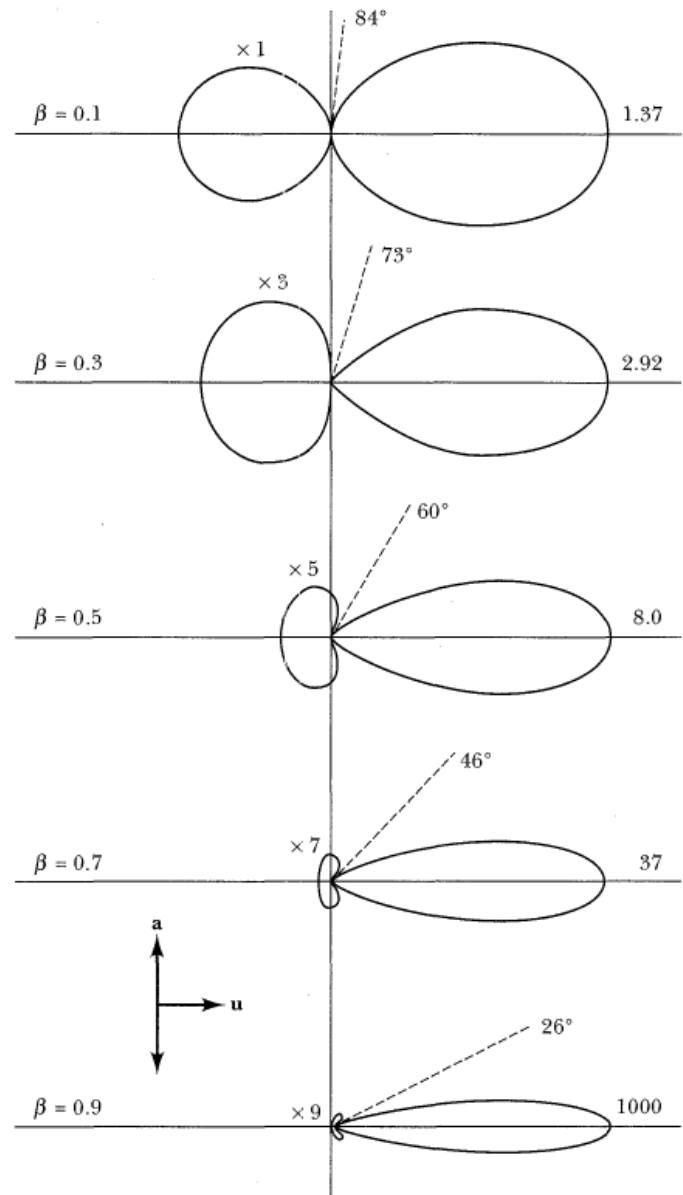
$$\langle \theta^2 \rangle^{1/2} = \frac{1}{\gamma} = \frac{mc^2}{E}$$

Aula passada

Velocidade e aceleração
perpendiculares

Dependência com a velocidade:

$$\langle \theta^2 \rangle^{1/2} = \frac{1}{\gamma} = \frac{mc^2}{E}$$

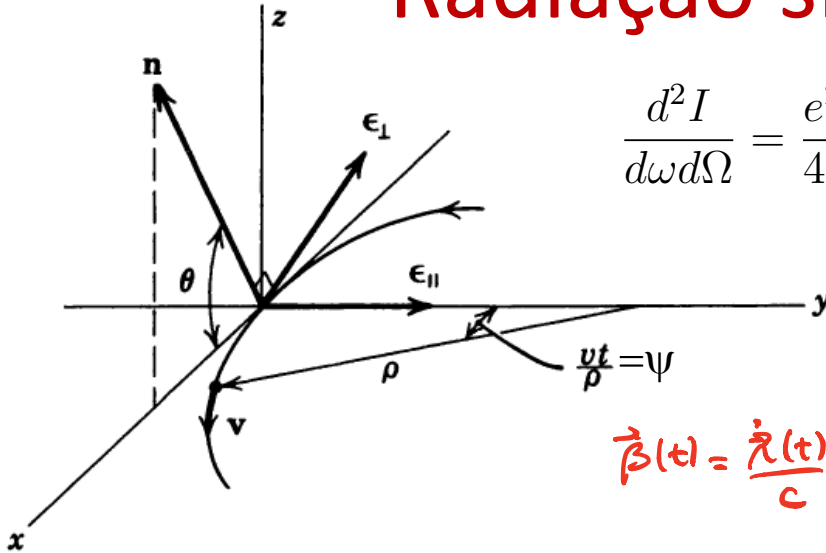


Distribuição espectral e angular

Energia irradiada por unidade de frequência por ângulo sólido:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} \hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta}) e^{i\omega[t - \hat{\mathbf{n}} \cdot \mathbf{r}(t)/c]} dt \right|^2$$

Radiação síncrotron



$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} \hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta}) e^{i\omega[t - \hat{\mathbf{n}} \cdot \mathbf{r}(t)/c]} dt \right|^2$$

$$\vec{\mathbf{r}}(t) = \rho \sin \psi(t) \hat{\mathbf{x}} + \rho(1 - \cos \psi(t)) \hat{\mathbf{y}}$$

$$\psi(t) = \omega_0 t = \frac{v}{\rho} t$$

$$\vec{\beta}(t) = \frac{\dot{\mathbf{r}}(t)}{c} = \frac{v}{\rho c} [\rho \cos \psi \hat{\mathbf{x}} + \rho \sin \psi \hat{\mathbf{y}}]$$

$$\vec{\beta}(t) = \frac{v}{c} [\cos \psi \hat{\mathbf{x}} + \sin \psi \hat{\mathbf{y}}]$$

$\hat{\mathbf{m}} \in \text{PLANO } xz$

$$\hat{\mathbf{m}} = \cos \theta \hat{\mathbf{x}} + \sin \theta \hat{\mathbf{z}}$$

UNITÁRIOS DE POLARIZAÇÃO:

$$\hat{\mathbf{e}}_{||} = \hat{\mathbf{y}}$$

$$\hat{\mathbf{e}}_{\perp} = -\sin \theta \hat{\mathbf{x}} + \cos \theta \hat{\mathbf{z}}$$

$$\hat{m} \times (\hat{m} \times \vec{\beta}) = \frac{\mu}{c} (\sin\theta \cos\psi \hat{E}_\perp - \sin\psi \hat{E}_\parallel)$$

$$\hat{m} \cdot \vec{\beta}(t) = \beta \cos\theta \sin\psi$$

$$t - \frac{1}{c} \hat{m} \cdot \vec{\beta}(t) = t - \frac{\beta}{c} \cos\theta \sin\psi$$

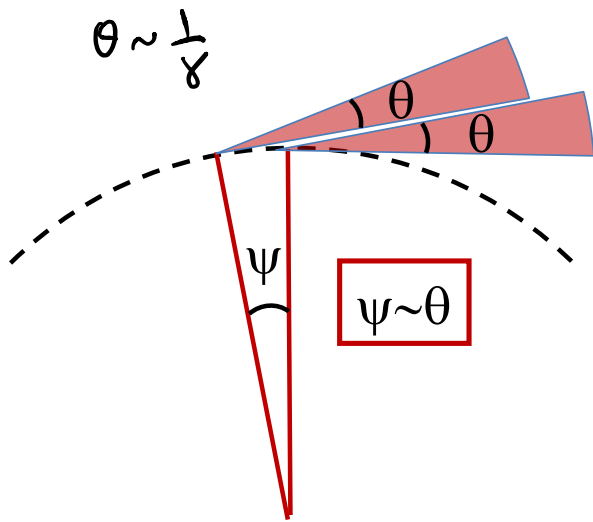
LEVANDO NA EXPRESSÃO PROCURADA:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| -A_\parallel(\omega) \hat{E}_\parallel + A_\perp(\omega) \hat{E}_\perp \right|^2$$

ONDE:

$$A_\parallel(\omega) = \frac{\mu}{c} \int_{-\infty}^{+\infty} e^{i\omega t} e^{-i\omega \frac{\beta}{c} \cos\theta \sin\psi(t)} \sin\psi(t) dt$$

$$A_\perp(\omega) = \frac{\mu}{c} \int_{-\infty}^{+\infty} e^{i\omega t} e^{-i\omega \frac{\beta}{c} \cos\theta \sin\psi(t)} \sin\theta \cos\psi(t) dt$$



$$t - \frac{\rho}{c} \cos \theta \sin \psi =$$

$$\approx t - \frac{\rho}{c} \left(1 - \frac{\theta^2}{2}\right) \left(\psi - \frac{\psi^3}{6}\right)$$

$$= t - \frac{\rho \psi}{c} \left(1 - \frac{\theta^2}{2}\right) \left(1 - \frac{\psi^2}{6}\right)$$

$$\approx t - \frac{\rho \psi}{c} \left(1 - \frac{\theta^2}{2} - \frac{\psi^2}{6}\right)$$

$$= t - \frac{\rho t}{c} \left(1 - \frac{\theta^2}{2} - \frac{\psi^2}{6}\right)$$

$$= t \left[\underbrace{1 - \frac{\rho}{c}}_{\approx 1} + \underbrace{\frac{\rho}{c}}_{\approx 1} \left(\frac{\theta^2}{2} + \frac{\psi^2}{6} \right) \right] =$$

$$\frac{1}{2\gamma^2}$$

$$= t \left[\frac{1}{2\gamma^2} + \frac{\theta^2}{2} + \frac{\psi^2}{6} \right] = \frac{t}{2} \left[\frac{1}{\gamma^2} + \theta^2 + \frac{c^2 t^2}{3\beta^2} \right]$$

$$\gamma^{-2} = (1 - \beta^2) = (1 - \beta)(1 + \beta)$$

$$\approx 2\beta(1 - \beta) \approx 2(1 - \beta)$$

$$\Rightarrow 1 - \beta \approx \frac{1}{2\gamma^2}$$

$$\psi = \frac{\rho t}{\beta} \approx \frac{c}{\beta} t$$

FORA DA EXPONENCIAL:

$$\frac{v}{c} \approx 1$$

$$\sin \theta \approx \theta$$

$$\sin \psi \approx \psi = \frac{vt}{s}$$

$$\cos \psi \approx 1$$

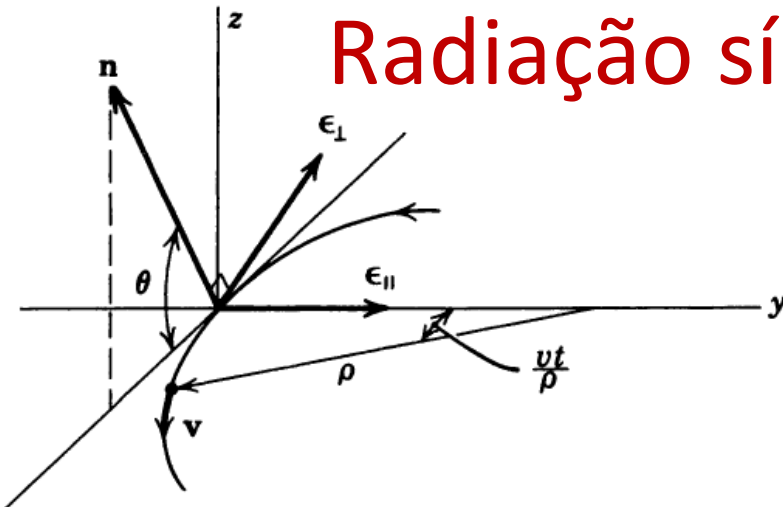
$$A_{||}(\omega) = \frac{C}{s} \int_{-\infty}^{+\infty} t \exp\left[\frac{i\omega t}{2} \left(\frac{1}{s^2} + \theta^2 + \frac{c^2 t^2}{3s^2} \right) \right] dt$$

$$A_{\perp}(\omega) = \theta \int_{-\infty}^{+\infty} \exp\left[\quad \right] dt$$

TROCA DE VARIÁVEIS:

$$\xi = \frac{\omega s}{3c} \left(\frac{1}{s^2} + \theta^2 \right)^{3/2}$$

Radiação síncrotron

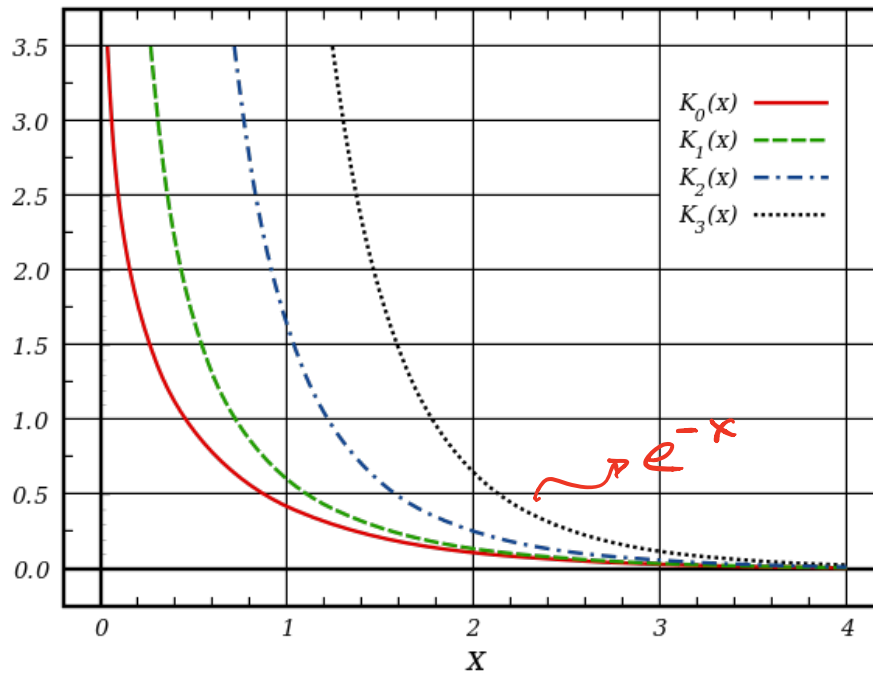


$$\xi = \frac{\omega \rho}{3c} \left(\frac{1}{\gamma^2} + \theta^2 \right)^{3/2}$$

$$^a \frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c} \left(\frac{\omega \rho}{c} \right)^2 \left(\frac{1}{\gamma^2} + \theta^2 \right)^2 \left[\underset{\mathbf{E}_{\parallel}}{\uparrow} K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2) + \theta^2} K_{1/3}^2(\xi) \underset{\mathbf{E}_{\perp}}{\uparrow} \right]$$

$K_{\nu}(x)$ É A FUNÇÃO DE BESSEL MODIFICADA DO
SEGUNDO TIPO DE ORDEM ν

Função de Bessel modificadas de segundo tipo $K_\nu(x)$



(i) COMO $K_V(x) \cong \sqrt{\frac{\pi}{2x}} e^{-x}$ PARA $x \gg 1$

A DISTRIBUIÇÃO $\frac{d^2 I}{d\omega d\theta}$ CAI EXPONENCIALMENTE COM O

ARGUMENTO ξ QUANDO $\xi \gg 1$.

$$\xi(\omega, \theta) = \frac{\omega \rho}{3c} \left(\frac{1}{\gamma^2} + \theta^2 \right)^{3/2}$$

QUE PODE CRESCER TANTO COM A FREQUÊNCIA ω QUANTO COM O ÂNGULO θ .

(ii) TOMANDO A DIREÇÃO $\theta=0$ COMO REFERÊNCIA

$$\xi = \frac{\omega \rho}{3\theta^3 c} \quad , \quad \frac{d^2 I}{d\omega d\theta} \propto e^{-2\xi}$$

DEFINIMOS UMA FREQUÊNCIA CRÍTICA ω_c :

$$\xi_c = \frac{1}{2} \left(\frac{d^2 I}{d\omega d\theta} \sim \frac{1}{c} \frac{d^2 I}{d\omega d\theta}(\omega=0) \right) \Rightarrow \boxed{\omega_c = \frac{3}{2} \left(\frac{c}{\rho} \right) \theta^3}$$

(iii) O MOVIMENTO É PERIÓDICO COM FREQUÊNCIA:

$$f_0 = \frac{c}{P_{ORB}} \quad \omega_0 = 2\pi f_0 = \frac{2\pi c}{P_{ORB}}$$

A RADIAÇÃO EMITIDA CONSISTIRÁ DE LINHAS DISCRETAS COM FREQUÊNCIAS QUE SÃO MÚLTIPLAS DA FUNDAMENTAL ω_0

HARMÔNICO CRÍTICO ω_c : $\omega_c = \frac{\omega_c}{\omega_0}$

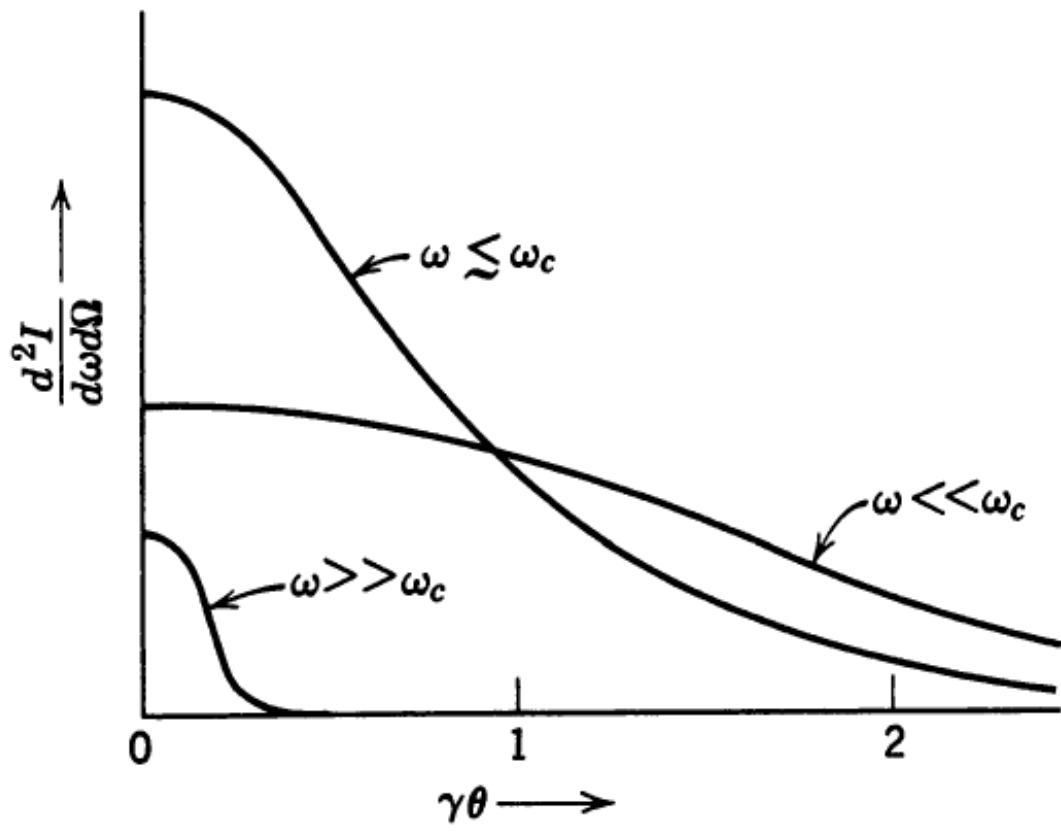
(iv) PARA $\theta \neq 0$ A DISTRIBUIÇÃO DE FREQUÊNCIA É MODIFICADA. ASSIM, PODEMOS DEFINIR ÂNGULOS CRÍTICOS PARA CADA VALOR DE FREQUÊNCIA:

$$\omega \ll \omega_c : \theta_c = \frac{1}{\gamma} \left(\frac{2\omega_c}{\omega} \right)^{1/3} \gg \frac{1}{\gamma}$$

APENAS PARA

$$\omega \approx \omega_c : \theta_c \sim \frac{1}{\gamma}$$

$$\omega \gg \omega_c : \theta_c = \frac{1}{\gamma} \left(\frac{2\omega_c}{3\omega} \right)^{1/2} \ll \frac{1}{\gamma}$$



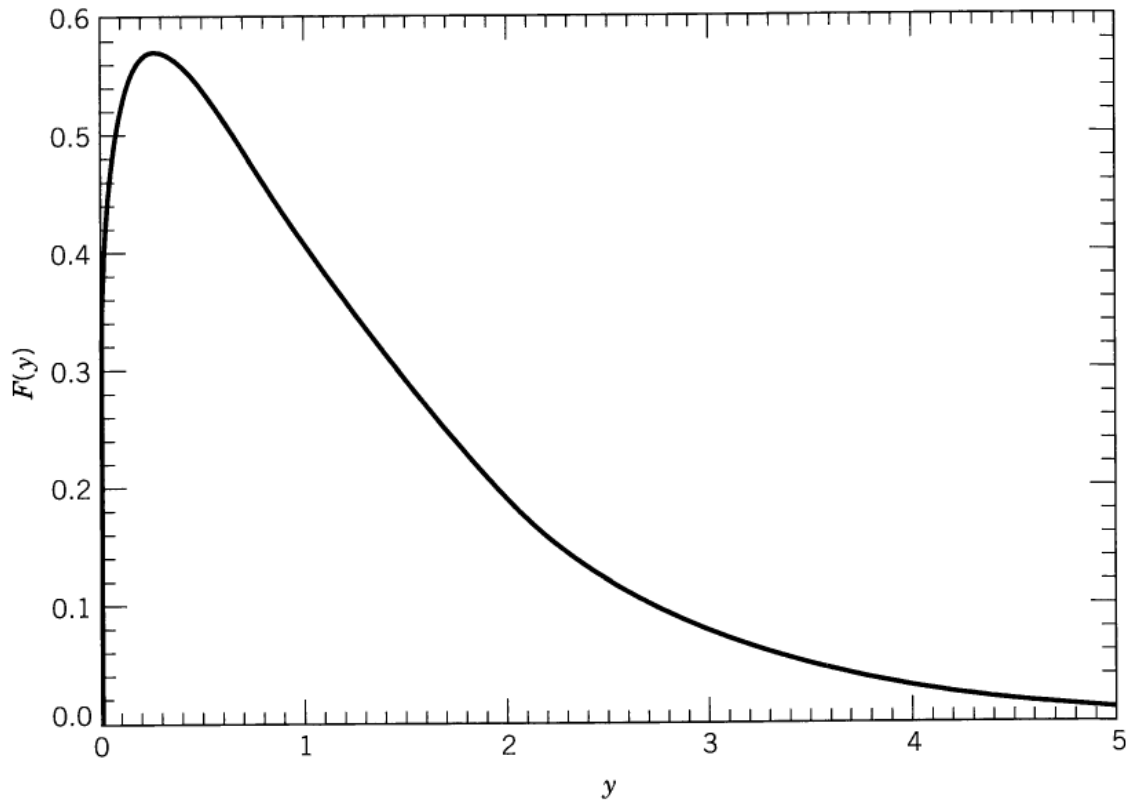
iv) INTEGRANDO SOBRE θ OBTÊMOS A DISTRIBUIÇÃO DE FREQUÊNCIAS APENAS:

$$\frac{dI}{d\omega} = \sqrt{3} \frac{e^2 \gamma}{c} \frac{\omega}{\omega_c} \int_{\omega/\omega_c}^{\infty} K_{5/3}(x) dx$$

$$= \begin{cases} 3.25 \frac{e^2 \gamma}{c} \left(\frac{3\omega}{2\omega_c}\right)^{1/3} & \omega < \omega_c \\ \sqrt{\frac{3\pi}{2}} \frac{e^2 \gamma}{c} \left(\frac{\omega}{\omega_c}\right)^{1/2} e^{-\omega/\omega_c} & \omega \gg \omega_c \end{cases}$$

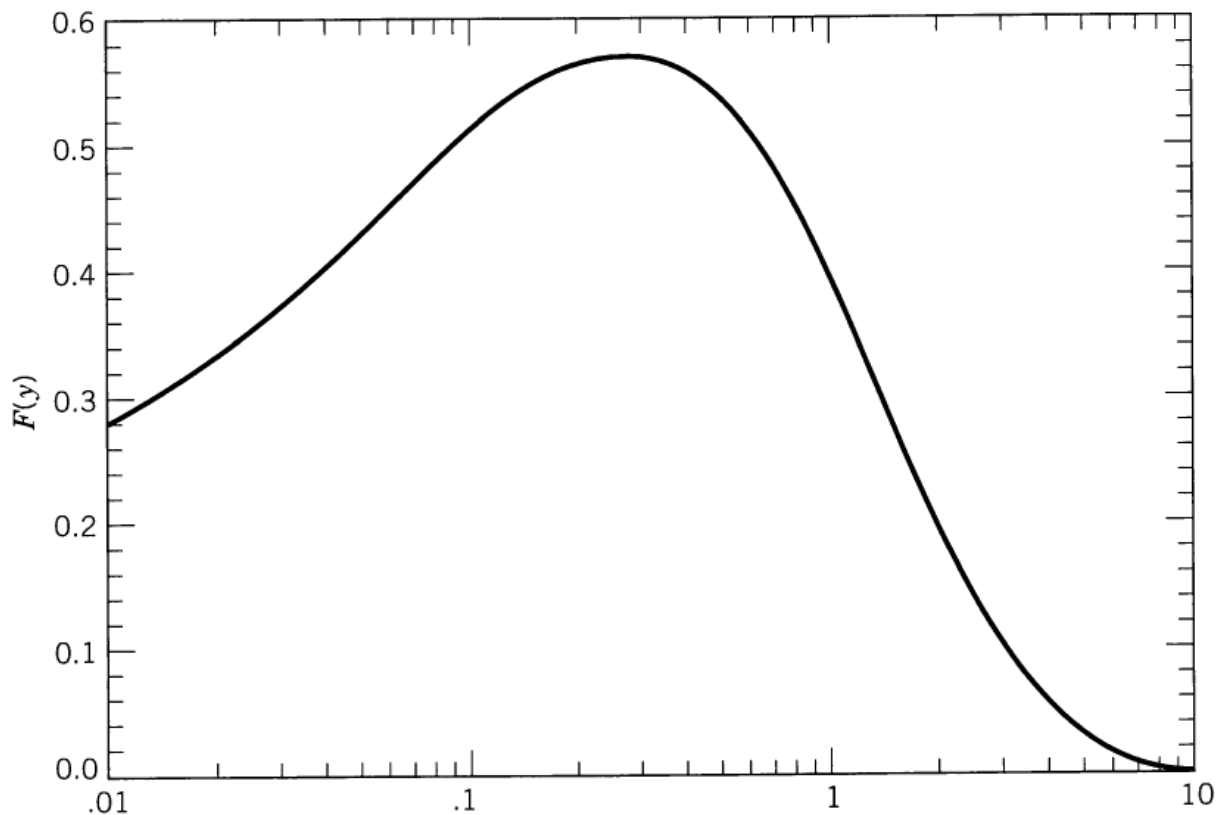
v) A RADIAÇÃO SÍNCROTRON É FORTEMENTE POLARIZADA NO PLANO DA TRAJETÓRIA (\hat{E}_{\parallel})

$$W_{\parallel} \approx 7 W_{\perp}$$



$$F(y) = (1/I)(dI/dy) = (9\sqrt{3}/8\pi)y \int_y^{\infty} K_{5/3}(x) dx$$

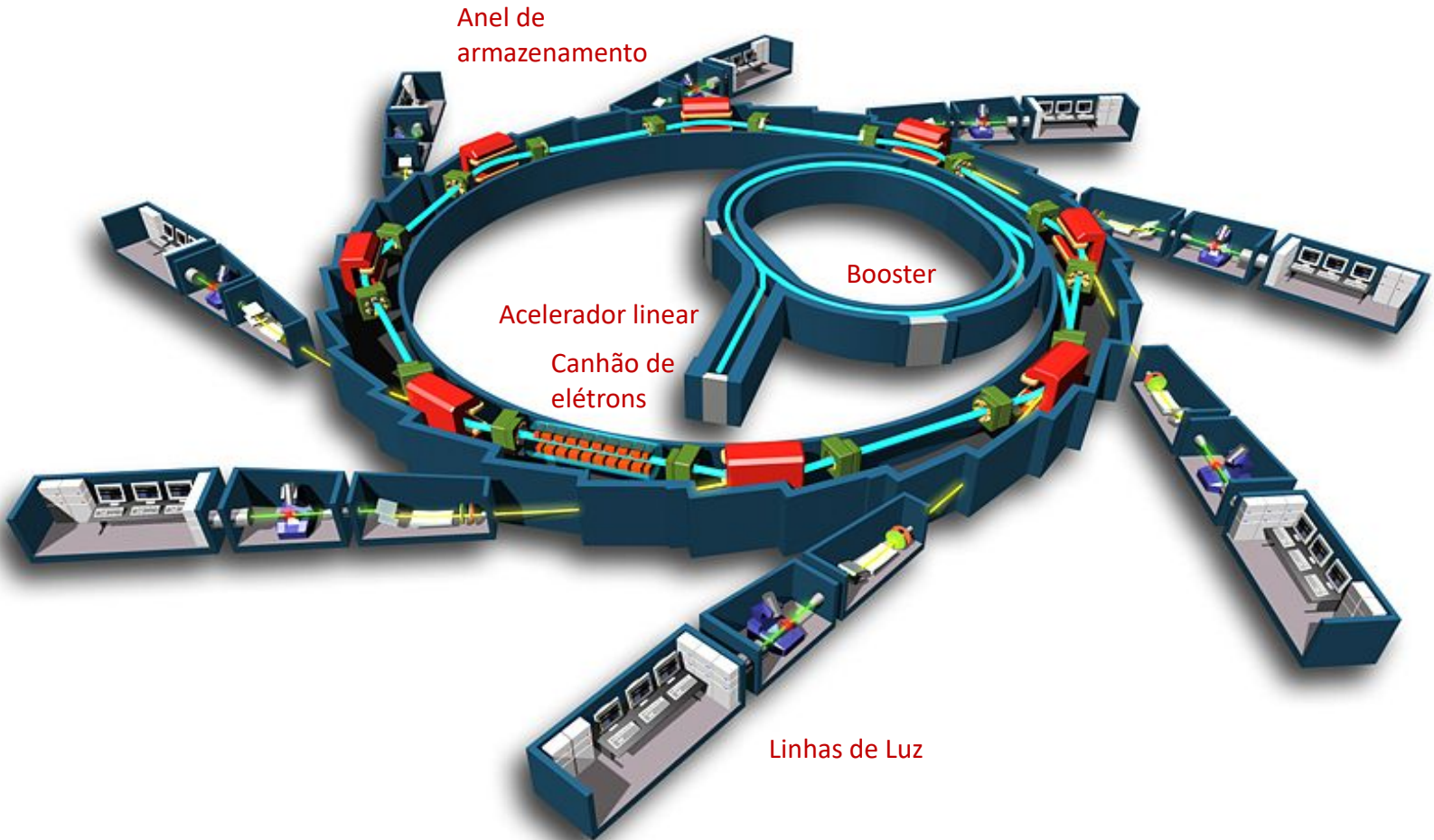
$$y = \omega/\omega_c \quad I = 4\pi e^2 \gamma^4 / 3\rho:$$



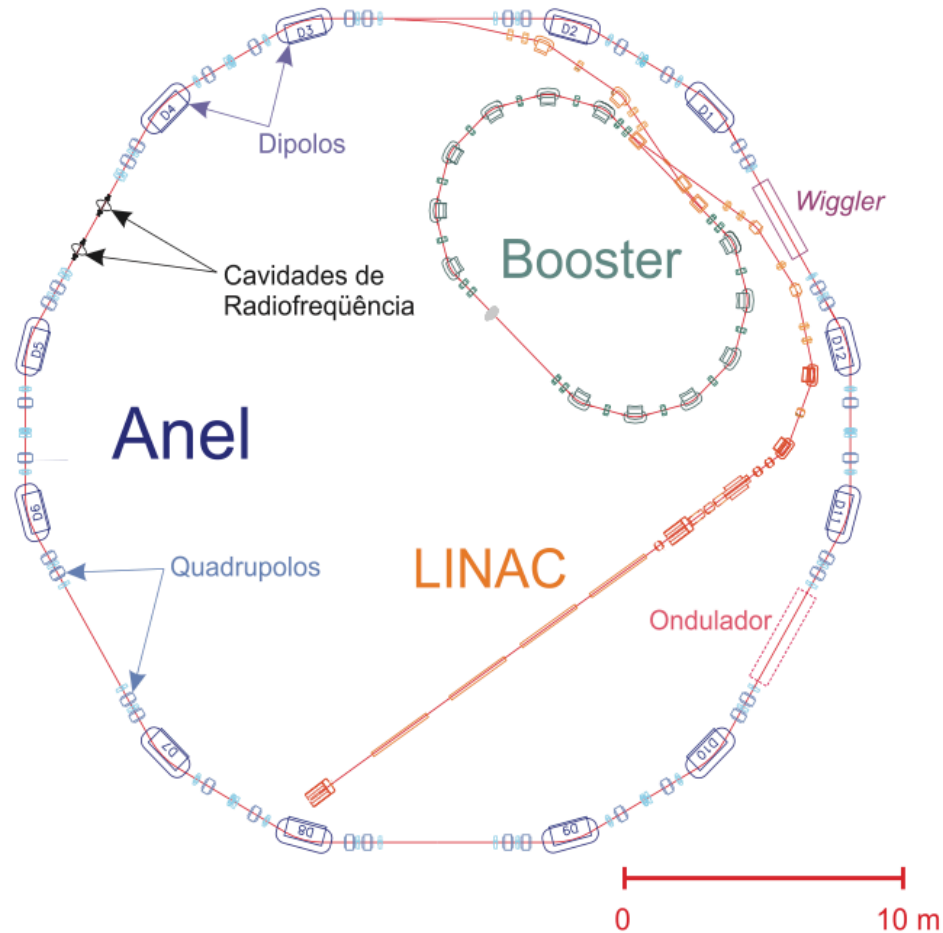
$$F(y) = (1/I)(dI/dy) = (9\sqrt{3}/8\pi)y \int_y^\infty K_{5/3}(x) dx$$

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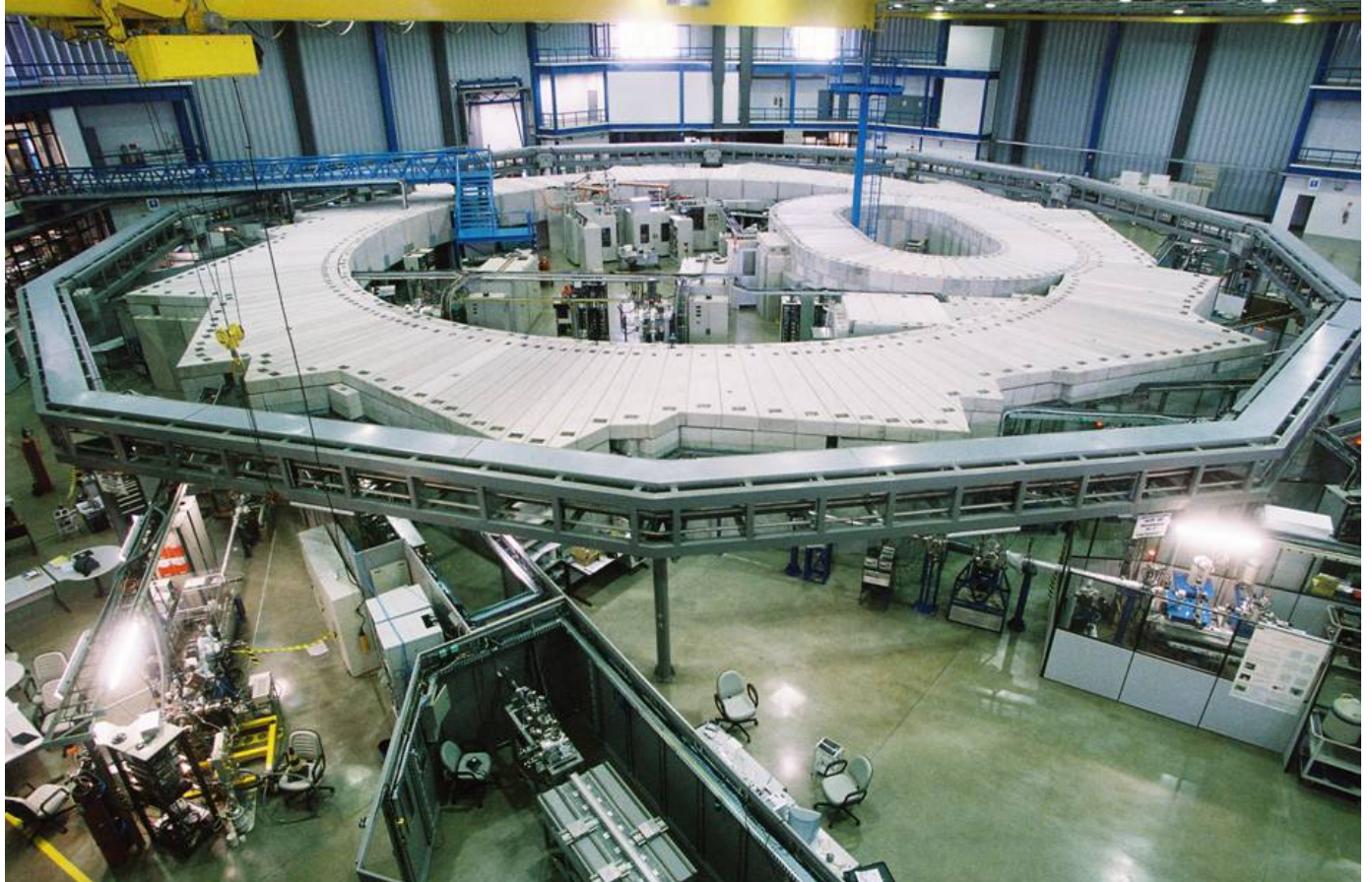
Fonte de Luz Síncrotron



UVX : o síncrotron “velho”



UVX : o síncrotron “velho”



Parâmetros do LNLS (UVX)

Energia	1.37 GeV
γ	2.7×10^3
γ^3	1.9×10^{10}
Corrente do feixe	250 mA
Energia de injeção (depois do booster)	500 MeV
γ de injeção	980
Circunferência (C)	93.2 m
Frequência de revolução ($f_0 = c/C$)	3.2 MHz
Número de dipolos	12
Raio nos dipolos (ρ)	2.735 m
Campo nos dipolos (B)	1.67 T
Energia crítica ($\hbar\omega_c$)	2.08 keV
Frequência crítica (ω_c)	3.2×10^{18} Hz
Perda de energia/elétron/volta nos dipolos	114 keV
Potência irradiada nos dipolos a 100 mA	11.4 kW

$$\gamma = \frac{E}{mc^2} = \frac{E}{0.51 \text{ MeV}}$$

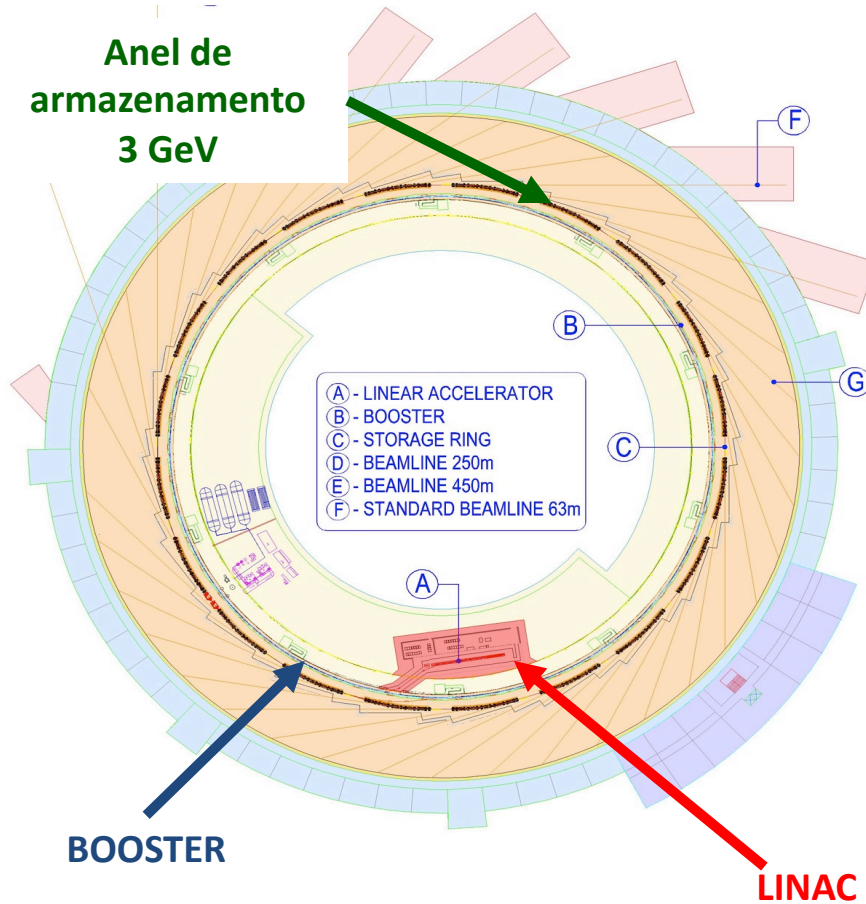
$$\rho = \frac{p}{eB} \approx \frac{E}{ceB} = \frac{\gamma mc}{eB} \Rightarrow$$

$$\Rightarrow \frac{\rho B}{\gamma} = \frac{mc}{e} = 1.7 \times 10^{-3} \text{Tm} \quad (\text{SI})$$

$$\omega_c = \frac{3c}{2\rho} \gamma^3$$

$$E_c = \hbar\omega_c = \frac{3\hbar c}{2} \frac{\gamma^3}{\rho} = 2.96 \times 10^{-10} \text{ keVm} \frac{\gamma^3}{\rho}$$

Sirius



Sirius



Parâmetros do Sirius	
Energia	3 GeV
γ	5871
γ^3	2.024×10^{11}
Corrente nominal do feixe	350 mA
Energia de injeção (após o booster)	0.15 GeV
γ de injeção	294
Circunferência (C)	518.4 m
Frequência de revolução ($\omega_0 = c/C$)	0.5783 MHz
Número de dipolos de baixo campo	120
Raio nos dipolos de baixo campo (ρ_l)	17.218 m
Campo nos dipolos de baixo campo (B_l)	0.5812 T
Energia crítica nos dipolos de baixo campo ($\hbar\omega_c$)	3.48 keV
Frequência crítica nos dipolos de baixo campo (ω_c)	5.3×10^{18} Hz
Número de dipolos de alto campo	20
Raio nos dipolos de alto campo (ρ_l)	3.127 m
Campo nos dipolos de alto campo (B_l)	3.2 T
Energia crítica nos dipolos de alto campo ($\hbar\omega_c$)	19.15 keV
Frequência crítica nos dipolos de alto campo (ω_c)	29×10^{18} Hz
Perda de energia/elétron/volta nos dipolos	475.3 keV
Potência irradiada nos dipolos a 500 mA	237.6 kW

$$\gamma = \frac{E}{mc^2} = \frac{E}{0.51 \text{ MeV}}$$

$$\rho = \frac{p}{eB} \approx \frac{E}{ceB} = \frac{\gamma mc}{eB} \Rightarrow$$

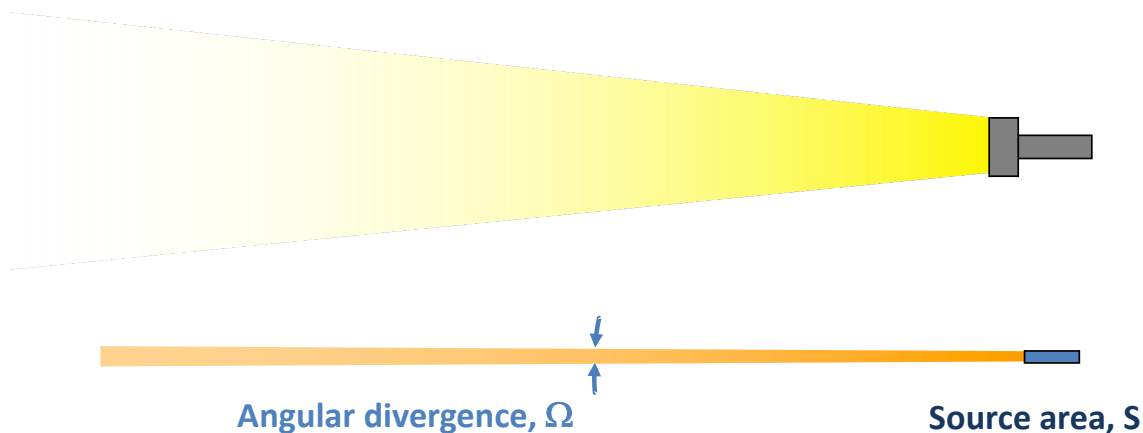
$$\Rightarrow \frac{\rho B}{\gamma} = \frac{mc}{e} = 1.7 \times 10^{-3} \text{Tm} \quad (\text{SI})$$

$$\omega_c = \frac{3c}{2\rho} \gamma^3$$

$$E_c = \hbar\omega_c = \frac{3\hbar c}{2} \frac{\gamma^3}{\rho} = 2.96 \times 10^{-10} \text{ keVm} \frac{\gamma^3}{\rho}$$

Alto Brilho – boa fonte de luz

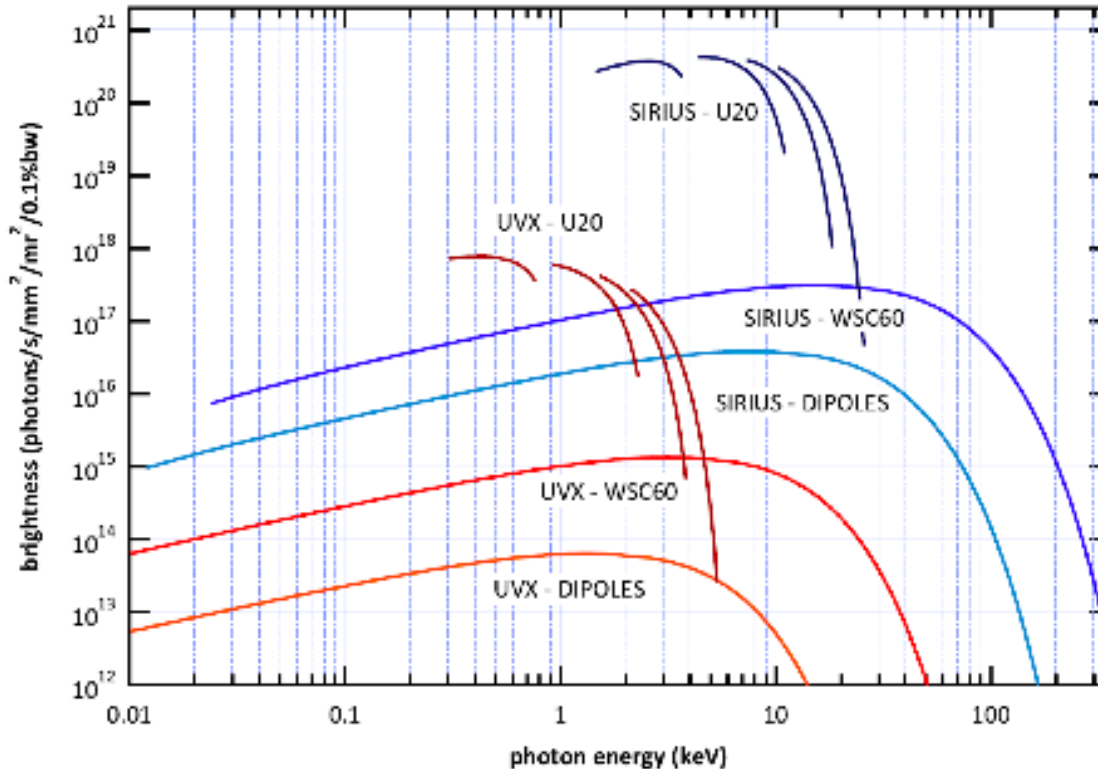
- Alta intensidade ou alto fluxo (fotons/s).
- Fonte pequena e colimada.



$$\text{Brilho} = \frac{\text{Flux}}{S \times \Omega}$$

Emittance

Sirius x UVX



Comparison of brightness of Sirius with that of the existing synchrotron radiation source (UVX) in operation at LNL for dipoles, superconducting wiggler (WSC) and an undulator with 20 mm period (U20).

Outros síncrotrons

