FI 008 – Eletrodinâmica I

1º Semestre de 2021
25/03/2021
Aula 4

Capacitância

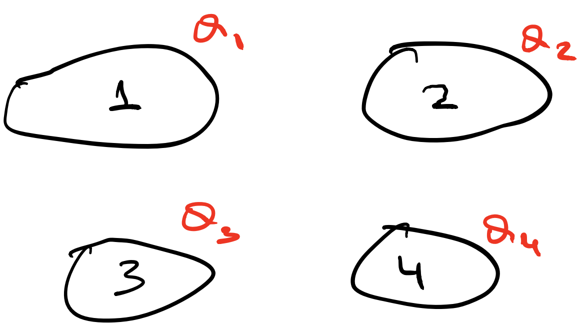
EQ. DE POISSON: $\nabla^2 \Phi = -\frac{\rho}{\epsilon_0}$ → EQ. LINEAR

SE Φ_1 É SOLUÇÃO DE $\nabla^2 \Phi_1 = -\frac{\rho_1}{\epsilon_0}$

E Φ_2 " " " $\nabla^2 \Phi_2 = -\frac{\rho_2}{\epsilon_0}$

ENTÃO A SOLUÇÃO PARA $c_1 \rho_1 + c_2 \rho_2$ É $c_1 \Phi_1 + c_2 \Phi_2$. A PROVA É IMEDIATA.

SE EU TIVER <u>N</u> CONDUTORES:



$$\Rightarrow \boxed{\Phi_i = \sum_{j=1}^{N} p_{ij} Q_j \quad (i=1,2,\dots N)}$$

p_{ij} DEPENDEM DA GEOMETRIA DOS CONDUTORES

$$\boxed{Q_i = \sum_{j=1}^{N} C_{ij}\, \Phi_j \quad (i=1,2,\dots N)}$$

$$C_{ij} = (P^{-1})_{ij}$$

(i) $C_{ii} \rightarrow$ AUTO-CAPACITÂNCIAS DOS CONDUTORES

(ii) C_{ij} $(i \neq j) \rightarrow$ COEFICIENTES DE INDUÇÃO

NO CASO PARTICULAR DE $N=2$ CONDUTORES:

SE EU PONHO $+Q$ NUM DELES E $-Q$ NO OUTRO

$$\Rightarrow \Delta\Phi = \Phi_1 - \Phi_2$$

C: CAPACITÂNCIA DESSE SISTEMA

$$C = \frac{|Q|}{|\Delta\Phi|}$$

DADOS $\underline{N}$ CONDUTORES COM CARGAS $Q_1, Q_2, \ldots, Q_N$

$$W = \frac{1}{2} \int_{T.E.} \rho(\vec{x}) \Phi(\vec{x}) d^3x$$

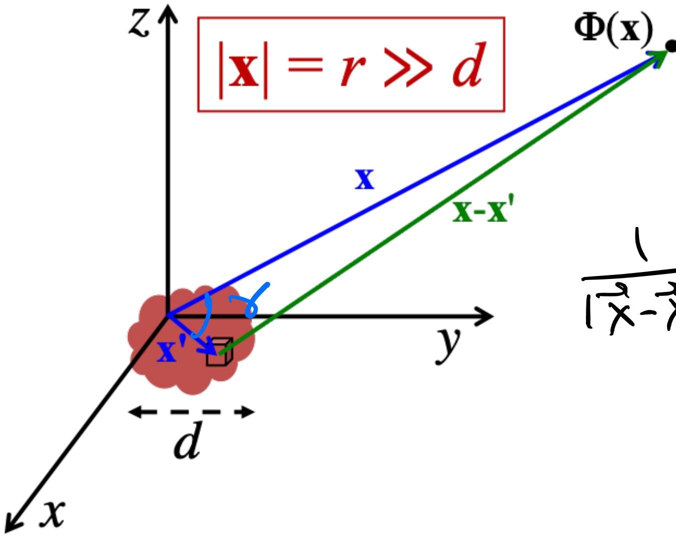
EM CADA CONDUTOR $\underline{i}$: $\Phi(\vec{x}) = \Phi_i = $ CONST.

$$\Rightarrow \int_{\text{CONDUTOR } \underline{i}} \rho(\vec{x}) \Phi(\vec{x}) d^3x = \Phi_i \underbrace{\int \rho(\vec{x}) d^3x}_{Q_i} = Q_i \Phi_i$$

$$\Rightarrow W = \frac{1}{2} \sum_{i=1}^{N} Q_i \Phi_i = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} \Phi_i \Phi_j$$

$$= \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} p_{ij} Q_i Q_j = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (C^{-1})_{ij} Q_i Q_j$$

Expansão multipolar



$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\int_{T.E.} \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x'$$

$$\frac{1}{|\vec{x}-\vec{x}'|} = \frac{1}{[r^2+r'^2-2\,\vec{x}\cdot\vec{x}']^{1/2}}$$

$$= \frac{1}{[r^2+r'^2-2rr'\cos\gamma]^{1/2}} = (*)$$

γ = ÂNGULO ENTRE $\vec{x}$ E $\vec{x}'$

$$\vec{x}\cdot\vec{x}' = rr'\cos\gamma \Rightarrow \cos\gamma = \frac{\vec{x}\cdot\vec{x}'}{rr'}$$

$s = r'/r \qquad r' < r$

$$(*) = \frac{1}{r\left[1+\frac{r'^2}{r^2}-2\frac{r'}{r}\cos\gamma\right]^{1/2}} = \frac{1}{r[1+s^2-2s\cos\gamma]^{1/2}} \qquad (s<1)$$

$s \ll 1$

DA FÍSICA MATEMÁTICA!

$$\frac{1}{\left[1+s^2-2s\underbrace{\cos\gamma}_{x}\right]^{1/2}} = \sum_{\ell=0}^{\infty} s^\ell P_\ell(\underbrace{\cos\gamma}_{x}) \qquad (s<1)$$

"FUNÇÃO GERATRIZ DOS $P_\ell(x)$"

ONDE $P_\ell(x)$ É O POLINÔMIO DE LEGENDRE DE ORDEM $\underline{\ell}$

$$\frac{1}{|\vec{x}-\vec{x}'|} = \frac{1}{r}\sum_{\ell=0}^{\infty}\left(\frac{r'}{r}\right)^\ell P_\ell(\cos\gamma)$$

$$= \sum_{\ell=0}^{\infty} \frac{r'^\ell}{r^{(\ell+1)}} P_\ell(\cos\gamma)$$

Polinômios de Legendre

Equação diferencial: $$\frac{d}{dx}\left[(1-x^2)\frac{dP}{dx}\right] + l(l+1)P = 0$$

Exemplos:

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \tfrac{1}{2}(3x^2 - 1)$$
$$P_3(x) = \tfrac{1}{2}(5x^3 - 3x)$$
$$P_4(x) = \tfrac{1}{8}(35x^4 - 30x^2 + 3)$$

$$P_\ell(-x) = (-1)^\ell P_\ell(x)$$

Ortogonalidade: $$\int_{-1}^{1} P_{l'}(x)P_l(x)\,dx = \frac{2}{2l+1}\delta_{l'l}$$

Base completa no intervalo [-1,1]:

$$f(x) = \sum_{l=0}^{\infty} A_l P_l(x)$$

$$A_l = \frac{2l+1}{2}\int_{-1}^{1} f(x)P_l(x)\,dx$$

Polinômios de Legendre

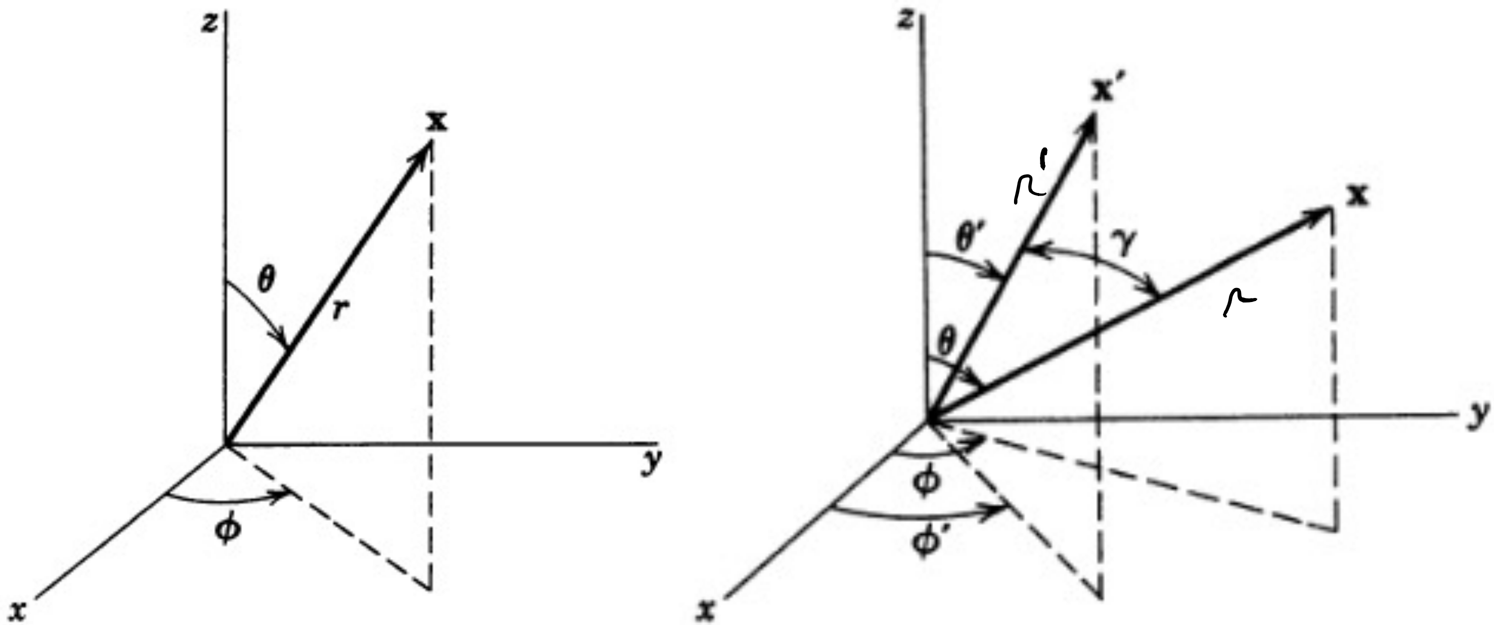
Relações de recorrência:

$$\frac{dP_{l+1}}{dx} - \frac{dP_{l-1}}{dx} - (2l + 1)P_l = 0$$

$$(l + 1)P_{l+1} - (2l + 1)xP_l + lP_{l-1} = 0$$

$$\frac{dP_{l+1}}{dx} - x\,\frac{dP_l}{dx} - (l + 1)P_l = 0$$

$$(x^2 - 1)\,\frac{dP_l}{dx} - lxP_l + lP_{l-1} = 0$$



$$\vec{x} = r\left[\sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}\right]$$

$$\vec{x}' = r'\left[\sin\theta'\cos\phi'\,\hat{x} + \sin\theta'\sin\phi'\,\hat{y} + \cos\theta'\,\hat{z}\right]$$

$$\cos\gamma = \frac{\vec{x}\cdot\vec{x}'}{r\,r'} = \sin\theta\cos\phi\sin\theta'\cos\phi' + \sin\theta\sin\phi\sin\theta'\sin\phi + \cos\theta\cos\theta'$$

$$= \sin\theta\sin\theta'\left[\cos\phi\cos\phi' + \sin\phi\sin\phi'\right] + \cos\theta\cos\theta'$$

$$\cos\gamma = \sin\theta\sin\theta'\cos(\phi-\phi') + \cos\theta\cos\theta'$$

Equação de Laplace em coordenadas esféricas

$$\boxed{\nabla^2\Phi = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\Phi) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2} = 0}$$

Separação de variáveis: $\Phi(\mathbf{x}) = \frac{U(r)}{r}P(\theta)Q(\phi)$

$$\frac{d^2Q}{d\phi^2} = -m^2Q \Rightarrow Q(\phi) = e^{im\phi}, (m\in\mathbb{Z})\,;$$ $Q(\phi+2\pi) = Q(\phi)$

$$\frac{d^2U}{dr^2} = l(l+1)\frac{U}{r^2} \Rightarrow U(r) = Ar^{(l+1)} + Br^{-l};$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right) + \left[l(l+1) - \frac{m^2}{\sin^2\theta}\right]P = 0, (l = 0, 1, 2, \ldots)\,.$$

Funções associadas de Legendre

Equação diferencial para $P(\theta)$: $\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right)+\left[l(l+1)-\frac{m^2}{\sin^2\theta}\right]P=0$

Solução: funções associadas de Legendre (em geral não são polinômios!):

POLINÔMIOS DE LEGENDRE

$$P_l^m(x) = (-1)^m(1-x^2)^{m/2}\frac{d^m}{dx^m}P_l(x) \quad (m>0)$$

$$P_l^{-m}(x) = (-1)^m\frac{(l-m)!}{(l+m)!}P_l^m(x)$$

$m = -l, -l+1, \ldots 0, 1, \ldots, l$

Ortogonalidade: $\int_{-1}^{1} P_{l'}^m(x)P_l^m(x)\,dx = \frac{2}{2l+1}\frac{(l+m)!}{(l-m)!}\delta_{l'l}$

$(2l+1)$ VALORES

Harmônicos esféricos

$P(\theta)Q(\phi)$: Harmônicos esféricos

$$Y_{lm}(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{im\phi}$$

$$Y_{l,-m}(\theta, \phi) = (-1)^m Y^*_{lm}(\theta, \phi)$$

Parte angular da eq. de Laplace em coord. esféricas

Ortogonalidade: $\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \, Y^*_{l'm'}(\theta, \phi) Y_{lm}(\theta, \phi) = \delta_{l'l}\delta_{m'm}$

Base completa:

$$g(\theta, \phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} A_{lm}Y_{lm}(\theta, \phi)$$

$$A_{lm} = \int d\Omega \, Y^*_{lm}(\theta, \phi) g(\theta, \phi)$$

Teorema de adição de harmônicos esféricos

$$P_l(\cos\gamma) = \frac{4\pi}{2l+1}\sum_{m=-l}^{l} Y^*_{lm}(\theta', \phi')Y_{lm}(\theta, \phi)$$

$$\cos\gamma = \sin\theta \sin\theta' \cos(\phi-\phi') + \cos\theta \cos\theta'$$

$$\frac{1}{|\bar{x}-\bar{x}'|} = 4\pi \sum_{l=0}^{\infty} \frac{1}{2l+1} \sum_{m=-l}^{l} \frac{r'^l}{r^{(l+1)}} Y^*_{lm}(\theta',\phi') Y_{lm}(\theta,\phi)$$

$(r'<r)$

$$= 4\pi \sum_{l=0}^{\infty} \frac{1}{2l+1} \sum_{m=-l}^{l} \frac{r^l}{(r')^{(l+1)}} Y^*_{lm}(\theta,\phi) Y_{lm}(\theta',\phi')$$

$(r<r')$

$$\frac{1}{|\bar{x}-\bar{x}'|} = 4\pi \sum_{l=0}^{\infty} \frac{1}{2l+1} \sum_{m=-l}^{m} \frac{r_<^l}{r_>^{(l+1)}} Y^*_{lm}(\theta',\phi') Y_{lm}(\theta,\phi)$$

$r_< = \min(r, r')$

$r_> = \max(r, r')$

SPHERICAL HARMONICS $Y_{lm}(\theta, \phi)$

$$l = 0 \qquad Y_{00} = \frac{1}{\sqrt{4\pi}}$$

$$l = 1 \begin{cases} Y_{11} = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi} \\ Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta \end{cases}$$

$Y_{1-1} = (-1)^1 Y^*_{1,1}$

$Y_{1,-1} = -Y^*_{1,1}$

$$l = 2 \begin{cases} Y_{22} = \frac{1}{4}\sqrt{\frac{15}{2\pi}} \sin^2\theta e^{2i\phi} \\ Y_{21} = -\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta e^{i\phi} \\ Y_{20} = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) \end{cases}$$

$Y_{2,-2} = Y^*_{2,2}$

$$l = 3 \begin{cases} Y_{33} = -\frac{1}{4}\sqrt{\frac{35}{4\pi}} \sin^3\theta e^{3i\phi} \\ Y_{32} = \frac{1}{4}\sqrt{\frac{105}{2\pi}} \sin^2\theta \cos\theta e^{2i\phi} \\ Y_{31} = -\frac{1}{4}\sqrt{\frac{21}{4\pi}} \sin\theta \left(5\cos^2\theta - 1\right) e^{i\phi} \\ Y_{30} = \sqrt{\frac{7}{4\pi}} \left(\frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta\right) \end{cases}$$

Momentos de multipolo elétrico

LEVANDO $\frac{1}{|\bar{x}-\bar{x}'|}$ NA SOL. DA EQ. POISSON:

$$\Phi(\bar{x}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\bar{x}')\,d^3x'}{|\bar{x}-\bar{x}'|} = \frac{1}{\epsilon_0}\int \rho(\bar{x}')\,d^3x' \left[\sum_{\ell=0}^{\infty}\frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell}\frac{r_<^{\ell}}{r_>^{\ell+1}}\,Y^*_{\ell m}(\theta',\phi')\times Y_{\ell m}(\theta,\phi)\right]$$

SE $r \gg r'$

$$= \frac{1}{\epsilon_0}\sum_{\ell,m}\frac{1}{r^{(\ell+1)}}\frac{Y_{\ell m}(\theta,\phi)}{2\ell+1}\underbrace{\left[\int (r')^{\ell}\,Y^*_{\ell m}(\theta',\phi')\,\rho(\bar{x}')\,d^3x'\right]}_{q_{\ell m}}$$

$q_{\ell m}$: MOMENTOS DE MULTIPOLO ELÉTRICO

$$\Phi(\bar{x}) = \sum_{\ell,m}\frac{q_{\ell m}}{\epsilon_0(2\ell+1)}\,\frac{Y_{\ell m}(\theta,\phi)}{r^{(\ell+1)}}$$

q = CARGA TOTAL

$$q_{00} = \frac{1}{\sqrt{4\pi}} \int \rho(\mathbf{x}')\, d^3x' = \frac{1}{\sqrt{4\pi}}\, q$$

$$\left.\begin{aligned}
q_{11} &= -\sqrt{\frac{3}{8\pi}} \int (x' - iy')\rho(\mathbf{x}')\, d^3x' = -\sqrt{\frac{3}{8\pi}}\,(p_x - ip_y) \\
q_{10} &= \sqrt{\frac{3}{4\pi}} \int z'\rho(\mathbf{x}')\, d^3x' = \sqrt{\frac{3}{4\pi}}\, p_z
\end{aligned}\right\} \quad \vec{p} = \int \vec{x}\, \rho(\vec{x})\, d^3x$$

$$\left.\begin{aligned}
q_{22} &= \frac{1}{4}\sqrt{\frac{15}{2\pi}} \int (x' - iy')^2\rho(\mathbf{x}')\, d^3x' = \frac{1}{12}\sqrt{\frac{15}{2\pi}}\,(Q_{11} - 2iQ_{12} - Q_{22}) \\
q_{21} &= -\sqrt{\frac{15}{8\pi}} \int z'(x' - iy')\rho(\mathbf{x}')\, d^3x' = -\frac{1}{3}\sqrt{\frac{15}{8\pi}}\,(Q_{13} - iQ_{23}) \\
q_{20} &= \frac{1}{2}\sqrt{\frac{5}{4\pi}} \int (3z'^2 - r'^2)\rho(\mathbf{x}')\, d^3x' = \frac{1}{2}\sqrt{\frac{5}{4\pi}}\, Q_{33}
\end{aligned}\right\}$$

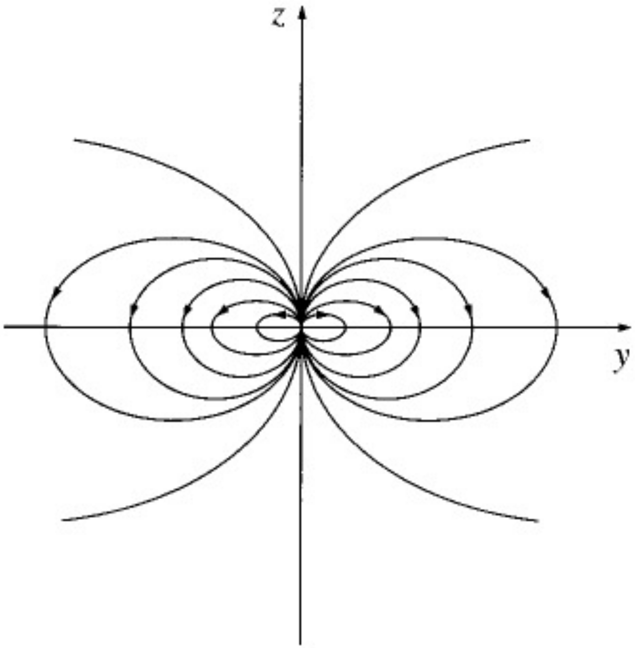
$$r' \sin\theta'\, e^{-i\phi'} = r' \sin\theta' [\cos\phi' - i \sin\phi'] = x' - iy'$$

$$r' \cos\theta' = z'$$

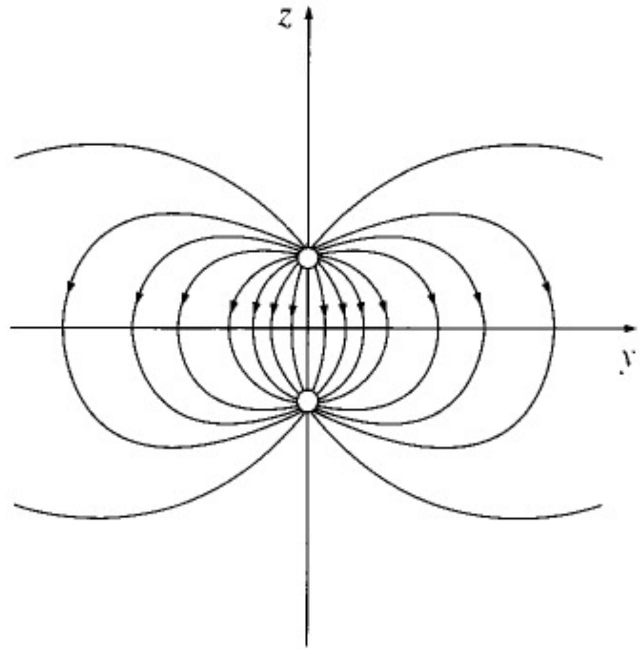
$$Q_{ij} = \int (3x'_i x'_j - r'^2 \delta_{ij}) \rho(\vec{x}') d^3x \qquad \begin{matrix} i = 1, 2, 3 \\ x, y, z \end{matrix} \qquad \begin{matrix} j = 1, 2, 3 \\ x, y, z \end{matrix}$$

MOMENTOS DE QUADRUPOLO ELÉTRICO CARTESIANOS

$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r} + \frac{\vec{p} \cdot \vec{x}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} + \cdots \right]$$



(a) Field of a "pure" dipole



(a) Field of a "physical" dipole

Magnetostática

$$\begin{cases} \vec{\nabla}\cdot\vec{B} = 0 \\ \vec{\nabla}\times\vec{B} = \mu_0 \vec{J} \end{cases} \Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0 = \mu_0 \vec{\nabla}\cdot\vec{J} \Rightarrow \boxed{\vec{\nabla}\cdot\vec{J} = 0}$$

CORRENTE ESTACIONÁRIA

CORRENTES QUE NÃO GERAM ACÚMULO OU DEPLEÇÃO DE CARGA EM REGIÕES DO ESPAÇO COMO FUNÇÃO DO TEMPO.

EQ. DA CONTINUIDADE DA CARGA: $\vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0$

$\vec{\nabla}\cdot\vec{J} \Rightarrow \frac{\partial \rho}{\partial t} = 0 \qquad \rho(\vec{x}) = \text{CONST.}$

$\vec{\nabla}\cdot\vec{B} = 0 \Longleftrightarrow \vec{B} = \vec{\nabla}\times\vec{A} \qquad \vec{A}$: POTENCIAL VETOR

LEVANDO NA EQ. DA LEI DE AMPÈRE:

$$\vec\nabla \times (\vec\nabla \times \vec A) = \mu_0 \vec J$$

$$\vec\nabla (\vec\nabla \cdot \vec A) - \nabla^2 \vec A = \mu_0 \vec J$$

A DEFINIÇÃO DE $\vec A$ NÃO O ESPECIFICA COMPLETAMENTE. PELO TEOREMA DE HELMHOLTZ, É PRECISO DAR, ALÉM DE $\vec\nabla \times \vec A = \vec B$, SEU DIVERGENTE $\vec\nabla \cdot \vec A$. DE FATO, DADO $\vec A$ TAL QUE

$$\vec B = \vec\nabla \times \vec A$$

SEGUE QUE $\vec A + \vec\nabla \Phi$, ONDE Φ É UM ESCALAR QUALQUER:

$$\vec B = \vec\nabla \times \left[\vec A + \vec\nabla \Phi \right] = \vec\nabla \times \vec A$$

$\vec B$ É INVARIANTE SOB A TRANSFORMAÇÃO:

$$\vec A \to \vec A + \vec\nabla \Phi$$

"TRANSFORMAÇÃO DE CALIBRE ("GAUGE")"

Liberdade de calibre ("gauge")

VOU USAR A LIBERDADE DE CALIBRE:

$$\vec{A} \to \vec{A} + \vec{\nabla}\Phi$$

PARA FIXAR UM CALIBRE CONVENIENTE:

$$\vec{\nabla}\cdot\vec{A} = 0 \quad \text{"CALIBRE DE COULOMB"}$$

$$\Rightarrow \nabla^2\vec{A} = -\mu_0\vec{J}$$

$$\begin{cases} \nabla^2 A_x = -\mu_0 J_x \\ \nabla^2 A_y = -\mu_0 J_y \\ \nabla^2 A_z = -\mu_0 J_z \end{cases}$$

$$A_x(\vec{x}) = \frac{\mu_0}{4\pi}\int_{T.E.} \frac{J_x(\vec{x}')}{|\vec{x}-\vec{x}'|}\,d^3x'$$

$$\vdots$$

$$\boxed{\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int_{T.E.} \frac{\vec{J}(\vec{x}')\,d^3x'}{|\vec{x}-\vec{x}'|}}$$

$$\vec{B}(\bar{x}) = \bar{\nabla}\times\bar{A}(\bar{x}) = \frac{\mu_0}{4\pi}\bar{\nabla}\times\int\frac{\bar{J}(\bar{x}')\,d^3x'}{|\bar{x}-\bar{x}'|}$$

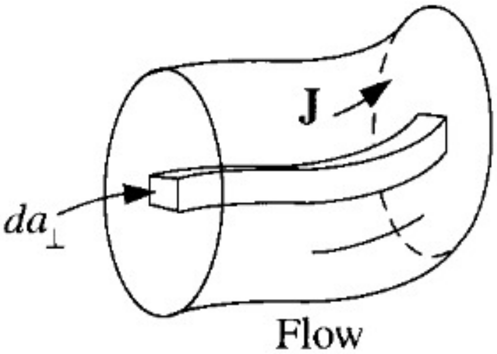
$$= \frac{\mu_0}{4\pi}\int\bar{\nabla}\times\left[\frac{\bar{J}(\bar{x}')}{|\bar{x}-\bar{x}'|}\right]d^3x'$$

$$= \frac{\mu_0}{4\pi}\int\underbrace{\bar{\nabla}\left[\frac{1}{|\bar{x}-\bar{x}'|}\right]}_{-\frac{\bar{x}-\bar{x}'}{|\bar{x}-\bar{x}'|^3}}\times\bar{J}(\bar{x}')\,d^3x'$$

$$\boxed{\vec{B}(\bar{x}) = \frac{\mu_0}{4\pi}\int\bar{J}(\bar{x}')\times\frac{(\bar{x}-\bar{x}')}{|\bar{x}-\bar{x}'|^3}\,d^3x'}$$

LEI DE BIOT-SAVART

Elementos de corrente



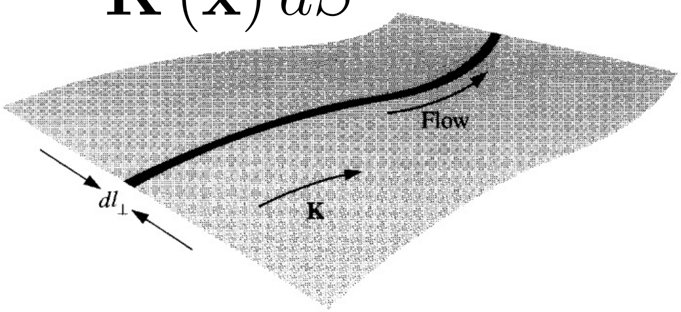
$$\mathbf{J}\left(\mathbf{x}\right)d^3x$$

$$\mathbf{A}\left(\mathbf{x}\right)=\frac{\mu_0}{4\pi}\int_{T.E.}\frac{\mathbf{J}\left(\mathbf{x}'\right)d^3x'}{\left|\mathbf{x}-\mathbf{x}'\right|}$$

$$\mathbf{B}\left(\mathbf{x}\right)=\frac{\mu_0}{4\pi}\int_{T.E.}\left[\mathbf{J}\left(\mathbf{x}'\right)d^3x'\right]\times\frac{\left(\mathbf{x}-\mathbf{x}'\right)}{\left|\mathbf{x}-\mathbf{x}'\right|^3}$$

Correntes superficiais:

$$\mathbf{K}\left(\mathbf{x}\right)dS$$

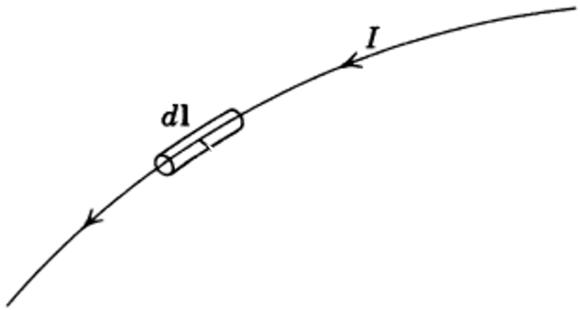


$$\mathbf{A}\left(\mathbf{x}\right)=\frac{\mu_0}{4\pi}\int_{T.E.}\frac{\mathbf{K}\left(\mathbf{x}'\right)dS'}{\left|\mathbf{x}-\mathbf{x}'\right|}$$

$$\mathbf{B}\left(\mathbf{x}\right)=\frac{\mu_0}{4\pi}\int_{T.E.}\left[\mathbf{K}\left(\mathbf{x}'\right)dS'\right]\times\frac{\left(\mathbf{x}-\mathbf{x}'\right)}{\left|\mathbf{x}-\mathbf{x}'\right|^3}$$

Elementos de corrente

Correntes lineares (fios):



$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0 I}{4\pi}\int_{T.E.} \frac{d\mathbf{l}'}{|\mathbf{x}-\mathbf{x}'|}$$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0 I}{4\pi}\int_{T.E.} d\mathbf{l}' \times \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}$$

$$\boxed{\mathbf{J}(\mathbf{x})\, d^3x \sim \mathbf{K}(\mathbf{x})\, dS \sim I d\mathbf{l}}$$