FI 008 – Eletrodinâmica I

1º Semestre de 2020
31/03/2020
Aula 7

Aula passada

$$\begin{aligned} \boldsymbol{\nabla} \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0}; \\ \boldsymbol{\nabla} \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}; \\ \boldsymbol{\nabla} \cdot \mathbf{B} &= 0; \\ \boldsymbol{\nabla} \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \end{aligned}$$

Usamos as equações sem fontes para definir os potenciais:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \iff \mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A};$$

$$\underline{\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}} = -\boldsymbol{\nabla} \times \frac{\partial \mathbf{A}}{\partial t} \iff \boldsymbol{\nabla} \times \underbrace{\left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right)}_{-\vec{\nabla} \Phi} = 0 \iff \mathbf{E} = -\boldsymbol{\nabla} \Phi - \frac{\partial \mathbf{A}}{\partial t}.$$

Aula passada

Levando nas equações com fontes obtemos: $\mu_0 \epsilon_0 = \frac{1}{c^2}$

$$\boxed{\begin{aligned} \nabla^2 \Phi + \frac{\partial \left(\boldsymbol{\nabla} \cdot \mathbf{A} \right)}{\partial t} &= -\frac{\rho}{\epsilon_0}; \\ \nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} - \boldsymbol{\nabla} \left[\boldsymbol{\nabla} \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \Phi}{\partial t} \right] &= -\mu_0 \mathbf{J}. \end{aligned}}$$

Mas existe enorme liberdade na definição dos potenciais:
invariância dos campos elétrico e magnético por <u>transf. de calibre</u>

$$\boxed{\begin{aligned} \mathbf{A} \;&\rightarrow\; \mathbf{A}' = \mathbf{A} + \boldsymbol{\nabla} \Lambda \\ \Phi \;&\rightarrow\; \Phi' = \Phi - \frac{\partial \Lambda}{\partial t} \end{aligned}}$$

Aula passada

Podemos impor a “condição de Lorenz”:

$$\boxed{\boldsymbol{\nabla} \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial \Phi}{\partial t} = 0}$$

Os potenciais então satisfazem a equação de onda não homogênea:

$$\boxed{\begin{aligned} \nabla^2 \Phi - \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} &= -\frac{\rho}{\epsilon_0}; \\ \nabla^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} &= -\mu_0 \boldsymbol{J}. \end{aligned}}$$

Aula passada

$$\boxed{\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = -f\left(\mathbf{x},t\right)}$$

Método das funções de Green:

$$\nabla^2 G\left(\mathbf{x}-\mathbf{x}',t-t'\right) - \frac{1}{c^2}\frac{\partial^2 G\left(\mathbf{x}-\mathbf{x}',t-t'\right)}{\partial t^2} = -\delta^{(3)}\left(\mathbf{x}-\mathbf{x}'\right)\delta\left(t-t'\right)$$

$$\nabla^2 G\left(\mathbf{x},t\right) - \frac{1}{c^2}\frac{\partial^2 G\left(\mathbf{x},t\right)}{\partial t^2} = -\delta^{(3)}\left(\mathbf{x}\right)\delta\left(t\right)$$

Usando transformadas de Fourier:

$$G\left(\mathbf{x},t\right) = \int \frac{d^3k\,d\omega}{\left(2\pi\right)^4} e^{i\left(\mathbf{k}\cdot\mathbf{x}-\omega t\right)} G\left(\mathbf{k},\omega\right);$$

$$\delta^{(3)}\left(\mathbf{x}\right)\delta\left(t\right) = \int \frac{d^3k\,d\omega}{\left(2\pi\right)^4} e^{i\left(\mathbf{k}\cdot\mathbf{x}-\omega t\right)}.$$

Aula passada

$$\boxed{G(\mathbf{x},t) = \int \frac{d^3k\,d\omega}{(2\pi)^4} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \frac{1}{k^2-\omega^2/c^2}}$$

Fazendo a parte angular da integral em **k** (em coord. esféricas)

$$G(\mathbf{x},t) = \int \frac{d\omega}{4\pi i r} e^{-i\omega t} \int_{-\infty}^{\infty} \frac{k\,dk}{(2\pi)^2} \frac{1}{k^2-\omega^2/c^2} \left(e^{ikr} - e^{-ikr}\right)$$

Precisamos de:

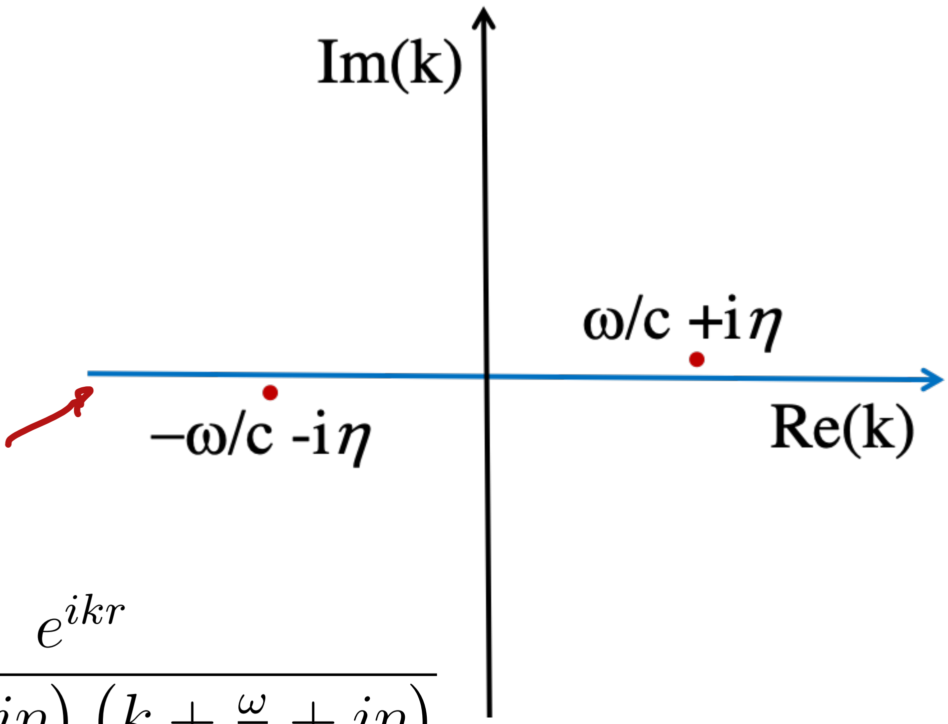
$$\boxed{I(\omega) = \int_{-\infty}^{\infty} \frac{k\,dk}{2\pi i} \frac{1}{\left(k-\frac{\omega}{c}\right)\left(k+\frac{\omega}{c}\right)} e^{\pm ikr}}$$

A integral diverge em $\pm\omega/c$. A maneira com que lidamos com a divergência determina as condições de contorno sobre $G(\mathbf{x},t)$.

Aula passada

Receita:

$$\boxed{\omega \to \omega + i\eta, \ \ \eta \to 0^+}$$



$$I_1^{(+)}(\omega) \ = \int_{-\infty}^{\infty} \frac{kdk}{2\pi i} \frac{e^{ikr}}{\left(k - \frac{\omega}{c} - i\eta\right)\left(k + \frac{\omega}{c} + i\eta\right)}$$

$$I_2^{(+)}(\omega) \ = \int_{-\infty}^{\infty} \frac{kdk}{2\pi i} \frac{e^{-ikr}}{\left(k - \frac{\omega}{c} - i\eta\right)\left(k + \frac{\omega}{c} + i\eta\right)}$$

Aula passada

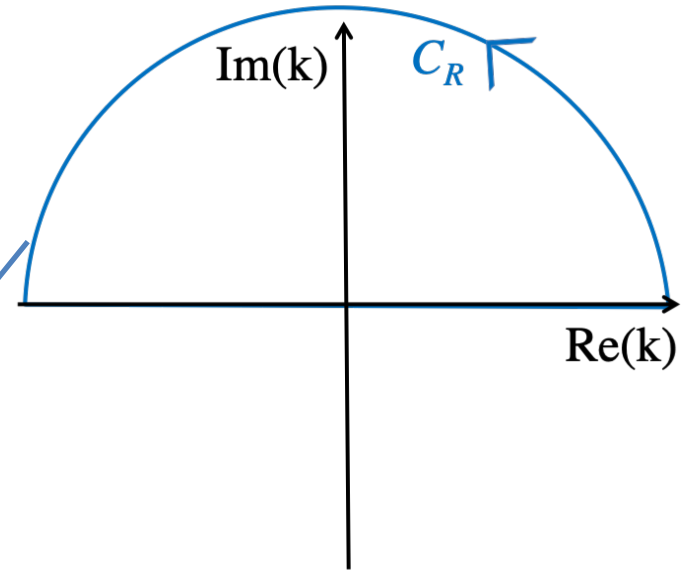
Lema de Jordan:

Se, no semi-plano superior,

a) $f(z)$ é analítica exceto em alguns pontos

b) $f(z) \to 0, \text{ se } |z| \to \infty$

$$\boxed{I(R \to \infty) = \lim_{R \to \infty} \int_{C_R} e^{i\alpha z} f(z)\, dz = 0, \ \alpha > 0}$$

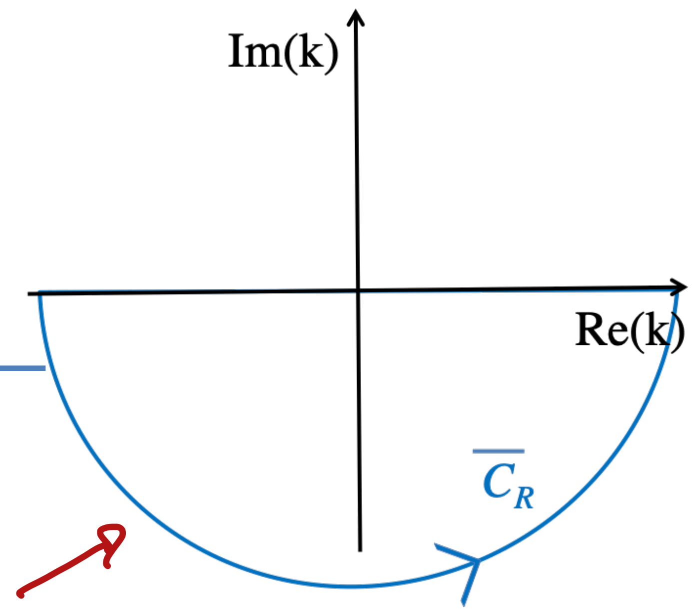


Se, no semi-plano inferior,

a) $f(z)$ é analítica exceto em alguns pontos

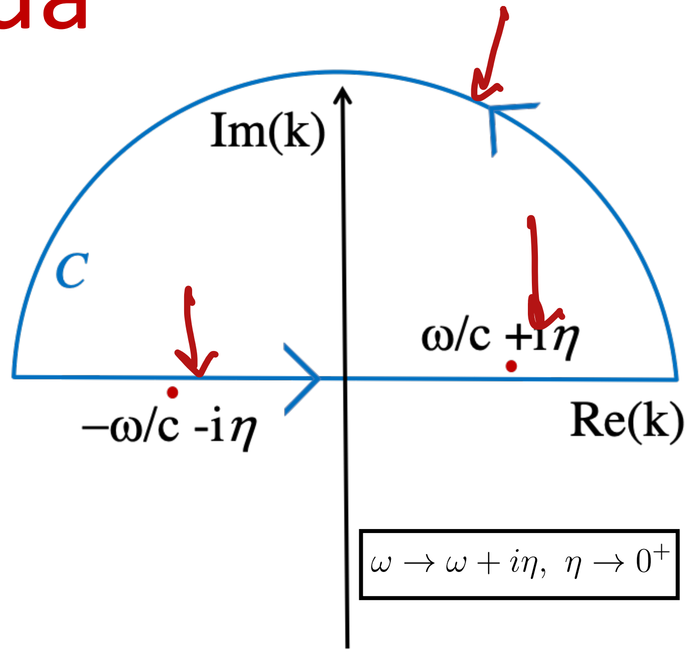
b) $f(z) \to 0, \text{ se } |z| \to \infty$

$$\boxed{I(R \to \infty) = \lim_{R \to \infty} \int_{\overline{C}_R} e^{i\alpha z} f(z)\, dz = 0, \ \alpha < 0}$$



Aula passada

$$I_1^{(+)}(\omega) = \oint_C \frac{kdk}{2\pi i}\frac{e^{ikr}}{\left(k-\frac{\omega}{c}-i\eta\right)\left(k+\frac{\omega}{c}+i\eta\right)}$$



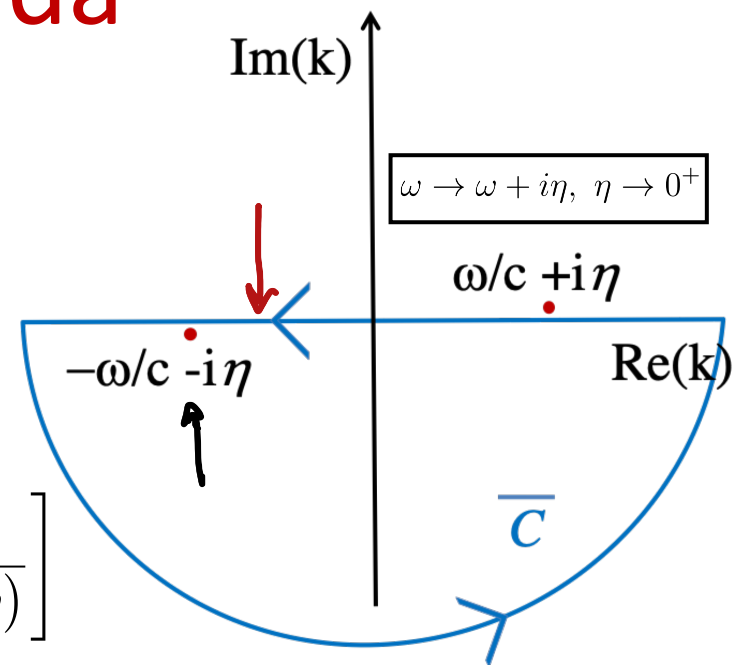
$$\boxed{\omega \to \omega + i\eta,\ \eta \to 0^+}$$

Usando o teorema dos resíduos:

$$\begin{aligned} I_1^{(+)}(\omega) &= 2\pi i \sum_{\text{res}}\left[\frac{1}{2\pi i}\frac{ke^{ikr}}{\left(k-\frac{\omega}{c}-i\eta\right)\left(k+\frac{\omega}{c}+i\eta\right)}\right] \\ &= \left.\frac{ke^{ikr}}{\left(k+\frac{\omega}{c}+i\eta\right)}\right|_{k=\frac{\omega}{c}+i\eta} \\ &= \frac{1}{2}e^{i\frac{\omega r}{c}} \end{aligned}$$

Aula passada

$$I_2^{(+)}(\omega) = -\oint_{\overline{C}} \frac{kdk}{2\pi i} \frac{e^{-ikr}}{\left(k - \frac{\omega}{c} - i\eta\right)\left(k + \frac{\omega}{c} + i\eta\right)}$$



Usando o teorema dos resíduos:

$$\begin{aligned} I_2^{(+)}(\omega) &= -2\pi i \sum_{\text{res}} \left[\frac{1}{2\pi i} \frac{ke^{-ikr}}{\left(k - \frac{\omega}{c} - i\eta\right)\left(k + \frac{\omega}{c} + i\eta\right)} \right] \\ &= -\left.\frac{ke^{-ikr}}{\left(k - \frac{\omega}{c} - i\eta\right)}\right|_{k=-\frac{\omega}{c}-i\eta} \\ &= -\frac{1}{2}e^{i\frac{\omega r}{c}} \end{aligned}$$

Aula passada

$$G(\mathbf{x},t) = \int_{-\infty}^{\infty} \frac{d\omega}{2(2\pi)^2 r} e^{-i\omega t}\left[I_1^{(+)}(\omega) - I_2^{(+)}(\omega)\right]$$

$$G(\mathbf{x},t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \boxed{\frac{e^{i\omega r/c}}{4\pi r}} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega t}\, G^{(+)}(\vec{x},\omega)$$

$$\boxed{G^{(+)}(\vec{x},\omega) = \frac{e^{i\omega r/c}}{4\pi r}}$$

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega(t - r/c)} = \delta\left(t - \frac{r}{c}\right)$$

$$\boxed{G^{(+)}(\vec{x},t) = \frac{1}{4\pi r}\,\delta\left(t - \frac{r}{c}\right)}$$

$$I^{(-)}(\omega) = \int_{-\infty}^{+\infty} \frac{dk\, k}{2\pi i} \frac{\left(e^{ikr} - e^{-ikr}\right)}{\left(k - \frac{\omega}{c} + i\eta\right)\left(k + \frac{\omega}{c} - i\eta\right)}$$

$\omega \to \omega - i\eta$

$\eta \to 0^+$

$$\boxed{G^{(-)}(\vec{x},\omega) = \frac{e^{-i\frac{\omega}{c}r}}{4\pi r}} = \left[G^{(+)}(\vec{x},\omega)\right]^*$$

$$G^{(-)}(\vec{x},t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega t}\, \frac{e^{-i\frac{\omega}{c}r}}{4\pi r}$$

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, e^{-i\omega\left[t + \frac{r}{c}\right]} = \delta\left(t + \frac{r}{c}\right)$$

$$\boxed{G^{(-)}(\vec{x},t) = \frac{\delta\left(t + \frac{r}{c}\right)}{4\pi r}} \qquad \boxed{G^{(\pm)}(\vec{x},t) = \frac{\delta\left(t \mp r/c\right)}{4\pi r}}$$

$t \to t - t' \qquad r = |\vec{x}| \to |\vec{x} - \vec{x}'|$

$$G^{(\pm)}(\vec{x}-\vec{x}', t-t') = \frac{\delta(t-t' \mp \frac{1}{c}|\vec{x}-\vec{x}'|)}{4\pi|\vec{x}-\vec{x}'|}$$

$$\psi^{(\pm)}(\vec{x},t) = \int G^{(\pm)}(\vec{x}-\vec{x}', t-t')\, f(\vec{x}', t')\, d^3x'\, dt'$$

$$= \int \frac{d^3x'\, dt'}{4\pi} \frac{\delta[t-t' \mp \frac{1}{c}|\vec{x}-\vec{x}'|]}{|\vec{x}-\vec{x}'|} f(\vec{x}', t')$$

$$t' = t \mp \frac{1}{c}|\vec{x}-\vec{x}'|$$

$$\psi^{(\pm)}(\vec{x},t) = \int \frac{d^3x'}{4\pi} \frac{f(\vec{x}', t \mp \frac{1}{c}|\vec{x}-\vec{x}'|)}{|\vec{x}-\vec{x}'|}$$

Resumo

$$G^{(\pm)}(\mathbf{x},\omega) = \frac{e^{\pm i\omega r/c}}{4\pi r}$$

$$\begin{aligned} G^{(\pm)}(\mathbf{x}-\mathbf{x}',t-t') &= \frac{1}{4\pi\left|\mathbf{x}-\mathbf{x}'\right|}\delta\left(t-t'\mp\left|\mathbf{x}-\mathbf{x}'\right|/c\right) \\ &= \frac{1}{4\pi\left|\mathbf{x}-\mathbf{x}'\right|}\delta\left[t'-\left(t\mp\left|\mathbf{x}-\mathbf{x}'\right|/c\right)\right] \end{aligned}$$

$$\begin{aligned} \psi^{(\pm)}(\mathbf{x},t) &= \int G^{(\pm)}(\mathbf{x}-\mathbf{x}',t-t')\, f(\mathbf{x}',t')\, d^3x'dt' \\ &= \int \frac{1}{4\pi\left|\mathbf{x}-\mathbf{x}'\right|}\delta\left[t'-\left(t\mp\left|\mathbf{x}-\mathbf{x}'\right|/c\right)\right] f(\mathbf{x}',t')\, d^3x'dt' \\ &= \int \frac{1}{4\pi\left|\mathbf{x}-\mathbf{x}'\right|}\underbrace{f\left(\mathbf{x}',t\mp\left|\mathbf{x}-\mathbf{x}'\right|/c\right)}_{}\, d^3x' \end{aligned}$$

Solução geral da eletrodinâmica

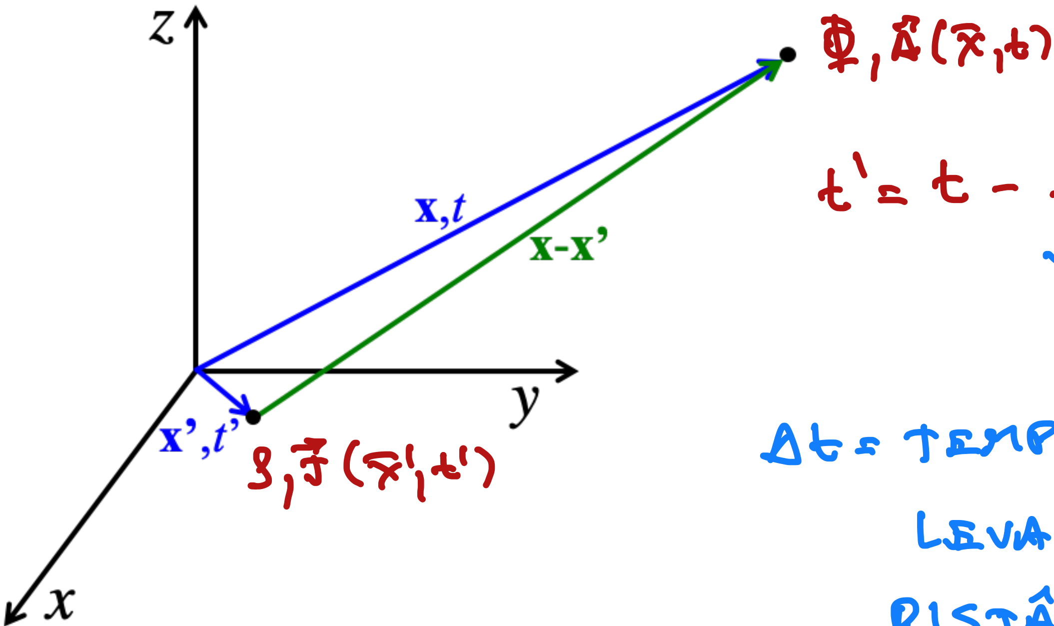
$$\boxed{\begin{aligned}\Phi^{(+)}(\mathbf{x},t) &= \frac{1}{4\pi\epsilon_0}\int \frac{1}{|\mathbf{x}-\mathbf{x}'|}\rho\left(\mathbf{x}', t-|\mathbf{x}-\mathbf{x}'|/c\right) d^3x' \\ \mathbf{A}^{(+)}(\mathbf{x},t) &= \frac{\mu_0}{4\pi}\int \frac{1}{|\mathbf{x}-\mathbf{x}'|}\mathbf{J}\left(\mathbf{x}', t-|\mathbf{x}-\mathbf{x}'|/c\right) d^3x'\end{aligned}}$$

$\Phi, \vec{A} \rightarrow \vec{E}, \vec{B}$

$$t_{ret} = t - |\mathbf{x}-\mathbf{x}'|/c$$

SOLUÇÃO GERAL

Interpretação física



$$t' = t - \underbrace{\frac{1}{c}|\vec{x}-\vec{x}'|}_{\Delta t} = t_{ret}$$

Δt = TEMPO QUE A LUZ LEVA PARA COBRIR A DISTÂNCIA $|\vec{x}-\vec{x}'|$

SE USARMOS $G^{(-)}(\vec{x},t)$:

$$t' = t_{av} = t + \frac{1}{c}|\vec{x}-\vec{x}'|$$

É COMO SE O EFEITO PRECEDESSE A CAUSA!

SOLUÇÃO GERAL ENVOLVE SOMAR UMA SOLUÇÃO DA EQ. HOMOGÊNEA

SOLUÇÃO DA HOMOGÊNEA

$$\psi^{(+)}(\bar{x},t) = \psi_{in}(\bar{x},t) + \int G^{(+)}(\bar{x}-\bar{x}', t-t')\, f(\bar{x}', t')\, d^3x'\, dt'$$

SUPONHAMOS QUE A FONTE $f(\bar{x},t)$ SÓ É DIFERENTE DE ZERO $|\vec{x}| < R$ $|t| < t_0$

NO PASSADO REMOTO $t \to -\infty$ O SEGUNDO TERMO SE ANULA DEVIDO À ESTRUTURA CAUSAL DE $G^{(+)}$

$$\psi^{(+)}(\vec{x},t) \xrightarrow[t \to -\infty]{} \psi_{in}(\bar{x},t)$$

SE EU USAR $G^{(-)}$:

$$\psi^{(-)}(\bar{x},t) = \psi_{out}(\bar{x},t) + \int G^{(-)}(\bar{x}-\bar{x}',t-t')\, f(\bar{x}',t')\, d^3x'\, dt'$$

NO FUTURO REMOTO $t \to +\infty$!

$$\psi^{(-)}(\bar{x},t) \to \psi_{out}(x,t)$$

COMO GERALMENTE SÓ CONTROLAMOS O PASSADO (CONDIÇÕES INICIAIS), A SEGUNDA SITUAÇÃO $[G^{(-)}]$ DIFICILMENTE OCORRERÁ NA PRÁTICA

Leis de conservação

CONSERVAÇÃO DE ENERGIA (TEOREMA DE POYNTING):

$W_{c\to m}$ = TRABALHO REALIZADO PELOS CAMPOS $\vec{E}$ E $\vec{B}$ NA MATÉRIA

$$\frac{dW_{c\to m}}{dt} = \sum_i \vec{F}_i \cdot \vec{v}_i = \sum_i q_i \left(\vec{E}(\vec{x}_i) + \vec{v}_i \times \vec{B}(\vec{x}_i) \right) \cdot \vec{v}_i$$

$$= \sum_i q_i \vec{E}(\vec{x}_i) \cdot \vec{v}_i \longrightarrow \int \rho(\vec{x})\, \vec{E}(\vec{x}) \cdot \vec{v}\, d^3x = (*)$$

DISTRIBUIÇÃO CONTÍNUA DE CARGA: $\sum_i q_i \to \int \rho(\vec{x})\, d^3x$

$$\rho \vec{v}\, d^3x = \vec{J}\, d^3x \Rightarrow (*) = \int \vec{E} \cdot \vec{J}\, d^3x$$

$$\frac{dW_{c\to m}}{dt} = \int_V \vec{E} \cdot \vec{J}\, d^3x = \frac{dE_{mec}}{dt}$$

E_{mec} = ENERGIA MECÂNICA DA MATÉRIA

$$\int_V \bar{J}\cdot\bar{E}\, d^3x = \int_V \frac{(\bar{\nabla}\times\bar{B})}{\mu_0}\cdot\vec{E}\, d^3x - \epsilon_0\int_V \vec{E}\cdot\frac{\partial\bar{E}}{\partial t}\, d^3x \quad (*)$$ (AMPÈRE-MAXWELL)

$$\bar{\nabla}\cdot(\bar{E}\times\bar{B}) = \bar{B}\cdot\underbrace{(\bar{\nabla}\times\bar{E})}_{-\frac{\partial B}{\partial t}} - \bar{E}\cdot(\bar{\nabla}\times\bar{B}) = -\bar{B}\cdot\frac{\partial\bar{B}}{\partial t} - \bar{E}\cdot(\bar{\nabla}\times\bar{B})$$

$$(*) = -\int_V \bar{\nabla}\cdot\frac{(\bar{E}\times\bar{B})}{\mu_0}\, d^3x - \int_V \left[\epsilon_0\,\bar{E}\cdot\underbrace{\frac{\partial\bar{E}}{\partial t}}_{\frac{1}{2}\frac{\partial}{\partial t}\underbrace{(\bar{E}\cdot\bar{E})}_{E^2}} + \frac{1}{\mu_0}\,\bar{B}\cdot\underbrace{\frac{\partial\bar{B}}{\partial t}}_{\frac{1}{2}\frac{\partial}{\partial t}\underbrace{(\bar{B}\cdot\bar{B})}_{B^2}}\right] d^3x$$

$$\int_V \bar{J}\cdot\bar{E}\, d^3x = -\int_V \frac{\partial}{\partial t}\left[\frac{\epsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2\right] d^3x - \int \bar{\nabla}\cdot\left(\frac{\bar{E}\times\bar{B}}{\mu_0}\right) d^3x$$

COMO V É GENÉRICO, POSSO IGUALAR OS INTEGRANDOS

$$\boxed{\bar{\nabla}\cdot\vec{S} + \frac{\partial u}{\partial t} = -\bar{J}\cdot\bar{E}}$$

$\vec{S} = \frac{1}{\mu_0}\hat{E}\times\hat{B}$ (VETOR DE POYNTING)

$u = \frac{\epsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2$ = DENSIDADE DE ENERGIA NOS CAMPOS

$\vec{S}$ = DENSIDADE DE CORRENTE DE ENERGIA ELETROMAGNÉTICA

$-\vec{J}\cdot\vec{E}$ = TAXA DE VARIAÇÃO DE ENERGIA E.M. POR UNIDADE DE VOLUME ("FONTE" OU "SORVEDOURO")

$$\int_V \vec{J}\cdot\vec{E}\, d^3x = \frac{dE_{mec}}{\partial t} = \frac{d}{dt}\int_V u_{mec}\, d^3x = \int \left(\frac{\partial u_{mec}}{\partial t}\right) d^3x$$

u_{mec} = DENSIDADE DE ENERGIA MECÂNICA

$$\vec{\nabla}\cdot\vec{S} + \frac{\partial u}{\partial t} = -\frac{\partial u_{mec}}{\partial t} \Rightarrow \boxed{\vec{\nabla}\cdot\vec{S} + \frac{\partial}{\partial t}\left[u + u_{mec}\right] = 0}$$

NA AUSÊNCIA DE MATÉRIA:

$$\boxed{\vec{\nabla}\cdot\vec{S} + \frac{\partial u}{\partial t} = 0}$$

CONSERVAÇÃO DE MOMENTO LINEAR:

$\vec{P}_{MEC}$ = MOMENTO LINEAR TOTAL DA MATÉRIA

$$\frac{d\vec{P}_{MEC}}{dt} = \sum_i \vec{F}_i = \sum_i q_i\left(\vec{E}(\vec{x}_i) + \vec{v}_i \times \vec{B}(\vec{x}_i)\right) = \int_V \left[\rho\vec{E} + \vec{J}\times\vec{B}\right] d^3x = (**)$$

$$[\,] = \epsilon_0(\vec{\nabla}\cdot\vec{E})\vec{E} + \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B})\times\vec{B} - \epsilon_0 \frac{\partial\vec{E}}{\partial t}\times\vec{B}$$

$$\frac{\partial\vec{E}}{\partial t}\times\vec{B} = \frac{\partial}{\partial t}(\vec{E}\times\vec{B}) - \vec{E}\times\frac{\partial\vec{B}}{\partial t} = \frac{\partial}{\partial t}(\vec{E}\times\vec{B}) + \vec{E}\times(\vec{\nabla}\times\vec{E})$$

$$[\,] = \epsilon_0(\vec{\nabla}\cdot\vec{E})\vec{E} + \frac{1}{\mu_0}(\vec{\nabla}\cdot\vec{B})\vec{B} - \frac{1}{\mu_0}\vec{B}\times(\vec{\nabla}\times\vec{B}) - \epsilon_0\,\vec{E}\times(\vec{\nabla}\times\vec{E})$$

$$-\epsilon_0\frac{\partial}{\partial t}(\vec{E}\times\vec{B})$$

$$(**) = \int_V \vec{\Pi}\, d^3x - \int_V \frac{\partial}{\partial t}\underbrace{\left(\epsilon_0\vec{E}\times\vec{B}\right)}_{\vec{p}} d^3x$$

$$\vec{p} = \epsilon_0\vec{E}\times\vec{B} = \epsilon_0\mu_0\vec{S} = \frac{1}{c^2}\vec{S}$$

$$\vec{\Pi} = \epsilon_0\vec{E}(\vec{\nabla}\cdot\vec{E}) + \frac{1}{\mu_0}\vec{B}(\vec{\nabla}\cdot\vec{B}) - \epsilon_0\vec{E}\times(\vec{\nabla}\times\vec{E}) - \frac{1}{\mu_0}\vec{B}\times(\vec{\nabla}\times\vec{B})$$

$$\frac{d\vec{P}_{MEC}}{dt} = \frac{d}{dt}\left[\int_V \vec{p}_{mec}\, d^3x\right] = \int_V \frac{\partial \vec{p}_{mec}}{\partial t} d^3x$$

COMO O VOLUME $\underline{V}$ É GENÉRICO, IGUALAMOS OS INTEGRANDOS

$$\frac{\partial}{\partial t}(\vec{p} + \vec{p}_{mec}) - \vec{\pi} = 0$$

$\vec{p}$ = DENSIDADE DE MOMENTO LINEAR DOS CAMPOS

PARA OBTERMOS UMA LEI DE CONSERVAÇÃO, $\vec{\pi}$ PRECISA SER UM DIVERGENTE:

$$\pi_i = \vec{\nabla} \cdot \vec{T}_i \qquad (i = 1, 2, 3;\ x, y, z)$$

COMO MOSTRADO NAS NOTAS:

$$\pi_i = \sum_j \frac{\partial T_{ij}}{\partial x_j}$$

TENSOR DE TENSÕES DE MAXWELL

$$T_{ij} = T_{ji} = \epsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{\delta_{ij}}{2}\left[\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right]$$

$$\frac{\partial}{\partial t}(p_i + P_{mec\,i}) - \vec{\nabla}\cdot\vec{T}_i = 0$$

LEI DE CONSERVAÇÃO DE MOM. LINEAR

p_i = DENSIDADE DE MOM. LINEAR DOS CAMPOS NA DIREÇÃO i

$-T_i$ = DENSIDADE DE CORRENTE DE MOM. LINEAR DOS CAMPOS NA DIREÇÃO i

$\vec{p} = \frac{1}{c^2}\vec{S} \Rightarrow$ DENS. MOM LINEAR $= \frac{1}{c^2}$ (DENS. DE CORRENTE DE ENERGIA)

$E = cp$ (LUZ) $\Rightarrow$ MOM. LINEAR $= \frac{1}{c}$ ENERGIA

CORRENTE DE ENERGIA : (DENSIDADE ENERGIA) c

DENS. M.L. $= \frac{1}{c}$ (DENS. EN.) $= \frac{1}{c}\frac{1}{c}$ (DENS. CORRENTE ENERGIA)

Conservação de energia: $$\frac{\partial u}{\partial t} + \frac{\partial u_{mec}}{\partial t} + \nabla \cdot \mathbf{S} = 0$$

$$u = \frac{\epsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2$$
$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$$

Gaussiano:
$$u = \frac{1}{8\pi}\left(E^2 + B^2\right)$$
$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B}$$

Conservação de momento linear: $$\frac{\partial \mathbf{p}}{\partial t} + \frac{\partial \mathbf{p}_{mec}}{\partial t} - \nabla \cdot \overleftrightarrow{\mathbf{T}} = 0$$

$$\mathbf{p} = \epsilon_0 \mathbf{E} \times \mathbf{B} = \frac{1}{c^2}\mathbf{S}$$
$$T_{ij} = \epsilon_0 E_i E_j + \frac{1}{\mu_0}B_i B_j - \delta_{ij}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)$$

Gaussiano:
$$\mathbf{p} = \frac{1}{4\pi c}\mathbf{E} \times \mathbf{B} = \frac{1}{c^2}\mathbf{S}$$
$$T_{ij} = \frac{1}{4\pi}\left[\left(E_i E_j + B_i B_j\right) - \frac{1}{2}\delta_{ij}\left(E^2 + B^2\right)\right]$$