FI 008 – Eletrodinâmica I

1º Semestre de 2020
07/04/2020
Aula 9

Monopolos magnéticos

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$\vec{\nabla} \cdot \vec{B} = \rho_m$ → DENSIDADE VOL. DE CARGA MAGNÉTICA

$$\rho_m = \lim_{\Delta V \to 0} \frac{\Delta q_m}{\Delta V}$$

$$\vec{\nabla} \times \vec{E} = -\vec{J}_m - \frac{\partial \vec{B}}{\partial t}$$

$\vec{J}_m = \rho_m \vec{v}$ DENS. VOL. CORRENTE MAGNÉTICA

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{E}) = 0 = -\vec{\nabla} \cdot \vec{J}_m - \frac{\partial}{\partial t}\underbrace{(\vec{\nabla} \cdot \vec{B})}_{\rho_m}$$

$$0 = -\vec{\nabla} \cdot \vec{J}_m - \frac{\partial \rho_m}{\partial t} \Rightarrow \boxed{\vec{\nabla} \cdot \vec{J}_m + \frac{\partial \rho_m}{\partial t} = 0}$$

CONSERVAÇÃO DA CARGA MAGNÉTICA

$$[\rho_m] = \frac{[B]}{L}$$

$$[\rho_m]/T = \frac{[J_m]}{L} \Rightarrow [J_m] = \frac{L}{T}[\rho_m]$$

ρ_m É PSEUDO-ESCALAR ÍMPAR SOB T

$\vec{J}_m$ É PSEUDO-VETOR PAR SOB T

$$\boxed{\begin{aligned} \nabla\cdot\mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla\times\mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t} - \mathbf{J}_m \\ \nabla\cdot\mathbf{B} &= \rho_m \\ \nabla\times\mathbf{B} &= \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t} \end{aligned}}$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\begin{aligned} \nabla\cdot\mathbf{E} &= \frac{1}{\sqrt{\epsilon_0}}\left(\frac{\rho}{\sqrt{\epsilon_0}}\right) \\ \nabla\cdot(c\mathbf{B}) &= \frac{1}{\sqrt{\epsilon_0}}\left(\frac{\rho_m}{\sqrt{\mu_0}}\right) \\ \nabla\times\mathbf{E} &= -\sqrt{\mu_0}\left[\sqrt{\epsilon_0}\frac{\partial(c\mathbf{B})}{\partial t} + \left(\frac{\mathbf{J}_m}{\sqrt{\mu_0}}\right)\right] \\ \nabla\times(c\mathbf{B}) &= \sqrt{\mu_0}\left[\sqrt{\epsilon_0}\frac{\partial\mathbf{E}}{\partial t} + \left(\frac{\mathbf{J}}{\sqrt{\epsilon_0}}\right)\right] \end{aligned}$$

$$\begin{aligned} c\mathbf{B} &\to \mathbf{B} \\ \frac{\rho}{\sqrt{\epsilon_0}} &\to \rho \\ \frac{\rho_m}{\sqrt{\mu_0}} &\to \rho_m \\ \frac{\mathbf{J}}{\sqrt{\epsilon_0}} &\to \mathbf{J} \\ \frac{\mathbf{J}_m}{\sqrt{\mu_0}} &\to \mathbf{J}_m \end{aligned}$$

$$\begin{aligned} \nabla\cdot\mathbf{E} &= \frac{1}{\sqrt{\epsilon_0}}\rho \\ \nabla\cdot\mathbf{B} &= \frac{1}{\sqrt{\epsilon_0}}\rho_m \\ \nabla\times\mathbf{E} &= -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} - \sqrt{\mu_0}\mathbf{J}_m \\ \nabla\times\mathbf{B} &= \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \sqrt{\mu_0}\mathbf{J} \end{aligned}$$

$$
\begin{aligned}
\boldsymbol{\nabla}\cdot\mathbf{E} &= \frac{1}{\sqrt{\epsilon_0}}\rho \\
\boldsymbol{\nabla}\cdot\mathbf{B} &= \frac{1}{\sqrt{\epsilon_0}}\rho_m \\
\boldsymbol{\nabla}\times\mathbf{E} &= -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} - \sqrt{\mu_0}\mathbf{J}_m \\
\boldsymbol{\nabla}\times\mathbf{B} &= \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \sqrt{\mu_0}\mathbf{J}
\end{aligned}
$$

$$
\boldsymbol{\Psi} = \begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix} \; ; \; \theta = \begin{pmatrix}\rho\\ \rho_m\end{pmatrix} \; ; \; \boldsymbol{\Gamma} = \begin{pmatrix}\mathbf{J}\\ \mathbf{J}_m\end{pmatrix}
$$

$$
\boxed{
\begin{aligned}
\boldsymbol{\nabla}\cdot\boldsymbol{\Psi} &= \frac{1}{\sqrt{\epsilon_0}}\theta \\
\boldsymbol{\nabla}\times\boldsymbol{\Psi} &= \frac{1}{c}\frac{\partial\left(\Upsilon\boldsymbol{\Psi}\right)}{\partial t} + \sqrt{\mu_0}\left(\Upsilon\boldsymbol{\Gamma}\right)
\end{aligned}
}
$$

$$
\Upsilon = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}
$$

$$
\Upsilon\vec{\Psi} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\vec{E}\\ \vec{B}\end{pmatrix} = \begin{pmatrix}-\vec{B}\\ \vec{E}\end{pmatrix}
$$

$$
\Upsilon\vec{\Gamma} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\vec{J}\\ \vec{J}_m\end{pmatrix} = \begin{pmatrix}-\vec{J}_m\\ \vec{J}\end{pmatrix}
$$

Transformações de dualidade

$$\begin{pmatrix} \mathbf{E}' \\ \mathbf{B}' \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix}$$

$$\begin{pmatrix} \rho' \\ \rho'_m \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} \rho \\ \rho_m \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{J}' \\ \mathbf{J}'_m \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} \mathbf{J} \\ \mathbf{J}_m \end{pmatrix}$$

$$\vec{E}' = \cos\xi\, \vec{E} + \sin\xi\, \vec{B}$$

$$\vec{B}' = -\sin\xi\, \vec{E} + \cos\xi\, \vec{B}$$

$$R = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix}$$

$$R^T = R^{-1}$$

$$\left\{ \begin{aligned} \boldsymbol{\Psi}' &= R\boldsymbol{\Psi} \\ \theta' &= R\theta \\ \boldsymbol{\Gamma}' &= R\boldsymbol{\Gamma} \end{aligned} \right.$$

$$\Upsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Transformações de dualidade

$$\begin{aligned}\nabla\cdot\boldsymbol{\Psi} &= \frac{1}{\sqrt{\epsilon_0}}\theta \quad (1)\\ \nabla\times\boldsymbol{\Psi} &= \frac{1}{c}\frac{\partial(\Upsilon\boldsymbol{\Psi})}{\partial t}+\sqrt{\mu_0}\,(\Upsilon\boldsymbol{\Gamma}) \quad (2)\end{aligned}$$

$$\boxed{R\Upsilon = \Upsilon R}$$

$$\begin{aligned}\nabla\cdot\boldsymbol{\Psi}' &= \frac{1}{\sqrt{\epsilon_0}}\theta' \quad (1')\\ \nabla\times\boldsymbol{\Psi}' &= \frac{1}{c}\frac{\partial(\Upsilon\boldsymbol{\Psi}')}{\partial t}+\sqrt{\mu_0}\,(\Upsilon\boldsymbol{\Gamma}') \quad (2')\end{aligned}$$

$R(1) \rightarrow \vec{\nabla}\cdot(\underbrace{R\vec{\Psi}}_{\vec{\Psi}'}) = \frac{1}{\sqrt{\epsilon_0}}\underbrace{R\theta}_{\theta'}$

$\Rightarrow \vec{\nabla}\cdot\vec{\Psi}' = \frac{1}{\sqrt{\epsilon_0}}\theta'$

$R(2) \Rightarrow \vec{\nabla}\times(R\vec{\Psi}) = \frac{1}{c}\frac{\partial}{\partial t}[R\Upsilon\vec{\Psi}] + \sqrt{\mu_0}\,(R\Upsilon\vec{\Gamma})$

$= \frac{1}{c}\frac{\partial}{\partial t}[\Upsilon R\vec{\Psi}] + \sqrt{\mu_0}\,(\Upsilon R\vec{\Gamma}) = \frac{1}{c}\frac{\partial}{\partial t}[\Upsilon\vec{\Psi}'] + \sqrt{\mu_0}\,(\Upsilon\vec{\Gamma}')$

NA AUSÊNCIA DE FONTES

$(\rho = \rho_m = 0 \; ; \; \vec{J} = \vec{J}_m = 0)$

$$\begin{cases}\vec{\nabla}\cdot\vec{\Psi} = 0\\ \vec{\nabla}\times\vec{\Psi} = \frac{1}{c}\frac{\partial}{\partial t}(\Upsilon\vec{\Psi})\end{cases}$$

* EQS. DE MAXWELL NO VÁCUO TÊM SIMETRIA DE DUALIDADE

$\boxed{\text{~~In~~Covariância por transformação de dualidade}}$

Quantidades quadráticas são invariantes

$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) = \frac{\epsilon_0}{2}\left(E^2 + c^2 B^2\right) \to \frac{\epsilon_0}{2}\left(E^2 + B^2\right) = \frac{\epsilon_0}{2}\boldsymbol{\Psi}^T \cdot \boldsymbol{\Psi}$$

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = \sqrt{\frac{\epsilon_0}{\mu_0}}\mathbf{E}\times(c\mathbf{B}) \to \sqrt{\frac{\epsilon_0}{\mu_0}}\mathbf{E}\times\mathbf{B} = -\frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}\underbrace{\boldsymbol{\Psi}^T \times \Upsilon\boldsymbol{\Psi}}$$

$$\vec{\Psi}^T \cdot \vec{\Psi} = (\vec{E} \quad \vec{B}) \cdot \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix} = \vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B} = E^2 + B^2$$

$$\xrightarrow{R} (\vec{\Psi}^T \underbrace{R^T)\cdot R}_{R^T R = 1} \vec{\Psi} = \vec{\Psi}^T \cdot \vec{\Psi}$$ INVARIANTE

$$\vec{\Psi}^T \times \Upsilon \vec{\Psi} = (\vec{E} \quad \vec{B}) \times \begin{pmatrix} -\vec{B} \\ \vec{E} \end{pmatrix} = -\vec{E}\times\vec{B} + \vec{B}\times\vec{E} = -2\,\vec{E}\times\vec{B}$$

$$\xrightarrow{R} \vec{\Psi}^T \times R^T \Upsilon R \vec{\Psi} = \vec{\Psi}^T \times R^T R \Upsilon \vec{\Psi} = \vec{\Psi}^T \times \Upsilon \vec{\Psi}$$ INVARIANTE

Quantidades quadráticas são invariantes

$$T_{\alpha\beta} \;=\; \epsilon_0\left[E_\alpha E_\beta + c^2 B_\alpha B_\beta - \frac{1}{2}\delta_{\alpha\beta}\left(E^2 + c^2 B^2\right)\right] \to \epsilon_0\left[E_\alpha E_\beta + B_\alpha B_\beta - \frac{1}{2}\delta_{\alpha\beta}\left(E^2 + B^2\right)\right]$$

$$T_{\alpha\beta} \;=\; \epsilon_0\left[\Psi_\alpha^T \Psi_\beta - \frac{1}{2}\delta_{\alpha\beta}\mathbf{\Psi}^T \cdot \mathbf{\Psi}\right]$$

TAMBÉM É INVARIANTE POR TRANS. DE DUALIDADE

Razão carga elétrica/magnética constante

Se todas as partículas tiverem a mesma razão carga elétrica/magnética

$$\boxed{q_m = rq} \Rightarrow \rho_m = r\rho, \;\; \mathbf{J}_m = r\mathbf{J}$$

$$\begin{pmatrix} \rho \\ \rho_m \end{pmatrix} = \rho \begin{pmatrix} 1 \\ r \end{pmatrix} \quad \begin{pmatrix} \mathbf{J} \\ \mathbf{J}_m \end{pmatrix} = \mathbf{J} \begin{pmatrix} 1 \\ r \end{pmatrix}$$

$$\theta' = R\theta \Rightarrow \begin{pmatrix} \rho' \\ \rho'_m \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \rho \begin{pmatrix} 1 \\ r \end{pmatrix} = \rho \begin{pmatrix} \cos\xi + r\sin\xi \\ -\sin\xi + r\cos\xi \end{pmatrix}$$

$\rho'_m = \rho\,(-\sin\xi + r\cos\xi)$ SE ESCOLHERMOS ξ TAL QUE

$r\cos\xi = \sin\xi \Rightarrow \boxed{\tan\xi = r} \Rightarrow \boxed{\rho'_m = 0 \qquad \mathbf{J}'_m = 0}$

NESSE CASO, A CARGA MAGNÉTICA NÃO É OBSERVÁVEL

Condição de quantização de Dirac

Covariância de calibre na mecânica quântica

MECÂNICA CLÁSSICA: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = m\vec{a}$

NÃO NECESSITA DE $\vec{A}$ E Φ

NA MECÂNICA QUÂNTICA, PRECISA DE TRABALHAR COM $\vec{A}$ E Φ

$$\hat{H} = \frac{\hat{\vec{p}}^2}{2m} + V(\hat{\vec{x}}) \xrightarrow[\text{C.E.M}]{} \frac{1}{2m}\left(\hat{\vec{p}} - q\hat{\vec{A}}\right)^2 + V(\hat{\vec{x}}) + q\,\Phi(\hat{\vec{x}}) \qquad H\psi(\vec{x},t) = \hbar i\,\partial_t\psi$$

COMO $\vec{A}$ E Φ PODEM SER MUDADOS POR TRANS. DE CALIBRE $\hat{H}$ TAMBÉM MUDA. O CONTEÚDO FÍSICO DA TEORIA MUDA TAMBÉM?

$$\boxed{\begin{aligned} \vec{A}' &= \vec{A} + \vec{\nabla}\Lambda \\ \Phi' &= \Phi - \frac{\partial\Lambda}{\partial t} \end{aligned}} \quad (*)$$

$(\vec{x},t)$

$$\Rightarrow \frac{1}{2m}\left[\frac{\hbar\vec{\nabla}}{i} - q\vec{A}\right]^2\psi(\vec{x},t) + \left[V(\vec{x}) + q\,\Phi(\vec{x},t)\right]\psi(\vec{x},t) = i\hbar\,\partial_t\psi(\vec{x},t)$$

$$\left(\frac{\hbar}{i}\bar{\nabla} - q\vec{A}\right) \longrightarrow \left(\frac{\hbar}{i}\bar{\nabla} - q\vec{A} - q\bar{\nabla}\Lambda\right) \quad (3)$$

$$\left[V(\bar{x}) + q\Phi\right] \rightarrow V(\bar{x}) + q\Phi(\bar{x}) - q\cancel{\frac{\partial \Lambda}{\partial t}} \quad (4)$$

SE IMPUSERMOS QUE $\psi(\bar{x},t)$ TAMBÉM SE TRANSFORMA SOB A TRANS. DE CALIBRE:

$$\boxed{\psi'(\bar{x},t) = e^{i\frac{q}{\hbar}\Lambda(\bar{x},t)}\,\psi(\bar{x},t)} \quad (*)$$

$$(3)\left(\frac{\hbar}{i}\bar{\nabla} - q\bar{A} - q\bar{\nabla}\Lambda\right) e^{i\frac{q}{\hbar}\Lambda(\bar{x},t)}\,\psi(\bar{x},t) =$$

$$\cancel{e^{i\frac{q}{\hbar}\Lambda}}\left(\frac{\hbar}{i}\bar{\nabla} + \cancel{q\bar{\nabla}\Lambda} - q\bar{A} - \cancel{q\bar{\nabla}\Lambda}\right)\psi(\bar{x},t)$$

$$i\hbar\frac{\partial}{\partial t}\left[e^{i\frac{q}{\hbar}\Lambda}\psi\right] = i\hbar\left(i\frac{q}{\hbar}\frac{\partial \Lambda}{\partial t}\right)e^{i\frac{q}{\hbar}\Lambda}\psi + i\hbar\, e^{i\frac{q}{\hbar}\Lambda}\frac{\partial \psi}{\partial t}$$

$$= \cancel{e^{i\frac{q}{\hbar}\Lambda}}\left[-q\cancel{\frac{\partial \Lambda}{\partial t}} + i\hbar\frac{\partial}{\partial t}\right]\psi$$

Dinâmica quântica de uma partícula carregada na presença de um monopolo magnético

MONOPOLO MAGNÉTICO NA ORIGEM: $\vec{\nabla}\cdot\vec{B} = \rho_m = g\,\delta^{(3)}(\vec{x})$

$$\vec{B}(\vec{x}) = \frac{g}{4\pi}\frac{\hat{r}}{r^2} \qquad \hat{r} = \frac{\vec{x}}{r} \qquad r = |\vec{x}|$$

DINÂMICA DE UMA PARTÍCULA DE CARGA ELÉTRICA $\underline{e}$ E MASSA $\underline{m}$ NA PRESENÇA DESSE MONOPOLO g

MAS $\vec{\nabla}\cdot\vec{B} \neq 0 \Rightarrow$ COMO DEFINIR O $\vec{A}$ PARA FAZER MEC. QU.?

É POSSÍVEL, ATÉ CERTO PONTO, DEFINIR UM $\vec{A}$ PARA ESSE $\vec{B}$:

$$\vec{A}(\vec{x}) = \frac{g}{4\pi r}\frac{(1-\cos\theta)}{\sin\theta}\,\hat{\phi} \quad \text{(COORDS. ESFÉRICAS)}$$

$$\vec{\nabla}\times\vec{A} = \frac{\hat{r}}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\, A_\phi\right] = \frac{g}{4\pi}\frac{\hat{r}}{r^2} = \vec{B}\,!$$

MAS ESSA $\vec{A}(\vec{x})$ NÃO É DEFINIDO NO EIXO z NEGATIVO $\theta = \pi$

$\theta \to \pi \qquad \vec{A}(\vec{x}) \to \infty$

NOTE QUE $\theta \to 0 \qquad \dfrac{1-\cos\theta}{\sin\theta} \to \dfrac{\theta^2/2}{\theta} = \dfrac{\theta}{2} \to 0$

$\vec{A}(\vec{x})$ É BEM COMPORTADO NO EIXO z POSITIVO $(\theta = 0)$

NA VERDADE, $\vec{A}'(\vec{x}) = -\dfrac{g}{4\pi r}\dfrac{(1+\cos\theta)}{\sin\theta}\hat{\phi}$ É TAL QUE

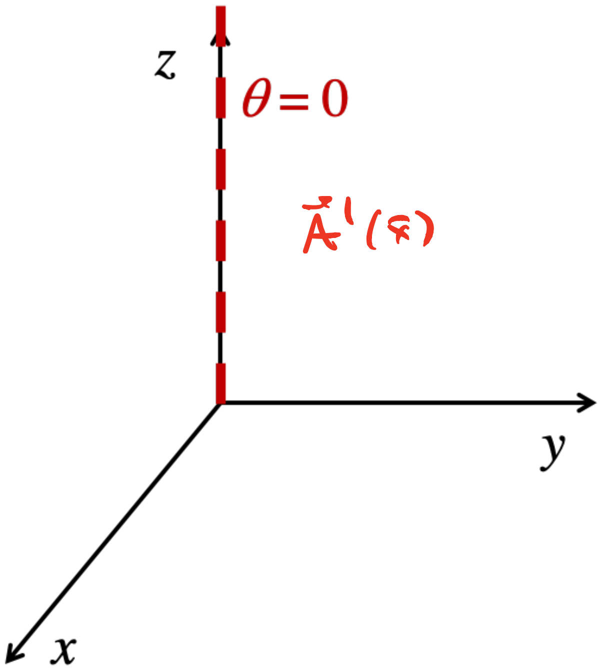
- $\vec{\nabla}\times\vec{A}' = \dfrac{g}{4\pi}\dfrac{\hat{r}}{r^2} = \vec{B}$

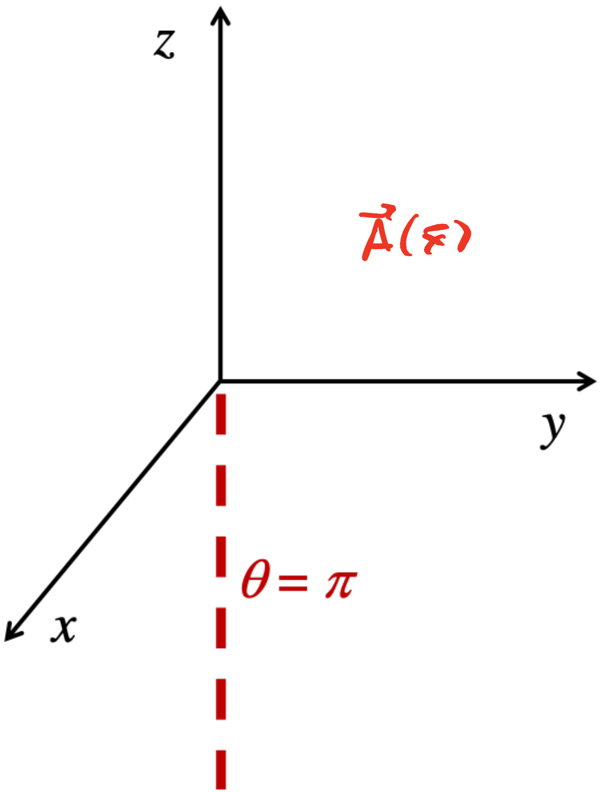
- É MAL DEFINIDO PARA $\theta = 0$ (EIXO z POSITIVO)

DOIS POTENCIAIS $\vec{A}$ E $\vec{A}'$ GERANDO O MESMO $\vec{B}$ EXCETO AO LONGO DAS DUAS CORDAS DE DIRAC

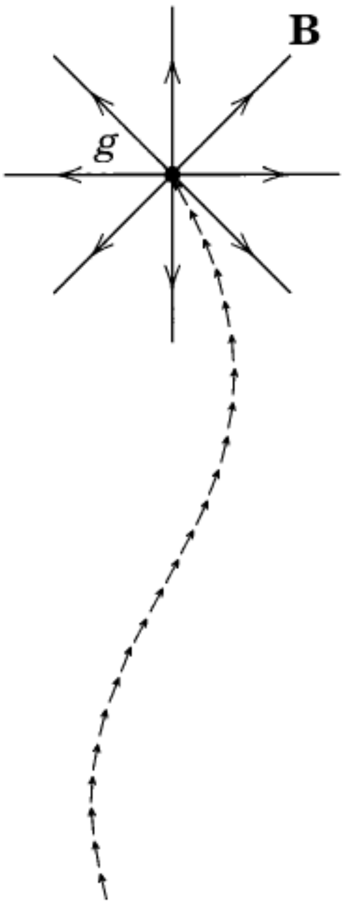
$$\vec{A}'(\vec{x}) = \vec{A}(\vec{x}) - \underbrace{\frac{2g}{4\pi r\sin\theta}\hat{\phi}}_{\hookrightarrow \vec{\nabla}\Lambda} \qquad \boxed{\Lambda = -\frac{g\,\phi}{2\pi}}$$

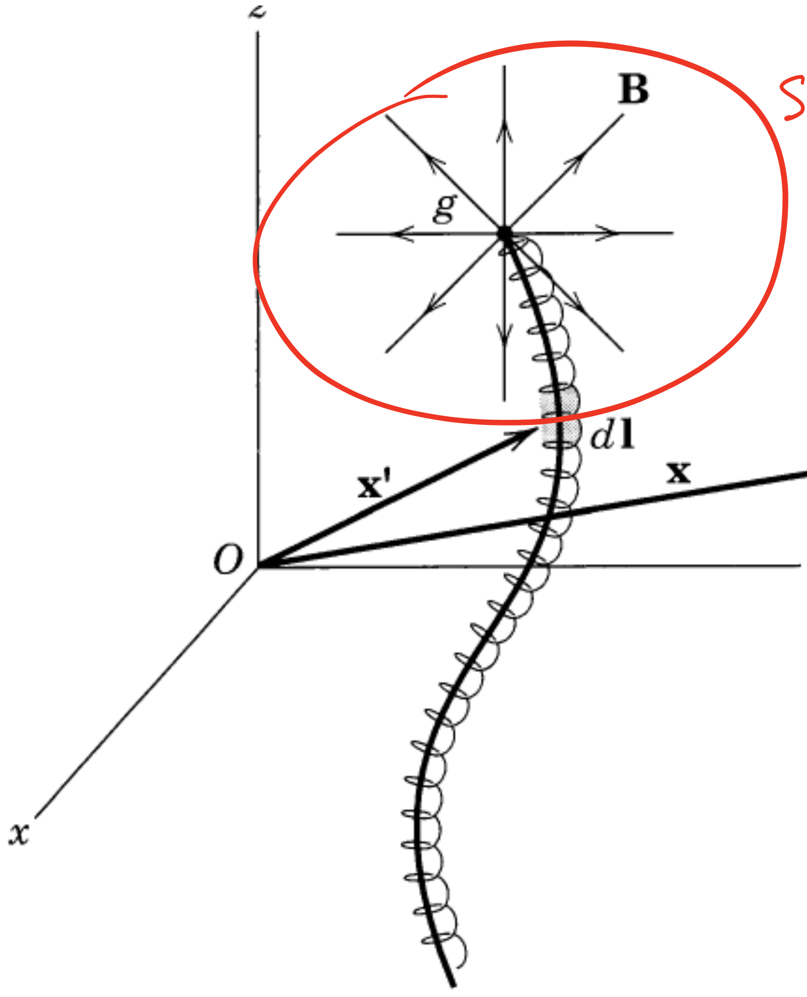
Duas possíveis cordas de Dirac





De maneira geral....





$\Phi_B(S) = 0$

$\vec{\nabla} \cdot \vec{B} = 0$

Mudança de calibre entre cordas de Dirac

$$\bar{A}' = \bar{A} + \bar{\nabla}\Lambda \qquad \Lambda = -\frac{g}{2\pi}\phi$$

$$\Rightarrow \psi'(\bar{x},t) = e^{\frac{ie}{\hbar}\Lambda}\psi(\bar{x},t) = e^{-i\frac{eg}{h}\phi}\psi(\bar{x},t) \quad (5)$$

NÃO SABEMOS COMO LIDAR COM $\psi(\bar{x},t)$ OU $\psi'(\bar{x},t)$ NOS EIXOS z NEGATIVO OU POSITIVO. MAS, LONGE DAS DUAS CORDAS DE DIRAC (FORA DO EIXO z) TEMOS (5)

A FUNÇÃO DE ONDA DEVE SER "SINGLE-VALUED"

SE OLHARMOS EM $\theta = \frac{\pi}{2}$ (PLANO XY) COMO FUNÇÃO DE ϕ:

$$\psi(r,\theta=\tfrac{\pi}{2},\phi,t) = \psi(r,\theta=\tfrac{\pi}{2},\phi+2\pi,t) \quad (6)$$

$$\psi'(r,\theta=\tfrac{\pi}{2},\phi,t) = \psi'(r,\theta=\tfrac{\pi}{2},\phi+2\pi,t) \quad (7)$$

LEVANDO (5) NA (7)

$$\cancel{e^{-i\frac{eg}{h}\phi}}\,\psi(r,\theta=\tfrac{\pi}{2},\phi,t) = \cancel{e^{-i\frac{eg}{h}\phi}}\,e^{-i\frac{eg}{h}(2\pi)}\,\psi(r,\theta=\tfrac{\pi}{2},\phi+2\pi,t)$$

USANDO (6) CANCELAMOS ψ:

$$1 = e^{-i\underbrace{\frac{eg}{h}(2\pi)}_{2\pi n}} \quad (n=0,\pm1,\pm2,\dots)$$

$$\Rightarrow \frac{eg}{h} = n \quad \Rightarrow \boxed{e = \frac{n}{hg} \quad (n=0,\pm1,\pm2,\dots)}$$

QUANTIZAÇÃO DA CARGA ELÉTRICA COMO CONSEQUÊNCIA APENAS DA EXISTÊNCIA DE MONOPOLOS MAGNÉTICOS