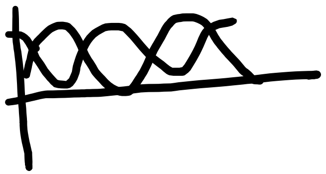
Aula 6:

$$\left.\begin{matrix} R = 1 \\ R = -1 \end{matrix}\right\} \text{ qdo } \beta^2 > 4\gamma$$

$$\underbrace{P_{n+1}} = \overset{= 1}{R} \underbrace{P_n}$$

$$P_{n+1} = -P_n$$

$\rho = 1$ quando $\underbrace{\beta^2 < 4\gamma}_{a+bi}$

$\sin\theta + \cos\theta$ 

$\begin{pmatrix} 10 \\ 5 \end{pmatrix}$

→ Programa no R

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}}_{A} \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

$\lambda > 1$

$$\vec{V}_n = \underset{-5}{C_+} \begin{pmatrix} 1 \\ 2 \end{pmatrix} 3^n + \underset{15}{C_-} \begin{pmatrix} 1 \\ 1 \end{pmatrix} 2^n$$

$x = y$

$C_+ = -5$ $\qquad$ $C_- = 15$

$$P_{n+1} = \overset{=0}{R} P_n$$

$P_0 = 100$

$P_0 = 1$

$$P_1 = 0\, P_0 = 0$$

$$P_2 = 0\, P_1 = 0$$

$$P_3 = 0\, P_2 = 0$$

R_+ e R_-

λ_+ e λ_-

$\rightarrow$ λ_+ ou $R_+ > 1$

λ_- ou R_- (0 e 1)

$R_+ = 0$

$R_- = -0{,}5$

$$P_n = C_+ \underbrace{R_+^n} + C_- \underbrace{R_-^n}$$

$$\vec{P}_n = C_+ \vec{v}_+ \underbrace{\lambda_+^n} + C_- v_- \underbrace{\lambda_-^n}$$

Aula 6:

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}}_{A} \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

autovalores
autovetores

$$Av = \lambda v$$

$$Av - \lambda v = 0$$

$$\underbrace{\left(A - \mathbb{I}\lambda \right)}_{=0} v = 0$$

$v \neq 0$

$$A - \lambda \mathbb{I} = 0$$

$$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)(3-\lambda) - 8 = 0$$

$$3 - \lambda - 3\lambda + \lambda^2 - 8 = 0$$

$$\lambda^2 - 4\lambda - 5 = 0$$

$$\lambda = \frac{4}{2} \pm \frac{1}{2}\sqrt{16+20}$$

$$\lambda = 2 \pm \frac{1}{2} 6 \begin{cases} \lambda = -1 \\ \lambda = 5 \end{cases}$$

$$(A - \lambda \mathbb{I})\vec{v} = 0$$

$\rightarrow Av = \lambda v$

$\rightarrow Av - \lambda v = 0$

$\lambda_+ = 5$

$\lambda_- = -1$

$\begin{pmatrix} 10 \\ 20 \end{pmatrix}$

$$\left[\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0$$

$$\cancel{\begin{pmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{pmatrix}}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = V_1 \begin{pmatrix} 1 \\ \frac{\lambda - 1}{2} \end{pmatrix}$$

$$(1-\lambda) V_1 + 2V_2 = 0$$

$$(1-\lambda)V_1 = -2V_2$$

$$V_2 = \frac{(1-\lambda)V_1}{-2} = \frac{(\lambda-1)V_1}{2}$$

$$\vec{P}_n = C_+\begin{pmatrix} 1 \\ 2 \end{pmatrix}(5)^n + C_-\begin{pmatrix} 1 \\ -1 \end{pmatrix}(-1)^n$$

$$\begin{pmatrix} 10 \\ 20 \end{pmatrix} = C_+\begin{pmatrix} 1 \\ 2 \end{pmatrix} + C_-\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$10 = C_+ + C_- \rightarrow C_- = 10 - C_+$$

$$20 = 2C_+ - C_- \Rightarrow 2C_+ - 10 + C_+$$

$$3C_+ = 30 \Rightarrow C_+ = 10$$

$$C_- = 10 - C_+ \Rightarrow C_- = 0$$

$$P_n = \boxed{10 \begin{pmatrix} 1 \\ 2 \end{pmatrix} 5^n} + 0 \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} (-1)^n$$

Aula 8 (ptos de eq)

$$x_{n+1} = \lambda x_n \left(1 + \alpha x_n\right)^{-b}$$

$$x_{n+1} = x_n = \bar{x}$$

$$\bar{x} = \lambda \bar{x} \left(1 + \alpha \bar{x}\right)^{-b}$$

$$\bar{x} - \lambda \bar{x} \left(1 + \alpha \bar{x}\right)^{-b} = 0$$

$$\bar{x} \underbrace{\left(1 - \lambda \left(1 + \alpha \bar{x}\right)^{-b}\right)}_{=0} = 0$$

$$\bar{x} = 0$$

$$1 - \lambda \left(1 + \alpha \bar{x}\right)^{-b} = 0$$

$$\lambda \left(1 + \alpha \bar{x}\right)^{-b} = 1$$

Modelo de saturação

$$x_{n+1} = \frac{K x_n}{c + x_n}$$

$$x_{n+1} = \frac{\frac{K}{c} x_n}{\frac{c + x_n}{c}}$$

$$= \frac{\frac{K}{c} x_n}{1 + \frac{1}{c} x_n}$$

$$= \frac{K}{c} x_n \left(1 + \frac{1}{c} x_n\right)^{-1}$$

$$\lambda(1+\alpha\bar{x})^{-b} = 1$$

$$(1+\alpha\bar{x})^{-b} = \frac{1}{\lambda}$$

$$1+\alpha\bar{x} = \sqrt[b]{\lambda}$$

$$\alpha\bar{x} = \sqrt[b]{\lambda} - 1$$

$$\bar{x} = \frac{1}{\alpha}\left(\sqrt[b]{\lambda} - 1\right)$$

$$x_{n+1} = R_0 e^{-x_n/k} x_n$$

$$\bar{x} = R_0 e^{-\bar{x}/k} \bar{x}$$

$$\bar{x} - \bar{x} R_0 e^{-\bar{x}/k} = 0$$

$$\bar{x} \left(\underbrace{1 - R_0 e^{-\bar{x}/k}}_{=0} \right) = 0$$

$$\bar{x} = 0 \,//$$

$$R_0 e^{-\bar{x}/k} = 1$$

$$e^{-\bar{x}/k} = \frac{1}{R_0}$$

$$-\frac{\bar{x}}{k} = \ln\left(\frac{1}{R_0}\right)$$

$$-\bar{x} = k \ln\left(\frac{1}{R_0}\right)$$

$$\parallel$$

$$k \ln(R_0^{-1})$$

$$-\bar{x} = k(-\ln(R_0))$$

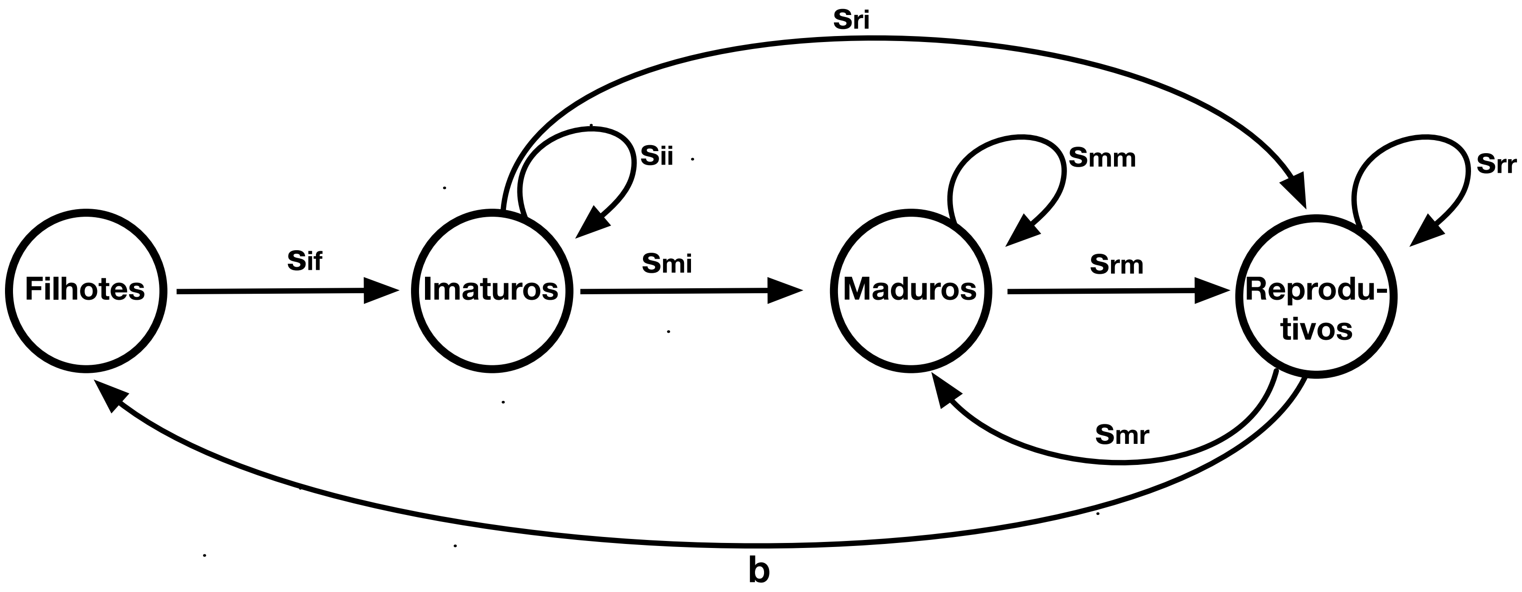
$$\bar{x} = k \ln R_0 \,//$$

$$e^{-5} = \frac{1}{e^5}$$

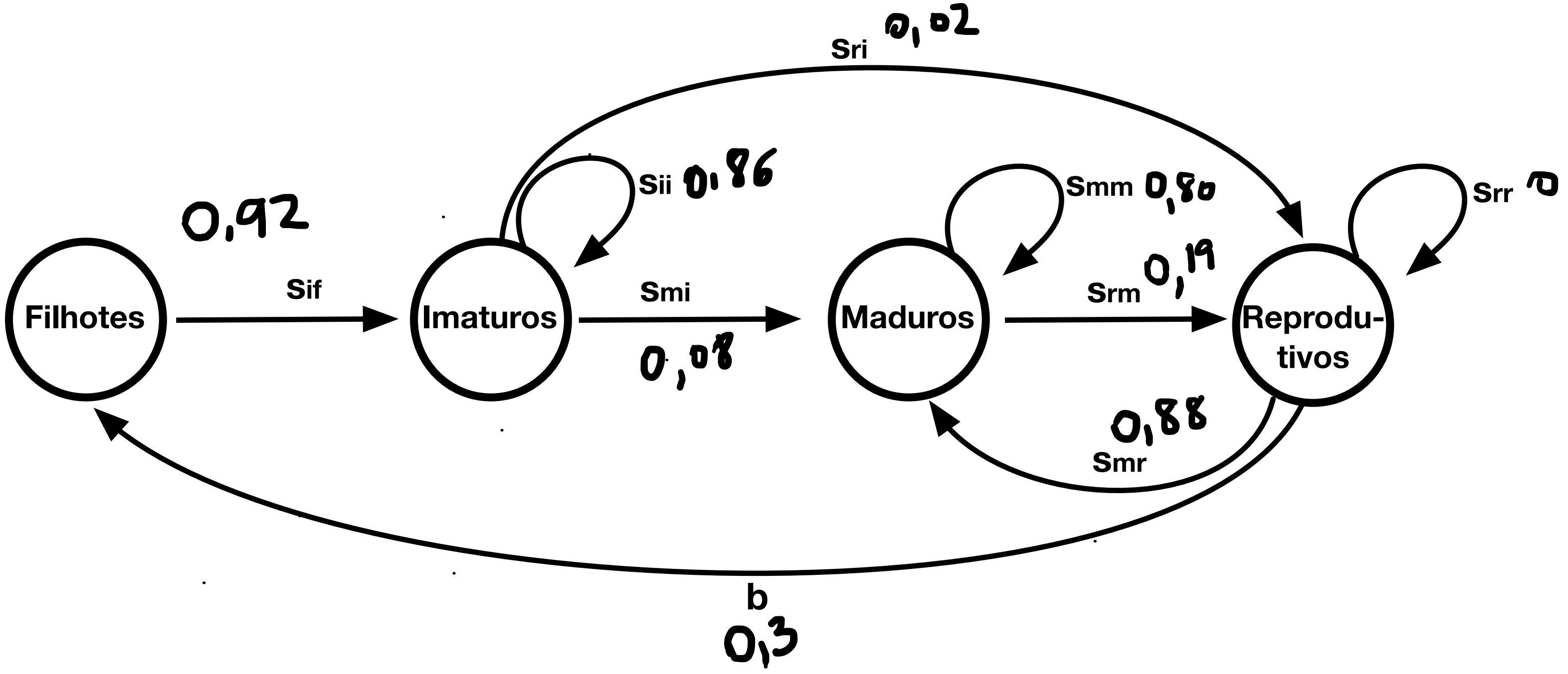
$$x_{n+1} = R_0 e^{-k \ln R_0/k} (k \ln R_0)$$

$$R_0 k \ln R_0 \, e^{-\ln R_0} = \cancel{R_0} k \ln R_0 \frac{1}{\underbrace{e^{\ln R_0}}_{\cancel{R_0}}} = k \ln R_0$$

Baleias-franca



Baleias-franca



$$F_{n+1} = \cancel{b} R_n$$

$$I_{n+1} = s_{if} F_n + s_{ii} I_n$$

$$M_{n+1} = s_{mi} I_n + s_{mm} M_n + S_{mr} R_n$$

$$R_{n+1} = S_{ri} I_n + S_{rm} M_n + s_{rr} R_n$$

$$\begin{pmatrix} 0 & 0 & 0 & \cancel{b} R_n \\ s_{if} F_n & s_{ii} I_n & 0 & 0 \\ 0 & s_{mi} I_n & s_{mm} M_n & s_{mr} R_n \\ 0 & s_{ri} I_n & s_{rm} M_n & s_{rr} R_n \end{pmatrix}$$

$$
\begin{pmatrix} F \\ I \\ M \\ R \end{pmatrix}_{n+1} = \underbrace{\begin{pmatrix} 0 & 0 & 0 & \sigma \\ S_{if} & S_{il} & 0 & 0 \\ 0 & S_{mi} & S_{mm} & S_{mr} \\ 0 & S_{ri} & S_{rm} & S_{rr} \end{pmatrix}}_{\text{Matriz deLeslie}} \begin{pmatrix} F \\ I \\ M \\ R \end{pmatrix}_{n}
$$

$$
\begin{pmatrix} 0 & 0 & 0 & \sigma R_n \\ S_{if} F_n & s_{ii} I_n & 0 & 0 \\ 0 & S_{mi} I_n & S_{mm} M_n & S_{mr} R_n \\ 0 & S_{ri} I_n & S_{rm} M_n & S_{rr} R_n \end{pmatrix}
$$

Aula 9:

Eq e Est

$$x_{n+1} = \overbrace{R_0 x_n e^{-x_n/k}}^{f(x)}$$

$\bar{x} = 0$ $\bar{x} = k \ln R_0$

$\left.\frac{df}{dx}\right|_{\bar{x}=0}$ $\left.\frac{df}{dx}\right|_{\bar{x}=1}$

$\left|\left.\frac{df}{dx}\right|_{\bar{x}}\right| < 1 \rightarrow$ Est

$\left|\left.\frac{df}{dx}\right|_{\bar{x}}\right| > 1 \rightarrow$ Inst

$$\frac{d}{dx} f(x) = \underbrace{R_0 x}_{f} \underbrace{e^{-x/k}}_{g}$$

Regra do Produto

$$\frac{dfg}{dx} = \frac{df}{dx} g + f \frac{dg}{dx}$$

$$R_0 e^{-x/k} - \frac{1}{k} R_0 x e^{-x/k}$$

$$\frac{d}{dx} e^{ax} = a e^{ax}$$

$$\left.\frac{df}{dx}\right|_{\bar{x}=0} = R_0 - 0 = R_0 \quad \bar{x}=0 \begin{cases} |R_0| > 1 \text{ instável} \\ |R_0| < 1 \text{ estável} \end{cases}$$

$R_0 > 1$

$\rightarrow 0 < R_0 < 1$

$$\left.\frac{df}{dx}\right|_{\bar{x}=k\ln R_0} = R_0 e^{-k\ln R_0/k} - \frac{1}{k} R_0 k \ln R_0 e^{-k\ln R_0/k}$$

$$= R_0 e^{-\ln R_0} - R_0 \ln R_0 e^{-\ln R_0}$$

$k>0$

$R_0>0$

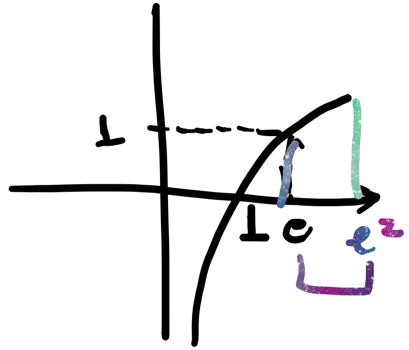
$$= R_0 e^{-\ln R_0} - R_0 \ln R_0 \, e^{-\ln R_0}$$

$$= \frac{R_0}{e^{\ln R_0}} - \frac{R_0 \ln R_0}{e^{\ln R_0}} = \frac{R_0}{R_0} - \frac{R_0 \ln R_0}{R_0}$$

$$= 1 - \ln R_0$$

$\left|\frac{df}{dx}\right| > 1 \rightsquigarrow -1 > \frac{df}{dx} > 1$ Inst

$\left|\frac{df}{dx}\right| < 1 \rightsquigarrow -1 < \frac{df}{dx} < 1$ Est



$$\left.\frac{df}{dx}\right|_{\bar{x}=k\ln R_0} = 1 - \ln R_0$$

$\bar{x}=k\ln R_0$

$$\begin{cases} R_0 < 1 \longrightarrow \text{Instável} \\ 1 < R_0 < e \longrightarrow \text{Estável} \\ e < R_0 < e^2 \longrightarrow \text{Estável} \\ R_0 > e^2 \longrightarrow \text{Instável} \end{cases}$$

$1 - \ln e^2$

$1 - 2\ln e$

$1 - 2 < -1$

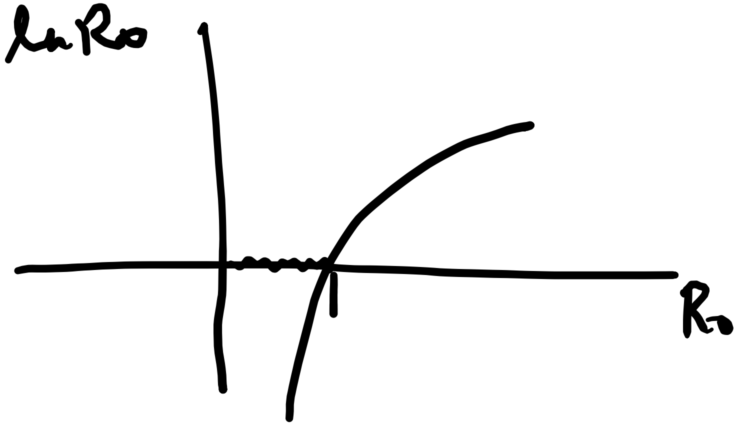
$$x_{n+1} = f(x_n) \quad — \quad x_{n+1} = x_n = \bar{x} \;\checkmark$$

$$\begin{cases} \Delta x = f(x) \\ \dfrac{df}{dx} = \dfrac{x(t)}{x'(t)} \end{cases} \quad — \quad f(x) = 0 \;\checkmark$$

$$\frac{dx}{dt} = 0 \;\checkmark$$

$$0 < R_0 < 1 \quad \frac{dx}{dt} \;\rightsquigarrow\; \underbrace{\left|\frac{df}{dx}\right| < 1}_{} \;\text{Estável}$$

$$\left|\frac{df}{dx}\right| > 1 \;\text{Inst}$$



$$-1 > \frac{df}{dx} > 1$$

$$-1 > 1 - \ln R_0 > 1$$

$$1 < -1 + \ln R_0 < 1$$

$$2 < \ln R_0 < 0$$

$$e^2 < R_0 < 1$$

$$|1 - \ln R_0| < 1$$

$$-1 < 1 - \ln R_0 < 1$$

$$1 > -1 + \ln R_0 > -1$$

$$2 > \ln R_0 > 0$$

$$e^2 > R_0 > 1$$