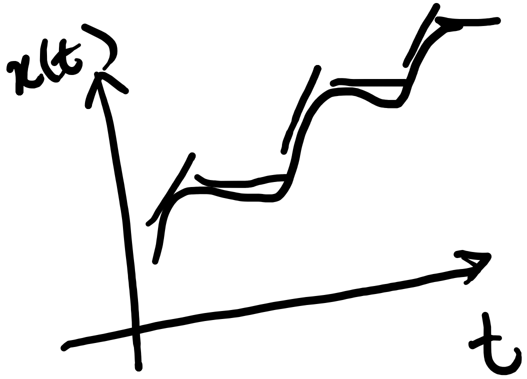
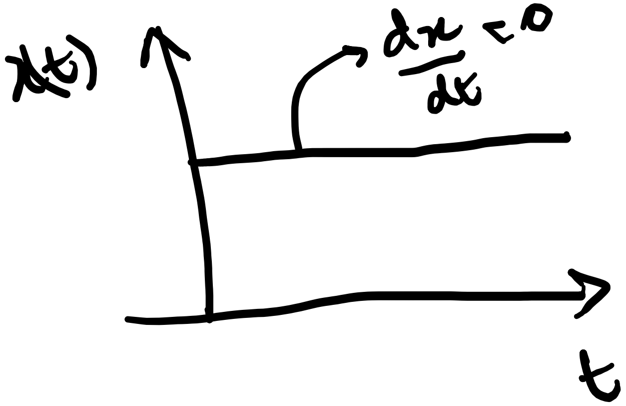
Eq e estabilidade $\frac{dx}{dt} = f(x)$

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

$$\frac{dN}{dt} = f(N)$$

$\frac{dx}{dt}$





Pontos de eq: $\frac{dx}{dt} = 0 = f(\bar{x})$

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right)$$

$$\frac{dx}{dt} = 0 \rightarrow rx\left(1 - \frac{x}{K}\right) = 0$$

$rx = 0$

$\bar{x} = 0$

$1 - \frac{x}{K} = 0$

$\frac{x}{K} = 1$

$\bar{x} = K$

$$\frac{dx}{dt} = f(x)$$

$$\boxed{x(t) = \bar{x} + x'(t)} \rightarrow \text{perturbação}$$

$$x'(t) = x(t) - \bar{x}$$

$$f(\bar{x}) = 0 = \frac{dx}{dt}$$

$$\frac{dx'(t)}{dt} = \frac{dx(t)}{dt}$$

$$\frac{dx}{dt} = f(x)$$

$$\frac{dx'}{dt} = \frac{dx}{dt} = f(\bar{x} + x') \approx f(\bar{x}) + \frac{df}{dx}(\bar{x})\,x'$$

$$\frac{dx'}{dt} = \cancel{f(\bar{x})}^{0} + \frac{df}{dx}\bigg|_{\bar{x}} x'$$

$$\boxed{\frac{dx'}{dt} = c\,x'}$$

$$c = \frac{df}{dx}\bigg|_{\bar{x}}$$

$$x'(t) = c' e^{ct}$$

$$\frac{dN(t)}{dt} = k\,N(t) \longrightarrow \boxed{N(t) = Ce^{kt}}$$

$C \to N_0$

$$\frac{dx'}{dt} = c\,x' \longrightarrow x'(t) = C'e^{ct}$$

$c \to \left.\frac{df}{dx}\right|_{\bar{x}}$

$c > 0 \to x'$ cresce c/o tempo → instável

$c < 0 \to x'$ decresce c/o tempo → estável

$$\frac{dx}{dt} = f(x) = rx\left(1 - \frac{x}{k}\right) = rx - \frac{rx^2}{k}$$

$$\bar{x} = 0 \qquad \bar{x} = k$$

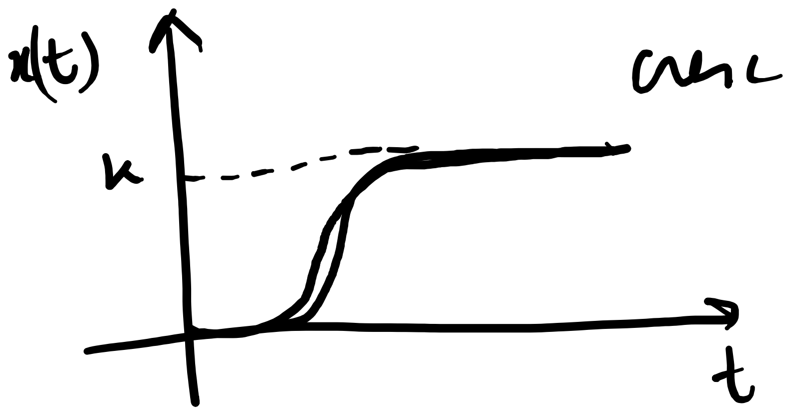
$$\frac{df}{dx} = r - \frac{2rx}{k}$$

$$\bar{x} = 0 \qquad \left.\frac{df}{dx}\right|_{\bar{x}=0} = r - \frac{2 \cdot r \cdot 0}{k} = r \qquad \begin{cases} r > 0 \rightarrow \text{instável} \\ r < 0 \rightarrow \text{estável} \end{cases}$$

$$\bar{x} = k \qquad \left.\frac{df}{dx}\right|_{\bar{x}=k} = r - \frac{2 \cdot rk}{k} = -r \qquad \begin{cases} r > 0 \rightarrow \text{estável} \\ r < 0 \rightarrow \text{instável} \end{cases}$$

Efeito Allee:

Pop pequenas.



cresc. logístico

$r>0$

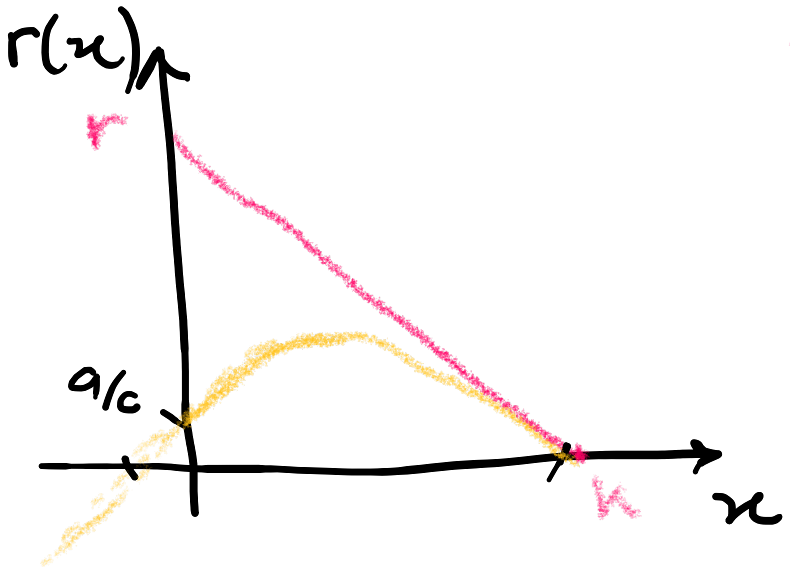
$$\frac{dx}{dt} = x\underbrace{\left(r - r\frac{x}{k}\right)}_{r(x)}$$

$$\frac{dx}{dt} = x\underbrace{\left(\frac{a+bx-x^2}{c}\right)}_{r(x)} = \underbrace{r(x)}\cdot x$$

$$r(x) = \frac{1}{c}\left(a + bx - x^2\right)$$

$r - \frac{r}{k}x = 0$

$x = \frac{rk}{r}$



$$\frac{dx}{dt} = \boxed{x\underbrace{(+2 - x - x^2)}_{r(x)}}$$

Pontos de eq.

$$\frac{dx}{dt} = 0 = \underset{\sim}{x}\underbrace{(+2 - x - x^2)} = 2x - x^2 - x^3 = f(x)$$

$\bar{x} = 0$ $\qquad -x^2 - x + 2 = 0$

$$x = \frac{+1}{2} \pm \frac{1}{2}\sqrt{1+8}$$

$\boxed{\bar{x} = 2}$ ✓

$\boxed{\bar{x} = -1}$ ✗

Estabilidade:

$$\frac{df}{dx} = 2 - 2x - 3x^2$$

$\bar{x} = 0$ $\quad \boxed{\left.\frac{df}{dx}\right|_{\bar{x}=0} = 2} \rightarrow$ instável

$\bar{x} = 2$ $\quad \left.\frac{df}{dx}\right|_{\bar{x}=2} = 2 - 2.2 - 3.2^2 = \underset{\sim}{-14} \rightarrow$ estável