Teoria dos jogos:

$$\begin{array}{c|cc} & A & B \\ \hline A & a & b \\ B & c & d \end{array}$$

1) Hawk-Dove

b = benefício

c = custo

J_2

		H	D
J_1	x H	$\frac{b-c}{2}$ a	b b
		∨ ∧	∨ ∨
	(1-x) D	0 c	$b/2$ d

$\bar{x} = 0$

$\bar{x} = 1$

$$\bar{x} = \frac{d-b}{a-b-c+d}$$

$$\bar{x} = \frac{b/2 - b}{\frac{b-c}{2} - b - 0 + \frac{b}{2}}$$

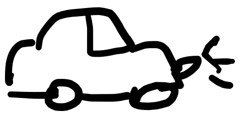
$$= \frac{-b/2}{-c/2} = \frac{b}{c}$$

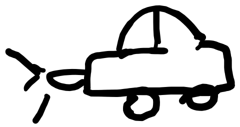
$b, c > 0$ $b > c$ → H

$b < c$ → H e D

2) Chicken

$$\begin{array}{c|c|c} & & J_2 \\ & A & B \rightarrow \text{sair da pista} \\ \hline J_1 \quad A & -c & b \\ & \wedge & \vee \\ \hline B & 0 & b/2 \end{array}$$





HD $(b<c)$

Coexistência das estratégias

3) Stag-hunt

$$\begin{array}{c|c|c} & \multicolumn{2}{c}{J_2} \\ & \text{Stag} & \text{Hare} \\ \hline J_1 \quad x \ \text{Stag} & a & b \\ & \vee & \wedge \\ \hline (1-x)\ \text{Hare} & c & d \end{array}$$

$\bar{x} = 1$

$\bar{x} < 0$

4) Dilema do prisioneiro

P2

		F	D
P1	Ficar silencio	-1	-10
	conf- delatar	$\hat{0}$	$\widehat{-7}$

	c	D
C	3	0
D	$\hat{5}$	$\hat{1}$

ε C

$(1-\varepsilon)$ D

fitness c

$3\varepsilon + 0(1-\varepsilon)$

fit defec

$5\varepsilon + 1(1-\varepsilon)$

$3\varepsilon < 4\varepsilon + 1$

$\varepsilon \to 0$

$(1-\varepsilon)$ C $\quad \varepsilon \to 0$

ε D

fitness coop:

$3(1-\varepsilon) + 0\varepsilon$

fitness defector

$5(1-\varepsilon) + 1\varepsilon$

$3-\varepsilon < 5-4\varepsilon$

Tempo Contínuo:

$$\begin{array}{c|cc} & A & B \\ \hline A & a & b \\ B & c & d \end{array}$$

$$\ln a.b = \ln a + \ln b$$

A e B

x $\quad$ $1-x$

$$f_A = x\,a + (1-x)\,b$$

$$f_B = x\,c + (1-x)\,d$$

$$\ln \frac{a}{b} < \ln a - \ln b$$

$$\ln a^b = b \ln a$$

$$\bar{w} = x\left(x a + (1-x) b\right) + (1-x)\left(x c + (1-x) d\right)$$

Eq Replicador

$$x(t+1) = \underbrace{\frac{f_A}{\bar{w}}}_{r}\, x(t)$$

$$x(t+\Delta t) = \left(\frac{f_A}{\bar{w}}\right)^{\Delta t} x(t)$$

$$\ln x(t+\Delta t) = \ln\left[\left(\frac{f_A}{\bar{w}}\right)^{\Delta t} x(t)\right]$$

$$\ln x(t+\Delta t) = \ln\left(\frac{f_A}{\bar{w}}\right)^{\Delta t} + \ln x(t)$$

$$\ln x(t+\Delta t) - \ln x(t) = \Delta t \ln \frac{f_A}{\bar{w}}$$

$$\frac{\ln x(t+\Delta t) - \ln x(t)}{\Delta t} = \ln f_A - \ln \bar{w}$$

$$\lim_{\Delta t \to 0} \frac{\overbrace{\ln x(t+\Delta t)}^{f(x,t+\Delta t)} - \overbrace{\ln x(t)}^{f(x,t)}}{\Delta t} = \ln f_A - \ln \overline{w}$$

$$\frac{df(x,t)}{dt} = \frac{d \ln x}{dt}$$

$$\frac{1}{x} \frac{dx}{dt}$$

$\Delta t \to 0$ (tempo contínuo)

$$\frac{d\overset{\ln x(t)}{f(x,t)}}{dt} = \ln f_A - \ln \overline{w}$$

$$\frac{1}{x} \frac{dx}{dt} = \ln f_A - \ln \overline{w}$$

$$\frac{dx}{dt} = x\left(\overbrace{\ln f_A}^{F_A} - \overbrace{\ln \overline{w}}^{\overline{W}}\right) = x\left(F_A - \overline{W}\right)$$

→ Equação do Replicador

$$\frac{dx}{dt} = x\,(F_A - \bar{w})$$

$$\bar{w} = x F_A + (1-x) F_B$$

$$= x\,(F_A - xF_A - (1-x)\,F_B)$$

$$F_A = ax + (1-x)b$$

$$F_B = cx + (1-x)d$$

$$= x\left(F_A\,(1-x) - F_B\,(1-x)\right)$$

$$= x\,((1-x)(F_A - F_B))$$

$$\frac{dx}{dt} = x\,(1-x)(F_A - F_B)$$ Eq replicador

$$\frac{dx}{dt} = 0 \qquad \bar{x} < 0 \qquad \bar{x} = 1 \qquad \text{ou} \qquad F_A = F_B \qquad \bar{x} = \frac{d-b}{a-b-c+d}$$

Alelos A e a q seleção natural?

A: p
a: q

	A	a
A	w_{AA}	w_{Aa}
a	w_{aA}	w_{aa}

fit A : $p w_{AA} + q w_{Aa}$

fit a : $p w_{Aa} + q w_{aa}$

freq A fit A + freq a fit a

$$\bar{w} = p(p w_{AA} + q w_{Aa}) + q(p w_{Aa} + q w_{aa}) =$$

$$= p^2 w_{AA} + 2pq w_{Aa} + q^2 w_{aa}$$

$$p(t+1) = p(t)\frac{f_A}{\bar{w}} = \frac{p(p w_{AA} + q w_{Aa})}{\bar{w}} = \frac{p^2 w_{AA} + pq w_{Aa}}{\bar{w}}$$